Microeconomics II - Problem Set 1

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EUI

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Exercise 1: Represent in extensive form the following variations of the highest-card example dicussed in class.

(a) Player 2 is now given the option of either passing or betting, *independently* of the prior choice made by player 1. Moreover, an initial amount has to be paid by both players at the start of the game, which is added to the players' bets if neither passes, it is received by the one who bets (together with her initial payment) if only one of them does, or is returned back to each if both pass.

(b) As in (a), with the additional possibility that player 1 may bet after having passed initially if player 2 bets.

Exercise 2: Two generals, A and B, whose armies are fortified in opposite hills have to decide whether to attack the enemy which is camped in a valley separating them. In order for the attack to be successful, General A must receive reinforcements. The arrival of these reinforcements on time has a prior probability of 1/2 and depends on weather conditions not observed by the generals. They have reached the agreement that if A receives the reinforcements, he will send an emissary to B. They both know, however, that in this case the emissary only has probability 1/3 of being able to cross the enemy lines. The payoffs have been evaluated as follows. They are equal to 50 for each general in case of victory. If they both refrain from attacking, their payoff is zero. If one attacks whereas the other does not, the former obtains a payoff of -50 and the latter a payoff of -10. Finally, if both generals attack but are defeated (because A has not received the reinforcements) each of them receives a payoff of -40.

(1) Represent the game described in extensive form.

(2) As in (1), for a modified game where General A considers the possibility of sending an emissary in any case.

(3) As in (1), for a modified game where General A always attacks but only sends an emissary in case he receives reinforcements.

(4) As in (1), for a modified game where General A always sends and emissary but only attacks if he receives reinforcements.

(5) Propose a prediction (perhaps in probabilistic terms) for the outcome of the battle in the latter two cases.

Exercise 3: Two individuals have to agree on how to divide 4 dollars. Two divisions are being considered: an even split that would give 2 dollars to each of them, and an asymmetric division that would leave 3 dollars with one of the players (labelled player 1) and one dollar with the other (player 2). The following allocation procedure is considered. First, player 1 has to make a proposal (i.e. one of the previous two possibilities), to which player 2 then has to respond with acceptance or rejection. If the proposal is accepted the four dollars are divided accordingly, whereas in the alternative case neither of them receives any money at all. Formulate the situation as a game in extensive form and discuss *informally* the likelihood of different outcomes.

Exercise 4: Consider the game described in Exercise 3. Define the players' strategy spaces formally, and specify in detail its strategic-form representation.

Exercise 5: Let there be two individuals, 1 and 2, who play the traditional Rock-Scissors-Paper (R-S-P) game: R beats S, S beats P, and P beats R. Suppose that "beating the opponent" implies receiving ten dollars from her, whereas if both individuals choose the same action no payment is made.

1. Represent the game in extensive and strategic forms. What would be your prediction on the strategy used by each player?
2. Consider now the following variation of the previous game: the order of movement is sequential (first one of them, then the other), each of the two possibilities being chosen with equal probability (say, by tossing a fair coin) at the beginning of the game.
   (a) Represent the game in extensive and strategic forms.
   (b) Propose a prediction for the strategy that will be played by each player and justify it informally.
   (c) Suppose that one of the individuals is given the option of choosing whether to play first or second in the sequential game. Which option will she prefer?

Exercise 6: Consider the game structure represented in Figure 1 and suppose that player 1 uses a mixed strategy σ_1 that assigns an equal weight of $1/3$ to the three following pure strategies: (T, X, Y), (T, Y, Y), (B, X, X), where the first component refers to her first information set, the second one to the information set that follows action T, and the third one to the information set that follows B. Determine the behavioral strategy equivalent to σ_1. Consider now the mixed strategy σ'_1 that associates an equal weight of $1/2$ to the strategies (T, X, Y) and (T, Y, Y). Determine the behavioral strategy equivalent to σ'_1.

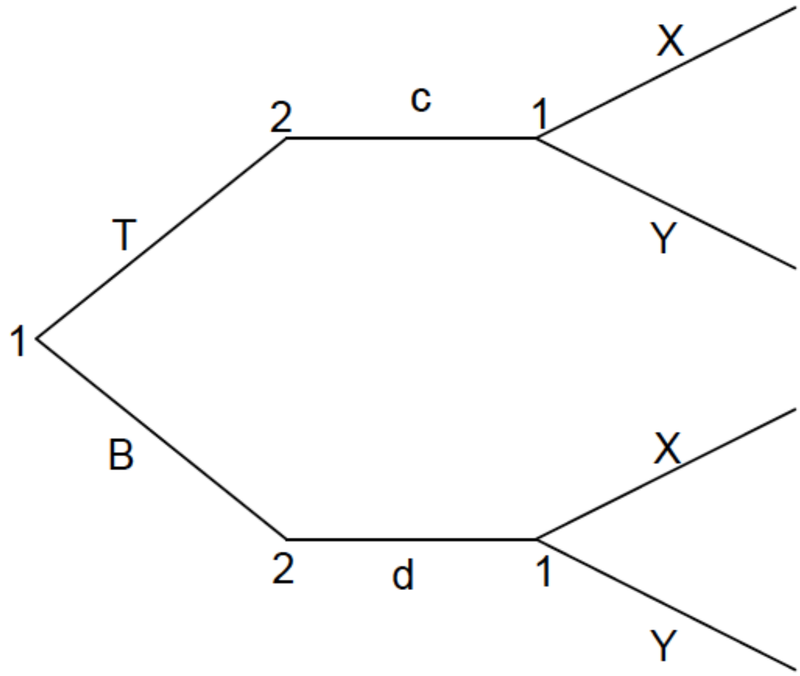

Figure 1: A game structure

In every problem set you will find at the end a question from previous years exams that you are expected to be able to solve at the time you get the problem set. This is for you to get used to the types, the length and the difficulty of the questions that you'll face in the final, and to check your level of understanding during the course.
At the end of the course I will give you some previous years exams with solutions; and we will have a class to solve together the last year exam.

Part a of question 1 of the 2009 exam:

Question 1: Consider the following situation. There are three players, indexed 1, 2, and 3. A valuable item owned by player 3 is found by player 1. He must then choose whether to pass it to player 2 or not. In the former case, it is then up to player 2 to pass it to player 3 or not. Player 3 is perfectly aware of the process but, at the end of it, he only learns whether he has been returned the item or not. Thus, if he does not receive it, he does not know whether it is in the hands of player 1 or 2. On the basis of this information alone, he must choose in either case (i.e. whether he receives the item or not) among three actions: punish player 1, punish player 2, or not punish either of them.

Let us suppose that the item is of no use to either player 1 or 2 but can be sold in the market for 100 €, which is therefore the value of it for players 1 and 2 if either of them keeps it. Instead, player 3 values it at 500 €. On the other hand, let us assume that the "punishment" that player 3 can inflict on either player 1 or 2 has a monetary cost to them that is estimated at 1000 €, i.e. punishment by player 3 leads to a payoff for the punished player (1 or 2), equal to -1000 €. Finally, let us suppose that the monetary value for player 3 of punishing *either* of the two players is 1 € (i.e. positive) if he does not receive the item and -1 € (i.e. negative) if he receives it. For each player, the payoff resulting from punishment is added to the monetary value of the item, or added to zero in case the player does not end up with the item.

(a, 10 points) Formalize the situation as a game in extensive and strategic forms. Note that the former can be done *diagrammatically*, but the latter requires the formal specification of the strategy sets and the payoff functions (possibly through payoff tables, if convenient).