二次函数的有关计算

2021 年 9 月 24 日

已知:二次函数$y=ax^2+bx+c$过三点$(x_1,y_1),(x_2,y_2),(x_3,y_3)$,求该函数的解析式.
解:将点$(x_1,y_1),(x_2,y_2),(x_3,y_3)$代入解析式,得

$$\begin{cases} y_1=ax_1^2+bx_1+c & (1) \\ y_2=ax_2^2+bx_2+c & (2) \\ y_3=ax_3^2+bx_3+c & (3) \end{cases}$$

$(1)-(2),(2)-(3)$,得

$$\begin{cases} y_1-y_2=a(x_1^2-x_2^2)+b(x_1-x_2) & (4) \\ y_2-y_3=a(x_2^2-x_3^2)+b(x_2-x_3) & (5) \end{cases}$$

$(4)\times(x_2-x_3)-(5)\times(x_1-x_2)$得

$$\begin{aligned} &(y_1-y_2)(x_2-x_3)-(y_2-y_3)(x_1-x_2) \\ =&a\times[(x_1^2-x_2^2)(x_2-x_3)-(x_2^2-x_3^2)(x_1-x_2)] \end{aligned}$$

即

$$a=\frac{(y_1-y_2)(x_2-x_3)-(y_2-y_3)(x_1-x_2)}{(x_1^2-x_2^2)(x_2-x_3)-(x_2^2-x_3^2)(x_1-x_2)}$$

$$=-\frac{\sum\limits_{cyc}x_1(y_2-y_3)}{\sum\limits_{cyc}x_1^2(y_2-y_3)}$$

$(4)\times(x_2^2-x_3^2)-(5)\times(x_1^2-x_2^2)$,得

$$\begin{aligned} &(y_1-y_2)(x_2^2-x_3^2)-(y_2-y_3)(x_1^2-x_2^2) \\ =&b\times[(x_1-x_2)(x_2^2-x_3^2)-(x_2-x_3)(x_1^2-x_2^2)] \end{aligned}$$

即

$$b=\frac{(y_1-y_2)(x_2^2-x_3^2)-(y_2-y_3)(x_1^2-x_2^2)}{(x_1-x_2)(x_2^2-x_3^2)-(x_2-x_3)(x_1^2-x_2^2)}$$

$$=\frac{\sum\limits_{cyc}x_1^2(y_2-y_3)}{\sum\limits_{cyc}x_1^2(x_2-x_3)}$$

$(1)\times x_2x_3,(2)\times x_3x_1,(3)\times x_1x_2$,得

$$\begin{cases}y_1x_2x_3=ax_1^2x_2x_3+bx_1x_2x_3+cx_2x_3 & (6)\\ y_2x_3x_1=ax_2^2x_3x_1+bx_2x_3x_1+cx_3x_1 & (7)\\ y_3x_1x_2=ax_3^2x_1x_2+bx_3x_1x_2+cx_1x_2 & (8)\end{cases}$$

$(6)-(7),(7)-(8)$,得

$$\begin{cases}x_3(y_1x_2-y_2x_1)=ax_1x_2x_3(x_1-x_2)+cx_3(x_2-x_1) & (9)\\ x_1(y_2x_3-y_3x_2)=ax_2x_3x_1(x_2-x_3)+cx_1(x_3-x_2) & (10)\end{cases}$$

$(9)\times(x_2-x_3)-(10)\times(x_1-x_2)$,得

$$x_3(x_2-x_3)(y_1x_2-y_2x_1)-x_1(x_1-x_2)(y_2x_3-y_3x_2)$$

$$=c\times[x_3(x_2-x_1)(x_2-x_3)-x_1(x_3-x_2)(x_1-x_2)]$$

即

$$c=\frac{x_3(x_2-x_3)(y_1x_2-y_2x_1)-x_1(x_1-x_2)(y_2x_3-y_3x_2)}{x_3(x_2-x_1)(x_2-x_3)-x_1(x_3-x_2)(x_1-x_2)}$$

$$=\frac{\sum\limits_{cyc}y_1x_2x_3(x_2-x_3)}{\sum\limits_{cyc}x_1x_2(x_1-x_2)}$$

所以,该二次函数的解析式为

$$y=-\frac{\sum\limits_{cyc}x_1(y_2-y_3)}{\sum\limits_{cyc}x_1^2(y_2-y_3)}x^2+\frac{\sum\limits_{cyc}x_1^2(y_2-y_3)}{\sum\limits_{cyc}x_1^2(x_2-x_3)}x+\frac{\sum\limits_{cyc}y_1x_2x_3(x_2-x_3)}{\sum\limits_{cyc}x_1x_2(x_1-x_2)}$$