Physical practical course part 1

(Mechanics and thermodynamics)

Viscosity

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1. Introduction

With the aid of these experiments the viscosity of water is measured with the help of a capillary or an Ubbelohde viscometer, which also uses a capillary based method. In the second part glass spheres are used to determine the viscosity of an unknown glycerol-water-mixture. The basics about viscosity from the instructions [1] are assumed and the results of these three procedures are compared to each other and values found in literature.

2. Falling sphere viscometer

2.1. Used methods

It is possible to determine the viscosity of an unknown liquid, by measuring the final velocity v of a sphere moving in the medium, the radius r of this sphere and the density of the liquid ρ_{fluid} and sphere ρ_{sphere}, using the relation

$$\eta = \frac{2r^2 g}{9v} \cdot (\rho_{fluid} - \rho_{sphere}) \tag{2.1}$$

called Stokes' Law. This assumes that three forces, gravitation, buoyancy and friction act on a sphere. The equation 2.1 is only valid for a Reynolds number $R_\mathrm{e} < 1$ (laminar flow), is idealized for a container with infinite volume and can only be used for spherical particles and homogeneous material. If we correct the formula to

$$\eta = \frac{2r^2 g}{9v \cdot (1 + 2.4\frac{r}{R})} \cdot (\rho_{fluid} - \rho_{sphere}) \tag{2.2}$$

where the sphere is moving in an cylinder with the inner radius R, we are now able to measure the viscosity of such a liquid, if the finite length of the cylinder is neglected.

As a criterion whether the flow is laminar or turbulent the Reynolds number, defined as

$$R_\mathrm{e} = \frac{L\rho_\mathrm{fluid} v}{\eta} \tag{2.3}$$

can be used. For a flow in a tube the characteristic dimension L results in $L = 2 \cdot R$. Generally if $R_\mathrm{e} < 1\,160$ the flow is laminar, between 1 160 and 2 300 some disturbances could be found and the flow resistance increases and it becomes more and more turbulent.

2.2. Experimental procedure

For this experiment a long cylinder with two "marks" is filled with a glycerol-water-mixture. As shown in the figure 2.1 the trajectory of the sphere is parallel to the cylinder axis and so equation 2.2 can approximately be used to calculate the viscosity, by measuring the necessary data. At the beginning the distance s between both marks was measured, using a folding meter. The radius of the ten almost equally large glass spheres was determined with a micrometer screw and the mass using a precision scale. Afterwards the time it took a sphere to fall down from the first mark below the surface to the other was measured with the help of a digital stopwatch. This procedure was repeated 20 times.

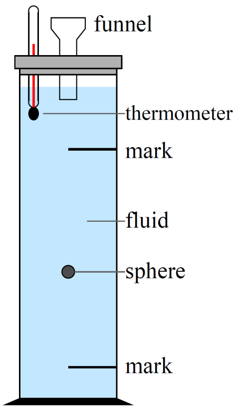

Figure 2.1.: Schematic representation of the falling sphere viscometer [1]

At the end of the experiment the temperature T_{liquid} was measured using a digital multimeter (GTH 175/Pt Pt1000), the density of the liquid with the help of an aerometer[1] and the inner radius R determined with a calliper.

2.3. Results

The measurements of the balls resulted, after conversion and error propagation, in the values in table 2.1. Therefore the diameters of the ten spheres d were averaged via the Student's t-distribution ($n = 10 \rightarrow t_{\text{Student}} = 1.06$) and apart from that statistical error of $\Delta\overline{d}_{\text{stat}} = 0.012\,\text{mm}$, the systematic one resulted from the measuring device was $\Delta\overline{d}_{\text{syst}} = 0.01\,\text{mm}$. Finally the density of the spheres were calculated with the formula of the volume V of a sphere $V = \frac{4}{3}\pi \cdot r^3$ and the fundamental relation $m = \rho \cdot V$. The errors by measuring the mass consists of a systematic linearity and scale error of 0.5 mg, a quasi statistical error $\Delta\overline{m}_{10_{\text{stat1}}} = 0.2\,\text{mg}$ and a reading error about $\Delta\overline{m}_{10_{\text{stat2}}} = 0.1\,\text{mg}$, whereas $\overline{m}_{10}$ is the mass of all ten spheres.

Table 2.1.: Physical quantities of the used glass spheres (averaged)

r_{sphere} mm	m_{sphere} mg	ρ_{sphere} kg/m^3
3.034 ± 0.011	286.19 ± 0.07	$2\,448 \pm 27$

The measurement of the geometrical dimension, the diameter D, of the cylinder was afflicted with a systematic error due to the calibration of $\Delta\overline{D}_{\text{syst}} = 0.01\,\text{mm}$ and a statistical error of

[1]The measuring and correction process of the density ρ_{liquid} can be found on the website `https://www.ph.tum.de/academics/org/labs/ap/ap1/`.

$\Delta\overline{D}_{\text{stat}} = 0.01\,\text{mm}$ due to the reading error (line width). The error of the temperature of the liquid consists of a systematic one of $0.1\,\% \cdot$ "measured value" $\pm\,2\,$digits and a statistical of $0.1\,°\text{C}$ due to the resolution of the digital multimeter. The statistical error of the measurement of the distance s between the to marks is $\Delta\overline{s}_{\text{stat}} = 0.14\,\text{mm}$ due to the reading error on both sides (line width). Furthermore there is a systematic one, because of the production accuracy of the folding ruler which can be calculated with $0.1\,\% \cdot$ "measured value". At last the density of the liquid was read off the scale and corrected to the temperature of the liquid. In addition the systematic error was neglected and the statistic one calculated with a quarter of the line space on the scale. The resulted values can be found in the table 2.2.

Table 2.2.: Characteristics of the cylinder and the glycerol-water-mixture

R mm	s mm	ρ_{liquid} kg/m^3	T_{liquid} °C
26.70 ± 0.01	376.0 ± 0.5	$1\,218 \pm 3$	22.0 ± 0.3

The time measurement resulted, after error propagation ($n = 20 \rightarrow t_{\text{Student}} = 1.03$) to $t = (2.16 \pm 0.22)\,\text{s}$, where the systematic part of the error is due to the digital stopwatch. With equation 2.1, $g = (9.806\,7 \pm 0.000\,5)\,\text{m/s}^2$ (45 degree of latitude and an estimated error) and the fundamental relation $v = \frac{\text{d}s}{\text{d}t}$ the viscosity is

$$\eta = (0.131 \pm 0.004)\,\text{Pa}\,\text{s}$$

Considering the correction with equation 2.2 this results in

$$\eta = (0.103 \pm 0.004)\,\text{Pa}\,\text{s}$$

at a temperature of $T_{\text{liquid}} = (22.0 \pm 0.3)\,°\text{C}$. Applying equation 2.3 the Reynolds number for the corrected version of the viscosity is

$$R_{\text{e}} = 110 \pm 15$$

2.4. Discussion

With this experiment a realistic value of the viscosity of a glycerol-water-mixture at $T_{\text{liquid}} = (22.0 \pm 0.3)\,°\text{C}$ was measured and according to the literature [1] it would correspond to a ratio of $\approx 82\,\%$ glycerol. But due to the high Reynolds number $R_{\text{e}} > 1$ the Stokes' Law is actually not valid and the equation 2.2 shouldn't be used, but in any case we can estimate that the flow is laminar. The big differences between the results of the viscosity indicate that the correction of the shape isn't negligible.

3. Capillary viscometer

3.1. Used methods

The aim of this experiment is to determine the viscosity of water, using the Hagen-Poiseuille's law

$$i = \frac{V}{t} = \frac{\pi r^4 \Delta p}{8\eta l} \tag{3.1}$$

Therefore i is the volume flowing through the pipe, also called current (like the current in an electrical circuit), l the length, r the inner radius of the pipe and Δp the pressure difference. Therefore the equation 3.1 will be solved for η and the volume V will be replaced by mass m and density ρ_{fluid} which can be measured more accurately. If we implement a new variable W, called flow resistance (like R in an electrical circuit) we can rewrite the law 3.1 to

$$\Delta p = W \cdot i \qquad \text{with } W = \frac{8\eta l}{\pi r^4} \tag{3.2}$$

As we see, for Newton liquids, where η is independent from v, W is the slope in an so called flux-pressure difference diagram $\Delta p(i)$. To calculate the pressure difference

$$\Delta p = \rho g h \tag{3.3}$$

is used. After all this, it is important to remember that Hagen-Poiseuille's law is only valid in idealized conditions. Fur further details take a look in the instructions. [1]

3.2. Experimental procedure

A small, opened water reservoir with a changeable height is connected to a cannula via a thin tube. A three-way valve at the end of the flexible tube is used to control the flow. To store the water, which leaves the system a small measuring cylinder is used. First of all the mass of the empty cup has to been measured to later calculate the mass of the water or better the volume, which we must determine for the viscosity. Therefore it's clear that we also had to measure the temperature of the water, where we used the same digital multimeter as in chapter 2.

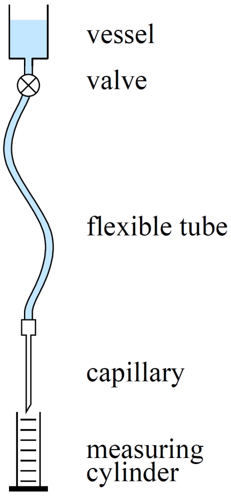

Figure 3.1.: Schematic representation of the capillary viscometer [1]

After this preparations the experiment can begin. Therefore the three-way valve will be opened for an "arbitrary" time, but it must be enough water to "exactly" measure the mass and with the digital stopwatch, also used in chapter 2, the time could be measured. For the first, small cannula (27Gx$^{3/4}$) we took ≈ 2.5 min and for the bigger one (25Gx^{1}) ≈ 1 min. After that the height of the reservoir will be measured with an folding ruler, changed and the same procedure starts again. In total we made ten measurements per cannula.

The last step contains the measurement of the length of the cannula, therefore it is important to separate the needle from the plastic and only measure the effective length l of the tube using a calliper, but more important is to determine the inside diameter (that's not the same as on the package) with the microscope. In this part errors shouldn't been made, as everyone can imagine if he knows Hagen-Poiseuille's law.

3.3. Results

The measurements of the cannula results after evaluation and error propagation to the values listed in table 3.1. For the length, the statistic reading error of 0.01 mm (line width) and the systematic calibration error of 0.01 mm have been considered. Furthermore the systematic error of the microscope was estimated to 0.1 Sktlen and the statistic reading error for a single measurement consists of 0.1 Sktlen (line width ...) and 0.25 Sktlen (1/4 space between two ticks) on the other side. For conversion

$$x[\mathrm{mm}] = \frac{x[\mathrm{Sktlen}]}{100\,\mathrm{Sktlen}} \cdot 1.058\,\mathrm{mm}$$

must be used.

Table 3.1.: Physical quantities of the used cannulas

cannula	$\frac{r}{\text{mm}}$	$\frac{l}{\text{mm}}$
1	0.1111 ± 0.0022	26.00 ± 0.02
2	0.1561 ± 0.0022	31.60 ± 0.02

During the whole measurements the temperature $T_{\text{liquid}} = (22.1 \pm 0.3)\,°\text{C}$ and the gravity $g = (9.8067 \pm 0.0005)\,\text{m/s}^2$ are regarded as constant. After researching the density of water $\rho_{\text{fluid}} = (997.75 \pm 0.01)\,\text{kg/m}^3$ [2] at 22.1 °C and 1 013 hPa the measured masses are listed in the table A.1, whereby principle the same error correction like in chapter 2 was executed.

The measurement of the time contains two important errors. First the quasi statistic error of the time was estimated to 0.2 s (reaction time) and due to the shutter release delay and other influences there are systematic one with about 0.2 ms. Regarding to the height there are two different errors. First the production accuracy of the folding ruler 0.1 % · "measured value" and secondly the statistical reading error on both sides of 1/4 space between two ticks or 1/2 of the line width depending on the height. After this the pressure differences can be calculated with equation 3.3 and the error propagation is performed separately for the statistic and systematic errors. Furthermore to determine the current with formula 3.1 the volume V must be calculated. All these is done with error propagation and leads to the linear fit considering the y- and x-error shown in figure 3.2.

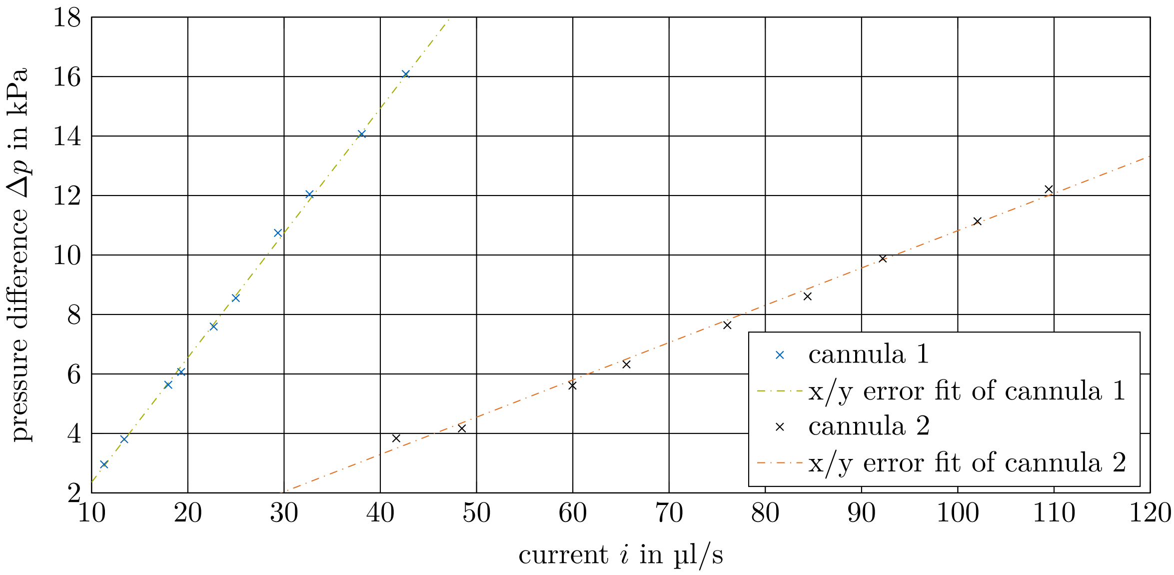

Figure 3.2.: Pressure difference as a function of the volume through the cannula 1 and 2. The two fits are calculated under consideration of the x/y-errors. The errors are in the order of the data points.

The slope now represents the resistance W of each cannula. The fit in Origin returns the values for each cannula, listed in the table 3.2. With the equation 3.1 we are able to determine the viscosity of water, whereby the error propagation can be found in section A.2 and the results are listed in table 3.2.

Table 3.2.: Determined flow resistance and viscosity of water by $T_{\text{liquid}} = (22.1 \pm 0.3)\,°\text{C}$

cannula	W kPa s/ml	η mPa s
1	419 ± 5	0.96 ± 0.08
2	125 ± 4	0.92 ± 0.06

3.4. Discussion

In contrast to the literature value[2] [3] of $\eta = 0.955\,\text{mPa s}$ at 22.1 °C. One can see that the measured values match with the literature within the tolerance range, but this method is highly sensitive against small errors in the radius and so it's very important to take some time to measure it carefully. It must be said, that we ignored the correction of the effective length, that means we used the whole length of the needle, but on one side the opening is diagonal truncated, but this only has a minor influence. Furthermore the experiment took a relative long time and the temperature was measured once a time, when we changed the cannula. Despite of this the result is realy close to the value given in the literature and the figure 3.2 shows that the current flowing through the cannula increases significantly by increasing the diameter of the needle as expected.

[2] The given literature value was linearised between the two given values 1.002 mPa s and 0.890 5 mPa s for the temperatures 20 °C and 25 °C.

4. Ubbelohde viscometer

4.1. Used methods

The aim of this experiment is to measure the viscosity of water using a Ubbelohde viscometer, which is quite similar to the capillary viscometer in chapter 3. The big difference is, that all geometrical dimensions of the measurement setup, i. e. the volume, mean hydrostatic pressure and so on, had been taken into account and combined to a device-dependent constant K. Therefore only the time t liquid flows down a certain length, the temperature and the density has to been measured to first calculate the kinematic viscosity

$$\nu = K \cdot t \tag{4.1}$$

and afterwards the dynamic viscosity can be determined using the defined relationship

$$\nu = \frac{\eta}{\rho_{\text{liquid}}} \tag{4.2}$$

4.2. Experimental procedure

Basically the time t, liquid runs down from the mark M_1 to M_2 (figure 4.1) is measured using the same digital stopwatches as in chapter 2. It must be said, that this experiment was performed together, also with the tutor and once only. In total all the three stopwatches were used by one person per group and the mean and statistical error from all values were calculated.

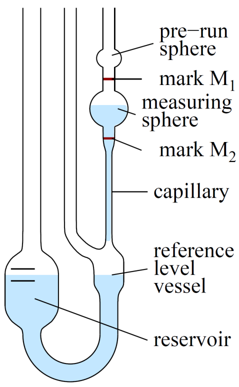

Figure 4.1.: Schematic representation of the Ubbelohde viscometer [1]

First one person locked the pressure compensation tube with his finger and the tutor sucked the water with the water-jet pump into the pre-run sphere. After al preparations the pressure compensation tube were unlocked and the flow time between the two marks were measured. Furthermore the temperature inside the device was measured with the help of temperature scale.

4.3. Results

The measured temperature of the water was $(21.85 \pm 0.04)\,°\mathrm{C}$, where the error correspondence to a statistical reading error $\Delta\overline{T}_{\mathrm{stat}} = 0.025\,°\mathrm{C}$ and an estimated systematic error $\Delta\overline{T}_{\mathrm{syst}} = 0.01\,°\mathrm{C}$ due to the calibration of the scale. The measured time results in $t = (340.5 \pm 0.4)\,\mathrm{s}$ where the statistical error was calculated via the Student's t-distribution ($n = 3 \rightarrow t_{\mathrm{Student}} = 1.32$) and the systematic one estimated, i. e. due to the shutter release delay.

After researching the density of water $\rho_{\mathrm{fluid}} = (997.80 \pm 0.01)\,\mathrm{kg/m^3}$ [2] at $21.9\,°\mathrm{C}$ and $1\,013\,\mathrm{hPa}$ and with the device-dependent constant $K = (0.002\,823 \pm 0.000\,001)\,\mathrm{cST/s}$, the equation 4.1 and 4.2 and error propagation this results in

$$\eta = (0.959\,1 \pm 0.001\,5)\,\mathrm{mPa\,s}$$

at $T = (21.85 \pm 0.04)\,°\mathrm{C}$ and "normal" pressure during this experiment.

4.4. Discussion

This shows, compared with the literature value[3] [3] $\eta = 0.961\,\mathrm{mPa\,s}$ at $21.85\,°\mathrm{C}$ that the measuring capability of the Ubbelohde is very precise, as expected. Differences between the values can be explained through the measurement method of the time and also the density and the viscosity are very dependent on temperature.

5. Conclusion

The falling sphere viscometer experiment demonstrates that Stokes' law is limited and a correction to infinite values is important. The reason is, that the flow must be viscously dominated and symmetrical, which only is given, when $R_{\mathrm{e}} < 1$. The results of the capillary viscometer in chapter 3 are not as close to the other values, like the one measured in chapter 4. This is due to the fact that the experimental procedure was much less accurate, but this one demonstrate how important it is to focus on variables, which have the maximum effect on the error, i. e. the inner radius r of the cannula.

[3] The given literature value was linearised between the two given values $1.002\,\mathrm{mPa\,s}$ and $0.890\,5\,\mathrm{mPa\,s}$ for the temperatures $20\,°\mathrm{C}$ and $25\,°\mathrm{C}$.

6. Questions

1. **Question:** Calculate the viscosity of air from the maximum velocity a parachuter reaches in free fall ($\approx 200\,\mathrm{km/h}$). Make additional assumptions. What do you notice? Calculate the Reynolds number!

   Antwort:

   First of all we make an assumption that the parachuter is rounded up like a ball, because this makes the first look much easier. Then we can say that a ball is falling down and due to the effect that the density of the air around is very low, the maximum velocity is reached at the time, when

   $$|\vec{F}_{\mathrm{R}}| = |\vec{F}_{\mathrm{G}}|$$

   With "Stokes" this correspondences to

   $$mg = 6\pi \cdot \eta \cdot r \cdot v$$

   and with an estimated person: $m = 80\,\mathrm{kg}$, $r = 0.4\,\mathrm{m}$ and a maximum speed given of approximately $v = 200\,\mathrm{km/h}$ the viscosity of air is

   $$\eta \approx 1.87\,\mathrm{Pa\,s}$$

   and that's far too much. The reason why this formula isn't correct in this situation is, that the flow is not laminar, instead it is highly turbulent as the Reynolds number shows, if we take a density of air $\rho_{\mathrm{air}} = 1.29 \cdot 10^{-3}\,\mathrm{g/cm^3}$ (next question) and estimate $L = 2R$ the equation 2.3 gives us

   $$R_{\mathrm{e}} = 3.23 \cdot 10^6$$

   if we take the literature value of $\eta = 17.74 \cdot 10^{-6}\,\mathrm{kg/ms}$ at approximately $10\,^\circ\mathrm{C}$. [3]

1. **Question:** Calculate the Reynolds number for normal respiration by nose by estimating radius of the nostrils. Normal breathing consists of approximately 15 breathes per minute, each breath approximately $0.5\,\mathrm{l}$ of air (density of air $\rho_{\mathrm{air}} = 1.29 \cdot 10^{-3}\,\mathrm{g/cm^3}$). Is it possible to achieve turbulent flow in the nostrils when intensifying your respiration?

   Antwort:

   First approximate the radius of a nostril with $10\,\mathrm{mm}$ and with $L = 2r$, $\eta \approx 18 \cdot 10^{-6}\,\mathrm{kg/ms}$ at $20\,^\circ\mathrm{C}$ [3] and $v = \frac{0.5\,\mathrm{l}}{\pi r^2 \cdot 4\,\mathrm{s}}$. Then the Reynolds number results in

   $$R_{\mathrm{e}} = 570$$

   This indicates that the flow is laminar and it would be very difficult to breath as fast as the flow becomes turbulent since we must reach $R_{\mathrm{e}} > 1160$.

A. Measurements and error calculation

A.1. Falling sphere viscometer

To calculate the error of the time measurement we used:

$$\bar{t} = \frac{1}{n}\sum_{i=1}^{n} t_i$$

$$\sigma_t = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(t_i - \bar{t})^2}$$

$$\Delta\bar{t}_{\text{stat}} = \frac{t_{\text{Student}}}{\sqrt{n}} \cdot \sigma_t$$

$$\Delta\bar{t} = \Delta\bar{t}_{\text{syst}} + \Delta\bar{t}_{\text{stat}}$$

For the physical quantities of the used glass spheres in table 2.1.

$$\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i$$

$$\sigma_d = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(d_i - \bar{d}\right)^2}$$

$$\Delta\bar{d}_{\text{stat}} = \frac{d_{\text{Student}}}{\sqrt{n}} \cdot \sigma_d$$

$$\Delta\bar{d} = \Delta\bar{d}_{\text{syst}} + \Delta\bar{d}_{\text{stat}}$$

$$\bar{r} = \frac{\bar{d}}{2} \qquad \Delta\bar{r} = \frac{\Delta\bar{d}}{2}$$

$$\Delta\overline{m}_{10_{\text{stat}}} = \sqrt{\Delta\overline{m}^2_{10_{\text{stat1}}} + \Delta\overline{m}^2_{10_{\text{stat2}}}}$$

$$\Delta\overline{m}_{10} = \Delta\overline{m}_{10_{\text{stat}}} + \Delta\overline{m}_{10_{\text{syst}}}$$

$$\overline{m} = \frac{\overline{m}_{10}}{10} \qquad \Delta\overline{m}_{\text{syst}} = \frac{\Delta\overline{m}_{10_{\text{syst}}}}{10} \qquad \Delta\overline{m}_{\text{stat}} = \frac{\Delta\overline{m}_{10_{\text{stat}}}}{10} \qquad \Delta\overline{m} = \Delta\overline{m}_{\text{stat}} + \Delta\overline{m}_{\text{syst}}$$

$$\overline{\rho} = \frac{3}{4\pi}\frac{\overline{m}}{\overline{r}^3}$$

$$\Delta\overline{\rho}_{\text{stat}} = \sqrt{\Delta\overline{m}_{\text{stat}}^2 \left(\frac{\partial\rho}{\partial m}\right)^2\Bigg|_{\overline{m},\overline{\rho}} + \Delta\overline{r}_{\text{stat}}^2 \left(\frac{\partial\rho}{\partial r}\right)^2\Bigg|_{\overline{m},\overline{\rho}}} \stackrel{(4)}{=} \frac{3}{4\pi}\frac{1}{\overline{r}^3}\sqrt{\Delta\overline{m}_{\text{stat}}^2 + \Delta\overline{r}_{\text{stat}}^2\left(\frac{3\overline{m}}{\overline{r}}\right)^2}$$

$$\Delta\overline{\rho}_{\text{syst}} = \left|\Delta\overline{m}_{\text{syst}}\left(\frac{\partial\rho}{\partial m}\right)\Bigg|_{\overline{m},\overline{\rho}}\right| + \left|\Delta\overline{r}_{\text{syst}}\left(\frac{\partial\rho}{\partial r}\right)\Bigg|_{\overline{m},\overline{\rho}}\right| \stackrel{(4)}{=} \frac{3}{4\pi}\frac{1}{\overline{r}^3}\left(|\Delta\overline{m}_{\text{syst}}| + \left|\Delta\overline{r}_{\text{syst}}\frac{3\overline{m}}{\overline{r}}\right|\right)$$

The next step is to calculate the average speed of the sphere and the error.

$$\overline{v} = \frac{\overline{s}}{\overline{t}}$$

$$\Delta\overline{v}_{\text{stat}} = \sqrt{\Delta\overline{s}_{\text{stat}}^2\left(\frac{\partial v}{\partial s}\right)^2\Bigg|_{\overline{t},\overline{s}} + \Delta\overline{t}_{\text{stat}}^2\left(\frac{\partial v}{\partial t}\right)^2\Bigg|_{\overline{t},\overline{s}}} \stackrel{(4)}{=} \frac{1}{\overline{t}}\sqrt{\Delta\overline{s}_{\text{stat}}^2 + \Delta\overline{t}_{\text{stat}}^2\left(\frac{\overline{s}}{\overline{t}}\right)^2}$$

$$\Delta\overline{v}_{\text{syst}} = \left|\Delta\overline{s}_{\text{syst}}\left(\frac{\partial v}{\partial s}\right)\Bigg|_{\overline{t},\overline{s}}\right| + \left|\Delta\overline{t}_{\text{syst}}\left(\frac{\partial v}{\partial t}\right)\Bigg|_{\overline{t},\overline{s}}\right| \stackrel{(4)}{=} \frac{1}{\overline{t}}\left(|\Delta\overline{s}_{\text{syst}}| + \left|\Delta\overline{t}_{\text{syst}}\left(\frac{\overline{s}}{\overline{t}}\right)\right|\right)$$

$$\Delta\overline{v} = \Delta\overline{v}_{\text{stat}} + \Delta\overline{v}_{\text{syst}}$$

For the error of the viscosity by equation 2.1 we get

$$\begin{aligned}\Delta\overline{\eta}_{\text{syst}} &= \left|\Delta\overline{r}_{\text{syst}}\left(\frac{\partial\eta}{\partial r}\right)\Bigg|_{\overline{x}}\right| + \left|\Delta\overline{g}_{\text{syst}}\left(\frac{\partial\eta}{\partial g}\right)\Bigg|_{\overline{x}}\right| + \left|\Delta\overline{v}_{\text{syst}}\left(\frac{\partial\eta}{\partial v}\right)\Bigg|_{\overline{x}}\right| + \left|\Delta\overline{\rho}_{\text{fluid}_{\text{syst}}}\left(\frac{\partial\eta}{\partial\rho_{\text{fluid}}}\right)\Bigg|_{\overline{x}}\right| \\ &\quad + \left|\Delta\overline{\rho}_{\text{sphere}_{\text{syst}}}\left(\frac{\partial\eta}{\partial\rho_{\text{sphere}}}\right)\Bigg|_{\overline{x}}\right| \qquad \text{with } \overline{x} = \left\{\overline{r}, \overline{g}, \overline{v}, \overline{\rho}_{\text{fluid}}, \overline{\rho}_{\text{sphere}}\right\} \\ &\stackrel{(4)}{=} \frac{2}{9}\frac{\overline{r}\,\overline{g}}{\overline{v}}\left(\left|\overline{\rho}_{\text{sphere}} - \overline{\rho}_{\text{fluid}}\right|\left(2\Delta\overline{r}_{\text{syst}} + \Delta\overline{g}_{\text{syst}}\overline{r} + \frac{\Delta\overline{v}_{\text{syst}}}{\overline{v}}\right) + \overline{r}\left(\Delta\overline{\rho}_{\text{sphere}_{\text{syst}}} + \Delta\overline{\rho}_{\text{fluid}_{\text{syst}}}\right)\right)\end{aligned}$$

The same can be done to get the statistic error, only by Gaussian (square) error propagation and of the viscosity by equation 2.2 and the Reynolds number. Afterwards to get the total error we only have to sum the statistical and systematic error.

[4] Be sure that "all" the variables are positive, otherwise this step isn't allowed.

A.2. Capillary viscometer

Table A.1.: Measured masses, time and height

cannula 1				cannula 2			
m_{total} g	m g	t s	h mm	m_{total} g	m g	t s	h mm
11.7272	1.7547	155.67	302.5	12.3688	2.3963	57.66	392.0
12.0435	2.0710	154.99	389.0	12.8310	2.8585	59.10	426.0
12.7268	2.7543	153.63	576.0	13.4868	3.5143	58.74	573.0
12.6466	2.6741	138.81	620.0	13.7267	3.7542	57.38	646.0
13.4788	3.5063	154.88	776.0	14.3221	4.3496	57.32	781.0
13.7535	3.7810	151.67	874.0	14.7880	4.8155	57.19	880.0
14.3513	4.3788	149.45	1098.0	15.2178	5.2453	57.02	1010.0
14.8374	4.8649	149.36	1231.0	15.8197	5.8472	57.43	1138.0
15.9209	5.9484	156.58	1438.0	16.0716	6.0991	55.85	1248.0
16.1096	6.1371	144.24	1644.0	17.0097	7.0372	56.00	1488.0

First for the errors of the masses take a look at section A.1. Secondly the two errors of the time, one systematic and one statistic only have to be summed up linearly. Furthermore for the statistical error of the height we used

$$\Delta\overline{h}_{\text{stat}} = \sqrt{\Delta\overline{h}_{\text{stat1}}^2 + \Delta\overline{h}_{\text{stat2}}^2}$$

whereby $\Delta\overline{h}_{\text{stat1/2}}$ is the error that occurs on one side when reading the scale. To get the total error, we summed up systematic and statistic ones linearly.
For the pressure difference we used following method:

$$\Delta\left(\overline{\Delta p}_{\text{syst}}\right) = \left|\Delta\overline{\rho}\cdot\overline{g}\cdot\overline{h}\right| + \left|\Delta\overline{g}\cdot\overline{\rho}\cdot\overline{h}\right| + \left|\Delta\overline{h}_{\text{syst}}\cdot\overline{\rho}\cdot\overline{g}\right|$$
$$\Delta\left(\overline{\Delta p}_{\text{stat}}\right) = \sqrt{\Delta\overline{h}_{\text{stat}}^2\left(\overline{\rho}\cdot\overline{g}\right)^2}$$
$$\Delta\left(\overline{\Delta p}\right) = \Delta\left(\overline{\Delta p}_{\text{stat}}\right) + \Delta\left(\overline{\Delta p}_{\text{syst}}\right)$$

Same process for the volume with the formula $V = \frac{m}{\rho}$ and then we can calculate the error of the current using equation 3.1 and Gaussion error propagation for the statistic error (square) and linear addition for the systematic one. To get the total error just sum up the systematic and statistic error and due to the reason that this is the same process as above it's left to the reader.

Next to determine the error of the length l of the cannula we just summed up the statistic and systematic one linearly. For the inner radius we used following method, whereby we chose needle one for demonstration.

$$\overline{d}[\mathrm{mm}] = \frac{\overline{d}[\mathrm{Sktlen}]}{100\,\mathrm{Sktlen}} \cdot 1.058\,\mathrm{mm} = \frac{21\,\mathrm{Sktlen}}{100\,\mathrm{Sktlen}} \cdot 1.058\,\mathrm{mm} = 0.222\,2\,\mathrm{mm}$$

$$\begin{aligned}\Delta\overline{d}_{\mathrm{stat}} &= \sqrt{\Delta\overline{d}_1^2 + \Delta\overline{d}_2^2} = \sqrt{\left(\frac{0.1\,\mathrm{Sktlen}}{100\,\mathrm{Sktlen}} \cdot 1.058\,\mathrm{mm}\right)^2 + \left(\frac{0.3\,\mathrm{Sktlen}}{100\,\mathrm{Sktlen}} \cdot 1.058\,\mathrm{mm}\right)^2} \\ &= 0.003\,3\,\mathrm{mm}\end{aligned}$$

$$\Delta\overline{d}_{\mathrm{syst}} = \frac{0.1\,\mathrm{Sktlen}}{100\,\mathrm{Sktlen}} \cdot 1.058\,\mathrm{mm} = 0.001\,1\,\mathrm{mm}$$

$$\Delta\overline{d} = \Delta\overline{d}_{\mathrm{syst}} + \Delta\overline{d}_{\mathrm{stat}} = 0.004\,4\,\mathrm{mm}$$

Finally to calculate the error of the viscosity we used equation 3.2, Gaussian or linear error propagation for the statistic or systematic errors and then we summed up the two types of errors linearly.

$$\begin{aligned}\Delta\overline{\eta}_{\mathrm{stat}} &= \sqrt{\Delta\overline{W}^2 \left(\frac{\partial\eta}{\partial W}\right)^2\bigg|_{\overline{W},\overline{l},\overline{r}} + \Delta\overline{r}_{\mathrm{stat}}^2 \left(\frac{\partial\eta}{\partial r}\right)^2\bigg|_{\overline{W},\overline{l},\overline{r}} + \Delta\overline{l}_{\mathrm{stat}}^2 \left(\frac{\partial\eta}{\partial l}\right)^2\bigg|_{\overline{W},\overline{l},\overline{r}}} \\ &\overset{(4)}{=} \frac{\pi\overline{r}^3}{8\overline{l}} \sqrt{\Delta\overline{W}^2\overline{r}^2 + \Delta\overline{r}_{\mathrm{stat}}^2 16\overline{W}^2 + \Delta\overline{l}_{\mathrm{stat}}^2 \left(\frac{\overline{W}\overline{r}}{\overline{l}}\right)^2}\end{aligned}$$

$$\begin{aligned}\Delta\overline{\eta}_{\mathrm{syst}} &= \left|\Delta\overline{r}_{\mathrm{syst}} \left(\frac{\partial\eta}{\partial r}\right)\bigg|_{\overline{W},\overline{l},\overline{r}}\right| + \left|\Delta\overline{l}_{\mathrm{syst}} \left(\frac{\partial\eta}{\partial l}\right)\bigg|_{\overline{W},\overline{l},\overline{r}}\right| \\ &\overset{(4)}{=} \frac{\pi\overline{r}^3\overline{W}}{8\overline{l}} \left(4\Delta\overline{r}_{\mathrm{syst}} + \Delta\overline{l}_{\mathrm{syst}}\frac{\overline{r}}{\overline{l}}\right)\end{aligned}$$

A.3. Ubbelohde viscometer

The average time was calculated as in section A.1. To get the total error of the kinematic viscosity we used the equation 4.1:

$$\Delta\overline{\nu}_{\mathrm{syst}} = \left|\Delta\overline{K} \left(\frac{\partial\nu}{\partial K}\right)\bigg|_{\overline{K},\overline{t}}\right| + \left|\Delta\overline{t}_{\mathrm{syst}} \left(\frac{\partial\nu}{\partial t}\right)\bigg|_{\overline{K},\overline{t}}\right| \overset{(4)}{=} \Delta\overline{K}\cdot\overline{t} + \Delta\overline{t}_{\mathrm{syst}}\cdot\overline{K}$$

$$\Delta\overline{\nu}_{\mathrm{stat}} = \sqrt{\Delta\overline{t}_{\mathrm{stat}}^2 \left(\frac{\partial\nu}{\partial t}\right)^2\bigg|_{\overline{K},\overline{t}}} \overset{(4)}{=} \Delta\overline{t}_{\mathrm{stat}}\cdot\overline{K}$$

$$\Delta\overline{\nu} = \Delta\overline{\nu}_{\mathrm{stat}} + \Delta\overline{\nu}_{\mathrm{syst}}$$

Now with equation 4.2 we can determine the searched errors:

$$\Delta\overline{\eta}_{\text{syst}} = \left| \Delta\overline{\nu}_{\text{syst}} \left(\frac{\partial\eta}{\partial\nu}\right)\Big|_{\overline{\nu},\overline{\rho}_{\text{fluid}}} \right| + \left| \Delta\overline{\rho}_{\text{fluid}_{\text{syst}}} \left(\frac{\partial\nu}{\partial\rho_{\text{fluid}}}\right)\Big|_{\overline{\nu},\overline{\rho}_{\text{fluid}}} \right| \overset{(4)}{=} \Delta\overline{\nu}_{\text{syst}} \cdot \overline{\rho}_{\text{fluid}} + \Delta\overline{\rho}_{\text{fluid}_{\text{syst}}} \cdot \overline{\nu}$$

$$\Delta\overline{\eta}_{\text{stat}} = \sqrt{\Delta\overline{\nu}^2_{\text{stat}} \left(\frac{\partial\eta}{\partial\nu}\right)^2\Big|_{\overline{\nu},\overline{\rho}_{\text{fluid}}}} \overset{(4)}{=} \Delta\overline{\nu}_{\text{stat}} \cdot \overline{\rho}_{\text{fluid}}$$

$$\Delta\overline{\eta} = \Delta\overline{\eta}_{\text{stat}} + \Delta\overline{\eta}_{\text{syst}}$$

B. Bibliography

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