Physical practical course part 3

(Optics and atomic physics)

Light diffraction and refraction

Course 3, Group 1, Team 2:
Eduard Koller
Michael Labenbacher

Department of Physics
Technical University of Munich

March 4, 2019

1. Introduction

The purpose of these experiments is to investigate diffraction properties and refraction of light. Prerequisite for diffraction is the wave property of light and refraction can be explained by Snell's law, a consequence of Huygens' principal. This means refraction is caused by the existence of different refraction indices and in matter the propagation speed of light changes. It should be noted that measurement results and error calculations can be found in the appendix.

2. Used methods

2.1. Huygens' principle and diffraction

Light diffraction occurs on all objects and is significant if their dimensions become smaller and are in the range of the wavelength. Huygens' principle applies: every point of a wavefront is the origin for another elementally wave, which interfere with each other. Furthermore a wave field, resulting from the interference of two (or more) waves, can only be stable over time, if these waves have a fixed phase relationship with each other. E. g. a grid can be such an instrument.

2.2. Diffraction at a grid

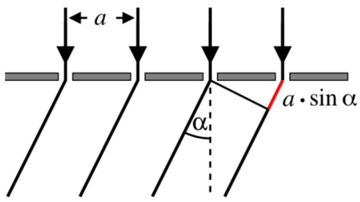

Figure 2.1.: Schematic figure of diffraction at a grid. [1]

First consider the grid, shown in figure 2.1, with a grating constant a and N columns. Incoming light will be diffracted and the difference in length that parallel and coherent waves of two adjacent columns need to a screen behind the grid (far field) is given by

$$\delta = 2\pi \frac{a}{\lambda} \sin(\alpha), \tag{2.1}$$

where λ is the wave length and for the angle α see figure 2.1. Constructive or destructive interference can be observed on a screen, in sufficient distance l, behind the grid. The angles of these two phenomena are

$$n\lambda = a \sin(\alpha) \qquad \text{for main maxima} \tag{2.2}$$

$$n\lambda = aN \sin(\alpha) \qquad \text{for minima,} \tag{2.3}$$

with $n \in \mathbb{Z}$. Geometric thought led to $\tan(\alpha) = s/l$ and equation (2.2) can be combined to

$$n\lambda = a \sin\left(\arctan\left(\frac{s}{l}\right)\right), \tag{2.4}$$

where s is the distance between the maximum n^{th}- and 0^{th}-order.

2.3. Diffraction at a slit

Considering a slit of the width d and it's intensity distribution

$$I \propto \left(\frac{\sin\left(\pi\frac{d}{\lambda}\cdot\sin(\alpha)\right)}{\pi\frac{d}{\lambda}\cdot\sin(\alpha)}\right)^2, \tag{2.5}$$

constructive and destructive interference can be observed at the angles

$$n\lambda = d\sin(\alpha) \qquad \text{for minima} \tag{2.6}$$

$$(n+1/2)\lambda \simeq d\sin(\alpha) \qquad \text{for maxima (and } \alpha = 0), \tag{2.7}$$

where $n \in \mathbb{Z}$. With equations (2.6) and (2.7), the small angle approximation and the geometric relation $\tan(\alpha) = s/l$ one will get

$$n\lambda \approx d \cdot \frac{s}{l} \tag{2.8}$$

2.4. Refraction

Whereas with diffraction the wave nature of light was very important, here one can concentrate on the different speed of lights in different mediums c_{m}, which defines the refraction index

$$n = \frac{c}{c_{\text{m}}} \tag{2.9}$$

If light crosses form one material to another at an angle Snell's law applies

$$\frac{\sin(\alpha_1)}{\sin(\alpha_2)} = \frac{n_2}{n_1} = \frac{c_1}{c_2}, \tag{2.10}$$

whereas α_1 is the angle of incidence of light from a medium n_1 into a medium with refraction index n_2 and the exit angle α_2.
To determinate the refraction index one can use the method of minimal deflection with a prism. Therefore n is given through

$$n = \frac{\sin\left(\frac{\delta_{\text{min}}+\varepsilon}{2}\right)}{\sin\left(\frac{\varepsilon}{2}\right)}, \tag{2.11}$$

whereas ε is the prism angle and δ_{min} the angle of minimal deflection.

3. Determination of the width of a slit

3.1. Experimental procedure

To determinate the expansion of the slit following setup will be used, composed of a laser, the slit and a screen in this order (see figure 3.1):

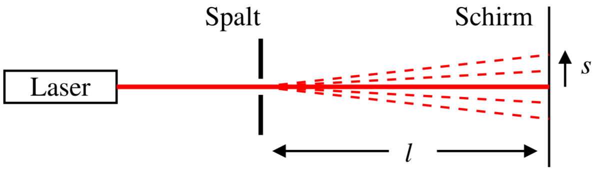

Figure 3.1.: Setup for the experiment to determinate the expansion of the slit. [1]

The wave length of the used laser is given by $(532 \pm 1)\,\mathrm{nm}$ (green). To get enough data the minima were marked up to order ten on a paper which was taped to the screen. Then the procedure was repeated for three times with different distances to the screen. After that the distance s between the main maximum ($s = 0$) and the minima where measured with a ruler.

3.2. Results

3.2.1. Error analysis

For the length measurements l a systematical error of $0.1\,\% \cdot$ "measurement value" due to manufacturing inaccuracies and a statistical of $0.25\,\mathrm{cm}$ due to reading error was used. For the distance s a systematic error of $1\,\mathrm{mm}$ was estimated as a zero point error and the statistical error is composed of an error through marking the minima ($0.5\,\mathrm{mm}$ or $1\,\mathrm{mm}$ depending on the distance s) on the paper and one by reading the value ($0.25\,\mathrm{mm}$). Error propagation were performed quadratic for statistical and linear for systematic errors and if it's not trivial it can be found in section A.2.

3.2.2. Approximation of small angles

First the maximum angle α observed in this experiment is given by the highest order ($n = 10$) at the lowest distance of $l_1 = 550\,\mathrm{nm}$ and amounts to approximately $2.57°$. Therefore one considers an interval of $(-2.6°, 2.6°)$ and with a taylor approximation of the tangent

$$\tan(x) = T_2 \tan(x, 0) + R_2 \tan(x, 0) = x + R_2 \tan(x, 0) = x + \mathcal{O}\left(x^3\right) \approx x$$

the relative error of the last step is

$$\frac{|T_2 \tan(x, 0) - \tan(x)|}{|\tan(x)|} = \left|\frac{x}{\tan(x)} - 1\right| \leq 0.07\,\% \qquad \text{for } x \in (-2.6°, 2.6°).$$

This is approximately 100 times smaller than the minimal relative uncertainty of s/l and an analogue calculation for the sine leads to the simplification $\tan(x) \approx \sin(x) \approx x$ (for $x \in (-2.6°, 2.6°)$) with an relative error, which can be neglected in this part.

3.2.3. Evaluation of measurement results

The three distances l_k between the slit and the screen are $l_1 = (550 \pm 4)$ mm, $l_2 = (950 \pm 5)$ mm and $l_3 = (1\,350 \pm 5)$ mm and the measurement values for the distance s_n to the n^{th} minima can be found in table A.1. Plotting the data points, showed in figure 3.2, a linear fit (slope p, intersection with y-axis q, specified in the figure 3.2) and equation (2.8) under the approximation of small angles leads to $d = \lambda/p$ and a width of the slit

$$d = (120.8 \pm 1.0)\,\mu\text{m}.$$

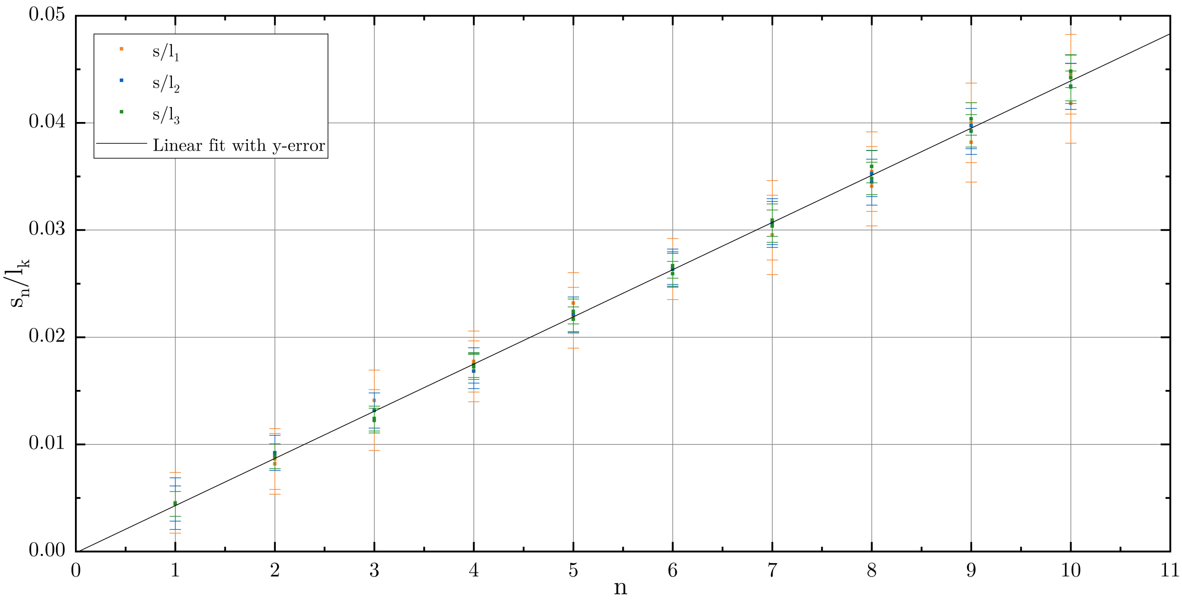

Figure 3.2.: Ratio of s/l over the order of minima n. A linear fit with all data points leads to a slope of $p = (4.404 \pm 0.029) \cdot 10^{-3}$ and an intersection with the y-axis of $q = (-0.12 \pm 0.13) \cdot 10^{-3}$. Within the tolerance range the graph hits the origin, deviations can be caused by an oblique position of the screen and the measuring process, but seems good that it's close to the origin.

3.3. Discussion

In figure 3.2 one can see, that no value is more than the tolerance range away from the linear fit. With increasing n one can observe increasing errors and variations. This is expected as higher orders of minima are more difficult to see at the screen. To sum up, the resulting value for the width $d = (120.8 \pm 1.0)\,\mu$m sounds realistic.

4. Determination of wave lengths with a grid

4.1. Experimental procedure

To determinate the wave length of three spectral lines ($m \in [1, 2, 3]$, where $m = 1$ corresponds to blue, 2. . . green and 3. . . orange) of the mercury lamp the following setup can be used (see figure 4.1).

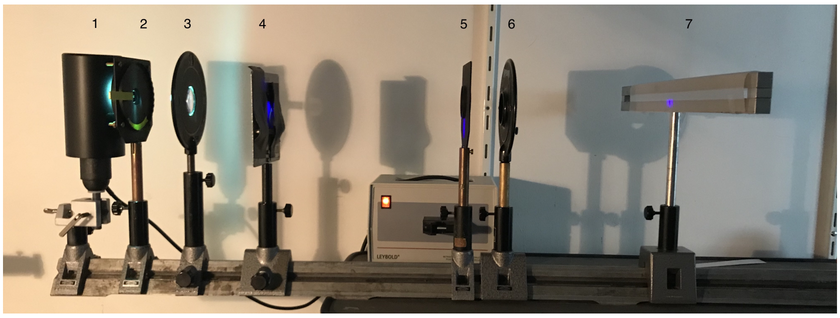

Figure 4.1.: Setup for the experiment to determinate three spectral lines of the mercury lamp.

1 . . . . mercury lamp
2, 3, 5 . . . lens system, built of lens at position 3 and 5 and a slit at position 2 (aperture), used to optimize the lightning of the grid
4 . . . . color filter, used to let only blue pass, makes it easier to measure the maxima on the screen
6 . . . . grid with grating constant $a = (10.000 \pm 0.020)$ µm
7 . . . . screen

The left and right edge of the maxima up to order five or four were marked on a paper, which was taped on the screen. This process was repeated for three ($k \in [1, 2, 3]$) different distances l_k between the grid and the screen and for all three colors blue, green and orange. Afterwards the distance s between the main maxima ($s = 0$) and the maxima of n^{th}-order on the paper was measured with a ruler.

4.2. Results

4.2.1. Error analysis

For the length measurements of l the errors are the same as in subsection 3.2.1. For the distance s a systematic error of 1 mm was estimated as a zero point error and the statistical error is composed of an error through marking the left and right edge of the maxima (0.5 mm) on the paper and one by reading the value (0.25 mm). Error propagation can be found in section A.2.

In this part the largest angle was approximately 16.6° and a small angle approximation would correspondence to large errors (analogous to subsection 3.2.2) and can not be neglected.

4.2.2. Evaluation of measurement results

The distances l_k between the grid and the screen are $l_1 = (220 \pm 4)$ mm, $l_2 = (470 \pm 4)$ mm and $l_3 = (720 \pm 4)$ mm and the measurement values for the distance s_{nm} to the n^{th} maxima of the color m can be found in table A.2. The following graphs in figure 4.2 have been fitted linear, to calculate the slope p and with equation (2.4) one gets the wave lengths $\lambda = a \cdot p$ in table 4.1.

Table 4.1.: Fit parameters (slope p, intersection with y-axis q) for figure 4.2, computed and literature wave lengths of a mercury lamp [2].

	computed values						literature values
m	p 10^{-3}	Δp 10^{-3}	q 10^{-3}	Δq 10^{-3}	λ nm	$\Delta\lambda$ nm	λ nm
blue	43.8	0.5	-1.4	1.2	438	5	436
green	53.8	0.3	-0.4	0.8	538	4	546
orange	56.8	0.4	0.6	1.0	568	5	579

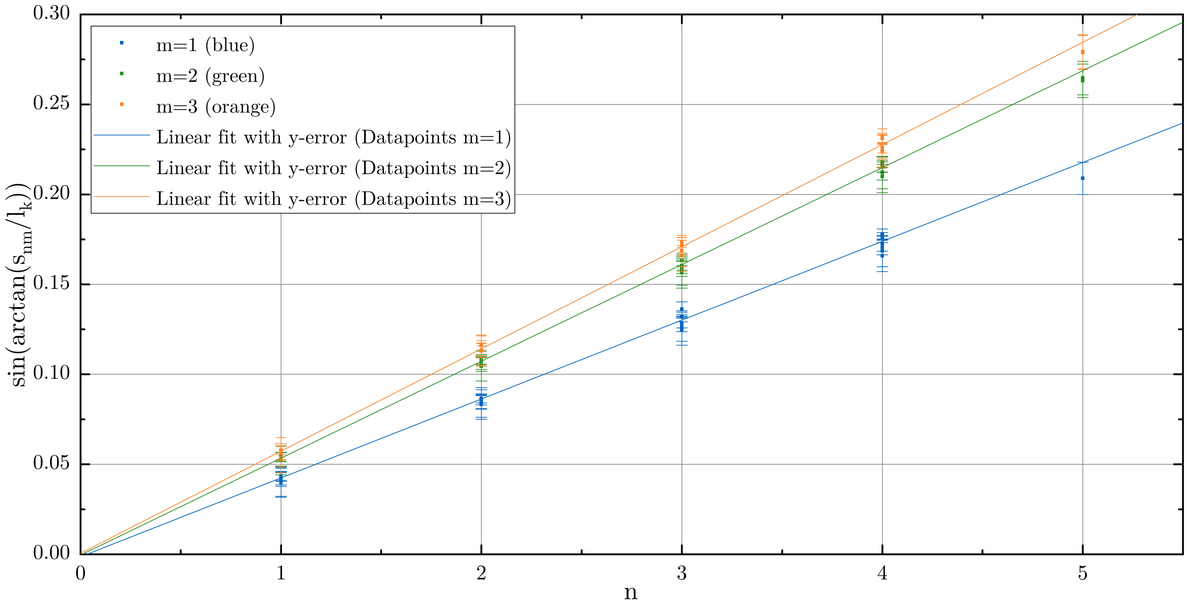

Figure 4.2.: sin(arctan(s/l)) over the order of minima n. A linear fit with each data points leads to a slope p and intersection with the y-axis given in table 4.1. Within the tolerance range the graph hits almost the origin, deviations can be explained analogous to chapter 3.

4.3. Discussion

As one can see in table 4.1, blue is very close to the literature value, green and orange are within tolerance range. Because of the good results for blue light included systematic errors can be assumed as minor and the variation with green and orange have to be of statistical nature (or not considered errors, which depends on the wave length). One reason for greater variation with green and orange light might be the overlay of maxima which made it difficult to find the right and left edge of them. This problem did not occur with blue light, due to the color filter. Another error source might be the used optical system. Parallel lightning of the grid is needed to have the optical path and phase difference named in equation (2.1), but in case of errors in the lens system not parallel light might variate the phase difference and led to a different distances between the maxima. This error is different for each length and color as the lens system had been adjusted for each measurement series.

5. Measurements at a prism spectroscope

5.1. Experimental procedure

The prism experiment has two parts. In the first one, the prism angle ε (see figure 5.1) and in the second one the reflection index $n(\lambda)$ will be determined for three different wave lengths. For both experiments following setup will be used:

lamp - slit - prism- telescope,

whereas the prism is mounted on a rotatable bracket, which has an angle scale. Also the telescope can be moved around the prism and the relative angle between prism and telescope can be seen on the angle scale.
In the first part the prism angle ε must be determined, in order to do so one has to orientate the prism with both polished sides towards the slit. Then one will find the reflected light, one on the left side (φ) and one the right (ψ) (see figure 5.1).

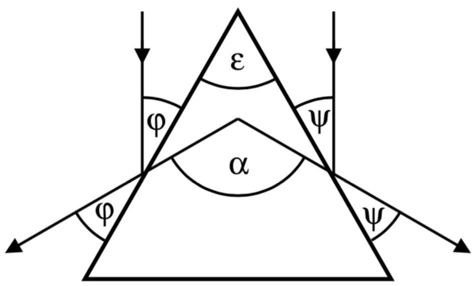

Figure 5.1.: Schematic sketch of the angles used to determinate the prism angle. [1]

To get the best results the size of the slit can be varied and the crosshair of the telescope should be in the center of the reflected light.
To determinate the reflection index $n(\lambda)$ with the second experiment one can use the method of minimal deflection. The same setup will be used, only with two lamps, first a mercury lamp and second a energy saving lamp. To determinate the angle of minimal deflection first the prism will be set up in a manner that allows white light to be divided into its components. Then the prism will be turned around and while one can observe the refracted light one can also see a turning point, when the light starts to move into another direction. This angle is (compared to the angle not refracted light) the angle of minimal deflection. This procedure is repeated for the blue, green and orange light of the mercury lamp and two clearly visible lines of the second lamp.

5.2. Results

5.2.1. Error analysis

For the angle measurements φ a systematical error of $0.1\,\% \cdot$ “measurement value” due to manufacturing inaccuracies was used. The statistical error is composed of an error through determination of the changing point (0.25°) and an error through reading the value (0.1°). The error which occurs when one focuses the crosshair in the telescope is negligible. Error propagation were performed quadratic for statistical and linear for systematic errors and if it's not trivial it can be found in section A.2.

5.2.2. Evaluation of measurement results

First to obtain the angle of the prism ϵ one can utilize

$$\epsilon = \varphi + \psi = \frac{\alpha}{2}$$

in figure 5.1. The measurements of the relative positions of the two reflected beams of light resulted in $\varphi = (6.00 \pm 0.27)\,°$ and $\psi = (126.00 \pm 0.27)\,°$ and therefore $\epsilon = (60.00 \pm 0.25)\,°$. Secondly the measured angles φ_i, to calculate (subtraction) the angle of minimal deflection δ_{min} are listed in table 5.1. With the literature values of the wave length of a mercury lamp in table 4.1 one gets the linear fit in figure 5.2.

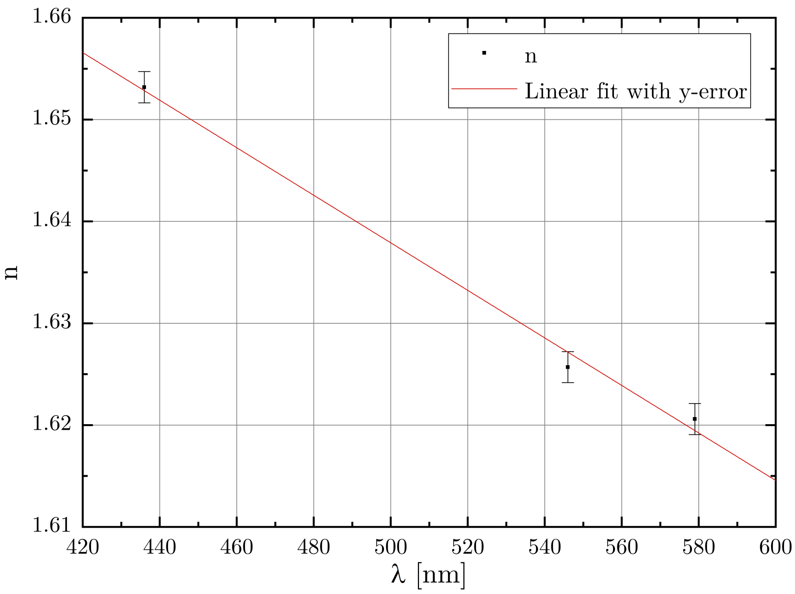

Figure 5.2.: Linear fit of the calculated refraction indices from the mercury lamp. The slope p is $(-233 \pm 18) \cdot 10^{-6}$ and the intersection with the y axis amounts to $q = 1.755 \pm 0.009$ with λ in the units nm. Deviations from a straight line are discernible, what's due to the fact, that for glass one normally expects a small increase in slope with decreasing wave length, but for the small range of the wave length the linearization is practicable.

For the unknown source of light, the refraction indices for the examined wave lengths can be calculated with the equation (2.11). Furthermore with the slope p and the y-intersection q of the linear fit one gets the wave length with $\lambda = (n - q)/p$, the result is listed in table 5.1.

Table 5.1.: Measured angles φ_i, the angle of minimal deflection, the computed refraction indices (with literature values of the wave length (see table 4.1) for the mercury lamp) and the calculated wave lengths of the unknown source of light.

m	φ_1 $^\circ$	φ_2 $^\circ$	δ_{min} $^\circ$	$\Delta\delta_{\text{min}}$ $^\circ$	n	Δn	λ nm
blue	2.00	310.50	51.5	0.4	1.6532	0.0015	
green	359.00	310.25	48.8	0.4	1.6257	0.0015	
orange	3.00	314.75	48.3	0.4	1.6206	0.0015	
“orange”	4.00	316.00	48.0		1.6180		585
“blue”	0.00	309.50	50.5		1.6433		477

5.3. Discussion

First, the measurements of the prism angle ϵ provides a reasonable value, since the prism seems to be equilateral. Secondly, as one can see in figure 5.2 the values are nearly within tolerance range with the expected values. One can comment, that although the fit in figure 5.2 was linear, the real refraction function of for example glass is more difficult, but can mostly be approximated as linear in the visible light region.

6. Questions

1. Which light source is best to demonstrate interference phenomena and why?

Best suited for this are lasers, due to almost monochromatic light and the high coherence length.

2. Do diffraction phenomena only occur on objects whose dimensions are in the order of magnitude of the light wave?

No, diffraction is the deflection of waves at an obstacle, but can be neglected if the wave length is much smaller than the object.

3. What’s the speed of light in glass?

The light speed can be obtained with the equation (2.9) and since the refraction index of glass is higher than 1, this means $c_{\text{glas}} < c$.

4. What’s meant by dispersion?

Dispersion describes the dependence of the refractive index on the wave length.

5. Whereby can the spectral resolution of a prism spectrometer be limited in practice?

Diffraction at the slit of the source limits the spectral resolution.

6. Does the refraction index of flint glass decrease or increase from the red to the blue spectral range?

Due to normal dispersion ($\mathrm{d}n/\mathrm{d}\lambda < 0$) in the visible range of the spectrum, the refraction index increases from red to blue.

7. Is the course of the function $n(\lambda)$ linear?

In general $n(\lambda)$ isn't linear, this can even be explained by classical view.

A. Measurements and error calculation

A.1. Measurement values

A.1.1. Slit

Table A.1.: Measured distance s to the minima n^{th}-order for the determination of the width of a slit.

	l_1		l_2		l_3	
n	s mm	Δs mm	s mm	Δs mm	s mm	Δs mm
1	2.50	1.56	4.25	1.56	6.00	1.56
2	4.75	1.56	8.75	1.56	12.00	1.56
3	7.75	1.56	12.50	1.56	16.75	1.56
4	9.75	1.56	16.50	1.56	23.50	1.56
5	12.00	1.56	21.00	1.56	30.25	1.56
6	14.50	1.56	25.00	1.56	36.00	1.56
7	17.00	2.03	29.00	2.03	41.75	2.03
8	19.50	2.03	32.75	2.03	48.50	2.03
9	22.00	2.03	37.25	2.03	54.50	2.03
10	24.50	2.03	41.25	2.03	60.50	2.03
1	2.50	1.56	4.25	1.56	6.00	1.56
2	4.50	1.56	8.75	1.56	12.00	1.56
3	6.75	1.56	12.50	1.56	16.50	1.56
4	9.25	1.56	16.00	1.56	23.25	1.56
5	12.75	1.56	21.00	1.56	29.25	1.56
6	14.50	1.56	25.25	1.56	35.00	1.56
7	16.25	2.03	29.25	2.03	41.00	2.03
8	18.75	2.03	33.50	2.03	47.00	2.03
9	21.00	2.03	37.75	2.03	53.00	2.03
10	23.00	2.03	42.00	2.03	58.50	2.03

A.1.2. Grid

Table A.2.: Measured distance s to the maxima n^{th}-order for the distances l_k, $k \in [1, 2, 3]$, for the determination of wave lengths with a grid

		blue		orange		green	
	n	s_{max} mm	s_{min} mm	s_{max} mm	s_{min} mm	s_{max} mm	s_{min} mm
l_1	1	10.00	7.50	13.50	10.00	13.00	10.00
	2	19.75	17.00	26.00	24.00	24.50	21.75
	3	28.75	26.50	38.50	35.50	36.25	33.50

	n	blue		orange		green	
		s_{max} mm	s_{min} mm	s_{max} mm	s_{min} mm	s_{max} mm	s_{min} mm
	4	38.25	35.75	51.50	49.75	48.75	45.75
	5	48.00	46.00	65.25	62.50	61.50	58.50
	1	10.00	7.75	14.00	11.00	13.50	10.00
	2	20.00	17.25	26.75	23.50	24.75	21.50
	3	29.50	26.75	39.00	36.25	36.75	33.75
	4	38.75	36.50	52.00	49.00	48.75	46.75
	5	48.00	46.00	65.50	62.50	61.75	59.00
l_2	1	22.50	17.25	28.25	25.00	26.50	23.00
	2	43.25	36.75	55.25	51.75	52.50	48.25
	3	62.75	58.25	84.25	79.75	76.75	74.25
	4	84.75	80.00	110.25	106.00	105.00	99.25
	1	21.25	17.75	28.00	24.75	27.00	22.75
	2	42.75	37.00	55.25	52.00	52.00	47.75
	3	62.00	67.25	84.00	80.00	78.25	74.25
	4	83.75	79.00	114.75	109.50	106.50	102.25
l_3	1	33.75	28.25	44.50	39.00	42.00	36.00
	2	64.50	59.25	84.50	79.50	80.25	75.00
	3	95.75	90.75	126.75	120.00	119.75	114.50
	4	129.75	127.75	169.75	164.25	161.00	156.25
	1	34.25	28.50	44.50	39.00	42.00	35.75
	2	65.00	60.25	87.00	81.25	81.25	75.50
	3	98.75	92.75	129.75	123.75	122.25	116.00
	4	131.00	129.50	174.00	168.00	164.00	157.50

A.2. Error calculation

The calculation of the error of a length $x = x_2 - x_1$, measured at two positions x_2 (max.) and x_2 (min.), is done by

$$\Delta x_{\text{sys}} = \Delta x_{\text{sys},2} - \Delta x_{\text{sys},1}$$
$$\Delta x_{\text{stat}} = \sqrt{2} \cdot \Delta x_{\text{stat},i}$$

in case of $\Delta x_{\text{stat},2} = \Delta x_{\text{stat},1} = \Delta x_{\text{stat},i}$. To calculate the error for the average value $x = 1/2 \cdot (x_1 + x_2)$ one will calculate

$$\Delta x_{\text{sys}} = 1/2 \cdot (\Delta x_{\text{sys},1} + \Delta x_{\text{sys},2})$$
$$\Delta x_{\text{stat}} = 1/\sqrt{2} \cdot \Delta x_{\text{stat},i}$$

A.2.1. Slit

To determine the width d of the slit use the relation (2.8). The errors can be calculated with the slope p of the fit and

$$d = \frac{\lambda}{p}, \text{ with } p = \frac{s/l}{n}$$
$$\Delta d_{\text{sys}} = \Delta\lambda_{\text{stat}} \frac{1}{p}$$
$$\Delta d_{\text{stat}} = \Delta a_{\text{stat}} \frac{\lambda}{p^2}$$

A.2.2. Grid

To determine the wave length equation (2.4) was used. With

$$f\left(s,l\right) = \sin\left(\arctan\left(\frac{s}{l}\right)\right)$$

one get as errors

$$\Delta f\left(s,l\right)_{\text{sys}} = A \cdot \left(\left|\frac{\Delta s_{\text{sys}}}{l}\right| + \left|\Delta l_{\text{sys}} \frac{s}{l^2}\right|\right)$$
$$\Delta f\left(s,l\right)_{\text{stat}} = A \cdot \sqrt{\left(\frac{\Delta s_{\text{stat}}}{l}\right)^2 + \left(\Delta l_{\text{stat}} \frac{s}{l^2}\right)^2}$$
$$\text{and } A = \cos\left(\arctan\left(\frac{s}{l}\right)\right) \cdot \frac{1}{1 + \left(\frac{s}{l}\right)^2}$$

With the linear fit of $f(s,l)$ over n in figure 3.2 one gets as errors for the wave length

$$\lambda = a \cdot p \text{ with } p = \frac{\sin\left(\arctan\left(\frac{s}{l}\right)\right)}{n}$$
$$\Delta\lambda_{\text{sys}} = \Delta a_{\text{sys}} \cdot p$$
$$\Delta\lambda_{\text{stat}} = \Delta p_{\text{stat}} \cdot a$$

A.2.3. Prism

The calculation for angle differences is analog to length calculation.

B. Bibliography

[1] *Lichtbeugung und Lichtbrechung (BUB)*. 2015. URL: https://www.ph.tum.de/academics/org/labs/ap/ap3/BUB.pdf (visited on March 1, 2019) (cit. on pp. 1, 3, 7).

[2] *Education in Micrsocopy and Digital Imaging*. 2018. URL: http://zeiss-campus.magnet.fsu.edu/articles/lightsources/mercuryarc.html (visited on March 5, 2019) (cit. on p. 6).