Math for the Social Sciences Module - Young Researchers Fellowship

Lecture 4 - Logarithms and Modelling in the Social Sciences

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Logarithms

- The logarithm of a number is the power to which a base must be raised to obtain that number.

$$\log_b(x) = y \iff b^y = x$$

* $\iff$ means "if and only if".

- The most common logarithms are base 10 and base e (Euler's number).
- The natural logarithm is the logarithm with base e.
  - It is denoted $\ln(x)$.
- Euler's number is approximately 2.71828.
  - It is an irrational number.
  - It was discovered by the Swiss mathematician Leonhard Euler while studying compound interest (percentages).

Properties of Logarithms

Product rule: $\log_b(xy) = \log_b(x) + \log_b(y)$

Quotient rule: $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$

Power rule: $\log_b(x^y) = y\log_b(x)$

Change of base formula: $\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$

Logarithms are our friends!

- As icky as they might seem, logarithms are our friends **because of their properties**.
- When dealing with variable exponentials, logarithms can help us simplify the problem.
  - Use the power rule to bring down the exponent and easily differentiate.
- Natural logarithms are particularly useful for the social sciences.
  - They are used in growth models because of the Euler's number relationship to percent growth.
- Exponential growth shows up in real life in many ways.
  - Population growth, compound interest, and the spread of diseases are all examples of exponential growth.
  - Logarithms are tools to deal with these phenomena.