

1 Introduction

We work on a hyper-cubic lattice ($D = 3, 4$ dimensions) with $N = L^3$ sites. We study the Euclidean scalar model defined by the following action

$$S = \sum_{n=1}^N \left[\frac{1}{2} B_4 (\square \phi_n)^2 + \frac{1}{2} B_2 \sum_{\mu=1}^D (\partial_\mu \phi_n) (\partial_\mu \phi_n) + \frac{1}{2} r \phi_n^2 + \frac{1}{4} \lambda \phi_n^4 \right] \quad (1)$$

The field index n is an integer valued array with D components, and the discretized derivatives are defined as follows

$$\partial_\mu \phi_n \equiv \phi_{n+\mu} - \phi_n \quad (2)$$

$$\square \phi_n \equiv \sum_{\mu} (\phi_{n+\mu} + \phi_{n-\mu} - 2\phi_n) \quad (3)$$

It is understood that all introduced variables and operators are dimensionless versions of their continuum counterparts, which are obtained by multiplying them by appropriate powers of the lattice spacing a , which does not appear explicitly. The B_2 parameter is dimensionless, while the dimensionful counterpart of the B_4 parameter has the dimensions of a^2 , or an inverse square mass.

We will also consider the interaction term with an external current:

$$S_J = \sum_{n=1}^N J_n \phi_n \quad (4)$$

where the current is in general not uniform. The Euclidean path integral involves the partition function

$$Z = \int [d\phi] \exp(-S[\phi]) \quad (5)$$

where the measure is defined by

$$[d\phi] \equiv \prod_{n=1}^N d\phi_n \quad (6)$$

The ground state expectation values of operators are then defined as

$$\langle A(\phi) \rangle \equiv \frac{1}{Z} \int [d\phi] A(\phi) \exp(-S[\phi]) \quad (7)$$

We are interested in studying the case

$$B_4 > 0, B_2 = -1 \quad (8)$$

which is supposed to give rise to a non-uniform field configuration for the ground state. In particular, we are interested in the so-called kinetic condensate

$$K(B_4, B_2, \lambda) \equiv \frac{1}{N} \sum_{n, \mu} \langle (\partial_\mu \phi_n)(\partial_\mu \phi_n) \rangle \quad (9)$$

or, rather, the quantity obtained by subtracting vacuum fluctuations of the field:

$$K_S(B_4, B_2, \lambda) \equiv K(B_4, B_2, \lambda) - K(0, 1, 0) \quad (10)$$

2 Vacuum fluctuations

In order to evaluate the vacuum field fluctuations of the field on the lattice, we consider the following partition function

$$Z(\alpha) \equiv \int [d\phi] \exp(-\alpha S_0[\phi]) \quad (11)$$

S_0 being the standard kinetic term of the action, implying that

$$K(0, 1, 0) = \frac{2}{N} \langle S_0 \rangle \quad (12)$$

where the expectation value is calculated with $S = S_0$ (i.e. $B_4 = 0, B_2 = 1, \lambda = 0$) at $\alpha = 1$. We have that

$$\langle S_0 \rangle = -\frac{\partial}{\partial \alpha} \ln Z(\alpha) \Big|_{\alpha=1} \quad (13)$$

Since S_0 is quadratic in the fields, we can define

$$\chi_n^2 \equiv \alpha \phi_n^2 \quad (14)$$

which gives

$$[d\phi] = \alpha^{-N/2} [d\chi], Z(\alpha) = \alpha^{-N/2} Z(1) \quad (15)$$

from which it follows that

$$K(0, 1, 0) = 1 \quad (16)$$

$$K_S(B_4, B_2, \lambda) = K(B_4, B_2, \lambda) - 1 \quad (17)$$

3 Simulation

We are employing Monte Carlo to evaluate path integral averages as averages over a set of configurations generated with the $\exp(-S)$ distribution. We use the Hybrid Monte Carlo (HMC) approach.

As a check for the algorithm, we use the Creutz criterion, i.e. we check that for the variations of the HMC Hamiltonian the following holds

$$\langle \exp(-\delta H) \rangle = 1 \tag{18}$$