

每日一题(10.2)

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1. 已知 x, y, z 为实数, 且满足

$$x + 2y - 5z = 3, x - 2y - z = -5,$$

求 $x^2 + y^2 + z^2$ 的最小值.

(门宇翎供题)

2. 因式分解:

$$(x^2 + 2y^2)^4 + 64y^8.$$

(程昊一命题)

$$\begin{aligned} & \frac{a^4 + 4b^4}{=} = (a^4 + 4b^4 + 4a^2b^2) - 4a^2b^2 \quad \left| \begin{array}{l} \text{令 } a = x^2 + y^2, b = y^2 \\ \text{原式} \end{array} \right. \\ & = (a^2 + 2b^2)^2 - (2ab)^2 \quad \left| \begin{array}{l} = a^4 + 64b^4 \\ = a^4 + 64b^4 + 16a^2b^2 - 16a^2b^2 \end{array} \right. \\ & = (a^2 - 2ab + 2b^2)(a^2 + 2ab + 2b^2) \quad \left| \right. \\ & \hline & = (a^2 + 8b^2)^2 - (4ab)^2 \\ & = (a^2 + 8b^2 - 4ab)(a^2 + 8b^2 + 4ab) \\ & = [(x^2 + 2y^2)^2 + 8y^4 - 4(x^2 + y^2)(y^2)] [\dots] \end{aligned}$$