Problem Set VI

Microeconomics II

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Exercise 1

The set of NE in pure strategies is $\{(M, l), (L, r)\}$.

1. $\gamma_1 = (0, 1, 0), \gamma_2 = (1, 0)$ Define $\gamma_{1k} = (\rho\varepsilon_{1k}, 1 - (1 + \rho)\varepsilon_{1k}, \varepsilon_{1k}), \gamma_{2k} = (1 - \varepsilon_{2k}, \varepsilon_{2k})$ such that $\lim_{k\to\infty} \varepsilon_{1k} = \lim_{k\to\infty} \varepsilon_{2k} = 0$, for some $\rho > 0$.

   Then, induced belief system is $\mu_{2k} = (\frac{1-(1+\rho)\varepsilon_{1k}}{1-\rho\varepsilon_{1k}}, \frac{\varepsilon_{1k}}{1-\rho\varepsilon_{1k}}) \to (1, 0)$ as $k \to \infty$.

   Given the belief system $\mu_2 = (1, 0)$, player 2 prefers to choose l and player 1's best response then is to choose M. Hence, $(\gamma_1, \gamma_2) = ((0, 1, 0), (1, 0))$ and $\mu_2 = (1, 0)$ constitute a sequential equilibrium.

2. $\gamma_1 = (1, 0, 0)$ and $\gamma_2 = (0, 1)$. Define $\gamma_{1k} = (1 - (1 + \rho)\varepsilon_{1k}, \rho\varepsilon_{1k}, \varepsilon_{1k}), \gamma_{2k} = (\varepsilon_{2k}, 1 - \varepsilon_{2k})$, where $\lim_{k\to\infty} \varepsilon_{1k} = \lim_{k\to\infty} \varepsilon_{2k} = 0$, for some $\rho > 0$.

   Induced belief system is $\mu_{2k} = (\frac{\rho\varepsilon_{1k}}{(1+\rho)\varepsilon_{1k}}, \frac{\varepsilon_{1k}}{(1+\rho)\varepsilon_{1k}}) = (\frac{\rho}{1+\rho}, \frac{1}{1+\rho}) \to (\frac{\rho}{1+\rho}, \frac{1}{1+\rho})$ as $k \to \infty$.

   $$\pi_2(l|\mu_2, H_2) = \frac{\rho}{1 + \rho} \leq \pi_2(r|\mu_2, H_2) = \frac{1}{1 + \rho} \iff \rho \leq 1$$

   Then, player 1's best response is to play L. Therefore, $(\gamma_1, \gamma_2) = ((1, 0, 0), (0, 1))$ and any belief system $\mu_2 = (\frac{\rho}{1+\rho}, \frac{1}{1+\rho})$ such that $\rho \in (0, 1]$ is a sequential equilibrium given.

3. $\gamma_1 = (1, 0, 0)$ and $\gamma_2 = (q, 1-q), \forall q \in [0, 1]$. Define $\gamma_{1k} = (1-2\varepsilon_{1k}, \varepsilon_{1k}, \varepsilon_{1k}), \gamma_{2k} = (q, 1-q)$, where $\lim_{k\to\infty} \varepsilon_{1k} = 0$.

   Induced belief system is $\mu_{2k} = (\frac{1}{2}, \frac{1}{2}) \to (\frac{1}{2}, \frac{1}{2})$ as $k \to \infty$.

   $$\pi_2(l|\mu_2, H_2) = \frac{1}{2} = \pi_2(r|\mu_2, H_2)$$

   Hence, $\gamma_2 = (q, 1 - q)$ is optimal for player 2 in his information set given the belief system $\mu_2 = (\frac{1}{2}, \frac{1}{2})$. Let's compare payoff of player 1 following strategy γ_1 and deviating to strategy $\hat{\gamma_1} = (1 - \nu, \nu, 0)$ for some $\nu > 0$:

   $$\pi_1(\gamma_1, \gamma_2|\mu_2) = 2 \geq 2 - \nu(1 - 2q) = \pi_1(\hat{\gamma_1}, \gamma_2|\mu_2) \iff q \leq \frac{1}{2}$$

   Hence, all strategy profiles $(\gamma_1 = (1, 0, 0), \gamma_2 = (q, 1-q)), \forall q \leq \frac{1}{2}$ given the belief system $\mu_2 = (\frac{1}{2}, \frac{1}{2})$ constitute sequential equilibria.

Exercise 2

Let's first find all NE in pure strategies. From Table 1, we can see that the set of NE in pure strategies is $\{(M, r, a), (R, l, a)\}$.

(M, r, a) Define completely mixed strategies of players $\gamma_{1k} = (\rho\varepsilon_{1k}, 1 - (1 + \rho)\varepsilon_{1k}, \varepsilon_{1k}), \gamma_{2k} = (\varepsilon_{2k}, 1 - \varepsilon_{2k}), \gamma_{3k} = (1 - \varepsilon_{3k}, \varepsilon_{3k})$ for some $\rho > 0$ such that $\varepsilon_{sk} \underset{k\to\infty}{\longrightarrow} 0, \forall s \in \{1, 2, 3\}$. Thus, $\lim_{k\to\infty} \gamma_{1k} = (0, 1, 0)$, $\lim_{k\to\infty} \gamma_{2k} = (0, 1)$, $\lim_{k\to\infty} \gamma_{3k} = (1, 0)$.

These strategies induce the following beliefs:

$$\mu_{2k} = (\frac{\rho\varepsilon_{1k}}{1 - \varepsilon_{1k}}, \frac{1 - (1 + \rho)\varepsilon_{1k}}{1 - \varepsilon_{1k}}) \underset{k\to\infty}{\longrightarrow} (0, 1) = \mu_2$$
$$\mu_{3k} = (\varepsilon_{2k}, 1 - \varepsilon_{2k}) \underset{k\to\infty}{\longrightarrow} (0, 1) = \mu_3$$

Player 3 plays a

1 \ 2	l	r
L	$(0, \underline{1}, \underline{0})$	$(0, 0, \underline{0})$
M	$(0, 1, \underline{1})$	$(\underline{2}, \underline{2}, \underline{1})$
R	$(\underline{1}, \underline{0}, \underline{0})$	$(1, \underline{0}, \underline{0})$

Player 3 plays b

1 \ 2	l	r
L	$(0, \underline{1}, \underline{0})$	$(0, 0, \underline{0})$
M	$(0, \underline{1}, 0)$	$(0, 0, 0)$
R	$(\underline{1}, 0, \underline{0})$	$(\underline{1}, 0, \underline{0})$

Table 1: Strategic form of the game

Given this belief system

$$\pi_3(a|\mu_2,\mu_3,H_3) = 1 > 0 = \pi_3(b|\mu_2,\mu_3,H_3) \Rightarrow \gamma_3^* = (1,0)$$
$$\pi_2((l,a)|\mu_2,\mu_3,H_2) = 1 < 2 = \pi_2((r,a)|\mu_2,\mu_3,H_2) \Rightarrow \gamma_2^* = (0,1)$$
$$\begin{cases}\pi_1((L,r,a)|\mu_2,\mu_3,H_1) = 0 < 2 = \pi_1((M,r,a)|\mu_2,\mu_3,H_1)\\ \pi_1((M,r,a)|\mu_2,\mu_3,H_1) = 2 > 1 = \pi_1((R,r,a)|\mu_2,\mu_3,H_1)\end{cases} \Rightarrow \gamma_1^* = (0,1,0)$$

Hence, a strategy profile $\gamma^* = (\gamma_1^*,\gamma_2^*,\gamma_3^*)$ and a belief system $\mu^* = (\mu_2,\mu_3)$ is a sequential equilibrium.

(R,l,a) Define completely mixed strategies of players $\gamma_{1k} = (\rho\varepsilon_{1k},\varepsilon_{1k},1-(1+\rho)\varepsilon_{1k}), \gamma_{2k} = (1-\varepsilon_{2k},\varepsilon_{2k}), \gamma_{3k} = (1-\varepsilon_{3k},\varepsilon_{3k})$ for some $\rho > 0$ such that $\varepsilon_{sk} \underset{k\to\infty}{\longrightarrow} 0, \forall s \in \{1,2,3\}$. Thus, $\lim\limits_{k\to\infty}\gamma_{1k} = (0,0,1), \lim\limits_{k\to\infty}\gamma_{2k} = (1,0), \lim\limits_{k\to\infty}\gamma_{3k} = (1,0)$.
These strategies induce following beliefs:

$$\mu_{2k} = (\frac{\rho}{1+\rho},\frac{1}{1+\rho}) \underset{k\to\infty}{\longrightarrow} (\frac{\rho}{1+\rho},\frac{1}{1+\rho}) = \mu_2$$
$$\mu_{3k} = (1-\varepsilon_{2k},\varepsilon_{2k}) \underset{k\to\infty}{\longrightarrow} (1,0) = \mu_3$$

Given this belief system

$$\pi_3(a|\mu_2,\mu_3,H_3) = 1 > 0 = \pi_3(b|\mu_2,\mu_3,H_3) \Rightarrow \gamma_3^* = (1,0)$$
$$\pi_2((l,a)|\mu_2,\mu_3,H_2) = 1 > \frac{2}{1+\rho} = \pi_2((r,a)|\mu_2,\mu_3,H_2) \Rightarrow \gamma_2^* = (1,0) \iff \rho > 1$$
$$\pi_1((L,l,a)|\mu_2,\mu_3,H_1) = \pi_1((M,l,a)|\mu_2,\mu_3,H_1) = 0 < 1 = \pi_1((R,l,a)|\mu_2,\mu_3,H_1) \Rightarrow \gamma_1^* = (0,0,1) \iff \rho > 1$$

Hence, a strategy profile $\gamma^* = (\gamma_1^*,\gamma_2^*,\gamma_3^*)$ and a belief system $\mu^* = (\mu_2,\mu_3)$ such that $\rho > 1$ is another sequential equilibrium.

Exercise 3

The game could be formalized as a Bayesian game $BG = \{N,T,A,P,\{\pi_i\}_{i\in N}\}$, where

$N = \{1,2\}$;

$T = T_1 \times T_2 = \{b_1,b_2\} \times \{t_2\}$, where b_1 refers to the first box and b_2 refers to the second box (notice that there's only one type of player 2);

$A = \{A,B\} \times \{C,D\}$;

$P : T \to [0,1], \begin{matrix}(b_1,t_2) \mapsto \frac{1}{2}\\ (b_2,t_2) \mapsto \frac{1}{2}\end{matrix};$

$\pi_i : T \times A \to \mathbb{R}$ specified by the table in the problem.

Suppose player 2 thinks that player 1's strategy is $\gamma_1(b_1) = (p_1, 1-p_1)$ and $\gamma_1(b_2) = (p_2, 1-p_2)$. Then,

$$\mathbb{E}\pi_2(C) = \frac{1}{2}[\pi_2((b_1,t_2),\gamma_1(b_1),C) + \pi_2((b_2,t_2),\gamma_1(b_2),C)] = \frac{1}{2}(p_1+2p_2+1-p_2) = \frac{1}{2}(1+p_1+p_2)$$
$$\mathbb{E}\pi_2(D) = \frac{1}{2}[\pi_2((b_1,t_2),\gamma_1(b_1),D) + \pi_2((b_2,t_2),\gamma_1(b_2),D)] = \frac{1}{2}(4p_2+3(1-p_2)) = \frac{1}{2}(3+p_2)$$
$$\mathbb{E}\pi_2(D) - \mathbb{E}\pi_2(C) = \frac{1}{2}(3+p_2-1-p_1-p_2) = \frac{1}{2}(2-p_1) > 0, \forall p_1 \in [0,1], \forall p_2 \in [0,1]$$

Hence, player 2 maximizes expected payoff by choosing D and $\gamma_2^* = (0,1)$.
Then, player 1 makes decision based on

$$\frac{1}{2}[\pi_1((b_1,t_2),A,\gamma_2^*) + \pi_1((b_2,t_2),A,\gamma_2^*)] = 0 < \frac{1}{2} = \frac{1}{2}[\pi_1((b_1,t_2),B,\gamma_2^*) + \pi_1((b_2,t_2),B,\gamma_2^*)] \Rightarrow \gamma_1^* = (0,1)$$

Therefore, a strategy profile $\gamma^* = \{\gamma_1^*, \gamma_2^*\}$ is a BNE in pure strategies.

Exercise 4

The game could be described as a Bayesian game $BG = \{N, T, A, P, \{\pi_i\}_{i\in N}\}$, where

$N = \{1,2\}$;

$T = T_1 \times T_2 = \{c^h, c^l\}^2$;

$A = \mathbb{R}^2_+$;

$$\begin{aligned} P : T &\to [0,1] \\ (c^h, c^h) &\mapsto p^2 \\ (c^h, c^l) &\mapsto p(1-p) \\ (c^l, c^h) &\mapsto (1-p)p \\ (c^l, c^l) &\mapsto (1-p)^2 \end{aligned}$$

$$\begin{aligned} \pi_i : T \times A &\to \mathbb{R} \\ (t_i, t_j, q_i, q_j) &\mapsto \max\{M - d(q_i + q_j), 0\}q_i - t_i q_i, \qquad \forall i \in N \end{aligned}$$

where $t_i \in T_i, \forall i \in N$ is a generic notation for a realization of the type of any player i and q_i is a typical element of A_i.

Since both firms could either be of high or low type and each firm only knows its own marginal cost, the equilibrium is symmetric, i.e., only depends on the realization of the type rather than the individual firm. Define strategy of each type of the firm as $\gamma_i : T_i \to \mathbb{R}+, c^h \mapsto q^h, c^l \mapsto q^l, \forall i \in N$. Hence, each type of the firm chooses output to maximize respective profit function given expectation of the other firm's strategy:

$$\max_{q^h} \quad \pi_i(q^h, \gamma_j | t_i = c^h) = \max\{M - d[q^h + pq^h + (1-p)q^l], 0\}q^h - c^h q^h \text{ s.t. } q^h \geq 0, \qquad \forall i \in N$$

$$\begin{aligned}
\mathcal{L}^h &= (M - d[q^h + pq^h + (1-p)q^l])q^h - c^h q^h + \mu(M - d[q^h + pq^h + (1-p)q^l]) + \lambda q^h \\
& M - d(1-p)q^l - c^h - \mu d(1+p) + \lambda = 2dq^h(1+p) \\
& \min\{\mu, M - d[q^h + pq^h + (1-p)q^l]\} = 0 \\
& \min\{\lambda, q^h\} = 0
\end{aligned}$$

Case I: $M - d[q^h + pq^h + (1-p)q^l] > 0 \Rightarrow \mu = 0$ and $q^h > 0 \Rightarrow \lambda = 0$

$$\begin{aligned}
& 2dq^h(1+p) = M - d(1-p)q^l - c^h \\
& q^h = \frac{M - d(1-p)q^l - c^h}{2d(1+p)} \text{ if } M - d(1-p)q^l - c^h > 0
\end{aligned}$$

Case II: $M - d(1-p)q^l > 0 \Rightarrow \mu = 0$ and $q^h = 0 \Rightarrow \lambda \geq 0$

$$\begin{cases} q^h = 0 \\ M - d(1-p)q^l - c^h + \lambda = 0 \end{cases} \qquad \begin{cases} q^h = 0 \\ \lambda = c^h - (M - d(1-p)q^l) \end{cases} \qquad \text{if } c^h \geq (M - d(1-p)q^l)$$

Case III: $M - d[q^h + pq^h + (1-p)q^l] = 0 \Rightarrow \mu \geq 0$ and $q^h > 0 \Rightarrow \lambda = 0$

$$\begin{cases} M - d(1-p)q^l - c^h - \mu d(1+p) = 2dq^h(1+p) \\ M - dq^h(1+p) - d(1-p)q^l = 0 \end{cases} \qquad \begin{cases} \mu = -\frac{c^h + M - d(1-p)q^l}{d(1+p)} \leq 0 \Rightarrow \lightning \\ q^h = \frac{M - d(1-p)q^l}{d(1+p)} \end{cases}$$

Case IV: $M - d(1-p)q^l = 0 \Rightarrow \mu \geq 0$ and $q^h = 0 \Rightarrow \lambda \geq 0$

$$\begin{cases} q^h = 0 \\ M - d(1-p)q^l = 0 \\ \lambda - \mu d(1+p) = c^h \end{cases}$$

Hence, the best response correspondence of the high cost firm is

$$\rho_h(q^l) = \begin{cases} \frac{M - d(1-p)q^l - c^h}{2d(1+p)} & \text{if } M - d(1-p)q^l - c^h > 0 \\ 0 & \text{if } c^h \geq M - d(1-p)q^l > 0 \text{ or } M - d(1-p)q^l = 0 \end{cases}$$

Similarly, for the low cost firm

$$\max_{q^l} \quad \pi_i(q^l, \gamma_j | t_i = c^l) = \max\{M - d[q^l + pq^h + (1-p)q^l], 0\}q^l - c^l q^l \text{ s.t. } q^l \geq 0, \qquad \forall i \in N$$

$$\mathcal{L}^l = (M - d[q^l + pq^h + (1-p)q^l])q^l - c^l q^l + \nu(M - d[q^l + pq^h + (1-p)q^l]) + \phi q^l$$

$$M - dpq^h - c^l - \nu d(2-p) + \phi = 2d(2-p)q^l$$

$$\min\{\nu, M - dpq^h - d(2-p)q^l\} = 0$$

$$\min\{\phi, q^l\} = 0$$

Case I: $M - dpq^h - d(2-p)q^l > 0 \Rightarrow \nu = 0$ and $q^l > 0 \Rightarrow \phi = 0$

$$M - dpq^h - c^l = 2d(2-p)q^l$$

$$q^l = \frac{M - dpq^h - c^l}{2d(2-p)} \text{ if } M - dpq^h - c^l > 0$$

Case II: $M - dpq^h > 0 \Rightarrow \nu = 0$ and $q^l = 0 \Rightarrow \phi \geq 0$

$$\begin{cases} M - dpq^h - c^l + \phi = 0 \\ q^l = 0 \end{cases} \quad \begin{cases} \phi = -(M - dpq^h - c^l) \\ q^l = 0 \end{cases} \quad \text{if } M - dpq^h \leq c^l$$

Case III: $M - dpq^h - d(2-p)q^l = 0 \Rightarrow \nu \geq 0$ and $q^l > 0 \Rightarrow \phi = 0$

$$\begin{cases} M - dpq^h - c^l - \nu d(2-p) = 2d(2-p)q^l \\ M - dpq^h - d(2-p)q^l = 0 \end{cases} \quad \begin{cases} -c^l - \nu d(2-p) = M - dpq^h \\ q^l = \frac{M - dpq^h}{d(2-p)} \end{cases} \quad \begin{cases} \nu = -\frac{c^l + M - dpq^h}{d(2-p)} < 0 \Rightarrow \lightning \\ q^l = \frac{M - dpq^h}{d(2-p)} \end{cases}$$

Case IV: $M - dpq^h = 0 \Rightarrow \nu \geq 0$ and $q^l = 0 \Rightarrow \phi \geq 0$

$$\begin{cases} -c^l - \nu d(2-p) + \phi = 0 \\ M - dpq^h = 0 \\ q^l = 0 \end{cases} \quad \begin{cases} \phi - \nu d(2-p) = c^l \\ M - dpq^h = 0 \\ q^l = 0 \end{cases}$$

Hence, the best response correspondence of the low cost firm is

$$\rho_l(q^h) = \begin{cases} \frac{M - dpq^h - c^l}{2d(2-p)} & \text{if } M - dpq^h - c^l > 0 \\ 0 & \text{if } c^l \geq M - dpq^h > 0 \text{ or } M - dpq^h = 0 \end{cases}$$

So, equilibrium strategy is found at the intersection of the two best responses. Suppose, first, that optimal strategies are interior solutions:

$$q^h = \frac{M - c^h}{2d(1+p)} - \frac{(1-p)}{2(1+p)} \left(\frac{M - c^l}{2d(2-p)} - \frac{pq^h}{2(2-p)} \right)^{[1]} \Rightarrow$$

$$\gamma^*(c^h) = \frac{(3-p)M - 2(2-p)c^h + (1-p)c^l}{d(8 + 3p - 3p^2)} \Rightarrow \gamma^*(c^l) = \frac{(2+p)M + pc^h - 2(1+p)c^l}{d(8 + 3p - 3p^2)}$$

For these to be optimal strategies the following condition needs to hold:

$$c^h < \frac{(3-p)M + (1-p)c^l}{2(2-p)} \tag{1}$$

Notice that $\gamma^*(c^l) > 0$ because $(2+p)M + pc^h - 2(1+p)c^l = (2+p)\underbrace{(M - c^l)}_{>0} + p\underbrace{(c^h - c^l)}_{>0} > 0$.

Suppose there exists an equilibrium such that $q^h = 0, q^l > 0$. From the best response correspondence of

[1] I hope you don't mind that I'm skipping the tedious calculation steps here. If not, I am sorry. I was too lazy to type it all up.

low-type firm, we infer that $q^l = \frac{M-c^l}{2d(2-p)}$. Given this strategy, optimal $q^h = 0$ if[2]

$$c^h \geq \frac{(3-p)M + (1-p)c^l}{2(2-p)} \tag{2}$$

Hence, whenever (2) holds, the equiliubrium strategies are $\gamma^*(c^l) = \frac{M-c^l}{2d(2-p)}, \gamma^*(c^h) = 0$.
Suppose there is another equilibrium such that $q^h > 0, q^l = 0$. From the best response of high-cost firm $q^h = \frac{M-c^h}{2d(1+p)}$. Then, $M - p\frac{M-c^h}{2(1+p)} - c^l = \frac{(2+p)(M-c^l)+p(c^h-c^l)}{2(1+p)} > 0 \Rightarrow q^{l*} = \frac{\frac{(2+p)(M-c^l)+p(c^h-c^l)}{2(1+p)}}{2d(2-p)}$ ↯
Suppose there is an equilibrium such that both $q^h = q^l = 0$. But the best response of the low-cost firm is to set $q^l = \frac{M-c^l}{2d(2-p)} > 0$ ↯
Therefore, there are two Bayesian equilibria:

$$\begin{cases} \gamma^*(c^h) &= \frac{(3-p)M-2(2-p)c^h+(1-p)c^l}{d(8+3p-3p^2)} \\ \gamma^*(c^l) &= \frac{(2+p)M+pc^h-2(1+p)c^l}{d(8+3p-3p^2)} \end{cases} \text{if } c^h < \frac{(3-p)M+(1-p)c^l}{2(2-p)}$$

$$\begin{cases} \gamma^*(c^h) &= 0 \\ \gamma^*(c^l) &= \frac{M-c^l}{2d(2-p)} \end{cases} \text{if } c^h \geq \frac{(3-p)M+(1-p)c^l}{2(2-p)}$$

Dependence on p

Exercise 5

The game could be described as a Bayesian game $BG = \{N, T, A, P, \{\pi_i\}_{i \in N}\}$, where

$N = \{1, 2\}$;

$T = V^2 = \{v_0, v_1\}^2$;

$A = \{v_0, v_1, \frac{v_0+v_1}{2}\}^2$;

$P : T \to [0, 1], \qquad (v, v') \mapsto \frac{1}{4}, \forall (v, v') \in V^2$;

$\pi_i : T \times A \to \mathbb{R}$ specified in the Table 2.

$v_0 \backslash v_0$	v_0	v_1	$\frac{v_0+v_1}{2}$
v_0	$(0,\ 0)$	$(0,\ v_0 - v_1)$	$(0,\ \frac{v_0-v_1}{2})$
v_1	$(v_0 - v_1,\ 0)$	$(\frac{v_0-v_1}{2},\ \frac{v_0-v_1}{2})$	$(v_0 - v_1,\ 0)$
$\frac{v_0+v_1}{2}$	$(\frac{v_0-v_1}{2},\ 0)$	$(0,\ v_0 - v_1)$	$(\frac{v_0-v_1}{4},\ \frac{v_0-v_1}{4})$

$v_0 \backslash v_1$	v_0	v_1	$\frac{v_0+v_1}{2}$
v_0	$(0,\ \frac{v_1-v_0}{2})$	$(0,\ 0)$	$(0,\ \frac{v_1-v_0}{2})$
v_1	$(v_0 - v_1,\ 0)$	$(\frac{v_0-v_1}{2},\ 0)$	$(v_0 - v_1,\ 0)$
$\frac{v_0+v_1}{2}$	$(\frac{v_0-v_1}{2},\ 0)$	$(0,\ 0)$	$(\frac{v_0-v_1}{4},\ \frac{v_1-v_0}{4})$

$v_1 \backslash v_0$	v_0	v_1	$\frac{v_0+v_1}{2}$
v_0	$(\frac{v_1-v_0}{2},\ 0)$	$(0,\ v_0 - v_1)$	$(0,\ \frac{v_0-v_1}{2})$
v_1	$(0,\ 0)$	$(0,\ \frac{v_0-v_1}{2})$	$(0,\ 0)$
$\frac{v_0+v_1}{2}$	$(\frac{v_1-v_0}{2},\ 0)$	$(0,\ v_0 - v_1)$	$(\frac{v_1-v_0}{4},\ \frac{v_0-v_1}{4})$

$v_1 \backslash v_1$	v_0	v_1	$\frac{v_0+v_1}{2}$
v_0	$(\frac{v_1-v_0}{2},\ \frac{v_1-v_0}{2})$	$(0,\ 0)$	$(0,\ \frac{v_1-v_0}{2})$
v_1	$(0,\ 0)$	$(0,\ 0)$	$(0,\ 0)$
$\frac{v_0+v_1}{2}$	$(\frac{v_1-v_0}{2},\ 0)$	$(0,\ 0)$	$(\frac{v_1-v_0}{4},\ \frac{v_1-v_0}{4})$

Table 2: Payoff table

Notice again that the definition of the game assumes symmetric solution, i.e., it only depends on the type of the buyer, not an individual buyer per se. Define $\forall i \in N$ strategy as $\gamma_i : T_i \to \{v_0, v_1, \frac{v_0+v_1}{2}\}$. Hence, a strategy profile is $\gamma = (\gamma(v_0), \gamma(v_1))$. Note that $\gamma^*(v_0) = v_0$ is strictly dominant strategy with a payoff of zero regardless of the types of the other player; otherwise, get negative payoff.

[2]Strictly speaking, there's another condition. But it holds regardless of the values of the model: $M - (1-p)\frac{M-c^l}{2(2-p)} = \frac{(3-p)M+(1-p)c^l}{2(2-p)} > 0$

Now, suppose $\gamma^*(v_1) = v_0$. Then, $\forall i \in N$ and $j \neq i$

$$\begin{aligned}
\mathbb{E}\pi_i(v_0|t_i = v_1) &= P(t_j = v_0|t_i = v_1)\pi_i(t_j = v_0, v_0, \gamma^*(v_0)|t_i = v_1) + P(t_j = v_1|t_i = v_1)\pi_i(t_j = v_1, v_0, \gamma^*(v_1)|t_i = v_1) \\
&= \frac{1}{2}\frac{v_1 - v_0}{2} + \frac{1}{2}\frac{v_1 - v_0}{2} = \frac{v_1 - v_0}{2} \\
\mathbb{E}\pi_i(\tfrac{v_0+v_1}{2}|t_i = v_1) &= P(t_j = v_0|t_i = v_1)\pi_i(t_j = v_0, \tfrac{v_0+v_1}{2}, \gamma^*(v_0)|t_i = v_1) \\
&+ P(t_j = v_1|t_i = v_1)\pi_i(t_j = v_1, \tfrac{v_0+v_1}{2}, \gamma^*(v_1)|t_i = v_1) = \frac{1}{2}\frac{v_1 - v_0}{2} + \frac{1}{2}\frac{v_1 - v_0}{2} = \frac{v_1 - v_0}{2} \\
\mathbb{E}\pi_i(v_1|t_i = v_1) &= P(t_j = v_0|t_i = v_1)\pi_i(t_j = v_0, v_1, \gamma^*(v_0)|t_i = v_1) + P(t_j = v_1|t_i = v_1)\pi_i(t_j = v_1, v_1, \gamma^*(v_1)|t_i = v_1) = 0
\end{aligned}$$

Hence, indeed $\gamma^*(v_1) = v_0$ is part of the BNE.
Now, suppose $\gamma^*(v_1) = \frac{v_0+v_1}{2}$. Then, $\forall i \in N$ and $j \neq i$

$$\begin{aligned}
\mathbb{E}\pi_i(v_0|t_i = v_1) &= P(t_j = v_0|t_i = v_1)\pi_i(t_j = v_0, v_0, \gamma^*(v_0)|t_i = v_1) + P(t_j = v_1|t_i = v_1)\pi_i(t_j = v_1, v_0, \gamma^*(v_1)|t_i = v_1) \\
&= \frac{1}{2}\frac{v_1 - v_0}{2} = \frac{v_1 - v_0}{4} \\
\mathbb{E}\pi_i(\tfrac{v_0+v_1}{2}|t_i = v_1) &= P(t_j = v_0|t_i = v_1)\pi_i(t_j = v_0, \tfrac{v_0+v_1}{2}, \gamma^*(v_0)|t_i = v_1) \\
&+ P(t_j = v_1|t_i = v_1)\pi_i(t_j = v_1, \tfrac{v_0+v_1}{2}, \gamma^*(v_1)|t_i = v_1) = \frac{1}{2}\frac{v_1 - v_0}{2} + \frac{1}{2}\frac{v_1 - v_0}{4} = \frac{3(v_1 - v_0)}{8} \\
\mathbb{E}\pi_i(v_1|t_i = v_1) &= P(t_j = v_0|t_i = v_1)\pi_i(t_j = v_0, v_1, \gamma^*(v_0)|t_i = v_1) + P(t_j = v_1|t_i = v_1)\pi_i(t_j = v_1, v_1, \gamma^*(v_1)|t_i = v_1) = 0
\end{aligned}$$

Thus, $\gamma^*(v_1) = \frac{v_0+v_1}{2}$ is also part of the BNE. More precisely, the set of BNE strategy profiles could be defined as $\{(v_0, v_0), (v_0, \frac{v_0+v_1}{2})\}$.

Exercise 6