UNIT - 3 (Interference)

Coherent Sources and Coherence

Two Sources of light are Said to be Coherent Source, if they emit light wave of Same frequency and Wavelength and there Should be a Constant Phase difference.

"Coherent waves give the Consistent Superposition"

The phenomenon of obtaining Coherent Sources are Known as Coherence.

There are two types of Coherence

1) Spatial Coherence

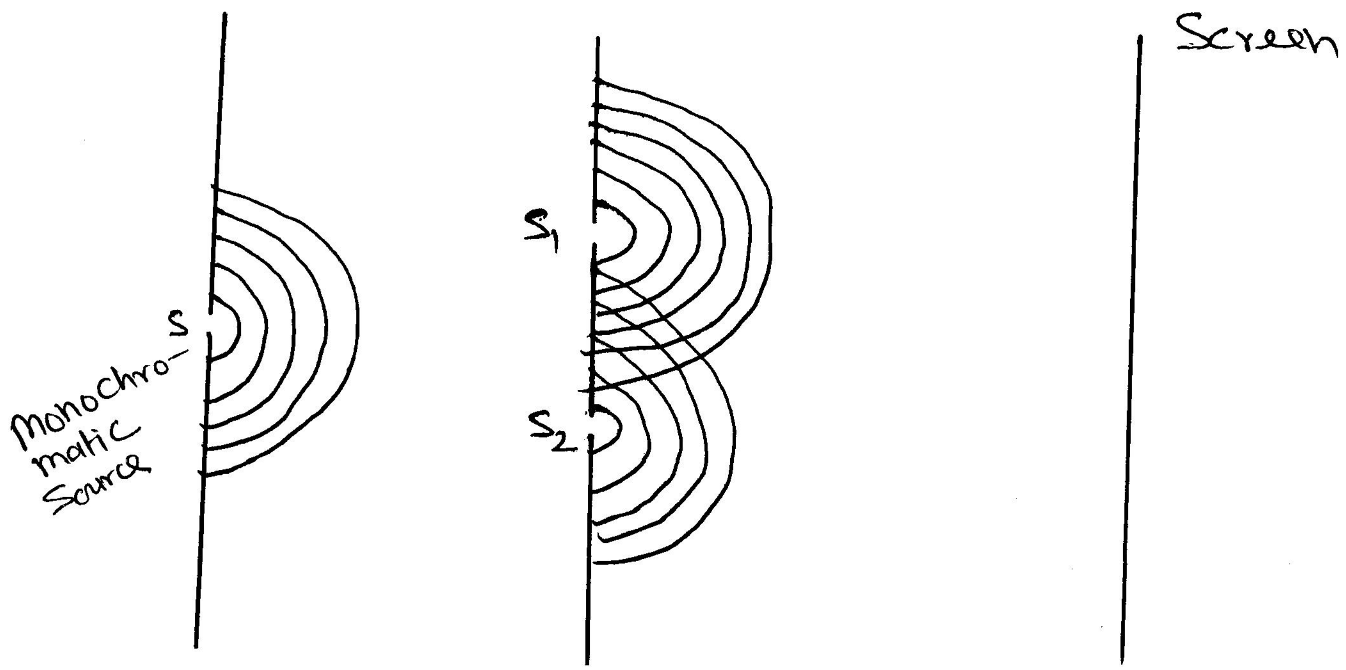
2) Temporal Coherence

Methods of obtaining Coherent Sources

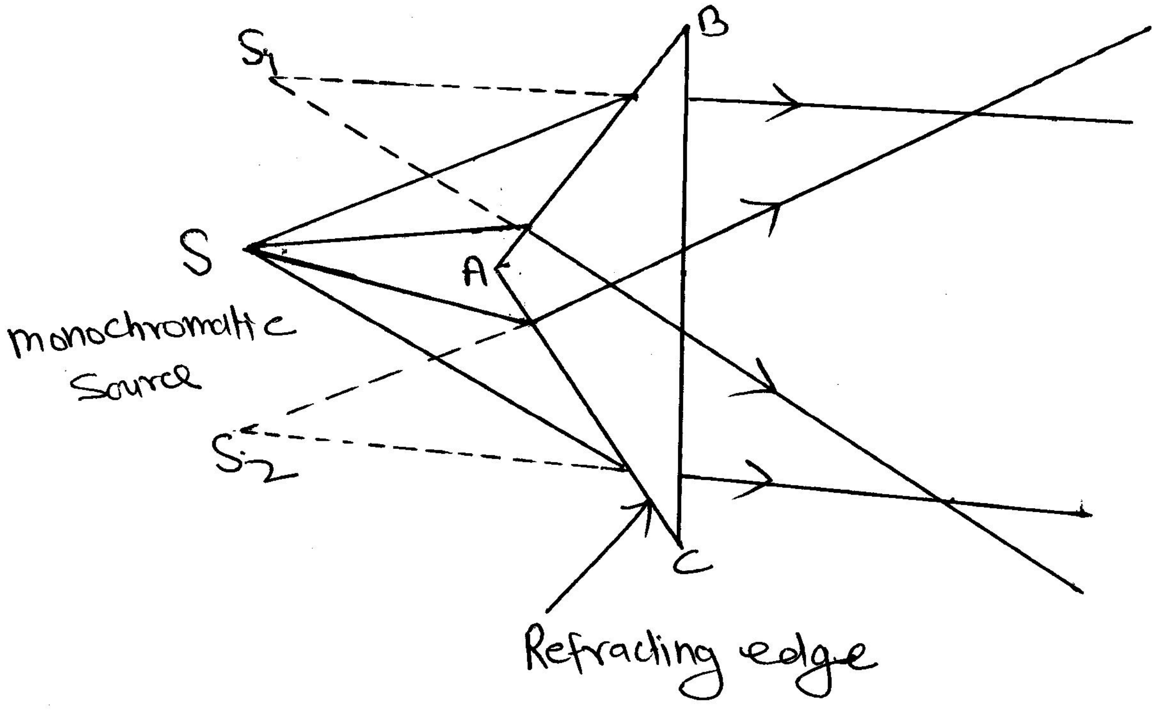
In actual practice it is not possible to have two independent Coherent Sources of light, but for Experimental purposes two Virtual Coherent Sources are derived from a Single Source by Some devices. Any Phase Change in one is Simultaneously accompained by the Same Phase Change in the other. So that the phase difference between the two Sources remains Constant.

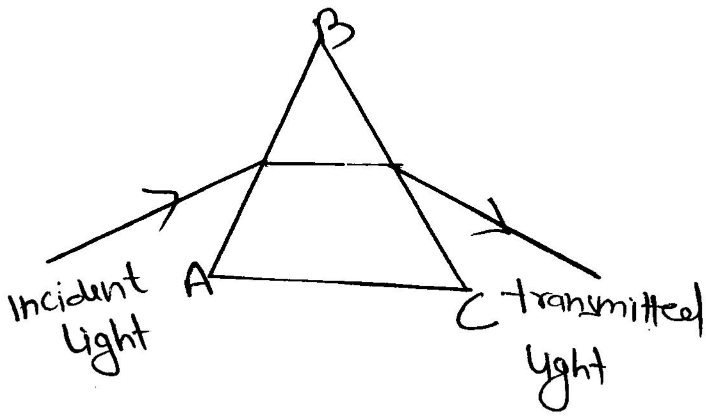
Some famous Experimental Set up the Coherent Sources are obtained

1) Young's Double Slit Experiment



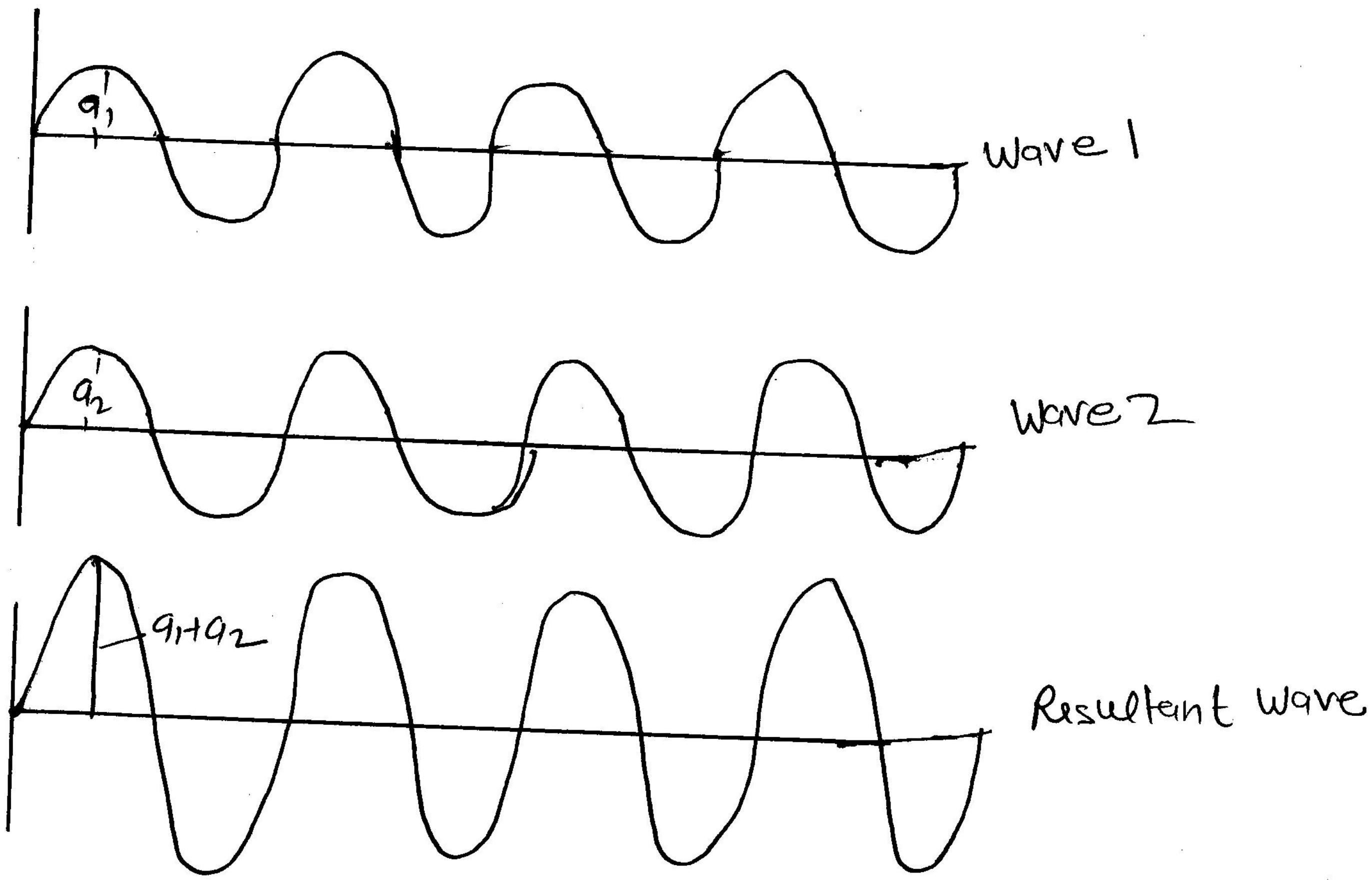
2) Freshel's Biprism Experiment





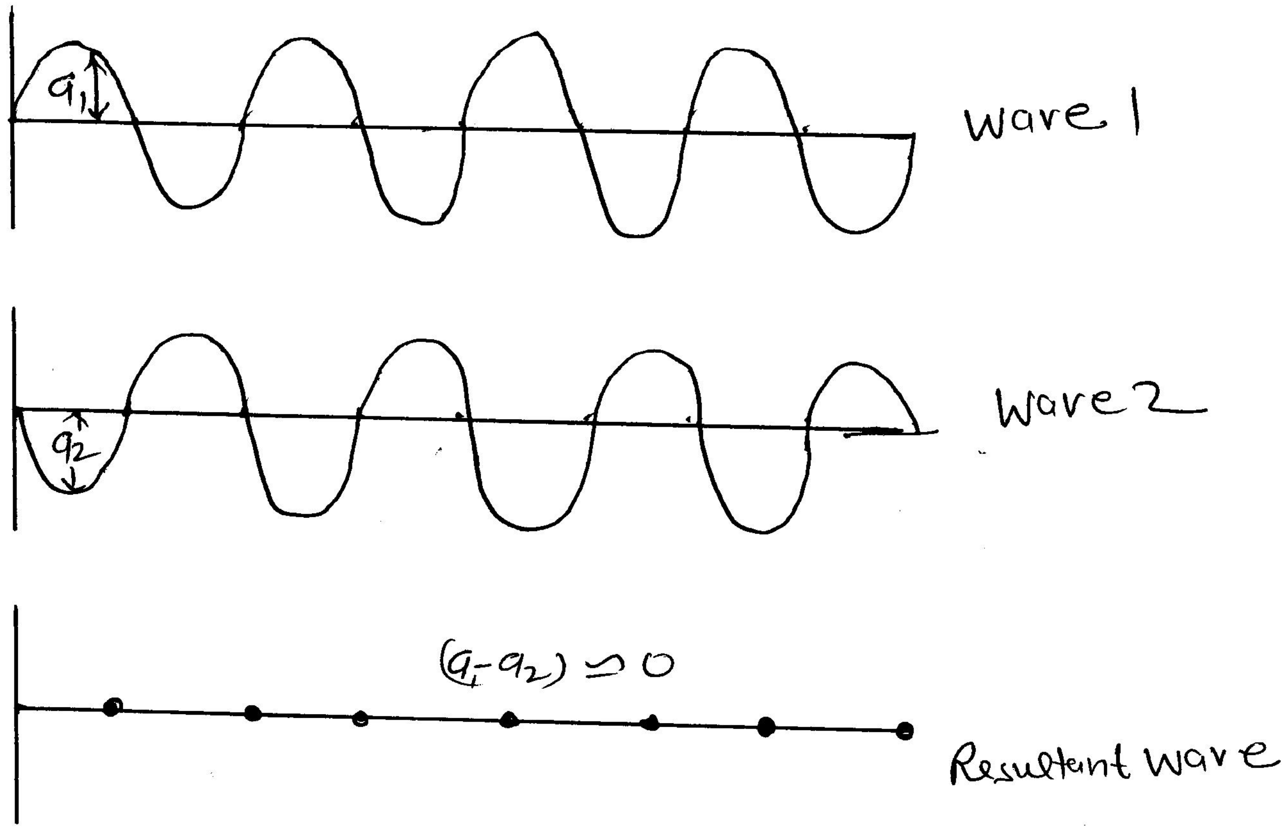
Interference

It is the phenomenon of redistribution of light Energy in the medium when light wave from two Coherent Sources Superimpose with each other.



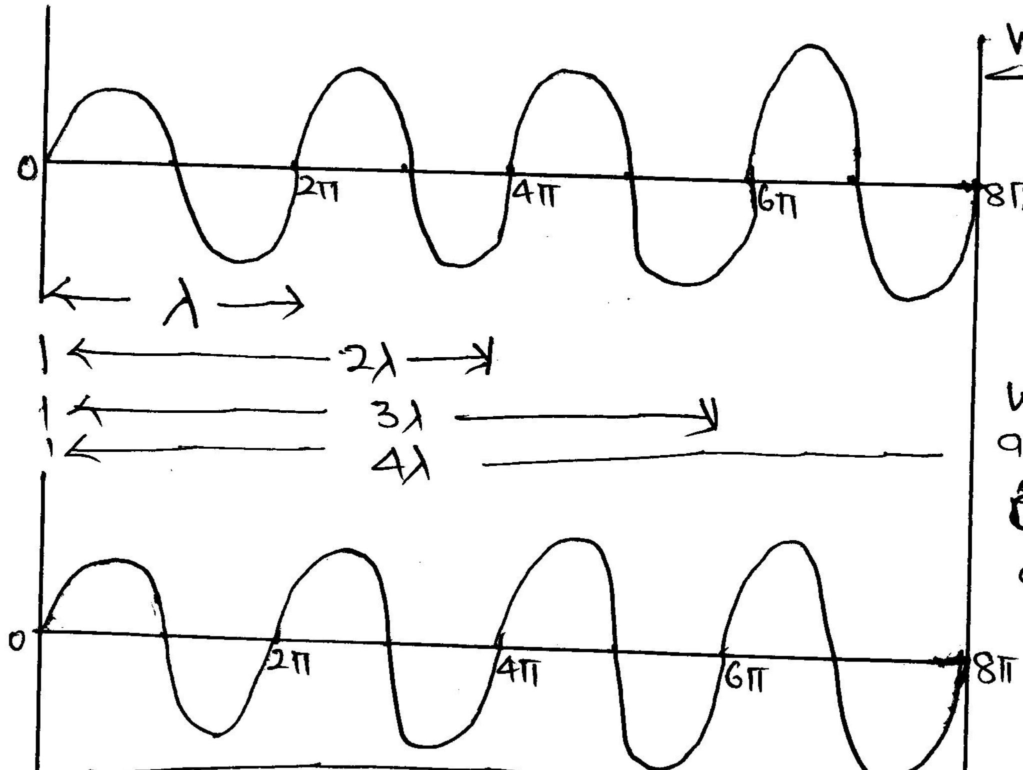
Constructive interference pattern

Destructive Interference



Destructive Interference pattern

Constructive Superposition of waves —
(By Graphically)



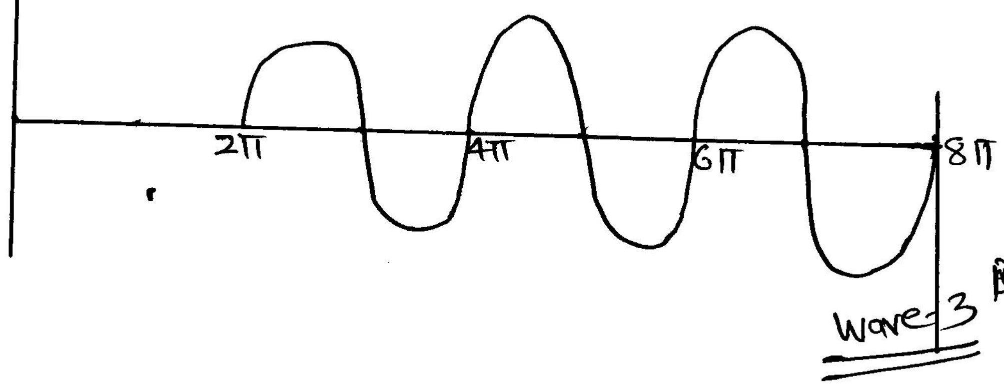
wave 1

Screen

When Phase diff = 0
and Path diff = 0
Condition for
Constructive Interference
Satisfy.

$\boxed{2\pi \text{ Phase diff} = 1\lambda \text{ Path diff}}$

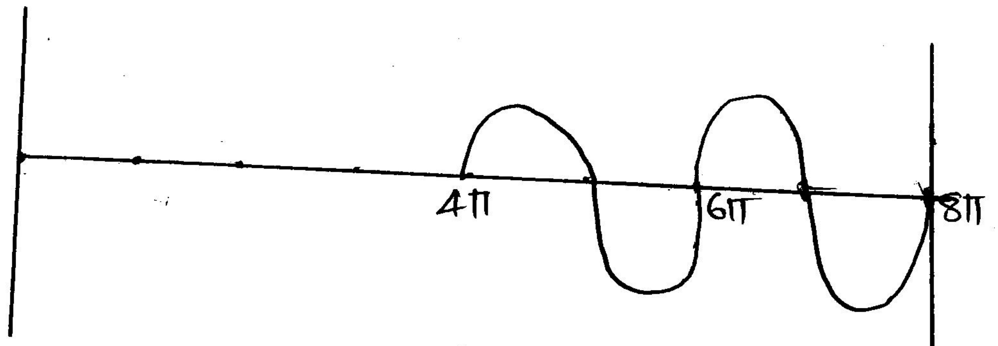
wave-2



Phase diff = 2π

Path diff = λ
w.r.to wave-1

wave-3



Phase diff = 4π

Path diff = 2λ

w.r.to wave 1

wave-4

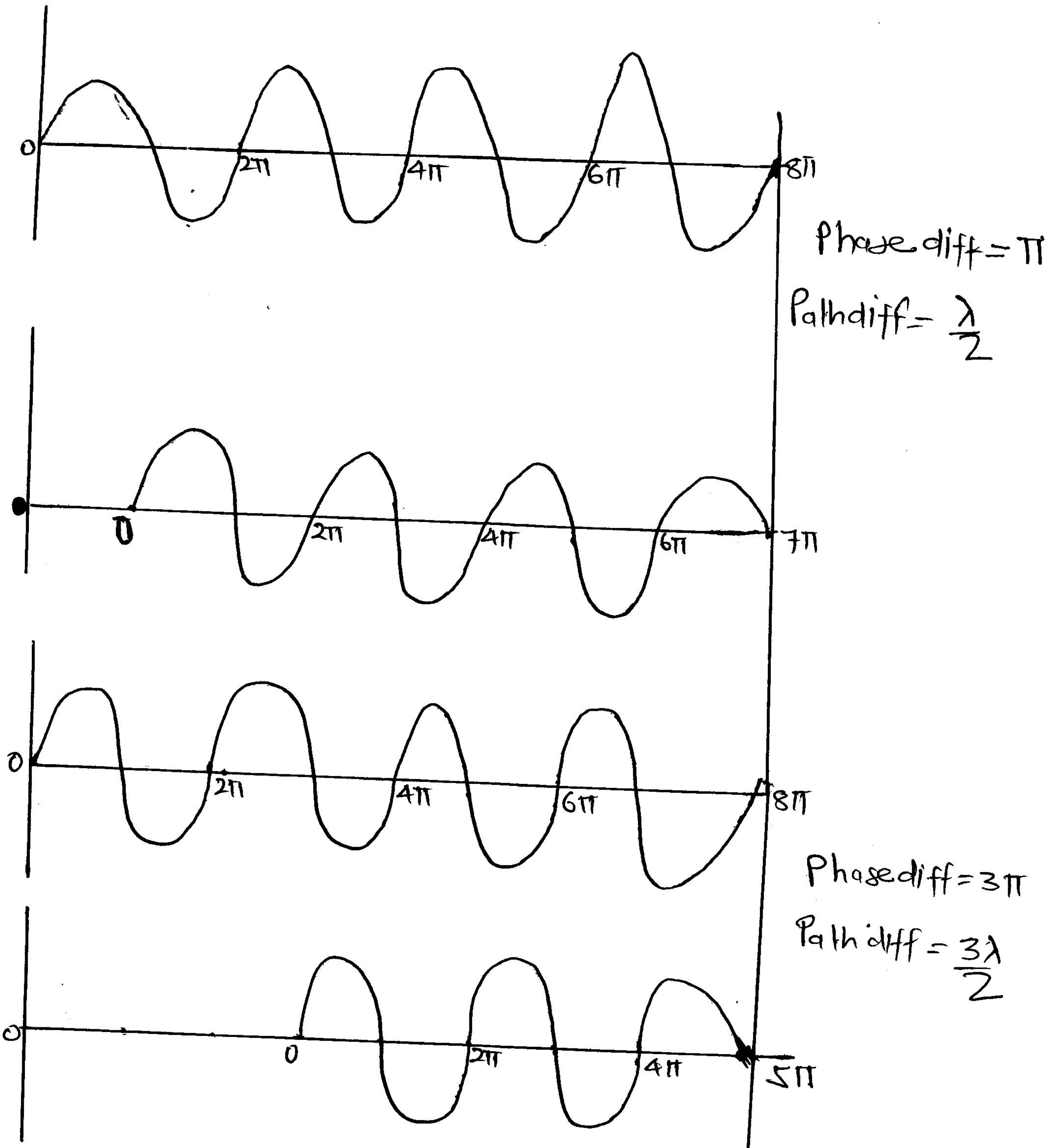
In general Condition for Constructive interference

Phase difference = 0, 2π, 4π, 6π . . . .

$\phi = 2n\pi \quad n = 0, 1, 2, 3 \ldots$

Path diff λ = 0, λ, 2λ, 3λ . . . $= n\lambda \quad n = 0, 1, 2, 3$

Destructive superposition of waves



In general condition for destructive interference

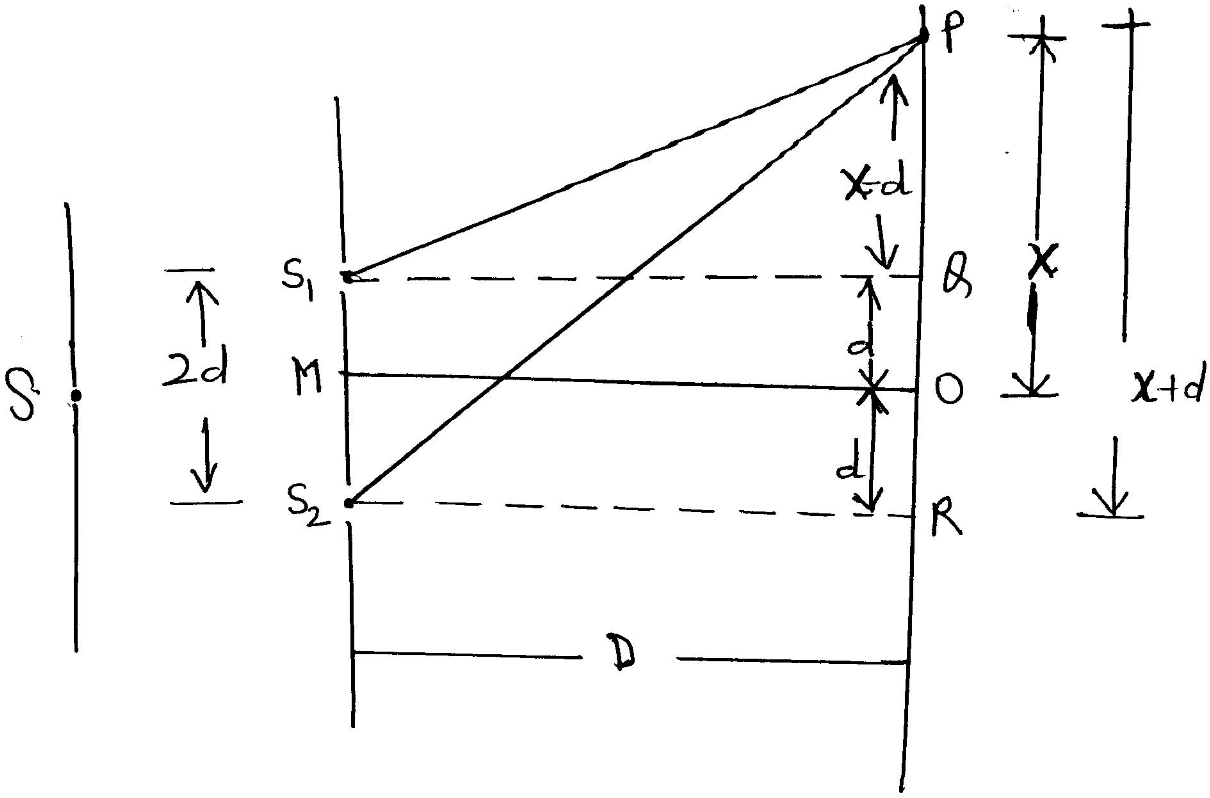
Phase diff $\phi = \pi, 3\pi, 5\pi \ldots$

$$\phi = (2n+1)\pi$$

Path diff $= \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \ldots$ $\quad n = 0, 1, 2, 3 \ldots$

$$\lambda = (2n+1)\frac{\lambda}{2} \quad n = 0, 1, 2, 3, \ldots$$

Young's Double Slit Experiment :—



From right angle triangle S_1QP

$$(S_1P)^2 = (S_1Q)^2 + (PQ)^2$$

$$(S_1P)^2 = D^2 + (x-d)^2 \quad \text{——} \quad ①$$

Similarly, in right angle triangle S_2RP

$$(S_2P)^2 = (PR)^2 + (S_2R)^2$$

$$= (S_2R)^2 + (PR)^2$$

$$(S_2P)^2 = D^2 + (x+d)^2 \quad \text{——} \quad ②$$

Subtracting eqn ① from eqn ②

$$(S_2P)^2 - (S_1P)^2 = D^2 + (x+d)^2 - D^2 - (x-d)^2$$

$$(S_2P)^2 - (S_1P)^2 = (x+d)^2 - (x-d)^2$$

$$(S_2P + S_1P)(S_2P - S_1P) = x^2 + d^2 + 2xd - x^2 - d^2 + 2xd$$

S_1 and S_2 are very close to each other

So, $S_1P \simeq S_2P = D$

$$(S_2P - S_1P)\,2D = 4xd$$

$$S_2P - S_1P = \frac{4xd}{2D}$$

$$S_2P - S_1P = \Delta = \frac{2xd}{D}$$

$$\Delta = \frac{2xd}{D} \quad \text{———} \quad (3)$$

1) Condition for Bright fringes: —

Point 'P' on the Screen will be bright when Path difference between two waves $= 0, \lambda, 2\lambda, 3\lambda, \ldots$ and Phase difference $= 0, 2\pi, 4\pi, 6\pi \ldots = 2n\pi$

or $S_2P - S_1P = n\lambda$ $n = 0, 1, 2, 3 \ldots$

$n = 0, 1, 2, 3 \ldots$

$$\frac{2xd}{D} = n\lambda$$

or $$x = \frac{n\lambda D}{2d} \quad \text{———} \quad (4)$$

The next bright fringes are formed when $n = 1, 2, 3 \dots$

When $n = 1$

$$x_1 = \frac{\lambda D}{2d} \quad \text{and} \quad x_2 = \frac{2\lambda D}{2d}$$

$$x_3 = \frac{3\lambda}{2d} \quad \text{and} \quad x_4 = \frac{4\lambda D}{2d}$$

In general

$$x_n = \frac{n\lambda D}{2d}$$

where $n = 0, 1, 2, 3 \dots$

The distance between any two consecutive bright fringe is

$$x_2 - x_1 = \frac{2\lambda D}{2d} - \frac{\lambda D}{2d}$$

$$\text{fringe width } (\beta) = \frac{\lambda D}{2d}$$

$$\boxed{\beta = \frac{\lambda D}{2d}}$$

2) Condition for Dark fringes:—

The point 'P' will be dark when the path difference is an odd number multiple of half wavelength i.e.

$$(S_2P - S_1P) = (2n+1)\lambda/2$$

where $n = 0, 1, 2, \ldots$

$$\frac{2xd}{D} = (2n+1)\frac{\lambda}{2}$$

$$x = \frac{(2n+1)\lambda D}{4d} \quad \text{——— (5)}$$

Dark fringes are formed as

when $n=0$

$$x_0 = \frac{\lambda D}{4d}$$

$n=1$

$$x_1 = \frac{3\lambda D}{4d}$$

$n=2$

$$x_2 = \frac{5\lambda D}{4d}$$

$n=3$

$$x_3 = \frac{7\lambda D}{4d}$$

In general

$$\boxed{x_n = \frac{(2n+1)\lambda D}{4d}}$$

The distance between any two consecutive dark fringes is

$$x_2 - x_1 = \frac{5\lambda D}{4d} - \frac{3\lambda D}{4d} = \frac{\lambda D}{2d}$$

fringe width $(\beta) = \dfrac{\lambda D}{2d}$

Intereference by Division of Amplitude :—

1) Thin Films

2) Newton's Rings

3) Michelson's Interferometer

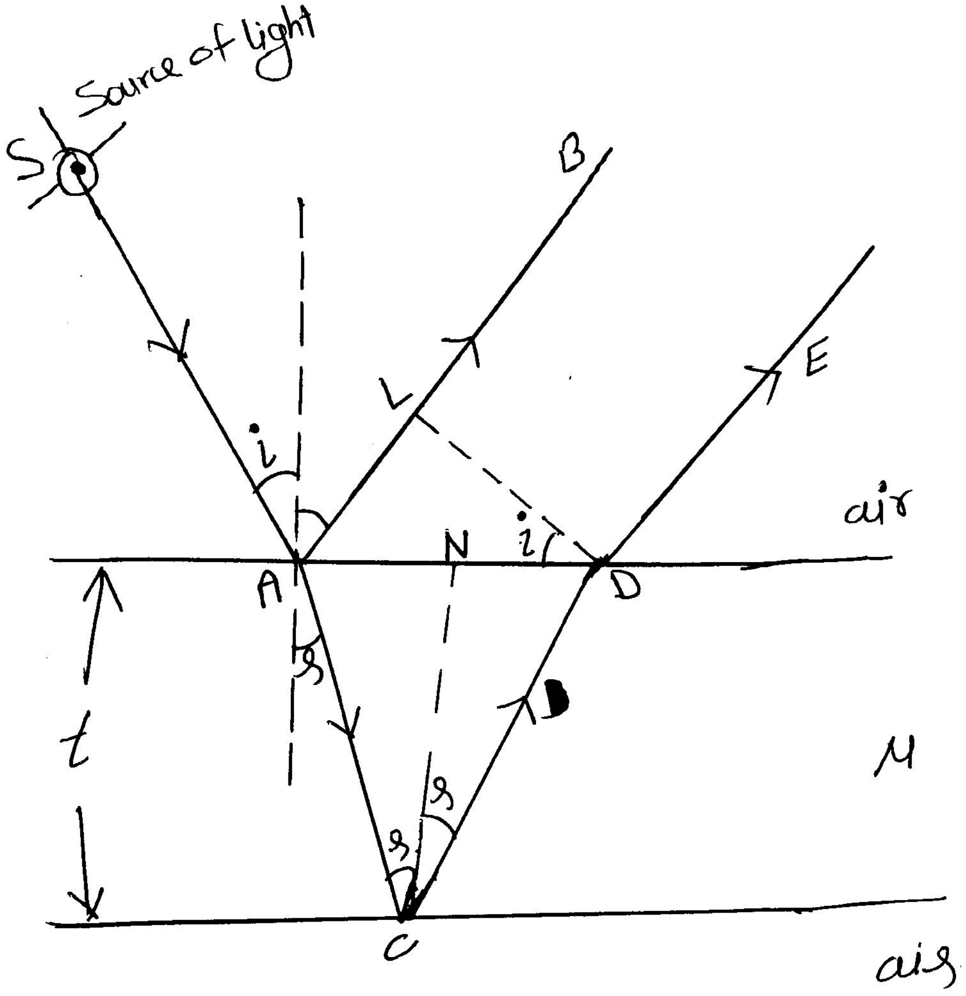
Thin film —

A film is said to be thin when its thickness is about the order of one wavelength of visible light ($\simeq 5500 \mathring{A}$). e.g. air film or a soap bubble.

A thin sheet of transparent material like glass, mica are also treated as thin films.

When a thin film of oil spreads on the surface of water and is exposed to white light beautiful colours are seen.

Interference Due to Reflected light :—



The phenomenon of interference in thin film due to reflected light, Consider a ray SA of ~~monochro~~ monochromatic light of wavelength λ be incident on the upper Surface of thin transparent film of Uniform thickness t and ~~index~~ index of refraction μ, at an angle i. The ray SA is ~~partially~~ partially reflected along AB and partially refracted along AC at an angle r. The refracted part AC is reflected from the point C on the lower surface of the film along CD and finally emerges out along DE.

As the rays AB and DE are derived from the same incident ray, they are coherent.

The optical path difference between these two rays is given by

$$\Delta = \text{path } (AC+CD) \text{ in film} - \text{path } AL \text{ in air}$$

$$\Delta = \mu(AC+CD) - AL \quad \text{———} \quad (1)$$

In right angle triangle ACN,

$$\frac{CN}{AC} = \cos r$$

$$AC = \frac{CN}{\cos r} = \frac{t}{\cos r} \quad \text{———} \quad (2)$$

Similarly, in right angle triangle CND

$$\frac{CN}{CD} = \cos r$$

$$CD = \frac{CN}{\cos r} = \frac{t}{\cos r} \quad \text{———} \quad (3)$$

In right angle triangle ADL

$$\frac{AL}{AD} = \sin i$$

$$AL = AD \sin i = (AN + ND) \sin i \quad \text{———} \quad (4)$$

Again in triangles ACN and CND

$$\frac{AN}{NC} = \tan r \text{ and } \frac{ND}{NC} = \tan r$$

$$AN = NC \tan r \quad \text{and} \quad ND = NC \tan r$$

$$AN = t \tan r \quad \text{and} \quad ND = t \tan r$$

Putting the value of AN and ND in Eqn (4)

$$AL = (t \tan r + t \tan r) \sin i$$

$$= 2t \tan r \sin i$$

$$= 2t \tan r \frac{\sin i}{\sin r} \cdot \sin r$$

$$= 2\mu t \tan r \sin r$$

$$= 2\mu t \frac{\sin r}{\cos r} \cdot \sin r$$

$$AL = 2\mu t \frac{\sin^2 r}{\cos r} \quad \text{—— (5)}$$

So, Path difference

$$\Delta = \mu (AC + CD) - AL$$

$$= \mu \left(\frac{t}{\cos r} + \frac{t}{\cos r} \right) - 2\mu t \frac{\sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r} (1 - \sin^2 r)$$

$$\boxed{\Delta = 2\mu t \cos r} \quad \text{—— (6)}$$

As the ray AB is the reflected ray from a denser medium, therefore, there occurs an addition path difference of $\lambda/2$ and Phase difference of π (according to Stoke's Law)

So, total Path difference $\Delta = 2\mu t \cos\theta \pm \lambda/2$ —— (7)

Condition for maximum intensities ——

x——————x——

For Constructive interference, path difference Should be an even multiple of $\lambda/2$

So $$\Delta = 2\mu t \cos\theta + \lambda/2 = 2n\lambda/2$$

$$2\mu t \cos\theta = 2n\frac{\lambda}{2} - \lambda/2 = (2n-1)\lambda/2$$

Where $n = 1, 2, 3 \ldots$

~~2μtCosθ~~

$$\boxed{2\mu t \cos\theta = (2n-1)\frac{\lambda}{2}} \quad \text{—— (8)}$$

When Condition (8) is Satisfied, the film appears bright.

For destructive interference, path difference should be an odd multiple of $\lambda/2$.

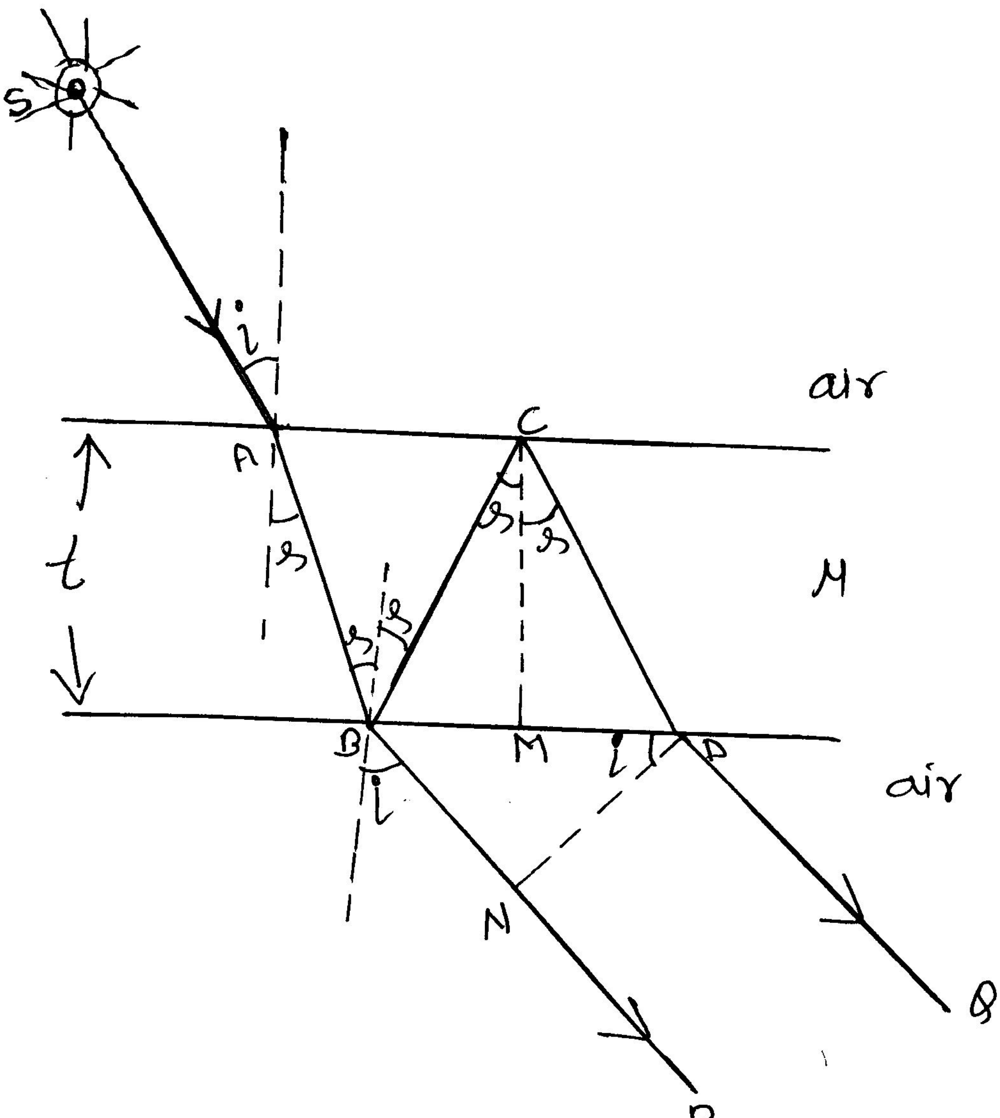
So $$2\mu t\cos r + \frac{\lambda}{2} = (2n+1)\lambda/2$$

$$2\mu t\cos r = \frac{2n\lambda}{2} + \frac{\lambda}{2} - \lambda/2 = n\lambda$$

$$\boxed{2\mu t\cos r = n\lambda} \quad \text{———} \quad (9)$$

When the above condition (9) is satisfied, film will appear dark.

Interference Due to Transmitted light



The optical path difference between transmitted rays BP and DQ is

$$\Delta = \text{path}(BC+CD) \text{ in film} - BN \text{ (in air)}$$

$$\Delta = \mu(BC+CD) - BN \qquad \text{——— (1)}$$

In right angle triangle BCM

$$\frac{MC}{BC} = \cos r$$

$$BC = \frac{MC}{\cos r} = \frac{t}{\cos r} \qquad \text{——— (2)}$$

In other right angle triangle MDC,

$$\frac{MC}{CD} = \cos r$$

$$CD = \frac{MC}{\cos r} = \frac{t}{\cos r} \quad \text{——— (3)}$$

In right angle triangle BND

$$\frac{BN}{BD} = \sin i$$

$$BN = BD \sin i = (BM + MD) \sin i \quad \text{——— (4)}$$

Again in right angle triangles BMC and CMD

$$\frac{BM}{MC} = \tan r \quad \text{and} \quad \frac{MD}{MC} = \tan r$$

$$BM = MC \tan r \quad \text{and} \quad MD = MC \tan r$$

$$BM = t \tan r \quad \text{and} \quad MD = t \tan r$$

putting the values of BM and MD in eqn (4)

$$BN = (t \tan r + t \tan r) \sin i$$

$$= 2t \tan r \sin i$$

$$= 2t \tan r \frac{\sin i}{\sin r} \cdot \sin r$$

$$= 2\mu t \tan r \cdot \sin r$$

$$BN = 2\mu t \frac{\sin^2 r}{\cos r} \quad \text{——— (5)}$$

Substituting values of BC, CD and BN in eqn 1

So
$$\Delta = \mu\left(\frac{t}{\cos r} + \frac{t}{\cos r}\right) - 2\,\frac{\mu t \sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r}\left(1 - \sin^2 r\right)$$

$$\Delta = 2\mu t \cdot \cos r \quad \text{———} \quad (6)$$

Condition for maximum and minimum intensities —

x ———— x

For constructive interference or maxima —

$$\Delta = 2n\frac{\lambda}{2}$$

$$\boxed{2\mu t \cos r = n\lambda} \quad \text{———} \quad (7)$$

where $n = 0, 1, 2, 3 \ldots$

For destructive interference or minima —

$$\Delta = (2n-1)\frac{\lambda}{2}$$

$$2\mu t \cos r = (2n-1)\frac{\lambda}{2}$$

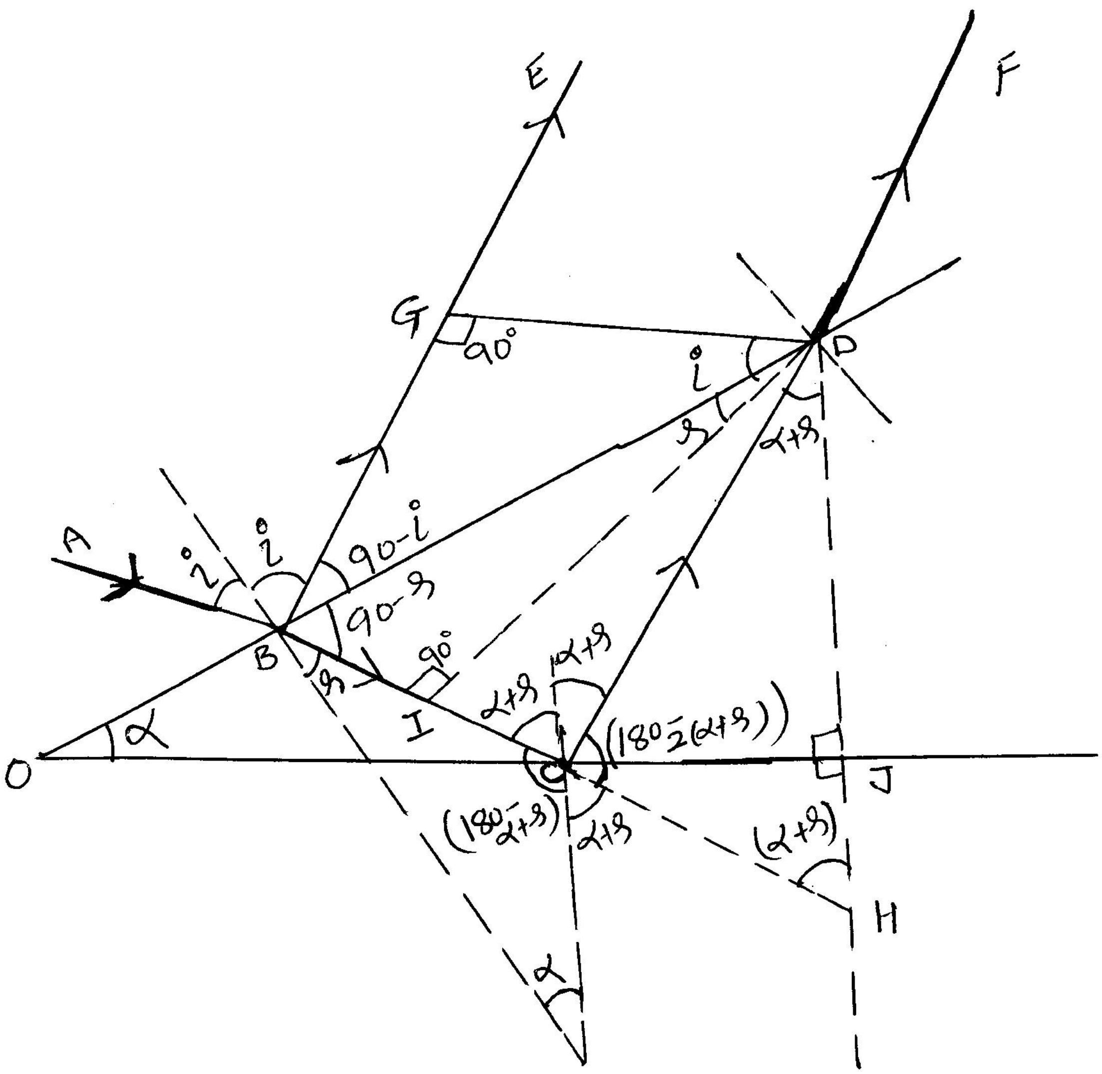
where $n = 1, 2, 3 \ldots$

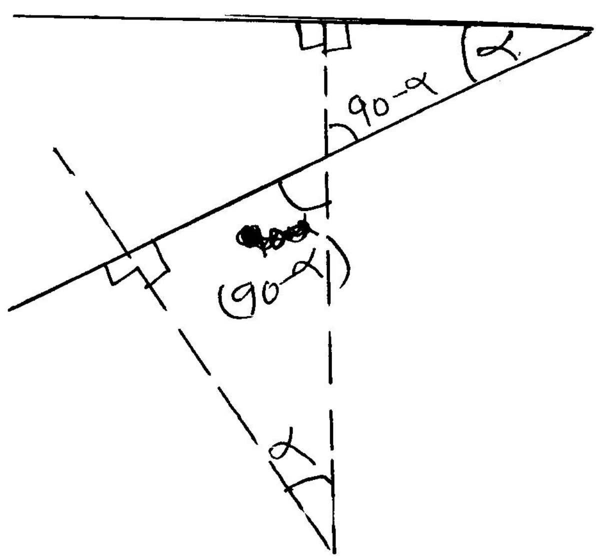
$$\boxed{2\mu t \cos r = (2n+1)\frac{\lambda}{2}}$$

where $n = 0, 1, 2, 3 \ldots$

Conditions of maxima and minima in transmitted light are just opposite to those for reflected light.
The point of the film which appears bright in reflected light, appears dark in transmitted light.

Interference By a Film with Two Non-Parallel Reflecting Surfaces (Wedge-Shaped Film)





The optical Path difference between rays BE and DF

$$\Delta = \mu(BC + CD) - BG$$

$$= \mu(BI + IC + CD) - BG \quad \text{——— (1)}$$

In triangles CDJ and CJH

$$CD = CH$$

$$\Delta = \mu(BI + IC + CH) - BG$$

$$= \mu(BI + IH) - BG$$

$$\Delta = \mu BI + \mu IH - BG \quad \text{——— (2)}$$

In ΔBGD

$$\sin i = \frac{BG}{BD}$$

Again in Δ BDI

$$\sin r = \frac{BI}{BD}$$

Therefore $\dfrac{\sin i}{\sin r} = \dfrac{BG/BD}{BI/BD} = \dfrac{BG}{BI}$

$$\mu = \frac{BG}{BI}$$

$$\text{So } BG = \mu BI \quad \text{——— (3)}$$

Put the value of BG in eqn (2)

$$\Delta = \mu BI + \mu IH - \mu BI$$

$$\Delta = \mu IH \quad \text{——— (4)}$$

In ΔIHD

$$\cos(\alpha+\theta) = \frac{IH}{DH}$$

$$IH = DH\cos(\alpha+\theta)$$

$$= (DJ+JH)\cos(\alpha+\theta)$$

$$= (t+t)\cos(\alpha+\theta)$$

$$IH = 2t\cos(\alpha+\theta)$$

Put the value of IH in eqn (4)

$$\boxed{\Delta = 2\mu t\cos(\alpha+\theta)} \quad \text{--- (5)}$$

Due to the reflection of ray from Denser medium there occurs an additional Path difference of $\lambda/2$

Therefore

$$\boxed{\Delta = 2\mu t\cos(\alpha+\theta) + \lambda/2} \quad \text{--- (6)}$$

1) Condition for Constructive interference

$$\Delta = n\lambda$$

$$2\mu t\cos(\alpha+\theta) + \lambda/2 = n\lambda$$

$$\boxed{2\mu t\cos(\alpha+\theta) = (2n-1)\lambda/2}$$

where $n = 1, 2, 3, \ldots$

2) Condition for destructive interference

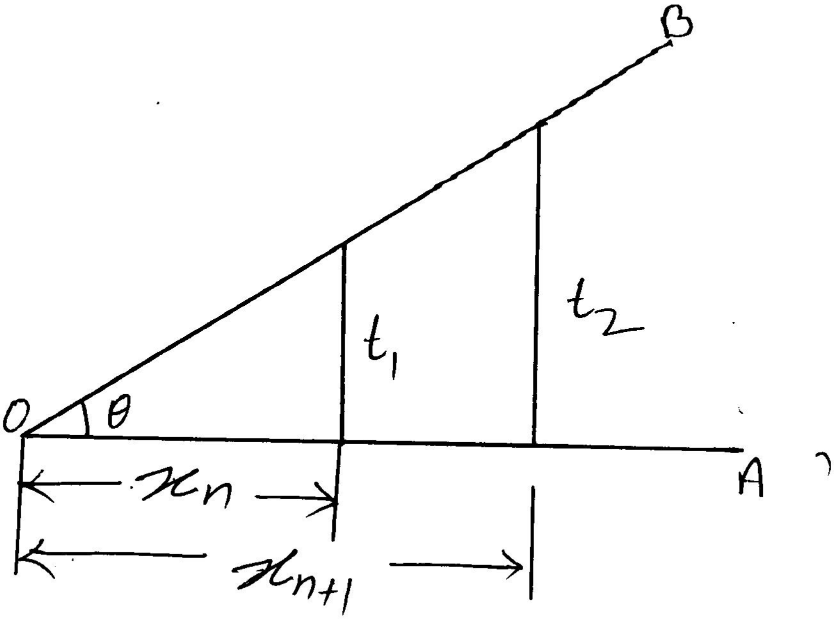
$$\Delta = (2n+1)\lambda/2$$

$$2\mu t\cos(\alpha+\theta) + \lambda/2 = (2n+1)\lambda/2$$

$$\boxed{2\mu t\cos(\alpha+\theta) = n\lambda}$$

where $n = 0, 1, 2, 3 \ldots$

Fringe width —



Fring width 'ω' or the Separation between two ~~Successive~~ Successive Bright fringes or between two Successive dark fringes may be obtained as

Let x_n be the distance of n^{th} dark fringe from the edge 'O' of the film.

Then

$$\tan\theta = \frac{t_1}{x_n}$$

or $$t_1 = x_n \tan\theta$$

Put the value of t_1 in the Condition for dark fringe

i.e. $$2\mu t \cos(\theta + r) = n\lambda$$
$$2\mu t = n\lambda$$

{ 'θ' is very small, 'r' is also very small for normal incident }

$$2\mu x_n \tan\theta = n\lambda$$

—— (1)

Similarly, if x_{n+1} is the distance of $(n+1)^{th}$ dark fringe.

Then

$$2\mu x_{n+1} \tan\theta = (n+1)\lambda \quad \text{——— (2)}$$

Subtracting eqn (1) from eqn (2) we get

$$2\mu (x_{n+1} - x_n) \tan\theta = \lambda$$

$$x_{n+1} - x_n = \frac{\lambda}{2\mu \tan\theta}$$

for very small value of θ, $\tan\theta = \theta$

Fringe width $\omega = x_{n+1} - x_n = \dfrac{\lambda}{2\mu\theta}$

$$\boxed{\omega = \frac{\lambda}{2\mu\theta}} \quad \text{——— (3)}$$

Similarly, we can obtain same formula for fringe width of bright fringes.

$$\boxed{\omega = \frac{\lambda}{2\mu\theta}} \quad \text{——— (4)}$$

Colours in Thin film

When an oil film on water or a soap film or wedge shaped air film between two glass plate is seen in reflected light it shows brilliant colours.

This coloured phenomenon was offered by young on the basis of interference of light wave.

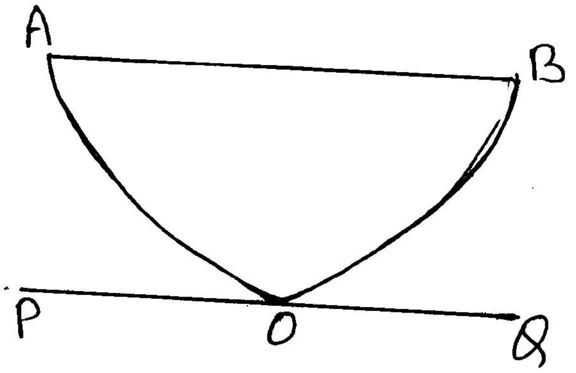
When a beam of ~~light~~ white light from the source is incident normally on a thin film of transparent material and seen in reflected light then coloured fringes will be observed. These colours arise due to the interference of light waves reflected from the upper and lower surface of the film. The path difference between these interfering rays depends upon the thickness of the film 't' and upon the inclination of the incident or reflected ray 'r'.

At a particular point of the film (t) and for a particular position of the eye (r) the waves of only certain wavelengths satisfy the condition of maxima (i.e. $2\mu t \cos r = (2n \pm 1)\lambda/2$). Hence only those colours (λ) which satisfy the condition of maxima will be present with maximum intensity. Other neighbouring colours (λ) will be present only with diminished intensity. While other wavelengths

Which Satisfy the Condition of minima ($2\mu t \cos\theta = n\lambda$) will be absent from the reflected System. Hence the point of the film will appear Coloured. Similarly, if the Same point of the film is observed with an eye in different positions or at different points by keeping the eye at the fixed position, a different Set of Colours is observed at each time.

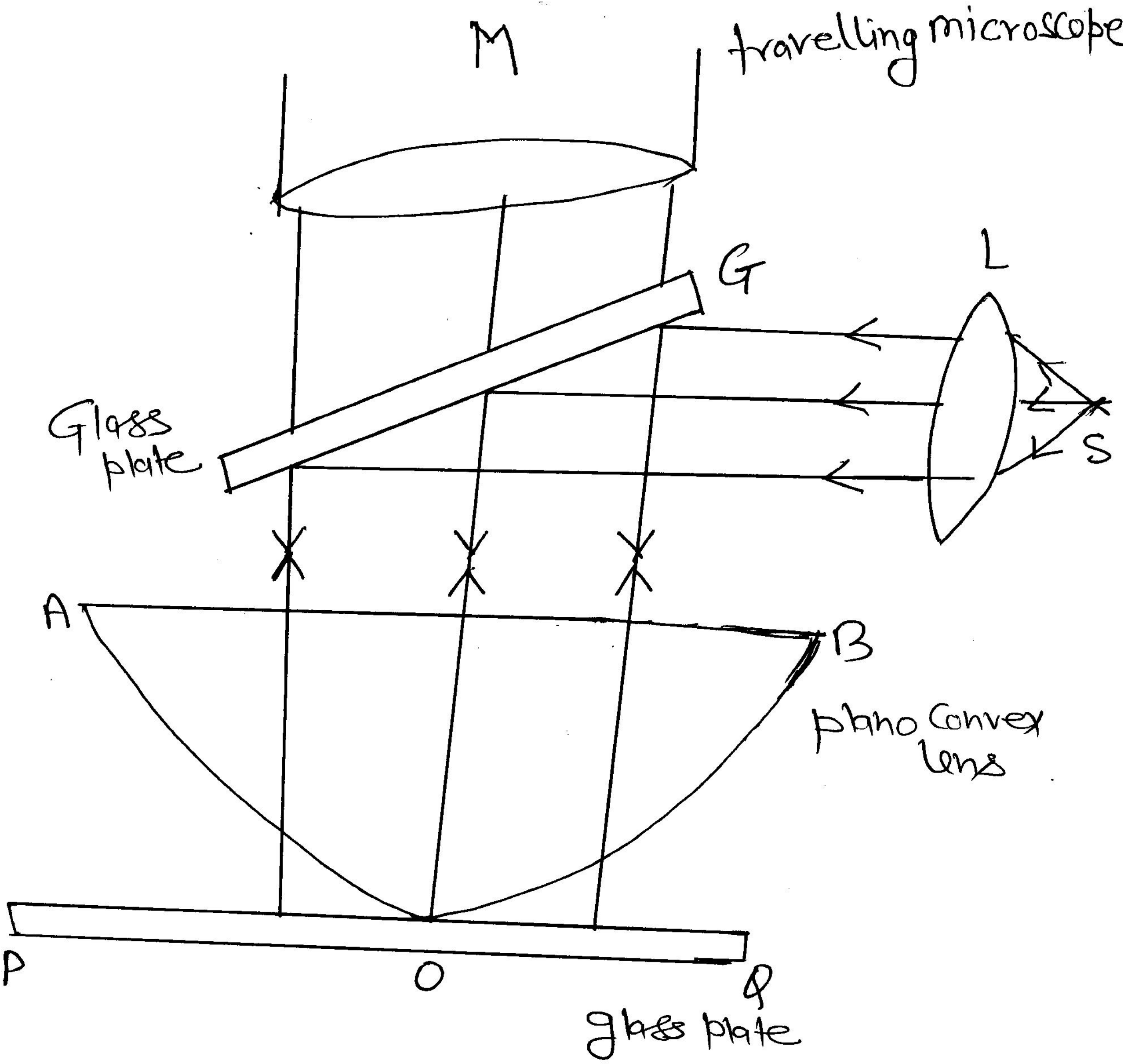
If white light falls as a parallel beam on a Uniformly thick film, then the path difference at each point of the film will be Same and film will appear Uniformly Coloured.

Newton's Rings

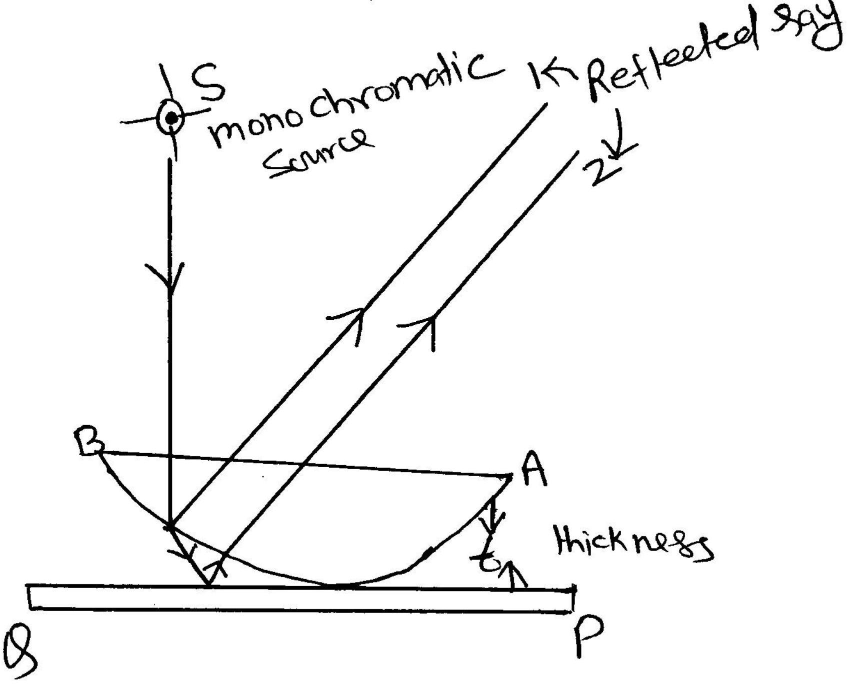


If we place a plano-convex lens of large focal length on a plane glass plate, a thin film of air is developed between the curved surface of the lens and the glass plate. The thickness of the air film is zero at the point of contact 'O' and gradually increases as we move away from the point of contact on either side. When the monochromatic light falls normally on the surface of lens, then the light reflected from the ~~lower~~ upper surface of the film AOB interferes with the light reflected from the lower surface of the film POQ. As a result, an alternate dark and bright circular rings concentric around the point of contact are seen. The interference rings of equal thickness so formed were first investigated Newton and hence called Newton's ring. If the incident light is white, a series of concentric coloured rings is seen around the point of contact in both reflected and transmitted system.

Experimental Arrangement



Explanation of the Formation of Circular Newton's Ring



Circular rings are formed due to the interference of light rays reflected from the upper and lower surfaces of the air film formed between the convex surface of a plano-convex lens and the glass plate.

The incident ray SF is divided into two coherent ray 1 and ray 2 by reflection from the upper and lower surfaces of the wedge shaped air film.

The reflected rays 1 and 2 interfere and produce bright and dark circular rings around the point of contact.

The effective path difference between the interfering rays in reflected light is $2\mu t \cos r + \lambda/2$ or $2\mu t \cos r - \lambda/2$.

For normal incidence $r = 0$, $\cos r = 1$

therefore, the Effective pathdifference $= 2\mu t + \lambda/2$

As the point of Contact, $t = 0$, the effective path difference $= \lambda/2$

This is the Condition of minimum intensity.

Hence the Central Spot of the rings System appears dark.

For Constructive interference or bright ring the Effective path difference $= 2n\lambda/2$

i.e. $$2\mu t + \lambda/2 = n\lambda$$

$$2\mu t = (2n \pm 1)\lambda/2 \quad \text{———} \quad (1)$$

For destructive interference or dark ring.

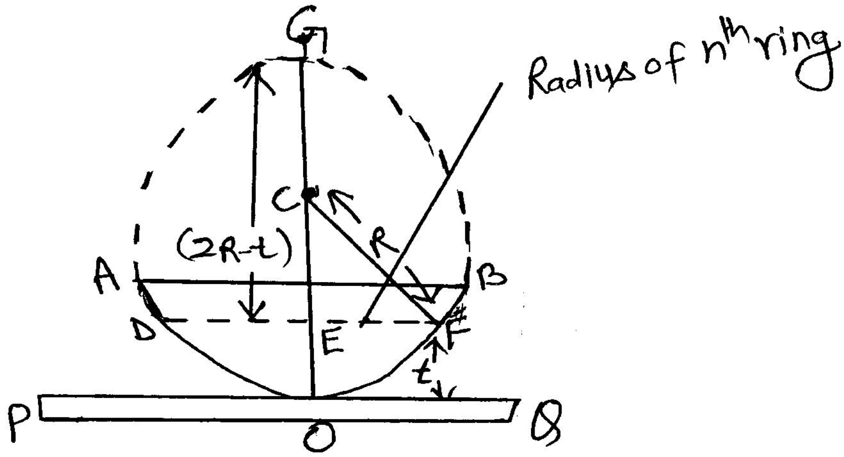
$$2\mu t + \lambda/2 = (2n+1)\frac{\lambda}{2}$$

or $$2\mu t = n\lambda \quad \text{———} \quad (2)$$

where $n = 0, 1, 2, 3, \ldots$

Eqn (1) and Eqn (2) indicate that for a particular bright or dark ring, 't' should be Constant.

Diameters of Bright and Dark Rings



Consider a plano-convex lens AOB placed on glass plate POQ. Let 'R' be the radius of curved surface AOB of the lens and 't' the thickness of film at any point F.

From the property of circle,

$$EF \times DE = OE \times GE \quad \text{——— (1)}$$

$$r \times r = t \times (2R - t)$$

But $EF = DE = r$ Radius of n^{th} ring

$$r^2 = 2Rt - t^2$$

$$t << R$$

Therefore, $r^2 = 2Rt$

$$t = \frac{r^2}{2R} \quad \text{——— (2)}$$

For Bright Rings —

The condition of constructive interference

$$2\mu t = (2n-1)\frac{\lambda}{2} \quad \text{——} (3)$$

Put the value of t from eqn (2) in eqn (3)

$$2\mu \frac{r^2}{2R} = (2n-1)\frac{\lambda}{2}$$

So $$r^2 = \frac{(2n-1)\lambda R}{2\mu} \quad \text{——} (4)$$

If n^{th} ring passes through the point F of the film, Then r should be the radius of n^{th} bright ring.

Therefore

$$r_n^2 = \frac{(2n-1)\lambda R}{2\mu} \quad \text{——} (5)$$

If 'D_n' is the diameter of n^{th} bright ring.

Then eqn (5) becomes

$$\left(\frac{D_n}{2}\right)^2 = \frac{(2n-1)\lambda R}{2\mu}$$

or $$D_n^2 = \frac{2(2n-1)\lambda R}{\mu}$$

$$D_n = \sqrt{\frac{2(2n-1)\lambda R}{\mu}}$$

$$\boxed{D_n = \sqrt{\frac{2(2n-1)\lambda R}{\mu}}} \quad \text{——} (6)$$

For air film $\mu = 1$

So

$$D_n = \sqrt{2(2n-1)\lambda R}$$

or $$D_n \propto \sqrt{(2n-1)}$$

$$D_n = k\sqrt{(2n-1)} \quad \text{---} \quad (7) \qquad \left\{ k = \sqrt{2\lambda R} \right\}$$

where $n = 1, 2, 3 \ldots$

Thus the diameters of bright rings are proportional to the square roots of the odd natural numbers.

For Dark Rings —

The condition for destructive interference is

$$2\mu t = n\lambda \quad \text{---} \quad (8)$$

Substituting the value of t in eqn (8)

So

$$2\mu \frac{r^2}{2R} = n\lambda$$

or $$r^2 = \frac{n\lambda R}{\mu}$$

If r_n is the radius of n^{th} dark ring which passes through the point F of the film.

Then $$r_n^2 = \frac{n\lambda R}{\mu}$$

or $$\left(\frac{D_n}{2}\right)^2 = \frac{n\lambda R}{\mu}$$

$$D_n^2 = \frac{4n\lambda R}{\mu}$$

$$D_n = \sqrt{\frac{4n\lambda R}{\mu}} \quad \text{———} \quad (9)$$

For air film, $\mu = 1$

$$D_n = \sqrt{4n\lambda R}$$

$$D_n \propto \sqrt{n}$$

$$D_n = k\sqrt{n} \quad \text{———} \quad (10) \qquad \left\{ k = \sqrt{4\lambda R} \right\}$$

Where $n = 0, 1, 2, 3 \ldots$

Thus, the diameters of dark rings are proportional to the square roots of the natural numbers.

If D_{n+1} and D_n are the diameters of ~~the~~ $(n+1)^{th}$ and n^{th} dark rings, Then the spacing between the successive rings.

$$D_{n+1} - D_n = k\left[\sqrt{(n+1)} - \sqrt{n}\right]$$

The spacing between consecutive dark rings

$$(D_2 - D_1) = K(\sqrt{2} - \sqrt{1})$$

$$(D_3 - D_2) = K(\sqrt{3} - \sqrt{2})$$

$$(D_4 - D_3) = K(\sqrt{4} - \sqrt{3})$$

Hence, it is concluded that the spacing decreases with the order of the rings and fringes get closer and closer as their order increases.

Determination of Wavelength of Sodium light Using Newton's Ring

Let D_n and D_{n+p} be the diameters of the n^{th} and $(n+p)^{th}$ dark Newton's rings in reflected System. respectively, then

$$D_n^2 = 4n\lambda R$$

and $$D_{n+p}^2 = 4(n+p)\lambda R$$

where p is any number

Thus

$$D_{n+p}^2 - D_n^2 = 4(n+p)\lambda R - 4n\lambda R$$

$$D_{n+p}^2 - D_n^2 = 4p\lambda R$$

So $$\boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}}$$

Numerical Problems

Q1 A man whose eyes are 150 Cm above the oil film on water Surface observed greenish Colour at a distance of 100Cm from his feet. Calculate the probable thickness of the film.

Given $\lambda_{green} = 5000\,\mathring{A}$

$\mu_{oil} = 1.4$

$\mu_{water} = 1.33$

Q.2 White light falls normally on a thin film of a Soapy water whose thickness is 1.5×10^{-5} Cm and refractive index 1.33. Which wavelength in the visible region will be reflected strongly.

Q.3 White light is incident normally on a thin film of thickness 0.50×10^{-6} m and index of refraction 1.50. Find the wavelength in visible region (400nm – 700nm) reflected most strongly.

Q4 A Soap Film of refractive index 1.43 is illuminated by white light incident at an angle of 30°. The refracted light is examined by a spectroscope in which dark band corresponding to the wavelengths 6×10^{-7} m is observed. Calculate the thickness of the film.

Q5 Light of wavelength 6000 Å falls normally on a thin wedge shaped film of refractive index 1.4 ~~forming~~ Forming fringes that are 2.0 mm apart. Find the angle of wedge in seconds.

Q6 Newton's rings are observed normally in reflected light of wavelength 6000 Å. The diameter of the 10^{th} dark ring is 0.50 cm. Find the radius of curvature of the lens and ~~keep~~ the thickness of the film.

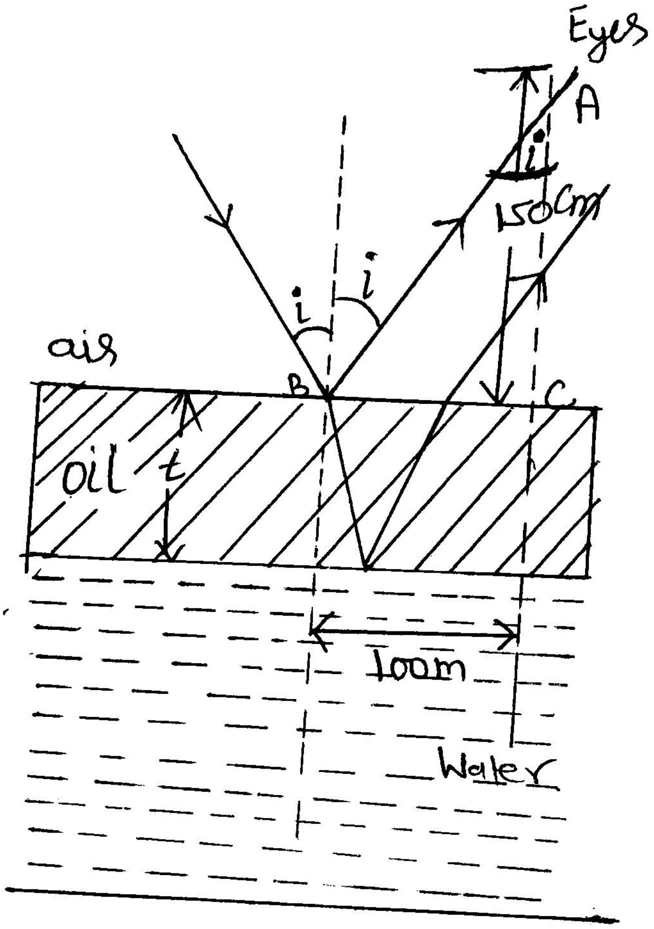
Q7 Newton's rings are observed by keeping a spherical surface of 100 cm radius on a plane glass plate. If the diameter of the 15^{th} bright ring is 0.590 cm and diameter of the 5^{th} ring is 0.336 cm. What is the wavelength of light used.

Q8 In Newton's ring experiment the diameter of 4^{th} and 12^{th} dark rings are 0.400 cm and 0.700 cm respectively. Deduce the diameter of 20^{th} dark ring.

Solution of Numerical Problems

Sol^n. 1

Let t is the thickness of film and r is the angle of refraction, then the condition of maxima in the reflected light is given by



$$2\mu t \cos r = (2n-1)\frac{\lambda}{2}$$

$n = 1, 2, 3 \ldots$

So $$t = \frac{(2n-1)\lambda}{4\mu \cos r} \quad \text{--- (1)}$$

from the fig.

$$\tan i = \frac{100}{150} = \frac{2}{3}$$

therefore $$\sin i = \frac{2}{\sqrt{13}}$$

So $\mu = \dfrac{\sin i}{\sin r}$ i.e. $\sin r = \dfrac{\sin i}{\mu} = \dfrac{2/\sqrt{13}}{1.4}$

or $$\sin r = \frac{2}{1.4 \times 3.605} = \frac{2}{5.047} = 0.3962$$

$$\cos r = \sqrt{1-\sin^2 r} = \sqrt{1-(.3962)^2} = \sqrt{0.8431}$$

$$\cos r = 0.918$$

Substituting this value of $\cos r$ in Eqn (1) we get

$$t = \frac{(2n-1)\times 5\times 10^{-5}\ \text{cm}}{4\times 1.4\times 0.918} = \underline{\underline{9.725\times 10^{-6}(2n-1)\ \text{cm}}}\ \text{Ans}$$

Sol^n. 2

When light falls normally ($\theta = 0°$) on the film the condition of maximia is

$$2\mu t \cos\theta = (2n+1)\frac{\lambda}{2} \qquad \text{Where } n = 0, 1, 2 \ldots$$

$$2\mu t = (2n+1)\lambda/2 \quad \text{——} ①$$

Given $t = 1.5 \times 10^{-5}$ cm and $\mu = 1.33$

So $\lambda = \dfrac{4\mu t}{(2n+1)}$ from eqn ①

$$\lambda = \frac{4 \times 1.33 \times 1.5 \times 10^{-5}}{(2n+1)}$$

for $n = 0$

$$\lambda = \frac{7.98 \times 10^{-5}}{1} = 7.98 \times 10^{-5}\,\text{cm}$$

for $n = 1$

$$\lambda = \frac{7.98 \times 10^{-5}}{3} = 2.66 \times 10^{-5}\,\text{cm}$$

for $n = 2$

$$\lambda = \frac{7.98 \times 10^{-5}}{5} = 1.596 \times 10^{-5}\,\text{cm}$$

for $n = 3$

$$\lambda = \frac{7.98 \times 10^{-5}}{7} = 1.14 \times 10^{-5}\,\text{cm}$$

so, out of above wavelength only 7.98×10^{-5} cm lie near the upper limit of visible region.
Hence, wavelength $\underline{7980\,Å}$ is most strongly reflected.

Sol^n.3

For normal incidence ($\delta = 0°$), the condition of maxima is given by.

$$2\mu t\cos\delta = (2n+1)\lambda/2$$

$$2\mu t\cos 0 = (2n+1)\lambda/2$$

$$2\mu t = \frac{(2n+1)\lambda}{2} \quad \text{where } n = 0,1,2\ldots \quad \text{—(1)}$$

Here

$\mu = 1.5$ and $t = 0.5\times10^{-6}$ m $= 5.0\times10^{-5}$ m

So from Eq^n (1)

$$\lambda = \frac{4\mu t}{(2n+1)} = \frac{4\times1.5\times5.0\times10^{-5}}{(2n+1)}$$

$$\lambda = \frac{30\times10^{-5}}{(2n+1)}$$

for $n = 0$

$$\lambda = \frac{30\times10^{-5}}{1} = 30\times10^{-5}\text{ m}$$

$$= 3000\text{ nm}$$

for $n = 1$

$$\lambda = \frac{30\times10^{-5}}{3} = 10\times10^{-5}\text{ m} = 1000\text{ nm}$$

for $n = 2$ $\quad \lambda = \frac{30\times10^{-5}}{5} = 6\times10^{-5}\text{ m} = 600\text{ nm}$

for $n = 3$ $\quad \lambda = \frac{30\times10^{-5}}{7} = 4.286\times10^{-5}\text{ m} = 428\text{ nm}$

for $n = 4$ $\quad \lambda = \frac{30\times10^{-5}}{9} = 3.333\times10^{-5}\text{ m} = 333\text{ nm}$

So, within the visible region (~~4000~~ 400nm – 700nm) The light of wavelengths 428.6 nm and 600nm are reflected most strongly.

Sol^n 4

The condition of destructive interference or dark band in reflected light is

$$2\mu t \cos r = 2n\frac{\lambda}{2}$$

So $t = \dfrac{n\lambda}{2\mu \cos r}$

Given $\angle i = 30^\circ$, $\mu = 1.43$, $n = 1$ and $\lambda = 6 \times 10^{-7} m$

We know $\mu = \sin i / \sin r$

So, $\sin r = \dfrac{\sin i}{\mu} = \dfrac{\sin 30}{1.43} = \dfrac{1/2}{1.43} = \dfrac{1}{2.86}$

$\sin r = 0.38$

So, $\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - (.38)^2} = 0.92$

Then $t = \dfrac{1 \times 6 \times 10^{-7}}{2 \times 1.43 \times 0.92}$

$t = 2.28 \times 10^{-7} m$

or $t = 2.28 \times 10^{-5} cm$

thickness of Film $t = 2.28 \times 10^{-5} cm$ Ans

Solⁿ. 5

If θ is the angle of the wedge formed by a medium of refractive index μ, then for normal incidence the fringe width for wavelength λ is given by

$$\omega = \frac{\lambda}{2\mu\theta}$$

or $$\theta = \frac{\lambda}{2\mu\omega}$$

Given $\lambda = 6000\,\mathring{A} = 6000 \times 10^{-8}\,cm$, $\mu = 1.4$

and $\omega = 2.0\,mm = 0.20\,cm$

$$\theta = \frac{6000 \times 10^{-8}}{2 \times 1.4 \times 0.20} = 10.71 \times 10^{-5}\,rad.$$

$$\theta = 10.71 \times 10^{-5} \times \frac{180^\circ}{\pi}$$

$$\theta = 0.0061^\circ$$

$$= 0.0061 \times 60 \times 60\ Sec$$

$\theta = 21.96$ Sec. Ans

Sol^n. 6

The diameter of n^{th} dark ring is given by

$$D_n^2 = 4n\lambda R$$

$$\text{or } R = \frac{D_n^2}{4n\lambda}$$

Given $\lambda = 6000\,\mathring{A}$, $D_n = 0.50\,cm$

and $n = 10$.

So

$$R = \frac{0.50 \times 0.50}{4 \times 10 \times 6 \times 10^{-5}}\ cm$$

$$R = 106\ cm$$

If t is the thickness of the film corresponding to a ring of 'D' diameter, then, $r^2 = 2Rt$

So

$$2t = \frac{D^2}{4R}$$

$$t = \frac{D^2}{8R} = \frac{0.50 \times 0.50}{8 \times 106}\ cm$$

$$t = 3 \times 10^{-4}\ cm$$

Sol^n. 7

If D_{n+p} and D_n be the diameters of $(n+p)^{th}$ and n^{th} bright ring, then

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

Given $D_{15} = 0.590$ cm and $D_5 = 0.336$ cm

$p = 10$ and $R = 100$ cm

So
$$\lambda = \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 100}$$

$$= \frac{(0.590 + 0.336)(0.590 - 0.336)}{4 \times 10 \times 100}$$

$$= 5.88 \times 10^{-5} \text{ cm}$$

$$\lambda = 5880 \, \mathring{A}$$

Solⁿ. 8

If D_{n+p} and D_n be the diameters of $(n+p)^{th}$ and n^{th} dark rings respectively, then

$$D_{n+p}^2 - D_n^2 = 4p\lambda R \quad \text{—— (1)}$$

Given

$n = 4$, $n+p = 12$, $D_4 = 0.400$ cm

and $D_{12} = 0.700$ cm, $p = 8$

$$D_{12}^2 - D_4^2 = 4 \times 8 \times \lambda \times R \quad \text{—— (2)}$$

Suppose the diameter of 20^{th} dark ring is D_{20}.

Then

$$D_{20}^2 - D_4^2 = 4 \times 16 \times \lambda \times R \quad \text{—— (3)}$$

dividing eqn (2) by (3), we get

$$\frac{D_{12}^2 - D_4^2}{D_{20}^2 - D_4^2} = \frac{4 \times 8}{4 \times 16} = \frac{1}{2}$$

$$2\left(D_{12}^2 - D_4^2\right) = D_{20}^2 - D_4^2$$

$$2D_{12}^2 - 2D_4^2 + D_4^2 = D_{20}^2$$

$$D_{20}^2 = 2D_{12}^2 - D_4^2 = 2 \times (0.700)^2 - (0.400)^2$$

$$D_{20}^2 = 0.98 - 0.16 = 0.82$$

or $D_{20} = 0.905$

16. Describe Newton's ring method for measuring the wavelength of monochromatic light. Give the necessary theory.
17. How Newton's rings are formed? Describe Newton's rings experiment to determine the wavelength of monochromatic light with necessary theory.
18. Derive an expression for the radius of a dark ring in Newton's rings formed by reflected light. Explain how the refractive index of a transparent liquid can be determined using Newton's rings.
19. Explain with necessary theory how you can determine the refractive index of a transparent liquid by an interference method (Newton's rings). Derive the formula used.
20. Discuss the phenomena of interference of light due to thin films of uniform thickness in reflected light and find the conditions of maxima and minima.
21. Discuss the formation of interference fringes due to a wedge shaped thin film seen by normally reflected sodium light and obtain an expression for the fringe width.
22. Describe Newton's ring method to determine the wavelength of sodium light. What will happen in fringes of air-film between the planoconvex lens and the glass plate is filled with a liquid of refractive index μ?
23. Find expressions for the diameters of n^{th} dark and bright Newton's ring.
24. Describe and explain the formation of Newton's ring in reflected monochromatic light. Prove that in reflected light diameters of the dark rings are proportional to the square root of natural numbers.
25. Show that the diameter D_n of the n^{th} Newton's ring, when two surfaces of radii R_1 and R_2 are placed in contact, is given by
$$\frac{1}{R_1} \pm \frac{1}{R_2} = \frac{4n\lambda}{D_n^2}$$
26. Describe and explain the formation of Newton's rings in reflected monochromatic light. Prove that in reflected light diameters of the dark rings are proportional to the square root of natural numbers.
27. Describe the construction of a Michelson's interferometer and explain its working. How wavelength of light is determined with it?
28. Describe the construction and working of Michelson interferometer. Explain how the instrument can be used to determine the wavelength of a monochromatic source of radiation.
29. Describe the principle, construction and working of a Michelson's interferometer. Explain how the wavelength of light is determined with it?
30. Describe a Michelson's interferometer. A Michelson interferomter is illuminated with sodium light. Why the rings are alternately clear and indistinct when the movable mirror is moved in one direction? How will you use this phenomenon to determine the difference between two wavelengths very close together?
31. Explain the occurrence of fringes in the Michelson interferometer. What change is observed in the fringe system when one of the mirrors is moved parallel to itself slowly?
32. Outline the theory of fringe shape in Michelson's interferometer and discuss the nature of interference pattern produced.

PROBLEMS

1. In a Young's double slit experiment the angular width of a fringe formed on a distant screen is 0.1°. The wavelength of light used is 6000 Å. What is the spacing between the slits ?

[Ans. 3.44×10^{-4} m]

[**Hint.** Angular fringe width $\beta_0 = \lambda/2\,d$ or $2\,d = \lambda/\beta_0$

Here, $\lambda = 6000$ Å $= 6 \times 10^{-7}$ m and $\beta_0 = 0.1° = \dfrac{0.1 \times \pi}{180} = \dfrac{0.1 \times 3.14}{180}$ radian]

2. At a certain point on a screen the path difference for the two interfering rays is $(1/8)^{th}$ of the wavelength. Find the ratio of the intensity at this point to that at the centre of a bright fringe. **[Ans.** 0.853]

[**Hint.** $I = 2a^2\,(1 + \cos\delta)$, where δ is phase difference between rays at the point.

At centre $\delta = 0°$ $\therefore$ $I_0 = 2a^2\,(1 + 1) = 4\,a^2$

when path difference is $\lambda/8$, then phase difference

$$= \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \frac{\pi}{4}$$

$$\therefore \quad I_1 = 2\,a^2\,(1 + \cos\pi/4) = 2\,a^2\,(1 + 0.707)$$

$$\frac{I_1}{I_0} = \frac{2\,a^2\,(1 + 0.707)}{4\,a^2}\Bigg]$$

3. Two coherent light sources of intensities ratio 25:4 are employed in an interference experiment. What is the ratio of the intensities of the maxima and minima in the interference pattern ? **[Ans.** 49/9]

[**Hint.** $a_1 / a_2 = \sqrt{(I_1 / I_2)} = \sqrt{(25/4)} = 5/2$

$\therefore$ $a_1 = 5$ units and $a_2 = 2$ units

$a_{max} = a_1 + a_2 = 7$ units

and $a_{min} = 5 - 2 = 3$ units

$$\frac{I_{max}}{I_{min}} = \frac{(a_{max})^2}{(a_{min})^2}\Bigg]$$

4. A Young's double slit arrangement produces interference fringes for sodium light ($\lambda = 5890$ Å) that are 0.20 cm apart. What is the angular fringe separation if the entire arrangement is immersed in water (refractive index of water = 4/3)? **[Ans.** 0.15°]

[**Hint.** Angular fringe separation $\theta = \lambda / d$ or $d = \lambda / \theta$ ...(1)

where d = separation between two slits

In water $d = \lambda' / \theta'$ ...(2)

$\therefore$ $\lambda/\theta = \lambda'/\theta'$ or $\theta' = (\lambda'/\lambda)\,\theta$ ...(3)

We know that $c/c' = \lambda/\lambda' = \mu$ (refractive index of water)

$\therefore$ $\lambda'/\lambda = 1/\mu = 3/4$]

5. In an interference pattern, at a point we observe 12^{th} order maximum for $\lambda_1 = 6000$ Å. What order will be visible here if the source is replaced by a light of wavelength 4800 Å? **[Ans.** 15]

[**Hint.** We know that $x_n = n\,\dfrac{\lambda\,D}{2\,d}$. When x_n, D and $2d$ are fixed, then

$$n_1\,\lambda_1 = n_2\,\lambda_2$$

or $$12 \times 6000 = n_2 \times 4800]$$

6. In a double slit arrangement, the slits are 0.015 mm apart and the screen is 0.75 m away. If the incident monochromatic light has a wavelength of 6000 Å, calculate the angular and linear separations of the second order bright fringe from the central maximum. What is the separation between the two bright fringes in the second order lying on either side of the central maximum? **[Ans.** 9.2°, 12 cm on the screen]

[**Hint.** The angular separation θ is given by

$$2d\sin\theta = n\lambda \quad \text{where} \quad 2d = 0.015\times10^{-3}\ \text{m},$$
$$n = 2 \quad \text{and} \quad \lambda = 6000\times10^{-10}\ \text{m}$$

$$\therefore \quad \sin\theta = \frac{n\lambda}{2d} = 0.08 \text{ or } \theta = 4.6^\circ$$

Also, $$x_n = n\frac{\lambda D}{2d}$$

$$\therefore \quad x_2 = \frac{2\times6000\times10^{-10}\times0.75}{0.015\times10^{-3}} = 0.06\ \text{m}$$

The angular separation between the two second order maxima

$$= 2\times4.6^\circ = 9.2^\circ$$

and linear separation $= 2\times0.06$ m]

7. When a thin slit illuminated by monochromatic source of light is placed at a distance of 50 cm from a Fresnel's biprism of refractive index 1.55 the distance between two consecutive bands on a screen placed at distance of 100 cm from the biprism is found to be 0.010 cm. If the wavelength of light is 5460 Å, find the magnitude of the obtuse angle of the biprism.

[**Ans.** 178.3°]

[**Hint.** $2d = 2a(\mu-1)A$ and $B = \lambda D/2d$ or $\lambda = \beta\times 2d/D$

$$\therefore \quad \lambda = \frac{\beta}{D}[2a(\mu-1)A]$$

Here,. $\mu = 1.55$, $D = 0.5+1 = 1.5$ metre, $\beta = 0.010\times10^{-2}$ metre

$\lambda = 5460\times10^{-10}$ m and $a = 0.5$ m

Putting these values, we get $A = 0.01419$ radian

$$\therefore \quad A = 0.01489\times\frac{180^\circ}{\pi} = 0.853^\circ$$

Now, obtuse angle of the prism $= 180^\circ - 2A$]

8. In an experiment with Fresnel's biprism, monochromatic light is used and bands 0.0202 cm in width are observed at a distance of 1 m from the slit. A convex lens is then put between the observer and the prism. The distance apart of the images is found to be 0.70 cm when the lens is at a distance of 35 cm from the slit. Calculate the wavelength of the monochromatic light used. [**Ans.** 7613.2×10^{-10} m]

9. Interference fringes are produced by Fresnel's biprism in the focal plane of a reading microscope which is 120 cm from the slit. A lens interposed between the biprism and the microscope gives two images of the slit in two positions. If the images of the slits are 4.08 mm in one position, 3.00 mm in other position and the wavelength of sodium light is 5896 Å, find the distance beween the consecutive interference bands. [**Ans.** 2.02×10^{-4} m]

10. Fringes are produced by Fresnel's biprism in focal plane of a reading microscope which is 100 cm from the slit. A lens inserted between the biprism and the eyepiece gives two images of the slit in two positions of the lens. In one case the two images of the slit are 4.05 mm apart while in the other case 2.90 mm apart. If sodium light of wavelength 5893 Å is used, find the width of interference fringes.

If the distance between the slit and biprism is 10 cm and refractive index of the material of the biprism is 1.5, calculate the angle in degrees which the inclined faces of the biprism make with its base. [**Ans.** 0.0172 cm, 2°]

[**Hint.** $$2d = \sqrt{(d_1 d_2)} = 0.342\ \text{cm}$$

$$\beta = \frac{\lambda D}{2d} = \frac{(5893\times10^{-8})(100)}{0.324} = 0.0172\ \text{cm}$$

Now, $2d = 2a(\mu-1)$, Here, $a = 10$ cm

$$2d = 0.342 \text{ cm and } \mu = 1.5$$

$$\therefore \quad A = 0.0342 \text{ radian} = 2°]$$

11. In a biprism experiment with sodium light ($\lambda = 5893$ Å), the micrometer reading is 2.32 mm when the eyepiece is placed at a distance of 100 cm from the source. If the distance between two virtual sources is 2 cm, find the new reading of micrometer when the eyepiece is moved such that 20 fringes cross the field of view. **[Ans.** 2.91 or 1.73 mm]

[Hint. $\beta = \dfrac{\lambda D}{2d} = 2.9465 \times 10^{-3}$ cm

Distance moved for 20 fringes = $20\beta = 0.59$ mm

reading of micrometer = 2.32 mm

$\therefore$ Final reading of micrometer = (2.32 ± 0.90)]

12. On introducing a thin sheet of mica (thickness 12×10^{-5} cm) in path of one of the interfering beams in a biprism arrangement, the central fringe is shifted through a distance equal to the spacing between successive bright fringes. Calculate the refractive index of mica ($\lambda = 6 \times 10^{-5}$ cm). **[Ans.** 1.5]

[Hint. $S = \dfrac{D}{2d}(\mu - 1)t, \quad \beta = \dfrac{\lambda D}{2d} \quad \text{or} \quad \dfrac{D}{2d} = \dfrac{\beta}{\lambda}$

$$\therefore \quad S = \frac{\beta}{\lambda}(\mu - 1)t$$

According to given problem $S = \beta$

$$\therefore \quad \beta = \frac{\beta}{\lambda}(\mu - 1)t \text{ or } \lambda = (\mu - 1)t]$$

13. A thin mica sheet ($\mu = 1.6$) of 7 microns thickness introduced in the path of one of the interfering beams in biprism arrangement shifts the central fringe to a position normally occupied by the 7th bright fringe from the centre. Find the wavelength of light used. (1 micron $= 10^{-4}$ cm). **[Ans.** 6000 Å]

[Hint. $S = \dfrac{\beta}{\lambda}(\mu - 1)t.$

Here, $S = 7\beta$ and $t = 7 \times 10^{-4}$ cm

$$\therefore \quad 7\lambda = (\mu - 1)t = (1.6 - 1) \times 7 \times 10^{-4}$$

$$\lambda = 0.6 \times 10^{-4} \text{ cm}]$$

14. When a glass plate of thickness 0.02 mm is placed in the path of one of the interfering beams, 20 fringes are shifted. If wavelength of light used is 6000 Å, find the refractive index of glass. **[Ans.** 1.6]

[Hint. $(\mu - 1)t = n\lambda$ or $(\mu - 1)\,0.0002 = 20 \times 6000 \times 10^{-8}$]

15. In Young's double slit experiment using monochromatic light, the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1.6 and thickness 1.964 micron is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the slit and the screen is doubled. It is found that the distance between successive maxima (or minima) now is the same as the observed fringe shift upon the introduction of mica sheet. Calculate the wavelength of monochromatic light used in the experiment. **[Ans.** 5892 Å]

[**Hint.** $S = (\mu - 1)\, t \,.\, \dfrac{D}{2d}$

The fringe width β is given by $\beta = (\lambda D/2d)$

When distance D is doubled, then $\beta_1 = (\lambda \cdot 2D/2d)$

Now, $$S = (\mu - 1)\, t \,.\, \frac{D}{2d} = \frac{\lambda (2D)}{2d}$$

or $$\lambda = (\mu - 1)\, t/2]$$

16. In an arrangement of double slit arrangement [Fig. (43)] the slits are illuminated by light of wavelength 600 nm. Find the distance of the first point on the screen from the centre maximum where intensity is 75% of central maxima. [**Ans.** 4.8×10^{-5} m]

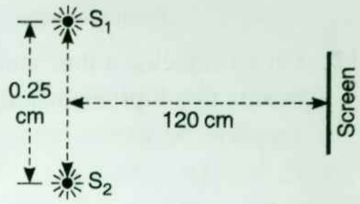


Fig. (43)

[**Hint.** Path difference $= \dfrac{2d}{D} x_n$

$\therefore$ Phase difference $$\delta = \frac{2\pi}{\lambda} \times \left(\frac{2d}{D} x_n\right) \quad \text{...(1)}$$

The intensity at any point I_x is expressed as

$$I_x = I_{\max} \cos^2 (\delta/2)$$

$$\frac{75}{100} I_{\max} = I_{\max} \cos^2 (\delta/2)$$

or $$\cos^2 (\delta/2) = \frac{75}{100} = \frac{3}{4}$$

or $$\cos\left(\frac{\delta}{2}\right) = \frac{\sqrt{3}}{2}$$

or $$\frac{\delta}{2} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = (\pi/6)$$

$\therefore$ $$\delta = (\pi/3) \quad \text{...(2)}$$

From eqs. (1) and (2),

$$\frac{\pi}{3} = \frac{2\pi}{\lambda}\left(\frac{2d}{D}\right) x_n$$

or $$x_n = \frac{\lambda D}{6 \times (2d)}$$

Here, $\lambda = 600 \times 10^{-19}$ m, $D = 120$ cm $= 1.2$ m and $2d = 0.25 \times 10^{-2}$ m]

17. Two coherent point sources S_1 and S_2 vibrating in phase emit light of wavelength λ. The separation between two sources is 2λ as shown in Fig. (44). Consider a line passing through S_2 as shown in the figure and perpendicular to line S_1S_2. What is the smallest distance from S_2 where a minimum of intensity occurs? [**Ans.** $\lambda = 71/2$]

[**Hint:** The smallest distance from S_2 where a minimum intensity occurs should be a point where the path difference between two wave reaching from S_1 and S_2 be $3\lambda/2$, *i.e.*, $(\lambda/2 + \lambda = 3\lambda/2)$.

From Fig. (40), $$S_1 P - S_2 P = 3\lambda/2$$

$\therefore$ $$\sqrt{[D^2 + (4\lambda^2)]} - D = 3\lambda/2$$

or $$D^2 + 4\lambda^2 = \left(D + \frac{3\lambda}{2}\right)^2$$

$$= D^2 + (9\lambda^2/4) + 3\lambda D$$

or $$D = (7\lambda/12)]$$

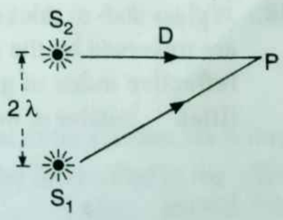


Fig. (44)

18. Consider the situation shown in Fig. (45). This shows analytically that fringes obtained on the screen S will be circular. Find the radius of nth bright fring.

[**Ans.** $r = D\sqrt{2\left(1-\frac{n\lambda}{d}\right)}$]

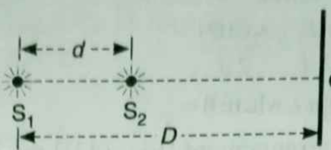


Fig. (45)

[**Hint:** The situation is shown in Fig. (46).

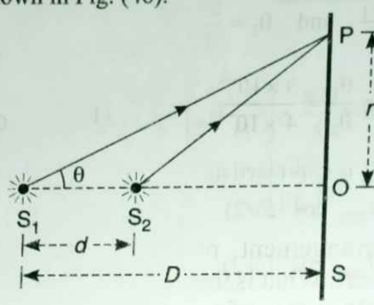


Fig. (46)

The optical path difference at a point P on the screen is given by

$$S_1P - S_2P = d\cos\theta$$

When θ is small, $\cos\theta = [1-(\theta^2/2)]$

$\therefore$ Path difference $= d\,[1-(\theta^2/2)]$

When $d \ll D$, then

$$d\left[1-\left(\frac{\theta^2}{2}\right)\right] = d\left[1-\frac{r^2}{2D^2}\right]$$

As the path difference will be the same for all points having same r and hence the fringes will be circular.

For n^{th} maximum, path difference $= n\lambda$

$$\therefore \quad d\left[1-\frac{r^2}{2D^2}\right] = n\lambda$$

$$\text{or} \quad \left[1-\frac{r^2}{2D^2}\right] = \frac{n\lambda}{d} \quad \text{or} \quad \left[1-\frac{n\lambda}{d}\right] = \frac{r^2}{2D^2}$$

$$\therefore \quad r = D\sqrt{\left[2\left(1-\frac{n\lambda}{d}\right)\right]}$$

19. A glass slab of thickness 8 cm contains the same number of waves as 9 cm of water when both are traversed by the same monochromatic light. If the refractive index of water is 4/3 find the refractive index of glass. [**Ans.** 1.5]

[**Hint.** Number of waves in 8 cm of glass = Number of waves in 9 cm water

$$\therefore \quad \frac{8}{\lambda_g} = \frac{9}{\lambda_w} \quad \text{or} \quad \frac{\lambda_w}{\lambda_g} = \frac{9}{8}$$

Now, $\dfrac{\mu_w}{\mu_g} = \dfrac{v_g}{v_w} = \dfrac{\lambda_g}{\lambda_w}$ or $\mu_g = \mu_w\left(\dfrac{\lambda_w}{\lambda_g}\right)$]

20. When sodium light $\lambda = 589$ nm is used in a double slit experiment, the first order maximum is found at an angle of 3×10^{-4} radian. When the light is replaced by a source of unknown wavelength, the second maximum occurs at 4×10^{-4} radian. Find the unknown wavelength. **[Ans.** 392.7 nm]

[Hint. $\dfrac{2x_n d}{D} = n\lambda$ or $2\theta d = n\lambda$ where $\theta = \dfrac{x_n}{D}$

$\therefore$ $$\theta = \frac{n\lambda}{2d}$$

Here, $\theta_1 = \dfrac{(1)\lambda_1}{2d}$ and $\theta_2 = \dfrac{2\lambda_2}{2d}$

$\therefore$ $$\frac{\lambda_1}{2\lambda_2} = \frac{\theta_1}{\theta_2} = \frac{3\times10^{-4}}{4\times10^{-4}}$$

or $$\lambda_2 = \lambda_1\left(\frac{2}{3}\right)$$]

21. A Young's double-slit arrangement produces interference fringes for sodium light ($\lambda = 5890$ Å) that are 0.20 apart. What is the angular fringe separation if the entire arrangement is immersed in water (refractive index of water = 4/3)? **[Ans.** 0.15°]

[Hint. Angular fringe separation $\theta = \lambda/d$ or $d = \lambda/\theta$ ...(1)

where d = separation between two slits

In water $d = \lambda'/\theta'$ ...(2)

From eqs. (1) and (2), $\lambda/\theta = \lambda'/\theta'$ or $\theta' = (\lambda'/\lambda)\,\theta$ ...(3)

If c and c' be the velocity of light in air and water respectively, then

$$\frac{c}{c'} = \frac{\lambda}{\lambda'} = n \text{ (refractive index of water)}$$

$\therefore$ $$\frac{\lambda'}{\lambda} = \frac{1}{n} = \frac{3}{4} \quad \text{...(4)}$$

Putting the value of (λ'/λ) from eq. (4) in equation (3), we get

$$\theta' = \frac{3}{4}\theta$$]

22. If a slit illuminated by sodium light (5893 Å) is placed 0.15 cm fiom the plane of the Lloyd's mirror, what will be the distance between consecutive bands formed on a screen 1.0 metre from the slit. **[Ans.** 0.02 cm]

[Hint. $\beta = \lambda D/2d$

Here, $\lambda = 5893\times10^{-8}$ cm, $D = 100$ cm and $2d = 2\times0.15 = 0.30$ cm]

23. Find the thickness of a soap film ($\mu = 1.33$) which gives constructive second order interference of reflected red light of $\lambda = 700$ mμ (millimicron). Assume normal incidence. Given that 1 mμ $= 10^{-9}$ metre. **[Ans.** 3.95×10^{-7} metre]

[Hint. $2\mu t = (3/2)\lambda$ or $t = 3\lambda/4\mu$

where $\lambda = 700$ m μ $= 700\times10^{-9}$ metre and $\mu = 1.33$]

24. A thin film 4×10^{-5} cm thick is illuminated by white light normal to its surface. Its refractive index is 1.5. What wavelengths within the visible spectrum will be intensified in the reflected beam? **[Ans.** 4800 Å (blue)]

[**Hint.** $2\mu\, t \cos r = (2n+1)\lambda/2$
where $\mu = 1.5, t = 4\times10^{-5}$ cm, $r = 0°$ and $n = 0, 1, 2, 3\ldots$

Taking $n = 0, 1, 2, 3\ldots$ we get
$\lambda = 24000$ Å, 8000 Å, 4800 Å, 3431 Å……

4800 Å lies in visible region]

25. A soap film 5×10^{-5} cm thick is viewed at an angle of 35° to the normal. Find the wavelengths of light in the visible spectrum which will be absent from the reflected light ($\mu = 1.33$).
[Ans. 6000 Å and 4000 Å]

[**Hint.** $2\mu\, t \cos r = n\lambda$

Here, $\mu = \dfrac{\sin i}{\sin r}$ or $1.33 = \dfrac{\sin 35°}{\sin r}$ or $r = 25° 33'$

Putting $n = 1, 2, 3, 4\ldots$ we get
$\lambda_1 = 12000$ Å, $\lambda_2 = 6000$ Å, $\lambda_3 = 4000$ Å $\lambda_4 = 3000$ Å…]

26. A parallel beam of sodium light of wavelength 5890×10^{-8} cm is incident at an angle of 30° on a film of olive oil ($\mu = 1.6$) on water. Calculate the smallest thickness of the film which will make it appear dark.
[Ans. 1938×10^{-8} cm]

[**Hint.** $2\mu\, t \cos r = n\lambda$

Here, $\mu = 1.6, n = 1, \lambda = 5890\times10^{-8}$ cm and $i = 30°$

Now, $\mu = \dfrac{\sin i}{\sin r}$ or $1.6 = \dfrac{\sin 30°}{\sin r}$ or $r = 18°15'$]

27. Two plane surfaces of glass in contact along one edge are separated at the opposite edge by a thin wire. If 200 fringes are formed between these edges when normally reflected sodium light is used, what is the thickness of wire ? ($\lambda = 5893\times10^{-8}$ cm)?
[Ans. 0.005893 cm]

28. A broad source of light ($\lambda = 5890$ Å) illuminates normally two glass plates 10 cm long which touch at one end and are separated by a wire 0.05 mm in diameter at the other end. How many bright fringes appear over the 10 cm distance?
[Ans. 170]

29. In an experiment on Newton's rings, the diameter of the tenth dark ring formed by yellow sodium light (589 nm) and seen in reflection is 3 mm. What is the radius of curvature of the lens surface?
[Ans. 0.382 m]

[**Hint.** $(D_n)^2 = 4\, n\lambda R$ or $R = \dfrac{D_n^{\ 2}}{4\, n\lambda}$

Given that $D_n = 3\times10^{-3}$ m for $n = 10$ and $\lambda = 5.89\times10^{-7}$ m]

30. Newton's rings are formed by reflection in the air-film between a plane glass surface and a spherical surface of radius 50 cm, and it is noticed that the centre of the system is bright. What do you conclude from a bright centre ? If the diameter of the 3rd bright ring is 0.181 cm and that of the 23rd bright ring is 0.501 cm calculate the wavelength of the light used.
[Ans. 5456 Å]

[**Hint.** Bright centre indicates that the lens and glass surfaces in contact are not perfectly clear, *i.e.*, there is some dust particle at the centre.

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

where $D_{n+p} = D_{23} = 0.501$ cm, $D_n = D_3 = 0.181$ cm
$\therefore\ p = 20$ and $R = 50$ cm]

31. If in a Newton's ring experiment, the air in the interspace is replaced by a liquid of refractive index 1.33, in what proportion would be diameters of the ring change?
[Ans. The rings are contracted to 0.867 of their previous diameter]

[**Hint.** $\mu = \dfrac{(D_n)^2_{\text{air}}}{(D_n)^2_{\text{liquid}}}$ or $\dfrac{D_{\text{liquid}}}{D_{\text{air}}} = \dfrac{1}{\sqrt{\mu}} = \dfrac{1}{\sqrt{1.33}}$]

32. The diameter of the tenth bright ring in a Newton's rings apparatus changes from 1.5 cm to 1.3 cm when a liquid is introduced between the lens and the plate. Find the refractive index of the liquid. [**Ans.** 1.33]

33. The diameter of 20th dark ring in a Newton's rings system viewed normally is 0.6 cm. Calculate the thickness of the corresponding air film. The wavelength of light used is 6×10^{-8} cm. [**Ans.** 0.0006 cm]

[**Hint.** $(D_n)^2 = 4\,n\,R\,\lambda$ or $R = \dfrac{D_n^{\ 2}}{4\,n\,\lambda}$

The thickness t of the air film at the n^{th} dark ring is given by

$$2t = \frac{D_n}{4R} = \frac{(D_n)^2 \times 4\,n\,\lambda}{4 \times (D_n)^2} = n\,\lambda$$

$$t = \frac{n\,\lambda}{2} = \frac{1}{2} \times 20 \times 6 \times 10^{-5}]$$

34. In an experiment with Michelson interferometer it is found that 40 rings to merge the centre, the mirror had to be moved through 0.01 mm. Calculate the wavelength of the light used. [**Ans.** 5×10^{-5} cm]

[**Hint.** $\lambda = \dfrac{2x}{N} = \dfrac{2 \times 0.001}{40}$]

35. In Michelson's interferometer the reading for a pair of maximum indistinctness were found to be 0.7873 mm and 1.0818 mm. If the mean wavelength of the two components of light be 5893 Å, deduce the difference between the wavelengths of the components. [**Ans.** 5.896 Å]

[**Hint.** $\Delta\lambda = \dfrac{\lambda_{av}^2}{2x} = \dfrac{(5893 \times 10^{-8})^2}{2\,(0.10818 - 0.07873)}$]

36. A thin film with refractive index $\mu = 1.58$ for light wavelength $\lambda = 5890$ Å is placed in one arm of a Michelson interferometer. If there is a shift of 20 fringes, calculate the thickness t of the film. [**Ans.** 0.001016 cm]

[**Hint.** $2(\mu - 1)t = N\,\lambda$ or $t = \dfrac{N\,\lambda}{2\,(\mu - 1)}$

$\mu = 1.58$, $N = 20$ and $\lambda = 5890 \times 10^{-8}$ cm]

37. In an experiment for determining the refractive index of air with Michelson's interferometer, a shift of 150 fringes is observed when all the air was removed from the tube. Fringes were obtained with light of wavelength 4000 Å. If the length of the the tube is 20 cm find the refractive index of air. [**Ans.** 1.00015]

[**Hint.** $2(\mu_{\text{air}} - 1)l = N\,\lambda$

where $N = 150$, $\lambda = 4000 \times 10^{-8}$ cm and $l = 10$ cm]

OBJECTIVE TYPE QUESTIONS

1. Two waves having the intensities in the ratio of 9 : 1 produce interference. The ratio of maximum to minimum intensity is equal to

(a) 10 : 8 (b) 9 : 1 (c) 4 : 1 (d) 2 : 1

2. In Young's double slit experiment, the two slits act as coherent sources of equal amplitude a and of wavelength λ. In other experiment with the same set up, the two slits are sources of equal amplitude a and wavelength λ but are incoherent. The ratio of the intensity of light at the mid-point of the screen in the first case to that in the second case is

(a) 2 : 1 (b) 1 : 2 (c) 3 : 4 (d) 4 : 3

3. Two coherent monochromatic light beams of intensities I and $4I$ are superposed. The maximum and minimum possible intensities in the resulting beam are
(a) $5I$ and I (b) $5I$ and $3I$ (c) $9I$ and I (d) $9I$ and $3I$

4. The intensity ratio of the two interfering beams of light is β. What is the value of $[(I_{max} - I_{min})/(I_{max} + I_{min})]$?
(a) $2\sqrt{\beta}$ (b) $2\sqrt{\beta}/(1+\beta)$ (c) $2/(1+\beta)$ (d) $(1+\beta)/2\sqrt{\beta}$

5. The intensity of light coming from one of the slits in Young's double slit experiment is double the intensity from the other slit. The ratio of maximum intensity to minimum intensity in the interference pattern will be
(a) 14 (b) 34 (c) 24 (d) 44

6. In a Young's double slit interference experiment, the fringe pattern is observed on a screen placed at a distance D. The slits are separated by d and are illuminated by light of wavelength λ. The distance from the central point where the intensity falls to half the maximum is
(a) $\dfrac{\lambda D}{3d}$ (b) $\dfrac{\lambda D}{2d}$ (c) $\dfrac{\lambda D}{d}$ (d) $\dfrac{\lambda D}{4d}$

7. In Young's double slit experiment, the intensity at a point is (1/4) of the maximum intensity. The angular position of this point is
(a) $\sin^{-1}\left(\dfrac{\lambda}{d}\right)$ (b) $\sin^{-1}\left(\dfrac{\lambda}{2d}\right)$ (c) $\sin^{-1}\left(\dfrac{\lambda}{3d}\right)$ (d) $\sin^{-1}\left(\dfrac{\lambda}{4d}\right)$

8. Four independent waves are expressed as $y_1 = a_1 \sin \omega t$, $y_2 = a_2 \sin 2\omega t$, $y_3 = a_3 \cos \omega t$ and $y_4 = a_4 \sin(\omega t + \pi/3)$. The interference is possible between
(a) (i) and (iii) (b) (i) and (iv)
(c) (iii) and (iv) (d) not possible at all

9. Two sources of light are said to be coherent if waves produced by them have the same
(a) wavelength
(b) amplitude
(c) wavelength and constant phase difference
(d) amplitude and same wavelength

10. In Young double slit experiment, an electron beam is used to obtain interference pattern. The slit width is d. If the speed of electron is increased, then
(a) no interference pattern is observed (b) fringe width increases
(c) fringe width decreases (d) fringe width remains same

11. In an experiment similar to Young's experiment, interference is observed using waves associated with electrons. The electrons are being produced in an electron gun. In order to increase the fringe width
(a) electron gun voltage be increased
(b) electron gun voltage be decreased
(c) the slits be moved away
(d) the screen be moved closer to interfering slits

12. A Young's double slit experiment uses a monochromatic source. The shape of the interference fringes formed on screen will be
(a) straight line (b) parabola (c) hyperbola (d) circle

13. In a double slit interference experiment the distance between the slits is 0.05 cm and screen is 2 m away from the slits. The wavelength of light is 6.0×10^{-5} cm. The distance between the fringes is

(a) 0.24 cm (b) 2.21 cm (c) 1.28 cm (d) 0.12 cm

14. In Young's double slit interference experiment if the slit is made 3 folds the fringe width becomes

(a) 1/3 fold (b) 3 fold (c) 3/6 fold (d) 6 fold

15. Two slits separated by a distance of 1 mm are illuminated with red light of wavelength 6.5×10^{-7} m. The interference fringes are observed on a screen placed 1 m from the slits. The distance between third dark fringe and the fifth bright fringe is equal to

(a) 0.65 mm (b) 1.63 mm (c) 3.25 mm (d) 4.88 mm

16. In Young's double slit experiment, the separation between the slits is halved and the distance between the slits and screen is doubled. The finge width is

(a) unchanged (b) halved (c) double (d) quadrupled

17. In Young's experiment the wavelength of red light is 7.8×10^{-5}cm and that of blue light is 5.2×10^{-5}cm. The value of n for which $(n+1)^{th}$ blue bright band coincides with n^{th} red band is

(a) 4 (b) 3 (c) 2 (d) 1

18. We shift Young's double slit experiment from air to water. Assuming that water is still and clear, it can be predicted that the fringe pattern will

(a) remain unchanged (b) disappear (c) shrink (d) be enlarged

19. In Young's double slit experiment, the fringe width is β. If the entire arrangement is now placed in a liquid of refractive index μ, the fringe width becomes

(a) $\mu\beta$ (b) $\frac{\beta}{\mu}$ (c) $\frac{\beta}{(\mu+1)}$ (d) $\frac{\beta}{(\mu-1)}$

20. In Young's double slit arrangement, the 7th maximum with wavelength λ_1 is at a distance d_1 and that with wavelength λ_2 is at a distance d_2. Then d_1/d_2 is

(a) $\frac{\lambda_1^2}{\lambda_2^2}$ (b) $\frac{\lambda_2^2}{\lambda_1^2}$ (c) $\frac{\lambda_1}{\lambda_2}$ (d) $\frac{\lambda_2}{\lambda_1}$

21. In the Young's experiment with sodium light, the slits are 0.589 m apart. What is the angular width of the fourth maximum? Given that $\lambda = 589$ mm.

(a) $\sin^{-1}(3 \times 10^{-6})$ (b) $\sin^{-1}(3 \times 10^{-8})$

(c) $\sin^{-1}(0.33 \times 10^{-6})$ (d) $\sin^{-1}(0.33 \times 10^{-8})$

22. In a double slit experiment, the distance between the slits is d. The screen is at a distance D from slits. If a bright fringe is formed opposite to a slit on the screen, the order of the fringe is

(a) $\frac{d^2}{2\lambda D}$ (b) $\frac{2\lambda D}{d^2}$ (c) $\frac{d}{\lambda D}$ (d) $\frac{2d}{\lambda D}$

23. A beam of light consisting of two wavelengths 650 mm and 520 nm is used to obtain interference fringes in Young's double slit experiment. The distance between slits is 2 mm and between the plane of slits and screen is 120 cm. The least distance from the central maximum where the bright fringes due to both wavelengths coincide is

(a) 1.17 mm (b) 3.34 mm (c) 3.12 mm (d) 156 mm

24. In Young's double slit experiment, the 8th maximum with wavelength λ_1 is at a distance d_1 from central maximum and 6th maximum with wavelength λ_2 is at a distance d_2. Then d_1/d_2 is

(a) $\frac{4}{3}\left(\frac{\lambda_2}{\lambda_1}\right)$ (b) $\frac{4}{3}\left(\frac{\lambda_1}{\lambda_2}\right)$ (c) $\frac{3}{4}\left(\frac{\lambda_2}{\lambda_1}\right)$ (d) $\frac{3}{4}\left(\frac{\lambda_1}{\lambda_2}\right)$

25. Two slits separated by a distance of 1 mm are illuminated with light of wavelength 6.0×10^{-7} m. The interference fringes are observed on a screen placed 1 m from the slits. The distance between third dark fringe and the fifth bright fringe is equal to

(a) 0.60 mm (b) 1.50 mm (c) 3.00 mm (d) 4.50 mm

26. In Young's double slit experiment, the slits are 0.5 mm apart and interference is observed on a screen placed at a distance of 100 cm from the slits. It is found that 9^{th} bright fringe is at a distance of 9.0 mm from the second dark fringe from the centre of the fringe pattern. The wavelength of light used as

(a) 3000 Å (b) 6000 Å (c) 9000 Å (d) 12000 Å

27. In the given arrangement, S_1 and S_2 are coherent sources [Fig. (47)]. The point P is point of

(a) bright fringe (b) dark fringe

(c) either dark or bright (d) none of these

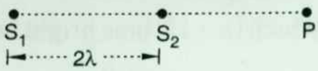


Fig. (47)

28. Two coherent sources S_1 and S_2 are situated on the X-axis. The screen is in Y-Z plane as shown in Fig. (48). The shape of the fringe on screen is

(a) straight line (b) elliptical

(c) circular (d) rectangular

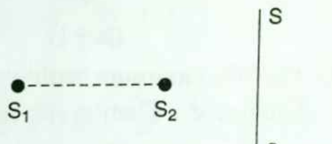


Fig. (48)

29. Ray optics is valid when characteristic dimensions are

(a) much smaller than wavelength of light (b) much larger than wavelength of light

(c) of the same order as wavelength of light (d) of the order of 1 millimeter

30. In Young's double slit experiment, the length of band is 1 mm. The fringe width is 0.021 mm. The number of fringes is

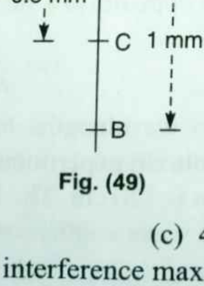


Fig. (49)

(a) 45 (b) 46 (c) 47 (d) 48

31. The maximum number of possible interference maxima for slit separation equal to twice the wavelength in Young's double slit experiment is

(a) infinite (b) 5 (c) 3 (d) zero

32. In Fig. (50), CP represents a wavefront and AO and BP are corresponding two parallel rays. AO strikes a plane minor QR at an angle θ. The wavelength of light used is λ. The condition on θ for constructive interference at P between the ray BP and reflected ray OP will be

(a) $\cos\theta = \dfrac{\lambda}{4d}$ (b) $\cos\theta = \dfrac{\lambda}{2d}$ (c) $\sin\theta = \dfrac{\lambda}{d}$ (d) $\sin\theta = \dfrac{\lambda}{2d}$

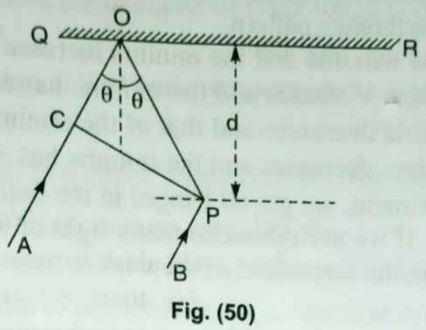


Fig. (50)

33. In Young's double slit experiment monochromatic light is used to illuminate the two slits A and B. Interference fringes are observed on a screen placed in front of the slits. Now, if a thin glass plate is normally placed in the path of the beam as shown in Fig. (51), then

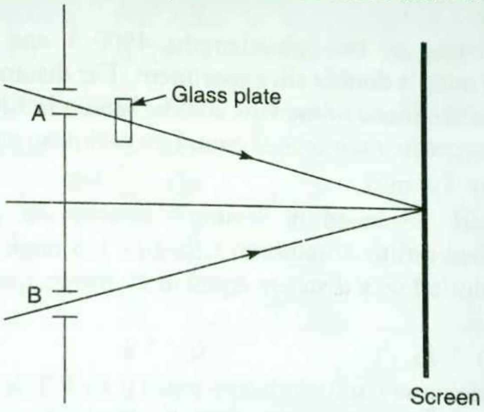


Fig. (51)

(a) the fringe will disappear
(b) the fringe width will increase
(c) the fringe width will decrease
(d) there will be no change in fringe width

34. In the ideal double slit experiment, when a glass-plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ) the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass-plate is

(a) 2λ (b) $2\lambda/3$ (c) $\lambda/3$ (d) λ

35. A double slit apparatus is immersed in a liquid of refractive index 1.33. It has slit separation of 1 mm and distance between the plane of slits and screen is 1.33 m. The slits are illuminated by a parallel beam of light whose wavelength in air is 6300 Å. What is the fringe width?

(a) (1.33×0.63) mm (b) $\frac{0.63}{1.33}$ mm

(c) $\frac{0.63}{(1.33)^2}$ (d) 0.63 mm

36. In a double slit experiment instead of taking slits of equal widths, one slit made twice as wide as the other. Then, in the interference pattern

(a) the intensities of both the maxima and the minima increase

(b) the intensity of the maxima increases and the minima has zero intensity

(c) the intensity of the maxima decreases and that of the minima increases

(d) the intensity of the maxima decreases and the minima has zero intensity

37. In Young's double slit experiment, we get 60 fringes in the field of view of monochromatic light of wavelength 4000 Å. If we use monochromatic light of wavelength 6000 Å, then the number of fringes obtained in the same field of view is

(a) 60 (b) 90 (c) 40 (d) 1.5

38. In an ideal double slit experiment, when a glass plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass-plate is

(a) 2λ (b) $2\lambda/3$ (c) $\lambda/3$ (d) λ

39. A beam of light consisting of two wavelengths 4500 Å and 7500 Å is used to obtain interference fringes in Young's double slit experiment. The distance between the slits is 1 mm and the distance between the plane of the slits and the screen is 120 cm. What is the minimum distance between two successive regions of complete darkness on the screen?

(a) 4.5 mm (b) 5.4 mm (c) 2.7 mm (d) 1.2 mm

40. Interference fringes were produced in Young's double slit experiment using light of wavelength 5000 Å. When a film of material 2.5×10^{-3} cm thick was placed over one of the slits, the fringe pattern shifted by a distance equal to 20 fringe widths. The refractive index of the material of the film is

(a) 1.25 (b) 1.33 (c) 1.4 (d) 1.5

41. If a thin mica sheet of thickness t and refractive index μ $(= 5/3)$ is placed in the path of one of the interfering beams as shown in Fig. (52), then the displacement of

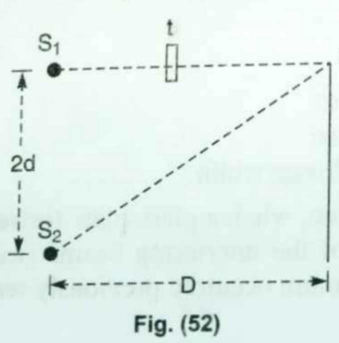


Fig. (52)

(a) $\frac{Dt}{3d}$ (b) $\frac{Dt}{5d}$ (c) $\frac{Dt}{4d}$ (d) $\frac{2Dt}{5d}$

42. Thin firms of oil and soapy water owe their brilliant colours due to

(a) interference (b) diffraction (c) polarization (d) none of these

43. Newton's rings are
(a) locii of points of equal thickness
(b) locii of points of equal inclination
(c) locii of points of equal thickness and equal inclination
(d) none of these

44. In Newton's ring arrangement the diameter of rings found is proportional to
(a) λ (b) λ^2 (c) $\sqrt{\lambda}$ (d) $1/\sqrt{\lambda}$

45. The diameters of bright rings in Newton's ring arrangement are proportional to
(a) n (b) n^2 (c) $(2n+1)$ (d) $\sqrt{(2n-1)}$

46. The diameters of dark rings in Newton's ring arrangement are proportional to
(a) n (b) $\sqrt{n}$ (c) n^2 (d) $(2n+1)$

47. In Newton's rings arrangement with air film in reflected light the diameter of n^{th} ring is D_n. If air is replaced by liquid film of refractive index μ, the diameter of n^{th} ring will become
(a) $\sqrt{\mu}$ times (b) $\frac{1}{\sqrt{\mu}}$ times (c) $\frac{1}{\mu}$ times (d) μ times

ANSWERS

1. (c)	**2.** (a)	**3.** (c)	**4.** (b)	**5.** (b)	**6.** (d)	**7.** (c)	**8.** (d)
9. (c)	**10.** (c)	**11.** (b)	**12.** (c)	**13.** (a)	**14.** (a)	**15.** (b)	**16.** (d)
17. (c)	**18.** (c)	**19.** (b)	**20.** (c)	**21.** (a)	**22.** (a)	**23.** (d)	**24.** (b)
25. (b)	**26.** (b)	**27.** (a)	**28.** (c)	**29.** (b)	**30.** (c)	**31.** (b)	**32.** (a)
33. (d)	**34.** (a)	**35.** (d)	**36.** (a)	**37.** (c)	**38.** (a)	**39.** (c)	**40.** (c)
41. (a)	**42.** (a)	**43.** (a)	**44.** (c)	**45.** (d)	**46.** (b)	**47.** (b)	

9. (a) Discuss the Fraunhofer diffraction at a double slit and deduce intensity distribution.
   (b) Explain the effect of slit width separation on the diffraction pattern.
10. Describe the feature of a double slit Fraunhofer's diffraction pattern. What are missing order?
11. Discuss Fraunhofer diffraction at a double slit. What is the effect of increasing the (i) slit-width (ii) slit separation and (iii) wavelength.
12. Describe Fraunhofer diffraction due to a double slit with necessary theory and discuss the intensity distribution. Why missing orders occur in this?
13. Give the construction and theory of a plane diffraction grating of the transmission type and explain the formation of spectra.
14. Give the theory of diffraction grating. How do you use it in the normal incidence method to measure the wavelengths of spectral lines in the mercury spectrum?
15. Explain the Fraunhofer type of diffraction at a grating and find out the ratio of intensities of principal and secondary maxima in the grating spectrum.
16. Distinguish between Fresnel and Fraunhofer diffraction. Discuss the Fraunhofer diffraction pattern due to N-slit and obtain intensity distribution, position of the maxima and minima and the width of the principal maximum.
17. Give the construction and theory of plane transmission grating and explain the formation of spectra by it. Explain what are absent spectra in the grating.
18. Show that the angular width of a principal maximum in a plane transmission grating does not depend upon the number of lines per unit length, but it does depend on the total number of lines present on the grating.
19. Explain the formation of spectra by plane diffraction grating. What are its chief characteristics?
20. What do you understand by missing orders spectrum? What particular spectra would be absent if the width of transparencies and opacities of the grating are equal? Show that only first order spectra is possible if the width of grating element is more than wavelength of light and less than twice the wavelength of light.
21. What is dispersive power of a plane transmission grating? Derive the expression for it.
22. Describe with necessary theory, how the wavelength of monochromatic light is determined using a plane diffraction grating. What is the expression for its resolving power ?
23. What is Rayleigh's criterion of the resolving power of optical instruments ? Deduce an expression for the resolving power of a plane transmission grating placed perpendicular to the path of the rays.
24. Explain Rayleigh's criterion of resolution. Define limit of resolution and resolving power.
25. What is meant by resolving power of an optical instrument? Explain Rayleigh's criterion for just resolution.
26. Define resolving power and dispersive power of grating. Derive an expression for the resolving power of a plane transmission grating.

PROBLEMS

1. In a single slit diffraction pattern seen on a screen 1 metre from the slit, the distance between the first minimum and the central maximum is 2.8 mm. The wavelength of light used is $\lambda = 546.1$ nm. Calculate the slit width. **[Ans.** 1.95×10^{-5} m]

   [Hint. Angular separation between central maximum and first minimum = λ/a.

   $\therefore$ Linear separation on screen at distance $D = (\lambda/a) \times D$

Here, $\frac{\lambda}{a} \times D = 2.8 \times 10^{-3}$ metre or $a = \frac{\lambda D}{2.8 \times 10^{-3}}$

$\lambda = 5.461 \times 10^{-7}$ m and $D = 1$ m]

2. A screen is placed 2 metre away from a narrow slit. Find the slit width if the first minima lie 5 mm on either side of the central maximum when plane waves of $\lambda = 5 \times 10^{-5}$ cm are incident on the slit. **[Ans.** 0.02 cm]

[Hint. Here, $e \sin\theta = \lambda$. If θ is small $\sin\theta = \theta$

$$\therefore \quad \theta = \frac{\lambda}{e} = \frac{5 \times 10^{-5}}{e} \qquad ...(1)$$

Further, $$\theta = \frac{0.5}{200} \text{ radian} \qquad ...(2)$$

$$\therefore \quad \frac{5 \times 10^{-5}}{e} = \frac{0.5}{200}\Bigg]$$

3. In a double slit Fraunhofer diffraction pattern, the screen is 160 cm away from the slits. The slit width are 0.08 mm and they are 0.4 mm apart. Calculate the wavelength of light if the fringe spacing is 0.25 cm. Also, find the missing orders.

[Ans. 6250×10^{-8} cm, 6th, 12th, 18th, order will be missing]

[Hint. The fringe spacing $\beta = \frac{\lambda D}{2d}$ or $\lambda = \frac{\beta \times 2d}{D}$

Here, $\beta = 0.25$, $2d = 0.04$, $2d = 0.04$ cm and $D = 160$ cm.

For missing order $\frac{e+d}{e} = \frac{n}{m}$

Here, $d = 0.04$ cm and $e = 0.008$ cm

$\therefore$ $n = 6$ m where $m = 1, 2, 3......$]

4. Light is incident normally on a grating with 250 lines per mm. A second order spectral line makes an angle of 18° with the central zero-order image. Calculate wavelength of spectral line. **[Ans.** 6.18×10^{-7} m]

[Hint. $(e+d)\sin\theta = n\lambda$, where $(e+d) = 10^{-3}$ m/250 $= 4 \times 10^{-6}$ m, $n = 2, \theta = 18°$]

5. Find the wavelength of monochromatic light falling normally on a diffraction grating element 2.2 μm. If the angle between the directions of Fraunhofer maxima of the first and second order is equal to 15°. **[Ans.** 5346×10^{-10} m]

[Hint. $(e+d)\sin\theta_1 = \lambda$ and $(e+d)\sin\theta_2 = 2\lambda$

$$\frac{\sin\theta_2}{\sin\theta_1} = 2$$

Adding and subtracting one on both sides and then dividing, we have

$$\frac{\sin\theta_2 + \sin\theta_1}{\sin\theta_2 - \sin\theta_1} = 3 \quad \text{or} \quad \frac{\sin\left(\frac{\theta_1+\theta_2}{2}\right)\cos\left(\frac{\theta_2-\theta_1}{2}\right)}{\cos\left(\frac{\theta_1+\theta_2}{2}\right)\sin\left(\frac{\theta_1-\theta_2}{2}\right)} = 3$$

$$\tan\frac{\theta_1+\theta_2}{2} = 3\tan\left(\frac{\theta_2-\theta_1}{2}\right) = 3\tan 7.5$$

$$= 3 \times 0.1317 = 0.3951$$

or $$\frac{\theta_1+\theta_2}{2} = 21°\,34' \quad \text{or} \quad \theta_1 + \theta_2 = 43°\,8' \qquad ...(1)$$

$$\theta_2 - \theta_1 = 15° \quad \therefore \quad \theta_1 = 14°\,4'$$

Now, $\lambda = (2.2 \times 10^{-6}) \sin 14°\,4'$]

6. Light of wavelength 530 nm falls on a transparent grating with period 1.5 μm. Find the angle relative to the grating normal, at which Fraunhofer maximum of highest order is observed provided the light falls on the grating (a) at right angles and (b) at an angle 60° to normal.

[**Ans.** (a) $\theta = 44°58'$, (b) $64°16'$]

[**Hint.** (a) $(1.5 \times 10^{-6}) \sin \theta = n\,(530 \times 10^{-9})$

Here, n cannot exceed 2.

Hence, $$\sin \theta = \frac{2 \times (530 \times 10^{-9})}{1.5 \times 10^{-6}}$$

(b) $(e + d)(\sin \theta + \sin i) = n\,\lambda$

$$(\sin \theta + \sin 60) = \frac{n\,(530 + 10^{-9})}{1.5 \times 10^{-6}}$$

Hence, n cannot exceed 5.

$$\therefore \quad \sin \theta + 0.866 = \frac{5 \times (530 \times 10^{-9})}{1.5 \times 10^{-6}}\Bigg]$$

7. A grating has slits that are each 0.1 mm wide. The distance between the centres of any two adjacent slits is 0.3 m. Which of the higher order maxima are missing ?

[**Ans.** $3^{\text{th}}, 6^{\text{th}}, 9^{\text{th}}, \ldots\ldots$]

[**Hint.** $(e + d)(\sin \theta + \sin i) = n\,\lambda$

and $e\,(\sin \theta + \sin i) = m\,\lambda$

$$\frac{e + d}{e} = \frac{n}{m}$$

or $$\frac{0.3 \text{ mm}}{0.1 \text{ mm}} = \frac{n}{m}$$

or $n = 3\,m$]

8. A plane transmission grating produces an angular separation of 0.01 radian between two wavelengths observed at an angle of 30°. If the mean value of the wavelength is 5000 Å and the spectrum is observed in second order, calculate the difference in the two wavelengths.

[**Ans.** 86.6 Å]

[**Hint.** $$d\theta = \frac{n\,d\lambda}{(e + d)\cos \theta} \quad \text{or} \quad d\theta = \frac{n\,d\lambda}{(n\,\lambda/\sin \theta)\cos \theta}$$

or $$d\theta = \frac{d\lambda}{\lambda \cot \theta}$$

or $d\lambda = (\lambda \cot \theta)\,d\theta$

$\lambda = 5000 \times 10^{-8}$ cm, $\theta = 30°$ and $d\theta = 0.01$ radian]

9. Two lines in the fourth order spectra formed by a plane transmission grating are resolved. If the lines are due to wavelengths 5890 and 5894 Å, find the number of lines on the grating.

[**Ans.** 368]

[**Hint.** $$N = \frac{1}{n}\frac{\lambda}{d\lambda} = \frac{1}{4} \times \frac{5892}{4}\Bigg]$$

10. Calculate the least width that a grating must have to resolve the components of D lines in the second order, the grating have 800 lines per cm. The components are 5896 and 5890 Å.

[**Ans.** 0.614 cm]

[**Hint.** $N = \frac{1}{n}\left(\frac{\lambda}{d\lambda}\right)$ and $n = 800$ lines per cm. width of the grating $= \frac{5893}{12 \times 800}$]

11. What should be the maximum number of lines in a grating which will fully resolve in the second order the lines whose wavelegths are 5890 and 5896 Å? [**Ans.** 492]

OBJECTIVE TYPE QUESTIONS

1. The main difference in the phenomenon of interference and diffraction is that
   (a) diffraction is due to interaction of light from the same wavefront whereas interference is the interaction of waves from two isolated sources
   (b) diffraction is due to interaction of light from wavefront, whereas the interference is the interaction of two waves derived from the same source
   (c) diffraction is due to interaction of waves derived from the same source, whereas the interference is the bending of light from the same wavefront
   (d) diffraction is caused by reflected waves from a source whereas interference caused is due to refraction of waves from a surface
2. To observe diffraction, the size of an obstacle
   (a) should be of the same order as wavelength
   (b) should be much larger than the wavelength
   (c) has no relation to wavelength
   (d) should be exactly $\lambda/2$
3. The condition for obtaining Fraunhofer diffraction from a single slit is that the light wavefrom incident on the slit should be
   (a) spherical (b) cylindrical (c) elliptical (d) plane
4. Diffraction pattern of a single slit consists of a central band which is
   (a) wider, brighter and accompanied with alternate dark and bright bands of decreasing intensity
   (b) narrow, bright and alternate dark and bright bands of equal intensity
   (c) wider, bright and alternate dark and bright bands of equal intensity
   (d) dark with alternate bright and dark bands of decreasing intensity
5. A diffraction pattern is obtained using a beam of red light. What happens if the red light is replaced by blue light
   (a) no change
   (b) diffraction bands become narrower and crowded together
   (c) bands become broader and farther apart
   (d) bands disappear
6. A parallel beam of fast moving electrons is incident normally on a narrow slit. A screen is placed at a large distance from the slit. If the speed of the electrons is increased, which of the following statements is correct?
   (a) diffraction pattern is not observed on the screen in the case of electron
   (b) the angular width of the central maximum of the diffraction pattern will increase
   (c) the angular width of the central maximum will decrease
   (d) the angular width of the central maximum will remain the same
7. A parallel beam of monochromatic light is incident normally on a slit. The diffraction pattern is observed on a screen placed at focal plane of convex lens. If the slit width is increased, the central maximum of the diffraction pattern will
   (a) become broader and fainter (b) become broader and brighter
   (c) become narrower and fainter (d) become narrower and brighter

   [**Hint.** $\beta_0 = \dfrac{2\lambda D}{d} = \dfrac{2\lambda f}{d}$ or $\beta_0 \propto \dfrac{1}{d}$ As d increases, β_0 decreases]
8. Angular width of central maximum of a diffraction pattern of a single slit does not depend upon
   (a) distance between slit and source (b) wavelength of light used

(c) width of the slit (d) frequency of light used

[**Hint.** Angular width, $\beta = \left(\frac{\lambda}{e}\right) = \left(\frac{c}{v\,e}\right)$

where, e = width of the slit, c = velocity of light and v = frequency.

Hence, angular width does not depend upon distance between slit and source]

9. The number of ruling (N) in grating is made larger then
(a) the principal and secondary (all) maxima will become sharp and intense
(b) the principal and secondary (all) maxima will become finite and wide
(c) the principal maxima will become intense and sharp while secondary maxima become weaker
(d) the principal maxima become weaker while secondary maxima become sharp and intense

10. When a monochromatic beam of light is passed through a plane transmission grating at normal incidence, the position of direct image is obtained at D as shown in Fig. (20). The diffracted image A and B correspond to first and second order diffraction. What shall happen if the source is replaced by a source of longer wavelength?
(a) A and B shall shift away from D (b) A and B shall shift towards D
(c) all the three shall shift to the right (d) all the three shall shift to the left

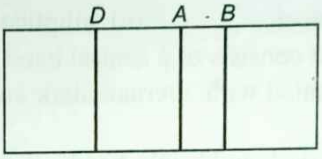


Fig. (20)

11. If N is the total number of rulings on the grating, n is the order of spectrum and λ is wavelength of light used, then resolving power of grating is given by
(a) $N\,n\lambda$ (b) $N\,n$ (c) $\frac{N\lambda}{n}$ (d) N/n

12. The Figure (21) shows Fraunhofer's diffraction due to a single slit. If first minimum is obtained in the direction shown, then the path difference between rays 1 and 3 is
(a) 0 (b) $\lambda/4$ (c) $\lambda/2$ (d) λ

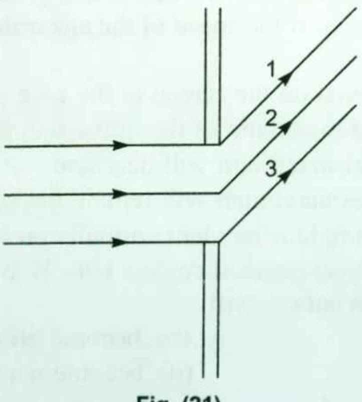


Fig. (21)

[**Hint.** The condition for minimum intensity in case of single slit is given by

$$e \sin\theta = \pm\, m\lambda$$

This equation gives the directions of first, second, third, … minima by putting m = 1, 2, 3, .., etc. For first minima m = 1)

$$e \sin \theta = \lambda$$

$\therefore$ **Path difference between rays Ist and 3rd is λ.]**

13. A screen is placed at a certain distance from a narrow slit which is illuminated by a parallel beam of monochromatic light. If the wavelength of light used in the experiment is λ and d is the width of the slit, then angular width of central maximum will be

(a) $\sin^{-1}(\lambda/d)$ (b) $2\sin^{-1}(\lambda/d)$ (c) $\sin^{-1}(2\lambda/d)$ (d) $\sin^{-1}(\lambda/2d)$

[**Hint.** In case of single slit, the intensity falls to zero on both sides of central maximum at an angle θ, given by $d \sin \theta = \lambda$ where d = slit width and θ is angular separation between central maximum and first minimum on either side.

So, 2θ will be the angular width of central maximum.

Hence, $\theta = \sin^{-1}(\lambda/d)$

or $2\theta = 2\sin^{-1}(\lambda/d)$]

14. A parallel beam of light of wavelength 6000 Å is incident normally on a slit of width 0.2 mm. The diffraction pattern is observed on a screen which is placed at the focal plane of a convex lens of focal length 50 cm. If the lens is placed close to the slit, the distance between the minima on both sides of central maximum will be

(a) 1 mm (b) 2 mm (c) 3 mm (d) 4 mm

[**Hint.** The angular separation of minima on both sides of central maximum of single slit is 2θ where θ is given by

$$e \sin \theta = \lambda \quad \text{or} \quad \sin \theta = \lambda/d$$

$$\therefore \quad \sin \theta = \frac{6000 \times 10^{-10}}{0.2 \times 10^{-3}} = 3 \times 10^{-3}$$

or $\theta \approx 3 \times 10^{-3}$ rad $\quad (\because \sin \theta \approx \theta)$

When the lens is placed close to the slit, then

$$x = f \tan \theta \approx f\,\theta$$

where x is the distance of first minimum from the central maximum.

The distance between two minima on both sides of central maximum is

$$2x = 2f\,\theta = 2 \times 0.5 \times (3 \times 10^{-3}) = 3 \times 10^{-3} \text{ m} = 3 \text{ m}]$$

15. If I_0 is the intensity of the principal maximum in the single slit diffraction pattern, then what will be its intensity when the slit width is doubled?

(a) $2I_0$ (b) $4I_0$ (c) I_0 (d) $I_0/2$

[**Hint.** In case of single slit,

$$I = I_0\left(\frac{\sin \theta}{\theta}\right)^2 \text{ and } \theta = \frac{\pi}{\lambda}\left(\frac{e\,y}{D}\right)$$

For principal maximum, $y = 0$, *i.e.*, $\theta = 0$

Hence, intensity will remain same.]

16. The first diffraction minima due to a single slit diffraction is at $\theta = 30°$ for a light of wavelength 5000 Å. The width of the slit is

(a) 5×10^{-5} cm (b) 1.0×10^{-4} cm (c) 2.5×10^{-5} cm (d) 1.25×10^{-5} cm

[**Hint.** The distance of first diffraction minimum from the central principal maximum, $x = \lambda D/d$

$$\therefore \quad \sin \theta = \frac{x}{D} = \frac{\lambda}{D} \quad \text{or} \quad d = \frac{\lambda}{\sin \theta}$$

$$= \frac{5000 \times 10^{-8}}{\sin 30°} = 2 \times 5 \times 10^{-5} = 1.0 \times 10^{-4} \text{ cm}]$$

17. Light of wavelength λ is incident on a slit of width d. The resulting diffraction pattern is observed on a screen at a distance D. The linear width of the principal maximum is then equal to the width of the slit if D equals

(a) d/λ (b) $2\lambda/d$ (c) $d^2/2\lambda$ (d) $2\lambda^2/d$

[Hint. Linear width of principal maximum $= \dfrac{2\lambda}{d} \times D$

According to the given problem,

$$d = \frac{2\lambda}{d} \times D \quad \text{or} \quad D = \frac{d^2}{2\lambda}]$$

18. A parallel beam of monochromatic light of wavelength 5000 Å is incident normally on a single narrow slit of width 0.001 mm. The light is focussed by convex lens on screen placed on focal plane. The first minimum will be formed for the angle of diffraction equal to

(a) 0° (b) 15° (c) 30° (d) 45°

[Hint. For a narrow slit, $e \sin\theta = n\lambda$

$$\therefore \quad \sin\theta = \frac{\lambda}{e} \quad (\because n = 1)$$

$$\text{or} \quad \sin\theta = \frac{5000 \times 10^{-10}}{0.001 \times 10^{-3}} = 0.5$$

$$\theta = \sin^{-1}(0.5) = \sin^{-1}(1/2) = 30°]$$

19. A parallel beam of monochromatic light falls normally on a plane grating having 5000 lines/cm. A second order spectral line is diffracted at an angle of 30°. The wavelength of spectral line is

(a) 5×10^{-7} cm (b) 5×10^{-6} cm (c) 5×10^{-5} cm (d) 5×10^{-4} cm

20. Green light of wavelength 5400 Å is diffracted by grating ruled 2000 lines/cm. The angular deviation of the third order image is

(a) $\sin^{-1}(0.324)$ (b) $\cos^{-1}(0.324)$ (c) $\tan^{-1}(0.324)$ (d) 82°

21. A grating which would be more suitable for constructing a spectrometer for the visible and ultraviolet regions should have

(a) 100 lines/cm (b) 1000 lines/cm (c) 10,000 lines/cm (d) 10^6 lines/cm

22. Light is incident normally on a diffraction grating through which the first order diffraction is seen at 32°. The second order diffraction will be seen at

(a) 48° (b) 84° (c) 80°

(d) There is no second order diffraction in this case

23. Which one of the following plane transmission grating of width e and number of lines per cm N will have the maximum resolving power in the first order?

(a) $e = 1$ cm, $N = 5000$ (b) $e = 1.5$ cm, $N = 4000$

(c) $e = 2$ cm, $N = 2400$ (d) $e = 3$ cm, $N = 1500$

ANSWERS

1. (b)	**2.** (a)	**3.** (d)	**4.** (a)	**5.** (b)	**6.** (c)	**7.** (d)	**8.** (a)
9. (c)	**10.** (a)	**11.** (b)	**12.** (d)	**13.** (b)	**14.** (c)	**15.** (c)	**16.** (b)
17. (c)	**18.** (c)	**19.** (c)	**20.** (a)	**21.** (c)	**22.** (d)	**23.** (b)	

26. Define specific rotation. Describe the construction and working of Laurent's half- shade polarimeter.
27. Write an essay on the production and analysis of polarised light.
28. Explain the Huygen's theory of double refraction in uniaxial crystals. How has theory been experimentally verified?
29. Thin plates are cut from a negative doubly refracting crystals with positions of the optic axis as described below. Explain giving diagrams of nature of refraction observed in each case:
    () Optic axis inclined to the upper face and lying in the plane of incidence.
    (ii) Optic axis parallel to the upper face and lying in the plane of incidence.
    (iii) Optic axis parallel to the upper face but perpendicular to the plane of incidence.
    (iv) Optic axis perpendicular to the upper face and lying in the plane of incidence.
30. Describe the phenomenon of double refraction in uniaxial crystals. Distinguish between negative and positive crystals. How is double refraction explained by Huygen's theory?
31. What is meant by plane-polarised, circularly polarised and elliptically polarised light? Show that the plane polarised and circularly polarised lights are the special cases of elliptically polarised light.
32. Describe the phenomenon of double refraction. Describe an experiment to determine the refractive indices of quartz for the ordinary and extraordinary rays for sodium yellow light using a properly cut quartz prism.
33. Write an essay on the production and analysis of linearly, circularly and elliptically polarised light.
34. Describe how quarter-wave and half wave plates are made and explain their use to study the different types of polarised light.
35. How will you find whether a given beam cf light is ordinary, plane polarised, circularly polarised or elliptically polarised?
36. State the laws of rotatory polarisation. Give Fresnel's hypothesis for rotatory polarisation and derive a formula for the rotation of quartz.
37. What is optical activity ? What are dextro-rotatory and laevorotatory substances ? Define specific rotatory power. On what factors does it depend ?
38. Define specific rotation. Describe the construction and working of Laurent's half-shade polarimeter. Explain how you would use it to determine the specific rotation of sugar solution.
39. Define specific rotation. Explain the working of a biquartz polarimeter. How would you use it to find the specific rotation of an optically-active substance ?

PROBLEMS

1. What is the angle of incidence for complete polarisation to occur on reflection at the boundary betwen water (refractive index 1.332) and densest flint (refractive index 1.963) if light is incident from the side of (a) water, (b) glass ? **[Ans.** (a) 55.84°, (b) 34.16°]

   [Hint. (a) $$\tan p = \frac{1.963}{1.332} = 1.474$$

   (b) $$\tan p = \frac{1.332}{1.963} = 0.6786 \Big]$$

2. A ray of light is incident on the surface of a glass plate of refractive index 1.55 at the polarising angle. Calculate the angle of refraction. **[Ans.** 32°50′]

   [Hint. $\mu = \tan p$ or $1.55 = \tan p$ or $p = 57^\circ\ 10'$

   Further, $r + p = 90$ or $r = 90^\circ - p$]

3. **The critical angle of incidence of water for total reflection is 45°. What is the polarising angle and the angle of refraction of light incident on water at an angle that gives maximum polarisation of reflected light?** **[Ans. 35°38′]**

[**Hint.** $$\mu = \frac{1}{\sin C} = \frac{1}{\sin 45^\circ} = 1.345$$

Now, $$\mu = \tan p$$

or $$1.345 = \tan p$$

or $$p = 53^\circ\, 22'$$

$\therefore$ $$r = 90^\circ - p]$$

4. Calculate the thickness of a quarter wave plate of a quartz for light of wavelength 5893 Å assuming the principal refractive indices as 1.5442 and 1.5553. **[Ans.** 0.001327 cm]

[**Hint.** $t = \dfrac{\lambda}{4\,(\mu_e - \mu_0)} = \dfrac{5893 \times 10^{-8}\ \text{cm}}{4\,(1.5553 - 1.5442)}$]

5. (a) Calculate the minimum thickness of a plate of calcite cut with its axis parallel to the plane faces to design a quarter wave plate for sodium light of wavelength 5893 nm. (Given $\mu_o = 1.6584$ and $\mu_e = 1.4864$ for calcite).

(b) Calculate the next two higher values of thickness for the plate still to act as quarter wave plate for the same light. [**Ans.** (a) 0.000857 mm, (b) 0.00257 mm and 0.000428 mm]

[**Hint.** $t = \dfrac{(2n-1)\,\lambda}{4\,(\mu_o - \mu_e)}$ where $n = 1, 2, 3, \ldots$]

6. Plane light is incident on a quartz plate cut parallel to the optic axis. Find the least thickness of the plate for which the ordinary and extraordinary rays combine to form plane polarised light on emergence.

$$\mu_o = 1.5442,\ \mu = 1.5533 \quad \text{and} \quad \lambda = 5893\ \text{Å}$$ **[Ans.** 0.0032 cm]

[**Hint.** Here, quartz plate must act as half wave plate, *i.e.,*

$$t = \frac{\lambda}{2\,(\mu_e - \mu_o)}]$$

7. A half wave plate is constructed for a wavelength of 6000 Å. For what wavelength does it work as a quarter wave plate ? **[Ans.** 12000 Å]

[**Hint.** For $\lambda/4$ plate $$t = \frac{\lambda_1}{4\,(\mu_e - \mu_o)}$$

For $\lambda/2$ plate $$t = \frac{\lambda_2}{2\,(\mu_e - \mu_o)}$$

$\therefore$ $$\frac{\lambda_1}{4} = \frac{\lambda_2}{2} \quad \text{or} \quad \lambda_1 = 2\,\lambda_2]$$

8. A plate of thickness 0.2 mm is cut from calcite with optic axis parallel to the face. Given $\mu_o = 1.648$ and $\mu_e = 1.481$ (neglecting the variation with wavelength). Find out those wavelengths for which the plate behaves as a half-wave plate and also wavelengths for which the plate behaves as a quarter wave plate.

[**Ans.** For half wave plate λ = 7422, 6073, 5138 and 4453 Å

For quarter wave plate λ = 7032, 6362, 5807, 5344, 4948, 4607, 4310, and 4048Å]

[**Hint.** For half wave plate $\lambda = \dfrac{2(\mu_o - \mu_e)}{(2n-1)}$

For $n = 5, 6, 7$ and 8 we get λ in visible region

For quarter wave plate $\lambda = \dfrac{4(\mu_o - \mu_e)}{(2n-1)}$

For $n = 10, 11, 12, 13, 14, 15, 16$ and 17 we get λ in visible region]

9. Calculate the thickness of a quarter wave plate when wavelength of light is 5890 Å and $\mu_0 = 1.55$ and $\mu_e = 1.54$. [**Ans.** 1.472×10^{-3} cm]

10. A beam of linearly polarised light is changed into a circularly polarised light by passing it through a sliced crystal of thickness 0.003 cm. Calculate the difference in refractive indices of the two rays in the crystal assuming this to be of minimum thickness that will produce the effect. The wavelength of light used is 6×10^{-7} m. [**Ans.** 5×10^{-3}]

11. Calculate the thickness of a quartz half wave plate for the Fraunhoffer *C*-line (wavelength of *C*-line is 6563 Å) for which the extraordinary and ordinary refractive indices for quartz are 1.55085 and 1.54181 respectively. [**Ans.** 3.63×10^{-3} cm]

12. Calculate the thickness of a calcite plate which would convert plane polarised light into circularly polarised light. The principal refractive indices are $\mu_o = 1.658$ and $\mu_e = 1.486$ at the wavelength of light used at 5890 Å.

[**Ans.** minimum thickness of the plate $= 8.56 \times 10^{-5}$ cm]

[**Hint.**
$$t = \frac{n\lambda}{4(\mu_o - \mu_e)} \quad \text{when } n = 1, 2, 3, \ldots$$
$$= \frac{n(5890 \times 10^{-8})}{4(1.658 - 1.486)} = 8.56 \times 10^{-5}\, n \text{ cm}$$

For minimum thickness, $n = 1$]

13. A sugar solution in a tube of length 20 cm produces an optical rotation of 13°. The solution is diluted to one-fourth of its previous concentration. Find the optical rotation produced by 30 cm long tube containing the dilute solution. [**Ans.** 4.87°]

14. A 200 mm long tube containing 48 cm^3 of sugar solution produces an optical rotation of 11° when placed in a saccharimeter. If the specific rotation of sugar solution is 66°, calculate the quantity of sugar contained in the tube in the form of a solution. [**Ans.** 3.998 g]

15. 80 gm of impure sugar is dissolved in a litre of water. The solution gives an optical rotation of 9.9° when placed in a tube of length 20 cm. If the specific rotation of pure sugar is 66° dm^{-1} (gm/cc)$^{-1}$, calculate the purity of the sugar sample. [**Ans.** 93.75%]

[**Hint.** $C = \dfrac{\theta}{lS}$ Here $\theta = 9.9°$, $l = 20$ cm $= 2$ dm and $S = 66°$ dm^{-1}

$$\therefore \quad C = 0.075 \text{ gm cm}^{-3} = 75 \text{ gm/litre}$$

The sugar sample dissolved in a of water is 80 gm.

In this sample 75 gm is pure sugar

$\therefore$ Purity $= \dfrac{75}{80} \times 100 = 93.75\%$]

16. The refractive indices of quartz for right handed and left handed circularly polarised light of wavelength 6300 Å are 1.53915 and 1.53921 respectively. Calculate the angle of rotation produced by quartz plate of thickness 0.5 mm. [**Ans.** 8.57°]

17. A tube of sugar solution 20 cm long is placed between crossed Nicols and illuminated with light of wavelength 6×10^{-5} cm. If the optical rotation produced is 13° and specific rotation is 65°, determine the strength of the solution. **[Ans.** 10%]

[Hint. $C = \dfrac{\theta}{l \times S} = \dfrac{13^\circ}{2.0 \times 65^\circ}$]

18. A 20 cm long tube containing 48 c.c. of sugar solution produces an optical rotation of 11° when placed in a saccharimeter. If the specific rotation of sugar is 66°, calculate the quantity of sugar contained in the tube in the form of solution. **[Ans.** 4 g]

[Hint. $C = \dfrac{\theta}{l \times S} = \dfrac{11}{2 \times 66}$

$\therefore$ Sugar contained in 48 cc. of solution is $C \times 48$]

19. The indices of refraction of quartz for right handed and left handed circularly polarised light of wavelength 7620 Å are 1.53914 and 1.53920 respectively. Calculate the rotation of the plane of polarisation of light in degree produced by a plate 0.5 mm thick. **[Ans.** 7.1°]

[Hint.
$$\theta = \frac{\pi\, d}{\lambda}(\mu_L - \mu_R)$$
$$= \frac{\pi \times 0.05}{7620 \times 10^{-8}}(1.53920 - 1.53914) \text{ radian}$$
$$= \frac{\pi \times 0.05 \times 0.00006}{7620 \times 10^{-8}} \times \frac{180^\circ}{\pi}$$
]

20. A sugar solution of specific rotation 52° per decimeter per g/c.c. causes rotation of 12° in a column of 10 cm long. What is the concentration of solution ? **[Ans.** 23%]

OBJECTIVE TYPE QUESTIONS

1. Polarisiation of light proves the
(a) corpuscular nature of light (b) quantum nature of light
(c) transverse wave nature of light (d) longitudinal wave nature of light

2. If a wave can be it must be
(a) a transverse wave (b) a stationary wave
(c) a longitudinal wave (d) an electromagnetic wave

3. Light from a denser medium-1 passes to a rarer medium-2 when the angle of incidence is θ, the reflected and refracted rays are mutually perpendicular. The critical angle will be
(a) $\sin^{-1}(\cot\theta)$ (b) $\sin^{-1}(\tan\theta)$
(c) $\sin^{-1}(\cos\theta)$ (d) $\sin^{-1}(\sec\theta)$

4. One of the devices to produce plane light is
(a) Nicol prism (b) a biprism
(c) a crystal (d) half wave plate

5. light falls on two so oriented that no light is transmitted. If a third Nicol is placed between them, not parallel to either of the two Nicols in question, then
(a) no light is transmitted (b) some light is transmitted
(c) light may or may not be transmitted (d) exactly 50% light is transmitted

6. Nicol prism is based on the phenomenon of
(a) reflection (b) refraction
(c) scattering (d) double refraction

7. A polarised beam falls on a Nicol such that its optic axis makes an angle θ with the electric vector of incident polarised beam. The intensity of emergent light is
   (a) I_0
   (b) $I_0/2$
   (c) $I_0 \cos^2\theta$
   (d) $I_0 \sin^2\theta$
8. Unpolarised light can be converted into a partially polarised or plane polarised light by several processes. Which of the following does not do this ?
   (a) reflection
   (b) diffraction
   (c) double refraction
   (d) scattering
9. When unpolarised light enters a doubly refracting crystal, we get two refracted rays called ordinary *O*-ray and extraordinary *E*-rays. Which of the following statement is true ?
   (a) only *O*-ray is polarised
   (b) only *E*-ray is polarised
   (c) both *O* and *E* rays are polarised
   (d) neither *O*-ray nor *E*-ray is polarised
10. Optically active substance among the following is/are
    (a) quartz
    (b) tourmaline
    (c) calcite
    (d) sodium chloride
11. When a plane polarised light is passed through a half wave plate, the emergent light is
    (a) elliptically
    (b) plane polarised
    (c) circularly
    (d) none of these
12. When a circularly polarised light, after passing through quarter wave plate is examined through a rotating Nicol, the emergent light would have shown
    (a) the variation of intensity with minimum not zero
    (b) no variation of intensity
    (c) the variation of intensity with minimum zero
    (d) the intensity is incident and emergent light is same
13. When elliptically polarised light, polarised after passing through quarter wave plate, is observed through a rotating Nicol, the emergent light would have shown
    (a) the variation of intensity with minimum not zero
    (b) no variation of intensity
    (c) the variation of intensity with minimum zero
    (d) the intensity of incident and emergent light is same
14. When a plane polarised light is incident on a quarter wave plate with its vibrations making an angle of 45° with the optic axis, the emergent light is
    (a) elliptically polarised
    (b) plane polarised
    (c) circularly polarised
    (d) a mixture of circularly and elliptically polarised light
15. For a quarter wave plate, the thickness *t* is
    (a) $t = \dfrac{\lambda}{4(\mu_o - \mu_e)}$
    (b) $t = \dfrac{\lambda}{2(\mu_o - \mu_e)}$
    (c) $t = \dfrac{4\lambda}{(\mu_o - \mu_e)}$
    (d) $t = \dfrac{2\lambda}{(\mu_o - \mu_e)}$
16. For a half wave plate, the thickness *t* is
    (a) $t = \dfrac{\lambda}{4(\mu_o - \mu_e)}$
    (b) $t = \dfrac{\lambda}{2(\mu_o - \mu_e)}$
    (c) $t = \dfrac{4\lambda}{(\mu_o - \mu_e)}$
    (d) $t = \dfrac{2\lambda}{(\mu_o - \mu_e)}$

17. If t be the minimum thickness of quarter wave plate needed to convert unpolarised light of wavelength 480 nm to circularly polarised light, then the corresponding thickness of a quarter wave plate for wavelength 600 nm is

(a) $0.56\ t$ (b) $0.75\ t$
(c) $1.25\ t$ (d) $1.44\ t$

18. The thickness of a quarter wave plate made of quartz for wavelength 5000 Å, refractive index $\mu_e = 1.553$ and $\mu_0 = 1.543$ is

(a) 1.25×10^{-3} cm (b) 2.50×10^{-3} cm
(c) 3.75×10^{-3} cm (d) 500×10^{-3} cm

19. Refractive indices of a crystal for ordinary and extraordinary rays are 1.5442 and 1.5533 respectively. The thickness of a quarter wave plate for light of wavelength 5893 is

(a) 1.00×10^{-6} m (b) 1.02×10^{-4} m
(c) 1.16×10^{-5} m (d) 1.61×10^{-5} m

20. In a dextrorotary substance, the rotation between the refractive indices of right-handed and left-handed circularly polarised light is

(a) $\mu_R = \mu_L$ (b) $\mu_R = \sqrt{\mu_L}$
(c) $\mu_R > \mu_L$ (d) $\mu_R < \mu_L$

21. The specific rotation of an optically active substance of length l_{cm} and concentration C with an optical rotation θ is

(a) $\dfrac{lC}{10\theta}$ (b) $\dfrac{\theta\, lC}{10}$ (c) $\dfrac{10\theta}{lC}$ (d) $\dfrac{10}{\theta\, lC}$

22. Polarimeter is the instrument designed for

(a) measure the angle of rotation produced by a substance
(b) to study the nature of polarised light
(c) to study the phase difference between circularly polarised light and elliptically polarised light
(d) none of these

23. A saccharimeter is the instrument designed for measurement of

(a) percentage of cane sugar in a solution
(b) to study the nature of polarised light
(c) to study the phase difference between circularly polarised light and elliptically polarised light
(d) none of these

24. A tube 2 decimeter long is filled with a cane sugar solution of 20 g in $100\ \text{cm}^3$. The specific rotation of cane sugar is 60°. The angle of rotation of plane polarised light passing through it is

(a) 15° (b) 20° (c) 24° (d) 30°

[**Hint.** $\theta = S\, l\, C = 60 \times 2 \times 0.2$]

25. A sugar solution in a tube of length 20 cm produces optical rotation of 13°. The solution is then diluted to one-third of its previous concentration. The optical rotation produced by a 30 cm long tube containing the diluted solution is

(a) 5.5° (b) 6.5° (c) 7.5° (d) 8.5°

[**Hint.** $\theta' = \dfrac{l' C' \theta}{l C}$. Here $l = 20$ cm $= 2$ decimeter, $l' = 3$ decimeter

$C' = C/3$ and $\theta = 13°$]

ANSWERS

1. (c)	2. (a)	3. (b)	4. (a)	5. (b)	6. (d)	7. (c)	8. (b)
9. (c)	10. (a)	11. (b)	12. (c)	13. (c)	14. (c)	15. (a)	16. (b)
17. (c)	18. (a)	19. (d)	20. (d)	21. (c)	22. (a)	23. (a)	24. (c)
25. (b)							

finds its applications in the study of time varying phenomena that occur in a certain region. This is not possible with ordinary microscopic techniques. Here, a hologram is recorded of the scene as and when it occurs. Thus, the event gets preserved in the hologram. Now, at our convenience, we can focus through the depths of the reconstructed image and study the phenomena in detail. Thus, we can study the transient microscopic events with the help of a hologram.

4. Other Applications

The other applications include holographic cinema, spatial filtration and character recognition, long distance holography using microwaves, rainbow holograms, focussed image holograms, holographic optical elements, etc.

SUMMARY

1. LASER is the abbreviation used for "***Light Amplification by Stimulated Emission of Radiation***".
2. The most important features of laser are:
   (i) high degree of coherence
   (ii) high directionality
   (iii) extraordinary monochromacity
   (iv) high intensity
3. Consider an atom in the excited state and a photon of correct-energy is allowed to incident on it. Now, the atom jumps to lower energy state, emitting an additional photon of same frequency. As a result, two photons move together. The process is called ***stimulated emission***.
4. The number of atoms in a given energy state is

$$N = N_0 \, e^{-E/kT}$$

   where N_0 = number of atoms in ground state.
5. Following are the Einstein's Coefficient

   B_{12} = Einstein's coefficient of absorption of radiation

   A_{21} = Einstein's coefficient of spontaneous emission of radiation

   B_{21} = Einstein's coefficient of stimulated emission of radiation

   Further, $B_{21} = B_{12}$ and $(A_{21}/B_{21}) = (8\pi h \nu^3/c^3) \propto \nu^3$
6. The population (*i.e.,* number of atoms per unit volume) decreases with increase of energy state, if by some mean, the number of atoms in any higher energy state becomes more than the number of atoms in lower energy state, then the state is said to be in ***population inversion***. The population inversion is possible only in those substances which have meta-stable states.
7. We have seen that for population inversion of atoms in excited state, the number of atoms in excited state should be more than that in ground state.

   Let N_1 and N_2 be the number of atoms in lower energy state E_1 and higher state E_2 respectively, then

$$N_2 = N_1 \, e^{-(E_2 - E_1)/kT}$$

   The condition of population inversion is $N_2 > N_1$.
8. The method of achieving population inversion by supplying external energy by any means is called ***pumping.***

   Following methods may be used for pumping:
   (i) optical pumping,
   (ii) electric discharge,
   (iii) inelastic atom-atom collision,
   (iv) direct-conversion, etc.

9. Following are the main components of a laser:
   (i) active medium,
   (ii) energy source and
   (iii) optical resonator.
10. The working substance in Ruby Laser is a crystal of Al_2O_3. The optical pumping is used for pumping. The Ruby Laser gives a pulsed laser beam.
11. In Helium-Neon laser, the working substance is a mixture of helium and neon gases. The optical pumping system is gaseous discharge. This excites helium atoms which in turn collide with unexcited neon-atom and force them to excited state. In this way, population inversion is achieved. It emits a continuous laser beam.
12. Laser beams are used in communications, computers, industry, medicine, military applications, chemical applications, etc.
13. Holography is a technique for recording and reproducing the three dimensional image of the object without using lenses and mirrors.
14. A long range coherent source is required for holography. Therefore, laser is used for holography.
15. A hologram is an interference/diffraction pattern of the object formed by the superposition of direct reference beam and the beam which is diffracted from the object to be hologramed.
16. Every smallest piece of a hologram is a hologram. It carries all information regarding the object.
17. The recording process of a hologram requires two beams : reference beam and beam diffracted from the object.

EXERCISES

1. What do you mean by laser? Define spontaneous and stimulated emission.
2. Differentiate spontaneous emission and stimulated emission.
3. What do you mean by population inversion in connection with laser?
4. Distinguish between spontaneous and stimulated emission. Give Einstein's theory of spontaneous and stimulated emission.
5. Define Einstein's coefficient of absorption, spontaneous emisson and induced emission. Obtain relationship between them.
6. What are Einstein's Coefficients A and B? Derive Einstein's relation between them.
7. What are differences between spontaneous and stimulated emission? Why is spontaneous radiation incoherent?
8. Explain spontaneous and stimulated emission of radiation. How stimulated emission takes place with the exchange of energy between Helium and Neon atoms?
9. Describe the principle and working of three-level laser system.
10. Explain the spontaneous and stimulated emission of radiation. Describe the working of a Ruby laser.
11. What is a laser? Discuss the construction and working of a Ruby laser, explaining the principle of population inversion.
12. What do you mean by population inversion? Describe the He-Ne laser. How the population inversion is achieved in He-Ne laser?
13. What are the characteristic properties of a laser beam? Describe its important applications.
14. Explain the principle of optical pumping and stimulated emission of radiation. Discuss the properties of laser radiation and mention some of its applications.

15. Draw a neat diagram of He-Ne laser and describe its method of working. What are the characteristics of laser beam? Discuss its important applications.
16. Describe the structure, operation and energy level diagram of a CO_2 laser. Mention its applications.
17. Describe in detail the operation of a CO_2 laser.
18. What are the three different modes of CO_2 molecules? Describe the features and the lasing transitions in CO_2 laser.
19. Describe in detail the operation of a solid state laser.
20. Discuss the applications of lasers.
21. What is holography? Describe the basic principle of holography.
22. Explain how a hologram is prepared and viewed.
23. Explain Gabor hologram and discuss its limitations.
24. Discuss the construction and reconstruction of image on a hologram.
25. What are the properties and applications of a hologram?

PROBLEMS

1. Determine the wavelength of radiation given out by a laser with an energy of 3 eV, given that $h = 6.63 \times 10^{-34}$ J-s and $c = 3 \times 10^{8}$ m/s. **[Ans.** 414×10^{9}m]

   [**Hint.** $E_2 - E_1 = 3$ eV $= 3 \times (1.6 \times 10^{-19})$ Joule

   Now, $E_2 - E_1 = h\nu = (hc/\lambda)$ or $E = (hc/\lambda)$

   $$\therefore \quad \lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34}) \times (3 \times 10^{8})}{35(1.6 \times 10^{-19})}\Bigg]$$

2. Calculate the energy and momentum of a photon of laser of wavelength 6328 Å. Given that $h = 6.63 \times 10^{-34}$ J-s and $C = 3 \times 10^{8}$ m/s. **[Ans.** 3.14×10^{-19} J. 1.047×10^{-27} kg m/s]
3. Calculate the coherence length of CO_2 laser whose line width is 1×10^{-5} nm at IR emission wavelength of 10.6 μm. **[Ans.** 11.2 km]

   [**Hint.** Coherence length $= \dfrac{\lambda^2}{\Delta\lambda} = \dfrac{(10.6 \times 10^{-6})^2 m^2}{10^{-5} \times 10^{-9} m}\Bigg]$

4. Find the intensity of a laser beam of 10 mW power and having a diameter of 1.3 m. Assume the intensity to be uniform across the beam. **[Ans.** 7.5 kW/m^2]

   [**Hint.** $I = \dfrac{\text{Power}}{\text{Area}} = \dfrac{P}{\pi(d/2)^2} = \dfrac{4P}{\pi d^2}$ $\quad P = 10\text{ mW} = 10 \times 10^{-3}$W and $d = 1\cdot 3\text{mm} = 1\cdot 3 \times 10^{-3}$ m]

5. A laser beam has wavelength of 7×10^{-7} m and aperture 5×10^{3} m. The laser beam is sent to moon. The distance of moon is 4×10^{5} km from the earth. Calculate (i) the angular spread of the beam and (ii) areal speed when it reaches to moon.

   [Ans. (i) $1\cdot 6 \times 10^{-4}$ radian, (ii) $4\cdot 09 \times 10^{9}$ m^2]

   [**Hint.** (i) $d\theta = \dfrac{\lambda}{d} = \dfrac{\theta \times 10^{-7}}{5 \times 10^{-3}} = 1\cdot 6 \times 10^{-4}$ radian (ii) Areal speed $= [D \times (d\theta)]^2 = [(4 \times 10^{8}) \times (1.6 \times 10^{-4})^2]$

OBJECTIVE TYPE QUESTIONS

1. LASER is abbreviation used for
   (a) name of scientist
   (b) light amplification by stimulated emission of radiation
   (c) light amplification by spontaneous emission of radiation

(d) light absorption by sun and earth radiation

2. A laser is a coherent source because it contains
(a) many wavelengths
(b) uncoordinated wave of a particular wavelength
(c) coordinated waves of a particular wavelength
(d) none of these

3. The ratio of probabilities of spontaneous emission and stimulated emission is directly proportional to
(a) frequency ν (b) ν^2 (c) ν^3 (d) ν^4

4. The directionality of a laser beam is measured by
(a) visibility of interference fringes
(b) the size and aperture of laser source
(c) the divergence angle of the beam with the distance from the source
(d) nature of the lasing medium

5. A laser beam is monochromatic. It means it has
(a) single frequency (b) narrow width (c) wide width (d) several colours

6. The population inversion in helium-neon laser is produced by
(a) photon excitation (b) chemical excitation
(c) inelastic atomic collisions (d) chemical reaction

7. In He-Ne laser, the most favourable ratio of helium to neon for satisfactory laser action is
(a) 1 : 7 (b) 7 : 1 (c) 1 : 10 (d) 10 : 1

8. Lasers are used in alignment of pipes because
(a) they are coherent (b) they are highly directional
(c) both (a) and (b) (d) none of these

9. Laser beam is highly coherent, so it can be used in
(a) interference (b) diffraction (c) polarisation (d) none of these

10. The laser used in cancer treatment is
(a) Ruby laser (b) He-Ne laser (c) CO_2 laser (d) solid state laser

11. A hologram is
(a) simply a photograph (b) one dimensional view
(c) two dimensional view (d) three dimensional view

12. Holography uses
(a) laser beam (b) coherence infrared beam
(c) coherent ultraviolet beam (d) incoherent beam

13. A small piece of a hologram carries
(a) information about a part of the object
(b) information whole of the object with full resolution
(c) information about whole of the object with less resolution
(d) no information

ANSWERS

1. (b)	2. (c)	3. (c)	4. (c)	5. (a)	6. (c)	7. (b)	8. (c)
9. (a)	10. (c)	11. (d)	12. (a)	13. (c)			

Now $E' = \dfrac{E - p\upsilon}{\left(1 - \dfrac{\upsilon^2}{c^2}\right)^{1/2}}$ and $p' = \dfrac{p - E\,\upsilon/c^2}{\left(1 - \dfrac{\upsilon^2}{c^2}\right)^{1/2}}$

$$\therefore \quad E'^2 - p'^2 c^2 = \frac{(E - p\upsilon)^2}{\left(1 - \dfrac{\upsilon^2}{c^2}\right)} - \frac{\left(p - \dfrac{E\upsilon}{c^2}\right)^2 c^2}{\left(1 - \dfrac{\upsilon^2}{c^2}\right)}$$

$$= \frac{(E^2 + 2E\,p\upsilon + p^2\upsilon^2) - \left(p^2 - \dfrac{2pE\upsilon}{c^2} + \dfrac{E^2\upsilon^2}{c^2}\right)c^2}{\left(1 - \dfrac{\upsilon^2}{c^2}\right)}$$

$$= \frac{(E^2 + 2E\,p\upsilon + p^2\upsilon^2 - p^2c^2 + 2pE\upsilon - E^2\upsilon^2)}{\left(1 - \dfrac{\upsilon^2}{c^2}\right)}$$

$$= \frac{E^2\left(1 - \dfrac{\upsilon^2}{c^2}\right) - p^2c^2\left(1 - \dfrac{\upsilon^2}{c^2}\right)}{\left(1 - \dfrac{\upsilon^2}{c^2}\right)}$$

$$= E^2 - p^2c^2$$

Hence, $E^2 - p^2c^2$ is invariant under Lorentz transformation.

SUMMARY

1. **Frame of reference:** Any system relative to which the motion of an object can be described is called as a frame of reference.
2. **Inertial and Non-inertial frames:** A frame of reference is said to be inertial when bodies in this frame obey Newton's law of inertial and other laws of Newtonian mechanics or the frames which have no acceleration of their own are called inertial frames. A frame of reference is said to be non-inertial frame when body, not acted by any external force, is accelerated. In this frame, Newton's laws are not valid. Such frames have their own acceleration.
3. **Earth is non-inertial frame:** The earth is a non-inertial frame because it has acceleration due to spin motion about its axis and orbital motion around the sun.
4. **Galilean transformations:** The equations relating the coordinates of a particle in two inertial frames (whose relative velocity is negligible in comparison of speed of light) are called as Galilean transformations.

   Consider an event happening at P at any particular time t. Let the coordinates of P with respect to frame S be (x, y, z, t) and with respect to frame S' be (x', y', z', t'). Then

   (i) $x' = (x - \upsilon t)$, $y' = y$, $z' = z$ and $t' = t$

   (ii) $u' = u - \upsilon$ (υ = velocity of frame S')

   (iii) $a' = a$

5. **Michelson-Morley experiment:** Michelson and Morley performed an experiment to estimate the velocity of earth relative to absolute frame ether. The main objective of conducting the experiment was to confirm the existence of a stationary frame of reference *i.e.*, ether frame. The experiment gave the negative results and concludes that there is no absolute frame like ether. The experiment implies that all frames are relative.

6. **Postulates of special theory of relativity:** Following are the two postulates of special theory of relativity.
   (i) All physical laws are the same in all inertial frame of reference which are moving with constant velocity relative to each other.
   (ii) The speed of light in vacuum is the same in every inertial frame.

7. **Lorentz transformation equations:** Following are the Lorentz transformation equations

$$x' = \frac{x - v\,t}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}, \qquad y' = y, \qquad z' = z \qquad \text{and} \qquad t' = \frac{t - vx/c^2}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

where symbols have their usual meanings.

8. **Lorentz-Fitzgerald contraction:** The length l of a moving rod with velocity v is contracted along the direction of motion by a factor $\sqrt{(1 - v^2/c^2)}$. If l' be the length of moving rod, then,

$$l' = l\sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

There is no contraction in length in perpendicular direction.

9. **Time dilation:** The time interval is dilated due to motion, *i.e.*,

$$\Delta t' = \frac{\Delta t}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

The moving clock runs slow. This is called apparent retardation of clocks.

10. **Velocity addition theorem:** Let u be the the velocity of a particle in frame S, u' is the velocity of particle in frame S' and v be the velocity of frame S' relative to S. Then

$$u' = \frac{u - v}{\left(1 - \frac{u\,v}{c^2}\right)}$$

or

$$u = \frac{u' + v}{\left(1 + \frac{u'\,v}{c^2}\right)}$$

We draw the following conclusions:
   (i) If one object moves with velocity c with respect to other, their relative velocity is always c, whatever may be the velocity of the other.
   (ii) The addition of the velocity of light to the velocity of light merely reproduces the velocity of light.

11. Variation of mass with velocity: The mass is a function of the velocity of a body. The mass of a body increases with increase of velocity. The relation is expresed as

$$m = \frac{m_0}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

12. Einstein's mass-energy relation: Einstein established that mass and energy are interconvertible *i.e.*, mass can be converted into energy and energy can be converted into mass. The mass-energy equivalence relation is given by

$$E = m c^2$$

Examples are:

(i) When an electron (charge e^-) and a positron (charge e^+) come close together, they annihilate each other and an equivalent amount of energy is produced in the form of a pair of γ-ray photons, *i.e.*,

$$e^- + e^+ = \gamma + \gamma$$

The phenomenon is known as ***pair annihilation***.

(ii) A photon of energy 1.02 MeV coming near the nucleus and under its intense electric field splits up into a pair of particles, an electron and a positron. The phenomenon is known as ***pair-production***.

13. Relativistic kinetic energy *E*: The relativistic kinetic energy is given by

$$E = m c^2 - m_0 c^2$$

where $m_0 c^2$ is the rest energy of the particle.

14. Relation between energy and momentum :

$$E^2 = m_0^2 c^4 + p^2 c^2$$

For a particle like neutrino, rest mass is zero. Therefore,

$$E^2 = p^2 c^2 \quad \text{or} \quad E = pc$$

EXERCISES

1. What is a frame of reference? Distinguish between inertial and non-inertial frames.
2. What do you understand by frame of reference? Is earth an inertial frame of reference? If not why?
3. Differentiate inertial and non-inertial frames of reference and show that inertial frames move with constant velocity relative to each other.
4. What is Galilean transformation? Derive Galilean transformation equations. Prove that the laws of mechanics are identical in all inertial frames.
5. Describe the Michelson's Morley experiment and explain the physical significance of negative results.
6. What was the objective of conducting the Michelson's Morley experiment? Describe the experiment. How is the negative result of the experiment interpreted?

27. Write down the relativistic expression for kinetic energy of a body and show that for smaller speeds it reduces to classical expression.

28. (a) Show that the relativistic kinetic energy of a particle is given by

$$K_E = (m - m_0)c^2 = m_0\, c^2\left[\left(1 - \frac{v^2}{c^2}\right)^{1/2} - 1\right]$$

(b) Show that for small velocities the relativistic energy reduces to classical kinetic energy.

29. What are consequences if momentum and energy are invariant under Lorentz transformations? Prove the relation

$$E^2 - p^2 c^2 = m^2 c^4$$

where the symbols have their usual meaning.

30. Show that momentum of a particle of rest mass m_0 and kinetic energy K_E is given by $p = \sqrt{\left[\frac{K_E{}^2}{c^2} + 2\, m_0\, K_E\right]}$

PROBLEMS

1. An event occurs at $x = 100$ km, $y = 10$ km and $z = 1.0$ km at $t = 2.0 \times 10^{-4}$ sec in a reference frame S. Another frame S' is moving with speed 0.95 c relative to S along the common X-X' axis, the origins coinciding at $t = t' = 0$. Compute the coordinates x', y', z' and t' of the event in S'. **[Ans.** $x' = \mathbf{137.8}$ km, $y' = 10$ km, $z' = 1.0$ km and $t' = 3.74 \times 10^{-4}$ sec**]**

[**Hint.** $$x' = \frac{x - v\,t}{\sqrt{\left\{1 - \left(\frac{v}{c}\right)^2\right\}}}$$

$$= \frac{100 - (0.95 \times 3 \times 10^5 \text{ km/sec})\,(2.0 \times 10^{-4} \text{ sec})}{\sqrt{\{1 - (0.95)^2\}}}$$

$y' = y = 10$ km, $z' = 1.0$ km

$$t' = \frac{t - \left(\frac{x\,v}{c^2}\right)}{\sqrt{\left\{1 - \left(\frac{v^2}{c^2}\right)\right\}}} = \frac{(2.0 \times 10^{-4}) - \frac{100 \times 0.95}{3 \times 10^5}}{\sqrt{\{1 - (0.95)^2\}}}$$

2. Calculate the expected fringe shift in Michelson-Morley experiment if the distance of each path is 7 metres and light is of wavelength 7000 Å. Velocity of earth is 3×10^4 m/s. **[Ans.** 0.2**]**

[**Hint.** Fringe shift $= \left(\frac{2\,l\,v^2}{\lambda\, c^2}\right)$

$$= \frac{(2 \times 7 \times (3 \times 10^4)^2)}{(7000 \times 10^{-10}) \times 9 \times 10^{16}}\Bigg]$$

3. What will be the fringe shift in Michelson-Morely experiment if the effective length of each path is 6 m and light wavelength of 6000 Å is used? Earth's velocity is 3×10^4 ms^{-1} and $c = 3 \times 10^8$ ms^{-1}. **[Ans.** 0.20**]**

[**Hint.** **Change in fringe shift is given by**

$$n = \frac{2\,l\,v^2}{\lambda\,c^2} = \frac{2\times 6\times(3\times10^4)^2}{(6000\times10^{-10})\times(3\times10^8)^2} = \mathbf{0.201}]$$

4. Find the proper length of the rod which appears to have a length of 1 m moving with a velocity of 0.5 c (c = velocity of light = 3×10^8 m/s). **[Ans.** 0.86 m]

[**Hint.** We know that, $l = l'\left[1-\dfrac{v^2}{c^2}\right]^{1/2}$

$$\therefore \quad l = 1\left[1-\frac{(0.5\,c)^2}{c^2}\right]^{1/2} = [1-(0.5)^2]^{1/2}$$

$$= [1-0.25]^{1/2} = (0.75)^{1/2}$$

$$= \mathbf{0.86}\text{ m]}$$

5. Find the velocity with which a body should travel so that its length becomes half of the rest length? **[Ans.** $\dfrac{\sqrt{3}}{2}c$]

[**Hint.** According to theory of relativity

$$l = l_0\sqrt{\left(1-\frac{v^2}{c^2}\right)}$$

Here, $\dfrac{l_0}{2} = l_0\sqrt{\left(1-\dfrac{v^2}{c^2}\right)}$ or $\dfrac{1}{2} = \sqrt{\left(1-\dfrac{v^2}{c^2}\right)}$

or $\dfrac{1}{4} = \left(1-\dfrac{v^2}{c^2}\right)$ or $\dfrac{v^2}{c^2} = 1-\dfrac{1}{4} = \dfrac{3}{4}$

or $\dfrac{v}{c} = \dfrac{\sqrt{3}}{2}$ or $v = \dfrac{\sqrt{3}}{2}c$]

6. Calculate the velocity of rod while its length will appear 80% of its proper length. ($c = 3\times10^8$ m/s). **[Ans.** 1.8×10^8 m/s]

[**Hint.** Let $l_0 = x$, then $l = \left(\dfrac{80}{100}\right)\times x = 0.8\,x$

Now $$l = l_0\sqrt{\left[1-\left(\frac{v^2}{c^2}\right)\right]}$$

$$\therefore \quad 0.8\,x = x\sqrt{1-\frac{v^2}{c^2}}$$

or $\sqrt{\left(1-\dfrac{v^2}{c^2}\right)} = 0.8$ or $\left(1-\dfrac{v^2}{c^2}\right) = 0.64$

or $$\frac{v^2}{c^2} = 1-0.64 = 0.36$$

or $$\frac{v}{c} = \sqrt{(0.36)} = 0.6$$

or $$v = 0.6\,c = 0.6\times(3\times10^8)$$

$$= \mathbf{1.8\times10^8}\text{ m/s]}$$

7. A rod has length 1 metre when the rod is in a satellite moving with velocity 0.8 c relative to laboratory. What is the length of the rod as determined by the observer (a) in the satellite and (b) in the laboratory? **[Ans.** (a) 1 metre, (b) 0.6 metre]

[Hint. (a) The observer is in the satellite which is at rest relative to rod. Hence, the length of the rod measured by the observer is 1 metre.

(b) $l = l'\sqrt{\left(1-\dfrac{v^2}{c^2}\right)} = 1\sqrt{(1-0.64)} = (0.36)^{1/2}$]

8. What is the area of a circular plate as observed by stationary observer when it moves with a velocity of $\left(\dfrac{\sqrt{3}}{2}\right)c$? **[Ans.** Half]

[Hint. Suppose that the radius of plate is r' when it moves along the x axis. Now

$$r = r'\sqrt{\left(1-\frac{v^2}{c^2}\right)} = r'\sqrt{\left(1-\frac{3}{4}\right)} = \frac{r'}{2}$$

Radius along Y-axis remains unchanged.

The new area is in the form of an ellipse $= \pi\, a\, b$

$$\therefore \quad \text{New area} = \pi.\frac{r'}{2}\cdot r' = \frac{\pi r'^2}{2}$$

$$= \text{Half (previous area)]}$$

9. Show that if $l_0{}^3$ is the rest volume of a cube, then $l_0{}^3\,(1-\beta^2)^{1/2}$ is the volume viewed from a reference frame moving with uniform velocity v in a direction parallel to an edge of the cube $\left(\beta = \dfrac{v}{c}\right)$.

[Hint. Length of the side of cube in the direction of motion

$$= l_0\,(1-\beta^2)^{1/2}$$

The length in other two directions will be still l_0.

$\therefore$ Volume of the cube $= l_0\,(1-\beta^2)^{1/2} \times l_0 \times l_0$]

10. A rod of 1 m length is moving with a velocity of 0.6×10^8 m/s with respect to a stationary observer. Find the length of the rod (in m) along its direction of motion as seen by the observer. **[Ans.** 1.0206 m]

[Hint. Given, $l = 1$ m and $v = 0.6\times10^8$ m/s

We know that, $$l' = \frac{l}{\sqrt{\left[1-\left(\dfrac{v^2}{c^2}\right)\right]}} = \frac{1}{\sqrt{\left[1-\left(\dfrac{0.6\times10^8}{3\times10^8}\right)^2\right]}}$$

$$= \frac{1}{\sqrt{(1-0.04)}} = \frac{1}{\sqrt{0.96}}$$

$$= \frac{1}{0.9798} = 1.0206 \text{ m]}$$

11. The proper mean life-time of π^+ meson is 2.5×10^{-8} second, when travelling with a velocity of 2.4×10^{10} cm/sec. Calculate distance travelled by it before disintegrating and distance it would travel if there were no relativity effect. **[Ans.** 1000 cm, 600 cm]

[Hint. $$\Delta t' = \frac{\Delta t}{\sqrt{\left\{1-\dfrac{v^2}{c^2}\right\}}} = \frac{2.5\times10^{-8}\text{ sec}}{\sqrt{\left\{1-\left(\dfrac{2.4\times10^{10}}{3\times10^{10}}\right)^2\right\}}}$$

$$= 4.17\times10^{-8}\text{ sec.}$$

Distance travelled = velocity × time

$$= (2.4 \times 10^{10} \text{ cm/sec})(4.17 \times 10^{-8})$$

$$= 1000 \text{ cm}$$

Distance travelled if their were no relativity

$$= (2.4 \times 10^{10} \text{ cm/sec})(2.5 \times 10^{-8} \text{ sec})]$$

12. A particle with a mean proper life time of 2 μ sec, moves through the laboratory with a speed of 0.2 c. Calculate its life time as measured by an observer in the laboratory.

[Ans. 2.0414×10^{-6} sec]

$$\left[\textbf{Hint. } \Delta t' = \frac{\Delta t}{\sqrt{[1-(v^2/c^2)]}} = \frac{2 \times 10^{-6}}{\sqrt{\{1-(0.2)^2\}}}\right]$$

13. A clock showing correct time when at rest and loses 2 hours per day when it is moving. What is its speed ?

[Ans. 1.199×10^8 m/sec]

[Hint. $$\Delta t' = \frac{\Delta t}{\sqrt{\left\{\left(1-\frac{v^2}{c^2}\right)\right\}}}$$

Here, $\Delta t' = 24$, $\Delta t = 24 - 2 = 22$ hours and $v = ?$]

14. At what speed the mass of an object will be double of its value at rest ?

[Ans. 2.6×10^8 m/s]

[Hint. We know that

$$m = \frac{m_0}{\sqrt{\left[1-\left(\frac{v^2}{c^2}\right)\right]}}. \text{ Here } m = 2\,m_0$$

$$\text{or } 2\,m_0 = \frac{m_0}{\sqrt{\left[1-\left(\frac{v^2}{c^2}\right)\right]}} \quad \text{or} \quad \left[1-\left(\frac{v^2}{c^2}\right)\right] = \frac{1}{4}$$

$$\text{or} \quad \left(\frac{v^2}{c^2}\right) = 1 - \frac{1}{4} = \frac{3}{4} \quad \text{or} \quad \frac{v}{c} = \sqrt{\left(\frac{3}{4}\right)}$$

$$\therefore \quad v = \left(\frac{\sqrt{3}}{2}\right)c = \left(\frac{\sqrt{3}}{2}\right) \times (3 \times 10^8)$$

$$= \mathbf{2.6 \times 10^8} \text{ m/sec}]$$

15. Find the mass of an electron moving with a velocity of 1×10^{10} cm/sec. The rest mass of the electron is 9.1×10^{-31} kg.

[Ans. 9.650×10^{-31} kg]

[Hint. $$m = \frac{m_0}{\sqrt{(1 - v^2/c^2)}}$$

$m_0 = 9.1 \times 10^{-31}$ kg, $v = 1 \times 10^8$ m/sec.

$c = 3 \times 10^8$ m/sec. and $m = ?$]

16. A particle of mass 10^{-24} kg is moving with a speed of 1.8×10^8 m/s. Calculate its mass when it is in motion.

[Ans. 1.25×10^{-24} kg]

$$\left[\textbf{Hint.}\ m = \frac{m_0}{\sqrt{\left\{1-\frac{v^2}{c^2}\right\}}} = \frac{10^{-24}}{\sqrt{\left[1-\left(\frac{1.8\times10^8}{3\times10^8}\right)^2\right]}}\right]$$

17. The density of gold is 19.3×10^3 kg m^{-3} in a frame *S* which is at rest. Calculate its density that an observer in a frame *S* would determine if the frame *S* is moving along the *x* axis with a speed 0.9 *c*. **[Ans. 101.6×10^3 kg m^{-3}]**

[**Hint.** Original density $\rho_0 = \left(\frac{m_0}{V_0}\right)$ and Final density $\rho = \left(\frac{m}{V}\right)$

$$\therefore \quad \frac{\rho}{\rho_0} = \left(\frac{m}{m_0}\right)\left(\frac{V_0}{V}\right)$$

But $$m = \frac{m_0}{\sqrt{[(1-0.9)^2]}} = \frac{m_0}{0.4359}$$

$$V_0 = I_0 = b\times d \quad \text{and} \quad V = l\times b\times d$$

But $$l = l_0\sqrt{\left(1-\frac{v^2}{c^2}\right)} = l_0\sqrt{[1-(0.9)^2]} = 0.4359\ l_0$$

Now, $V = 0.4359\ l_0\times b\times d$ and $V_0 = l_0\times b\times d$

$$\therefore \quad \frac{V_0}{V} = \frac{1}{0.4359} \quad \text{and} \quad \frac{m}{m_0} = \frac{1}{0.4359}\ ]$$

18. Two particles are travelling in opposite directions with speed 0.9 *c* relative to laboratory. What is their relative speed? **[Ans. 0.995 c]**

$$\left[\textbf{Hint.}\ u = \frac{u'+v}{1+\frac{u'v}{c^2}} = \frac{0.9\,c+0.9\,c}{1+\frac{0.9\,c\times0.9\,c}{c^2}}\right]$$

19. A particle of mass m_0 is moving with a velocity 0.9 *c*. Calculate its relativistic mass and its kinetic energy. **[Ans. $2.294\ m_0, 1.294\ m_0\ c^2$]**

[**Hint.** $$m = \frac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} = \frac{m_0}{\sqrt{\left\{1-\left(\frac{0.9\,c}{c}\right)^2\right\}}} = 2.294\ m_0$$

and $$K.E. = mc^2 - m_0c^2 = 2.294\ m_0c^2 - m_0\ c^2 = 1.294\ m_0c^2\ ]$$

20. The total energy of a particle is exactly twice its rest energy. Find its speed. **[Ans. 0.866 c]**

[**Hint.** $m\,c^2 = 2\,m_0\,c^2$ or $m = 2\,m_0$

where $$m = \frac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}$$

$$\therefore \quad \frac{m_0}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} = 2\,m_0 \quad \text{or} \quad \sqrt{\left(1-\frac{v^2}{c^2}\right)} = \frac{1}{2}\ ]$$

21. It is assumed that sun gets energy by fusion of 4 hydrogen atoms to a helium atom. Rest mass of hydrogen and helium are 1.0081 and 4.0039 atomic mass unit respectively. Calculate the energy released in the process. 1 a.m.u. $= 1.66 \times 10^{-27}$ kg. **[Ans.** 26.53 MeV]

22. What is the velocity of nuclear particles whose mean life time is 2.5×10^{-7} sec? The proper life time is 2.5×10^{-8} sec. **[Ans.** 0.995 c]

23. Calculate the ratio of the mass of electron moving with K.E. 20 MeV to its rest mass which is 0.51 MeV. **[Ans.** 40.22]

[Hint. K.E. $= (m - m_0)c^2 = 20$ MeV, $m_0 c^2 = 0.5$ MeV

$$\therefore \quad \frac{m - m_0}{m_0} = \frac{20}{0.51} \quad \text{or} \quad \frac{m}{m_0} = \frac{20}{0.51} + 1 = \frac{20.51}{0.51}\Bigg]$$

24. Calculate the velocity that one atomic mass unit will have if it has a kinetic energy equal to twice the rest mass energy. **[Ans.** 0.941 c]

[Hint. $2 m_0 c^2 = (m - m_0) c^2$ or $3 m_0 c^2 = m c^2$ or $m = 3 m_0$

$$\text{Now, } m = \frac{m_0}{\sqrt{\left\{\left(1 - \frac{v^2}{c^2}\right)\right\}}} \quad \text{or} \quad 3 m_0 = \frac{m_0}{\sqrt{\left\{\left(1 - \frac{v^2}{c^2}\right)\right\}}}$$

$$\therefore \quad 1 - \frac{v^2}{c^2} = \frac{1}{9} \quad \text{or} \quad \frac{v^2}{c^2} = \frac{8}{9} \quad \text{or} \quad v^2 = \left(\frac{8}{9} c^2\right)\Bigg]$$

25. When a positron and an electron annihilate, what is the wavelength of the two photons emitted? **[Ans.** 2.482×10^{-12} m]

[Hint. $E = m c^2 + m c^2 = 2 m c^2 = 2 h\nu = \left(\dfrac{2 h c}{\lambda}\right)$

$$\therefore \quad \lambda = \frac{h}{m c} = \frac{6.62 \times 10^{-34}}{(9.1 \times 10^{-31}) \times (3 \times 10^8)}\Bigg]$$

26. If the total energy of a particle is exactly thrice its rest energy, what is the velocity of the particle? **[Ans.** 0.94 c]

[Hint. We know that total energy of a particle of mass m is given by

$$E = m c^2$$

If m_0 be the rest mass, then rest mass energy $= m_0 c^2$

Given that, $m c^2 = 3 m_0 c^2$ or $m = 3 m_0$

$$\text{Now, } m = \frac{m_0}{\sqrt{\left[1 - \left(\frac{v^2}{c^2}\right)\right]}} \quad \text{or} \quad 3 m_0 = \frac{m_0}{\sqrt{\left[1 - \left(\frac{v^2}{c^2}\right)\right]}}$$

$$\text{or} \quad \left[1 - \left(\frac{v^2}{c^2}\right)\right]^{1/2} = \frac{1}{3} \quad \text{or} \quad \left(1 - \frac{v^2}{c^2}\right) = \frac{1}{9}$$

$$\therefore \quad \frac{v^2}{c^2} = 1 - \frac{1}{9} = \frac{8}{9} \quad \text{or} \quad v = \sqrt{\left(\frac{8}{9}\right)} \times c = \mathbf{0.94}\, c\Bigg]$$

27. Show that the velocity of a relativistic particle is given by $v = \dfrac{p c}{\sqrt{(p^2 + m_0{}^2 c)^2}}$, where p is momentum of the particle.

$$\Bigg[\textbf{Hint.} \quad p = m v = \frac{m_0 v}{\sqrt{\{(1 - v^2/c^2)\}}} \quad \therefore \quad p^2 = \frac{m_0{}^2 v^2 c^2}{c^2 - v^2}.$$

or $\quad p^2 c^2 - p^2 v^2 = m_0^2 v^2 c^2 \quad$ or $\quad p^2 c^2 = v^2 [p^2 + m_0^2 c^2]$

$$\therefore \qquad v = \frac{p\,c}{\sqrt{(p^2 + m_0^2 c^2)}}$$

28. Show that the momentum of a particle is $m_0\, c$ when its velocity is $c/\sqrt{2}$ (where c is velocity of light and m_0 is the rest mass of the particle).

Hint. $m = \dfrac{m_0}{\sqrt{(1 - v^2/c^2)}} = \dfrac{m_0}{\sqrt{1 - (c/\sqrt{2}/c^2)^2}}$

$= \dfrac{m_0}{\sqrt{(1/2)}} = \sqrt{2}\, m_0$

$p = \text{mass} \times \text{velocity} = \sqrt{2}\, m_0 \times \dfrac{c}{\sqrt{2}}$

$= m_0\, c$]

OBJECTIVE TYPE QUESTIONS

1. A reference frame attached to the earth
   (a) is an inertial frame by definition
   (b) cannot be an inertial frame because the earth is revolving round the sun
   (c) is an inertial frame because Newton's laws are applicable in this frame
   (d) cannot be an inertial frame because the earth is rotating about its own axis
2. An experiment that led to the theory of relativity was performed by
   (a) Einstein (b) Michelson and Morley
   (c) Rutherford (d) Lorentz
3. Theory of relativity shows that Newtonian mechanics is correct
   (a) approximately for all velocities
   (b) approximately for velocity approaching the velocity of light
   (c) approximately only for velocities much smaller than that of light
   (d) wholly
4. The length contraction
   (a) predicts that the length of an object approaches zero as its speed approaches the speed of light in vacuum
   (b) predicts that there is no change in the length of an object when its speed approaches the speed of light in vacuum
   (c) predicts that the length of an object reduces to half when its speed approaches the speed of light in vacuum
   (d) predicts that the length of the object is directly proportional to its velocity
5. A space ship in space will have
   (a) clocks running slower than a stationary clock by a factor $\sqrt{1-(v^2/c^2)}$
   (b) its length shrunk in the direction of the relative motion by a factor of $\sqrt{1-(v^2/c^2)}$
   (c) its mass increased by a factor $1/\sqrt{1-(v^2/c^2)}$
   (d) all the above

6. The rest mass of a particle is defined as
   (a) mass when absolutely at rest
   (b) mass when moving with speed of light
   (c) mass of the particle moving at a speed very small compared with the speed of light
   (d) mass of the particle moving at a speed which is half the speed of light
7. Einstein's mass-energy relation ($E = mc^2$) shows that
   (a) mass disappears to reappear as energy
   (b) energy disappears to reappear as mass
   (c) mass and energy are two different forms of the same entity
   (d) all the above statements are correct
8. Given the relativistic mass of particle
$$m = \frac{m_0}{\sqrt{1-(v^2/c^2)}}$$
   where m_0 = its rest mass, v = its velocity and c = velocity of light, which of the following statements is true?
   (a) increase in mass is due to its increase in potential energy
   (b) increase in mass is equal to the increase in kinetic energy divided by c^2
   (c) there is no increase in mass
   (d) mass increase when $v = 0$
9. Which of the following statements is not a prediction of the special theory of relativity?
   (a) the mass of the electron increases with its speed
   (b) the kinetic energy of an electron cannot be converted into matter
   (c) the electrons in an atom have certain quantised or discrete energies
   (d) photons whose rest mass is zero have momentum
10. A body with a charge q starts from rest and acquires a velocity $v = c/2$, then the new charge on it is
   (a) q
   (b) $q\sqrt{1-(1/2)^2}$
   (c) $q/\sqrt{1-(1/2)^2}$
   (d) $q\sqrt{(1/2)^2-1}$
11. A source of light moves with a velocity $c/2$ towards the observer, then speed of light is
   (a) $3c/2$
   (b) $c/2$
   (c) c
   (d) $2c$
12. A rod has a proper (or rest) length L_0. The length of the same rod, as measured by an observer moving with a velocity v relative to the rod, is given by
   (a) $l = l_0\left(1-\frac{v^2}{c^2}\right)$
   (b) $l = l_0\left(1-\frac{v^2}{c^2}\right)^{1/2}$
   (c) $l = \frac{l_0}{(1-v^2/c^2)}$
   (d) $l = \frac{l_0}{(l-v^2/c^2)^{1/2}}$
13. The rest mass of a particle is m_0. The mass of the particle as measured by an observer with respect to which the particle moves with a velocity v is given by

(a) $m = m_0\left(1-\frac{v^2}{c^2}\right)$ (b) $m = m_0\left(1-\frac{v^2}{c^2}\right)^{1/2}$

(c) $m = \frac{m_0}{(1-v^2/c^2)}$ (d) $m = \frac{m_0}{(1-v^2/c^2)^{1/2}}$

14. The special theory of relativity shows that the Newtonian mechanics is valid at
(a) all velocities
(b) velocity nearer to that of light
(c) velocities much smaller than that of light
(d) velocities in the ultra relativistic range

[**Hint.** The special theory of relativity shows that the Newtonian mechanics is valid at velocities much smaler than the velocity of light.]

15. Which of the following are invariant under Galilean transformations?
(a) velocity (b) acceleration (c) speed (d) none of the above

[**Hint.** Acceleration is invariant under Galilean transformation.]

16. Which of the following is not invariant under Galilean transformations?
(a) space interval (b) time interval (c) mass (d) momentum

[**Hint.** Momentum is not invariant under Galilean transformation.]

17. The apparent length of a meter stick measured an observer at rest when the stick is moving along its length with a velocity equal to c is
(a) zero (b) infinite (c) 1 m (d) none of the above

[**Hint.** $l = l_0\sqrt{[(1-(v^2/c^2)]} = l_0\sqrt{(1-1)} = 0.$

18. The volume of a cube, the proper length of each edge of which is l_0 when it is moving with velocity v along one of its edge can be given as …

[**Hint.** $V' = l_x\, l_y\, l_z = l_0\left(1-\frac{v^2}{c^2}\right)^{1/2}\cdot l_o\, l_0 = l_0^{\,3}\left(1-\frac{v^2}{c^2}\right)^{1/2}$]

19. A rod of length one metre is moving with a velocity of 0.8 c. What is the contraction in its length as measured by an observer at rest?
(a) 0.04 cm (b) 0.4 cm (c) 4 cm (d) 60 cm

[**Hint.** $l = l_0\sqrt{(1-v^2/c^2)} = l_0\sqrt{[1-(0.8\,c/c)^2]} = 100\times 0.6 = 60$ cm]

20. A rod of length 2 m moves with a velocity of 10^8 m / s relative to an observer at rest on the earth. What is the apparent length of the rod appearing to the observer?
(a) 1.885 m (b) 8.115 m (c) 5.118 m (d) 11.58 m

[**Hint.** $l = l_0\sqrt{\left(1-\frac{v^2}{c^2}\right)} = 2\sqrt{\left\{1-\left(\frac{10^8}{3\times10^8}\right)^2\right\}} = 2\sqrt{\left(1-\frac{1}{9}\right)} = 2\times\frac{2\sqrt{2}}{3} = 1.885$ m]

21. An electron has a rest mass 9.11×10^{-31} kg when its speed is 0.90 c, its mass will be
(a) 10.5×10^{-31} kg (b) 64.4×10^{-31} kg
(c) 20.9×10^{-31} kg (d) 6.31×10^{-31} kg

[**Hint.** $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{9.11\times10^{-31}\text{ kg}}{\sqrt{\left[1-\left(\frac{0.90\,c}{c}\right)^2\right]}} = \frac{9.11\times10^{-31}}{\sqrt{0.19}} = 10.5\times10^{-31}$ kg]

22. The speed at which a particle moves in order to double its rest mass is

(a) 2.60×10^8 m/sec (b) 2.20×10^8 m/sec

(c) 2.60×10^8 m/sec (d) 1.30×10^8 m/sec

[**Hint.** $m = m_0\sqrt{(1-v^2/c^2)}$ or $2\,m_0 = \dfrac{m_0}{\sqrt{(1-v^2/c^2)}}$ or $4\left(1-\dfrac{v^2}{c^2}\right)=1$

or $3 = 4\dfrac{v^2}{c^2}$ or $v^2 = \dfrac{3c^2}{4}$ or $v = \sqrt{\left(\dfrac{3}{4}\right)}\cdot c$]

23. The velocity of π-mesons whose proper mean life is 2.5×10^{-8} sec. and observed mean life is 2.5×10^{-7} sec will be

(a) 0.995 c (b) c (c) 2 c (d) 0

[**Hint.** $t = \dfrac{t_0}{\sqrt{\left\{1-\left(\dfrac{v^2}{c^2}\right)\right\}}}$

where t_0 = proper life time and v = speed of particle.

$$\therefore \quad \sqrt{\left\{1-\left(\frac{v^2}{c^2}\right)\right\}} = \frac{t_0}{t}$$

$$\text{or} \quad \sqrt{\left\{1-\left(\frac{v^2}{c^2}\right)\right\}} = \frac{2.5\times10^{-8}}{2.5\times10^{-7}} = 0.1$$

$$1-\frac{v^2}{c^2} = 0.01 \quad \text{or} \quad \frac{v^2}{c^2} = 1-0.01 = 0.99$$

$$v = \sqrt{(0.99)}.c = 0.995\,c.]$$

24. At what speed the mass of a body will be almost doubled?

(a) 0.97 c (b) 0.87 c (c) 0.77 c (d) none of the above

[**Hint.** We know that, $m = \dfrac{m_0}{[1-(v^2/c^2)]}$

When $m = 2\,m_0$, then

$$2m_0 = \frac{m_0}{\sqrt{[1-(v^2/c^2)]}} \quad \text{or} \quad \left[1-\frac{v^2}{c^2}\right] = \frac{1}{4}$$

$$\text{or} \quad \frac{3}{4} = \frac{v^2}{c^2} \quad \text{or} \quad v = \frac{\sqrt{3}}{c}\,c = 0.87\,c]$$

25. A particle of rest-mass m_0 moves with speed $c/\sqrt{2}$. Its mass can be given ...

[**Hint.** $m = \dfrac{m_0}{\sqrt{\left(1-\dfrac{v^2}{c^2}\right)}} = \dfrac{m_0}{\sqrt{\left[1-\left(\dfrac{c^2}{2c^2}\right)\right]}}$

$$\therefore \quad m = \frac{m_0}{\sqrt{\left(1-\dfrac{1}{2}\right)}} = \frac{m_0}{\sqrt{(1/2)}} = 2m_0]$$

26. At what velocity the kinetic energy of a particle is equal to the rest mass energy?

(a) $\dfrac{c}{2}$ (b) $\dfrac{\sqrt{3}}{2}$ c (c) $\dfrac{\sqrt{5}}{2}$ c (d) none of the above

[**Hint.** $mc^2 - m_0c^2$ or $mc^2 = 2m_0c^2$ or $m = 2m_0$

Using the formula, $m = \dfrac{m_0}{\sqrt{[1-(v^2/c^2)]}}$ and putting $m = 2\,m_0$, we get $v = \dfrac{\sqrt{3}}{2}\,c$]

27. **The kinetic energy of a particle moving with relativistic speed v is given by (m_0 = rest mass)**

(a) $\dfrac{1}{2} m_0 v^2$
(b) $\dfrac{1}{2}\dfrac{m_0 v^2}{\sqrt{(1-v^2/c^2)}}$
(c) $\left[\dfrac{m_0}{\sqrt{(1-v^2/c^2)}} - m_0\right]c^2$
(d) $\dfrac{m_0 c^2}{\sqrt{(1-v^2/c^2)}}$

[**Hint.** K.E. $= mc^2 - m_0 c^2 = m_0 c^2\left[\dfrac{1}{\sqrt{(1-v^2/c^2)}}\right]$]

28. On the surface of the earth, the mass of a man is 100 kg. When he is in a rocket moving with a speed of 4.2×10^7 m/s relative to the earth will be

(a) 100 kg
(b) 101 kg
(c) 200 kg
(d) infinite

$$\left[\textbf{Hint.}\quad m = \frac{m_0}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}} = \frac{100\text{ kg}}{\sqrt{\left\{1-\left(\frac{4.2\times10^7}{3.0\times10^6}\right)^2\right\}}} = \frac{100\text{ kg}}{\sqrt{1-(0.14)^2}} = \frac{100\text{ kg}}{\sqrt{(0.9804)}} = 101\text{ kg}\right]$$

29. The rest mass of an electron is 9×10^{-31} kg. When it is moving with (4/5)th the speed of light, its mass will be

(a) 9×10^{-31} kg
(b) 3×10^{-31} kg
(c) 15×10^{-31} kg
(d) 91×10^{-31} kg

[**Hint.** $m = \dfrac{m_0}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}} = \dfrac{9\times10^{-31}\text{kg}}{\sqrt{\left\{1-\left(\frac{4}{5}\right)^2\right\}}} = \dfrac{9\times10^{-31}}{\left(\frac{3}{5}\right)} = 15\times10^{-31}$ kg]

30. One kilogram of mass is completely converted into heat energy. The heat produced in kilocalories will be

(a) 2.1×10^{10}
(b) 2.1×10^{3}
(c) 2.1×10^{16}
(d) none of the above

[**Hint.** $E = mc^2 = 1\times(3\times10^8)^2 = 9\times10^{16}\,\text{J} = \dfrac{9\times10^{16}}{4.2}$ cal $= 2.1\times10^{16}$ cal $= 2.1\times10^{13}$ k.cal]

31. At what velocity the kinetic energy of a particle is equal to the rest mass energy?

(a) c
(b) $c/2$
(c) $2\,c/3$
(d) $\sqrt{3}\,c/2$

[**Hint.** We know that, $E_K = mc^2 - m_0 c^2$

Given that, $mc^2 - m_0 c^2 = m_0 c^2$

$\therefore$ $m = 2m_0$

Further, $m = \dfrac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ or $2m_0 = \dfrac{m_0}{\sqrt{\left\{1-\frac{v^2}{c^2}\right\}}}$

or $1-\dfrac{v^2}{c^2} = \dfrac{1}{4}$ or $\dfrac{3}{4} = \dfrac{v^2}{c^2}$ or $v = \dfrac{\sqrt{3}}{2}\,c$.]

32. Two space crafts *A* and *B* are approaching an observer on the moon from two opposite directions with speeds of 0.7 *c* and 0.6 *c* respectively. The relative speed of the space craft as observed by the observers in the two crafts will be nearly equal to

(a) 0.90 *c* (b) 0.92 *c* (c) 0.82 *c* (d) 0.80 *c*

[**Hint.** We know that,
$$u' = \frac{u - v}{\left(1 - \frac{uv}{c^2}\right)}$$

Substituting the given values, we get
$$u' = \frac{0.7\,c - (-\,0.6\,c)}{\left[1 - \frac{(0.7\,c) \times (-\,0.6\,c)}{c^2}\right]} = \frac{1.3\,c}{1.42} = 0.9155 \approx 0.92\,c\,]$$

33. A proton is accelerated to a kinetic energy of 1.60×10^{-9} J in a modern synchrotron. If the rest mass of proton is 1.67×10^{-27} kg, then increase in mass of proton is

(a) 0.89×10^{-26} kg (b) 1.78×10^{-26} kg

(c) 3.58×10^{-26} kg (d) 3.5×10^{-26} kg

[**Hint.**
$$E_K = mc^2 - m_0\,c^2 = \Delta m c^2$$
where
$$(m - m_0) = \Delta m$$
$$\therefore \quad \Delta m = \frac{E_K}{c^2} = \frac{1.6 \times 10^{-19}}{(3 \times 10^8)^2} = 1.78 \times 10^{-26} \text{ J.kg]}$$

34. A stick of 1 m is moving with velocity of 2.7×10^8 ms^{-1}. What is the apparent length of the stick ($c = 3 \times 10^8$ ms^{-1})?

(a) 2.4 m (b) 0.44 m (c) 0.22 m (d) 10 m

[**Hint.** We know that,
$$l = l_0 \sqrt{\left(1 - \frac{v^2}{c^2}\right)} = 1 \sqrt{\left[1 - \left(\frac{2.7 \times 10^8}{3 \times 10^8}\right)^2\right]} = 0.44 \text{ m.}$$

35. According to the special theory of relativity, which of the following has same value in all inertial frames?

(a) mass of an object (b) length of an object

(c) velocity of sound (d) velocity of light

[**Hint.** The velocity of light has same value in all inertial frames.

ANSWERS

1. (a)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (d)	**6.** (c)	**7.** (d)	**8.** (b)
9. (c)	**10.** (a)	**11.** (c)	**12.** (b)	**13.** (d)	**14.** (c)	**15.** (b)	**16.** (d)
17. (a)	**18.** $l_0^3\sqrt{\left(1 - v^2/c^2\right)}$	**19.** (d)	**20.** (a)	**21.** (a)	**22.** (a)	**23.** (a)	**24.** (d)
25. $2m_0$	**26.** (b)	**27.** (c)	**28.** (b)	**29.** (c)	**30.** (b)	**31.** (d)	**32.** (b)
33. (b)	**34.** (b)	**35.** (d)					

SUMMARY

1. **Equation of Continuity :**

$$\nabla \cdot \boldsymbol{J} + \frac{\partial \rho_f}{\partial t} = 0$$

2. **Maxwell's equation :**

$$(i)\ \nabla \cdot \boldsymbol{D} = \rho$$

$$(ii)\ \nabla \cdot \boldsymbol{B} = 0$$

$$(iii)\ \nabla \cdot \boldsymbol{E} = \frac{-\partial \boldsymbol{B}}{\partial t}$$

$$(iv)\ \nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$

3. **Displacement Current :** This is a current which flows in the gap of the capacitor and is represented by $\boldsymbol{J}_D = \dfrac{\partial \boldsymbol{D}}{\partial t}$

4. **Poynting Vector :** Represented by $\boldsymbol{S}$ and gives the rate of energy flow, is given by $\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H}$.

5. **Poynting Theorem**

$$\oint (\boldsymbol{E} \times \boldsymbol{H}) \cdot d\boldsymbol{S} = \frac{-\partial}{\partial t} \int_v \left(\frac{\mu}{2} \boldsymbol{H}^2 + \frac{\varepsilon}{2} \boldsymbol{E}^2 \right) dV - \int_v (\boldsymbol{E} \cdot \boldsymbol{J})\, dV$$

6. **Wave equations :**

$$\nabla^2 \boldsymbol{E} - \sigma \mu \frac{\partial \boldsymbol{E}}{\partial t} - \varepsilon \mu \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = 0$$

and

$$\nabla^2 \boldsymbol{H} - \sigma \mu \frac{\partial \boldsymbol{H}}{\partial t} - \varepsilon \mu \frac{\partial^2 \boldsymbol{H}}{\partial t^2} = 0$$

7. **Skin Depth :** Represent by δ and measures the depth at which the electromagnetic waves entering a conducting medium is damped to $1/e$ of its initial amplitude at its surface.

$$\therefore \quad \delta = 1/\beta = \frac{1}{\omega \sqrt{\mu \varepsilon}} \left[\frac{\sigma}{2 \omega \varepsilon} \right]^{-1/2}$$

$$= \sqrt{\frac{2}{\omega \sigma \mu}}$$

8. **Polarization :** This is time varying behaviour of electric field vector at any point in space. For example, if the electric field vector E_y and E_z are in phase, then the tip of electric field vector will pass through a straight line and the wave is said to be **linearly polarised**. Likewise we can have **circular or elliptical polarised** light depending when E_y and E_x vectors are of same magnitudes or different magnitudes and 90° phase difference in between them.

SHORT ANSWER QUESTIONS

1. The rate of decrease of stored energy in an electromagnetic cavity is given by

(a) $u = \frac{-\partial}{\partial t}\left(\frac{1}{2}\varepsilon \boldsymbol{E}^2\right)$ (b) $u = \frac{-\partial}{\partial t}\left(\frac{1}{2}\mu \boldsymbol{H}^2\right)$

(c) $u = \frac{-\partial}{\partial t}\left(\frac{1}{2}\varepsilon \boldsymbol{E}^2 - \frac{1}{2}\mu \boldsymbol{H}^2\right)$ (d) $u = \frac{-\partial}{\partial t}\left(\frac{1}{2}\mu \boldsymbol{H}^2 - \frac{1}{2}\varepsilon \boldsymbol{E}^2\right)$

2. Continuity equation is
(a) integral form of charge conservation law
(b) differential form of charge conservation law
(c) both of the above (d) none of the above.

3. Between the plates of a capacitor, when connected to an AC source
(a) no current flows.
(b) current flows due to motion of charge carriers.
(c) current flows but no charge is transported between the plates.
(d) conduction current as well as displacement current flows.

4. Which of the following is not a Maxwell equation

(a) $\nabla \times \boldsymbol{E} = \frac{-\partial \boldsymbol{B}}{\partial t}$ (b) $\nabla \cdot \boldsymbol{B} = 0$

(b) $\nabla \times \boldsymbol{D} = \boldsymbol{J}$ (d) $\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$

5. Velocity of light in free space is given by

(a) $c = \sqrt{\mu_o \varepsilon_o}$ (b) $c = \sqrt{\frac{\mu_o}{\varepsilon_o}}$

(c) $c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$ (d) $c = \sqrt{\frac{\varepsilon_o}{\mu_o}}$

6. Poynting vector is expressed as
(a) $\boldsymbol{E} \times \boldsymbol{H}$ (b) $\boldsymbol{H} \times \boldsymbol{E}$
(c) $\boldsymbol{H} \cdot \boldsymbol{E}$ (d) $(\boldsymbol{H} \times \boldsymbol{E}) \cdot dS$

7. The ratio of the phase velocity and the velocity of light is
(a) unity (b) more than unity
(c) less than unity (d) sometimes more sometimes less.

8. The field vector for electromagnetic wave in a conducting medium $\boldsymbol{B}$ and $\boldsymbol{H}$ have a phase difference ϕ then
(a) $\phi = \pi/2$ (b) $\phi = 0$ (c) $\phi = \pi$ (d) $\phi = \theta\ (\theta \neq 0)$

9. Skin depth δ is expressed as

(a) $\sqrt{\frac{2}{\omega \sigma \mu}}$ (b) $\sqrt{\frac{1}{\omega \sigma \mu}}$

(c) $\sqrt{\frac{2\omega}{\sigma \mu}}$ (d) $\sqrt{\frac{2\mu}{\omega \sigma}}$

10. What does $\nabla \cdot \boldsymbol{B} = 0$ signify ?
   (*a*) Faraday's law
   (*b*) Ampere's circuital law.
   (*c*) Gauss's law.
   (*d*) Monopoles does not exist.

Answers :

1. (*d*)	2. (*b*)	3. (*c*)	4. (*c*)	5. (*c*)
6. (*a*)	7. (*c*)	8. (*d*)	9. (*a*)	10. (*d*)

$$\frac{BC}{AD'} = \frac{c_1 t}{c_2 t} = \frac{c_1}{c_2}$$

$$\frac{\sin i}{\sin r} = \frac{c_1}{c_2} = \frac{\mu_2}{\mu_1} = \text{constant.}$$

This is Snell's law of refraction.

SUMMARY

1. **Wave motion :** This is a disturbance in material medium which travels from one plane to another by means of the medium.
2. **Transverse waves:** Wave motion in which the particles in a medium vibrate about their mean position at right angles to the motion.
3. **Longitudinal waves:** Wave motion in which the particles in a medium vibrate about their mean position along the direction of the motion .
4. **Equation of wave motion :** $\frac{d^2y}{dx^2} = 1/v \frac{d^2y}{dt^2}$ where 'v' is the velocity with which the wave propagates.
5. **Superposition principle :** According to this principle, when a medium is disturbed simultaneously by any number of waves, the instantaneous resultant displacement at any instant is the algebraic sum of the resultant displacement of the medium due to the individual waves in the absence of others.
6. **Interference of waves :** Two waves of same frequency moving in the same direction produces interference of waves.
7. **Beats :** Two waves of slightly different frequencies moving in the same direction produces waxing and waning of sound called beats.
8. **Stationary waves :** Two waves of same frequency moving in the opposite direction gives rise to stationary waves.
9. **Laws of string :** $\nu = \frac{1}{2l}\sqrt{T/m}$ where l is the string length T is the tension and m is the mass per unit length.
10. **Huygen's wave theory :** Each particle in a source of light sends, out waves in all direction in a hypothetical medium called ether.
11. **Wavefront :** This is a continuous locus of all neighbouring particles vibrating in the same phase.

SHORT ANSWER QUESTIONS

1. Which of the following expressions is that of a simple harmonic progressive wave
   (*a*) $a \sin \omega t$ (*b*) $a \sin \omega t \cos kt$
   (*c*) $a \sin (\omega t - kx)$ (*d*) $a \cos kx$

2. Velocity of wave motion is .... the velocity of the particle of the medium.

3. The equation of $y = a \sin\left(\frac{2\pi}{\lambda}\right)(vt - x)$ is expression for

(a) Stationary wave of single frequency along x-axis.

(b) a simple harmonic motion.

(c) a progressive wave of single frequency along x-axis.

(d) The resultant of two SHM's of slighlty different frequencies.

4. A wave equation which gives the displacement along the Y-direction is given by $y = 10^{-4} \sin(60t + 2x)$ where x and y are in metres and t is time in seconds. This represents a wave

(a) travelling with a velocity of 30 m/s in negative x direction.

(b) of wavelength π meters.

(c) of frequency $30/\pi$ hertz.

(d) of amplitude 10^{-4} meter travelling along the negative x-direction.

5. The equation of the transverse wave is $y = 10 \sin \pi(0.01x - 2t)$, where y and x are in cm and t in sec. Its frequency is

(a) 10 sec^{-1} (b) 2 sec^{-1} (c) 1 sec^{-1} (d) 0.01 sec^{-1}

6. A tuning fork originally in motion with another fork of frequency 260 produces 4 beats per second, when a little wax is attached to it. What is the frequency now ?

(a) 260 (b) 264 (c) 256 (d) 258

7. Two sounding bodies producing progressive waves given by :

$$y_1 = 4 \sin 400\,\pi t$$

and

$$y_2 = 3 \sin 404\,\pi t$$

are situated very near to the ears of a person who will hear.

(a) 2 beats per second with intensity ratio (4/3) between maxima and minima.

(b) 2 beats per second with intensity ratio (49/1) between maxima and minima.

(c) 4 beats per second with intensity ratio (7/1) between maxima and minima.

(d) 4 beats per socond with intensity ratio (4/3) between maxima and minima.

8. In stationary wave

(a) strain is maximum at antinodes. (b) strain is maximum at nodes.

(c) strain is minimum at nodes. (d) amplitude is zero at all points.

9. With the increase in streching force of a wire, its frequency.

(a) increases (b) decreases (c) may increase or decrease (d) no change

10. What happens to the frequency of a stretched wire when its length and diameter are increased.

(a) decreases (b) increases

(c) no change (d) may increase or decrease.

11. Transverse waves are generated in two uniform steel wires A and B by attaching their free ends to a vibrating source of frequency 500 Hz. The diameter of wire A is half that of wire B and the tension on wire A is half that on wire B. What is the ratio of the velocities of waves in wires A and B.

(a) 1:2 (b) 2:1 (c) $1 : \sqrt{2}$ (d) $\sqrt{2:1}$

12. There are two strings of equal length and diameter but the densities are in the ratio 1:2. They are stretched by a tension T. The ratio of frequencies will be

(a) $\sqrt{2}:1$ (b) 1:4 (c) 1:2 (d) 2:1

2. Velocity of wave motion is .... the velocity of the particle of the medium.

3. The equation of $y = a \sin\left(\frac{2\pi}{\lambda}\right)(vt - x)$ is expression for

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(b) a simple harmonic motion.
(c) a progressive wave of single frequency along x-axis.
(d) The resultant of two SHM's of slighlty different frequencies.

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12. There are two strings of equal length and diameter but the densities are in the ratio 1:2. They are stretched by a tension T. The ratio of frequencies will be

(a) $\sqrt{2}:1$ (b) 1:4 (c) 1:2 (d) 2:1

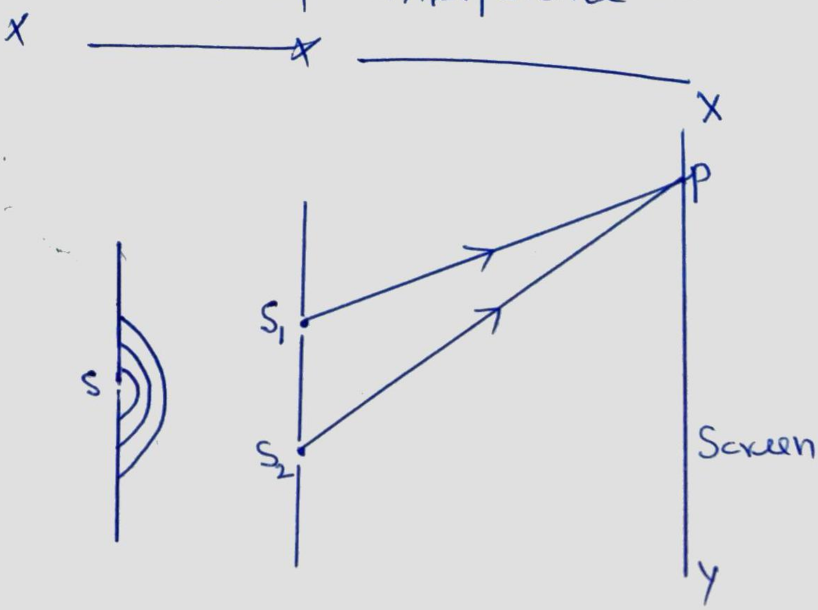
13. Huygen's conception of secondary waves
    (*a*) allows us to find the focal length of a thick lens
    (*b*) gives us the magnifying power of a microscope.
    (c) is a geometrical method to find the position of a wavefront.
    (*d*) is used to determine the velocity of light.

14. In Huygen's wave theory, the locus of all point in the same state of vibration is called
    (*a*) half period zone
    (*b*) wavefront
    (*c*) a ray
    (*d*) vibrator

Answers :

1. (*c*)	2. quite different from	3. (*c*)	4. (a,b,c,*d*)
5. (*c*)	6. (*c*),	7. (*b*),	8. (*b*)
9. (*a*)	10. (*a*)	11. (*d*)	12. (*a*)
13. (*c*)	14. (*b*)		

Analytical Treatment of Interference —



Consider a monochromatic Source ~~S~~ of light 'S' emitting waves and S_1 and S_2 be the two narrow Slits equidistant from the Source S. We shall Calculate or Investigate the resultant intensity of light at a point P of the Screen XY placed parallel to S_1 and S_2 as a result of interference. Let a_1 and a_2 be the amplitudes of the waves from S_1 and S_2 and δ be the phase difference between the two waves reaching at point P. If y_1 and y_2 are the displacement of the waves, then

$$y_1 = a_1 \sin \omega t \quad \text{——— (1)}$$

$$y_2 = a_2 \sin(\omega t + \delta) \quad \text{——— (2)}$$

According to the principle of Superposition, the resultant displacement

$$y = y_1 + y_2$$

$$y = a_1 \sin\omega t + a_2 \sin(\omega t + \delta)$$

$$= a_1 \sin\omega t + a_2 \sin\omega t \cos\delta + a_2 \cos\omega t \sin\delta$$

$$y = \sin\omega t(a_1 + a_2 \cos\delta) + \cos\omega t(a_2 \sin\delta) \quad \text{--- (3)}$$

Let $a_1 + a_2 \cos\delta = R\cos\theta$ --- (4)

$a_2 \sin\delta = R\sin\theta$ --- (5)

Substituting Eqn (4) and Eqn 5 in Eqn (3) we get

$$y = \sin\omega t \, R\cos\theta + \cos\omega t \, R\sin\theta$$

$$y = R\sin(\omega t + \theta) \quad \text{--- (6)}$$

Hence, the resultant vibration at P is simple harmonic vibration of amplitude R.. The resultant amplitude can be obtained by squaring Eqn (4) and Eqn (5) and adding.

$$R^2\cos^2\theta + R^2\sin^2\theta = (a_1 + a_2\cos\delta)^2 + (a_2\sin\delta)^2$$

$$R^2(\cos^2\theta + \sin^2\theta) = a_1^2 + a_2^2\cos^2\delta + 2a_1a_2\cos\delta + a_2^2\sin^2\delta$$

$$R^2 = a_1^2 + a_2^2(\sin^2\delta + \cos^2\delta) + 2a_1a_2\cos\delta$$

$$R^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta \quad \text{--- (7)}$$

The resultant intensity at P is given by square of amplitud

$$I = R^2 = (a_1)^2 + (a_2)^2 + 2a_1a_2\cos\delta$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta \quad \text{--- (8)}$$

The phase difference δ between the two waves at point P is

$$\delta = \frac{2\pi}{\lambda} \times \text{Path diff.}$$

$$\delta = \frac{2\pi}{\lambda} \cdot (S_2P - S_1P)$$

1) Condition for maximum intensity —

I is maximum when $\cos\delta = +1$

or $\delta = 2n\pi$ $\quad n = 0, 1, 2, \ldots$

$\delta = 0, 2\pi, 4\pi \ldots$

~~or Path difference~~

or Path difference $= n\lambda$

So $I = (a_1)^2 + (a_2)^2 + 2a_1a_2 = (a_1+a_2)^2$

$I_{max} = I_1 + I_2 + 2\sqrt{I_1I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$ (9)

This shows that the resultant intensity is greater than the sum of two separate intensities.

2) Condition for minimum intensity —

The intensity I is maximum when ~~...~~ $\cos\delta = -1$

Phase diff $\delta = (2n+1)\pi$

$\delta = \pi, 3\pi, 5\pi \ldots$ $\quad n = 0, 1, 2, \ldots$

or Path difference $= (2n+1)\frac{\lambda}{2}$

$$I_{min} = (a_1)^2 + (a_2)^2 - 2a_1a_2$$
$$I_{min} = (a_1 - a_2)^2$$
$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

When $a_1 = a_2 = a$ then

$$I_{max} = a_1^2 + a_2^2 + 2a_1a_2$$
$$= a^2 + a^2 + 2a \cdot a$$
$$I_{max} = 4a^2$$

or $$I_{min} = a_1^2 + a_2^2 - 2a_1a_2$$
$$I_{min} = 0$$

Q1 Two Coherent Sources whose intensity ratio is 81 : 1 produce interference fringes. Deduce the ratio of maximum intensity to minimum intensity.

Sol^n Given $\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{81}{1}$

So $\frac{a_1}{a_2} = \frac{9}{1}$

$a_1 = 9a_2$

Now $\frac{I_{max}}{I_{min}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \frac{(9a_2+a_2)^2}{(9a_2-a_2)^2}$

$\frac{I_{max}}{I_{min}} = \frac{(10a_2)^2}{(8a_2)^2}$

$\frac{I_{max}}{I_{min}} = \frac{100}{64} = \frac{25}{16}$

$I_{max} : I_{min} = 25 : 16$ Ans

Q2 The ratio of intensity of the maximum and minimum of interference fringes is 25 : 9. Determine ~~Deduce~~ the ratio between the amplitude and intensities of the two interference beam.

Soln Given $\frac{I_{max}}{I_{min}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \frac{25}{9}$

So $\frac{a_1+a_2}{a_1-a_2} = \frac{5}{3}$

$5a_1 - 5a_2 = 3a_1 + 3a_2$

$2a_1 = 8a_2$

$\frac{a_1}{a_2} = \frac{8}{2} = 4$

$\frac{a_1}{a_2} = \frac{4}{1}$

$a_1 : a_2 = 4 : 1$

If I_1 and I_2 be the intensities of the two interfering beams then

$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{4^2}{1} = \frac{16}{1}$

$I_1 : I_2 = 16 : 1$

Q3 Two Coherent Sources of intensity ratio α interfere. Prove that in the interference pattern.

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{\alpha}}{1+\alpha}$$

Sol^n Let I_1 and I_2 be the intensities and a_1 and a_2 the amplitudes of the waves from the two coherent sources.

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} \quad \text{or} \quad \frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\alpha}$$

(given)

$$I_{max} = (a_1+a_2)^2 \quad \text{and} \quad I_{min} = (a_1-a_2)^2$$

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{(a_1+a_2)^2 - (a_1-a_2)^2}{(a_1+a_2)^2 + (a_1-a_2)^2}$$

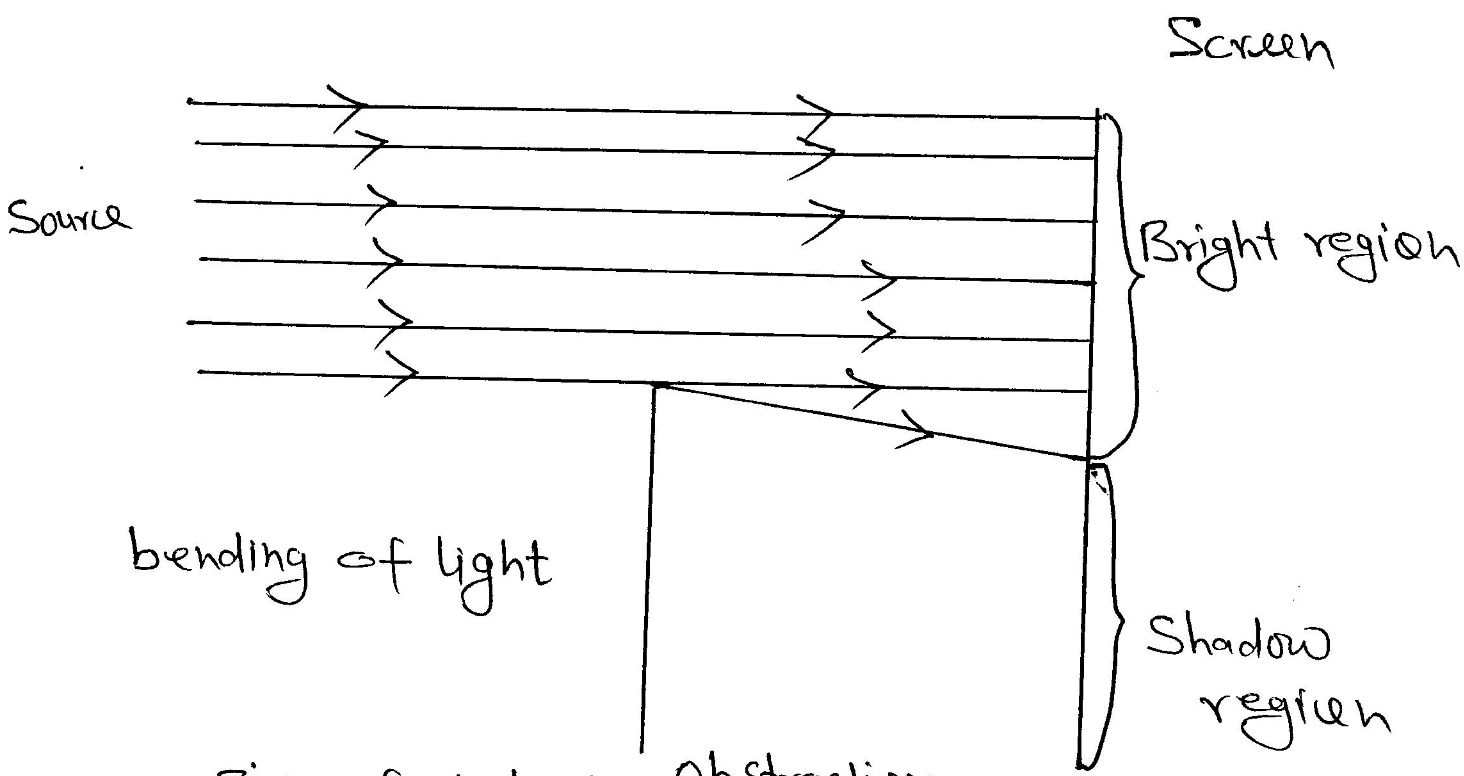
$$= \frac{2a_1a_2}{a_1^2 + a_2^2} = \frac{2(a_1/a_2)}{\left(\frac{a_1}{a_2}\right)^2 + 1}$$

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{\alpha}}{\alpha+1}$$

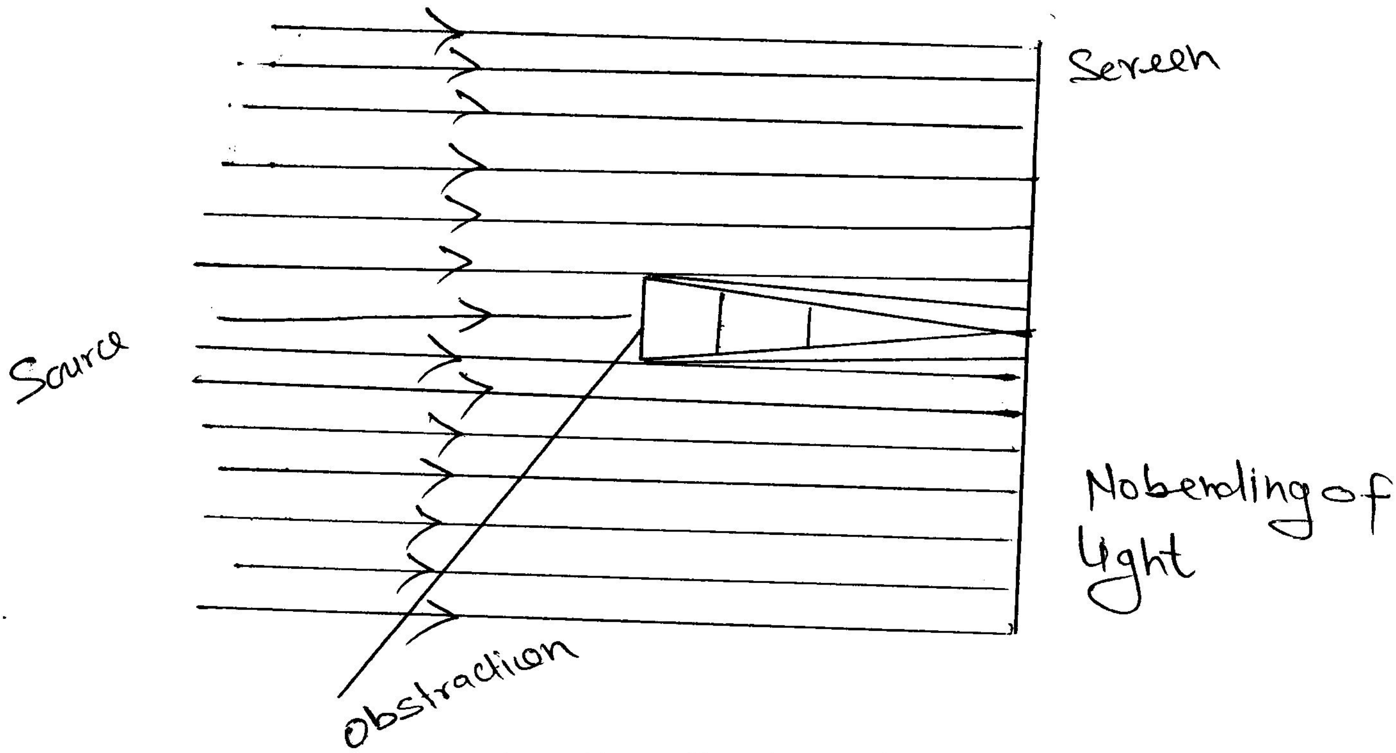
$$= \frac{2\sqrt{\alpha}}{1+\alpha}$$

Diffraction of light

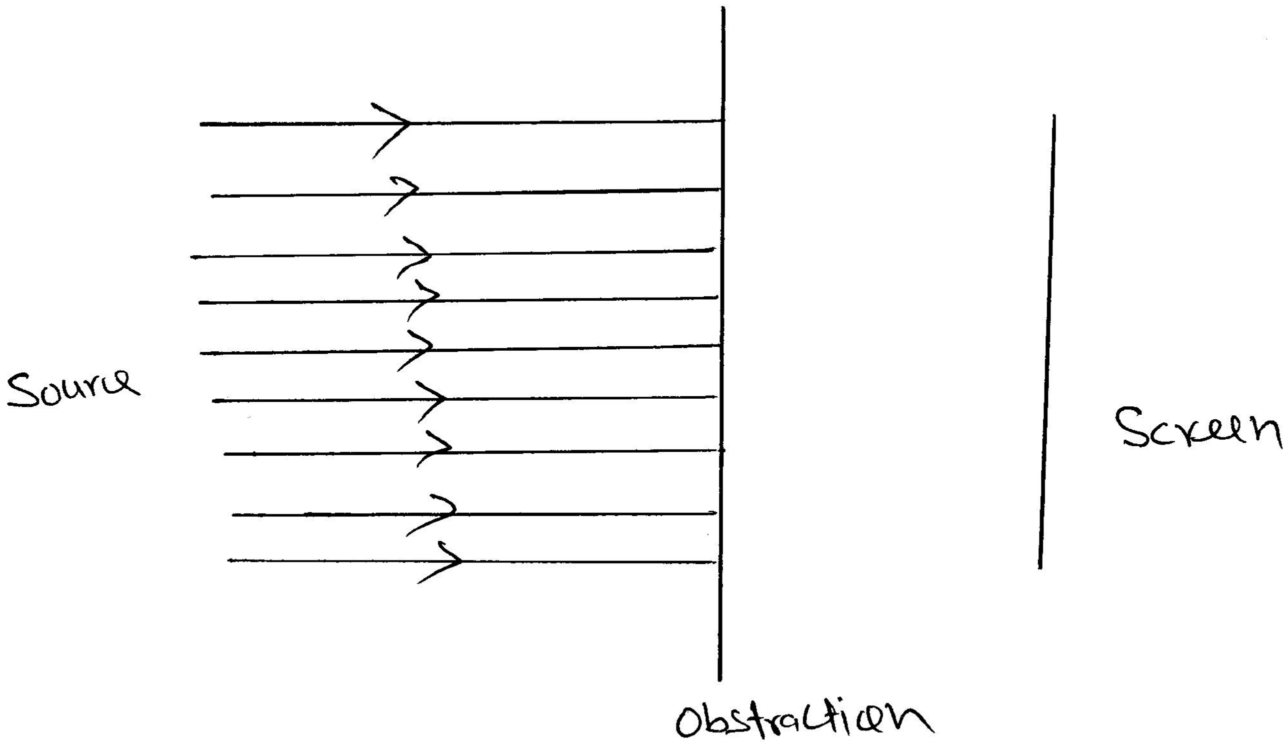
Diffraction —



Size of obstraction is Comparable the wave length of light



Size of obstraction is less than the wave length of light



Size of obstraction is greater than the Wave length of light

No bending of light

When light falls on obstacles or small apertures whose size is comparable with the wavelength of light, there is a departure from straight lines propagation, the light bends round the corners of the obstacles or apertures and enters in the geometrical shadow. The bending of light is called diffraction.

Types of Diffraction

1) Fresnel's diffraction

2) Fraunhofer's diffraction

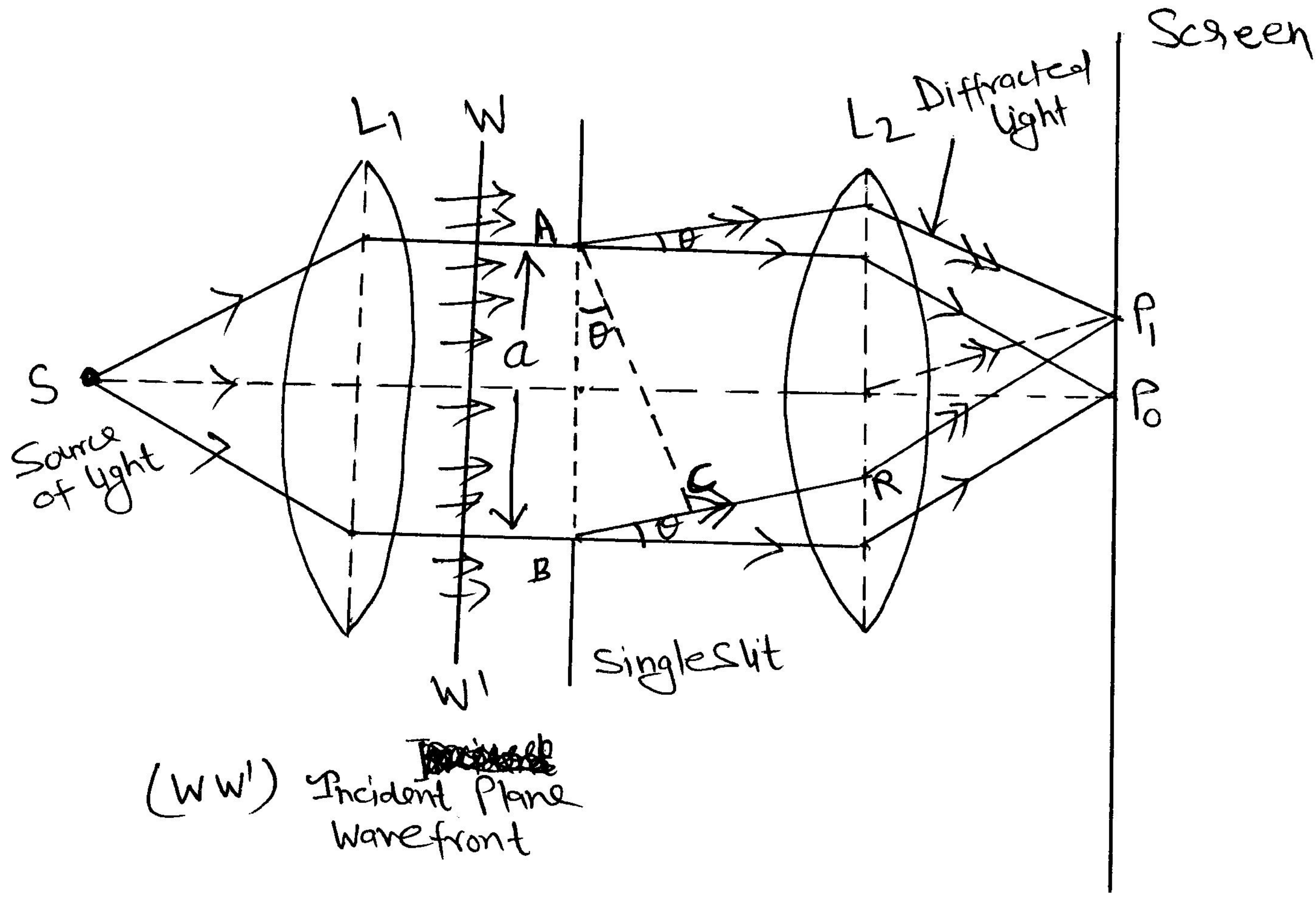
Fresnel diffraction —

The Source and the Screen are at finite distances from the obstacle.

Fraunhofer diffraction —

Source and the Screen are at infinite distane from the obstacle.

Fraunhofer diffraction for Single Slit



The path difference between Secondary wavelets from A and B in direction $\theta = BC = AB \sin\theta = a \sin\theta$

and Corresponding Phase difference $\phi = \frac{2\pi}{\lambda} \times$ Path difference

$$\phi = \frac{2\pi}{\lambda} \cdot a\sin\theta$$

Consider that the width of slit is divided into n equal parts, each parts being the source of Secondary wavelets. The amplitude of the wave due to each part is equal to a but their phase will vary gradually from 0 to $\frac{2\pi}{\lambda}(a\sin\theta)$, as the path difference between the rays originating from points in AB vary from 0 to $a \sin\theta$.

Thus, the phase difference between the waves from any two successive parts of the slit AB would be,

$$\frac{1}{n}\left(\frac{2\pi}{\lambda} a \sin\theta\right) = \delta \text{ (let)}$$

According to the theory of composition of n simple harmonic motions of equal amplitude (a) and common phase difference between successive vibrations,
The resultant amplitude at P is given by,

$$R = a \frac{\sin(n\delta/2)}{\sin(\delta/2)}$$

$$= a \frac{\sin\left\{(\pi a \sin\theta)/\lambda\right\}}{\sin\left\{(\pi a \sin\theta)/n\lambda\right\}}$$

$$= a \frac{\sin\alpha}{\sin\left(\frac{\alpha}{n}\right)} \qquad \left\{\text{let } \alpha = \frac{\pi a \sin\theta}{\lambda}\right\}$$

for small angle $\sin\frac{\alpha}{n} = \frac{\alpha}{n}$

So

$$R = a \frac{\sin\alpha}{\frac{\alpha}{n}}$$

$$R = n a \frac{\sin\alpha}{\alpha}$$

$$R = A \frac{\sin\alpha}{\alpha} \quad \text{——— (1)} \qquad \text{where } \{na = A\}$$

The resultant intensity at P, which is proportional to the square of resultant amplitude R, is given by

$$I = R^2 = \left(A\frac{\sin\alpha}{\alpha}\right)^2$$

$$I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{—— (2)}$$

Eqn (2) is the required expression for the intensity distribution due to Fraunhofer diffraction at a single slit.

Positions of maxima and minima ——

principal maxima ——

The resultant amplitude can be expanded as

$$R = \frac{A}{\alpha}\sin\alpha$$

$$= \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{\lfloor 3} + \frac{\alpha^5}{\lfloor 5} - \frac{\alpha^7}{\lfloor 7} + \cdots\right]$$

$$= \frac{A}{\alpha}\cdot\alpha\left[1 - \frac{\alpha^2}{\lfloor 3} + \frac{\alpha^4}{\lfloor 5} - \frac{\alpha^6}{\lfloor 7} + \cdots\right]$$

$$R = A\left[1 - \frac{\alpha^2}{\lfloor 3} + \frac{\alpha^4}{\lfloor 5} - \frac{\alpha^6}{\lfloor 7} + \cdots\right]$$

So R will be maximum, when $\alpha = 0$

or $\frac{\pi}{\lambda} a\sin\theta = 0$

$\sin\theta = 0$

$\boxed{\theta = 0}$

Thus the maximum value of resultant amplitude R at C is 'A' and the corresponding maximum intensity $\boxed{I_{max} \propto A^2}$

positions of Minima —

From Eqn ② it is clear that the intensity is minimum when $\sin\alpha = 0$ and $\alpha \neq 0$

i.e $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi \ldots \pm n\pi$

or $\alpha = \pm n\pi$ where $n = 1, 2, 3 \ldots$

$$\alpha = \frac{\pi a \sin\theta}{\lambda} = n\pi$$

$$a \sin\theta = n\lambda$$

$$\boxed{a \sin\theta = \pm n\lambda} \quad \text{——— ③}$$

Where $n = 1, 2, 3 \ldots$ gives the direction of first, second, third ... minima

Here $n \neq 0$ because $n = 0$ or $\theta = 0$ give the direction of principal maxima.

Secondary Maxima

In addition to principal maxima, there are less intense secondary maxima between equally spaced minima.

For finding the positions of secondary maxima, differentiate eqn (2) with respect to α, equate to zero

$$I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$\frac{dI}{d\alpha} = A^2\, 2\,\frac{\sin\alpha}{\alpha}\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right] = 0$$

i.e. $\frac{\sin\alpha}{\alpha} = 0$ or $\sin\alpha = 0$

~~or~~ and $\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$

$$\alpha\cos\alpha - \sin\alpha = 0$$

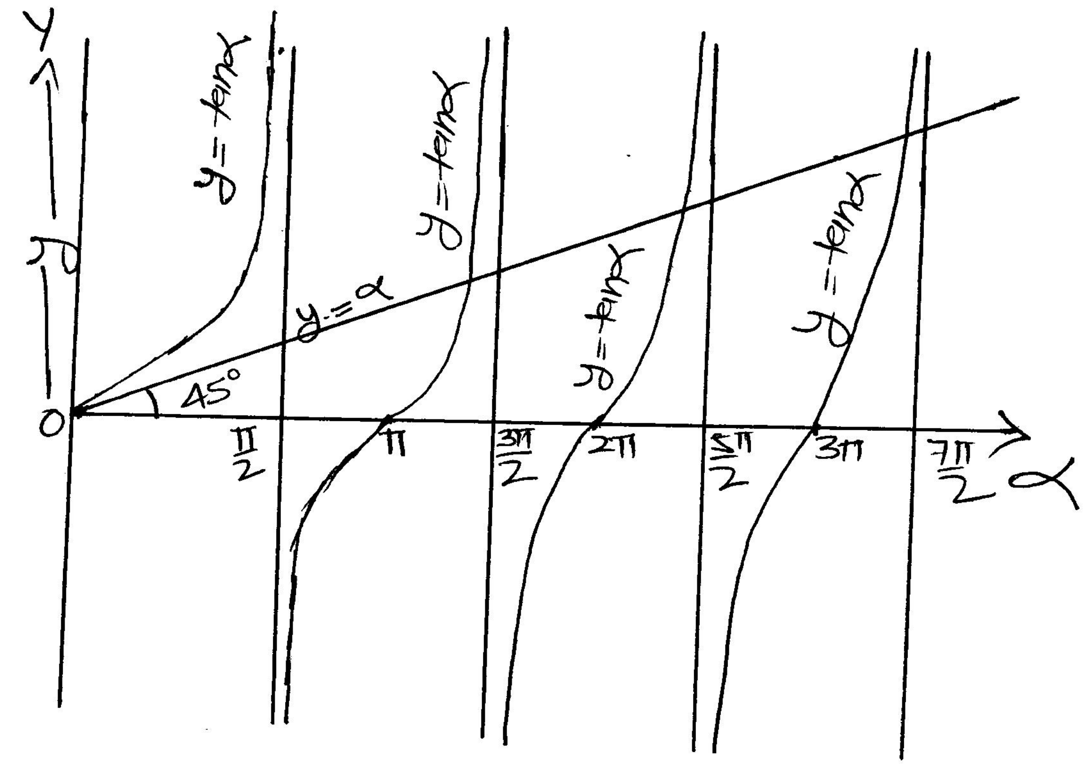
or $\alpha = \tan\alpha$

The eqn $\sin\alpha = 0$ gives the positions of minima except when $\alpha = 0$.

Thus the position of secondary maxima is given by eqn $\alpha = \tan\alpha$ ——— (4)

The Eqn (4) Can be Solved Graphically by plotting the Curves according to the Simultaneous Eqn

$$y = \alpha \text{ and } y = \tan\alpha$$



The first value of $\alpha = 0$ gives principal maxima. The remaining value of α which give the Secondary maxima are $\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2} \ldots\ldots$

The direction of Secondary maxima are given by the relation

$$\alpha = \pm \frac{(2n+1)\pi}{2}$$

where $n = 1, 2, 3, \ldots$

or $\frac{\pi}{\lambda} a\sin\theta = \pm \frac{(2n+1)\pi}{2}$

$\left\{ \alpha = \frac{\pi}{\lambda} a\sin\theta \right\}$

or $\boxed{a\sin\theta = \pm (2n+1)\lambda/2}$

1) for principal maxima, $\alpha = 0$

$$I = I_0$$

2) Intensity of first secondary maxima

$$\alpha = 3\pi/2$$

$$I_1 = A^2 \frac{\left(\sin \frac{3\pi}{2}\right)^2}{\left(\frac{3\pi}{2}\right)^2} = \cancel{4}$$

$$= \frac{4}{9\pi^2} A^2 = \frac{A^2}{22} = \frac{I_0}{22}$$

$$I_1 = \frac{I_0}{22}$$

3) Intensity of second secondary maxima.

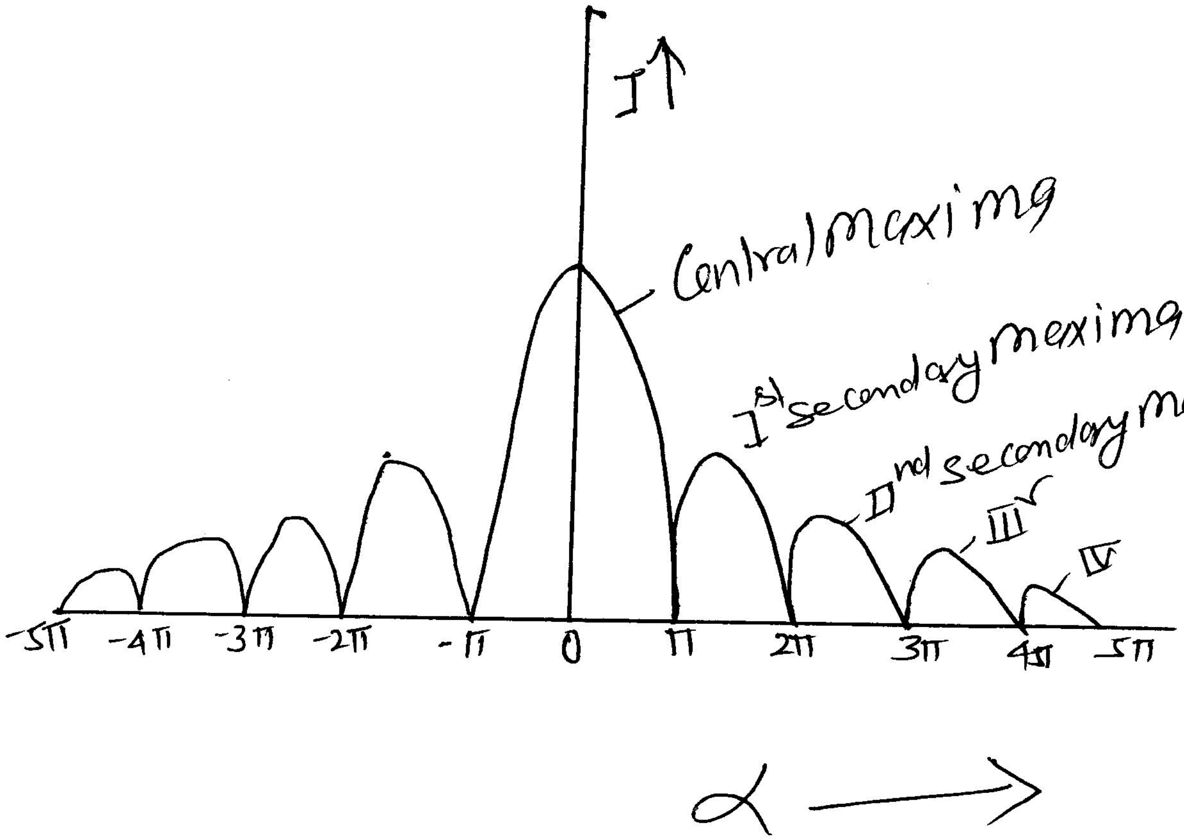
$$\alpha = 5\pi/2$$

$$I_2 = A^2 \frac{\left(\sin 5\pi/2\right)^2}{\left(\frac{5\pi}{2}\right)^2}$$

$$= A^2 \cdot \frac{4}{25\pi^2}$$

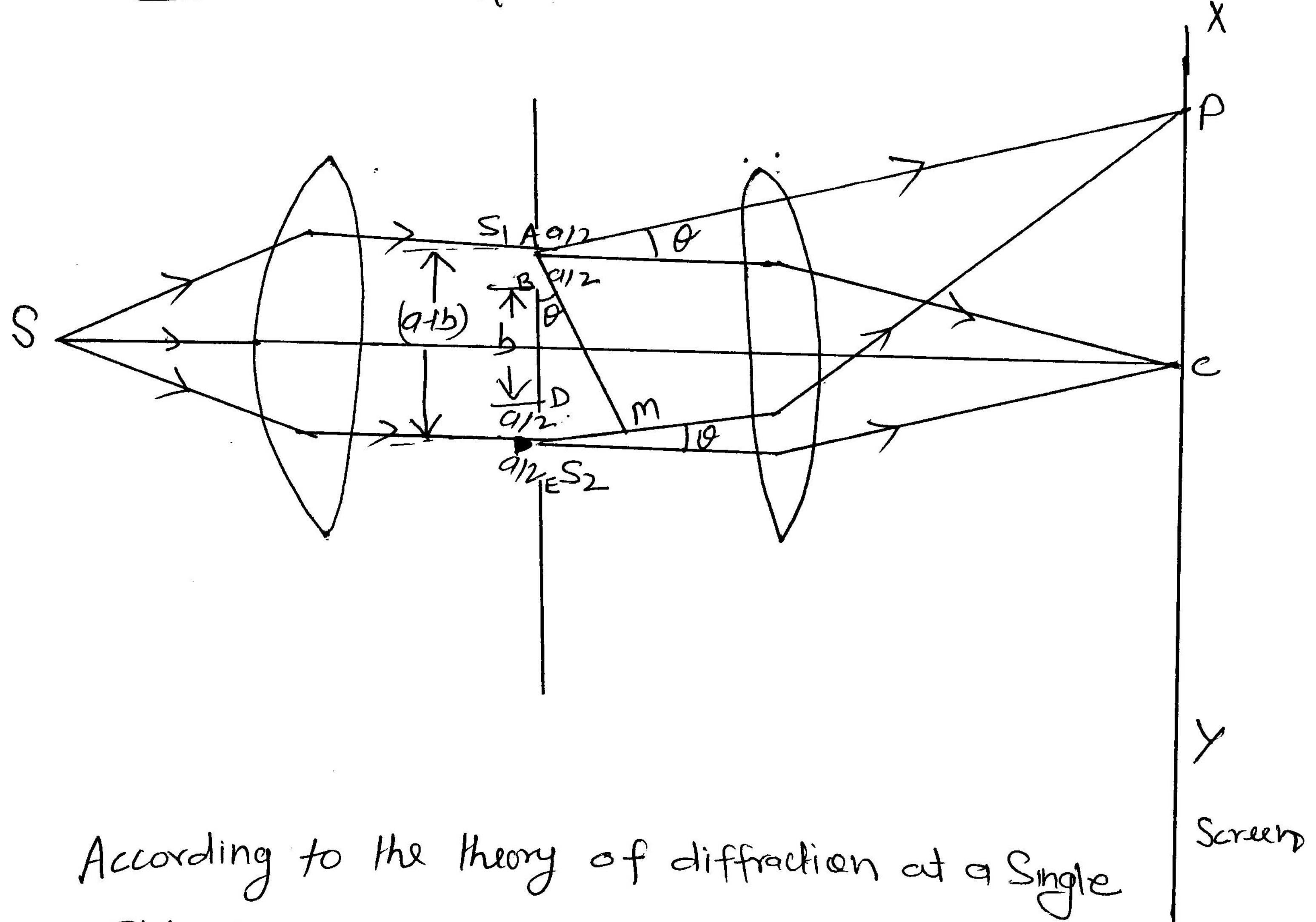
$$= \frac{A^2}{62} = \frac{I_0}{62}$$

$$I_2 = \frac{I_0}{62}$$



I
Central maxima
Ist secondary maxima
IInd secondary maxi
III
IV
-5π
-4π
-3π
-2π
-π
0
π
2π
3π
4π
5π
α

Fraunhofer's Diffraction at a Double Slit —



According to the theory of diffraction at a single slit, the resultant amplitude due to all the wavelets diffracted from each slit is given by

$$R = A\frac{\sin\alpha}{\alpha} \qquad (1)$$

and the resultant phase in this direction is

$$\alpha = \frac{\pi a \sin\theta}{\lambda} \qquad (2)$$

We can therefore, replace all the secondary wavelets originate from the slit AB by a single wave from its middle point S_1. Similarly all the secondary wavelets from its middle point S_2.

Hence resultant amplitude at point P on the Screen will be due to inteference between two waves originate from S_1 and S_2.

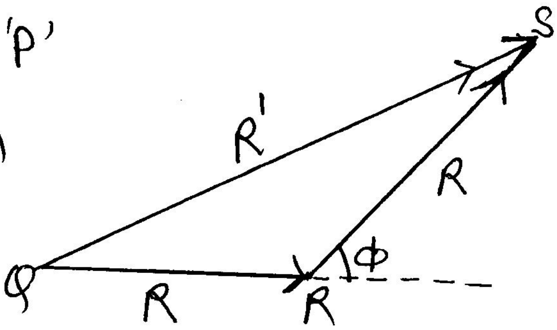
So, the path difference between these two waves at point P will be.

$$S_2m = (a+b)\sin\theta \qquad \text{—— (3)}$$

Therefore, the phase difference between them is

$$\phi = \frac{2\pi}{\lambda}(a+b)\sin\theta \qquad \text{—— (4)}$$

So, the resultant amplitude at point 'P' due to these two wave originating from S_1 and S_2 ~~waves~~ having Phase difference ϕ.



$$QS^2 = QR^2 + RS^2 + 2(QR)(RS)\cos\phi$$

$$= R^2 + R^2 + 2R\cdot R\cos\phi$$

$$R'^2 = 2R^2(1+\cos\phi)$$

$$R'^2 = 2R^2\left(1 + 2\cos^2\frac{\phi}{2} - 1\right)$$

$$R' = 4R^2\cos^2\frac{\phi}{2} \qquad \text{—— (5)}$$

Put the value of R and ϕ from eqn (1) and (4) in Eqn (5)

$$R'^2 = 4A^2 \frac{\sin^2\alpha}{\alpha^2} \cos^2\beta$$

Where $\beta = \frac{\phi}{2} = \frac{2\pi}{2\lambda}(a+b)\sin\theta$

$$\beta = \frac{\pi}{\lambda}(a+b)\sin\theta$$

Therefore the resultant intensity at 'P' is

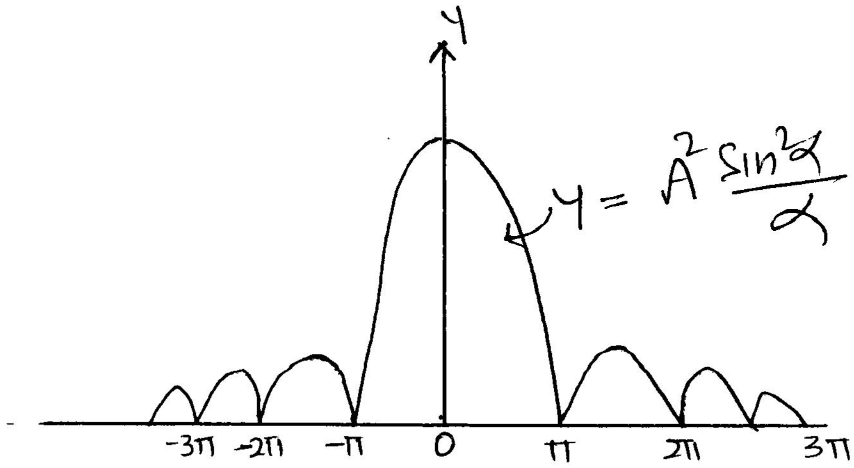
$$I = R'^2 = 4A^2 \frac{\sin^2\alpha}{\alpha^2} \cos^2\beta \quad \text{——— (6)}$$

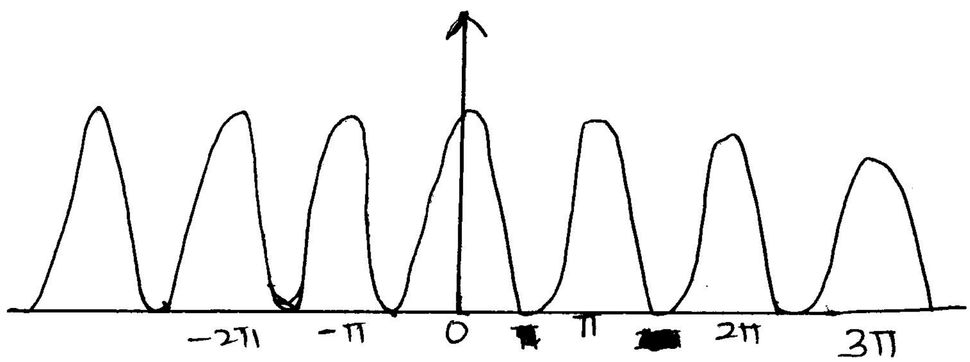
This is the required Expression for the intensity distribution due to a Fraunhofer diffraction at a double slit.

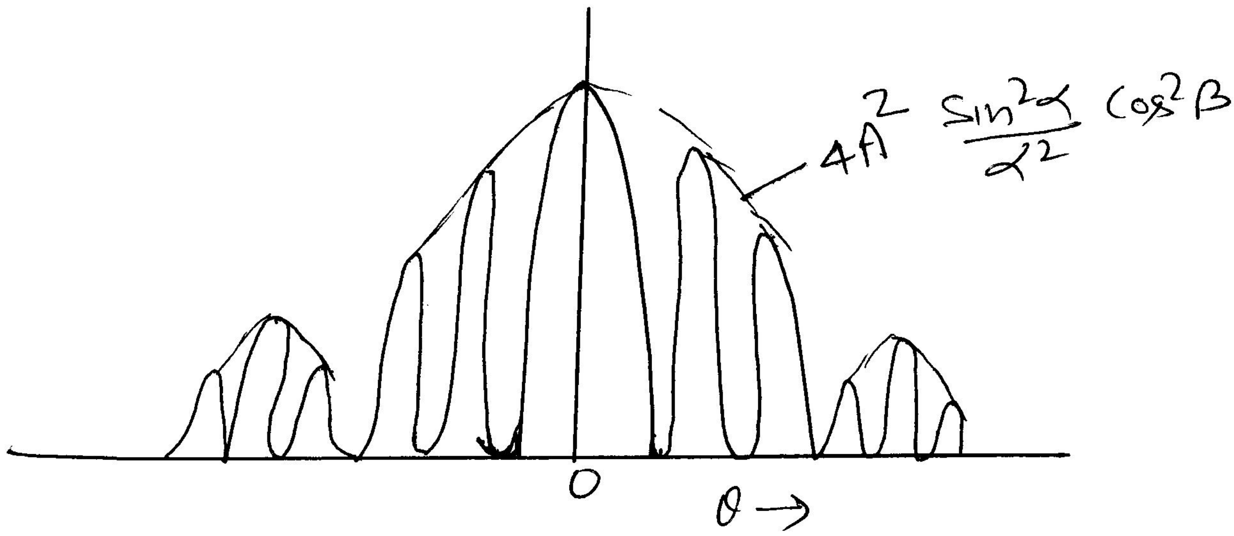
So Resultant intensity depends upon two factors

1) $\frac{\sin^2\alpha}{\alpha^2}$ which gives diffraction pattern due to each Single Slit.

2) $\cos^2\beta$ which gives the interference pattern due to two waves of Same amplitude originating from mid points of Slit S_1 and S_2.







Intensity will be maximum, when $\cos^2\beta = 1$

or $\beta = \pm n\pi$ where $n = 0, 1, 2 \ldots$

$$\frac{\pi(a+b)\sin\theta}{\lambda} = n\pi$$

$$\boxed{(a+b)\sin\theta = n\lambda} \quad \text{——} \; (1)$$

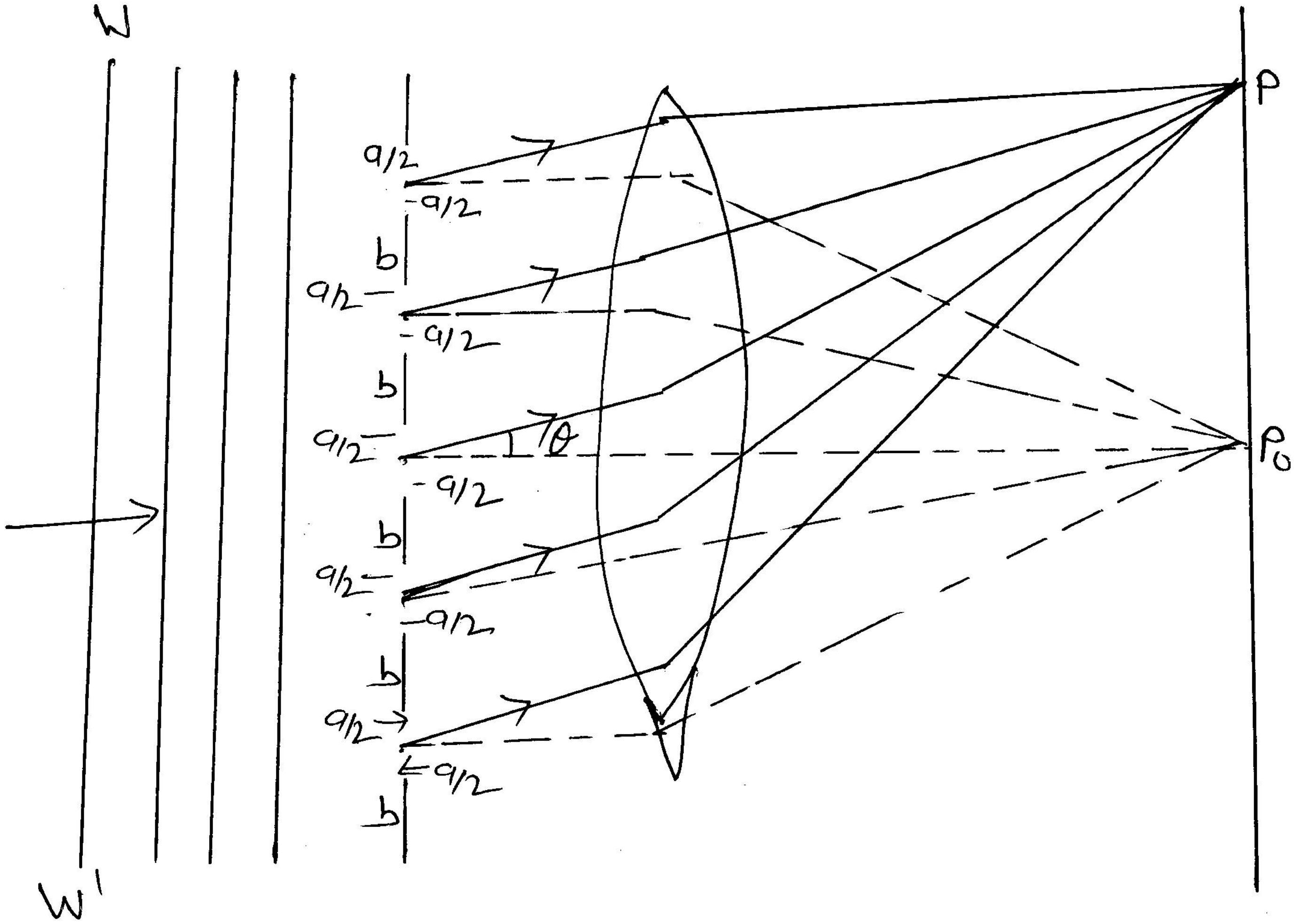
And intensity will be minimum, when $\cos^2\beta = 0$

or $\beta = (2n+1)\frac{\pi}{2}$

$$\frac{\pi(a+b)\sin\theta}{\lambda} = (2n+1)\pi/2$$

$$\boxed{(a+b)\sin\theta = \pm(2n+1)\frac{\lambda}{2}}$$

A Plane Transmission Diffraction Grating —
(N-Slits Diffraction)



An arrangement consisting of a large number of parallel, equidistant narrow rectangular slits of the same width is known as <u>diffraction grating</u>.

Intensity Distribution

Consider a beam of monochromatic light of wavelength λ is incident on a plane transmission grating has 'N' parallel lines or slits with each of width 'a' and opaque space in between two slits each of width 'b'. Light diffracted from all these slits reach the screen with the same amplitude but with different phases.

Now consider a point P on the screen where waves diffracted at an angle θ superimpose.

From the theory of diffraction at single slit, we know that the disturbance originate from all points of the slits can be summed up into single wavelet A.

$$A = A_0 \frac{\sin\alpha}{\alpha}$$

Where $\alpha = \frac{\pi}{\lambda} a \sin\theta$

Thus the path difference between the waves from any two ~~[illegible]~~ consecutive slit is $(a+b)\sin\theta$.

Therefor the corresponding phase diff is $\frac{2\pi}{\lambda}(a+b)\sin\theta$.

i.e. $\beta = \frac{2\pi}{\lambda}(a+b)\sin\theta$

The resultant amplitude of N waves of Equal amplitude $\left(\frac{A\sin\alpha}{\alpha}\right)$ in the direction of θ is A'

$$A' = A\frac{\sin\alpha}{\alpha}\frac{\sin N\beta}{\sin\beta} \quad \text{——} \quad (1)$$

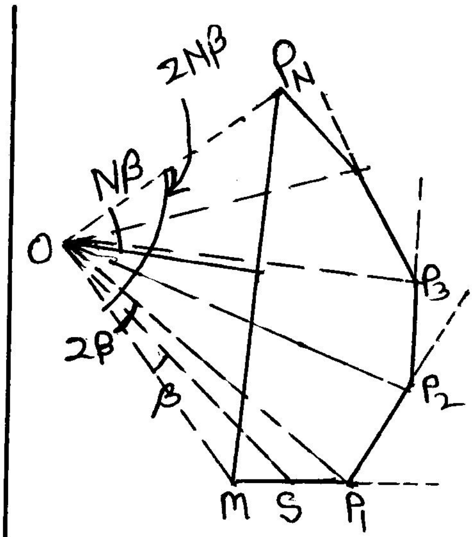
Corresponding intensity at point 'P' is

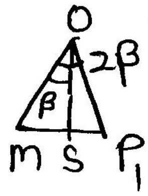
$$I = A'^2 = \left(A\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

$$I = A^2\frac{\sin^2\alpha}{\alpha^2}\frac{\sin^2 N\beta}{\sin^2\beta} \quad \text{——} \quad (2)$$

The factor $A^2\frac{\sin^2\alpha}{\alpha^2}$ in eqn (2) represents the intensity distribution due to diffraction at a single slit. Where as the factor $\frac{\sin^2 N\beta}{\sin^2\beta}$ gives the distribution of intensity in the diffraction pattern due to the interference in the waves from all the N slits.

both effect combine together give intensity pattern of light diffracted by plane transmission grating.





$$\frac{MS}{OM} = \sin\beta$$

$$MS = OM\sin\beta$$

$$MP_1 = 2MS$$

$$MP_1 = 2OM\sin\beta$$

Similarly

$$MP_N = 2OM\sin N\beta$$

$$MP_N = 2\frac{MP_1}{2}\frac{\sin N\beta}{\sin\beta}$$

$$MP_N = MP_1\frac{\sin N\beta}{\sin\beta}$$

MP_1 is the amplitude due to single slit $= A\frac{\sin\alpha}{\alpha}$

So,

$$MP_N = \frac{A\sin\alpha}{\alpha}\frac{\sin N\beta}{\sin\beta}$$

that is resultant amplitude due to N vibrations.

Principal maxima

Intensity would be maximum when

$\sin\beta = 0$ or $\beta = \pm n\pi$

also where $n = 0, 1, 2 \ldots$

$\sin N\beta = 0$

Thus $\frac{\sin N\beta}{\sin\beta} = \frac{0}{0}$ i.e. undefined

$$\left\{\begin{array}{l} \beta = \frac{\pi}{\lambda}(a+b)\sin\theta \\ \text{if } \beta = 0 \\ \sin\theta = 0 \\ \theta = 0 \end{array}\right\}$$
Principal maxima

So, by using L Hospital's Rule

$$\lim_{\beta \to \pm n\pi} \frac{\sin N\beta}{\sin\beta}$$

$$\lim_{\beta \to \pm n\pi} \frac{N\cos N\beta}{\cos\beta}$$

$$= \frac{N\cos N(\pm n\pi)}{\cos(\pm n\pi)}$$

$$= N$$

So, Intensity of principal maxima

$$\boxed{I_p = A^2 \frac{\sin^2\alpha}{\alpha^2} \cdot N^2}$$

Thus if we increase the number of Slits (N), the intensity of principal maxima increase.

Secondary Minima

When $\sin N\beta = 0$ but $\sin\beta \neq 0$

Then intensity $I = A^2 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta} = 0$

(Which is minimum)

So

$$N\beta = \pm m\pi$$

$$N \frac{\pi}{\lambda}(a+b)\sin\theta = \pm m\pi$$

$$N(a+b)\sin\theta = \pm m\lambda$$

$$\boxed{N(a+b)\sin\theta = \pm m\lambda}$$

where

$m = 1, 2, 3 \cdots (N-1)$

If $m=0$ gives principal maxima and $m=N$ also gives principal maxima. So $m = 1, 2, 3 \cdots (N-1)$ gives minima. There is $(N-1)$ minima between two maxima

Secondary Maxima —

Since there are $(N-1)$ minima between two maxima then there must be $(N-2)$ maxima between two principal maxima. To find the position of these secondary maxima we differentiate intensity eqn $\left[I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\left(\frac{\sin N\beta}{\sin\beta}\right)^2\right]$ w.r.t. β and equating it to zero.

i.e.

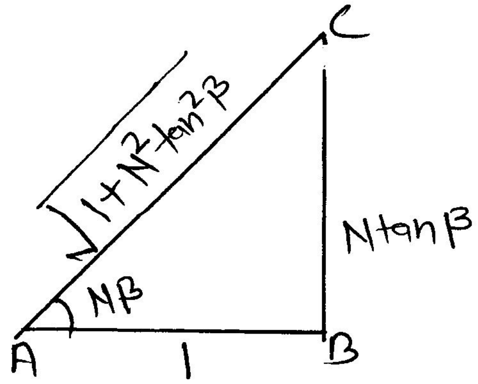
$$\frac{dI}{d\beta} = 0 \quad \text{—————} \quad (1)$$

$$\frac{dI}{d\beta} = \frac{A^2\sin^2\alpha}{\alpha^2}\cdot 2\,\frac{\sin N\beta}{\sin\beta}\left[\frac{N\sin\beta\cos N\beta - \sin N\beta\cos\beta}{\sin^2\beta}\right] = 0$$

$$N\sin\beta\cos N\beta - \sin N\beta\cos\beta = 0$$

$$\cancel{\tan N\beta = \tan\beta}$$

$$\tan N\beta = N\tan\beta \quad \text{———} \quad (2)$$



from ΔABC

$$\sin N\beta = \frac{N\tan\beta}{\sqrt{1+N^2\tan^2\beta}}$$

So

$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2\tan^2\beta}{\sin^2\beta\,(1+N^2\tan^2\beta)} = \frac{N^2\tan^2\beta}{\sin^2\beta\left(\frac{\cos^2\beta+N^2\sin^2\beta}{\cos^2\beta}\right)}$$

$$= \frac{N\tan^2\beta}{\tan^2\beta\,(\cos^2\beta+N^2\sin^2\beta)}$$

$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{\cos^2\beta+N^2\sin^2\beta)}$$

$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{1-\sin^2\beta + N^2\sin^2\beta} = \frac{N^2}{1+N^2\sin^2\beta - \sin^2\beta}$$

$$= \frac{N^2}{1+(N^2-1)\sin^2\beta}$$

So, Intensity of Secondary maxima is

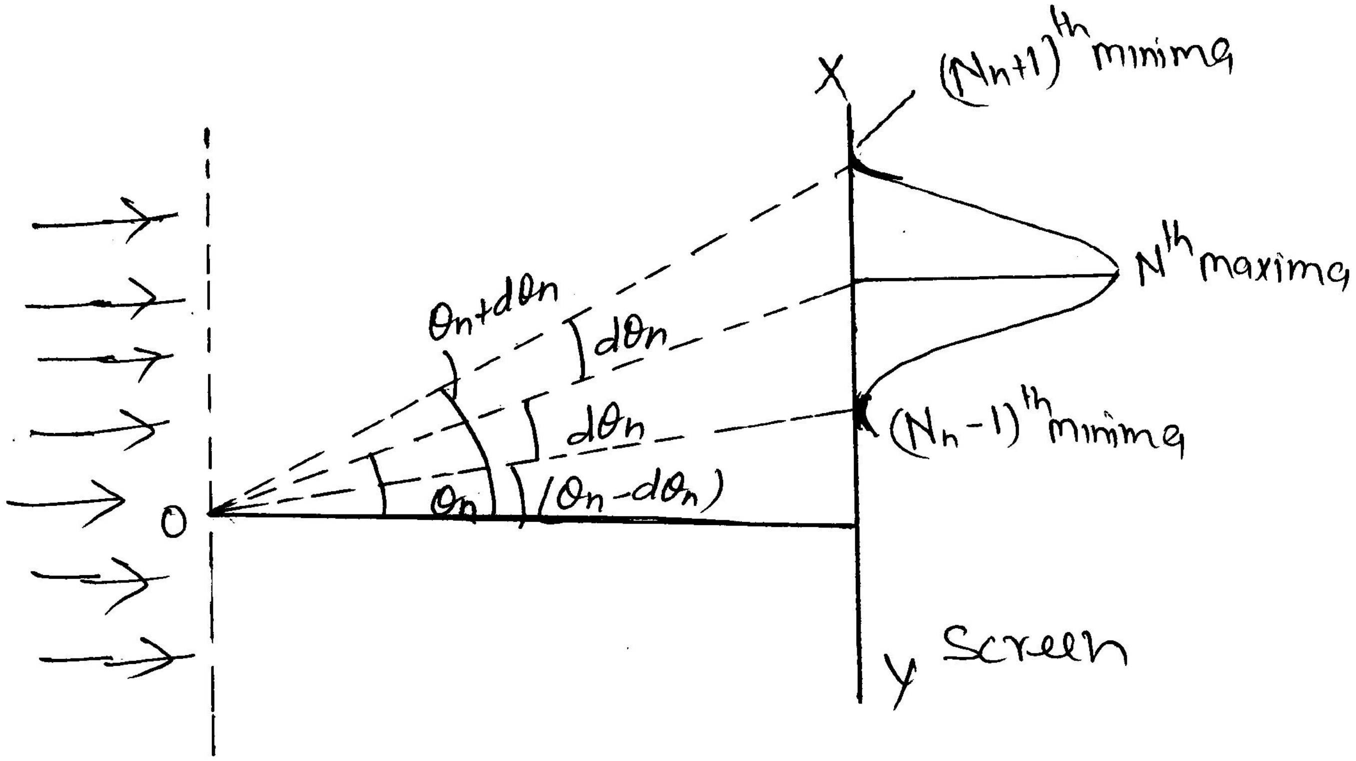
$$I_s = A^2 \frac{\sin^2\alpha}{\alpha^2}\left[\frac{N^2}{1+(N^2-1)\sin^2\beta}\right]$$

$$I_s = A^2 \frac{\sin^2\alpha}{\alpha^2} \times N^2 \left(\frac{1}{1+(N^2-1)\sin^2\beta}\right)$$

$$I_s = \text{Intensity of Principal maxima} \times \frac{1}{1+(N^2-1)\sin^2\beta}$$

if N is large, the intensity of Secondary maxima is less.

Angular Half width or width of the principal maxima



The angular separation between the first two minima lying on either side of the principal maxima is called the angular width of principal maxima of any order.

n^{th} order principal maxima lies in the direction

$$(a+b)\sin\theta_n = n\lambda \quad \text{——— (1)}$$

Let $(\theta_n+d\theta)$ and $(\theta_n-d\theta)$ give the direction of first secondary minima on two sides of n^{th} order principal maxima

Then

$$(a+b)\sin(\theta_n \pm d\theta) = n\lambda \pm \frac{\lambda}{N} \quad \text{——— (2)}$$

dividing eqn ② by eqn ①

$$\frac{(a+b)\sin(\theta_n \pm d\theta)}{(a+b)\sin\theta_n} = \frac{n\lambda \pm \frac{\lambda}{N}}{n\lambda}$$

$$\frac{\sin(\theta_n \pm d\theta)}{\sin\theta_n} = 1 \pm \frac{1}{nN}$$

$$\frac{\sin\theta_n \cos d\theta \pm \cos\theta_n \sin d\theta}{\sin\theta_n} = 1 \pm \frac{1}{nN}$$

For the small value of $d\theta$,

$\cos d\theta = 1$ and $\sin d\theta = d\theta$

$$\frac{\sin\theta_n + \cos\theta_n d\theta}{\sin\theta_n} = 1 \pm \frac{1}{nN}$$

$$1 + \cot\theta_n d\theta = 1 + \frac{1}{nN}$$

$$\boxed{d\theta = \frac{1}{nN\cot\theta_n}}$$

$d\theta$ is the half the angular width of principal maxima
$d\theta$ is inversely proportional to N i.e. for large number of slit, $d\theta$ is small, then sharpness increases.

Absent Spectra or Missing Order Spectra —

As the resultant intensity due to N-parallel slits is given by

$$I = A^2 \frac{\sin^2\alpha}{\alpha^2} \cdot \frac{\sin^2 N\beta}{\sin^2\beta} \quad \text{—— (1)}$$

where $\alpha = \frac{\pi a \sin\theta}{\lambda}$ and $\beta = \frac{\pi (a+b)\sin\theta}{\lambda}$

Now the direction of principal maxima in grating spectra is

$$(a+b)\sin\theta = n\lambda \quad \text{—— (2)}$$

The direction of minima in single slit pattern

$$a\sin\theta = m\lambda \quad \text{—— (3)}$$

where $m = 1, 2, 3, \ldots$

If both the conditions are simultaneously satisfied, a particular maxima of order n will be absent in grating spectrum, these are known as absent spectra or missing order spectra.

dividing eqn (2) by eqn (3)

$$\frac{(a+b)\sin\theta}{a\sin\theta} = \frac{n\lambda}{m\lambda}$$

$$\frac{(a+b)}{a} = \frac{n}{m} \quad \text{—— (4)}$$

Eqn (4) is the condition for the spectrum of the order n to be absent.

1) if $b = a$

$$\frac{2a}{a} = \frac{n}{m}$$

$$n = 2m$$

where $m = 1, 2, 3, \ldots$

$$n = 2, 4, 6 \ldots$$

Hence, when the width of slit and the opaque portion are equal i.e $a = b$, then $2^{nd}, 4^{th}, 6^{th} \ldots$ order spectrum are missing.

2) if $b = 2a$

then $n = 3m$

where $m = 1, 2, 3, \ldots$

$$n = 3, 6, 9, 12, \ldots$$

When the width of slit is double the opaque portion then $3^{rd}, 6^{th}, 9^{th} \ldots$ order spectra are missing.

Dispersive Power of a Plane Diffraction Grating

The dispersive power of grating is defined as the rate of change of the angle of diffraction with change in the wavelength of light used.

Thus, if the wavelength changes from λ to $(\lambda + d\lambda)$ and corresponding angle of diffraction changes from θ to $(\theta + d\theta)$, then the ratio $\frac{d\theta}{d\lambda}$ is called the dispersive power of the grating.

The grating equation for normal incidence is

$$(a+b)\sin\theta = n\lambda \quad \text{——— (1)}$$

differentiating eqn (1) w.r.to λ

$$(a+b)\cos\theta \frac{d\theta}{d\lambda} = n$$

Therefore

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta} \quad \text{——— (2)}$$

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)(1-\sin^2\theta)^{1/2}} \quad \text{——— (3)}$$

from eqn (1)

$$\sin\theta = \frac{n\lambda}{(a+b)}$$

Put the Value of $\sin\theta$ in Eqn (2) we get

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\sqrt{1-\left(\frac{n\lambda}{a+b}\right)^2}}$$

$$\frac{d\theta}{d\lambda} = \frac{1}{\sqrt{\left(\frac{a+b}{n}\right)^2 - \lambda^2}} \quad \text{———} \quad (4)$$

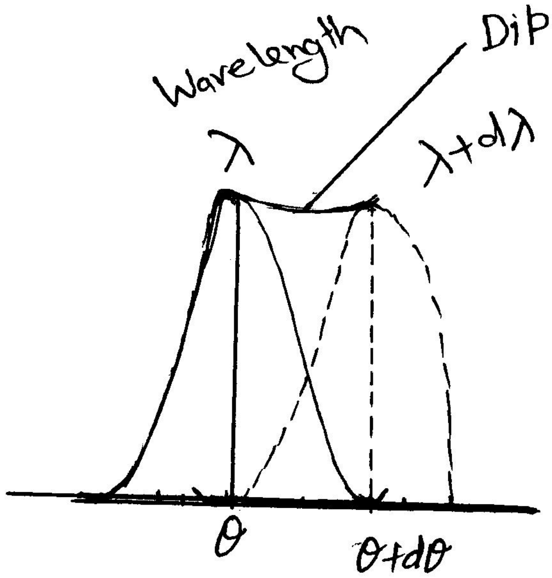
from Eqn (2)

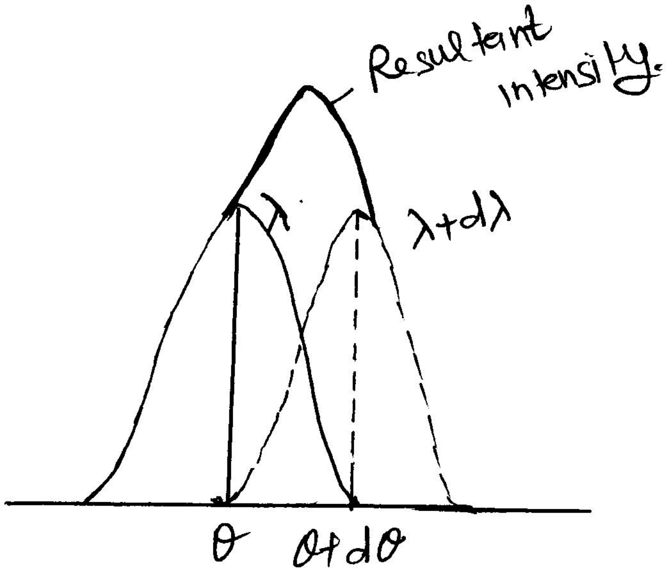
1) dispersive power is directly proportional to the order of the Spectrum (n) i.e. higher is the order greater is the dispersive power.

2) dispersive power is inversely propertional to the grating element i.e. the dispersive power of two given lines is greater for a grating having Larger number of lines per cm.

3) dispersive power is inversely proportional to $\cos\theta$ i.e. Larger the Value of θ higher is the dispersive power.

Rayleigh Criterion for the Limit of Resolution





According to Rayleigh, the two point Sources or two Equally intense Spectral lines are just resolved by an optical Instrument When the Central maxima of the diffraction pattern due to one Source falls ~~exactly~~ Exactly on the first Minima of the diffraction pattern of the other and Vice-Versa.

Resolving Power of an optical Instrument

When two objects are very close together they may appear as one object and it may be difficult for the naked eye to see them as separate.

So, the capacity of an optical Instrument to show two close objects separately is called resolution and the ability of an optical Instrument to resolve the images of two close point objects is called its resolving power.

"The eye can see two objects as separately only if the angle subtended by them at the eye is greater than one minute, which is the resolving limit of the normal eye."

The minimum angle of resolution provided by a lens of diameter D at a wavelength λ is

$$\boxed{\theta_{min} = 1.22\frac{\lambda}{D}}$$

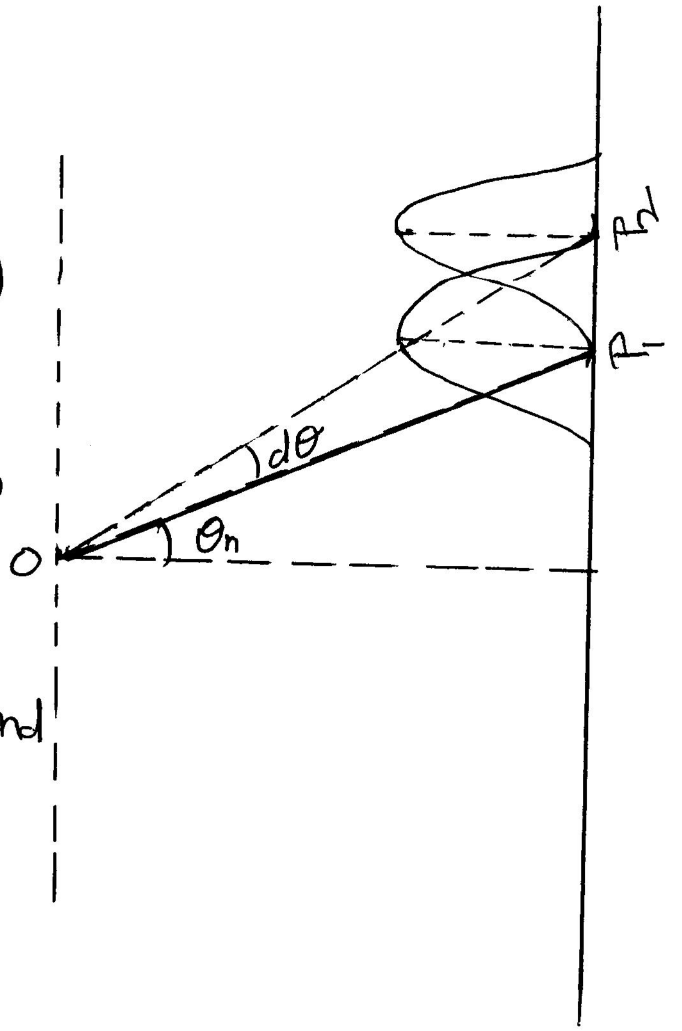
Resolving power of Grating

The resolving power of grating is defined as its ability to show the two neighbouring spectral lines in a spectrum as separate.

It can also be defined as the ratio of the wavelength of any spectral line to the smallest wavelength between neighbouring lines for which spectral lines can be just resolved at the wavelength λ.

It can expressed as $\lambda / d\lambda$.

Let P_1 be the n^{th} maxima of spectral line of wavelength λ and P_2 be the n^{th} primary maxima for $(\lambda + d\lambda)$ of diffraction angle $(\theta_n + d\theta)$.



According to Rayleigh, the two spectral lines will appear resolved if position of P_2 also correspond to the first minima at P_1.

For n^{th} primary maxima for wave length λ

$$(a+b)\sin\theta_n = n\lambda \quad \text{——— (1)}$$

For n^{th} primary maxima for wave length $(\lambda+d\lambda)$

$$(a+b)\sin(\theta_n+d\theta) = n(\lambda+d\lambda) \quad \text{——— (2)}$$

For just resolved the $(\theta_n+d\theta)$ Corresponds to the direction of first secondary minimum after n^{th} primary maximum at P_1 of wavelength λ.
So this introduce extra path difference i.e.

Extra path difference $= \dfrac{\lambda}{N}$

Where N = number of lines on grating Surface.

So

$$(a+b)\sin(\theta_n+d\theta) = n\lambda + \frac{\lambda}{N} \quad \text{——— (3)}$$

From eqn (2) and (3)

$$n(\lambda+d\lambda) = n\lambda + \frac{\lambda}{N}$$

$$n\lambda + nd\lambda = n\lambda + \frac{\lambda}{N}$$

$$nd\lambda = \frac{\lambda}{N}$$

or

$$\frac{\lambda}{d\lambda} = nN \quad \text{———} \quad (4)$$

Eqn (4) is the expression for resolving power of grating, from Eqn (4) it is clear that resolving power is directly proportional to order of Spectrum and number of lines in the grating Surface.

For Central maxima $n=0$, hence resolving power is zero.

The resolving power of grating may also be expressed in terms of dispersive power

$$\frac{\lambda}{d\lambda} = N\,(a+b)\cos\theta \cdot \frac{d\theta}{d\lambda}$$

So

Resolving power of grating = Total aperture × dispersive power

$$\left\{ \because \frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta} \right\}$$

Numerical Problems

Q1. Light of wavelength 5500Å falls normally on a slit of width 22×10^{-5} cm. Calculate the angular position of the first two minima on either side of the central maxima.

Q.2. In Fraunhofer diffraction due to a narrow slit, a screen is placed 2 m away from the lens to obtain the pattern. If the slit width is 0.2 mm and first minima lie 5 mm on either side of the central maxima, find the wavelength of light.

Q3. Light of wavelength 5000 Å is incident normally on a slit. The first minimum of diffraction pattern is observed to lie at a distance of 5 mm from the central maxima on a screen placed at a distance of 2 m from the slit. Calculate the width of slit.

Q4. A light of wavelength 6000Å falls normally on a slit of width 0.10 mm. Calculate the total angular width of the central maxima and also the linear width as observed on a screen placed 1 metre away.

Q.5. Calculate the angle at which the first dark band and the next bright band are formed in the fraunhofer diffraction pattern of a Slit 0.3 mm wide ($\lambda = 5890 \mathring{A}$)

Q.6 A Single Slit of width 0.14 mm is illuminated normally by monochromatic light and diffraction bands are observed on the Screen 2 m away. If the Centre of Second dark band is 1.6 cm from the middle of the Central bright band, deduce the wavelength of light.

Q.7. A Single Slit is illuminated by light Composed of two wavelength λ_1 and λ_2. One observes that due to fraunhofer diffraction, the first minima obtained λ_1 Coincides with the Second diffraction minima of λ_2. What is the relation between λ_1 and λ_2.

Q.8 In a double Slit fraunhofer diffraction pattern the Screen is 170 cm away from the Slits. The Slits widths are 0.8 mm and they are 0.4 mm apart. Calculate the Wavelength of light if the fringe Spacing is 0.25 cm. Also deduce the missing order.

Q9. A Parallel beam of monochromatic light is allowed to be incident normally on a plane grating having 1250 lines per Cm and a Second order Spectral Line is observed to ~~be~~ be deviated through 30°. Calculate the Wavelength of the Spectral Line.

Q10. Calculate the angle between the Central image of a lamp filament and its first diffracted image produce by a fabric with 160 threads per Cm.

(Given $\lambda = 6 \times 10^{-5}$ Cm)

Q11. Find the angular Separation of 5048 Å and 5016 Å Wavelength in Second order Spectrum obtain by a plane diffraction grating having 15000 lines per inch.

Q12. A diffraction grating used at normal ~~incidence~~ incidence give a yellow line ($\lambda = 6000$ Å) in a Certain spectral order superimposed on a blue line ($\lambda = 4800$ Å) of next higher order. If the angle of diffraction is $\sin^{-1}(3/4)$ Calculate the grating element.

Q13. A diffraction grating used at normal incidence gives a green line (5400Å) in a certain order n superimposed on the violet line (4050 Å) of the next higher order. If the angle of diffraction is 30°. Calculate the value of n. Also find how many lines per cm are there in the grating.

Q14 How many orders will be visible if the wavelength of incident radiation is 5000 Å and the number of lines on the grating is 2620 to an inch.

Q15. How many orders will be observed by a grating having 4000 lines per cm, if it is illuminated by light of wavelength in the range 5000 Å to 7500 Å.

Q16 What ~~must~~ must be minimum number of lines per cm in a half inch ~~width~~ width grating to resolve the wavelength 5890 and 5896 Å.

Solutions

Ans 1. The angular positions of minima in a Single Slit diffraction pattern are given by

$$a \sin\theta = \pm n\lambda$$

$$\sin\theta = \pm \frac{n\lambda}{a} \quad \text{where } n = 1, 2, 3, \ldots$$

Given $a = 22 \times 10^{-5}$ cm and $\lambda = 5500 \times 10^{-8}$ cm

For the first order minima, $n = 1$

$$\therefore \sin\theta_1 = \frac{\lambda}{a} = \frac{5500 \times 10^{-8}}{22 \times 10^{-5}}$$

$$\sin\theta_1 = 0.25$$

$$\text{or } \theta_1 = \sin^{-1}(0.25) = 14^\circ 29'$$

For Second order minima, $n = 2$

$$\sin\theta_2 = \frac{2\lambda}{a} = 2 \times .25 = 0.5$$

$$\sin\theta_2 = 0.5$$

$$\theta_2 = \sin^{-1}(0.5) = 30^\circ$$

Therefore, the first two minima will occur at an angle $14^\circ 29'$ and 30° on either side of Central maxima.

Ans2. In the fraunhofer diffraction pattern due to a Single Slit of width a, the direction of minima are given by

$$a \sin\theta = \pm n\lambda \quad \text{where } n = 1, 2, 3 \ldots$$

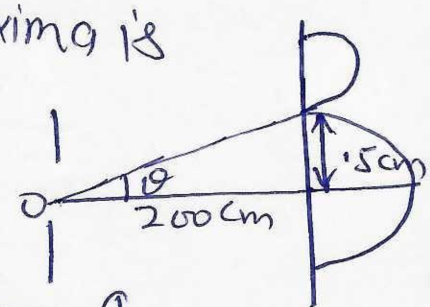
$$\sin\theta = \frac{n\lambda}{a}$$

If θ is measured in radian, then

$$\sin\theta = \theta = \frac{n\lambda}{a}$$

The angular separation θ between the first minima on either side of the central maxima is

$$\theta = \frac{\lambda}{a} \quad \text{for } n = 1$$



$$\theta = \left(\frac{\lambda}{0.02}\right) \text{rad} \quad \text{——— (1)}$$

Linear separation between the first minima and central maxima is equal to 0.5 cm

$$\theta = \frac{0.5}{200} \text{ rad} \quad \text{——— (2)}$$

from eqn (1) and eqn (2)

$$\frac{\lambda}{0.02} = \frac{0.5}{200}$$

$$\lambda = \frac{0.5 \times 0.02}{200} = 5 \times 10^{-5} \text{ cm}$$

$$\lambda = 5000 \text{ Å}$$

Ans 3. In the fraunhofer diffraction due to Single Slit of width 'a' the directions of minima are given by

$$a \sin\theta = \pm n\lambda$$

where $n = 1, 2, 3 \ldots\ldots$

for $n = 1$

$$a \sin\theta = \lambda$$

$$\sin\theta = \lambda / a$$

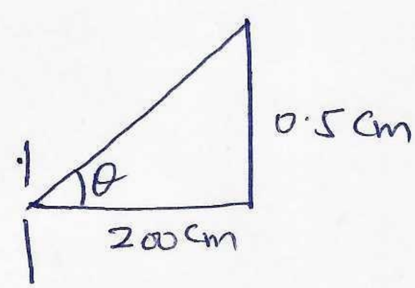
for small angle $\sin\theta = \theta$

$$\theta = \frac{\lambda}{a} \quad \text{———} \; (1)$$

The linear Separation between the first minima and Central maxima $= 5\,mm = 0.5\,cm$

and distance between slit and screen $D = 2m = 200\,cm$

So $\theta = \dfrac{0.5}{200}$ rad ——— (2)



From eqn (1) and (2)

$$\frac{0.5}{200} = \frac{\lambda}{a}$$

So $a = \lambda \left(\dfrac{200}{0.5} \right) = \dfrac{5000 \times 10^{-8} \times 200}{0.5}$

Here $\lambda = 5000\,\mathring{A}$

$$\boxed{a = 0.02\,cm}$$

Ans.

Ans 4

In a Single Slit diffraction pattern, the directions of minima are given by

$$a \sin\theta = \pm n\lambda$$

for $n=1$ Where $n = 1, 2, 3 \ldots$

$$a \sin\theta = \lambda$$

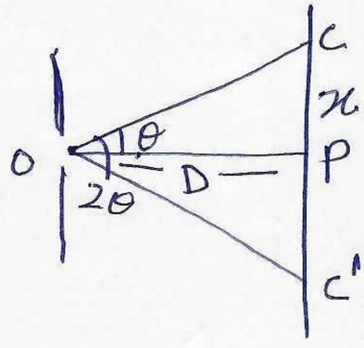
or $\theta = \dfrac{\lambda}{a}$ $\left\{ \begin{array}{l} \text{for Small angle} \\ \sin\theta = \theta \end{array} \right\}$

Given

$a = 0.10\,mm = 0.01\,cm$

$\lambda = 6000\,Å = 6000 \times 10^{-8}\,cm$

So Angular half width



$$\theta = \frac{6000 \times 10^{-8}}{0.01}$$

$$\theta = 6 \times 10^{-3}\,rad$$

Hence total angular width $= 2\theta = 2 \times 6 \times 10^{-3}$

$2\theta = 1.2 \times 10^{-2}\,rad$

the Linear half width $CP = x = \theta . D$

the total linear width $= 2x = (2\theta) . D \times 2$

Here $D = 1\,m = 100\,cm$

So $2x = 2 \times 6 \times 10^{-2} \times 100 \times 2$

$\boxed{\text{Linear width} = 2.4\,cm}$ Ans

Ans 5

In fraunhofer differaction pattern due to a Single Slit of width 'a' the directions of minima are given by

$$a \sin\theta = \pm n\lambda \quad \text{where } n = 1, 2, 3, \ldots$$

for $n = 1$

$$a \sin\theta = \lambda$$

$$\sin\theta = \frac{\lambda}{a} = \frac{5890 \times 10^{-8}}{0.03}$$

$$\sin\theta = 0.00196$$

$$\theta = \sin^{-1}(0.00196)$$

$$\theta = 0.112^\circ$$

Angle of diffraction θ' Corresponding to the first bright band on either Side of the Central maxima is given by

$$a \sin\theta' = \frac{3}{2}\lambda = 1.5\lambda$$

$$\sin\theta' = 1.5\frac{\lambda}{a} = 1.5 \times 0.00196$$

$$\sin\theta' = 0.00294$$

$$\theta' = \sin^{-1}(0.00294) = 0.168^\circ$$

$$\boxed{\theta' = 0.168^\circ}$$ Ans

Ans 6. For Single Slit diffraction pattern, the direction of minima is

$$a\sin\theta = \pm n\lambda$$

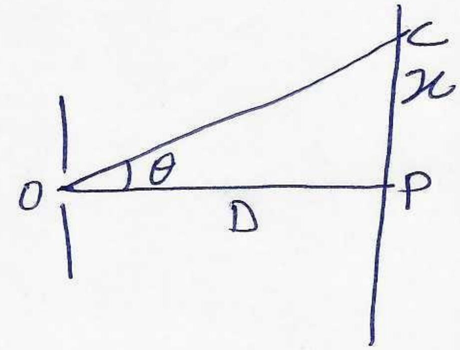
$$\sin\theta = \frac{n\lambda}{a} \qquad \text{where } n = 1, 2, 3, \ldots$$

for small angle

$$\theta = \frac{n\lambda}{a}$$

If D is the distance of the Screen from the Slit and x is the distance of Centre of Second dark band from the middle of the ~~[illegible]~~ Central bright band

$$\theta = \frac{x}{D}$$



So

$$\frac{x}{D} = \frac{n\lambda}{a}$$

$$\lambda = \frac{x \cdot a}{nD} = \frac{1.6\times10^{-2} \times 0.14\times10^{-3}}{2\times2}$$

$$\lambda = 5.6\times10^{-7}\,m$$

or $\boxed{\lambda = 5600\,\text{Å}}$

Ans 7.

In the Fraunhofer diffraction pattern due to Single Slit of width a, the direction of minima

$$a \sin\theta = \pm n\lambda$$

For wavelength λ_1, the direction of first minima is
$n=1$

$$a \sin\theta_1 = \lambda_1 \quad \text{——— (1)}$$

Similarly, for wavelength λ_2, the direction of Second minima is
$n=2$

$$a \sin\theta_2 = 2\lambda_2 \quad \text{——— (2)}$$

direction of first minima of λ_1 Coincides with the direction of Second minima of λ_2.

So $\theta_1 = \theta_2 = \theta$

Therefore, from eqn (1) and eqn (2)

$$a \sin\theta = \lambda_1 = 2\lambda_2$$

$$\boxed{\lambda_2 = \frac{\lambda_1}{2}}$$

Ans 8.

The fringe Spacing in the Case of interference pattern is given by

$$\omega = \frac{D\lambda}{2b}$$

or $$\lambda = \frac{\omega \cdot 2b}{D}$$

Here 2b is the Separation between the Slits

$2b = 0.4\,mm = 0.04\,cm$, $\omega = 0.25\,cm$
and $D = 170\,cm$

$$\lambda = \frac{0.25 \times 0.04}{170} = 5.88 \times 10^{-5}\,cm$$

$$\boxed{\lambda = 5880\,\text{Å}}$$

If 'a' be the Slit width and 'b' is the Slit Separation, Then the Condition of missing order is

$$\frac{a+b}{a} = \frac{n}{m}$$

Here $a = 0.08$ mm
$b = 0.4$ mm

$$\frac{0.08 + 0.4}{0.08} = \frac{n}{m}$$

$n = 6m$ where $m = 1, 2, 3, \ldots$

$n = 6, 12, 18, \ldots$

Hence $6^{th}, 12^{th}, 18^{th} \ldots$ order will be missing.

Ans 9. The grating Eqn for normal incidence is

$$(a+b)\sin\theta = n\lambda \quad \text{——— (1)}$$

Where $(a+b)$ is the grating element

given

$$(a+b) = \frac{1}{1250}\ cm$$

$$\theta = 30^\circ \text{ and } n = 2$$

So

$$(a+b)\sin\theta = n\lambda$$

$$\lambda = \frac{(a+b)\sin\theta}{n}$$

$$= \frac{\frac{1}{1250} \times \sin 30^\circ}{2}$$

$$= \frac{1}{1250 \times 2 \times 2}$$

$$= \frac{1}{5000}$$

$$= 2 \times 10^{-4}\ cm$$

$$\boxed{\lambda = 2 \times 10^{-4}\ cm}$$

Ans 10

The grating equation is

$$(a+b)\sin\theta = n\lambda$$

$$\sin\theta = \frac{n\lambda}{(a+b)}$$

Here

$$(a+b) = \frac{1}{160}\text{ cm}, \quad n=1$$

$$\lambda = 6\times10^{-5}\text{ cm}$$

So

$$\sin\theta = \frac{1\times6\times10^{-5}}{1/160}$$

$$= 160\times6\times10^{-5}$$

$$\sin\theta = 0.0096$$

$$\theta = \sin^{-1}(0.0096)$$

$$\boxed{\theta = 33\text{ min}}$$

Ans

Ans 11.

The angular Separation $d\theta$ is

$$d\theta = \frac{d\lambda}{\sqrt{\left(\frac{a+b}{n}\right)^2 - \lambda^2}}$$

$$d\theta = \frac{n\,d\lambda}{\sqrt{(a+b)^2 - n^2\lambda^2}}$$

$$d\theta = \frac{n\,d\lambda}{(a+b)} \quad \text{(approxi)}$$

Given

$$d\lambda = 5048 - 5016 = 32 \times 10^{-8}\,\text{cm}$$

$$a+b = \frac{2.54}{15000} = 1.69 \times 10^{-4}\,\text{cm} \quad \text{and } n = 2$$

$$d\theta = \frac{2 \times 32 \times 10^{-8}}{1.69 \times 10^{-4}}$$

$$\boxed{d\theta = 3.787 \times 10^{-4}\ \text{radian}}$$

Ans 12.

The direction of principal maxima is

$$(a+b)\sin\theta = n\lambda \quad \text{——— (1)}$$

Let n^{th} order maxima of λ_1 Coincide with $(n+1)^{th}$ order maxima of λ_2 then

$$(a+b)\sin\theta = n\lambda_1 \quad \text{——— (2)}$$

$$(a+b)\sin\theta = (n+1)\lambda_2 \quad \text{——— (3)}$$

So $(a+b)\sin\theta = n\lambda_1 = (n+1)\lambda_2$

$$n\lambda_1 = n\lambda_2 + \lambda_2$$

$$n = \frac{\lambda_2}{\lambda_1 - \lambda_2}$$

Thus eqn (2) becomes

$$(a+b)\sin\theta = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2}$$

$$(a+b) = \frac{\lambda_1\lambda_2}{(\lambda_1 - \lambda_2)\sin\theta}$$

Given $\lambda_1 = 6000 \times 10^{-8}$ cm, $\lambda_2 = 4800 \times 10^{-8}$ cm

$\theta = \sin^{-1}(3/4)$

$$(a+b) = \frac{6000 \times 10^{-8} \times 4800 \times 10^{-8}}{} = 3.2 \times 10^{-4}$$

Ans 13.

The direction of principal maxima is

$$(a+b)\sin\theta = n\lambda \quad \text{———} \quad (1)$$

Let n^{th} order maxima of λ_1 coincide with $(n+1)^{th}$ order maxima of λ_2, then

$$(a+b)\sin\theta = n\lambda_1 \quad \text{———} \quad (2)$$

and

$$(a+b)\sin\theta = (n+1)\lambda_2 \quad \text{———} \quad (3)$$

from eqn (2) and eqn (3)

$$(a+b)\sin\theta = n\lambda_1 = (n+1)\lambda_2$$

$$n\lambda_1 = n\lambda_2 + \lambda_2$$

or $$n = \frac{\lambda_2}{(\lambda_1 - \lambda_2)}$$

So $$(a+b)\sin\theta = \frac{\lambda_1\lambda_2}{(\lambda_1-\lambda_2)}$$

$$(a+b) = \frac{\lambda_1\lambda_2}{(\lambda_1-\lambda_2)\sin\theta}$$

Here $\lambda_1 = 5400\times10^{-8}\,cm$, $\lambda_2 = 4050\times10^{-8}\,cm$, $\theta = 30^\circ$

So $$(a+b) = \frac{5400\times10^{-8}\times4050\times10^{-8}}{(5400-4050)\times10^{-8}\times\sin30} = \frac{5400\times4050\times10^{-8}\times2}{1350}$$

Therefore number of lines per cm $= \dfrac{1}{a+b}$

$$= 1350\times10^{8}$$

Ans 14.

The direction of principal maxima

$$(a+b)\sin\theta = n\lambda \quad \text{—— (1)}$$

maximum value of $\theta = 90^\circ$

from eqn (1)

$$n = \frac{(a+b)\sin\theta}{\lambda}$$

Here

$$(a+b) = \frac{2.54}{2620}\ \text{cm} \quad \text{and } \lambda = 5000\,\text{Å}$$
$$= 5000\times10^{-8}\ \text{cm}$$

$$n = \frac{2.54 \times \sin 90}{2620 \times 5000\times10^{-8}}$$

$$= \frac{2.54\times1}{2620\times5000\times10^{-8}}$$

$$= 19.4$$

$$n > \underline{\underline{19}} \text{ Ans}$$

Hence, the maximum number of orders visible in the Spectrum are 19.

Ans 15.

The number of orders visible with grating.

$$n = \frac{(a+b)\sin\theta}{\lambda}$$

For maximum order visible with grating

$\theta = 90^\circ$ or $\sin 90 = 1$

Therefore

$$n_{max} = \frac{(a+b)}{\lambda}$$

Here

$(a+b) = \frac{1}{4000}$ cm $\quad \lambda = 5000 \times 10^{-8}$ cm

So

$$n_{max} = \frac{1}{4000 \times 5000 \times 10^{-8}}$$

$$n_{max} = \frac{10}{2} = \underline{\underline{5}}$$

for $\lambda = 7500 \times 10^{-8}$ cm

$$n_{max} = \frac{1}{4000 \times 7500 \times 10^{-8}}$$

$$n_{max} = \frac{10}{3} = \underline{\underline{3.3}}$$

Hence, In the wavelength range 5000 Å to 7500 Å, the observed number of orders range between 3 to 5.

Ans 16.

The resolving power of grating is given by

$$\frac{\lambda}{d\lambda} = nN \quad \text{———} \quad (1)$$

Where N is the number of lines per inch in the grating

$$\lambda = \frac{\lambda_1 + \lambda_2}{2} = \frac{5890 + 5896\,\text{Å}}{2}$$

$$\lambda = 5893\,\text{Å}$$

$$d\lambda = \lambda_2 - \lambda_1$$

$$d\lambda = 5896 - 5890 = 6\,\text{Å}$$

if order is not given in any problem Consider $n = 1$

$$N = \frac{1}{n} \times \frac{\lambda}{d\lambda}$$

$$N = \frac{5893}{6} = 982$$

Since grating is half inch wide, therefore number of lines per inch $= 2 \times 982 = 1964$

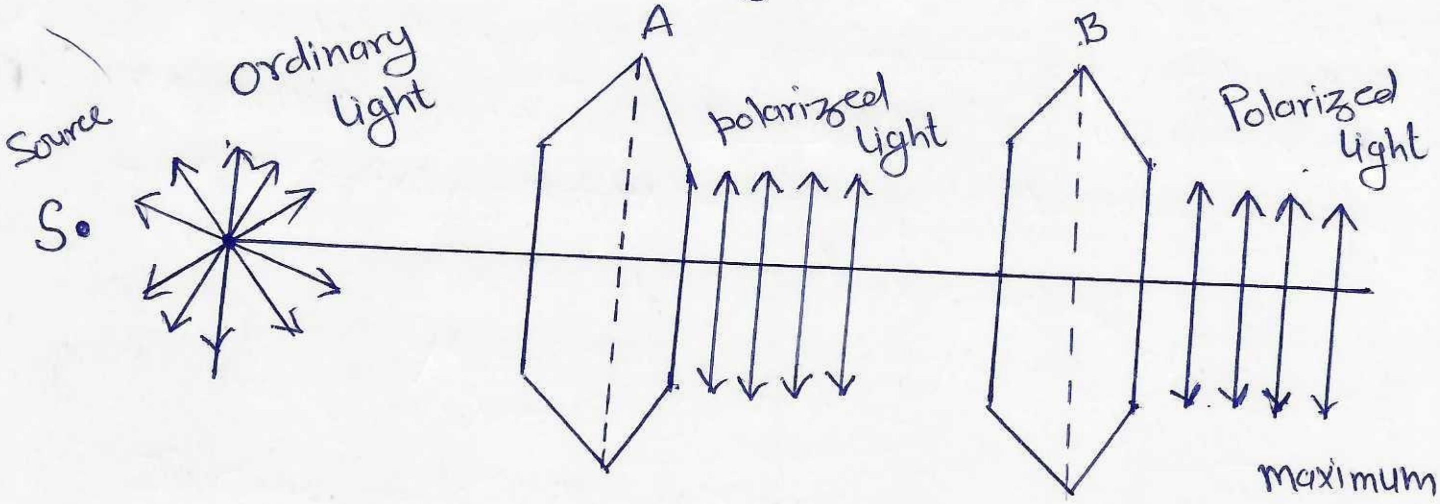
Hence, the minimum number of lines per cm is

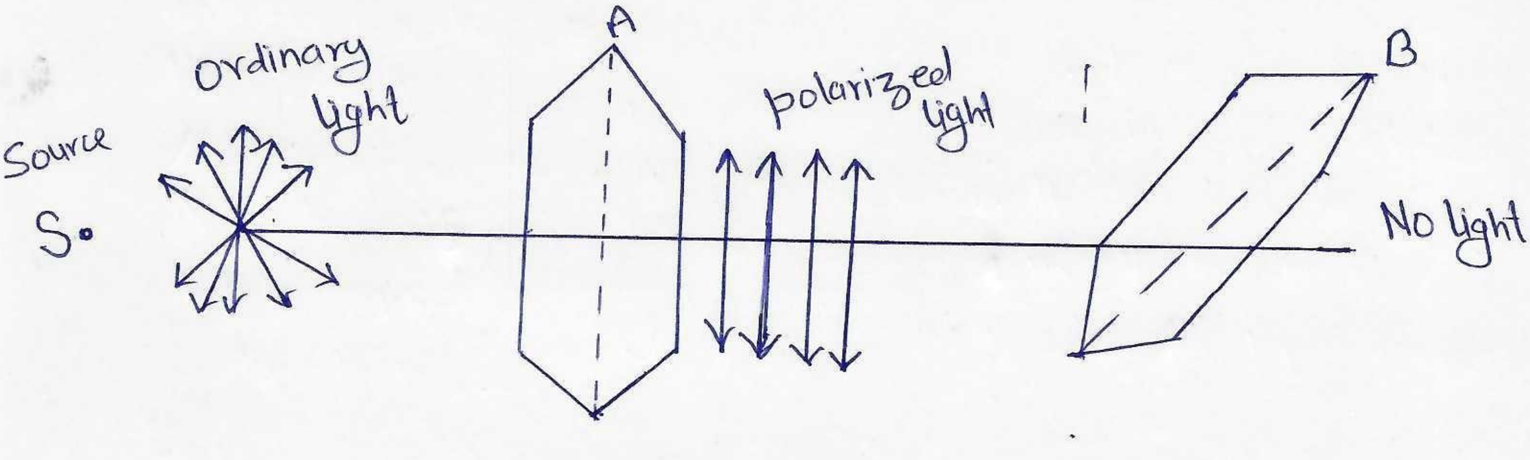
$$= \frac{1964}{2.54} =$$

$$= 773$$

Polarization

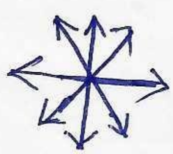
Polarization — The process of transforming unpolarised light wave to polarized light wave called the polarization of light.



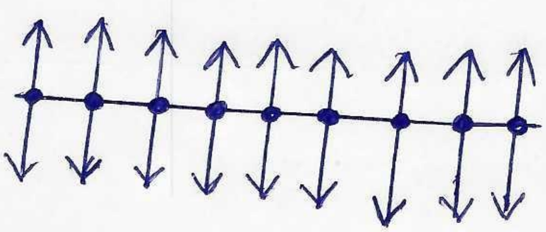


When ordinary light is incident normally on a pair of parallel tourmaline crystal plate A and B cut parallel to their crystallographic axis the emergent light shows a variation as B is rotated. The intensity is maximum when the axis of B is parallel to that of A and minimum when at right angle. This shows that the light emerging from A is not symmetrical about the direction of propagation of light, but its vibrations are confined only a single line in a plane perpendicular to direction of propagation. Such light is called plane polarized light.

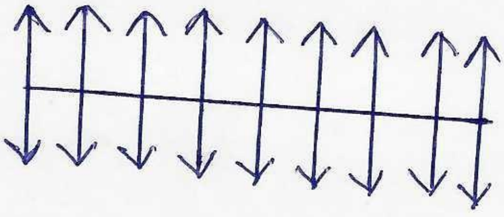
Pictorial Representation of light Vibrations



unpolarized light



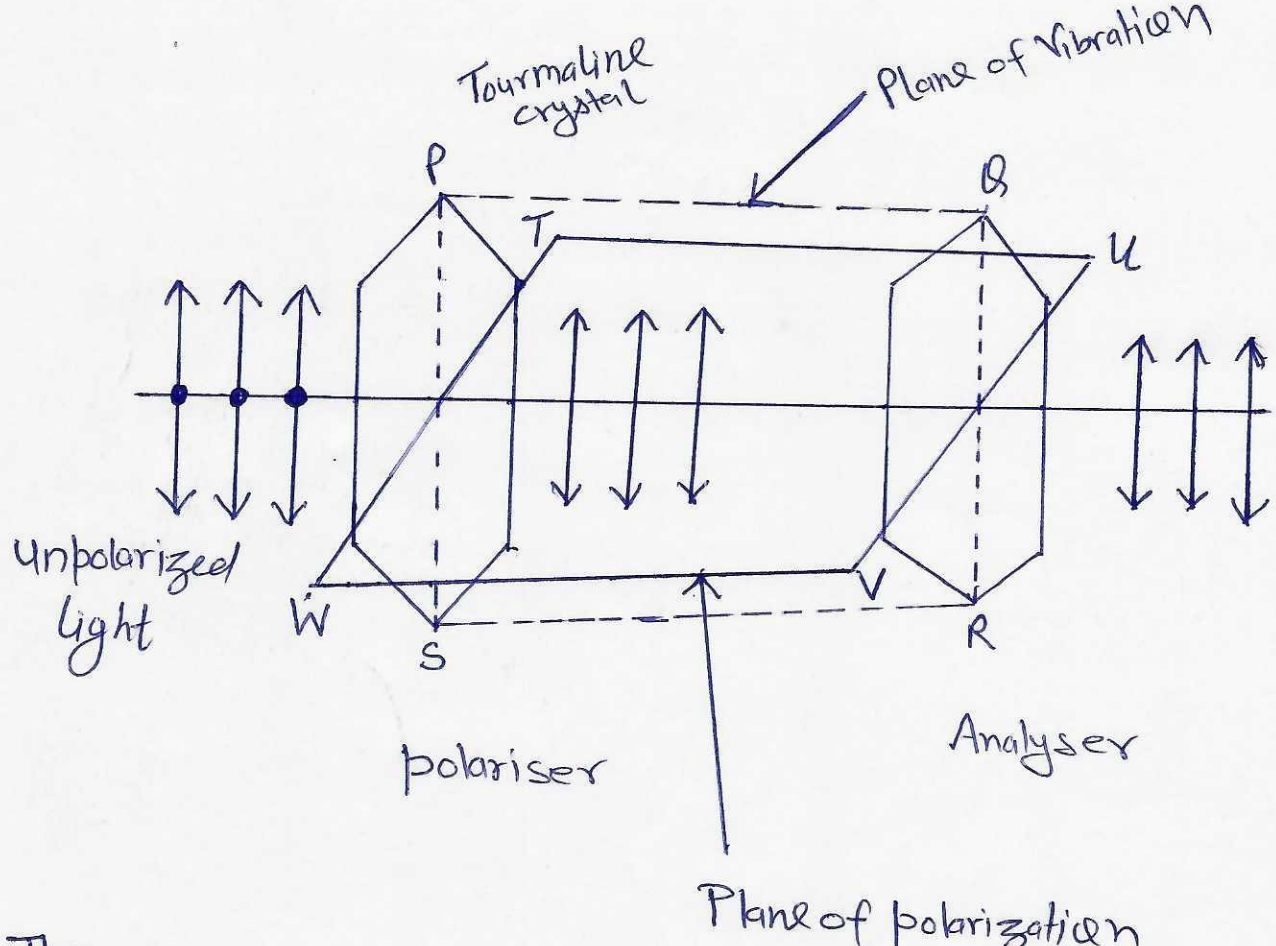
Unpolarized light



Plane polarized Vibrations perpendicular to plane of paper

plane polarized light Vibrations paralle to plane of paper

Plane of Vibration and Plane of Polarization



The plane in which the vibrations take place i.e. the plane containing the direction of vibration and the direction of propagation, is called plane of vibration. Plane PQRS is the plane of vibration.

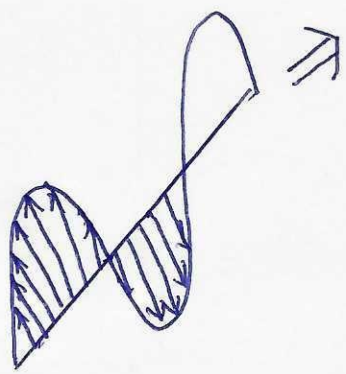
A plane perpendicular to the plane of vibration is called the plane of polarization. Thus the plane of polarization is the plane passing through the direction of propagation and contain no vibrations. The plane TUVW is the plane of polarization.

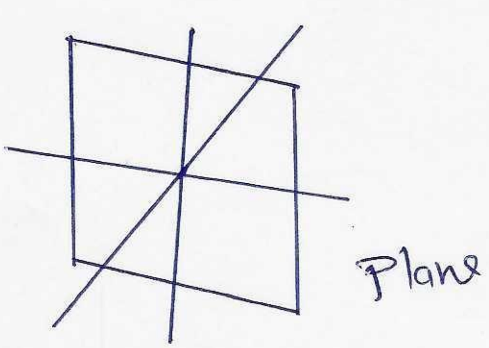
Types of polarization —

— x —

1- Plane polarized light
2- Circularly polarized light
3- Elliptically polarized light

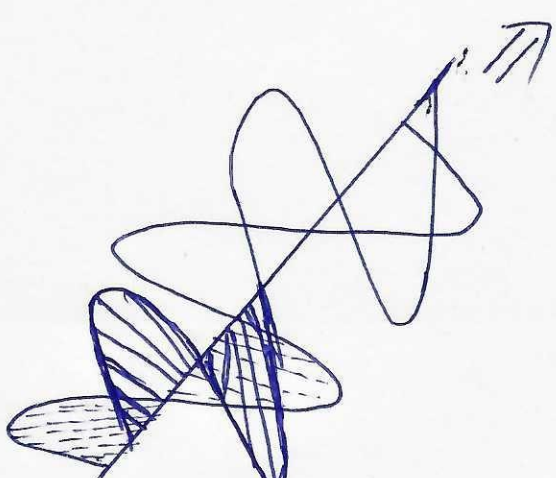
Plane polarized light — If in a polarized light, the electric Vector Vibrates in a fixed Straight line perpendicular to direction of propagation of light, it is Said to be plane polarized light (PPL).

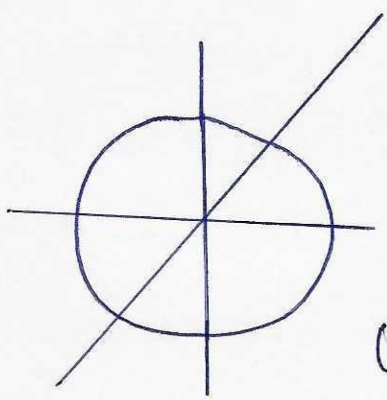




Circularly polarized light —

When two plane polarised light waves are superimposed Under Certain Conditions, the resultant light Vector rotates with a Constant magnitudes in a plane perpendicular to the direction of propagation of light; the tip of a Vector Space traces a Circle and the light is Said to be Circularly polarised light. Circularly polarised light Consists of two perpendicular electromagnetic plane Waves of equal amplitude and with 90° difference in Phase.

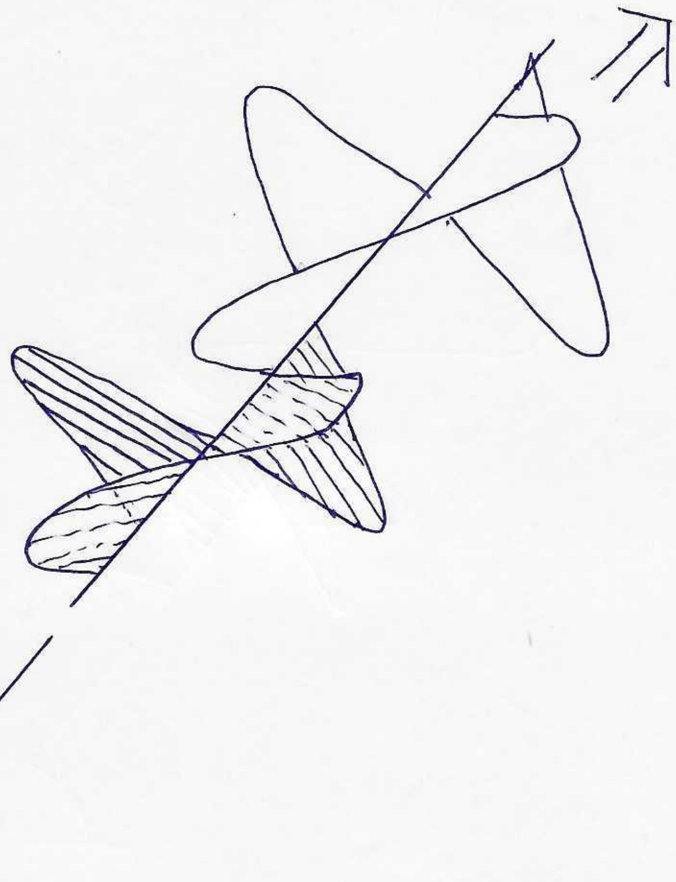




Circular

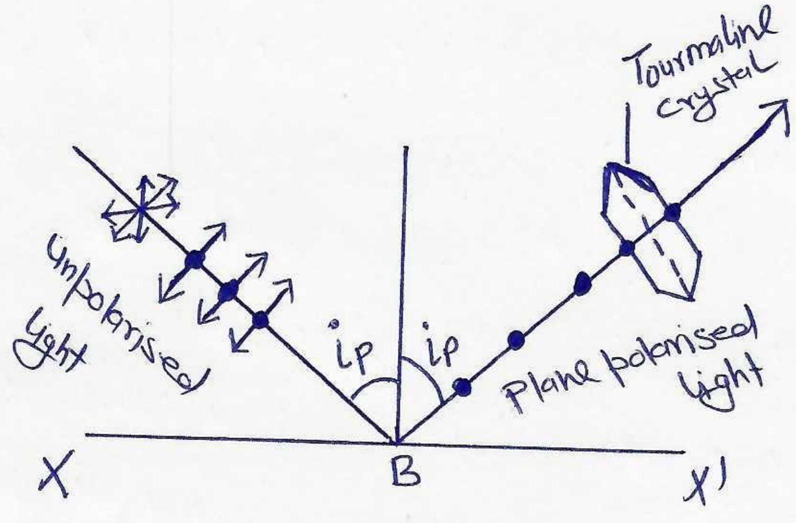
Elliptically polarized light —

When two plane polarised light waves are superimposed, then under certain condition, the resultant light vector rotates the a plane perpendicular to the direction of propagation of light; the tip of vector traces an ellipse and the light is said to be elliptically polarised light.
Elliptically polarised light consists of two perpendicular waves of unequal amplitude which differ in phase by 90°.



Elliptical

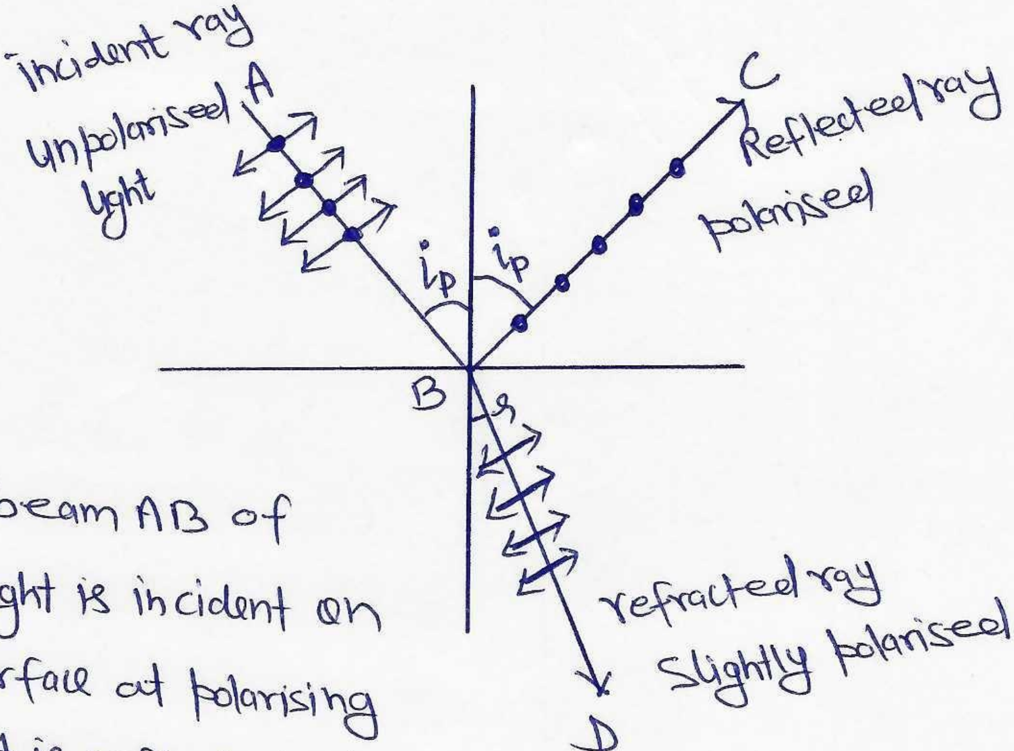
Polarisation by Reflection —



Ordinary light when reflected from a plane sheet of glass gets partially polarised. The degree of polarisation varies with angle with incidence. At a particular angle (i_p) known as angle of polarisation. The percentage of polarisation is maximum. The angle of polarisation slightly depends upon the nature of reflecting surface and wavelength of light.

Brewster's Law — Brewster performed number of experiments to study the polarisation of light by reflection. He found that at a perticular angle i_p the light is completely polarised in the plane of incident ray. He also found that the value of (i_p) depend upon the refractive index of reflecting medium. He discovered a relation

Which is known as Brewster's law. It also came into picture that reflected and refracted rays are perpendicular to each other.



Suppose a beam AB of unpolarised light is incident on the glass surface at polarising angle i_p. It is reflected along BC and refracted along BD. Then from Brewster's law

$$\mu = \tan i_p = \frac{\sin i_p}{\cos i_p}$$

From Snells Law

$$\mu = \frac{\sin i_p}{\sin r}$$

$$\frac{\sin i_p}{\cos i_p} = \frac{\sin i_p}{\sin r}$$

$$\frac{\sin i_p}{\sin\left(\frac{\pi}{2} - i_p\right)} = \frac{\sin i_p}{\sin r}$$

$$\frac{\pi}{2} - i_p = r$$

$$i_p + r = \frac{\pi}{2}$$

As $i_p + \angle CBD + r = \pi$

$$\angle CBD = \pi - (i_p + r)$$

$$= \pi - \frac{\pi}{2}$$

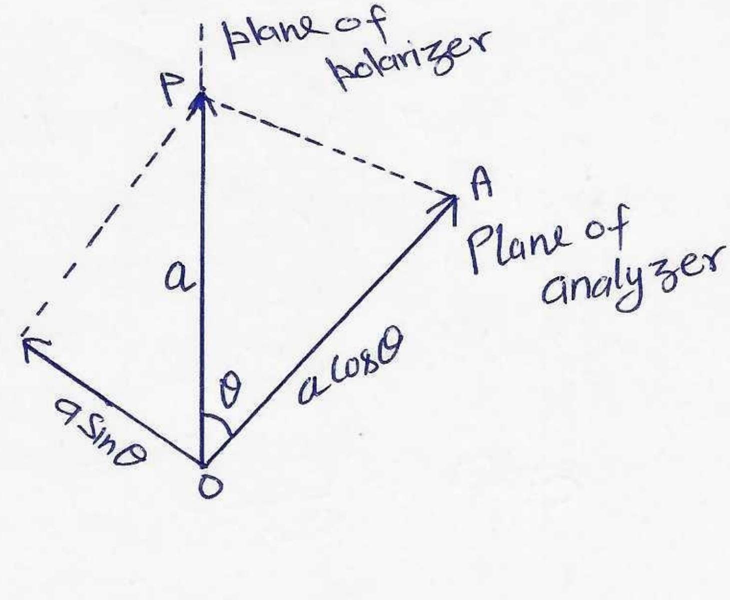
$$\boxed{\angle CBD = \frac{\pi}{2}}$$

i.e the reflected ray is at right angle to the refracted ray.

Polarisation by Refraction —

It is found that when ordinary light get refracted through any transparent medium, the refracted ~~ray~~ ray is partially ~~polarised~~ polarised. In order to obtain completely polarised light it is refracted through piles of plates which consists of adequate number of glass plates separated by airgaps. after multiple refraction through this arrangement the emerging light gets completely polarised.

Malus Law — According to malus when a completely plane polarised light is incident on an analyser, the intensity of the emergent light varies as the square of cosine of the angle between the plane of transmission of analyser and the polariser.



Let OP = a be the amplitude of the incident plane polarised light from a polariser and θ is the angle between plane of polariser and plane of analyser.

So the amplitude of incident plane polarised light can be resolved into two components one parallel to the plane of transmission of analyser ($a\cos\theta$) and other perpendicular to it ($a\sin\theta$)

The component $a\cos\theta$ is transmitted through the analyser.

The intensity of transmitted light through analyser

$$I = (a\cos\theta)^2 = a^2\cos^2\theta$$

If I_0 be the intensity of incident polarised light

then $I_0 = a^2$

So, $I = I_0\cos^2\theta$

or $\boxed{I \propto \cos^2\theta}$

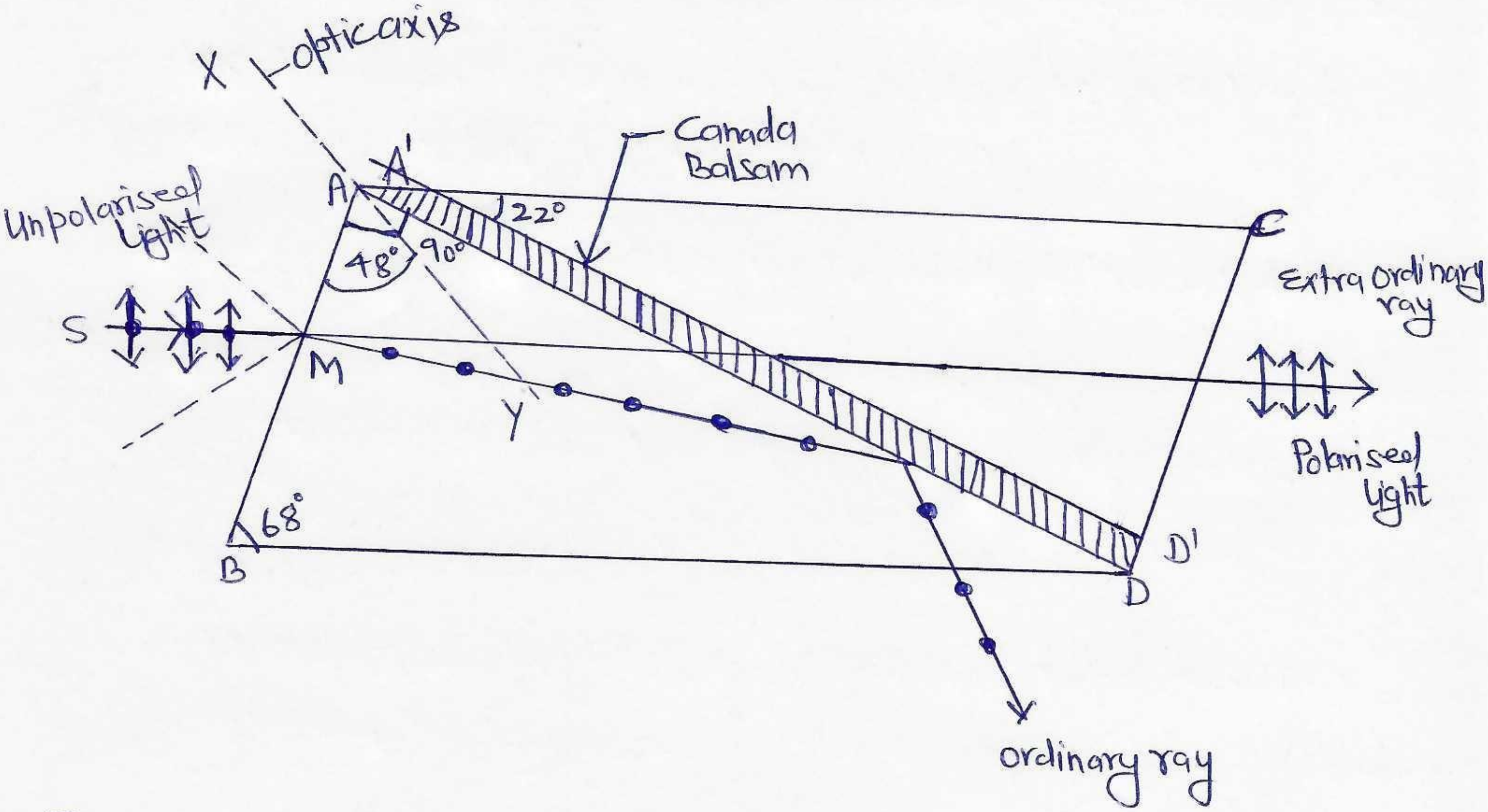
1) When $\theta = 0^\circ$, two planes are parallel.

$$I = I_0$$

2) When $\theta = 90^\circ$, two planes are perpendicular

$$I = 0$$

Nicol Prism — Nicol Prism is an optical device, invented by William Nicol for producing and analysing Plane polarised light.



Principle — It is based on the phenomenon of double refraction. When an Unpolarised light is passed through a doubly refracting Uniaxial Crystal, it is broken up into two rays.

1) ordinary ray

2) Extraordinary ray.

Both are polarised, having their Vibrations at right angle to each other. If by any suitable means one of the two rays is eliminated, the remaining ray coming out from the Crystal will be plane polarised.

Construction — Nicol prism is constructed from a calcite crystal whose length is nearly three times of its width ($l : b = 3 : 1$). The end faces of the crystal are cut down so as to reduce the angle of the principal section to a more acute angle of 68°. The crystal is then cut along a diagonal and the two cut surfaces after polishing, cemented back together with a special cement called Canada balsam, which is transparent substance. It is optically more denser than calcite for the e-ray and less denser for o-ray

for Sodium light $\mu_o = 1.6583$

$$\mu_{cb} = 1.55$$

$$\mu_e = 1.486$$

Action — A ray of light SM is incident nearly parallel to BD' on the face A'B of the Nicol prism, it split into e-ray and o-ray whose vibrations are respectively, perpendicular and parallel to the principal section of the Nicol prism. The o-ray suffers total internal reflection at the Canada balsam surface for nearly normal incidence, because Canada Balsam is optically more denser than Calcite for the e-ray and less denser than Calcite for the o-ray.

The e-ray is refracted through Canada balsam and is transmitted but o-ray, moving from a denser Calcite medium to the rarer of incidence greater than the Critical angle.

The Value of Critical angle i_c for the o-ray for Calcite to Canada balsam is

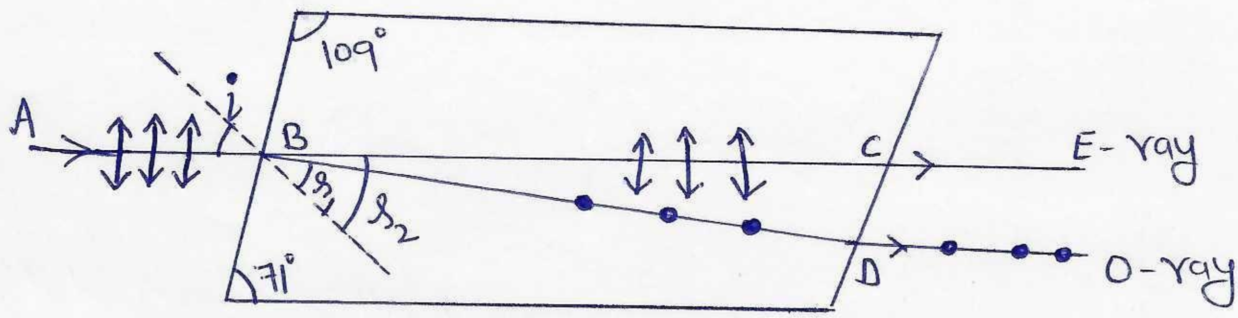
$$i_c = \sin^{-1} \frac{1.550}{1.658} = 69^\circ$$

Limitation — The Nicol prism works only when the incident ray is slightly Convergent or slightly divergent.

1) If the incident ray makes an angle much smaller than ∠BMS with the surface A'B the o-ray will Strike the Canada balsam layer at an angle less than the Critical angle and hence will be transmitted and light emerging from the Nicol prism will not be plane polarised.

2) If the incident ray make an angle greater than ∠BMS, the e-ray become more and more parallel to the optic axis and hence its refractive index will become nearly Equal to the Calcite for the o-ray. This will Suffer total internal reflection like o-ray. Hence no light will emerge out of the Nicol prism,

Double Refraction



When a beam of ordinary Unpolarised light is passed through a Calcite Crystal, the refracted light is split up into two refracted rays. The one which always obeys the ordinary Laws of refraction and having vibrations perpendicular to the principal Section is known as ordinary ray. the other, doesnot obey the Laws of refraction and having vibrations in the principal Section is Called as extra ordinary ray. Both the rays are plane polarised. This phenomenon is known as double refraction. The crystals Showing this phenomenon are Called ~~double~~ doubly refracting crystals.

There are two types of doubly refracting Crystals! 1) Uniaxial and 2) biaxial. In Uniaxial crystal there is only one direction (optic axis) along which the two refracted rays travel with the same ~~veloci~~ Velocity (Exp. Calcite, tourmaline and quartz). ~~In biaxial~~ In biaxial Crystals there are two such directions along which the velocities are the same ...

Consider abeam AB of unpolarised light incident on the Calcite Crystal at an angle of incident i. Inside the Crystal the ray breaks up into ordinary and extraordinary rays. The ordinary ray travelling along BD makes an angle of refraction(r_1) While the Extraordinary travelling along BC makes an angle of refraction(r_2). Since the two opposite faces of the Crystal are always parallel, both the rays emerge parallel to the incident rays. The refractive index of ordinary and extraordinary rays can be expressed as

$$\mu_o = \frac{\sin i}{\sin r_1} \quad \text{and} \quad \mu_e = \frac{\sin i}{\sin r_2}$$

In Case of Calcite $\mu_o > \mu_e$ because $r_1 < r_2$, Therefore the Velocity of light for ordinary ray inside the Crystal will be less than the extraordinary ray. It is observed that μ_o is Same for all the angle of incidence while μ_e Varies with angle of incidence. Therefore, ordinary ray travels with the Same Speeds in all directions while Extraordinary has different speeds in different directions.

Negative Crystals and Positive Crystals

Negative Crystals —

In Crystals Such as Calcite the angle of refraction for ordinary ray (r_o) is Less than the angle of refraction for Extra-Ordinary ray (r_e) hence the refractive index for ordinary ray (μ_o) is greater than the refractive index for Extraordinary ray (μ_e) and Velocity of ordinary ray is less than the Velocity of Extra ordinary ray. Such Crystals are Called negative Crystals. The ordinary image remains Stationary and e-image revolves around the o-imge.

Positive Crystals —

In positive Crystals $r_o > r_e$, hence $\mu_o < \mu_e$ and $v_o > v_e$. In Such Crystals the ordinary image remains fixed but the extraordinary image revolves in between the ordinary image, the line joining the two being parallel to the longer diagonal of the Emergent Face i.e. quartz Crystal

Retarders/Wave Plates

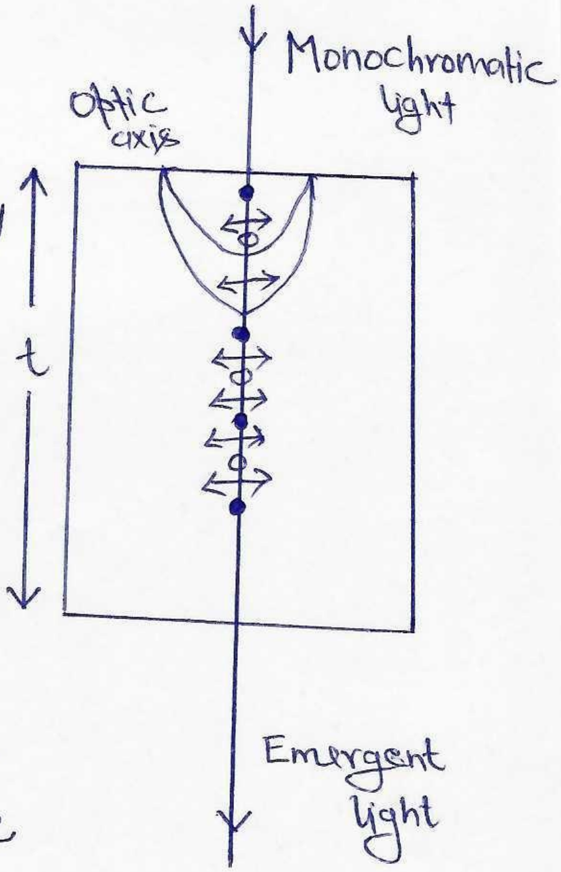
Retardation Plates —

The Simplest device for producing and detecting Circularly and Elliptically polarised light is Known as retardation plate. A plate cut from a doubly refracting Crystal So as to produce a definite ~~Valueof~~ Value of path difference or phase diffrence between e-ray and o-ray is Known as retardation plate. In generally, a retarding plate is cut from a doubly refracting Crystal with its face parallel to the optic axis. There are two types of retarding plates.

1) Quarter Wave Plate 2) Half Wave Plate

Quarter Wave Plate — A Plate of doubly refracting Uniaxial Crystal cut with its optic axis parallel to the refracting faces and Capable of producing a path difference of $\lambda/4$ or a phase difference of $\pi/2$ between the ordinary and extraordinary ~~rays~~ waves is Called a quarter wave plate or $\lambda/4$ plate.

When a beam of Monochromatic light of wavelength λ is incident normally on such a plate, it is broken up into o-wave and e-wave inside the plate. Both these waves travel in the same direction perpendicular to the faces but with different velocity



If 't' is the thickness of the plate, then the path t in the crystal plate is equivalent to $\mu_o t$ and $\mu_e t$ for o- and e- rays.

Hence the path difference between the two waves on emerging in case of negative crystal is

$$\Delta = \mu_o t - \mu_e t$$

$$\Delta = (\mu_o - \mu_e)t \quad \text{———} \quad (1)$$

If the plate acts as a quarter wave plate, the path difference must be equal to $\lambda/4$.

$$\Delta = \lambda/4$$

$$(\mu_o - \mu_e)t = \lambda/4$$

$(\mu_o > \mu_e)$

$$t = \frac{\lambda}{4(\mu_o - \mu_e)} \quad \text{———} \quad (2)$$ for negative crystal

for +ve crystal

λ ——— (3)

A quarter wave plate is used in the production of Circularly and Elliptically polarised light. If the angle of incidence of plane polarised light is 45° with optic axis the emergent light is circularly polarised. If the incident plane polarised light meets the optic axis at an angle not equal to 45° then the emergent ray is elliptically polarised.

<u>Half Wave plate</u> — A plate of doubly refracting uniaxial crystal cuts with its optic axis parallel to the refracting faces and capable of producing a path difference of $\lambda/2$ and phase difference of π between e- and o-rays is called Half wave plate or $\lambda/2$ plate.

If 't' is the thickness of such a plate, the in case of negative crystal ($\mu_o > \mu_e$) the path difference between o-ray and e-ray is

$$\Delta = (\mu_o - \mu_e)t = \lambda/2$$

$$t = \frac{\lambda}{2(\mu_o - \mu_e)} \quad \text{——} (1)$$

for positive crystal, $\mu_e > \mu_o$

$$t = \frac{\lambda}{2(\mu_e - \mu_o)} \quad \text{——} (2)$$

Numericals (Polarisation)

Q.1 When the angle of incidence on a certain material is 60°, the reflected light is completely polarised. Find the refractive index for the material and also the angle of refraction.

Q.2 A Glass plate is to be used as a polariser. Find the angle of polarisation. Also find the angle of refraction.

Given μ for glass $= 1.54$

Q3. Two polarising sheets have their directions parallel so that the intensity of transmitted light is maximum. Through what angle must either sheet be ~~turned~~ turned so that the intensity become one half of the initial value.

Given $I = I_0/2$

Q4. Intensity of light through a polariser and analyser is maximum, when their principal planes are parallel. Through what angle the analyser ~~[illegible]~~ must be rotated so that the intensity get reduced to 1/4 of the maximum value.

Ans 1

By Brewster's Law

$\mu = \tan i_p$

Here $i_p = 60^\circ$

Therefore Refractive index of the material

$\mu = \tan 60 = \sqrt{3} = 1.732$

$\mu = 1.732$

Angle of refraction $= 180^\circ - (90^\circ + 60^\circ)$

$= 180^\circ - 150^\circ$

Angle of refraction $= 30^\circ$

Ans

Ans 2

We know that from Brewster's Law

$\tan i_p = \mu$

$i_p = \tan^{-1}(\mu) = \tan^{-1}(1.54) = 57^\circ$

$i_p = 57^\circ$

If r is the angle of refraction, then from Brewster's Law

$i_p + r = \pi/2$

$r = \frac{\pi}{2} - i_p = 90^\circ - 57^\circ = 33^\circ$

$\angle r = 33^\circ$

Ans

Ans 3

We know that Malus Law,

$$I = I_0 \cos^2\theta$$

Given $I = I_0/2$

Therefore $\frac{I_0}{2} = I_0 \cos^2\theta$

$$\cos^2\theta = \frac{1}{2}$$

$$\cos\theta = \frac{1}{\sqrt{2}} = \cos 45^\circ$$

$$\theta = 45^\circ \text{ or } 135^\circ$$

Ans

Ans 4

From Law of Malus

$$I = I_0 \cos^2\theta$$

Given $I = I_0 - \frac{I_0}{4} = \frac{3}{4} I_0$

So $\frac{3}{4} I_0 = I_0 \cos^2\theta$

$$\cos^2\theta = \frac{3}{4}$$

$$\cos\theta = \frac{\sqrt{3}}{2} = \cos 30^\circ$$

$$\theta = 30^\circ$$

Ans

Ans 5 Given

$C_1 = 5\% = 0.05,\ l_1 = 40\,cm,\ \theta_1 = 20^\circ$

$C_2 = 10\% = 0.1,\ l_2 = ?\ ,\ \theta_2 = 35^\circ$

We Know that Specific rotation

$$S = \frac{\theta}{l \times C}$$

$$S = \frac{\theta_1}{l_1 \times C_1} = \frac{\theta_2}{l_2 \times C_2}$$

$$l_2 = \left(\frac{\theta_2}{\theta_1}\right)\left(\frac{C_1}{C_2}\right) \times l_1$$

$$= \frac{35}{20} \times \frac{0.05}{0.1} \times 40$$

$$l_2 = 35\ cm$$

Ans