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Sensitivity of the Wolf's and Rosenstein's Algorithms to Evaluate Local Dynamic Stability from Small Gait Data Sets

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Abstract—The Wolf's (*W*-algorithm) and Rosenstein's (*R*-algorithm) algorithms have been used to quantify local dynamic stability (largest Lyapunov exponent, λ_1) in gait, with prevalence of the latter one that is considered more suitable for small data sets. However, such a claim has never been investigated. To address it, the λ_1 of the Lorenz attractor was estimated using small data sets and varied delays and embedding dimensions. Overall, the λ_1 estimates from the *R*-algorithm got closer to the theoretical exponent than those from the *W*-algorithm. The *W*-algorithm also overestimated λ_1 while the *R*-algorithm underestimated it, overlooking the attractor convergences and divergences, respectively. Local dynamic stability was then examined from 1-, 2- and 3-min long gait time series of younger (YA) and older adults (OA). The OA were found more locally unstable than the YA regardless of time series length with the *W*-algorithm but only for the longest time series with the *R*-algorithm. The lack of sensitivity to capture age-related decline in local dynamic stability from shorter time series is proposed to result from a drawback of the *R*-algorithm that overlooks the expansion of the attractor trajectories. The *W*-algorithm is advocated for use when examining local dynamic stability with small gait data sets.

Keywords—Largest Lyapunov exponent, Walking, Aging, Delay embedding.

INTRODUCTION

The most popular approach to quantify the presence of chaos in dynamical systems is to examine the property of sensitivity to initial conditions by means of the Lyapunov exponents. These exponents, noted λ_i, reflect the rate at which infinitesimally close trajectories of an attractor converge or diverge within a *n*-dimensional phase space ($i = 1, 2,\ldots,n$). A positive exponent reflects exponential divergence of the trajectories (i.e., sensitivity to initial conditions) and diagnoses chaos.[12,28,29,33] However, it is common to only refer to the largest Lyapunov exponent (λ_1) when exploring chaos and strange attractors since two initial conditions diverge exponentially at a rate given by this exponent (with probability 1), the effect of the other exponents being obliterated over time.[13,14]

In gait studies, search for deterministic chaos has become popular and much attention has been given to λ_1 calculation. The main reason is that λ_1 reflects local instability in a particular direction of the phase space and can be used as a direct measure of movement (in)stability.[8–11] However, it is important to note that for the existence of an attractor, which is the case for every movement, the overall dynamics must be dissipative, i.e., globally stable, the total rate of contraction outweighing the total rate of expansion of the attractor. Thus, even though a positive λ_1 reflects a movement locally unstable, its global dynamics is still stable with the sum of the Lyapunov exponents being negative across the entire spectrum $\{\lambda_1, \lambda_2,\ldots,\lambda_n\}$. Practically, even though many algorithms are available to estimate λ_1 from experimental time series,[1,7,15,29] only the algorithm of Wolf *et al.*[33] (*W*-algorithm) and the algorithm of Rosenstein *et al.*[28] (*R*-algorithm) have been used in gait studies, with prevalence of the *R*-algorithm. Overall, both algorithms work similarly, tracking the exponential divergence of nearest neighbors of the attractor over time. However, they also show dissemblances. The *W*-algorithm focuses on a reference trajectory of the attractor, with a single nearest neighbor being followed and repeatedly replaced when its separation from the reference trajectory grows beyond a

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certain limit. On the contrary, the R-algorithm focuses on subsequent nearest neighbors on two separate trajectories of the attractor and repeats the tracking procedure over all points in the phase space.

It has been assumed that the R-algorithm is more suitable for λ_1 estimation than the W-algorithm, especially for small data sets, since it takes advantage of all the attractor data points and avoids approximations by disregarding the procedure of neighbor replacement.[28] However, although this algorithm has been designed for studying small data sets, the number of data points is yet suggested to be higher than 10^D, with D the attractor dimension. This requirement is in fact similar to the one of the W-algorithm and impossible to meet by using kinematic gait data, especially with clinical populations. Indeed, attractors in gait are at least 5-dimensional,[5,8,9,16] leading to consider a minimum of 10^5 data points, i.e., time series of 30-min duration assuming a 60 Hz sampling rate. Studies on the R-algorithm have confirmed that long gait time series need to be used to obtain λ_1 values that are reliable and able to capture walking modulations between experimental conditions.[4,20]

Consequently, although the R-algorithm appears to be the most popular method used to determine λ_1 from small data sets, there is no experimental evidence that it is better suited than the W-algorithm to examine these series. The aim of the present study was then to compare the performance of the two algorithms and figure out whether the R-algorithm is more appropriate than the W-algorithm for estimating λ_1 when considering small data sets. Data from a chaotic system, the Lorenz attractor, were first considered and the λ_1 exponents from both algorithms were examined while varying the attractor characteristics (i.e., embedding dimension and reconstruction delay) and the size of the data set. It was hypothesized that both algorithms would give the most accurate λ_1 exponent (i.e., closest to the expected value of 1.5; see Rosenstein *et al.*[28]) when using input parameters that unfold the attractor in the phase space. It was also expected a strong dependence on the size of the data set for the λ_1 estimation, with the worst estimation to be for time series that have a number of data points lower than the theoretical recommendation (i.e., 10^3 points for the Lorenz attractor). Following this first step, hip and ankle local dynamic stability of younger and older adults (OA) was investigated for 1-, 2-, and 3-min walking trials. These trials did not meet the theoretical recommendation of both algorithms in terms of number of data points. Nevertheless, it was hypothesized that both algorithms would be able to separate the two groups using proper attractor reconstruction (i.e., unfolded attractor), with a larger λ_1 value in the OA reflecting more local instability as previously demonstrated.[5,21,22,25]

METHODS

Lorenz Data

The Lorenz system[30] is defined by three coupled nonlinear differential equations:

$$\begin{aligned} \dot{x} &= \sigma(y - x) \\ \dot{y} &= x(\rho - z) - y \\ \dot{z} &= xy - \beta z \end{aligned} \tag{1}$$

where the parameters σ, ρ, and β were set to 16.0, 45.92 and 4.0, respectively, so that the system exhibits chaotic dynamics.[18] These equations were solved in Matlab using a fourth-order Runge–Kutta method (ode45) with a step size equal to 0.01 s. Sets including 5 x-coordinate time series of 10, 15, 25 and 45 s were generated (Fig. 1). The first 5 s were subsequently removed to eliminate transients. Sets with time series counting 500, 1000, 2000, and 4000 data points (i.e., 5, 10, 20, and 40 s) were considered for attractor reconstruction.

Multi-dimensional attractors were reconstructed from each original time series $[x(t)]$ and its time-delayed copies $[x(t + \tau), x(t + 2\tau),\ldots,x(t + (d_E - 1)\tau)]$[31]:

$$X(t) = [x(t), x(t+\tau), x(t+2\tau), \ldots, x(t+(d_E-1)\tau)] \tag{2}$$

where $X(t)$ is a d_E-dimensional vector that defines the attractor, τ is the reconstruction delay and d_E is the embedding dimension (Fig. 1). In order to test for the robustness of the two algorithms, different delays ($\tau = 1, 11, 21, 31$) and embedding dimensions ($m = 3, 5, 7, 9$) were examined. The parameters that unfold the Lorenz attractor in phase space are $\tau = 11$ and $m = 3$.

Gait Data

Seven healthy OA (age 65–80) and seven height- and gender-matched healthy younger adults (YA) (age 21–34) took part in the experiment after signing an institutionally approved informed consent (Table 1). The subjects were free of lower extremity injuries or disabilities that might have influenced their walking ability.

Reflective markers were placed on anatomic locations of each subject's lower limb according to Nigg *et al.*[27] and Vaughan *et al.*[32] Subjects then walked on a motorized treadmill (312-C, Bodyguard, Canada) while wearing a safety harness (LiteGait®, Mobility Research, LLC, Tempe, AZ) and their own walking shoes. The LiteGait® supported the subjects only if

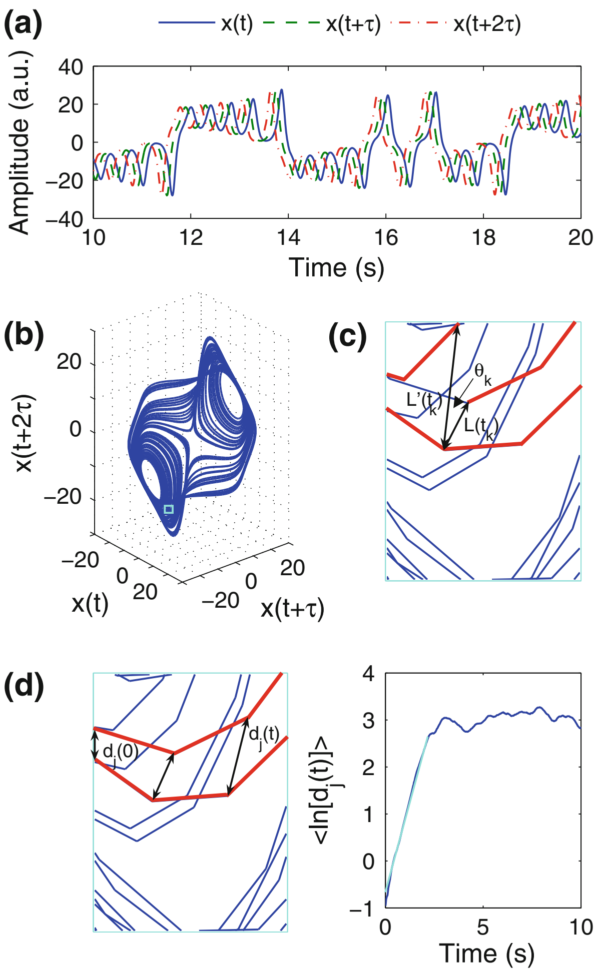

FIGURE 1. Illustration of the working procedures of the *W*- and *R*-algorithms for λ_1 exponent (largest Lyapunov exponent) estimation using a Lorenz data set. (a) An original Lorenz time series, $x(t)$, and two time delayed copies, $x(t+\tau)$ and $x(t+2\tau)$, as obtained from the Lorenz system (x-coordinate in Eq. (1)) with the parameterization $\sigma = 16$, $\rho = 45.92$, and $\beta = 4$. τ is set here at 11 frames (i.e., 0.11 s). (b) The Lorenz attractor embedded in a 3D phase space (i.e., $d_E = 3$ in Eq. (2)). (c) The *W*-algorithm focuses on a reference trajectory of the attractor, with a single nearest neighbor being followed and repeatedly replaced when its separation $L'(t_k)$ from the reference trajectory becomes large. The new neighbor is chosen to minimize both the replacement length $L(t_k)$ and the orientation change θ_k. Once the reference trajectory has gone over the entire data sample, the λ_1 exponent is estimated from the distances between the vectors at the beginning [$L(t_k)$] and end [$L'(t_k)$] of the replacement steps on the basis of Eq. (3). (d) The *R*-algorithm tracks the distance $d_j(t)$ between nearest neighbors on two separate trajectories of the attractor and repeats the tracking procedure over all points in phase space. The λ_1 exponent is then estimated from the slope of the mean log divergence curve as defined by Eq. (4).

balance was lost during walking. Each subject's preferred walking speed (PWS) was determined using the protocol established by Jordan *et al.*[19] The PWS determination allowed subjects to acclimate to the treadmill. Subjects then completed a 3-min walking trial at their PWS.

TABLE 1. Subjects' characteristics.

	Younger adults	Older adults	*p*-Value
Gender (M/F)	4/3	5/2	0.5[a]
Age (years)	25 ± 4.86	70.28 ± 5.08	0.001[b]
Body mass (kg)	69.9 ± 11.53	85.62 ± 13.54	0.02[b]
Height (m)	1.76 ± 0.07	1.73 ± 0.08	0.65[b]
PWS (m/s)	0.95 ± 0.21	0.85 ± 0.11	0.29[b]

[a]Fisher's exact test.
[b]Mann–Whitney *U* test.

The three-dimensional positions of the markers were acquired at 60 Hz with an eight-camera motion capture system using EVART software (Motion Analysis Corp, Santa Rosa, CA). The three-dimensional angular displacements of the hip and ankle joints were then calculated using the algorithms described by Vaughan *et al.*[32] Only the angular displacements in the sagittal plane were considered (i.e., plantarflexion/dorsiflexion of the ankle and flexion/extension of the hip) since data from the other planes collected *via* skin markers are associated with increased measurement error.[6] From the 3-min time series (i.e., 10,800 data points), 1- and 2-min time series (i.e., 3600 and 7200 data points) were subsequently generated. No filtering was applied to avoid altering the stride-to-stride fluctuations present in the time series.[23,26] Attractors from all time series were then reconstructed from Eq. (2). The delay τ was determined using the first minimum of the average mutual information (AMI) function.[17] The embedding dimension d_E was selected where the percentage of the global false nearest neighbors (GFNN) approached zero (Fig. 2).[24]

The λ_1 Calculation

The λ_1 exponents were calculated using the *W*- and *R*-algorithms implemented as previously recommended.[28,33] For both algorithms, the first two steps were similar. An embedded point in the attractor was randomly selected, which was a delay vector with d_E elements $[x(t), x(t+\tau), x(t+2\tau),\ldots,x(t+(d_E-1)\tau)]$. This vector generated the reference trajectory. Its nearest neighbor vector $[x(t_0), x(t_0+\tau), x(t_0+2\tau),\ldots,x(t_0+(d_E-1)\tau)]$ was then selected on another trajectory by searching for the point that minimizes the distance to the particular reference point. For the *R*-algorithm, we imposed the additional constraint that the nearest neighbor has a temporal separation greater than the mean period of the time series defined as the reciprocal of the mean frequency of the power spectrum.[28]

The two procedures then differed. For the *W*-algorithm, the divergence between the two vectors was computed and as the evolution time was higher than

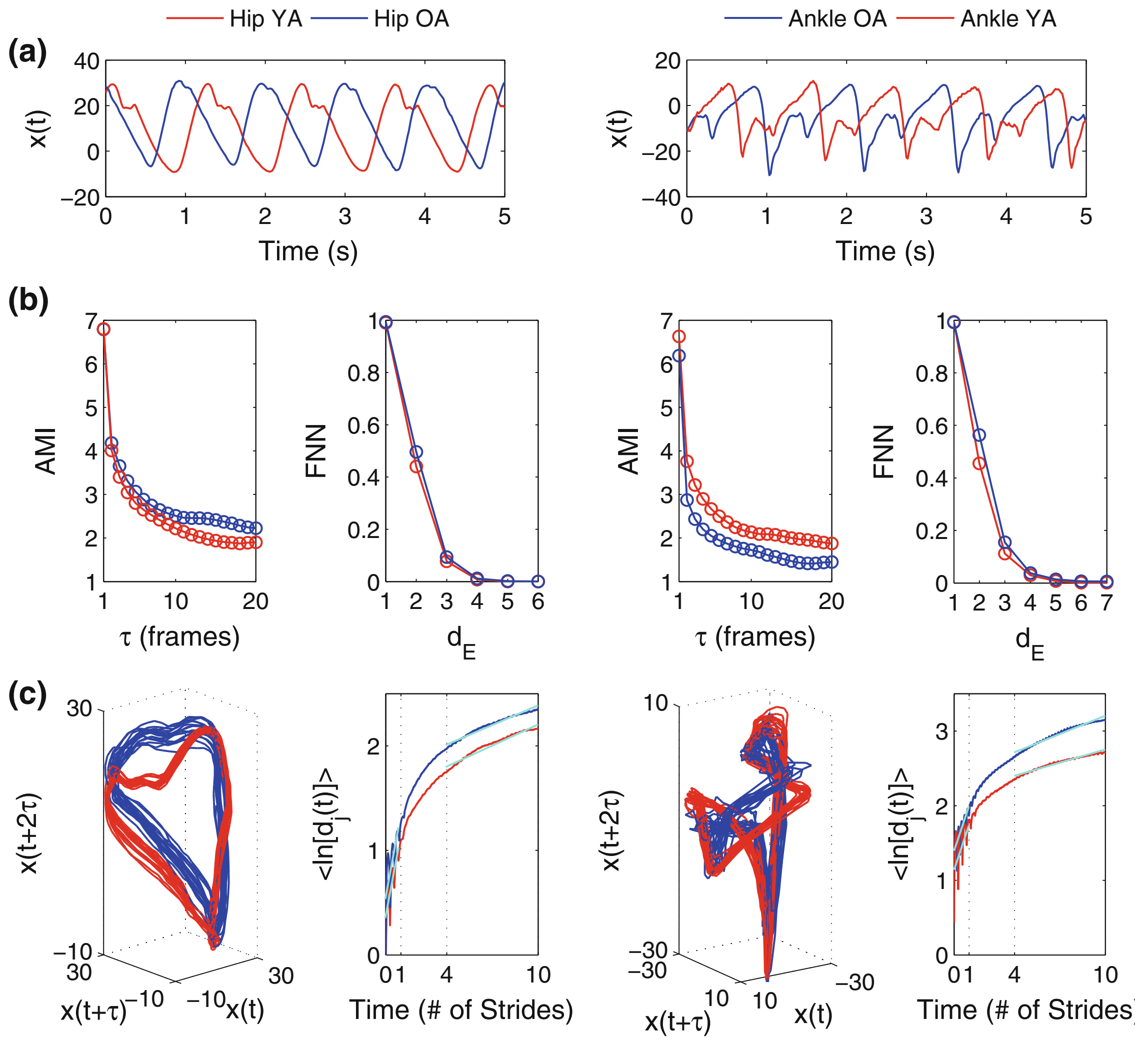

FIGURE 2. Illustration of the attractor reconstruction using the time delay method. Hip and ankle data from a YA and an OA are presented. (a) The original time series. (b) Calculation of the reconstruction delay τ using the AMI function and determination of the number of embedding dimensions d_E using the GFNN analysis. The delay obtained with the AMI function maximizes the information content of the time series used to reconstruct the attractor. The GFNN analysis determines an optimal number of dimensions so that the attractor is completely unfolded in phase space. The τ values for the hip and ankle were respectively found at 19 and 11 frames (i.e., 0.32 and 0.18 s) for the YA and 18 and 17 frames (i.e., 0.3 and 0.28 s) for the older adult. The d_E values for the hip and ankle were both of 5 for the YA and respectively of 5 and 7 for the older adult. (c) The hip and ankle attractors embedded in a 3D phase space (by convenience). One complete orbit around the attractor constitutes one cycle of movement. Rate of divergence, λ_1 (largest Lyapunov exponent), was calculated with the *R*-algorithm from the slope of the mean log divergence curve between 0–1 stride and 4–10 strides.

three sample intervals, a new neighbor vector was considered. This replacement restricted the use of trajectories that shrunk through a folding region of the attractor. The new vector was selected to minimize the length and angular separation with the evolved vector on the reference trajectory (see Wolf *et al.*[33] for complete algorithm implementation). This procedure was repeated until the reference trajectory has gone over the entire data sample and λ_1 was estimated as:

$$\lambda_1 = \frac{1}{t_M - t_0} \sum_{k=1}^{M} \ln \frac{L'(t_k)}{L(t_{k-1})} \tag{3}$$

where $L(t_{k-1})$ and $L'(t_k)$ are the distance between the vectors at the beginning and end of a replacement step, respectively, and M is the total number of replacement steps (Fig. 1). Note that Eq. (3) uses the natural logarithm function and not the binary logarithm function as presented by Wolf *et al.*[33] This change makes λ_1 exponents more comparable between the two algorithms.

For the *R*-algorithm, the divergence $d(t)$ between the two vectors was computed at each time step over the data sample. Considering that $N-(d_E-1)\tau$ embedded points (delay vectors) composed the attractor, the above procedure was repeated for all of them and λ_1 was then estimated from the slope of linear fit to the curve defined by:

$$y(t) = \frac{1}{\Delta t}\langle \ln d_j(t) \rangle \quad (4)$$

where $\langle \ln d_j(t) \rangle$ represents the mean logarithmic divergence for all pairs of nearest neighbors over time (Fig. 1). For the gait data, λ_1 was estimated from the slopes of linear fits to the curves between 0-1 stride and 4-10 strides (Fig. 2). These short- and long-term regions have been consistently used in the literature as regions of interest to estimate λ_1.[11,20–22] To do so, the time axes of the curves were normalized by multiplying by the average stride frequency for each subject. Note that another procedure sometimes used to estimate λ_1 over the two regions consists in computing the divergence curves when re-sampling the original time series so that they have the same average frequency.[3,4,16] However, studies that have addressed similar issues, as the effect of walking speed or the effect of time series length on λ_1,[3,4,11,16,20] have reported nearly similar results using either procedure. We therefore preferred re-scaling the time axes of the divergence curves that is more straightforward. Importantly, we did not find a statistical difference between the PWS of the YA and OA (Table 1). Thus, any group difference in λ_1 reflects an aging effect without the confounding effect of walking speed which has been shown to affect the measure.[2,3,11,16]

Statistical Analysis

Differences between expected and calculated λ_1 exponents from the Lorenz attractor was quantified using a percentage index error $[(\lambda_{1\ \mathrm{estimated}} - \lambda_{1\ \mathrm{expected}})/\lambda_{1\ \mathrm{estimated}}] \times 100$, with $\lambda_{1\ \mathrm{expected}} = 1.5$. For each size of data set (3600, 7200 and 10,800 data points), differences in the λ_1 exponents from the gait data were tested using two-way ANOVAs with the between-subject factors being joint (hip and ankle) and group (YA and OA). *Post hoc* Tukey tests tallied differences between the factors' modalities. The critical level for statistical significance was set to 0.05. Effect sizes are reported as $\eta^2 = \mathrm{SS_{explained}}/\mathrm{SS_{total}}$.

RESULTS

Lorenz Data

The λ_1 exponent obtained with both algorithms was closer to the expected value when the attractor is unfolded (i.e., $\tau = 11$, $d_E = 3$) and when a larger number of data points was considered, with very satisfactory results for $N = 4000$ (% error: ~1%). Importantly, any improper reconstruction increased the error, with larger errors found for the *W*-algorithm. Larger λ_1 exponents were also found when using the *W*-algorithm as compared to the *R*-algorithm, overestimating and underestimating the exponent, respectively (Fig. 3).

Gait data—W-Algorithm

The ANOVA results showed that the λ_1 exponents for the OA were higher than those of the YA for the time series with 3600 ($F_{1,24} = 5.83$, $p = 0.023$, $\eta^2 = 0.08$), 7200 ($F_{1,24} = 5.86$, $p = 0.023$, $\eta^2 = 0.08$) and 10,800 ($F_{1,24} = 6.15$, $p = 0.021$, $\eta^2 = 0.08$) data points. The ankle exponent values were also found higher than those of the hip with 3600 ($F_{1,24} = 37.31$, $p < 10^{-6}$, $\eta^2 = 0.54$), 7200 ($F_{1,24} = 52.06$, $p < 10^{-7}$, $\eta^2 = 0.63$) and 10,800 ($F_{1,24} = 47.36$, $p < 10^{-7}$, $\eta^2 = 0.59$) data points (Fig. 4).

Gait data—R-Algorithm

The results from the ANOVA indicated a higher exponent for OA as compared to YA between 0-1 stride ($F_{1,24} = 4.43$, $p = 0.045$, $\eta^2 = 0.14$) and 4-10 strides ($F_{1,24} = 9.29$, $p < 0.01$, $\eta^2 = 0.23$) for the time series with 10,800 data points. The ankle exponents obtained over 4-10 strides were also found lower than the hip exponents for the time series with 7200 ($F_{1,24} = 7.49$, $p = 0.011$, $\eta^2 = 0.14$) and 10,800 ($F_{1,24} = 7.71$, $p = 0.01$, $\eta^2 = 0.19$) data points (Fig. 4).

DISCUSSION

The λ_1 estimation is used to quantify local dynamic stability of the locomotor system.[8] In terms of methods, only the *W*- and *R*-algorithms have been used in gait studies, with a prevalence of the latter one. This is due to the fact that this algorithm has been shown to be robust to changes in attractor characteristics and size of data sets, and has been reported to work well with small data sets.[28] However, long gait time series (~5-min) are usually considered using this algorithm and studies have demonstrated that λ_1 exponents obtained using shorter time series are weakly reliable and weakly able to capture walking modulations between experimental conditions.[4,20] Therefore, the aim of the present study was to examine whether the *R*-algorithm is more appropriate than the *W*-algorithm for estimating λ_1 when considering small data sets.

The results from the Lorenz data showed that a more accurate λ_1 exponent was obtained using larger number of data points with both algorithms. In addition, the percent errors from the theoretical λ_1 value

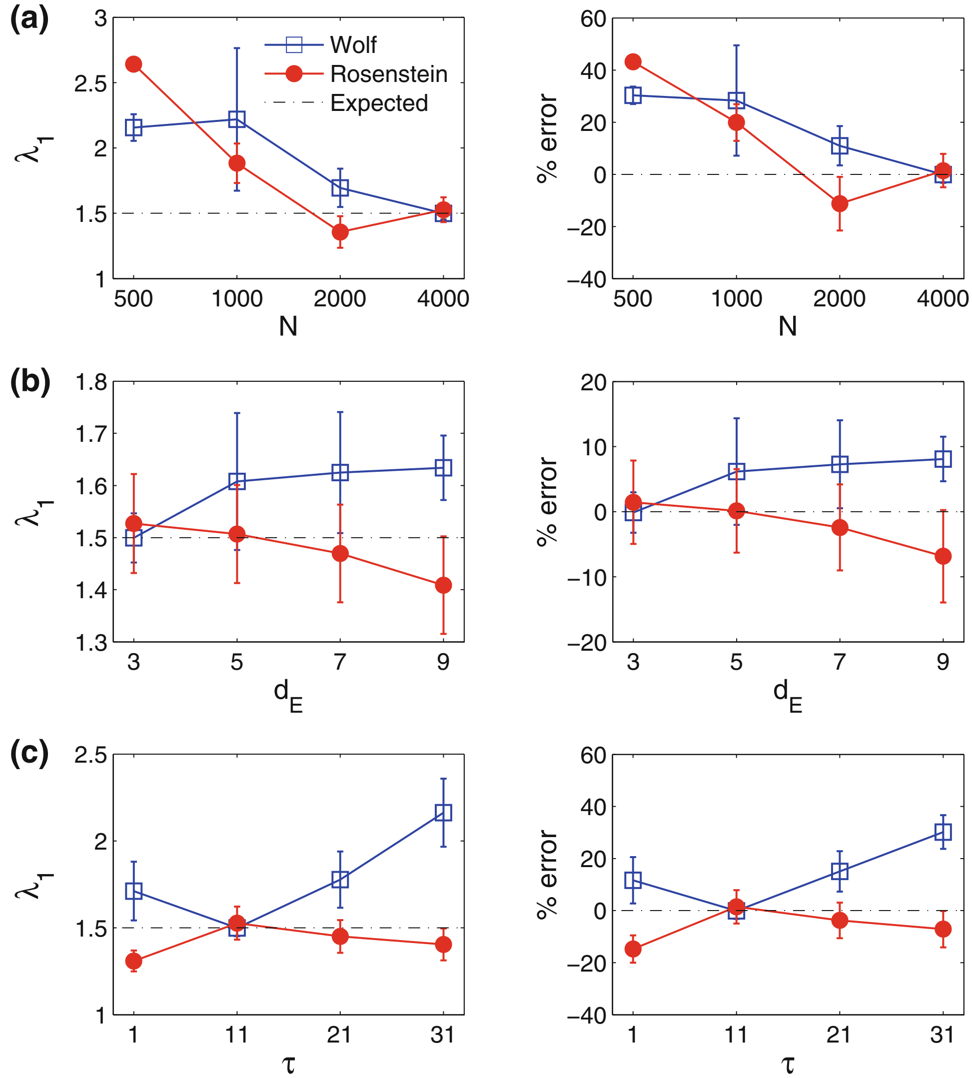

FIGURE 3. Mean ± SD largest Lyapunov exponent (λ_1) and percentage error (% error) obtained with the *W*- and *R*-algorithms from the Lorenz attractor while varying (a) the number of data points *N* [τ = 11, d_E = 3], (b) the embedding dimension d_E [τ = 11, *N* = 4000], and (c) the reconstruction delay τ [d_E = 3, *N* = 4000]. The dash-dot line represents the expected λ_1 exponent of 1.5 and 0% error.

were equivalent using both algorithms for the smaller data sets. Hence, as hypothesized, both algorithms depend on the size of data sets; the more the number of points, the more accurate the λ_1 exponent. Also, although the *R*-algorithm takes advantage of all the attractor data points, it does not appear better designed to study small data sets as originally proposed.[28] Another expected result was that proper attractor reconstruction led to a better estimation of the λ_1 exponent using either algorithm, meaning that any loss or redundancy of information in phase space affects significantly the measure. However, estimates from the *R*-algorithm were less affected by changes in the embedding dimension and reconstruction delay than those from the *W*-algorithm. This indicated that the former algorithm is quite robust to variations in these quantities as concluded by Rosenstein *et al.*[28] Lastly, an important result was that the *W*-algorithm overestimated the λ_1 exponent while the *R*-algorithm underestimated it. This means that the divergence of neighboring trajectories is minimized with the *R*-algorithm, while it is magnified with the *W*-algorithm. As the Lorenz attractor trajectories are contracting or expanding depending on the regions of the phase space,[33] the expanding character of the trajectories with the *R*-algorithm is overlooked due to the arithmetic averaging of the divergences and convergences. Inversely, the replacement procedure of the *W*-algorithm tends to magnify the expansion, overlooking the convergences. Keeping in mind that the λ_1 exponent evaluates the exponential rate of divergence of neighboring trajectories of the attractor, the complete inclusion of

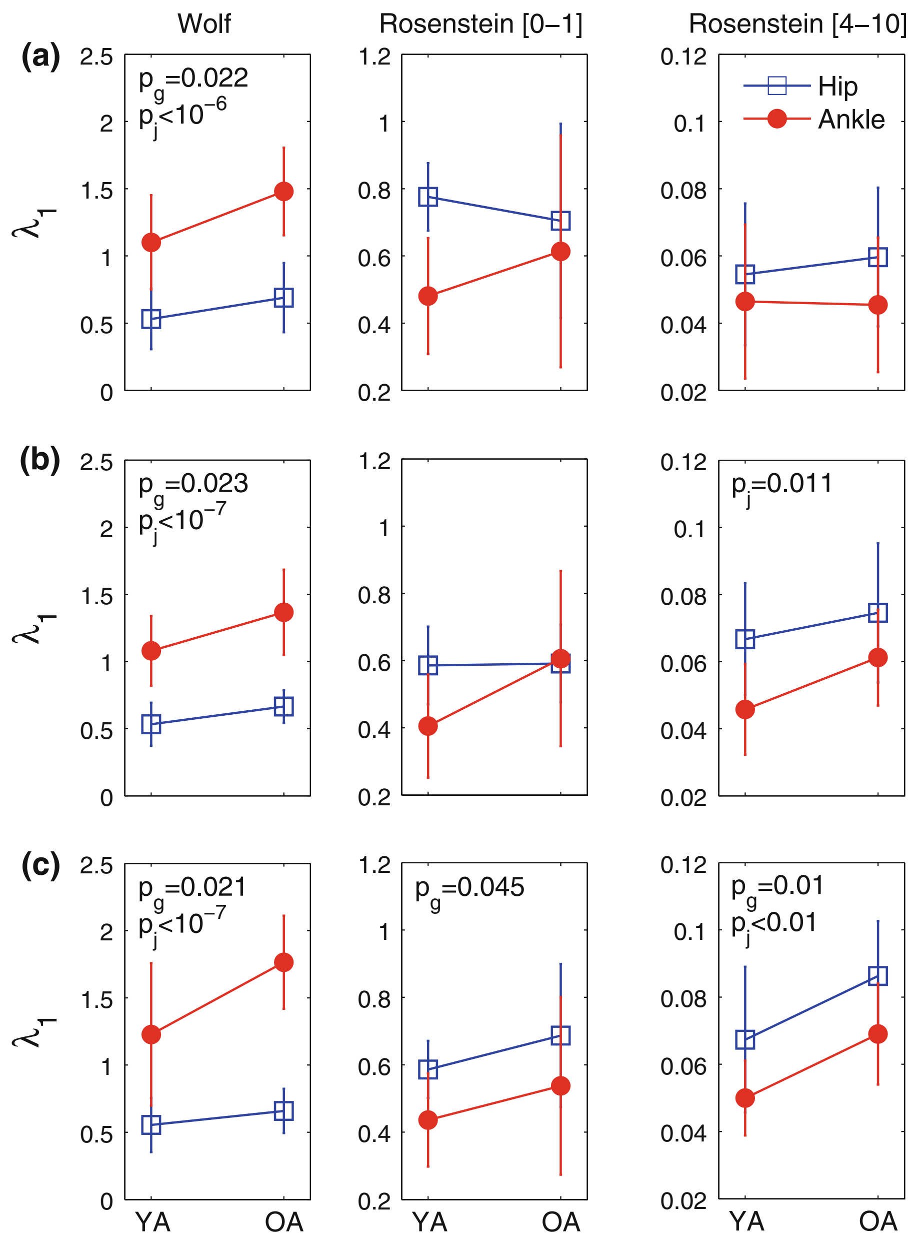

FIGURE 4. Mean ± SD largest Lyapunov exponent (λ_1) obtained using the *W*-algorithm and the *R*-algorithm between 0-1 stride and 4-10 strides with gait data counting (a) 3600, (b) 7200 and (c) 10,800 data points. YA denotes younger adults and OA older adults. ANOVA results for differences between groups (p_g) and joints (p_j) are shown when significant.

the convergences in its calculation appears as a drawback to the *R*-algorithm, while it is not a concern for the *W*-algorithm.

The λ_1 exponents obtained from gait data with the *W*-algorithm were found significantly higher in the OA as compared to the YA. This means that the former group was more locally unstable as already shown in the literature.[5,21,22,25] Importantly and as hypothesized, this differentiation was present whatever the size of the data set. This result demonstrated that the λ_1 estimate obtained even from our smaller data set can accurately detect decline in gait function induced by physiological aging. On the other hand, exponents obtained with the *R*-algorithm revealed more local instability in the OA only for the longest time series. Such a lack of sensitivity of the exponents to dissociate between the two groups for the shorter time series may arise from the 'convergence drawback' of the *R*-algorithm discussed above, with convergences present in many parts of the gait attractors (Fig. 2). It is likely that this drawback is lessened for longer size of data sets as the probability of finding very close nearest neighbors that can diverge far apart increases as the number of points increases in state space, thus increasing the total rate of divergence.[4] It is however important to remind that the sample size of seven YA and seven OA was small, so that the probability of not rejecting the null hypothesis with the ANOVAs, and thus not finding the OA more locally unstable than the YA, was likely to be important (i.e., a low statistical power). As the *W*-algorithm dissociated between the two groups whatever the size of gait data sets, increasing the sample size would have only affected the effect size of that result (i.e., the magnitude of the difference in local dynamic stability between the two groups). On the other hand, an increase in the sample size might have revealed a difference between YA and OA with the *R*-algorithm for the smaller data sets. Therefore, null findings obtained with the *R*-algorithm should be interpreted with caution.

Furthermore, the hip was found to be less locally unstable than the ankle with the *W*-algorithm. This reiterates findings from studies that have either used the *W*- or the *R*-algorithm.[22,25] Kang and Dingwell[22] have proposed that the greater inertia of the proximal segments may attenuate the effect of a given perturbation on segment motion so that their local instability is reduced. This interpretation can also be used here to explain for the difference between the hip and the ankle local instability. On the other hand, an opposite result was found when using the *R*-algorithm with the hip more locally unstable than the ankle. Although these opposite outcomes between the two algorithms are puzzling, this latter result fits in with previous findings obtained with the *R*-algorithm.[8,16] England and Granata[16] suggested that the lower local instability about the ankle reflects a greater neuromuscular stabilizing control of this joint. However, the narrower trajectories of the hip attractor as compared to those of the ankle attractor argue against this interpretation and question the result (Fig. 2). A more likely explanation is that the *R*-algorithm underestimates λ_1 for attractors with narrow trajectories, in which divergences occur on large time scales. Such an underestimation has been observed by Rosenstein *et al.*[28] for the Rössler attractor (when using short data sets), where chaos generation typically occurs on a large time scale.[33] Moreover, since the hip trajectories are stretching and folding in many parts of the hip attractor, the exponent underestimation is most likely magnified as previously explained.

CONCLUSIONS

In summary, while the λ_1 estimates from both algorithms were nearly equal for small Lorenz data sets, the *R*-algorithm provided less sensitive λ_1 estimates than the *W*-algorithm to capture age-related differences in local dynamic stability from small gait data sets. The data supported the idea that this latter outcome results from the ability and inability of the *W*-algorithm and *R*-algorithm, respectively, to estimate adequately λ_1 of attractors with an important rate of convergence as those in gait. Indeed, it was found that the *W*-algorithm makes an excellent use of the attractor divergences for estimating λ_1 while the *R*-algorithm overlooks the attractor expansion. Therefore, the *W*-algorithm appears to be more appropriate than the *R*-algorithm to evaluate local dynamic stability from small gait data sets. Increase in the size of data set has been shown to make the results of the *R*-algorithm more suitable, although other means as increasing the sample size might have a similar effect.

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CONFLICT OF INTEREST

The authors disclose any financial and personal relationships with other people or organizations that could inappropriately influenced (bias) this work.

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