Math for the Social Sciences Module - Young Researchers Fellowship

Lecture 2 - Equation Systems and Graphing

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Equation systems

- A set of of equations that share the same variables is called an *equation system*.
- For example:

$$x + y = 3 \tag{1}$$
$$2x - y = 1 \tag{2}$$

- Because both (1) and (2) share x and y, they form an equation system.
- We usually want to *solve* the system, i.e., find the values of x and y that satisfy both equations.

Solving equation systems

- There are several methods to solve equation systems.
  - Substitution
  - Elimination
  - Graphing
  - Matrices (we will see this later)
- Substitution is typically the most "mechanical" method.
  - Express one variable in terms of the other and substitute in the other equation.
- Elimination is more algebraic.
  - Add or subtract the equations to eliminate one variable.
  - Might involve multiplying one or both equations by a constant.

Solving the example system

- Let's solve the example system:

$$
\begin{aligned}
x + y &= 3 \\
2x - y &= 1
\end{aligned}
$$

- We can solve this system by substitution.
    - From (1), we have $y = 3 - x$.
    - Substitute this into (2):

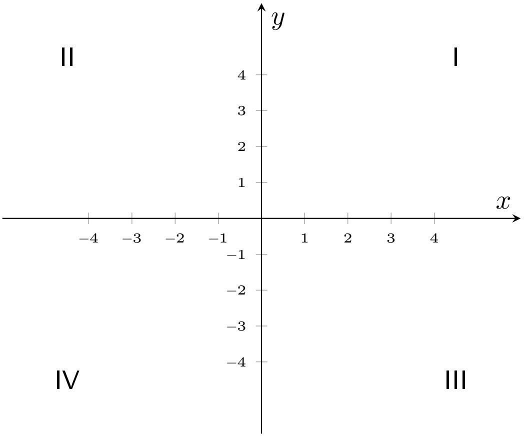
$$2x - (3 - x) = 1$$

- Solve for x and then substitute back to find y.

The Cartesian plane

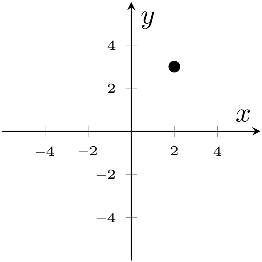
- The Cartesian plane is a two-dimensional space where we can plot points.
- It is formed by two perpendicular lines, the *x-axis* and the *y-axis*.
- The point where the axes intersect is called the *origin*.
- The axes divide the plane into four *quadrants*.

The Cartesian plane



Plotting points

- To plot a point, we use an ordered pair (x, y).
    - x is the distance from the y-axis.
    - y is the distance from the x-axis.
- For example, the point $(2, 3)$ is 2 units to the right and 3 units up from the origin. See below:



Linear equations

- The equations we've seen so far are *linear* equations.
    - They represent straight lines in the Cartesian plane.
- Linear equations can be written in the form $y = mx + b$.
    - m is the *slope* of the line.
    - b is the *y-intercept*.

The Slope

- The ratio of the vertical change to the horizontal change.
  - It tells us how steep the line is.
  - The bigger the slope, the steeper the line.
- Given by $m = \dfrac{y_2 - y_1}{x_2 - x_1}$
- Requires two points (call them P_1 and P_2) on the line, with coordinates (x_1, y_1) and (x_2, y_2).

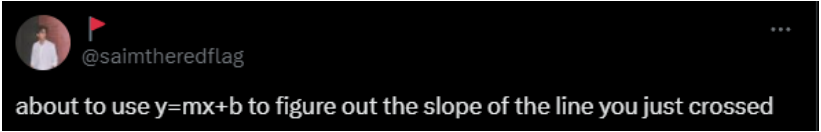


Figure 1: A meme

Intercepts

- The *y-intercept* is the point where the line crosses the y-axis.
  - This happens when $x = 0$.
  - So, we set $x = 0$ in the equation and solve for y.
  - In the equation $y = mx + b$, the y-intercept is $(0, b)$.
- The *x-intercept* is the point where the line crosses the x-axis.
  - This happens when $y = 0$.
  - So, we set $y = 0$ in the equation and solve for x.

Graphing linear equations

- To graph a linear equation, we need to find two points on the line.
  - The easiest points are the intercepts.
  - We can also use the slope to find a second point.
- Example: graph the line $y = 2x + 1$.
  - It might be useful to draw a table of values.

x	y
0	1
1	3
-1	-1

Graphing the line

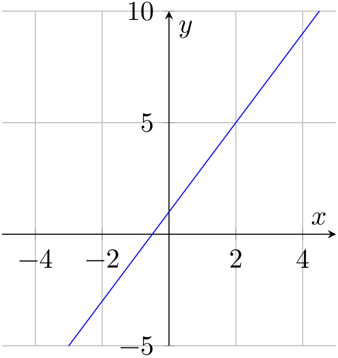


Figure 2: Plot of the equation $y = 2x + 1$

Upward-sloping and downward-sloping lines

- If $m > 0$, the line is "upward-sloping" or increasing.
  - As x increases, y also increases.
- If $m < 0$, the line is "downward-sloping" or decreasing.
  - As x increases, y decreases.

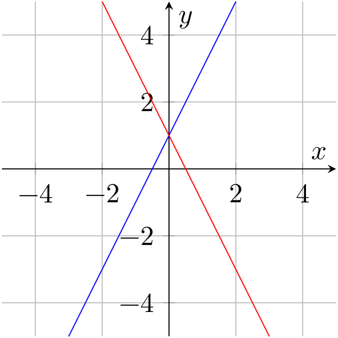


Figure 3: Upward-sloping and downward-sloping lines

Properties of slopes

- If $m = 0$, the line is horizontal.
  - y does not change as x changes.
- If $m = \infty$, the line is vertical.
  - x does not change as y changes.
- If $m = 1$, the line has a 45-degree angle.
- Lines with the same slope are parallel.
- Lines with slopes that multiply to -1 are perpendicular.
  - This means that $m_1 \cdot m_2 = -1$, or that $m_1 = -\dfrac{1}{m_2}$ (the negative reciprocal).