A relational view of quantum mechanics

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The Copenhaguen interpretation provides a mathematical framework that has proved useful to evaluate the probabilities for future events in terms of our previous knowledge on a system. However, it is a source of controversy since any physical intuition we want to give on the processes yields paradoxes like instantaneous collapse, spooky action at a distance... The goal of the relational interpretation is to solve some of them without changing the actual formalism, purely by focusing our attention to what is really the intrinsic property of Quantum Mechanics: discreteness.

I. INTRODUCTION

> Experimental science was born. But experiment is a tool. The aim remains: to understand the world. To restrict quantum mechanics to be exclusively about piddling laboratory operations is to betray the great enterprise. A serious formulation will not exclude the big world outside the laboratory.
>
> John S. Bell, "Against Measurement"

Quantum Mechanics (QM) ever since it was born came with a lot of conceptual problems which made it difficult to understand even to its fathers. The intrinsic properties of the new theory put most of the greatest physics in the XX century against the ropes because their classical intuition was no longer valid. QM introduced superposed states, long ranged correlations, wave-particle dualism and some others, neither with a classical analogue. Profound discussions end up building the orthodox interpretation of QM or the *Copenhaguen* interpretation, constructed as a set of 5 mathematical postulates describing the space, evolution and measurement of quantum states as well as the rules to add an evaluate probabilities of events [1].

However, the development of the theory showed that the latter interpretation was not sufficient to explain the under-laying behaviour of reality. Specially, one of the most important questions deals with the measurement and collapse of the wave function [see ref. 2, Chapter 2]. In the orthodox interpretation, the space of observers is classical and unaffected by quantum effects, those interact with quantum states producing a collapse from a superposition of all possible outcomes to just one instantly [3, 4]. It introduces the notion of an external observer (which can be an apparatus, a human being or a god) invisible to QM rules but capable of measuring its effects. How can we make this division if all the elements in the universe are made upon the same microscopic pieces which individually live in the QM domain? Where is the limit between the classical and quantum world? What does it mean that the wave function collapses instantly?

The community has dealt with these open problems by creating interpretations which tried to give a physical reason for what we see in experiments either by introducing hidden-variables, creating an infinite number of worlds, adding non-linear terms to the Schrodinger equation... [2] These are some examples which also come with drawbacks, non of them completely solve the problem without introducing extra difficulties to the theory.

Right now, we are continuously seeing how Quantum Technologies are being developed basing their approach in the Copenhaguen interpretation so it is certainly not wrong in what it predict only that it is just a theory to compute probabilities of events [5], it misses a physical explanation of the processes [6].

The Relational interpretation is a logical next step of the orthodox interpretation which, without changing the formalism and at low costs, is able to answer some of the open problems.

This letter is organised as follows, in section II we develop the interpretation as it was originally formulated by Carlo Rovelli, then we perform a discussion stating the puzzles solved and some of the main objections in section III to give a final conclusion in section IV.

II. RELATIONAL QUANTUM MECHANICS

> The issue is thus not to replace or fix [the Copenhaguen interpretation], but rather to understand what precisely it says about the world.
>
> Carlo Rovelli, "Relational Quantum Mechanics"

We must understand Relational Quantum Mechanics (RQM) as a democratisation of the orthodox view in which systems and observers have the same weight in the theory such that we can conceive every physical system as being observed or observing [7]. Rovelli's desire, as a theoretical physicist specialised in Quantum Gravity, was to understand QM in a way compatible with special relativity (SR), a theory where two observers can give different explanations of the same event [8]. RQM bases its approach on making quantum states relative to an observer system, just like in classical theory where velocities are only meaningful when defined as relative to an observer, but obviously, we must be capable of reversing the observed and observer role without changing the physics.

Although it is attributed to him the birth of the RQM, the relative state formulation was already introduced by Huge Everett [9], who also considered that observers have to be explained within the framework of QM. This same paper led to the development of the Many-Worlds interpretation [10] which has many conceptual differences from RQM that we will discuss in the following section.

In RQM, it is not feasible to ask about the quantum state of a system, in other words, there is no absolute quantum state, it only makes sense to say that $\mathcal{S}$ has a state ψ with respect to $\mathcal{O}$: $|\psi\rangle_{S:O}$. In fact, QM can be formulated without the need of quantum states (actually, QM was born before quantum states) [11] but this is not Rovelli's approach since states provide a convenient tool for calculations[1] [5, 13].

A first consequence of the relative state formulation is that quantum states have no ontological meaning [14] just like in Quantum Bayesianism (QBism) [12], they are just mathematical tools employed to keep track of the probabilities of events[2], encoding the information we can extract from the given system.

Furthermore, superpositions appear only in the formalism and should never be understood as multiple things happening at the same time. So, Schrodinger's cat is not dead or alive, QM is simply telling us the probabilities of finding it either dead or alive.

The relational approach to QM can be summarised in two hypothesis:

H1: All systems are equivalent.

H2: Quantum Mechanics is complete to our present level of experimental observations.

A theory that has **H1** as a postulate automatically solves one of the problems in the orthodox interpretation, there is no distinction between classical and quantum world. Therefore, there is never a collapse, only interactions between quantum elements occur.

The second hypothesis is simply saying that the mathematical formulation is good as it is, there is no need to include extra formalism. The Born rule [3] allows an observer to calculate the probabilities for a future event by taking into account only his previous history.

Rovelli's making their deductions empirical, from the knowledge acquired in experiments, his goal is to deduce a formal theory of QM and solve some of the puzzles open but by no means he will try to explain where does uncertainty and randomness come from, those are just extra requirements in the theory.

The reader may argue where does the Schrodinger equation fit in this interpretation, for that we must recall an implicit hypothesis that is usually assumed to be true: a quantum system cannot interact with itself[3]. Thus, a system $\mathcal{S}$ observed by $\mathcal{O}$ may be seen by an external observer $\mathcal{R}$ as a coupled system $\mathcal{S}+\mathcal{O}$ where if one knows the full Hamiltonian (free Hamiltonians for $\mathcal{S}$ and $\mathcal{O}$ as well as the interaction Hamiltonian between both) then it is possible to use the Schrodinger equation to perform the time evolution of probabilities.

On the other hand, $\mathcal{O}$ is not allowed to perform this analysis because she[4] is interacting with the system and her only picture is $\mathcal{S}$, she cannot take herself into account because she is part of the interaction. In the latter case, the evolution is seen as a discrete actualisation of a value in $\mathcal{O}$'s memory. Essentially, value assignment in a measurement is not inconsistent with unitary evolution of the $\mathcal{S}+\mathcal{O}$, because value assignment refers to the properties of the system with respect to the apparatus, while the unitary evolution refers to properties with respect to an external system.

An important remark is that only our previous information about a system allows us to calculate future probabilities. Therefore, the outcomes of a measurement are also subjective. This leads us to the most important aspect in his interpretation, information as the basis of interactions. All systems can be understood as observers and observed but it is only when one gains information about the other that we can say there has been an interaction (measurement in standard language), after that the two systems become correlated. It is even possible to define an operator whose output tell you if an interaction occurred or not [7].

Information is correlation and by such $\mathcal{O}$ cannot have a full description of the interaction of $\mathcal{S}$ with himself ($\mathcal{O}$) and there is no meaning in being correlated with oneself.

From an information theoretic perspective, RQM and other interpretations [16] propose two postulates[5] to reconstruct QM:

[1] Qbism's goal is to formulate QM without using states, only with probability assignments of events, by replacing the Born rule with a modified expression that takes into account the dimensionality of the Hilbert space [12]. For completeness, classical probability functions are defined upon a Boolean algebra while quantum probability functions are defined on orthoalgebras [5].

[2] This point of view is taken to the extreme in QBism where the notion of probability is also made subjective. Christopher Fuchs performs a very illuminating experiment in [15, minutes 29 to 40] to explain why we must not regard quantum states as physical, this is in the context of QBism but the same reasoning applies to RQM.

[3] To the extend of our knowledge, a spin is not influenced by its own value neither a charged particle is not influenced by its own electric field.

[4] The usage of personal pronouns may not be associated with a system being a human being, its only goal is to easily distinguish observer systems in the text.

[5] Together with Everett's idea, Rovelli was inspired by the way Einstein derived the Theory of Special Relativity from the Lorentz transformation [8] which prior to 1905 only look like abstract equations. He aims to derive a smaller set of physical postulates from which the abstract mathematical postulates of the Copenhaguen interpretation can be derived. This view is shared by other scientist which see the current interpretation as a useful mathematical model that lacks from physical meaning [17].

P1: There is a maximum amount of relevant information that can be extracted from a system.

P2: It is always possible to acquire new information about a system.

Although at first they might seem contradictory, they are not due to the non-commutative nature of QM. That is, the first postulate asserts that the amount of information stored in a system is finite. For instance, finite d-dimensional Hilbert spaces have at most $N = \log_2 d$ bits of information.

The second is a consequence of the non-commuting algebra, we can always perform a measurement of an operator that doesn't commute with a previous observed quantity. The previous information stored, although it remains true (we can continue saying that we measured that value), it becomes irrelevant in favour of the outcome of the new interaction, that is, the previous result is not any more useful to compute future probabilities.

Formally, a set containing the maximal amount of information in a system $\mathcal{S}$ can be written as a string $s = [e_1, \ldots, e_N]$ with respect to $\mathcal{O}$ which correspond to the measurement of N commuting observables. Any latter interaction can be in a repeated observable, which both Copenhagen and RQM assert that the same outcome will be obtained and thus s will remain unchanged; or in a different non-commuting observable, in this case a combination of bits in s will have to be overwritten in order to account for the new information.

Discreetness is revitalised, not only in the finite number of outcomes, but also in the amount of information about a system. Basically, we return to Heisenberg idea that QM doesn't tell us what's happening in between measurements, it is a theory of interactions and observed values.

For instance, let's take a 2^N-dimensional system, the maximum amount of relevant information is at most $N = \log_2 2^N$. It is always possible to find a set of N commuting observables $A_1, \ldots, A_N$ [3] with eigenvalues $\{a_j^k\}$ that completely determine a state by assigning a string $s = [a_1^{k_1}, \ldots, a_N^{k_N}]$. Clearly, it is also possible to find an operator that does not commute with any of the $\{A_j\}$, say B, thus it is always possible to acquire new information by measuring B but the information is limited to n bits at a time.

Suppose that initially, $\mathcal{O}$ has the knowledge that $\mathcal{S}$ is in the state $|\psi\rangle_{S:O} = \sum_{k=1}^N \alpha_k |a_k\rangle$[6]. A posterior measurement of A will take the state from $|\psi\rangle_{S:O}$ to any of the N possibilities a_k and her memory will be updated accordingly. Suppose $\mathcal{O}$ saw a_1 then the state of $\mathcal{S}$ with respect to $\mathcal{O}$ is

$$|\psi'\rangle_{S:O} = |a_1\rangle \tag{1}$$

Note that the state didn't collapse because there is nothing to collapse, quantum states are not physical.

Now consider an external observer $\mathcal{R}$ who sees the interaction from outside, he is also aware of the initial state of $\mathcal{S}$ because he perform measurement on his own, also he is aware that $\mathcal{O}$'s state is $|o\rangle$. The total initial state for $\mathcal{R}$ is $|\psi\rangle_{SO:R} = \sum_{k=1}^N \alpha_k |a_k\rangle |o\rangle$ which after interaction the state under unitary evolution becomes

$$|\psi'\rangle_{SO:R} = \sum_{k=1}^{N} \alpha_k |a_k\rangle |o_k\rangle \tag{2}$$

where the $\mathcal{O}$'s part in this state denotes the update produced in her memory when she observe the corresponding state on that system. Taking the partial trace of the last stat with respect to $\mathcal{O}$ yields $\rho_{S:R} = \sum_{k=1}^N |\alpha_k|^2 |a_k\rangle\langle a_k|$ which is the state of $\mathcal{S}$ with respect to $\mathcal{R}$, expressing the fact that if $\mathcal{R}$ where to measure $\mathcal{S}$ by interaction he will see a_k with probability $|\alpha_k|^2$.

This example clearly show the power of RQM, the state of a system depend on the observer, the external observer can use unitary evolution as derived from the Schrodinger equation because he did not interact with the system, in contrast, the internal observer only sees her memory being update simply because she is not giving a full dynamical description of the interaction.

III. DISCUSSION

> The question raised by EPR "Can quantum-mechanical description of physical reality be considered complete?" has a positive answer. However, reality may be different for different observers.
>
> A. Peres, "Einstein, Podolsky, Rosen, and Shannon"

Before heading into the discussion we should stress the purely mathematical meaning of a quantum state, it mustn't be regarded as having physical support but only a tool to keep track of the probabilities of events as seen by a given observer.

A. Puzzles closed and puzzles remaining

Previously, we already saw how some of the open problems in the standard interpretation are solved by **H1** and removing all ontological meaning to the quantum state. The latter removes the possibility of a collapse, that of a Schrodinger's cat while the former implies the fall of the wall between classical and quantum worlds while solving the problem of different views of an event in the orthodox interpretation.

Also, the Wigner's friend paradox [18], which is essentially the example posed above, is no longer a paradox since it is part of the interpretation. von Neuman's infinite chain can also be explained by noting that only the

[6] Either because $\mathcal{O}$ measured many times $\mathcal{S}$ in order to compute the probabilities $|\alpha_k|^2$ or she measured some other set and used standard QM to change the basis of the state.

apparent superposition of the results in an interaction can only be seen by an external system but not within the observer-system itself, that would imply having self-measurement states which are illegal in the theory. Finally, the Frauchiger–Renner paradox for which Waaijer and van Neerven [19] recently gave a solution using the relative state formulation and showing that within this framework no contradiction arises.

Furthermore, it removes the notion of "spooky action at a distance" introduced by Einstein *et al.* [20] although multiple authors give different explanations: Laudisa [21] solve the problem by slightly modifying the definition of the words *reality* and *locality* to fit them in RQM, Van Fraassen [22] introduces a third postulate to relate the views of three different observers of an event and Smerlak and Rovelli [14] assert that the original version of the problem is ill-posed in RQM because it requires a super-observer able to measure in two space-like separated regions. Essentially, the state vectors in hands of the two agents do not represent the physical spin but the information those agents have on their value. Moreover, the outcome of the local measurements is an observer-dependent element of physical reality and attributing an absolute view is what introduces conceptual problems to QM.

It is also possible to determine the time at which a measurement occurred by an external observer, never by the systems that are interacting (see [7] or [23] for a more extensive discussion). In contrast, it doesn't explain how are values actualised in a measurement as Laudisa [24] points out. Rovelli answers that in RQM the question is meaningless since this process is taken as an intrinsic property[7]. A similar answer is given to the question about where does indeterminism come from? This certainly appears due to the non-commutative nature of QM [7] as a mathematical consequence of his hypothesis, "trying to fill in the sparse ontology of Nature with our classical intuition about continuity [...] In the history of physics much progress has happened by realising that some naively realist expectation was ill founded, and therefore by dropping these kind of questions" [25].

B. Criticism

There are two main objections to RQM: (i) the relationship between observer views (a.k.a the "third-person problem") and (ii) the weakening of realism.

The first objection is posed by Brown [26] who argues that from the form of eq. (2), a measurement of $\mathcal{S}$ might give a_k with $k \neq 1$ in contradiction to what eq. (1) asserts[8]. After $\mathcal{O}$'s interaction the only information that $\mathcal{R}$ has is that both $\mathcal{S}$ and $\mathcal{O}$ are correlated and this remains true after $\mathcal{R}$ interacts with $\mathcal{S}$ [7]. Nevertheless, it seems as if $\mathcal{O}$ and $\mathcal{R}$ where leaving in two different branches as the many-world interpretation considers [10] but this is a precipitate conclusion. We should analyse the problem from an external observer (call it $\mathcal{P}$) which has all dynamical knowledge of $\mathcal{S}+\mathcal{O}+\mathcal{R}$, we must do so because among comparing directly the results of $\mathcal{O}$ and $\mathcal{R}$ we will be exposing their absolute state which has no meaning in RQM. Essentially, see appendix A, the paradox is solved once the full dynamical description is given: if $\mathcal{P}$ ensures that the states must be correlated then the previous process as stated by Brown [26] is forbidden by the same QM [14, 27, 28] but if $\mathcal{P}$'s information allows such a scenario then there is no contradiction at all.

A more subtle discussion is how can we relate two views, this is also a problem in the Coherent Histories interpretation [2] where it is the choice of the framework that determines the probabilities as well as the outcomes and those can not be related without giving a contradiction. In RQM, a similar problem arises if we do not take into account an external observer, some work tackling this problem is done by Van Fraassen [22] by introducing a third postulate. Those are simplified by [28] by simply stating that if something is "true-for-$\mathcal{O}$" then it must not be "false-for-$\mathcal{R}$" which is in agreement with the previous example and it is a consequence of **H2** (the completeness of QM).

Still, it seems as if something else without being part of the system can have more information about it than its elements [26]. Of course, this paradox is only apparent, the external observer has more information in the sense that it can give a full description but it cannot access the bits of information gained during interactions, so in terms of Shannon it is completely ignorant.

This leads us to the second objection, the weakening of realism due to the probabilistic and not ontological interpretation of quantum states. Realism is the notion that there is something real out there which exists independent from us but strong realism, as the Copenhaguen interpretation considers, assumes that the world at all possible times has a definite value, a state. This strong version is not supported in RQM as values are only acquired at discrete instants of time, namely at interactions, and those are relative to the observer-system [11]. If I do nothing, then I can explain my surroundings as a unitary evolution driven by the Schrodinger equation (I am the external observer).

Despite seeming a harmless assumption it leads to enormous consequences, specially for cosmologists which are firm defendants of the Many-Worlds interpretation. Due to the relativisation of states and discreteness of interactions, there is no universal flow of time [27], it

[7] That would be like asking why is the velocity of light constant in SR?

[8] "Consider a case where $\mathcal{O}$ is Schrodinger's cat [, $\mathcal{S}$ its heart] and $\mathcal{R}$ is the evil experimenter. The cat could be dead for the cat, while it's alive for $\mathcal{R}$. If you think that there is a problem with spontaneous reanimation, then you might think there is a problem with the relational interpretation", Brown [26].

is observer-dependent, just like in SR. But most importantly, it is stating that even if we are the observers of the universe, cosmology is about the large degrees of freedom of it and by interacting with them (through telescopes, interferometers...) we are not affecting those but only the knowledge we have about them. Hence, cosmology understood as the study of the large dof still makes sense, what is not clear is if "totology", the study of the universe as a whole, is well defined within RQM because there is nothing we can relativise $|\psi\rangle_{universe:?}$ on [29, 30]. There is nothing else than the universe, therefore there can't exist a state of the universe [25].

Cosmologist must content themselves upon seeing the part of the universe outside them, Brown [26] introduces the notion of a *canonical cut* to separate the part of the universe being observed from the observer part which is seen again as a dualism between observers and systems just like in orthodox QM we had classical and quantum worlds. Dorato [27] makes sense of this by noting that there must be a reduction of this dualism to account of interference effects but it distinguishes external vs. internal systems that have two different views of the interaction.

Experimentalists may also be worried since we can only access quantum properties of systems through apparatus and RQM manifests that they are measuring the composed system particle+apparatus. There should be no controversy at all, it is just stating that as an external observer you should see the device as a quantum system as well and act in consequence.

IV. CONCLUSION

> So far, QM has been unquestionably winning for nearly a century, beyond all expectations.
>
> Carlo Rovelli,
> "Space is blue and birds fly through it"

RQM offers a new perspective of QM without changing the formalism, it treats observers as quantum systems which allows to express measurements as purely quantum interactions. The ontological weight of quantum states is removed leaving them as mathematical tools to keep track of the probabilities for events as seen from the observer perspective. Those fewer assumptions are sufficient to explain most of the paradoxes in QM although some are fixed because they are taken as granted and unquestionable.

Besides the difficulties, it introduces aspects which are already natural in other theories like SR, constituting a logical next step from the Copenhagen Interpretation. Nevertheless, there is still room for improvement before finally replacing it.

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Appendix A: Third person problem as seen by a fourth

Take the example at the end of section II, in section III we saw that a posterior measurement done by $\mathcal{R}$ might take the state of $\mathcal{S}+\mathcal{O}$ to

$$|\psi''\rangle_{SO:R} = |a_k\rangle\,|o_k\rangle \tag{A1}$$

according to the probability distribution $\{|\alpha_k|^2\}$.

As said, there is no contradiction between eq. (1) and the previous result since $\mathcal{O}$ information about the state $\mathcal{S}$ is that it is in $\uparrow$ but she doesn't know that a third interaction occurred with $\mathcal{R}$ since that would imply self-measurement and the notion of absolute state. At the same time, $\mathcal{R}$ knowledge prior to the measurement of $\mathcal{S}$ was that $\mathcal{S}$ and $\mathcal{O}$ were correlated and this remains true in eq. (A1).

A third observer $\mathcal{P}$ who knows the dynamical Hamiltonian of $\mathcal{S}+\mathcal{O}+\mathcal{R}$ as well as their interaction computes the final state of the previous process and sees that for him it is

$$|\psi'\rangle_{SOR:P} = \sum_{k=1}^{N}\alpha_k\,|a_k\rangle\,|o_k\rangle\,|p_k\rangle \tag{A2}$$

noting that their state is always correlated. Therefore, this external observer is the only one capable of concluding that the previous scenario never occurs without reaching a contradiction. In eq. (2), the state simply manifest the uncertainty in $\mathcal{O}$'s knowledge and not the real state of the system+observer.

Despite this, we can consider a non-ideal interaction with a small error probability q in the second interaction leading to the state

$$\begin{aligned}|\psi'\rangle_{SOR:P} = \sqrt{1-q}\sum_{k=1}^{N}\alpha_k\,|a_k\rangle\,|o_k\rangle\,|p_k\rangle \\ + \sqrt{q}\sum_{k\neq l}\alpha_k\,|a_k\rangle\,|o_k\rangle\,|p_l\rangle\end{aligned} \tag{A3}$$

The previous certainly allows a process like the previous too occur with probability $q|\alpha_k|^2$ without any contradiction.