Chair of High-Power Converter Systems
Technical University of Munich Prof. Marcelo Lobo Heldwein

Arcisstr. 21 Email: hlu@ed.tum.de Tel.: +49 (0)89 289 28358
80333 Munich Web: www.epe.ed.tum.de/hlu Fax: +49 (0)89 289 28336

Practical Course

Simulation and Optimization of Mechatronic Drive Systems for MSPE

Summer Semester 2024

Haoheng Li / 03797669
Haojun Cao /

Experiment 4
Field Oriented Control of the AC Drive

4.4 Implementation of the Field Oriented Control

4.4.1 Current Control Loop

1.) From equatin in instrruction, we have the differential equation of the rotor current as:

$$\mathbf{i}_r = \frac{\mathbf{\Psi}_r - L_m \mathbf{i}_s}{L_r}, \quad \frac{d\mathbf{i}_r}{dt} = \frac{1}{L_r}\left(\frac{d\mathbf{\Psi}_r}{dt} - L_m \frac{d\mathbf{i}_s}{dt}\right) \tag{4.1}$$

and the differential equation of the rotor flux can be derived as:

$$\frac{d\Psi_r}{dt} = -\frac{R_r}{L_r}\mathbf{\Psi}_r + \frac{L_m R_r}{L_r}\mathbf{i}_s - (\omega_K - \omega)\mathbf{J}\mathbf{\Psi}_r \tag{4.2}$$

Substituting the equation of rotor flux into the equation of stator flux, we have:

$$\frac{d\mathbf{\Psi}_s}{dt} = L_s \frac{d\mathbf{i}_s}{dt} + L_m \frac{d\mathbf{i}_r}{dt} = \left(L_s - \frac{L_m^2}{L_r}\right)\frac{d\mathbf{i}_s}{dt} - \frac{L_m R_r}{L_r^2}\mathbf{\Psi}_r - \frac{L_m}{L_r}(\omega_K - \omega)\mathbf{J}\mathbf{\Psi}_r + \frac{L_m^2 R_r}{L_r^2} \tag{4.3}$$

$\omega_K \mathbf{J}\Psi_s$ can be expressed as:

$$\omega_K \mathbf{J}\mathbf{\Psi}_s = \omega_K \left(L_s - \frac{L_m^2}{L_r}\right)\mathbf{J}\mathbf{i}_s + \omega_K \frac{L_m}{L_r}\mathbf{J}\mathbf{\Psi}_r \tag{4.4}$$

Finally, the differential equation of the stator voltage can be derived as:

$$\mathbf{u}_s = \left(R_s + \frac{L_m^2}{L_r^2}R_r\right)\mathbf{i}_s + \left(L_s - \frac{L_m^2}{L_r}\right)\frac{d\mathbf{i}_s}{dt} + \omega_K L'\mathbf{J}\mathbf{i}_s + \left(-\frac{L_m R_r}{L_r^2}\mathbf{I} + \frac{\omega L_m}{L_r}\mathbf{J}\right)\mathbf{\Psi}_r \tag{4.5}$$

To simplify the equation, we can define the following parameters:

$$\begin{aligned} R' &= R_s + \left(\frac{L_m}{L_r}\right)^2 R_r = 4.966\Omega \\ L' &= L_s - \frac{L_m^2}{L_r} = \sigma L_s = 27.424mH \\ \sigma &= 1 - \frac{L_m^2}{L_s L_r} \end{aligned} \tag{4.6}$$

The final equation of the stator voltage in d-q frame is:

$$
\begin{aligned}
u_s^d &= R' i_s^d + L' \frac{di_s^d}{dt} + (-\omega_K L')\, i_s^q + \left(-\frac{L_m R_r}{L_r^2}\right) \Psi_r^d + \frac{\omega L_m}{L_r}(-\Psi_r^q) \\
u_s^q &= R' i_s^q + L' \frac{di_s^q}{dt} + (+\omega_K L')\, i_s^d + \left(+\frac{\omega L_m}{L_r}\right) \Psi_r^d + \left(-\frac{L_m R_r}{L_r^2}\right) \Psi_r^q
\end{aligned}
\tag{4.7}
$$

Therefore, the coefficients a_1, a_2, b_1, b_2 are:

$$
\begin{aligned}
a_1 &= -\frac{L_m R_r}{L_r^2}, \quad a_2 \frac{\omega L_m}{L_r} \\
b_1 &= -\omega_K L', \quad b_2 = \omega_K L'
\end{aligned}
\tag{4.8}
$$

2.) The Laplace transformation of the differential equations of the stator voltage in d-q frame is:

$$
\begin{aligned}
U_s^d(s) &= (R' + L's)\, I_s^d(s) + a_1 \Psi_r^d(s) + b_1 I_s^q(s) \\
U_s^q(s) &= (R' + L's)\, I_s^q(s) + a_2 \Psi_r^d(s) + b_2 I_s^d(s)
\end{aligned}
\tag{4.9}
$$

The signal flow graph of the above equations is shown in Fig. 4.1.

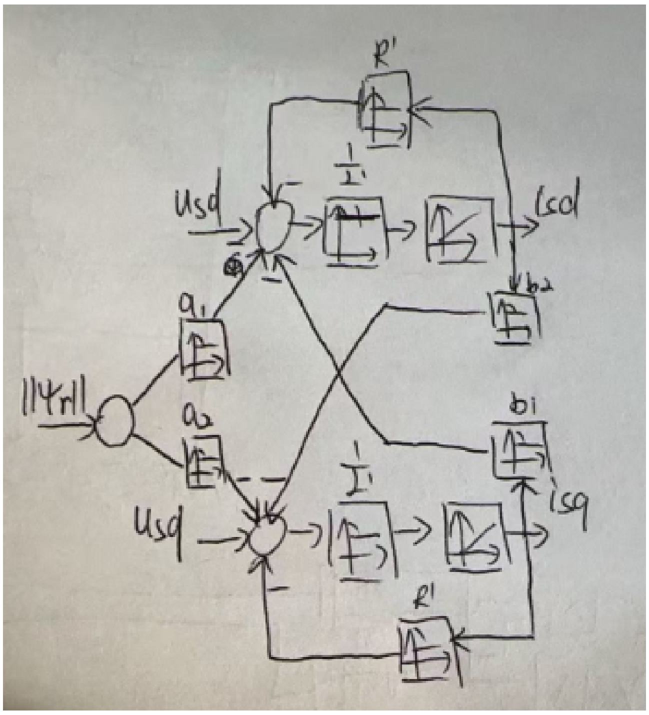

Figure 4.1: Signal Flow Graph of the Stator Voltage Equations in d-q Frame

The values of $a_1 \Psi_r^d + b_1 I_s^q$ and $a_2 \Psi_r^d + b_2 I_s^d$ can be considered as disturbance inputs to the system. To eliminate the effects of these disturbances, we can use the feedforward control method. The compansated stator voltages are given in the next question.

3.) The trnasfer functions $F_i(s)$ of the current and the desired voltages is:

$$\begin{aligned} F_i(s) &= \frac{I_s^s}{U_s^{s*}(s)} = \frac{1}{R' + L's} \cdot G_U(s) \\ &= \frac{V_{i_s} V_{PE}}{(1 + T_{i_s} s)(1 + T_{PE} s)} \end{aligned} \tag{4.10}$$

where $V_{i_s} = \frac{1}{R'} = 0.2014$, $T_{i_s} = \frac{L'}{R'} = 5.522ms$, $V_{PE} = \frac{U_{DC}}{2} = 280V$ and $T_{PE} = 250\mu s$ are the gain and time constant of the power electronics.

and the desired voltages $U_s^{s*}(s)$ are:

$$\begin{aligned} U_s^{d*\prime}(s) &= U_s^{d*}(s) - a_1 \Psi_r^d(s) - b_1 I_s^q(s) \\ U_s^{q*\prime}(s) &= U_s^{q*}(s) - a_2 \Psi_r^d(s) - b_2 I_s^d(s) \end{aligned} \tag{4.11}$$

Therefore, the parameters of the plant transfer function $F_i(s)$ are:

$$\begin{aligned} V_i &= V_{i_s} \cdot V_{PE} = \frac{1}{R'} \cdot \frac{U_{DC}}{2} = 56.38 \\ T_{1,i} &= T_{i_s} = \frac{L'}{R'} = 0.0055\text{s} = 5.52\text{ms} \\ T_{\sigma,i} &= T_{PE} = 250\mu\text{s} \end{aligned} \tag{4.12}$$

4.) With information above, we can design the PI controller for the current control loop. It's a PT2 systm. Therefore, the parameters of the PI controller can be designed as:

$$\begin{aligned} G_{PI,i}(s) &= V_{\text{C,i}} \frac{1 + sT_{\text{n,i}}}{sT_{\text{n,i}}} \\ V_{C,i} &= \frac{T_{1,i}}{2T_{\sigma,i} \cdot V_i} = 0.196 \\ T_{n,i} &= T_{1,i} = 5.52\text{ms} \end{aligned} \tag{4.13}$$

4.4.2 Flux and Rotor Speed Control

9.) The transfer function of the closed loop current control system is:

$$F_{sub,i}(s) = \frac{I_s^s(s)}{I_s^{s*}(s)} = \frac{F_i(s) G_{PI,i}(s)}{1 + F_i(s) G_{PI,i}(s)} = \frac{1}{1 + s2T_{PE}s + s^2 T_{PE}^2} \tag{4.14}$$

where $T_{PE}^2 \ll 1$ can be neglected. Therefore, the simplified transfer function of the closed loop current control system is:

$$F_{sub,i}(s) = \frac{1}{1 + sT_{equi,i}} \tag{4.15}$$

where $T_{equi,i} = 2T_{PE} = 500\mu s$ is the equivalent time constant of the closed loop current control system.

10.) From the differential equation of the rotor flux, we can derive the transfer function of the rotor flux as:

$$F_{\Psi i}(s) = \frac{\Psi_r^d(s)}{i_s^d(s)} = \frac{L_m}{s + \frac{R_r}{L_r}} = \frac{V_\Psi}{1 + sT_\Psi} \tag{4.16}$$

where $V_\Psi = L_m = 0.326$ and $T_\Psi = \frac{L_r}{R_r} = 0.1172s$ are the gain and time constant of the rotor flux.

Considering the closed loop current control system, the transfer function from the desired d-axis current to the rotor flux is:

$$F_{\Psi_r}(s) = \frac{\Psi_r^d(s)}{i_s^{d*}(s)} = \frac{V_\Psi}{(1 + sT_\Psi)(1 + sT_{equi,i})} \tag{4.17}$$

The small and large time constants are $T_{\sigma,\Psi_r} = T_{equi,i} = 500\mu s$ and $T_{1,\Psi_r} = T_\Psi = 0.1172s$ respectively.

11.) It's a PT2 system. The parameters of the PI controller for the flux control loop can be designed as:

$$\begin{aligned} G_{PI,\Psi}(s) &= V_{C,\Psi}\frac{1 + sT_{n,\Psi}}{sT_{n,\Psi}} \\ V_{C,\Psi} &= \frac{T_{1,\Psi}}{2T_{\sigma,\Psi}V_\Psi} = 359.5 \\ T_{n,\Psi} &= T_{1,\Psi} = 0.1172s \end{aligned} \tag{4.18}$$

14.) The transfer function of the speed loop can be derived as:

$$\begin{aligned} F_\omega(s) = \frac{\omega(s)}{i_s^{q*}(s)} &= \frac{\hat{\omega}_m(s)}{\omega_m(s)} \cdot \frac{\omega_m(s)}{M_M(s)} \cdot \frac{M_M(s)}{i_s^q(s)} \cdot \frac{i_s^q(s)}{i_s^{q*}(s)} \\ &= \frac{1}{1 + sT_{f,\omega}} \cdot \frac{3}{2}p\frac{L_m}{L_r}\psi_{r,k} \cdot \frac{1}{\Theta_m s} \cdot \frac{1}{1 + T_{equi,i}s} \\ &= \frac{V_{S,\omega_M}}{T_{1,\omega_M}s(1 + T_{\sigma,\omega_M}s)} \end{aligned} \tag{4.19}$$

where $V_{S,\omega_M} = \frac{3\pi f_N L_m \psi_{rN}}{M_{MN}L_r} = 59.05$, $T_{1,\omega_M} = \frac{\Theta_M 2\pi f_N}{pM_{MN}} = 0.0951$ and $T_{\sigma,\omega_M} = T_{equi,i} + T_{f,\omega} = 2.5ms$ are the gain, large and small time constants of the speed control loop.

15.) It's a IT1 system. The M_L is disturbance input. we want SO optimization. Therefore, the parameters of the PI controller for the speed control loop can be designed as:

$$
\begin{aligned}
G_{PI,\omega}(s) &= V_{C,\omega} \frac{1 + sT_{n,\omega}}{sT_{n,\omega}} \\
V_{C,\omega} &= \frac{T_{1,\omega}}{2T_{\sigma,\omega} V_{S,\omega}} = 0.322 \\
T_{n,\omega} &= 4T_{\sigma,\omega} = 0.01s
\end{aligned}
\tag{4.20}
$$