## Advanced lab course

# Particle physics with the computer*


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# Contents

# 1. Introduction

The purpose of the experiment *Particle physics with the computer* is to examine data sets from HADES[1]. HADES is a large spectrometer which is known for his high acceptance and a covering of 85 % of the azimuthal angle and a covering from (15 to 85) degrees of the polar angles.[1]

Examined cases of this lab course were typical proton-proton collisions. As a result of those collisions new particles can be created. Indeed, the focus of this lab course is on elastic proton-proton collisions. The aim of this experiment is to analyze and finally interpret the given data. The data for this analysis was preselected and only contains events where two particles were detected.

[1] **H**igh **A**cceptance **D**i-**E**lectron **S**pectrometer (**HADES**) in Darmstadt

# 2. Physical Background

## 2.1. Collision

A collision is an event during which particles interact. Theoretical calculations of collision processes are based on an infinitely rapid transport of energy and momentum. For an ideal collision in an inertial system the sum of following physical values must be conserved: energy $E$, momentum $\mathbf{p}$, angular momentum $\mathbf{L}$, electric charge $q$, baryon-number $B$, lepton-number $L$ and total spin $S$.

The conservation of energy and momentum can be written with four-vectors

$$\sum_i P_i = \sum_i P'_i, \tag{2.1}$$

whereas $P = (E, p_{\mathrm{x}}, p_{\mathrm{y}}, p_{\mathrm{z}})^\top$ is before and $P'$ after the collision. Furthermore it is important to distinguish between elastic and inelastic collisions:

- elastic: The particles conserve their identity, charge and mass of each particle doesn't change and no new particles are created.
- inelastic: New particles might be created. But still all physical values are conserved.

## 2.2. Possible collisions

This experiment focuses on the proton-proton collision, in the elastic case:

$$p + p \to p + p \tag{2.2}$$

In case of inelastic collisions different particles might be produced, for example

$$p + p \to p + p + \pi^0 \tag{2.3}$$

with the pion $\pi^0$ with $m_{\pi^0} = 134.98\,\mathrm{MeV}$.[2]

## 2.3. Inertial systems

In order to simplify the problem different inertial systems (reference frames) are useful.

- **LAB**oratory - / **T**arget **S**ystem (**LAB** / **TS**): The frame in which the experiment is measured, all data refer to this inertial system. In case of an elastic proton-proton collision there remain tow degrees of freedom, due to the four-momentum conservation and the two fixed masses, which have to be measured.

- **C**enter of **M**ass **S**ystem (**CMS**): This is the frame where the momentum vectors of the two protons sum up to zero (marked with an asterisk *). In case of an elastic collision two degrees of freedom remain. When using spherical coordinates with an azimuthal angle $\varphi$ ($x$-$y$-plain) and a polar angle $\theta$, the four-vector conservation of a proton-proton collision translates to

$$\theta_1^* = \pi - \theta_2^* \tag{2.4}$$
$$|\varphi_1^*| = \pi - |\varphi_2^*|, \tag{2.5}$$

whereas the measurable quantities are $\theta$ and $\varphi$.

## 2.4. Transformation

In the labor system the angles

$$\varphi = \arctan\left(\frac{p_\mathrm{x}}{p_\mathrm{y}}\right) \qquad\qquad \theta = \arccos\left(\frac{p_\mathrm{z}}{|\mathbf{p}|}\right) \tag{2.6}$$

can be measured. In order to use the conditions (2.4) and (2.5) to determinate whether the event is of elastic or inelastic nature one has to transform the conditions back to the labor system. By choosing the $z$-axis as the beam direction (before the collision) the Lorentz boost has no effect on the $x$-$y$-plain:

$$|\varphi_1| = \pi - |\varphi_2| \tag{2.7}$$

As $\theta$ is the angle to the $z$-axis, it transforms according to the Lorentz boost. Transforming the condition back to the laboratory system results in

$$\tan(\theta_1)\tan(\theta_2) = \frac{1}{\gamma_\mathrm{pp}^2}, \tag{2.8}$$

derived in chapter B.

## 2.5. Missing mass method

Elementary reactions event topology can be reconstructed by the so called missing mass method. For this the Lorentz vectors of the incoming beam (B) and the target (T) are used. The missing particle finally can be identified by its missing mass which can be determined in the following way[1]:

$$M_\mathrm{missed}^2 = (P_\mathrm{B} + P_\mathrm{T} - P_1 - P_2 \cdots - P_n)^2 \tag{2.9}$$

# 3. Analysis of the data and results

The analysis of the data shall determinate whether the working hypothesis “the event is an elastic $p + p \to p + p$ collision” is justified. To get a statistically significant result more events were analyzed and plotted. Therefore histograms of the observables are used to represent there distribution. To get an estimation of the total number of pp collisions the challenge is to find appropriate cut conditions, in this case one might use the conditions (2.7) and (2.8).

## 3.1. Analysis of the azimuthal observable

The raw distribution of $\Delta\varphi := \varphi_1 - \varphi_2$ (eq. (2.7)) is plotted in figure 3.1.

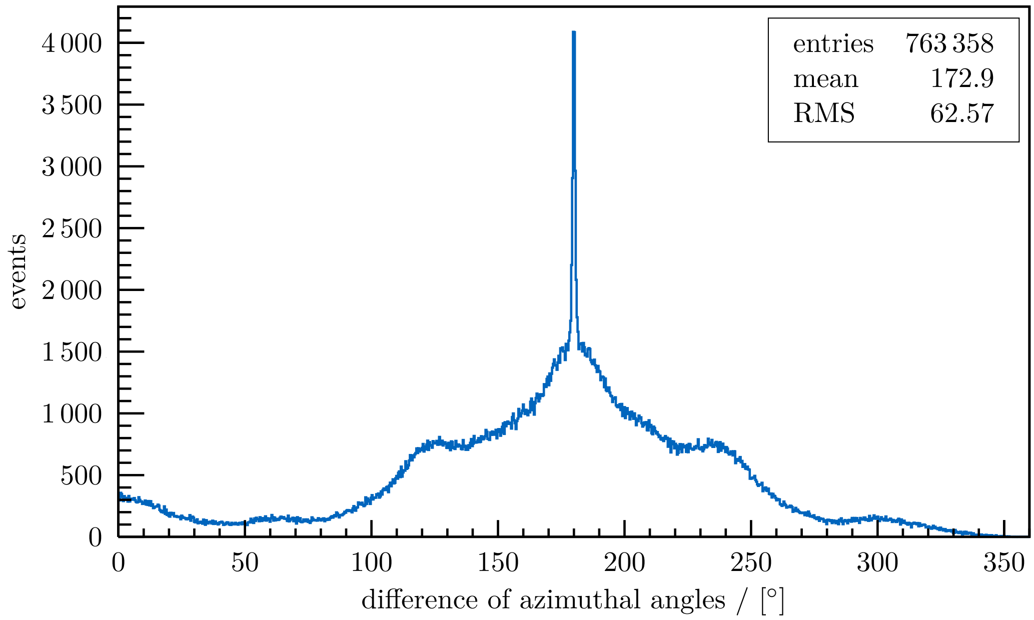


**Figure 3.1.:** Distribution of the observable $\Delta\varphi$ without any filter conditions for $\Delta\varphi > 0$.

According to the condition (2.7) applying for the $\varphi$-angle, all events that are of elastic nature have to have a $\varphi$-angle sum of $\pi$. As can be seen in figure 3.1 there is a peak with $N'_{\text{pp}} \approx 4\,100$ events which have a angle sum of $|\varphi_1| + |\varphi_2| = \pi$. One has to take an uncertainty and errors in the measurement process into account, so to get the total number of events one might integrate form $\pi - \varepsilon$ to $\pi + \varepsilon$ whereas $\varepsilon > 0$ is the tolerance range of the error. To put the pp events in perspective one might compare them to the number of background events. The same process must be done for $\Delta\varphi < 0$, because there is also a peak at $-180$ degrees. A reduction of the background noise can be done by applying a constraint to the observable $\Delta\varphi$ by using the condition (2.8).

## 3.2. Analysis of the polar angle-condition

Second the condition (2.8) for the laboratory system was analyzed to get one filter condition for the $\varphi$-plot in section 3.1. The raw distribution of this polar angle-condition is plotted in figure 3.2.

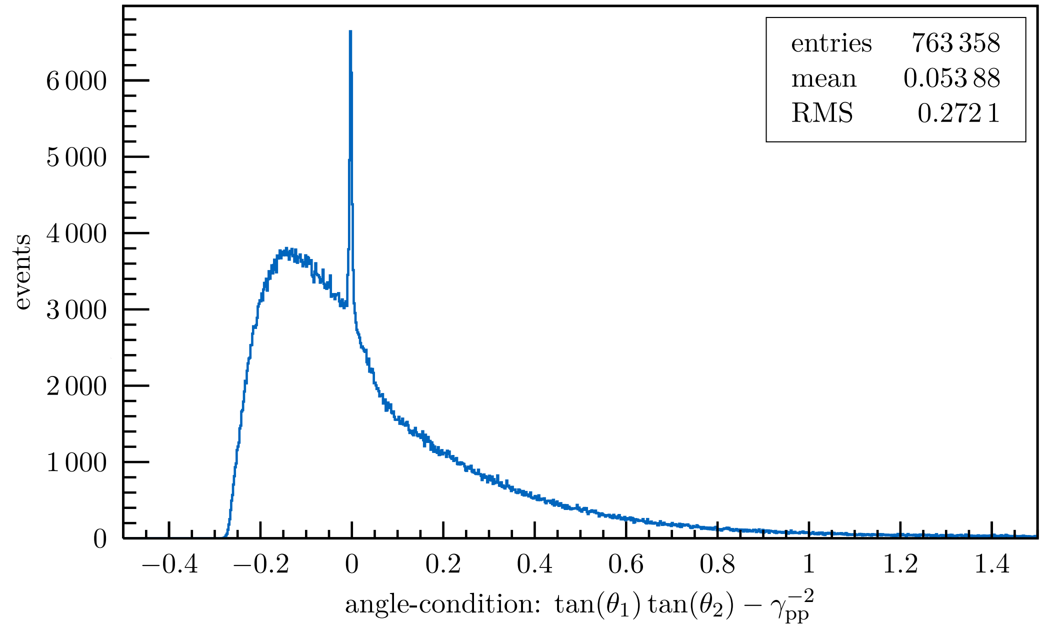


**Figure 3.2.:** Distribution of the angle-condition for $\theta_1$ and $\theta_2$.

As one can see the course is not symmetric at zero due to the tangent-functions and there is a peak at an angle of about 0, which represents the elastic pp events. But also, there is background noise in the area form $-0.3$ to 0 with event numbers in approximately $\mathcal{O}(10^5)$. By considering this background and filtering it one can drastically improve the results of the $\varphi$-distribution 3.1.

## 3.3. Combination of azimuthal observable and polar angle-condition

Next, both conditions (2.7) and (2.8) will be analyzed simultaneously. Therefore the conditions were calculated and filled into a two-dimensional histogram 3.3. One can see a peak where both, the $\theta$-condition ($\tan(\theta_1)\tan(\theta_2) - \gamma_{\mathrm{pp}}^{-2} \approx 0$) and the $\varphi$-condition ($|\varphi_1| + |\varphi_2| \approx \pi$) are fulfilled. This means that the working hypotheses, the assumption of an elastic collision, was right. The total number of elastic events can be calculated by integration of an ellipse with center $(180, 0)$ and the halfaxsis as the $\theta$ or the $\varphi$ uncertainty of the conditions.

Afterwards only events which fulfill equation (2.8) with a fixed maximum deviation, estimated with 0.01, and where both outgoing particles have a minimum mass of around the half of the proton mass $\approx 500\,\mathrm{MeV}$ where counted and the resulting distribution of the observable $\Delta\varphi$ is plotted in figure 3.4.

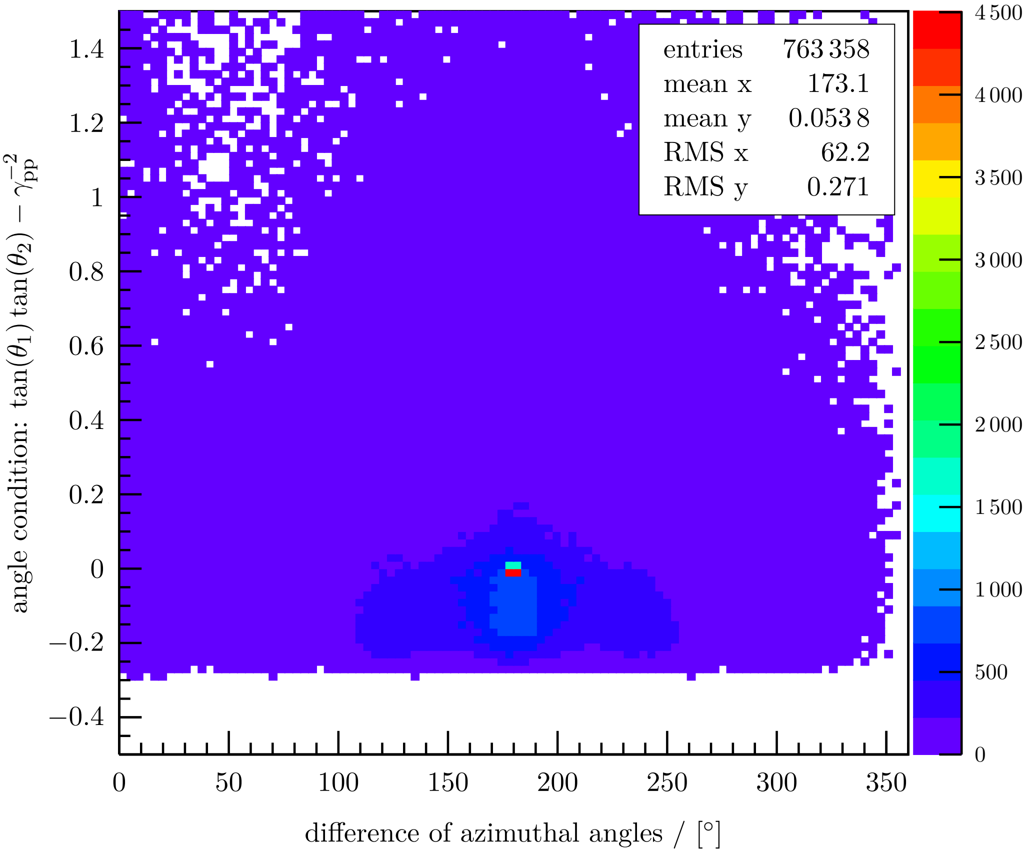


**Figure 3.3.:** 2-D Histogram with the polar angle-condition on the y-axis and the observable $\Delta\varphi$ on the x-axis. The color describes the total number of events.

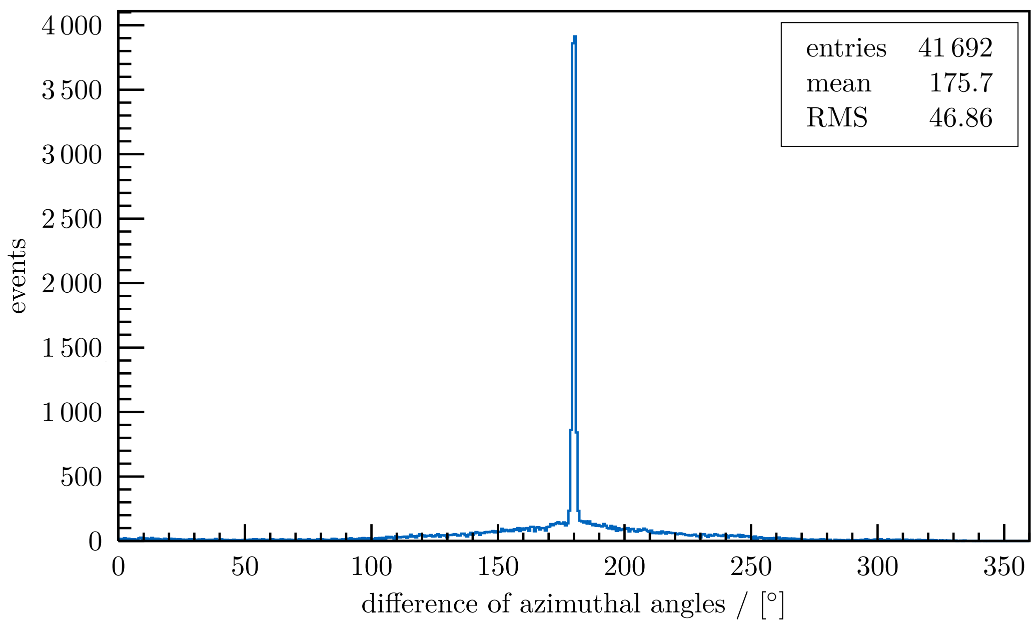


**Figure 3.4.:** Distribution of the chosen observable $\Delta\varphi$ with the condition for the polar angle and the two outgoing particles.

Like one can see in figure 3.4, there is only left a fraction of events and a sharper peak around 180 degrees can be observated, which confirms the assumption of an elastic proton-proton collision, which was made in the beginning. Integration over $(180\pm3)$ degrees results in $N'_{\mathrm{pp}} = 10\,072$ proton-proton collisions for $\Delta\varphi > 0$ and in the same way it must be done for $\Delta\varphi < 0$ to get an estimation of the total number of pp collisions.

## 3.4. Missed mass

Finally the missing mass method, described in section 2.5 was applied on the given data and filled in a histogram 3.5.

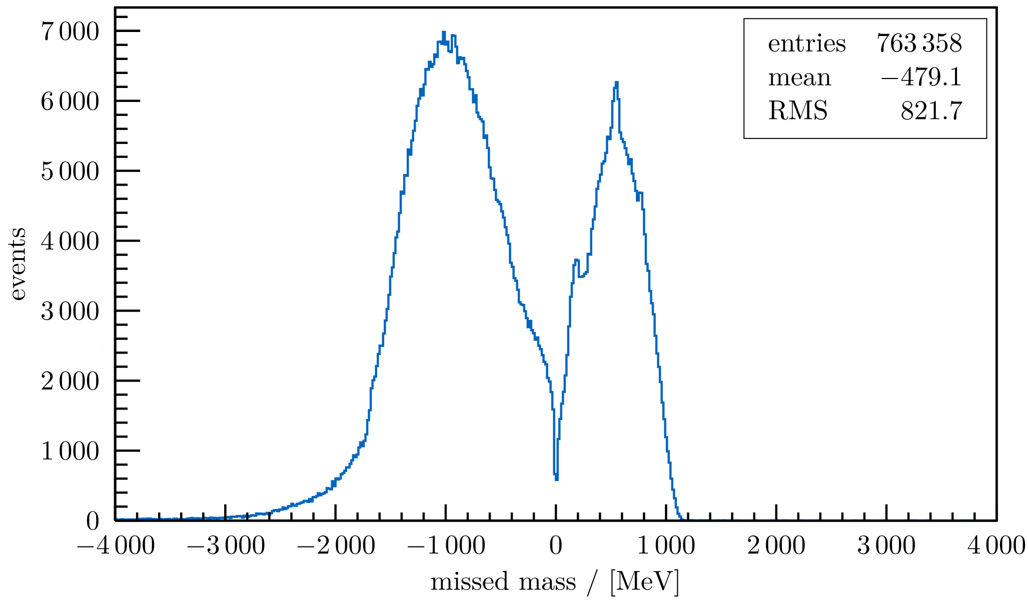


**Figure 3.5.:** Distribution of the missing mass for all considered events.

As mentioned in chapter B a peak at about the mass of the pion $\pi^0$ can be identified. If one uses the polar angle-condition the background decreases and the distribution will be symmetrically around zero, as expected. Figure 3.5 indicates that without any filter conditions there are a lot of inelastic collisions where particles were produced, e.g. $\pi$, $K$ and so on.

## 3.5. Results and interpretation

Due to the fact that only the positive range of the observable $\Delta\varphi$ was examined, the total number of proton-proton collisions can only be estimated to about twice the amount of the filtered result in 3.3, which leads to $N_{\mathrm{pp}} \approx 2N'_{\mathrm{pp}} = 20\,144$, assuming symmetry. Concerning this matter it would be improvable to take $|\varphi_1| + |\varphi_2|$ as azimuthal observable.

The number of selected bins can be considered as good, since a further reduction of the bins due to the background, especially shown in figure 3.3, would not have yielded any significant gain. An optimum between efficiency and purity found by trial and error provides a strong reduction in the number of background events. If the cut conditions applied were chosen more strict the amount of signal would have decreased strongly, so a balance was found. Furthermore a third condition, the missing mass, could be added, to further reduce the background or to make the other conditions less strict and also the structure and efficiency of the code could be even better.

The main focus of this lab course was the data analysis of the given data sets from HADES, whereby systematic errors in the experiment were neglected, because the theoretically expected difference value $\pi$ of the azimuthal angles showed good congruence and primarily at the beginning the background predominated. Next step, would be the analysis of the amount of $p + p + \pi^0$ events and an estimation of the cross-sections. Due to the facts that the detected particles remain unchanged and only one pion is created, which takes a relatively small part of the energy (lightest hadron), the acceptance and efficiency is almost similar for both reactions elastic pp and pp$\pi_0$. Thus means that the ratio between the number of events and the cross sections of the reactions are almost the same.

# A. Code

For the provided C++-template program refer to the advanced lab course *Particle physics with the computer*. The main part, where modifications and extensions have been made, can be found in A.1.

**Listing A.1:** Main part of the total (final) C++-program code.

```
//----------------------------------------
//DEFINING THE BEAM AND TARGET PARTICLES
Double_t M_Proton =  HPidPhysicsConstants::mass(14);

Float_t E_kin_beam = 3500.0; // MeV/Nukleon
Float_t E_kin_target = 0.0; // MeV/Nukleon
Float_t E_total_beam = E_kin_beam + M_Proton;
Float_t E_total_target = E_kin_target + M_Proton;
Float_t pz_beam = sqrt(pow(E_total_beam,2) - pow(M_Proton,2));

TLorentzVector Vec_pp35_beam = TLorentzVector(0,0,pz_beam,E_total_beam);
TLorentzVector Vec_pp35_target = TLorentzVector(0,0,0,E_total_target);
TLorentzVector Vec_pp35_pp = Vec_pp35_beam + Vec_pp35_target;

//----------------------------------------
//DEFINING PARTICLE 1 AND 2
Float_t px1, px2, py1, py2, pz1, pz2;
Float_t charge1, charge2;
Float_t beta1, beta2;

TLorentzVector Vec_pp35_1;
TLorentzVector Vec_pp35_2;
TLorentzVector Vec_pp35_missed;
float gamma_pp = Vec_pp35_pp.Energy()/Vec_pp35_pp.M();

//----------------------------------------
//DEFINING VARIABLES
float condition_theta;
float sum_phi, theta_correlation;

//----------------------------------------
//HISTOGRAM DEFINITION (FIT DEFINITION)
//1D histo:
TH1F* Histo_theta_condition = new
    TH1F("Histo_theta_condition","Histo_theta_condition",1000,-0.5,1.5);
TH1F* Histo_sum_phi = new
    TH1F("Histo_sum_phi","Histo_sum_phi",1000,0,360);
TH1F* Histo_sum_phi_filtered = new
    TH1F("Histo_sum_phi_filtered","Histo_sum_phi_filtered",500,0,360);
TH1F* Histo_missed = new
    TH1F("Histo_missed","Histo_missed",500,-4000,4000);
```

```
//2D histo:
TH2F* Histo_2d = new TH2F("Histo_2d","Histo_2d",100,0,360,100,-0.5,1.5);

for(Int_t counter; counter<Statistic; counter++){
    if(counter!=1.00 && counter%3000 == 0) cout<<"."<<flush;
    if(counter!=0 && counter%50000==0 ){
        Float_t event_percent = 100.0*counter/Statistic;
        cout << " " << counter << "("<< event_percent <<"%)" << "\n" <<
            "==> 1) Processing data" << flush;
    }
    Event_tree->GetEntry(counter);
    // Lorentz-Vectors for the particle 1 and 2
    Vec_pp35_1 = TLorentzVector(px1,py1,pz1,sqrt(M_Proton*M_Proton +
        px1*px1 + py1*py1 + pz1*pz1));
    Vec_pp35_2 = TLorentzVector(px2,py2,pz2,sqrt(M_Proton*M_Proton +
        px2*px2 + py2*py2 + pz2*pz2));

    if(Vec_pp35_1.P() < 500) continue; // Condition for the particle 1
    if(Vec_pp35_2.P() < 500) continue; // Condition for the particle 2

    // CALCULATE CONDITIONS
    sum_phi = (Vec_pp35_1.Phi() - Vec_pp35_2.Phi())*180.0/TMath::Pi();
    theta_correlation = tan(Vec_pp35_1.Theta()) * tan(Vec_pp35_2.Theta())
        - pow(gamma_pp,-2);
    // Histograms 1d
    Histo_sum_phi->Fill(sum_phi);
    Histo_theta_condition->Fill(theta_correlation);
    // Missed mass
    Vec_pp35_missed = Vec_pp35_beam + Vec_pp35_target - Vec_pp35_1 -
        Vec_pp35_2;
    Histo_missed->Fill(Vec_pp35_missed.M());
    // Histogram 2d
    Histo_2d->Fill(sum_phi,theta_correlation);

    condition_theta = tan(Vec_pp35_1.Theta()) * tan(Vec_pp35_2.Theta()) -
        pow(gamma_pp,-2);
    if(fabs(condition_theta) < 0.01){
        Histo_sum_phi_filtered->Fill((Vec_pp35_1.Phi() -
            Vec_pp35_2.Phi())*180.0/TMath::Pi());
    }
}// END OVER EVENT LOOP
```

# B. Questions and Tasks

**1. Identify the particles on the plot 3 in [1].**

In the figure one can see the energy loss of the particles in the MDC chambers plotted against the particles momentum times their charge. As the energy loss is unique for each particle or more specifically for each mass one can see three different particles, whereby two only differ in there charge. Furthermore, if the mass is larger, a faster increase in energy loss occurs with smaller momentum, as one can see in the Beth-Bloch formula.[1] As we analyze proton-proton elastic collisions the particle with the higher mass and a positive charge can be identified as a proton. The other two with a smaller mass can be determined as the same particle with different charge. As the probability of forming new particles decreases with increasing mass the two curves can be identified as pions $\pi^+$ and $\pi^-$, the lightest hadrons. Furthermore the energy loss as a function of the momentum has most commonly a minimum at $\gamma\beta \approx 3$ to 4 (depends on the mean excitation potential of the material).[3] As one can see in the plot the minimum of the two lighter particles is located at $p \approx \pm 450\,\mathrm{MeV}/c$ and these leads to $m \approx (113 \text{ to } 150)\,\mathrm{MeV}/c^2$ whereby the magnitude matches with the identification as pions.

**2. Which of these reactions (see [1], chapter 4) are not possible and why?**

Due to the conservation laws, listed in 2.1, the following reactions are not possible, whereby one not fulfilled conservation is specified.

| | |
|---|---|
| $p + p \to \pi^0 + \pi^+ + \pi^+$ | baryon-number ($2 \neq 0$) |
| $p + p \to p + n + e^+$ | lepton-number ($0 \neq 1$) |
| $p + p \to \sum^0 + K^+ + p + \pi^+$ | electric charge ($2 \neq 3$) |

**3. Why in case of ion collisions, it is complicated, or even impossible to apply the missing mass technique?**

Like already mentioned in section 2.5, event topologies of elementary events can be reconstructed by the missing mass method. Since the number of undetected particles increases with the number of particles created during a collision and the method is only applicable if there is one missing particle, the missing mass technique is not suitable for ion collisions.

**4. Connect $\tan(\theta_1)$ and $\tan(\theta_2)$ (TS) without any dependence on angles.**

First we generally want to consider a collision of two particles, the beam $P_\mathrm{B}$ and target particle $P_\mathrm{T}$ and two particles 1 and 2 in the final state, whereas the TS frame is defined so that the beam is collinear with the z-axis and the second particle rests. Therefore with a Lorentz boost in z-direction (from TS to CMS, $\beta_\mathrm{pp}$... speed of the beam particle)

$$\Lambda_\mathrm{pp} = \begin{pmatrix} \gamma_\mathrm{pp} & 0 & 0 & -\gamma_\mathrm{pp}\beta_\mathrm{pp} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma_\mathrm{pp}\beta_\mathrm{pp} & 0 & 0 & \gamma_\mathrm{pp} \end{pmatrix} \qquad \gamma_\mathrm{pp} = \frac{1}{\sqrt{1-\beta_\mathrm{pp}^2}} \tag{B.1}$$

only the component parallel to the beam direction changes. This can be expressed with:

$$|\mathbf{p}_1|\sin(\theta_1) = |\mathbf{p}_1^*|\sin(\theta_1^*) \tag{B.2}$$

$$|\mathbf{p}_1|\cos(\theta_1) = \gamma_\mathrm{pp}|\mathbf{p}_1^*|\cos(\theta_1^*) + \gamma_\mathrm{pp}\beta_\mathrm{pp}E_1^* \tag{B.3}$$

Dividing (B.2) by (B.3) results, with the relation (2.4) and $\beta = \frac{|\mathbf{p}|}{E}$, in a transformation of the $\theta$ angles:

$$\tan(\theta_1) = \frac{\sin(\theta_1^*)}{\gamma_\mathrm{pp}\left(\cos(\theta_1^*) + \frac{\beta_\mathrm{pp}}{\beta_1^*}\right)} \tag{B.4}$$

$$\tan(\theta_2) = \frac{\sin(\theta_1^*)}{\gamma_\mathrm{pp}\left(-\cos(\theta_1^*) + \frac{\beta_\mathrm{pp}}{\beta_1^*}\right)} \tag{B.5}$$

First we want to prove that for elastic scattering $\beta_1^* = \beta_\mathrm{pp}$. Therefore we define the combined vector

$$P_\mathrm{pp} = P_\mathrm{B} + P_\mathrm{T} = \begin{pmatrix} E_\mathrm{pp} \\ \mathbf{p}_\mathrm{pp} \end{pmatrix} = \begin{pmatrix} E_\mathrm{B} + m_\mathrm{T} \\ \mathbf{p}_\mathrm{B} \end{pmatrix}. \tag{B.6}$$

With the conservation (2.1) of the four-momentum follows that the energy in the CMS frame doesn't change and to calculate $E_\mathrm{pp}$ consider

$$\begin{aligned}
&P_\mathrm{pp}^2 = m_\mathrm{pp}^2 = (P_\mathrm{B} + P_\mathrm{T})^2 = m_\mathrm{B}^2 + m_\mathrm{T}^2 + 2E_\mathrm{B}m_\mathrm{T} \\
&\Rightarrow 2E_\mathrm{B}m_\mathrm{T} = m_\mathrm{pp}^2 - m_\mathrm{B}^2 - m_\mathrm{T}^2 \\
&\Rightarrow (P_\mathrm{pp} + P_\mathrm{T})^2 = m_\mathrm{pp}^2 + m_\mathrm{T}^2 + 2E_\mathrm{pp}m_\mathrm{T} \\
&\qquad = m_\mathrm{pp}^2 + m_\mathrm{T}^2 + 2(E_\mathrm{B} + m_\mathrm{T})m_\mathrm{T} \\
&\qquad = m_\mathrm{pp}^2 + m_\mathrm{T}^2 + m_\mathrm{pp}^2 - m_\mathrm{B}^2 - m_\mathrm{T}^2 + 2m_\mathrm{T}^2 \\
&\qquad = 2m_\mathrm{pp}^2 + 2m_\mathrm{T}^2 - m_\mathrm{B}^2
\end{aligned}$$

$\Rightarrow$

$$E_\mathrm{pp} = \frac{1}{2m_\mathrm{T}}\left(m_\mathrm{pp}^2 + m_\mathrm{T}^2 - m_\mathrm{B}^2\right) \tag{B.7}$$

With the energy momentum relation we can now calculate $|\mathbf{p}_{\mathrm{pp}}|$:

$$\begin{aligned}
\mathbf{p}_{\mathrm{pp}}^2 &= E_{\mathrm{pp}}^2 - m_{\mathrm{pp}}^2 = \frac{1}{4m_{\mathrm{T}}^2}\left(m_{\mathrm{pp}}^2 + m_{\mathrm{T}}^2 - m_{\mathrm{B}}^2\right)^2 - m_{\mathrm{pp}}^2 \\
&= \frac{1}{4m_{\mathrm{T}}^2}\left(m_{\mathrm{pp}}^4 + m_{\mathrm{T}}^4 + m_{\mathrm{B}}^4 - 2m_{\mathrm{pp}}^2 m_{\mathrm{T}}^2 - 2m_{\mathrm{pp}}^2 m_{\mathrm{B}}^2 - 2m_{\mathrm{B}}^2 m_{\mathrm{T}}^2\right) \\
&= \frac{1}{4m_{\mathrm{T}}^2}\left(\left(m_{\mathrm{T}}^2 - m_{\mathrm{B}}^2 - m_{\mathrm{pp}}^2 + 2m_{\mathrm{B}}m_{\mathrm{pp}}\right)\left(m_{\mathrm{T}}^2 - m_{\mathrm{B}}^2 - m_{\mathrm{pp}}^2 - 2m_{\mathrm{B}}m_{\mathrm{pp}}\right)\right)
\end{aligned}$$

$\Rightarrow$

$$|\mathbf{p}_{\mathrm{pp}}| = \frac{1}{2m_{\mathrm{T}}}\left[\left(m_{\mathrm{T}}^2 - (m_{\mathrm{pp}} - m_{\mathrm{B}})^2\right)\left(m_{\mathrm{T}}^2 - (m_{\mathrm{pp}} + m_{\mathrm{B}})^2\right)\right]^{1/2} \tag{B.8}$$

To calculate $E_1^*$ consider ((1′)... Lorentz invariant, (2′)... CMS, (3′)... elastic collision):

$$\begin{aligned}
m_{\mathrm{pp}}^2 &= P_{\mathrm{pp}}^2 = (P_{\mathrm{B}} + P_{\mathrm{T}})^2 \overset{(1')}{=} (P_{\mathrm{B}}^* + P_{\mathrm{T}}^*)^2 \overset{(2')}{=} (E_{\mathrm{B}}^* + E_{\mathrm{T}}^*)^2 \overset{(3')}{=} (E_1^* + E_2^*)^2 \overset{(2')}{=} (P_1^* + P_2^*)^2 \\
&\overset{(2')}{=} m_1^2 + m_2^2 + 2(E_1^* E_2^* + \mathbf{p}_1^{*2}) \\
&= m_1^2 + m_2^2 + 2(E_1^* E_2^* + E_1^{*2} - m_1^2) \\
&= m_1^2 + m_2^2 + 2(E_1^* m_{\mathrm{pp}} - m_1^2)
\end{aligned}$$

$\Rightarrow$

$$E_1^* = \frac{1}{2m_{\mathrm{pp}}}\left(m_{\mathrm{pp}}^2 + m_1^2 - m_2^2\right) \tag{B.9}$$

$$|\mathbf{p}_1^*| = \frac{1}{2m_{\mathrm{pp}}}\left[\left(m_{\mathrm{pp}}^2 - (m_1 - m_2)^2\right)\left(m_{\mathrm{pp}}^2 - (m_1 + m_2)^2\right)\right]^{1/2} \tag{B.10}$$

Wherein the equation (B.10) follows exactly in the same way from (B.9) like (B.8) from (B.7). Finally we arrived for an elastic scattering of two protons ($m_i = m_{\mathrm{p}}, \forall i \in \{1, 2, \mathrm{T}, \mathrm{B}\}$):

$$\begin{aligned}
\frac{\beta_1^*}{\beta_{\mathrm{pp}}} &= \frac{E_1^*}{|p_1^*|}\frac{|p_{\mathrm{pp}}|}{E_{\mathrm{pp}}} = \left.\frac{\left[(m_{\mathrm{T}}^2 - (m_{\mathrm{pp}} - m_{\mathrm{B}})^2)(m_{\mathrm{T}}^2 - (m_{\mathrm{pp}} + m_{\mathrm{B}})^2)\right]^{1/2}}{\left[\left(m_{\mathrm{pp}}^2 - (m_1 - m_2)^2\right)\left(m_{\mathrm{pp}}^2 - (m_1 + m_2)^2\right)\right]^{1/2}}\right|_{m_i = m_{\mathrm{p}}} \\
&= \frac{\left[\left(m_{\mathrm{p}}^2 - (m_{\mathrm{pp}} - m_{\mathrm{p}})^2\right)\left(m_{\mathrm{p}}^2 - (m_{\mathrm{pp}} + m_{\mathrm{p}})^2\right)\right]^{1/2}}{\left[m_{\mathrm{pp}}^2\left(m_{\mathrm{pp}}^2 - 4m_{\mathrm{p}}^2\right)\right]^{1/2}} \\
&= \frac{\left[\left(-m_{\mathrm{pp}}^2 + 2m_{\mathrm{pp}}m_{\mathrm{p}}\right)\left(-m_{\mathrm{pp}}^2 - 2m_{\mathrm{pp}}m_{\mathrm{p}}\right)\right]^{1/2}}{\left[m_{\mathrm{pp}}^4 - 4m_{\mathrm{p}}^2 m_{\mathrm{pp}}^2\right]^{1/2}} = \frac{\left[m_{\mathrm{pp}}^4 - 4m_{\mathrm{p}}^2 m_{\mathrm{pp}}^2\right]^{1/2}}{\left[m_{\mathrm{pp}}^4 - 4m_{\mathrm{p}}^2 m_{\mathrm{pp}}^2\right]^{1/2}} = 1
\end{aligned}$$

Multiplying equations (B.4) and (B.5) results then in (2.8): $\tan(\theta_1)\tan(\theta_2) = \gamma_{\mathrm{pp}}^{-2}$.

**5. What is the value for $\gamma$ for a $p+p$ reaction with a beam kinetic energy of $E_{\mathrm{B,kin}} = 3.5\,\mathrm{GeV}$?**

We can derive $\gamma_{\mathrm{pp}}$ from $\beta_{\mathrm{pp}}$ and therefore we consider the Lorentz boost in $z$-direction:

$$
\begin{aligned}
&\mathbf{p}_{\mathrm{B}}^{*} = \gamma_{\mathrm{pp}}(\mathbf{p}_{\mathrm{B}} - \beta_{\mathrm{pp}} E_{\mathrm{B}} \hat{\mathbf{e}}_{\mathrm{z}}) \overset{(2')}{=} -\mathbf{p}_{\mathrm{T}}^{*} = -\gamma_{\mathrm{pp}}(-\beta_{\mathrm{pp}} m_{\mathrm{T}} \hat{\mathbf{e}}_{\mathrm{z}}) \\
&\Rightarrow \beta_{\mathrm{pp}} = \frac{|\mathbf{p}_{\mathrm{B}}|}{m_{\mathrm{T}} + E_{\mathrm{B}}} \\
&\Rightarrow \gamma_{\mathrm{pp}} = \frac{1}{\sqrt{1-\beta_{\mathrm{pp}}^2}} = \frac{m_{\mathrm{T}} + E_{\mathrm{B}}}{\sqrt{(m_{\mathrm{T}} + E_{\mathrm{B}})^2 - \mathbf{p}_{\mathrm{B}}^2}} = \frac{E_{\mathrm{pp}}}{\sqrt{m_{\mathrm{T}}^2 + m_{\mathrm{B}}^2 + 2 m_{\mathrm{T}} E_{\mathrm{B}}}} \\
&\qquad\;\; = \frac{E_{\mathrm{pp}}}{\sqrt{(P_{\mathrm{B}} + P_{\mathrm{T}})^2}} \overset{(1')}{=} \frac{E_{\mathrm{pp}}}{\sqrt{(P_{\mathrm{B}}^{*} + P_{\mathrm{T}}^{*})^2}} \overset{(2')}{=} \frac{E_{\mathrm{pp}}}{\sqrt{(E_{\mathrm{B}}^{*} + E_{\mathrm{T}}^{*})^2}}
\end{aligned}
$$

With $W = E_{\mathrm{B}}^{*} + E_{\mathrm{T}}^{*}$ the total CMS energy or

$$
W^2 = (E_{\mathrm{B}}^{*} + E_{\mathrm{T}}^{*})^2 \overset{(1',2')}{=} (P_{\mathrm{B}} + P_{\mathrm{T}})^2 = P_{\mathrm{PP}}^2 = m_{\mathrm{pp}}^2
$$

one gets:

$$
\gamma_{\mathrm{pp}} = \frac{E_{\mathrm{pp}}}{m_{\mathrm{pp}}} \tag{B.11}
$$

For an elastic scattering of two protons we can calculate $\gamma_{\mathrm{pp}}$ using $E_{\mathrm{B}} = E_{\mathrm{B,kin}} + m_{\mathrm{p}}$ and $m_{\mathrm{p}} = 938.27\,\mathrm{MeV}$[2]:

$$
\gamma_{\mathrm{pp}} = \frac{m_{\mathrm{p}} + E_{\mathrm{B}}}{\sqrt{2m_{\mathrm{p}}^2 + 2 m_{\mathrm{p}} E_{\mathrm{B}}}} = \sqrt{1 + \frac{E_{\mathrm{B,kin}}}{2m_{\mathrm{p}}}} \approx 1.69
$$

# C. Bibliography


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# D. List of figures