

# Single and multiscale entropy

## - Regularity and complexity of human movement

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# Content

- Relative input parameter consistency
- Discrete vs. continuous data
- $m$  – vector length
- $r$  – tolerance limit
- $N$  – number of data points
- Recommended reading

# Sample entropy – relative input parameter consistency

- Many (if not all) nonlinear mathematical tools have input parameters.
- Changing the input parameters affects the outcome of the calculation.
- Any observed biological phenomenon should remain the same when changing input parameters (within a range) to be considered valid.
  - Differences between tasks
  - Differences between groups
  - Differences before/after intervention

**Always test and report the relative input parameter consistency of your data!**

- For sample entropy:  $m$ ,  $r$  and  $N$

# Sample entropy – input parameters

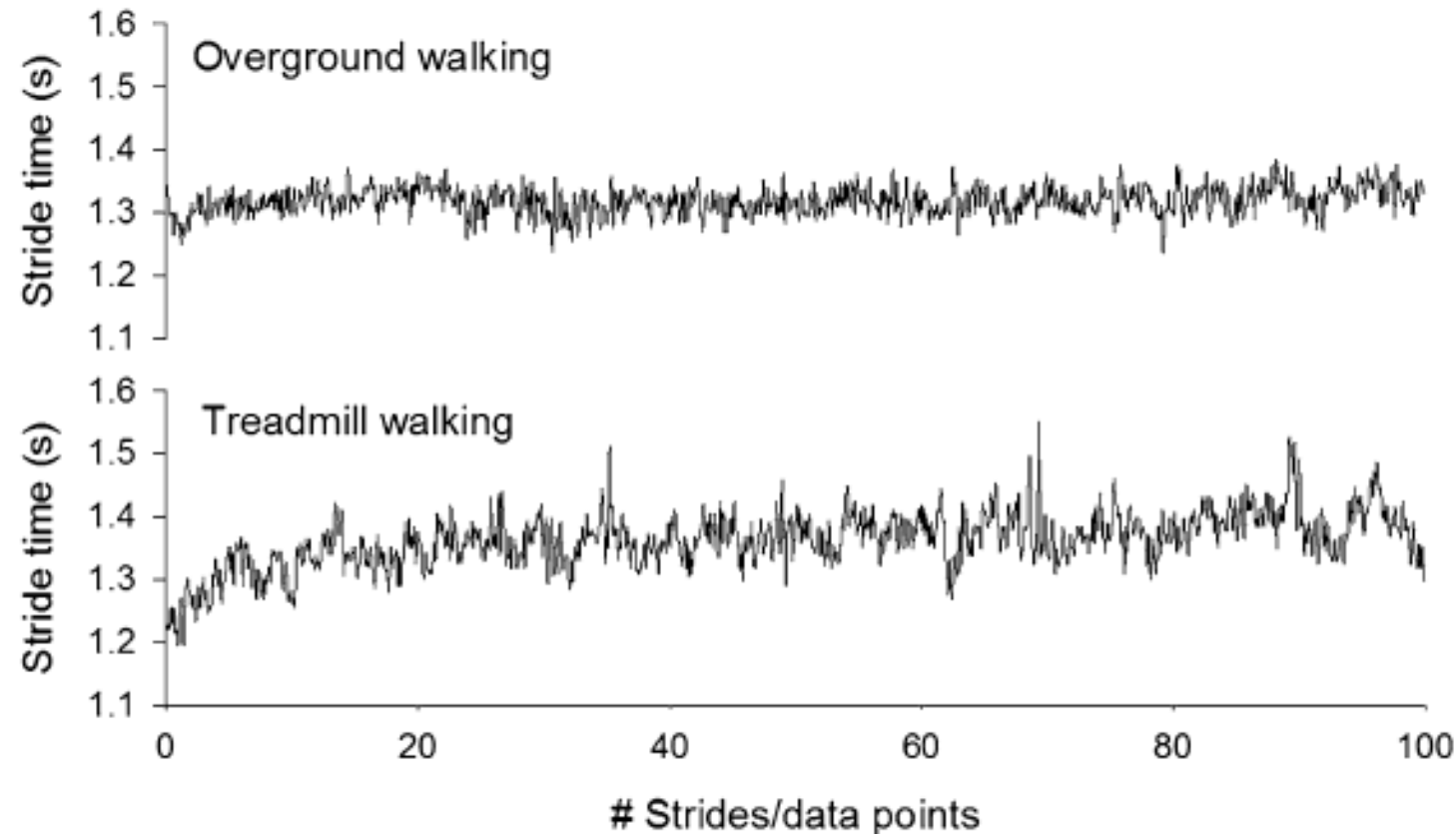
Three studies:

- Yentes et al. 2012 (discrete data)
  - Treadmill walking of young adults for 3 minutes.
  - Motion capture quantified step time, step width and step length.
  - Comparisons of ApEn and SaEn with different input parameters.
- Yentes et al. 2018 (discrete data)
  - Overground and treadmill walking in young adults for one hour.
  - Step time quantified using heel switches.
  - Comparisons of ApEn and SaEn with different input parameters.
- Raffalt et al. 2023 (continuous data)
  - Isometric submaximal force in young adults for 180 seconds.
  - Investigate relative input parameter consistency of  $r$ ,  $m$ , and  $N$ .

# Sample entropy – input parameters

$$SaEn(m, r, N) = -\ln \left[ \frac{A^{m+1}(r)}{B^m(r)} \right]$$

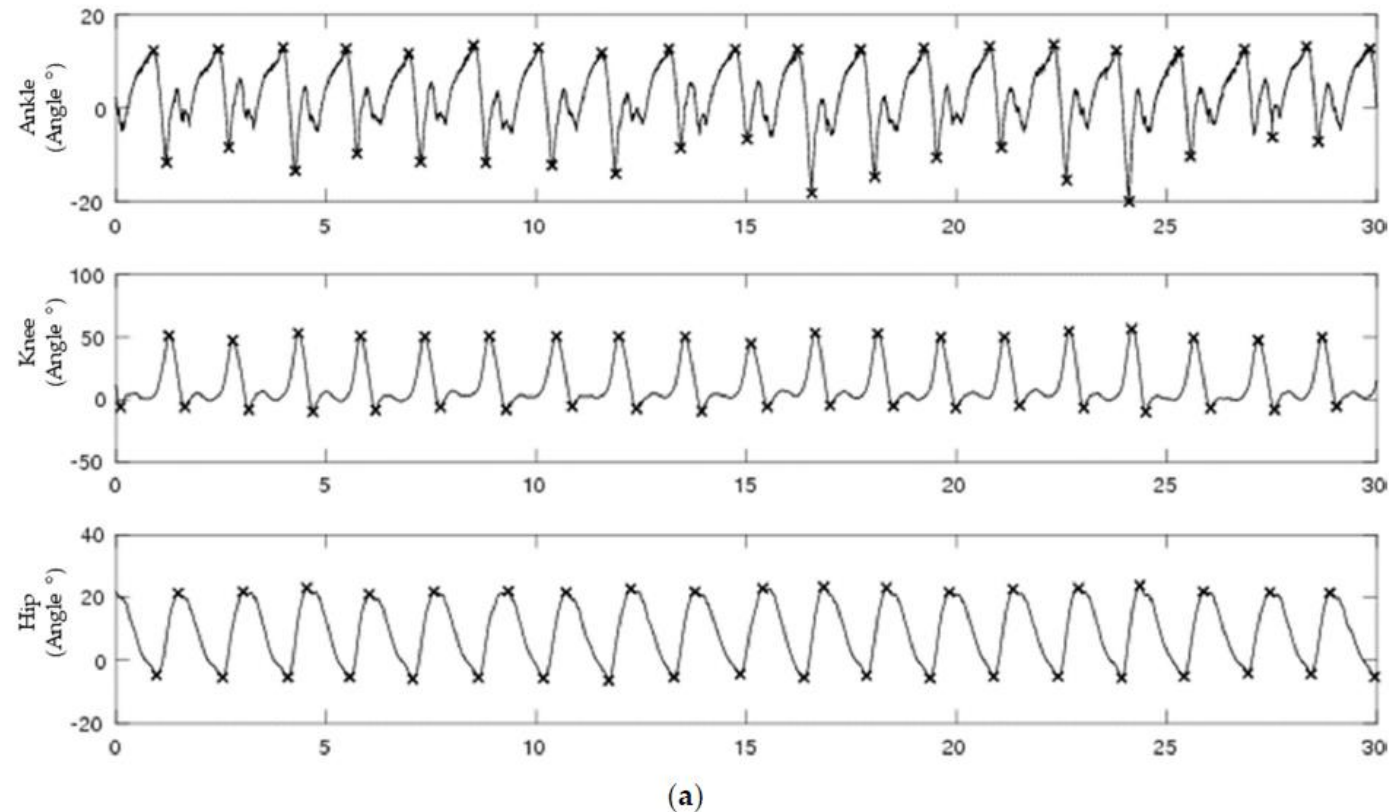
- **Discrete** vs continuous data – N is the number of observations of a particular event e.g., strides/steps



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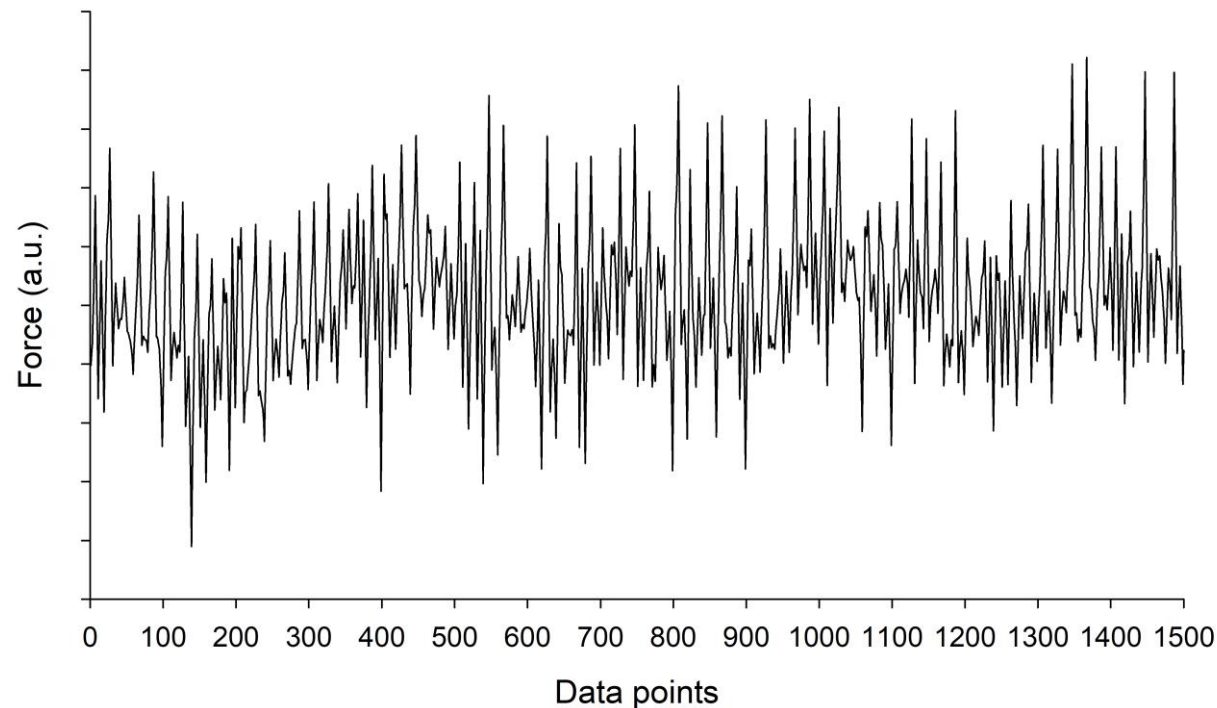
- Discrete vs **continuous** data – N is the number of data points given by the observation time and the sampling frequency



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$$SaEn(m, r, N) = -\ln \left[ \frac{A^{m+1}(r)}{B^m(r)} \right]$$

- Discrete vs **continuous** data – N is the number of data points given by the observation time and the sampling frequency

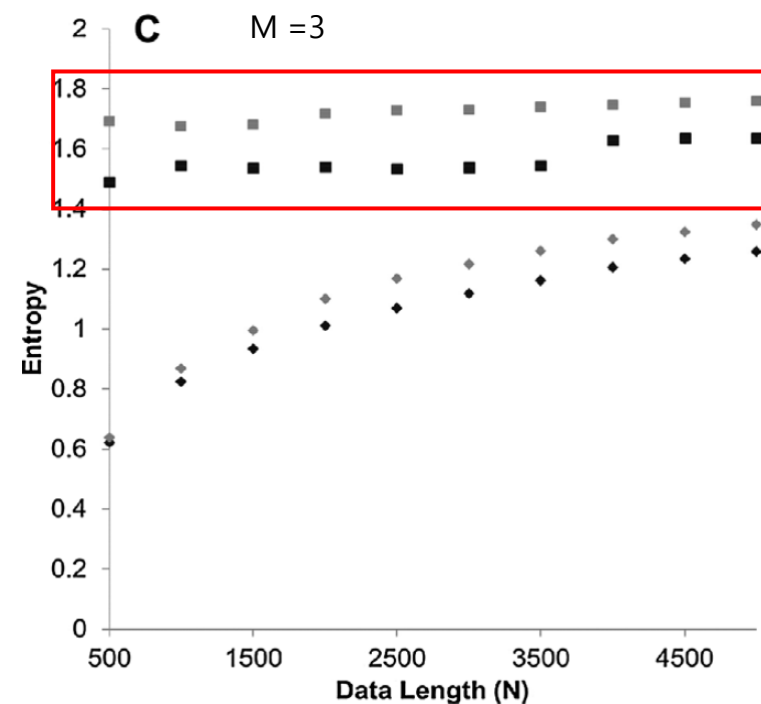
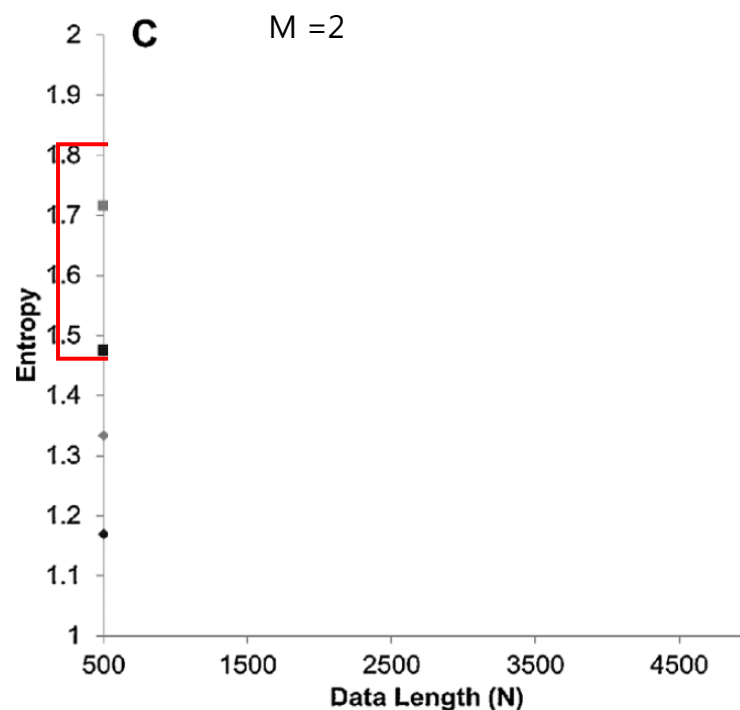


# Sample entropy – input parameter m

$$SaEn(m, r, N) = -\ln \left[ \frac{A^{m+1}(r)}{B^m(r)} \right]$$

- m is the vector length
- ApEn vs SaEn across different N (from 500 to 5000) with r=0.25 and m=2 or 3.

Gray: overground walking  
Black: treadmill walking  
ApEn: diamond  
SaEn: square

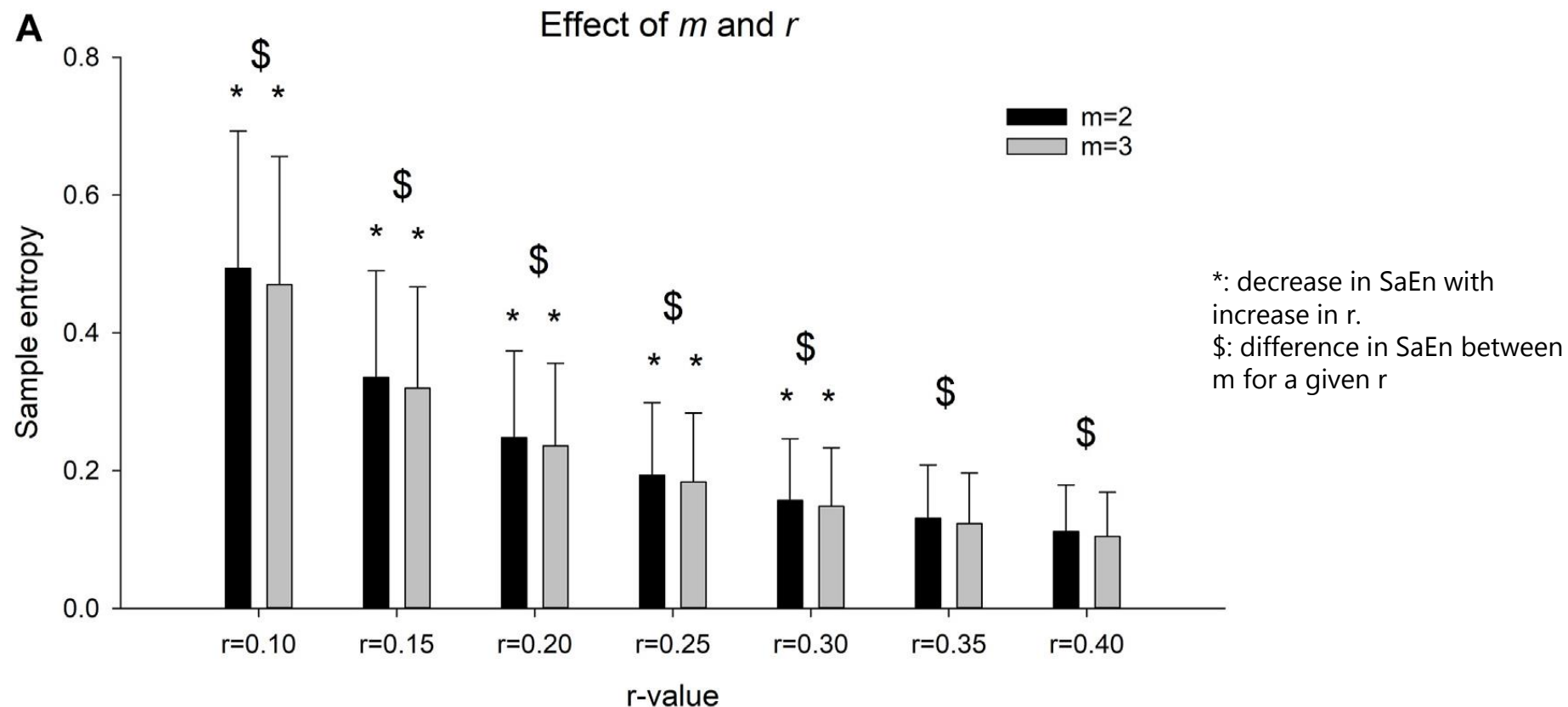
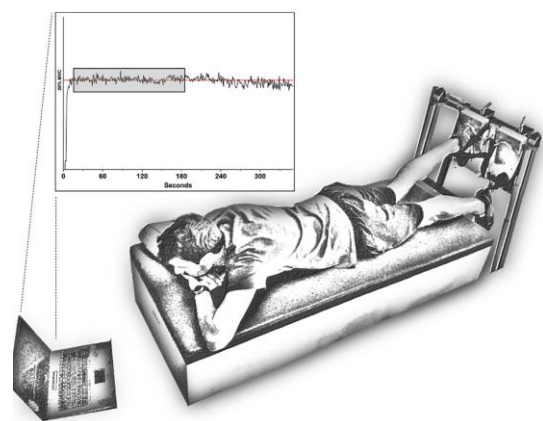




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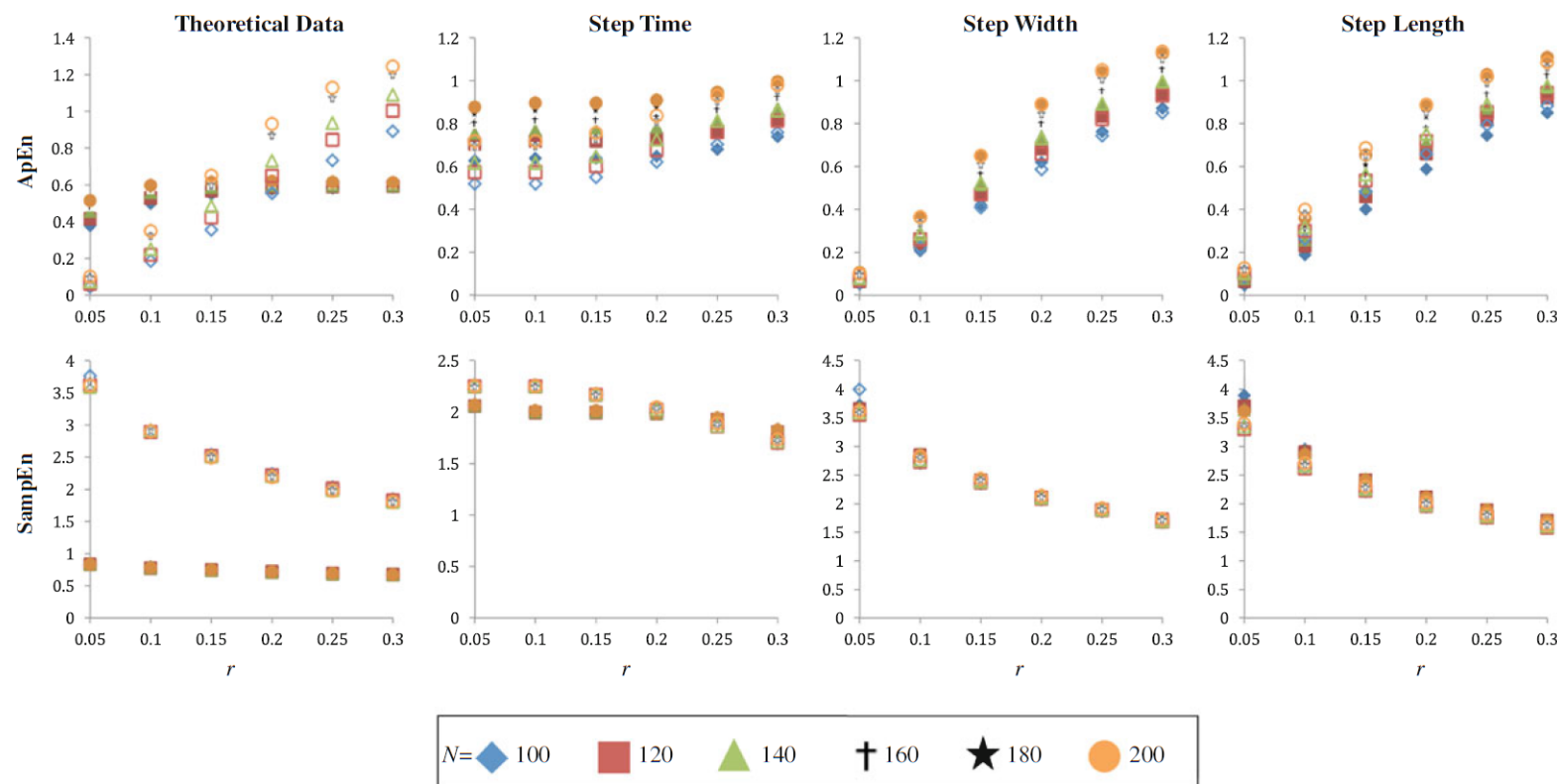
- m is the vector length
- SaEn across different r with m=2 and 3, sampling frequency=100Hz and observation time = 180s



# Sample entropy – input parameter $r$

$$SaEn(m, r, N) = -\ln \left[ \frac{A^{m+1}(r)}{B^m(r)} \right]$$

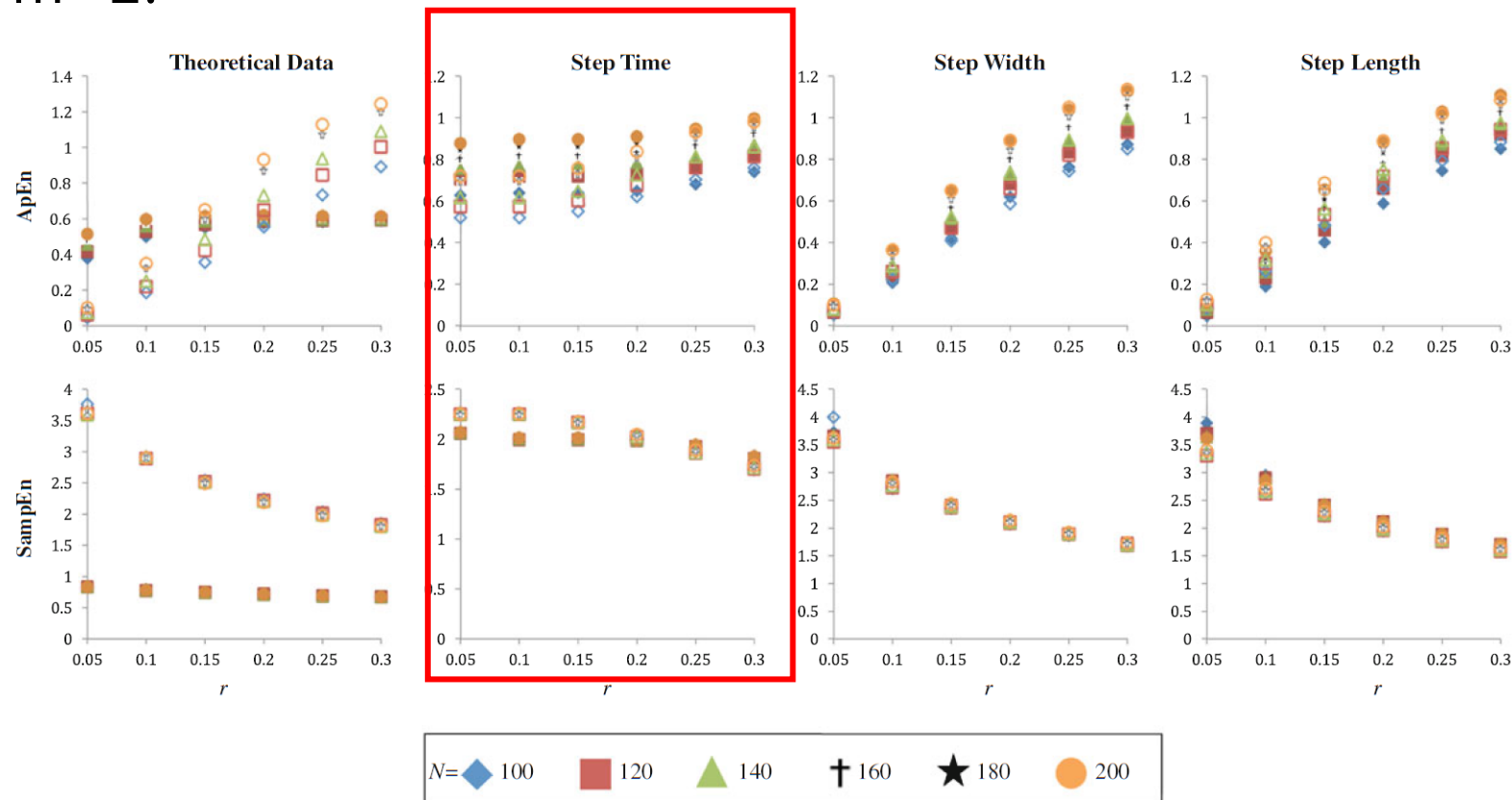
- $R$  is the tolerance limit ( $r \times SD$ )
- ApEn vs SaEn across different  $N$  (from 100 to 200) and different  $r$  (from 0.05 to 0.3) and  $m=2$ .



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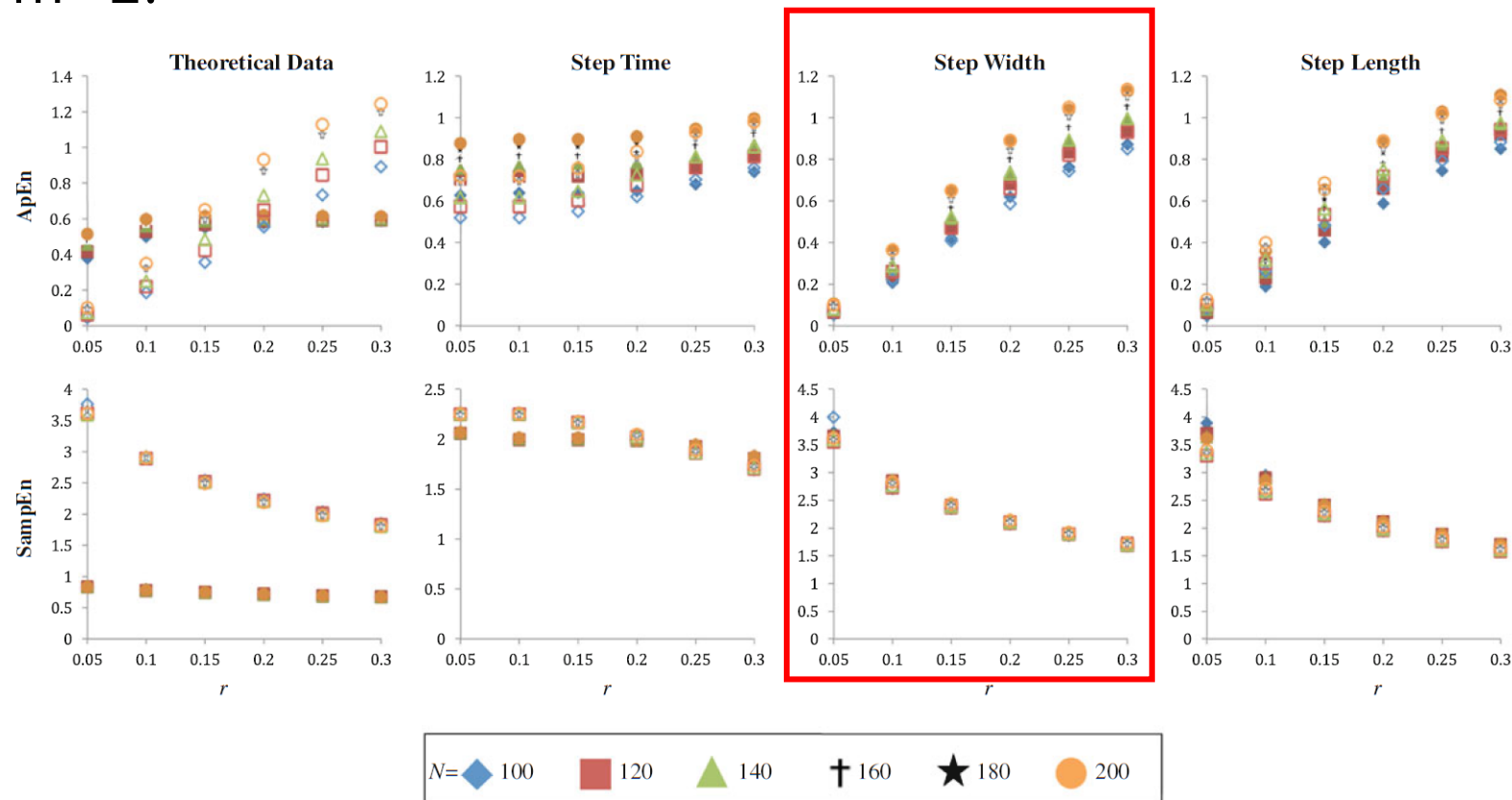
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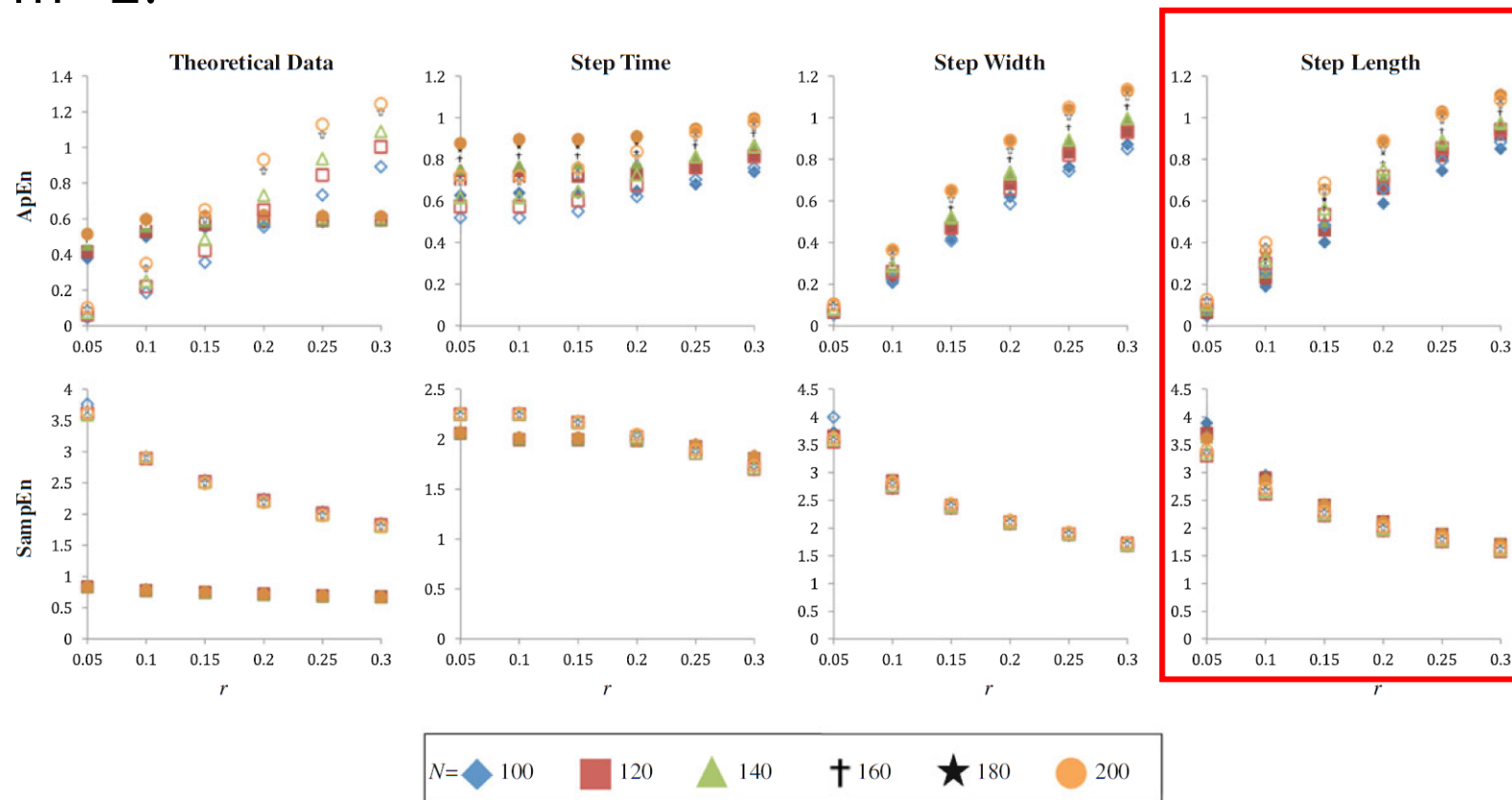
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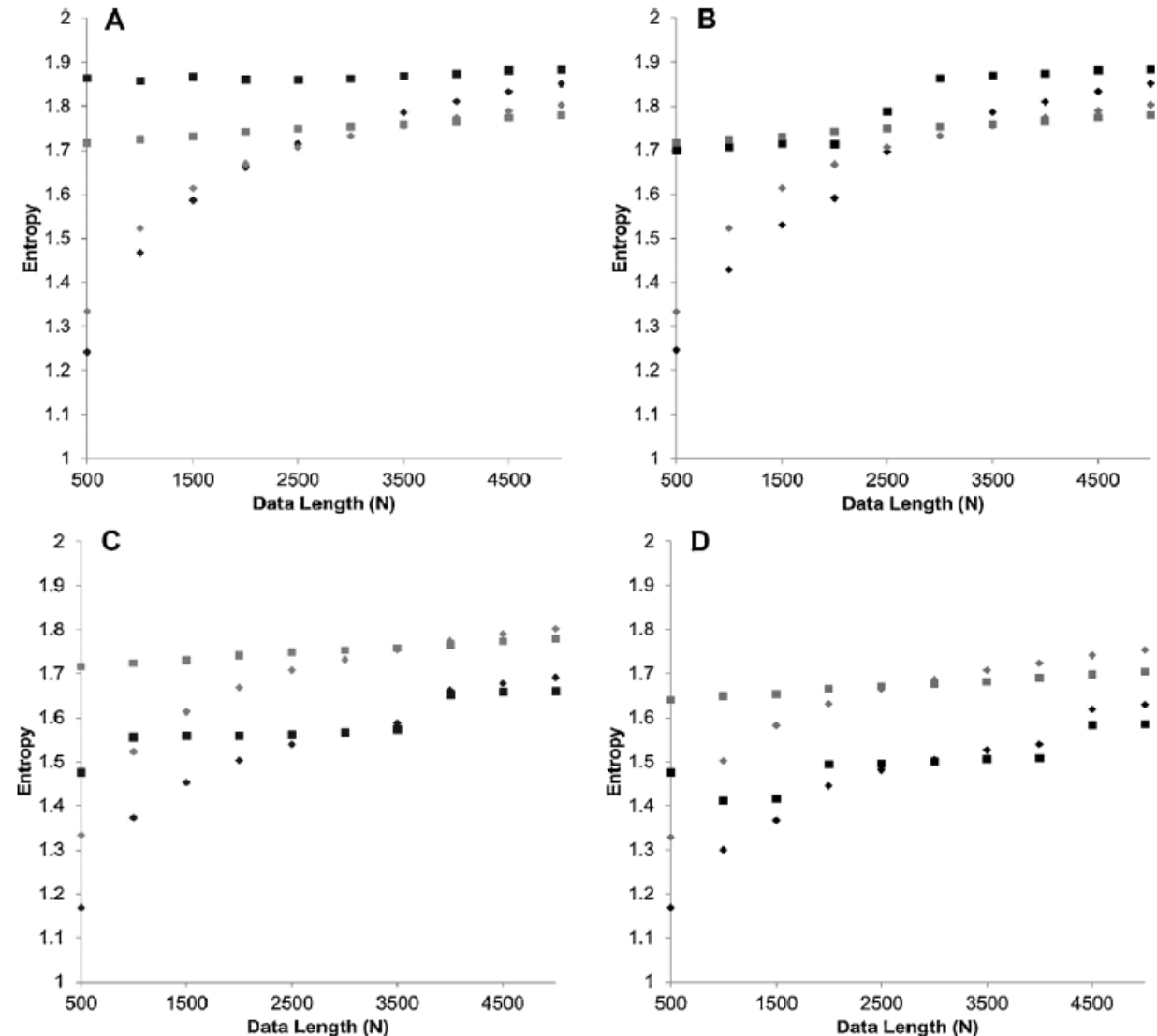
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# Sample entropy – input parameter $r$

- $R$  is the tolerance limit ( $r \times \text{SD}$ )
- ApEn vs SaEn across different  $N$  (from 500 to 5000)
- $r=0.15$  (A),  $r=0.2$  (B),  $r=0.25$  (C) and  $r=0.3$  (D)
- $m=2$

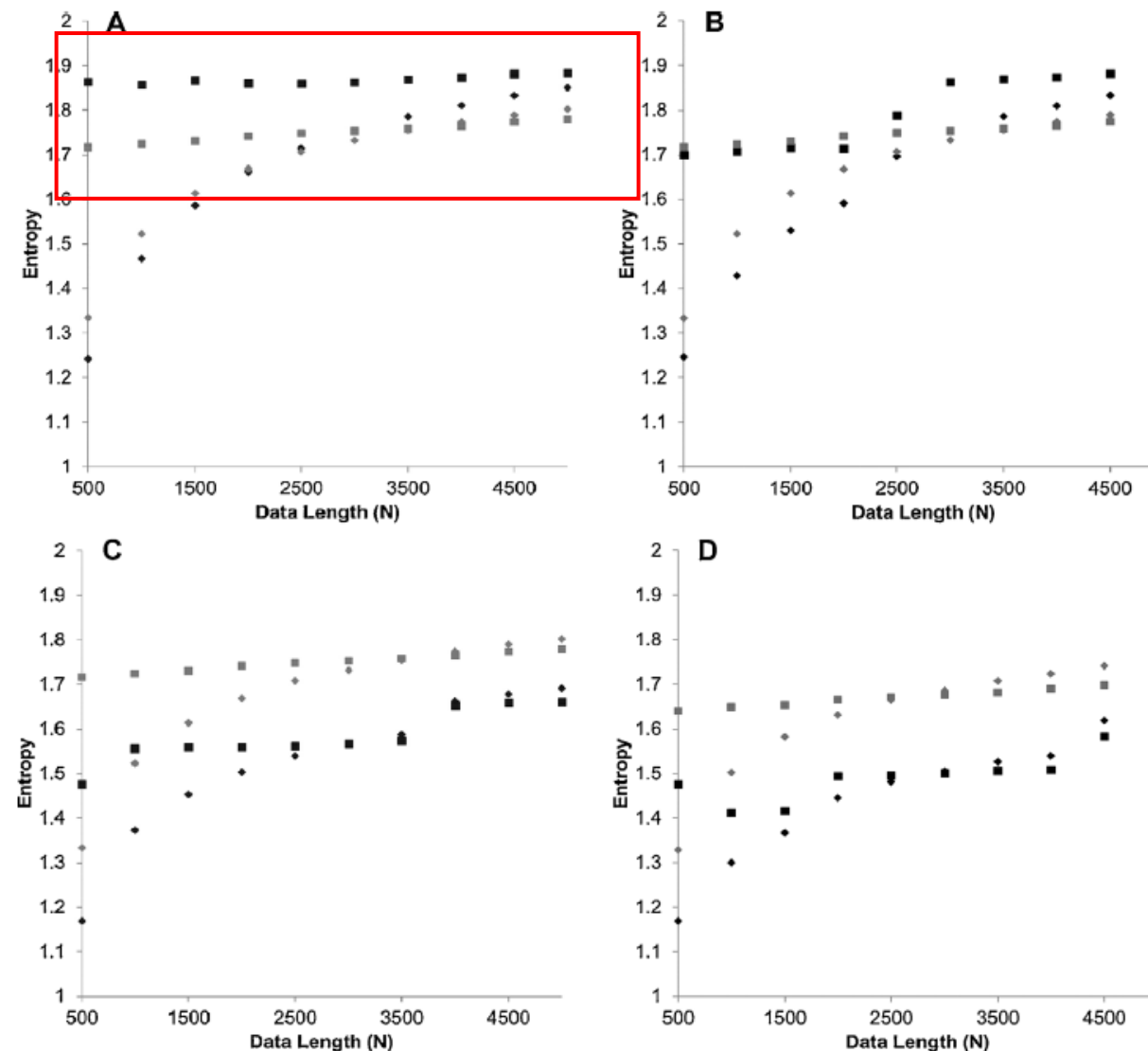
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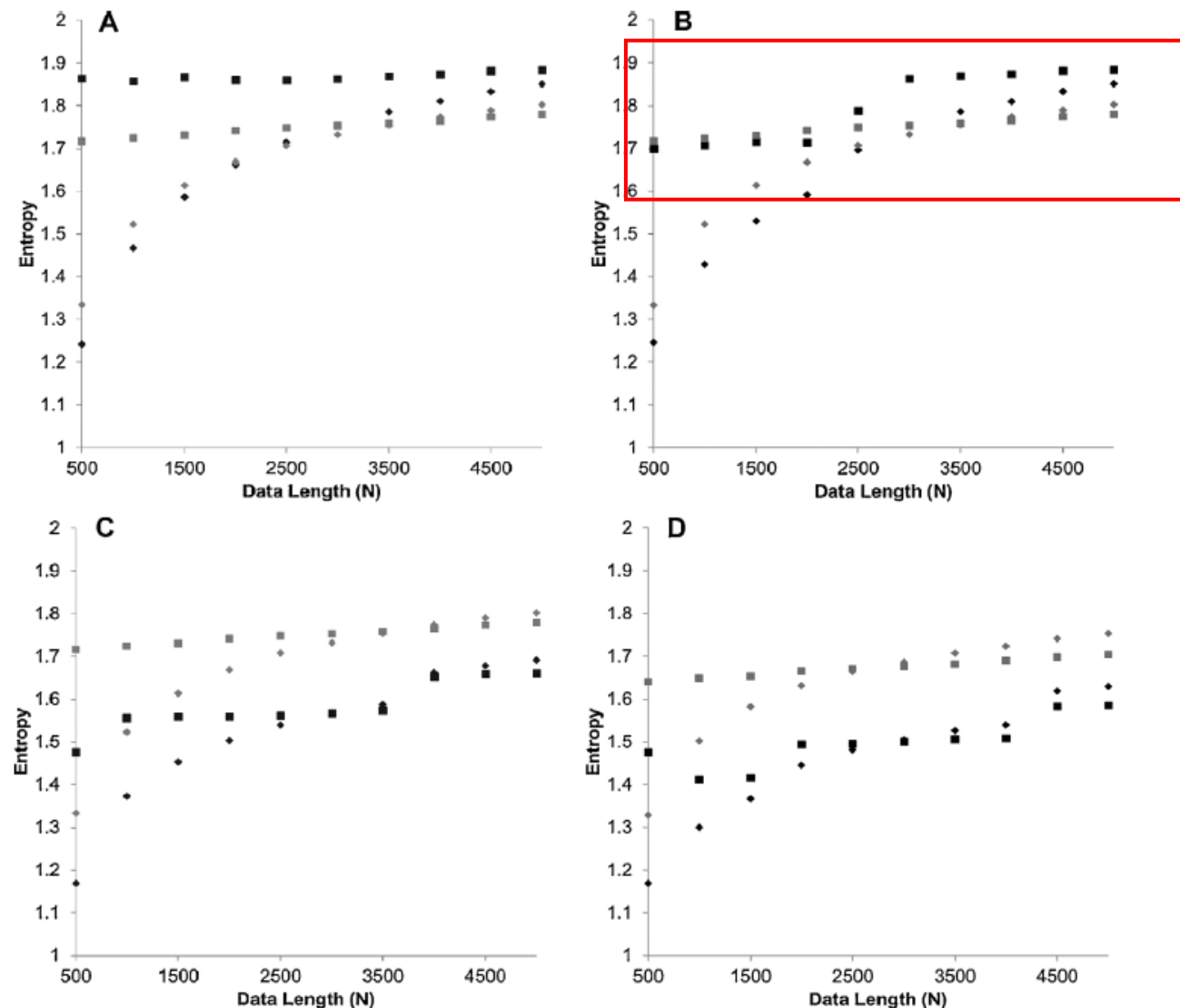
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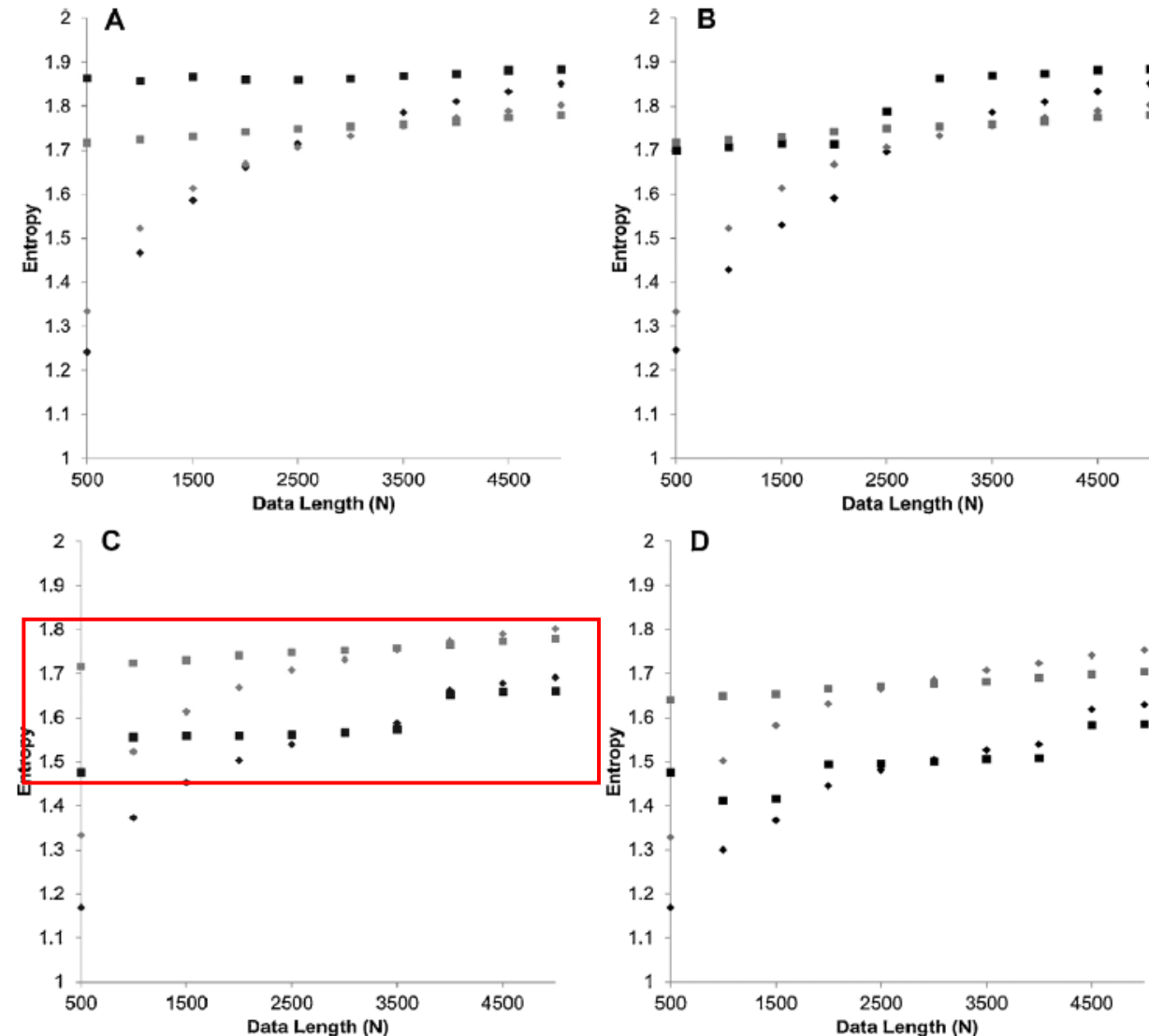




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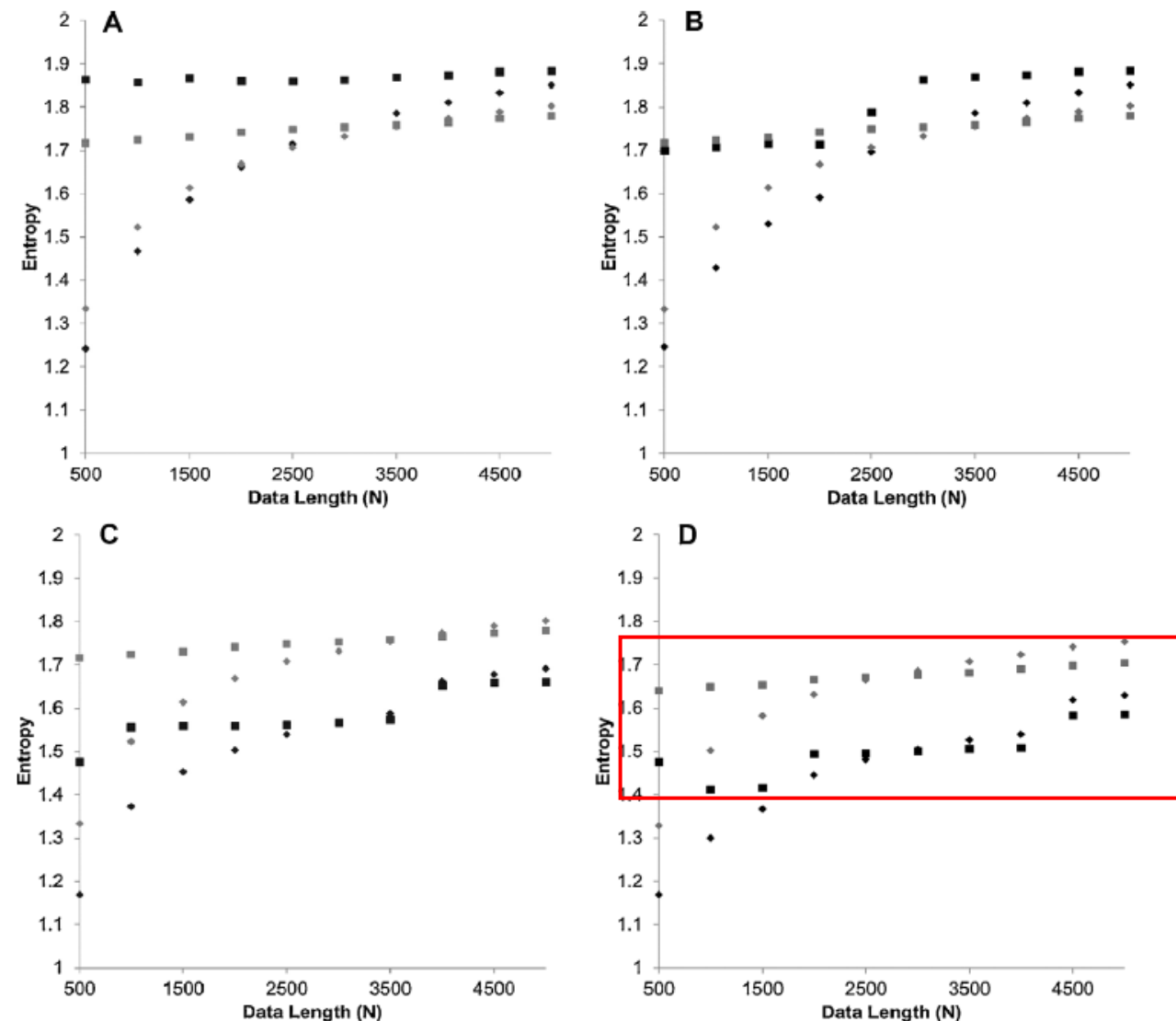


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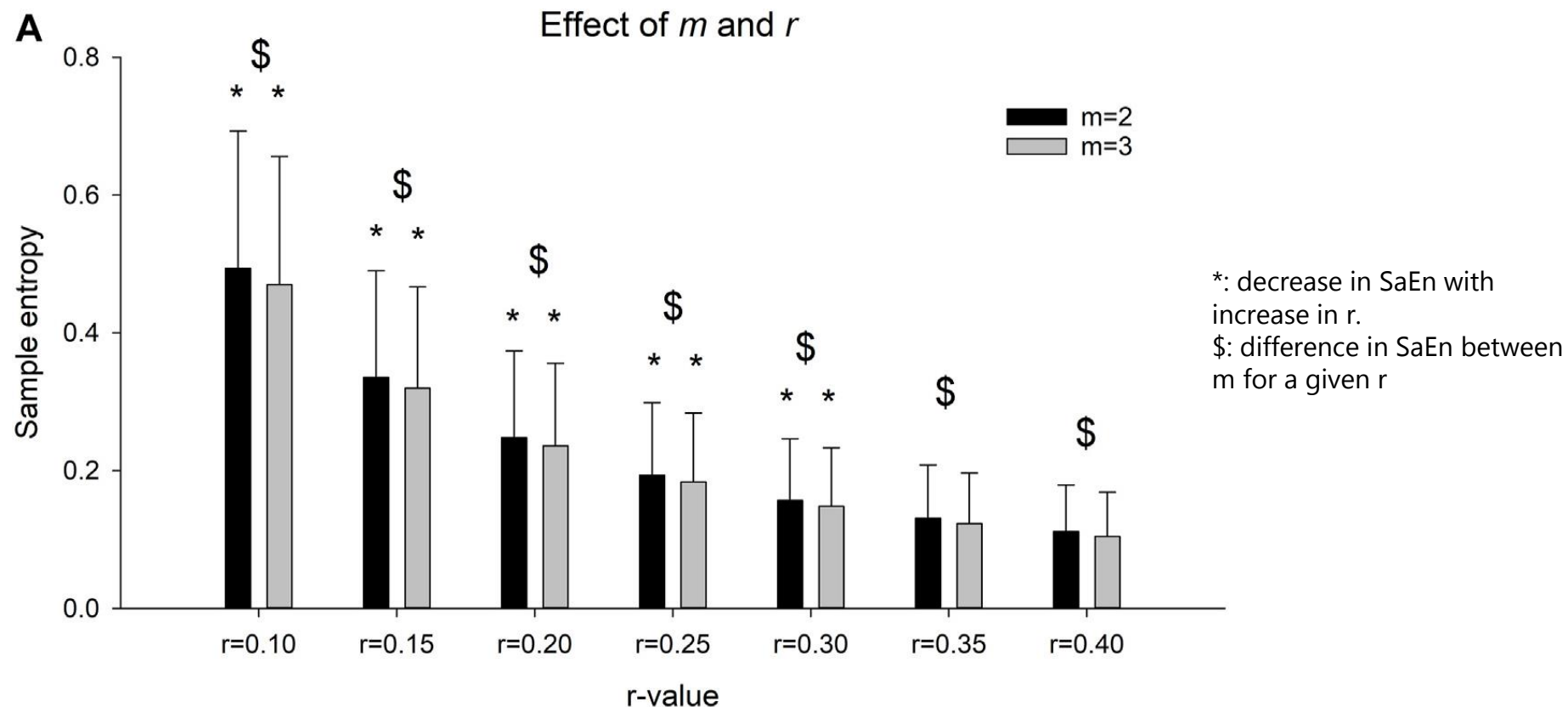
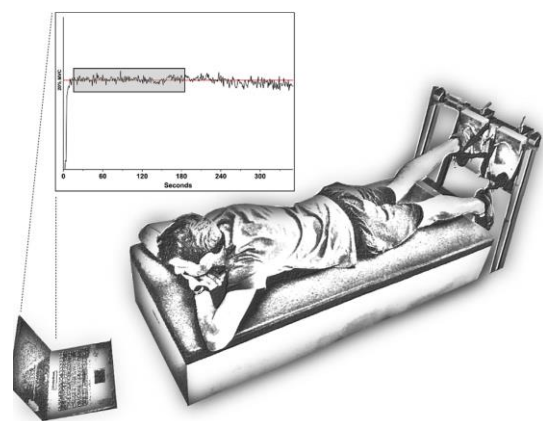


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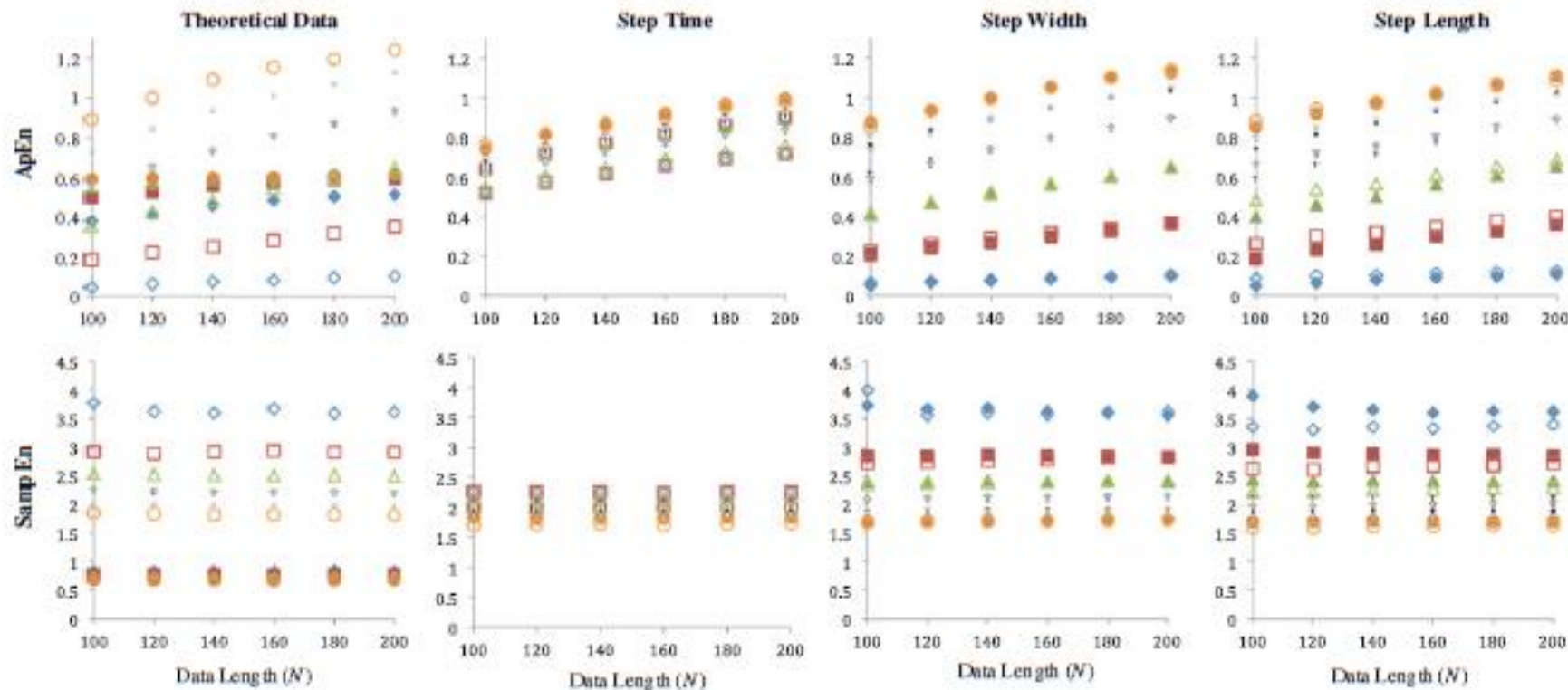
- $R$  is the tolerance limit ( $r \times SD$ )
- SaEn across different  $r$  with  $m=2$  and  $3$ , sampling frequency=100Hz and observation time = 180s



# Sample entropy – input parameter N

$$SaEn(m, r, N) = -\ln \left[ \frac{A^{m+1}(r)}{B^m(r)} \right]$$

- When N is the number of walking steps
- ApEn vs SaEn across different N (from 100 to 200), different rxSD and m=2

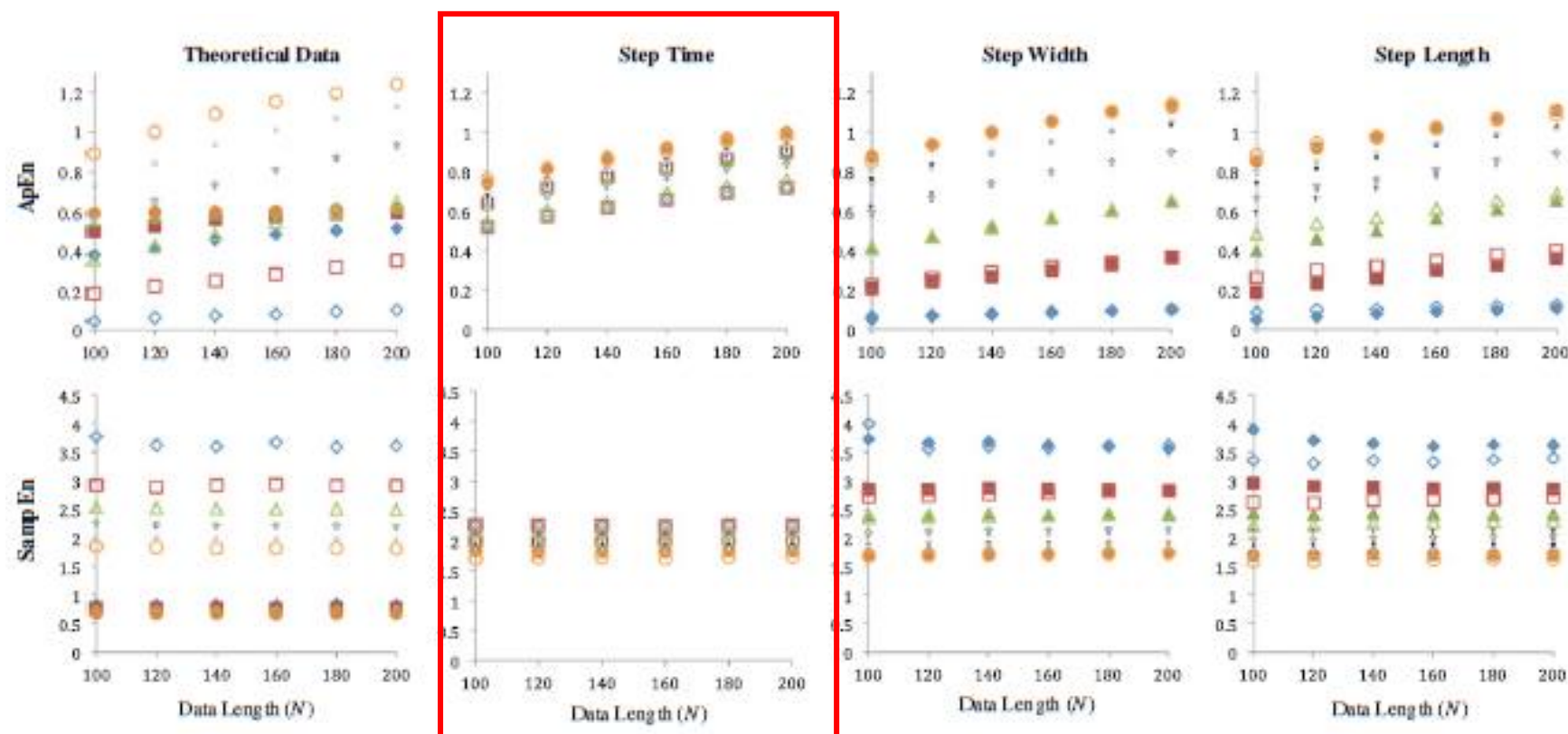


Filled symbols: young adults  
Open symbols: older adults

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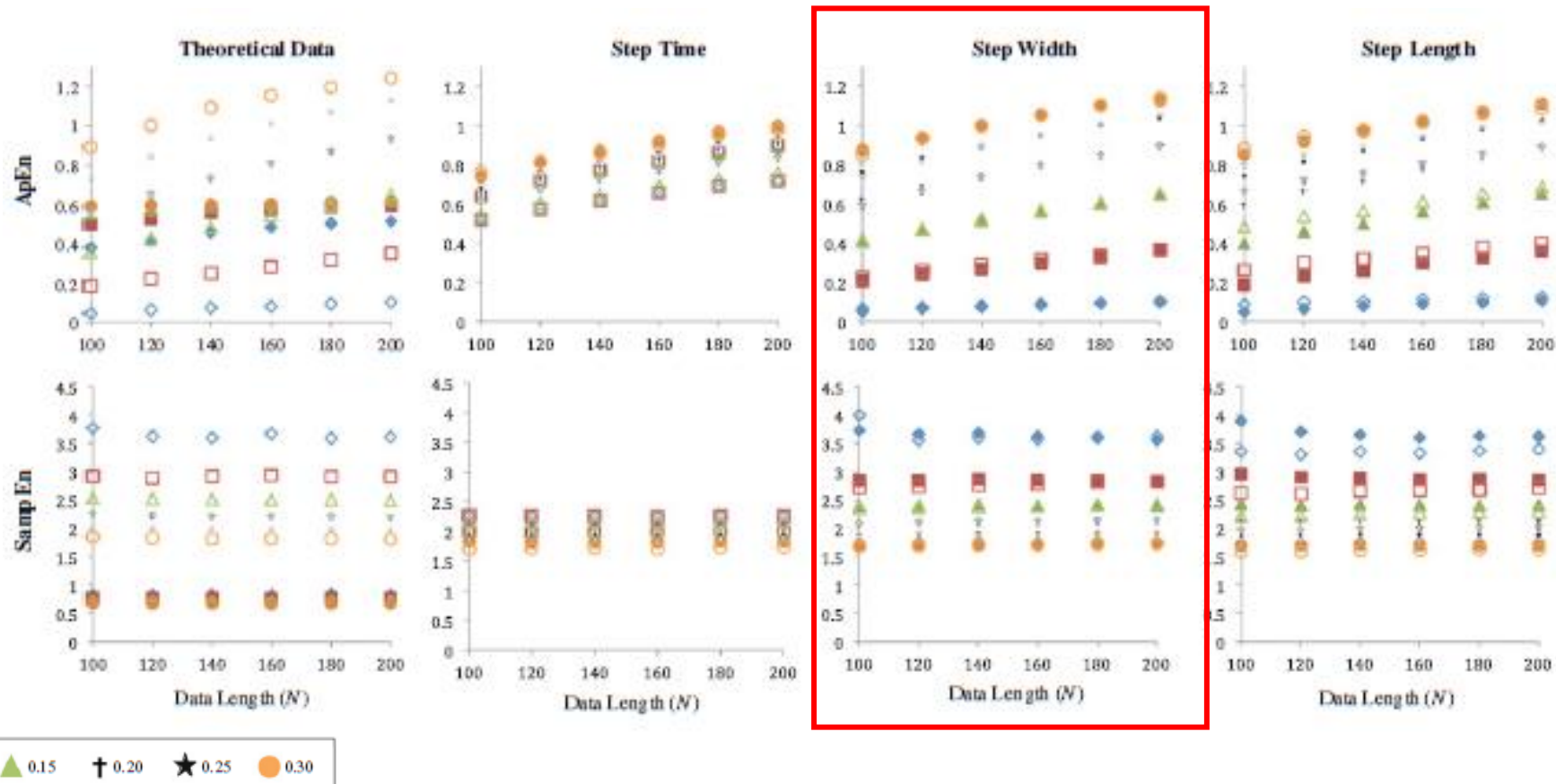


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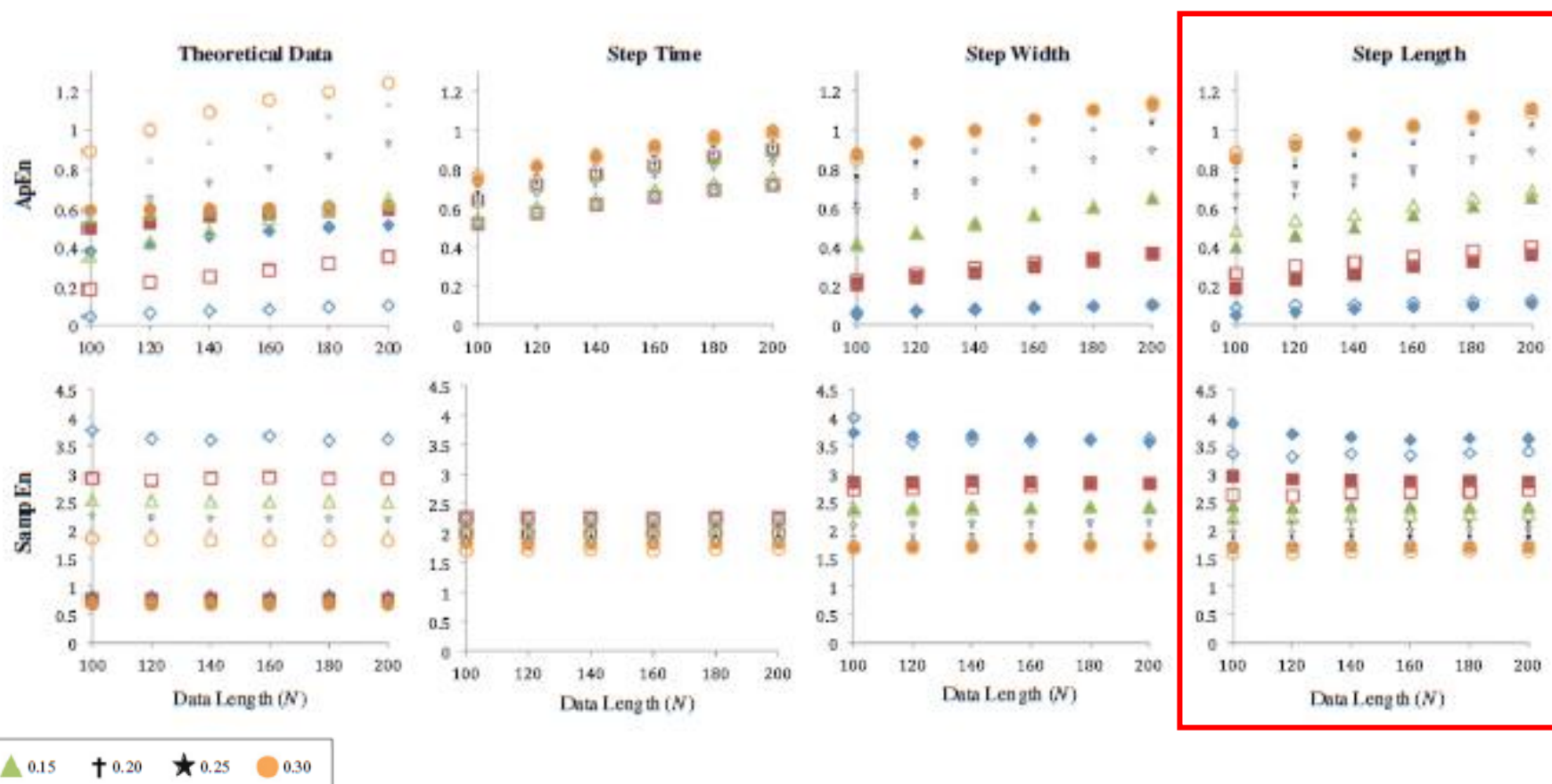
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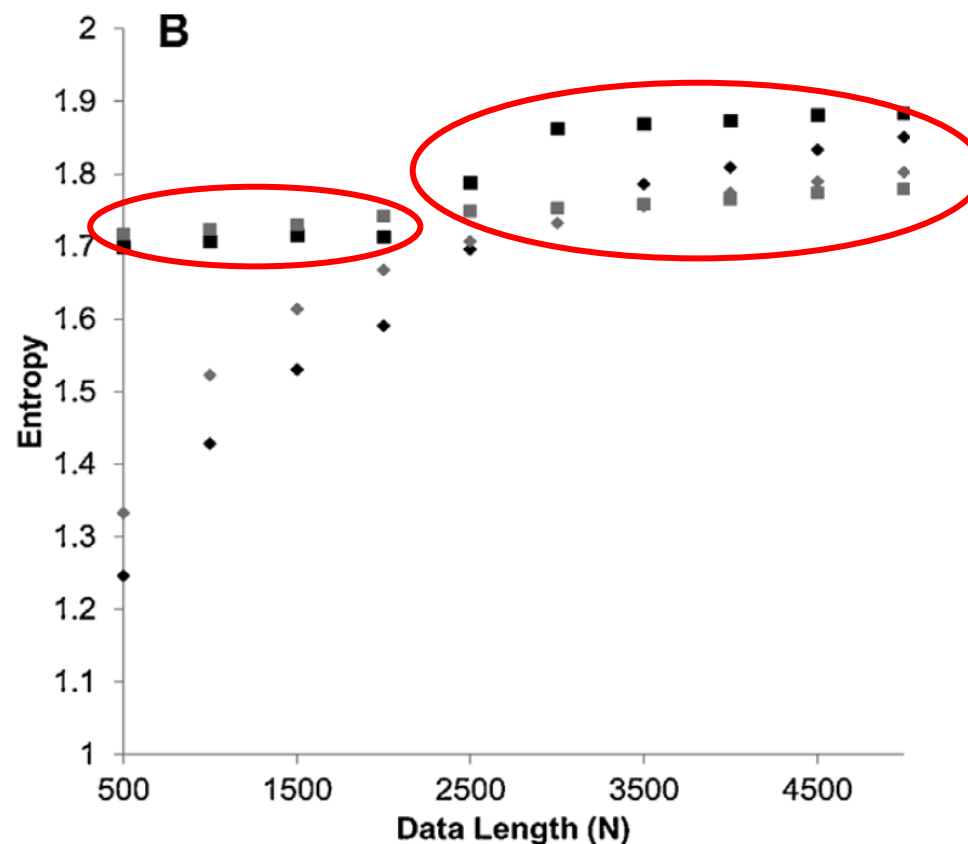
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$$SaEn(m, r, N) = -\ln \left[ \frac{A^{m+1}(r)}{B^m(r)} \right]$$

- When N is the number of walking steps
- ApEn vs SaEn across different N (from 500 to 5000) for  $r=0.2 \times SD$  and  $m=2$

Gray: overground walking  
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ApEn: diamond  
SaEn: square



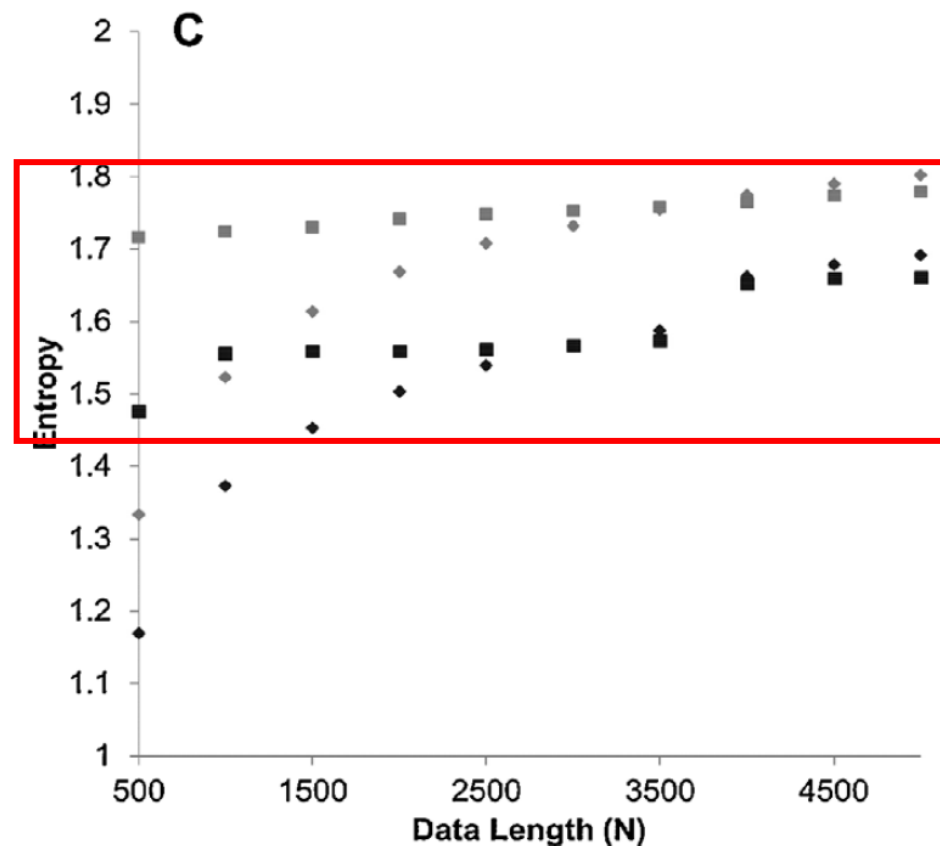


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$$SaEn(m, r, N) = -\ln \left[ \frac{A^{m+1}(r)}{B^m(r)} \right]$$

- When N is the number of walking steps
- ApEn vs SaEn across different N (from 500 to 5000) for  $r=0.25 \times SD$  and  $m=2$

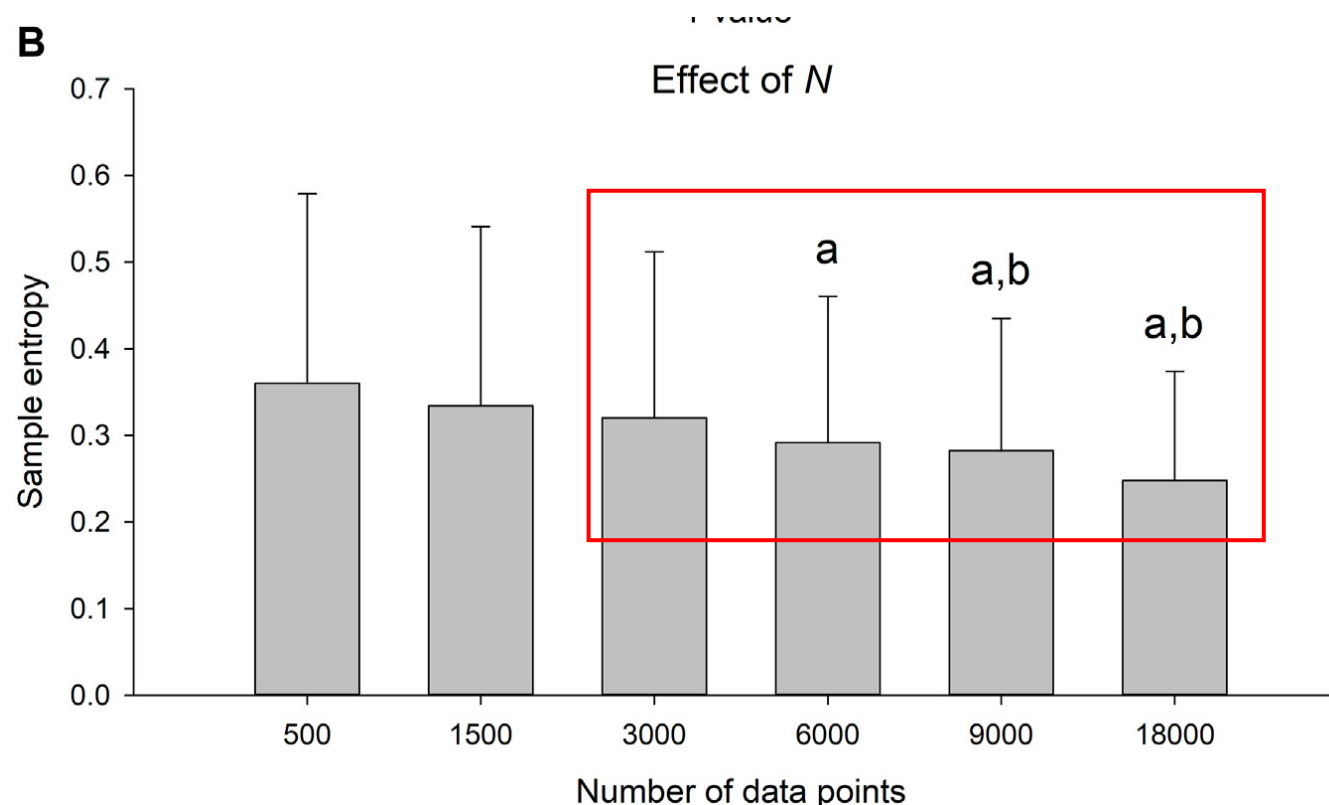
Gray: overground walking  
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ApEn: diamond  
SaEn: square



# Sample entropy – input parameter N

$$SaEn(m, r, N) = -\ln \left[ \frac{A^{m+1}(r)}{B^m(r)} \right]$$

- When N is the length of the force time series
- SaEn across different N (from 500 to 18000) for  $r=0.2 \times SD$ ,  $m=2$  and sampling frequency = 100Hz



A: different from SaEn of 500 data points.  
B: different from SaEn of 1500 data points

## Sample entropy – input parameter N with discrete data

- N is the number of observations of a particular event e.g., strides/steps.
- The used N should provide relative input parameter consistency.
- Changing the sampling frequency affects the binning of vectors in SaEn calculation. McCamley et al. 2018
- A high sampling frequency enables accurate detection of events.
- To capture the temporal evolution of any given biological phenomenon through **discrete** data requires *sufficient observations* and *adequate observation frequency* for the detailed dynamics of the phenomenon to unfold.

# Sample entropy – input parameter N with continuous data

- N is the number of data points.
- The used N should provide relative input parameter consistency.
- $N = \text{sampling time} \times \text{sampling frequency}$ .
- Choose sampling frequency based on:
  - A priori power spectral analysis
  - The neuromuscular system operates on a millisecond level
- To capture the temporal evolution of any given biological phenomenon through **continuous** data requires *sufficient observation time* and *adequate observation frequency* for the detailed dynamics of the phenomenon to unfold.

# Methodological recommendations

Annals of Biomedical Engineering (© 2021)  
https://doi.org/10.1007/s10439-020-02616-8

**BMES** BIOMEDICAL  
ENGINEERING  
SOCIETY



Review

## Entropy Analysis in Gait Research: Methodological Considerations and Recommendations

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<sup>1</sup>Center for Research in Human Movement Variability, University of Nebraska at Omaha, 6160 University Drive South, Omaha, NE 68182-0860, USA; <sup>2</sup>Department of Physical Performance, Norwegian School of Sport Sciences, Sognsveien 220, 0806 Oslo, Norway; and <sup>3</sup>Department of Biomedical Sciences, University of Copenhagen, Blegdamsvej 3B, 2200 Copenhagen N, Denmark

(Received 12 February 2020; accepted 8 September 2020)

European Journal of Applied Physiology  
https://doi.org/10.1007/s00421-021-04684-7

LETTER TO THE EDITOR



## Methodological considerations for a non-linear analysis of running in the heavy and severe intensity domains

Jennifer M. Yentes<sup>1</sup> · Peter C. Raffalt<sup>2</sup> · João R. Vaz<sup>3</sup>

Received: 7 March 2021 / Accepted: 8 April 2021  
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Frontiers in **Physiology**

TYPE Original Research  
PUBLISHED 01 June 2023  
DOI 10.3389/fphys.2023.1173702

## Calculating sample entropy from isometric torque signals: methodological considerations and recommendations

Peter C. Raffalt<sup>1,2</sup>, Jennifer M. Yentes<sup>3</sup>, Sandro R. Freitas<sup>4</sup> and João R. Vaz<sup>5\*</sup>

<sup>1</sup>Department of Biology, University of Southern Denmark, Odense, Denmark, <sup>2</sup>Department of Nutrition, Exercise and Sports, University of Copenhagen, Copenhagen, Denmark, <sup>3</sup>Department of Kinesiology & Sport Management, Texas A&M University, College Station, TX, United States, <sup>4</sup>Faculdade de Motricidade Humana, Universidade de Lisboa, Lisboa, Portugal, <sup>5</sup>Egas Moniz Center for Interdisciplinary Research (CiEM), Egas Moniz School of Health & Science, Caparica, Almada, Portugal

## Time for some Matlab!

- Try to implement sample entropy calculation to different time series
  - Investigate the effect of filtering
  - Investigate relative parameter consistency
  - Try to implement multiscale entropy
- 
- Two scripts
  - Five data sets