Math for the Social Sciences Module - Young Researchers Fellowship

Lecture 2 - Equation Systems and Graphing

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Equation systems

- A set of of equations that share the same variables is called an *equation system*.
- For example:

$$x + y = 3 \tag{1}$$
$$2x - y = 1 \tag{2}$$

- Because both (1) and (2) share x and y, they form an equation system.
- We usually want to *solve* the system, i.e., find the values of x and y that satisfy both equations.

Solving equation systems

- There are several methods to solve equation systems.
  - Substitution
  - Elimination
  - Graphing
  - Matrices (we will see this later)
- Substitution is typically the most "mechanical" method.
  - Express one variable in terms of the other and substitute in the other equation.
- Elimination is more algebraic.
  - Add or subtract the equations to eliminate one variable.
  - Might involve multiplying one or both equations by a constant.

Solving the example system

- Let's solve the example system:

$$x + y = 3$$
$$2x - y = 1$$

- We can solve this system by substitution.
  - From (1), we have $y = 3 - x$.
  - Substitute this into (2):

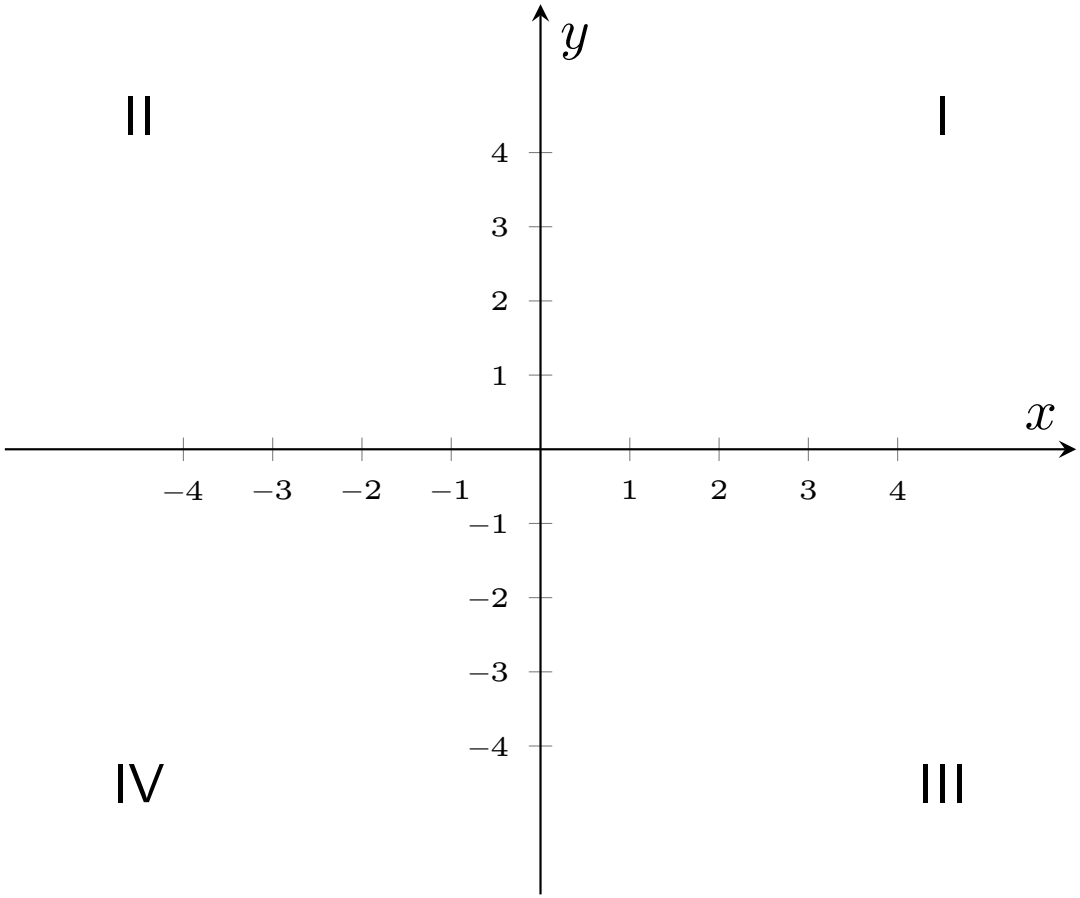
$$2x - (3 - x) = 1$$

- Solve for x and then substitute back to find y.

The Cartesian plane

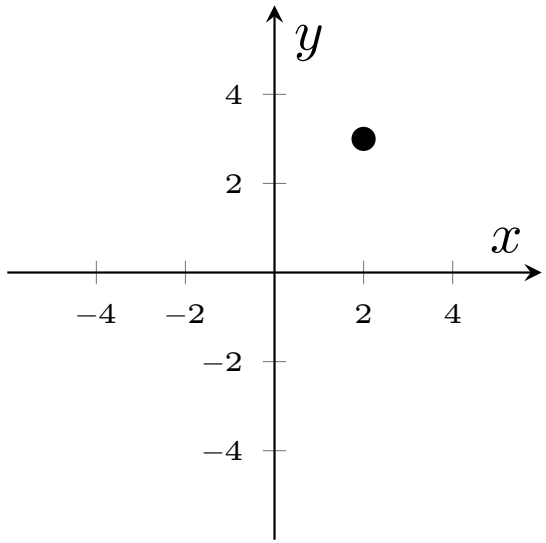
- The Cartesian plane is a two-dimensional space where we can plot points.
- It is formed by two perpendicular lines, the *x-axis* and the *y-axis*.
- The point where the axes intersect is called the *origin*.
- The axes divide the plane into four *quadrants*.

The Cartesian plane



Plotting points

- To plot a point, we use an ordered pair (x, y).
  - x is the distance from the y-axis.
  - y is the distance from the x-axis.
- For example, the point $(2, 3)$ is 2 units to the right and 3 units up from the origin. See below:



Linear equations

- The equations we've seen so far are *linear* equations.
  - They represent straight lines in the Cartesian plane.
- Linear equations can be written in the form $y = mx + b$.
  - m is the *slope* of the line.
  - b is the *y-intercept*.

The Slope

- The ratio of the vertical change to the horizontal change.
  - It tells us how steep the line is.
  - The bigger the slope, the steeper the line.
- Given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
- Requires two points (call them P_1 and P_2) on the line, with coordinates (x_1, y_1) and (x_2, y_2).

Figure 1: A meme

For extra credit: It's never fun to explain memesIn a private Slack