Exam 2015 - 2016

Microeconomics II

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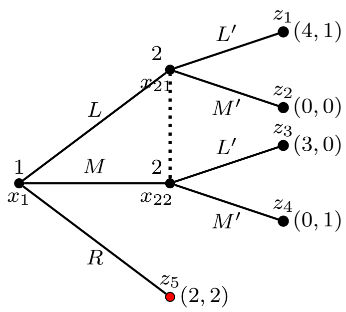
Question 1

1

The game in extensive form:



There are two players, i.e., $N = \{1, 2\}$. $K = \{x_1, x_{21}, x_{22}, z_1, z_2, z_3, z_4, z_5\}$ and $P(x_1) = \emptyset$, $x_{21}Rz_1$, $x_{21}Rz_2$, $x_{22}Rz_3$, $x_{22}Rz_4$. The set of nodes is partitioned between the two players in the following way: $K_1 = \{x_1\}$ and $K_2 = \{x_{21}, x_{22}\}$. The information sets are defined as follows: $H_1 = K_1$ and $H_2 = K_2$. The strategy spaces of the two players are: $S_1 = \{L, M, R\}$ and $S_2 = \{L', M'\}$. The profit functions are

$$\pi_1 : S_1 \times S_2 \to \{0, 2, 3, 4\}$$
$$\pi_2 : S_2 \times S_1 \to \{0, 1, 2\}$$

and could be tabulated as

	L'	M'
L	(4, 1)	(0, 0)
M	(3, 0)	(0, 1)
R	(2, 2)	(2, 2)

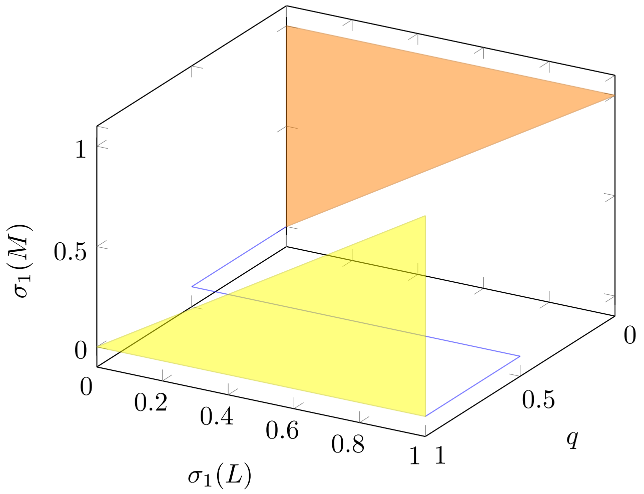
2

To find Nash equilibria, need to find intersection of best responses of the two players. Those are underlined in the table above. Hence, there are two NE in pure strategies, $\{(L, L'), (R, M')\}$.
Since there's only one proper subgame, the entire game itself, the set of NE in pure strategies is the same as the set of SGPE in pure strategies.

3

Let μ denote the system of beliefs of player 2 in his information set, $\mu = (\mu(x_{21}, 1 - \mu(x_{21})))$.

Player 2 chooses L' if	Player 2 chooses M' if
$\mu(x_{21}) > 1 - \mu(x_{21})$	$\mu(x_{21}) < 1 - \mu(x_{21})$
$\mu(x_{21}) > \frac{1}{2}$	$\mu(x_{21}) < \frac{1}{2}$



Suppose the belief system μ is such that $\mu(x_{21}) > \frac{1}{2}$, i.e., player 2 chooses L'. Then, by sequential rationality, player 1 chooses L. This implies a strategy $\gamma = (L, L')$. Given this strategy, $P^\gamma(H_2) = 1$ and $P^\gamma(x_{21}) = 1$. Thus, to be statistically consistent, $\mu(x_{21}) = \frac{P^\gamma(x_{21})}{P^\gamma(H_2)} = 1$, which also satisfies the initial condition on the belief system for player 2 to choose L'. Hence, a strategy (L, L') and a belief system $\mu = (1, 0)$ constitute a WPBE.
Now, consider another case, where $\mu(x_{21}) < \frac{1}{2}$, i.e., player 2 chooses M'. Then, by sequential rationality, player 1 chooses R. This implies that the information set H_2 is never reached, and any belief system of player 2 is consistent. Therefore, a strategy (R, M') and any belief system such that $\mu(x_{21}) < \frac{1}{2}$ constitute a WPBE.
Therefore, the set of WPBE pure strategies is $\{(L, L'), (R, M')\}$, i.e., the same as the set of NE.

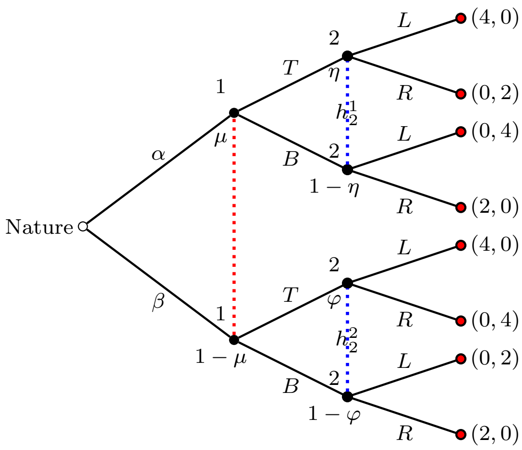
Question 2

1

Let the nature define the type of player 2. Player 1 has one information set as he/she does not observe the type of player 2. Player 2 has two information sets because player 2 observes his/her own type. Thus, the strategy sets of the two players are

$$S_1 = \{T, B\}$$
$$S_2 = \{L, R\}^2 = \{(L, L), (L, R), (R, L), (R, R)\}$$

2



Let the belief system of player 1 be described as $\mu = (\mu(\alpha), 1 - \mu(\alpha)$. Here, we are given that $\mu = (\frac{1}{2}, \frac{1}{2})$. Then, let the belief system of player 2 at the information set h_2^1 be $\eta = (\eta(T), 1 - \eta(T))$ and at the information set h_2^2 be $\varphi = (\varphi(T), 1 - \varphi(T))$.

Player 2 chooses L at h_2^1 if	Player 2 chooses R at h_2^1 if
$4(1-\eta) > 2\eta$	$4(1-\eta) < 2\eta$
$1-\eta > \frac{1}{2}\eta$	$1-\eta < \frac{1}{2}\eta$
$\frac{3}{2}\eta < 1$	$\frac{3}{2}\eta > 1$
$\eta < \frac{2}{3}$	$\eta > \frac{2}{3}$
Player 2 chooses L at h_2^2 if	Player 2 chooses R at h_2^2 if
$2(1-\varphi) > 4\varphi$	$2(1-\varphi) < 4\varphi$
$1-\varphi > 2\varphi$	$1-\varphi < 2\varphi$
$\varphi < \frac{1}{3}$	$\varphi > \frac{1}{3}$

Suppose that $\eta < \frac{2}{3}$ and $\varphi < \frac{1}{3}$. Then, player 2 chooses L at both information sets. Then, by sequential rationality, player 1 chooses T at his information set regardless of μ. Given the strategy $\gamma = (T, (L, L))$, statistically consistent belief η should be $\eta = \frac{1}{\frac{1}{2}} = 2$ ↯.

Suppose that $\eta < \frac{2}{3}$ and $\varphi > \frac{1}{3}$. Given such a belief system, player 2 chooses L at h_2^1 and R at h_2^2. Then,

$$\pi_1(T;\mu) = 4\mu = 2$$
$$\pi_1(B;\mu) = 2(1-\mu) = 1$$

Given the belief system $\mu = (\frac{1}{2}, \frac{1}{2})$, by sequential rationality, player 1 chooses T. This again implies that statistically consistent beliefs of player 2 should satisfy $\eta = \frac{1}{\frac{1}{2}} = 2$ and $\varphi = \frac{1}{\frac{1}{2}} = 2$ ↯.

Suppose that $\eta > \frac{2}{3}$ and $\varphi < \frac{1}{3}$. Given such a belief system, player 2 chooses R at h_2^1 and L at h_2^2. Then,

$$\pi_1(T;\mu) = 4(1-\mu) = 2$$
$$\pi_1(B;\mu) = 2\mu = 1$$

Given the belief system $\mu = (\frac{1}{2}, \frac{1}{2})$, by sequential rationality, player 1 again chooses T. This again implies that statistically consistent beliefs of player 2 should satisfy $\eta = \frac{1}{\frac{1}{2}} = 2$ and $\varphi = \frac{1}{\frac{1}{2}} = 2$ ↯.

Suppose that $\eta > \frac{2}{3}$ and $\varphi > \frac{1}{3}$. Given such a belief system, player 2 chooses R at both information sets. Then, player 1 gets payoff 2 regardless of the type of player 2 if he/she chooses B, which is strictly better than 0 if he/she chooses T. Hence, by sequential rationality, player 1 chooses B. This implies that statistically consistent beliefs of player 2 should satisfy $\eta = \frac{0}{\frac{1}{2}} = 0$ and $\varphi = \frac{0}{\frac{1}{2}} = 0$, which contradicts sequential rationality of (B, R).

Hence, the set of WPBE pure strategies is empty.