Pufferfish Differential Privacy

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Disclaimer

Any findings are not to be considered as official statistics and the opinions and conclusions expressed are those of authors, not the ABS.

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Background and context

- *Statistics Determination 2018* describes circumstances that enable information can be disclosed by the ABS.
- The Determination allows for greater utility, but also requires the ABS provides passive confidentiality under certain circumstances.
- Consequential suppression:
  - is the traditional confidentiality method, but
  - causes information loss.
- Proposing a new approach that:
  - uses a Differential Privacy framework,
  - facilitates privacy risk-utility trade off discussion,
  - enables the ABS's ability to meet user needs, and
  - reduces the costs of managing statistical risks.

Consequential suppression causes utility loss

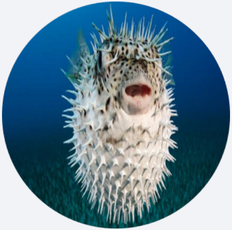
Regions	Contributors	Sales ($000)
A	12	2608
B	3	2562
C	5	2608
D	1	n.p.
E	2	n.p.
Total	23	13427

- D – Primary suppression.
- E – Secondary suppression to protect D.
- Further suppressions on outputs with contributors in E (and D).

Regions	Contributors	Sales ($000)
A	12	2608
B	3	2562
C	5	2608
D	1	2727
E	2	2240
Total	23	12745

- Sensitive record in D is protected by DP noise.
- Perturbed value is used in all outputs.
- D is protected in all outputs that it contributes to.

Why it is called Pufferfish?



$\Longrightarrow$

$\Longrightarrow$

- Pufferfish
  - tetrodotoxin
- Data
  - sensitive info (secrets)

- Certified chef
  - remove toxin
- Formal privacy definition algorithm
  - protect sensitive info (secrets)

- Fugu sashimi
  - no toxin
- Safe data
  - minimum sensitive info (secrets)

Source: adapted from [Machanavajjhala et al., 2017]

Pufferfish DP Framework

Kifer and Machanavajjhala [2014] define given set of potential secrets $\mathbb{S}$, a set of discriminative pairs $\mathbb{S}_{pairs}$, a set of all plausible data distributions $\mathbb{D}$, and a privacy parameter $\epsilon > 0$, a publishing method M satisfies ϵ-Pufferfish($\mathbb{S}$, $\mathbb{S}_{pairs}$, $\mathbb{D}$) privacy if

- for all possible outputs $\omega \in \text{range}(M)$,
- for all pairs $(s_i, s_j) \in \mathbb{S}_{pairs}$ of potential secrets,
- for all distributions $\theta \in \mathbb{D}$ which $P(s_i \mid \theta) \neq 0$ and $P(s_j \mid \theta) \neq 0$

the following holds

$$P(M(\mathfrak{Data}) = \omega \mid s_i, \theta) \leq e^{\epsilon} P(M(\mathfrak{Data}) = \omega \mid s_j, \theta) \quad (1)$$

$$P(M(\mathfrak{Data}) = \omega \mid s_j, \theta) \leq e^{\epsilon} P(M(\mathfrak{Data}) = \omega \mid s_i, \theta) \quad (2)$$

Our q% interval Pufferfish DP

Inspired by the $p\%$ rule. Choose a fixed $q \in (0, 1)$. The set of secrets is defined as

$$\mathbb{S} = \left\{\sigma_{[(1-q)y,(1+q)y)} : y > 0\right\} \cup \left\{\sigma_{((1+q)y,(1-q)y]} : y < 0\right\} \tag{3}$$

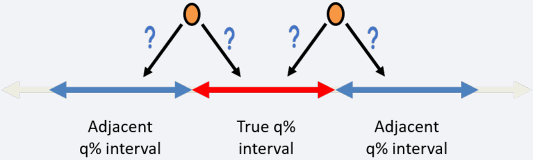
where $\sigma_{[(1-q)y,(1+q)y)}$ is a statement that the record's value is within the interval ${}_{[(1-q)y,(1+q)y)}$. The set of discriminative pairs is defined as

$$\begin{aligned}\mathbb{S}_{pairs} = &\left\{\left(\sigma_{[(1-q)y,(1+q)y)}, \sigma_{[(1+q)y, \frac{(1+q)^2}{(1-q)}y)}\right) : y > 0\right\} \cup \\ &\left\{\left(\sigma_{[\frac{(1+q)^2}{(1-q)}y,(1+q)y)}, \sigma_{[(1+q)y,(1-q)y)}\right) : y < 0\right\}.\end{aligned} \tag{4}$$

The data evolution scenarios $\mathbb{D}$ is defined as the set of probability distributions where $\theta \in \mathbb{D}$ if and only if $P(y > 0 \mid \theta) + P(y < 0 \mid \theta) = 1$, with θ corresponds to a data user's prior knowledge about the data.

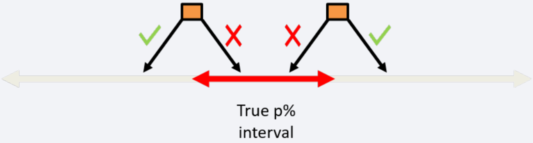
Main ideas

Pufferfish Protection



- Probabilistic (Inferential).
- All sensitive values are protected with noise.

p% Rule



- Deterministic.
- Values that violate the rule must be suppressed.

Log-Laplace multiplicative noise

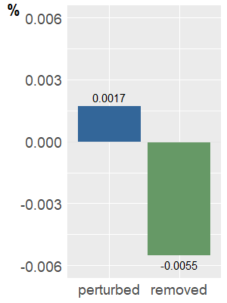
Given the set of potential secrets $\mathbb{S}$, the set of discriminative pairs $\mathbb{S}_{pairs}$, the set of data evolution scenarios $\mathbb{D}$, and privacy parameter $\epsilon > 0$, the log-Laplace multiplicative perturbation mechanism

$$M(\mathfrak{Data} = y) = ce^{X}y$$

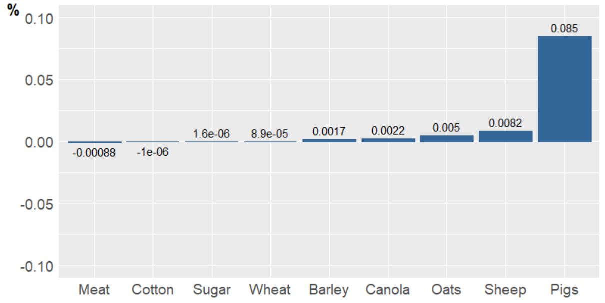
satisfies $(\mathbb{S}, \mathbb{S}_{pairs}, \mathbb{D}, \epsilon)$-Pufferfish, where $X \sim Laplace(0, b)$ with $b = -\frac{4}{\epsilon}\ln(1 - q)$ and $c = 1 - b^2$ (bias correction factor, equal to $1/E(e^X)$. Note that e^X has a log-Laplace distribution if X has a Laplace distribution. Note proof in the upcoming paper.

Case 1 : Impact to Aggregate Estimates

% differences for the total barley production



% difference for the total production for principle commodities



Case 2 : Tuning parameters using sugarcane data

Australian Bureau of Statistics

- Explore utility and risk trade-off under different privacy parameters setting for ϵ and q
- The case study design:
  - Set an arbitrary $p\%$ at 15%
  - 3 contributors in a small area from the sugarcane administrative data

$$Y = y_1 + y_2 + y_3 \tag{5}$$

$$Y^{\ddagger} = y_1 + ce^X y_2 + y_3 \tag{6}$$

    where Y is the true total of sugarcane production, $Y^{\ddagger}$ is the perturbed total of sugarcane production and y_1, y_2 and y_3 are the largest, second and third contributors.
  - The risk scenario is the largest contributor wants to estimate the second largest contributor's true value.

Case study 2 : risk and utility assessment

We perform $M = 1000$ simulations for each combination of arbitrary selected ϵ and q for the following:

- Measure utility loss as

$$RSE = \frac{\sqrt{\sum_{m=1}^{M}(Y_m^{\ddagger} - Y)^2}}{Y}$$

where $Y_m^{\ddagger}$ is the perturbed total of sugarcane production from simulation run m and Y is the true total of sugarcane production.

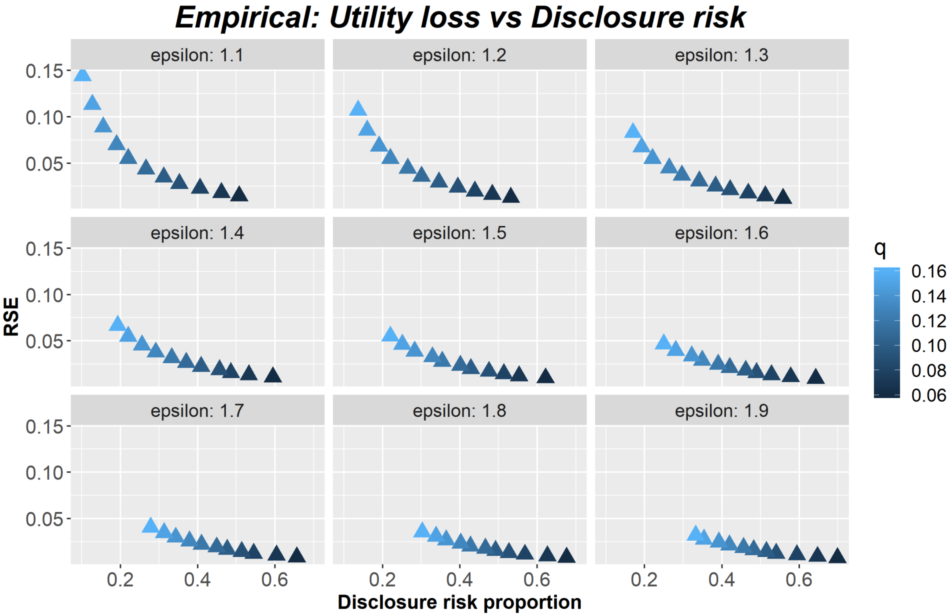
- Assess risk of disclosure as

$$P = \frac{\sum_{M}^{m=1} I_m}{M}$$

where P is the proportion of $p\%$ violation and $I_m =$

$$\begin{cases} 1, & \text{if}(Y_m^{\ddagger} - y_1) \in [y_2(1-p\%), \\ & y_2(1+p\%)], \\ 0, & \text{otherwise.} \end{cases}$$

Case study 2 : empirical results



Conclusion and future directions

- DP Pufferfish offers a nice framework to explore utility and risk trade-off.
- many areas of future research
  - explore a test case for producing experimental statistics.
  - application on two or more passive claimants cases.
  - explore an input based disclosure risk measure.
  - determine an optimal choice of ϵ and q under different settings.
  - explore truncated noise distribution.

References I

Kifer, D. and Machanavajjhala, A. (2014). Pufferfish: A framework for mathematical privacy definitions. *ACM Transactions on Database Systems (TODS)*, 39(1):1–36.

Machanavajjhala, A., He, X., and Hay, M. (2017). Differential privacy in the wild: A tutorial on current practices & open challenges. In *Proceedings of the 2017 ACM International Conference on Management of Data*, pages 1727–1730.