

# Math for the Social Sciences Module - Young Researchers Fellowship

## Lecture 2 - Equation Systems and Graphing

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# Equation systems

- A set of equations that share the same variables is called an *equation system*.
- For example:

$$x + y = 3 \tag{1}$$

$$2x - y = 1 \tag{2}$$

- Because both (1) and (2) share  $x$  and  $y$ , they form an equation system.
- We usually want to *solve* the system, i.e., find the values of  $x$  and  $y$  that satisfy both equations.

# Solving equation systems

- There are several methods to solve equation systems.
  - Substitution
  - Elimination
  - Graphing
  - Matrices (we will see this later)
- Substitution is typically the most “mechanical” method.
  - Express one variable in terms of the other and substitute in the other equation.
- Elimination is more algebraic.
  - Add or subtract the equations to eliminate one variable.
  - Might involve multiplying one or both equations by a constant.

# Solving the example system

- Let's solve the example system:

$$x + y = 3$$

$$2x - y = 1$$

- We can solve this system by substitution.
  - From (1), we have  $y = 3 - x$ .
  - Substitute this into (2):

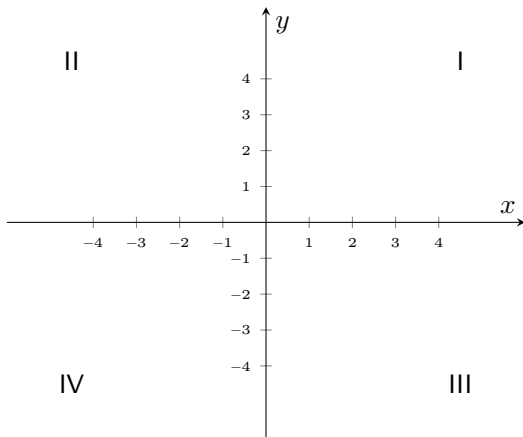
$$2x - (3 - x) = 1$$

- Solve for  $x$  and then substitute back to find  $y$ .

# The Cartesian plane

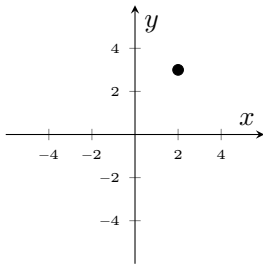
- The Cartesian plane is a two-dimensional space where we can plot points.
- It is formed by two perpendicular lines, the *x-axis* and the *y-axis*.
- The point where the axes intersect is called the *origin*.
- The axes divide the plane into four *quadrants*.

# The Cartesian plane



# Plotting points

- To plot a point, we use an ordered pair  $(x, y)$ .
  - $x$  is the distance from the  $y$ -axis.
  - $y$  is the distance from the  $x$ -axis.
- For example, the point  $(2, 3)$  is 2 units to the right and 3 units up from the origin. See below:



# Linear equations

- The equations we've seen so far are *linear* equations.
  - They represent straight lines in the Cartesian plane.
- Linear equations can be written in the form  $y = mx + b$ .
  - $m$  is the *slope* of the line.
  - $b$  is the *y-intercept*.



# The Slope

- The ratio of the vertical change to the horizontal change.
  - It tells us how steep the line is.
  - The bigger the slope, the steeper the line.
- Given by  $m = \frac{y_2 - y_1}{x_2 - x_1}$
- Requires two points (call them  $P_1$  and  $P_2$ ) on the line, with coordinates  $(x_1, y_1)$  and  $(x_2, y_2)$ .

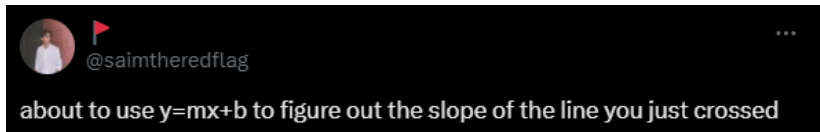


Figure 1: A meme

# Intercepts

- The *y-intercept* is the point where the line crosses the *y*-axis.
  - This happens when  $x = 0$ .
  - So, we set  $x = 0$  in the equation and solve for  $y$ .
  - In the equation  $y = mx + b$ , the *y*-intercept is  $(0, b)$ .
- The *x-intercept* is the point where the line crosses the *x*-axis.
  - This happens when  $y = 0$ .
  - So, we set  $y = 0$  in the equation and solve for  $x$ .

# Graphing linear equations

- To graph a linear equation, we need to find two points on the line.
  - The easiest points are the intercepts.
  - We can also use the slope to find a second point.
- Example: graph the line  $y = 2x + 1$ .
  - It might be useful to draw a table of values.

| $x$ | $y$ |
|-----|-----|
| 0   | 1   |
| 1   | 3   |
| -1  | -1  |

# Graphing the line

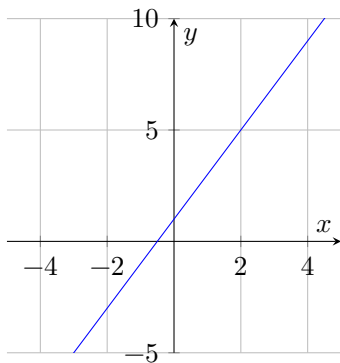


Figure 2: Plot of the equation  $y = 2x + 1$

# Upward-sloping and downward-sloping lines

- If  $m > 0$ , the line is “upward-sloping” or increasing.
  - As  $x$  increases,  $y$  also increases.
- If  $m < 0$ , the line is “downward-sloping” or decreasing.
  - As  $x$  increases,  $y$  decreases.

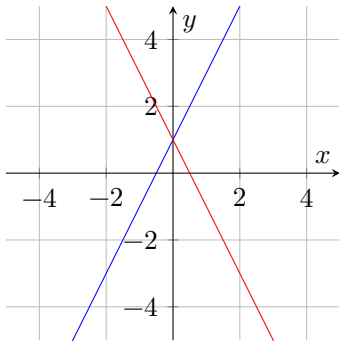


Figure 3: Upward-sloping and downward-sloping lines

# Properties of slopes

- If  $m = 0$ , the line is horizontal.
  - $y$  does not change as  $x$  changes.
- If  $m = \infty$ , the line is vertical.
  - $x$  does not change as  $y$  changes.
- If  $m = 1$ , the line has a 45-degree angle.
- Lines with the same slope are parallel.
- Lines with slopes that multiply to -1 are perpendicular.
  - This means that  $m_1 \cdot m_2 = -1$ , or that  $m_1 = -\frac{1}{m_2}$  (the negative reciprocal).

# Graphing equation systems

- To solve an equation system graphically, we graph both equations and find the point where they intersect.
- The point of intersection is the solution to the system.
- Example: graph the system

$$x + y = 3$$

$$2x - y = 1$$

# Graphing the system

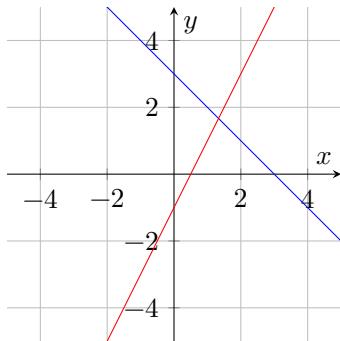


Figure 4: Graph of the system