On the Secure Compilation of the Constant-Time Policy

Quantitative Information Flow

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Side Channels

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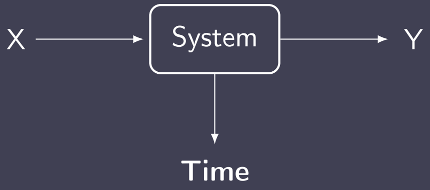

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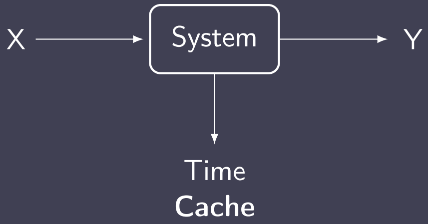

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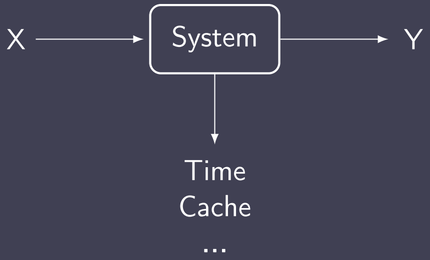

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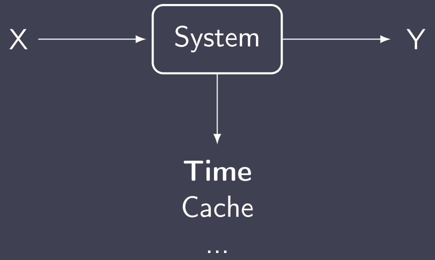

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check' : Vect n Bit -> Vect n Bit -> Result
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check' (a :: g) (b :: pw) = ctsel (r && r') Accept Reject
    where r  = check' g pw == Accept
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- Function ctsel stands for constant-time selector
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- **How to prove that the constant-time property is preserved?**

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- A program P is composed by a sequence of commands
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$$a \xrightarrow{t}{}^{n} a',$$

where a and a' are states, t is the leakage and n is the number of steps

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$$a \xrightarrow{t}{}^{n} b \wedge a' \xrightarrow{t'}{}^{n} b' \wedge a \,\phi\, a' \implies t = t' \wedge (b \in \mathcal{S}_f \iff b' \in \mathcal{S}_f).$$

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3. $\forall$ source and target states b and β such that $b \approx \beta$, we have that b is a final source state iff β is a final target state

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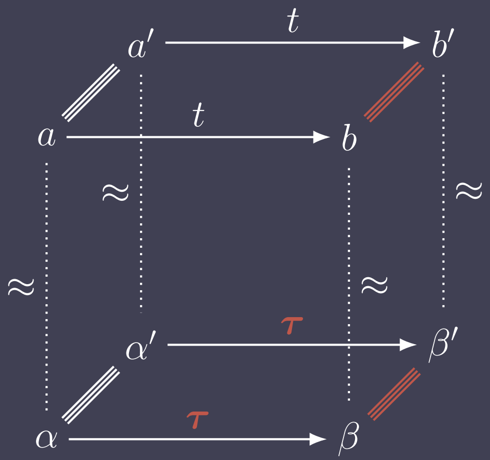

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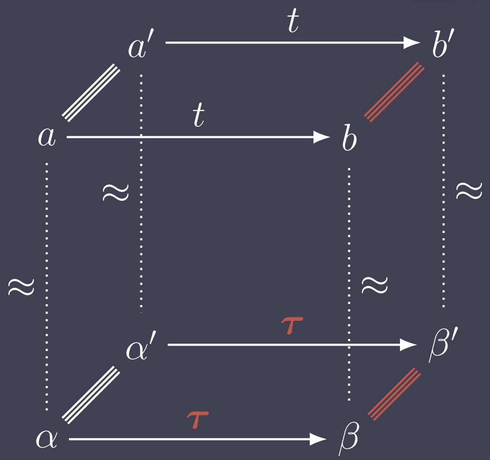

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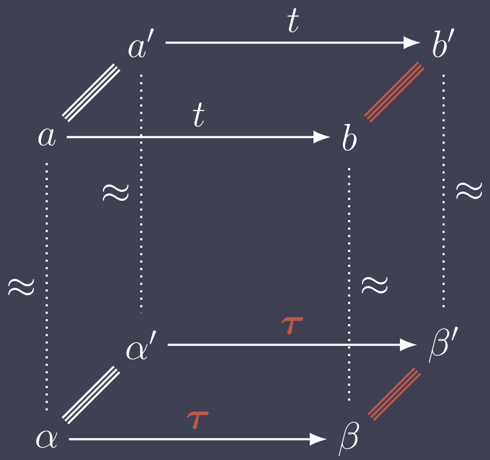
- Let a.cmd and a'.cmd be $y := A[i] * k$



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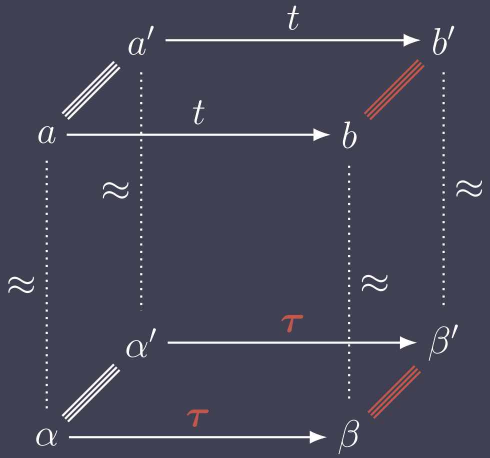

Secure Compilation

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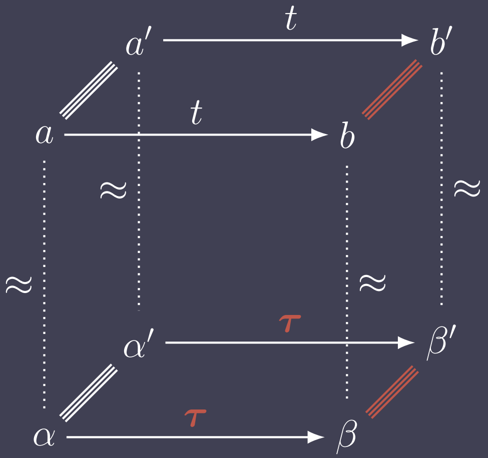

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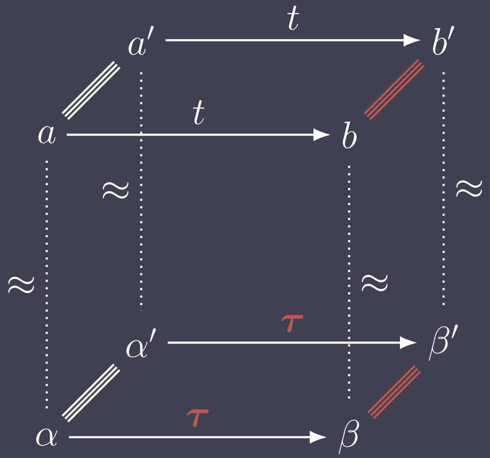

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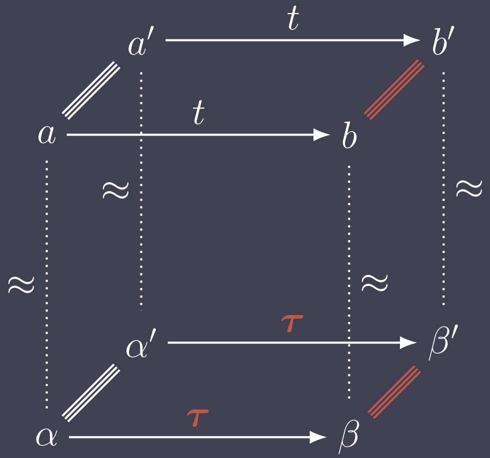

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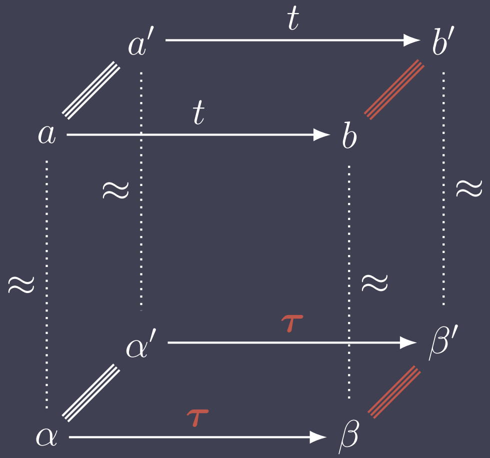
- $t = t' \cdot (A, [i]_{\rho_a})$ and $[i]_{\rho_a} = [i]_{\rho_{a'}}$



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- Then $\alpha.\mathrm{cmd}$ $\alpha'.\mathrm{cmd}$ are $y := 0$
- Similarly, $\beta.\mathrm{cmd}$ and $\beta'.\mathrm{cmd}$ are $z := x$
- $t = t' \cdot (A, [i]_{\rho_a})$ and $[i]_{\rho_a} = [i]_{\rho_{a'}}$
- What is the leakage τ and is it the same in both steps?

$$
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a \arrow[ur, equal] \arrow[rr, "t"] \arrow[dd, dotted, no head, "\approx"'] & & b \arrow[ur, equal] \arrow[dd, dotted, no head, "\approx"] & \\
 & \alpha' \arrow[rr, "\tau"] & & \beta' \\
\alpha \arrow[ur, equal] \arrow[rr, "\tau"] & & \beta \arrow[ur, equal] &
\end{tikzcd}
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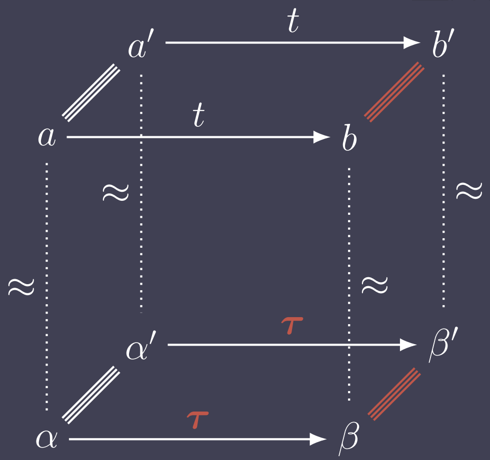
- $\tau = \tau'$



Relation to QIF

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- Labelled transitions
- Information-theoretic channels

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- Labelled transitions
- **Leakage as a trace of events**

- Information-theoretic channels
- **Leakage as a real number**

Relation to QIF

- Labelled transitions
- Leakage as a trace of events
- **Constant-time simulation**

- Information-theoretic channels
- Leakage as a real number
- **Refinement**

References

- Alvim, Mário S et al. (2020). *The Science of Quantitative Information Flow*. Springer.
- Barthe, Gilles, Benjamin Grégoire, and Vincent Laporte (2018). "Secure Compilation of Side-Channel Countermeasures: The Case of Cryptographic "Constant-Time"". In: *2018 IEEE 31st Computer Security Foundations Symposium (CSF)*, pp. 328–343. DOI: 10.1109/CSF.2018.00031.