常用导数与积分公式

1 常用导数与积分对照表

序号	导数	积分
1	$(c)' = 0$	$\int 0\mathrm{d}x = C$
2	$(x^\alpha)' = \alpha x^{\alpha-1} (\alpha \neq 0)$	$\int x^\alpha \mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1} + C$
3	$(a^x)' = \ln a \cdot a^x$	$\int a^x \mathrm{d}x = \frac{a^x}{\ln a} + C$
4	$(\sin x)' = \cos x$	$\int \cos x \mathrm{d}x = \sin x + C$
5	$(\cos x)' = -\sin x$	$\int \sin x \mathrm{d}x = -\cos x + C$
6	$(\tan x)' = \frac{1}{\cos^2 x} = \sec^2 x$	$\int \frac{1}{\cos^2 x} \mathrm{d}x = \tan x + C$
7	$(\cot x)' = \frac{1}{\sin^2 x}$	$\int \frac{1}{\sin^2 x} \mathrm{d}x = \cot x + C$
8	$(\ln \lvert x \rvert)' = \frac{1}{x}$	$\int \frac{1}{x} \mathrm{d}x = \ln \lvert x \rvert + C$
9	$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}} \mathrm{d}x = \arcsin x + C$
10	$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{1-x^2}} \mathrm{d}x = -\arccos x + C$
11	$(\arctan x)' = \frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} \mathrm{d}x = \arctan x + C$
12	$(\mathrm{arccot} x)' = -\frac{1}{1+x^2}$	$\int \frac{1}{1+x^2} \mathrm{d}x = -\mathrm{arccoy} x + C$
13	-	$\int \tan x \mathrm{d}x = -\ln \lvert \cos x \rvert + C$
14	-	$\int \cot x \mathrm{d}x = \ln \sin x + C$
15	-	$\int \frac{\mathrm{d}x}{x^2+a^2} = \frac{1}{a} \arctan \frac{x}{a} + C (a > 0)$
16	-	$\int \frac{\mathrm{d}x}{x^2-a^2} = \frac{1}{2a} \ln \left\lvert \frac{x-a}{x+a} \right\rvert + C (a > 0)$

17	-	$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$
18	-	$\int \frac{\mathrm{d}x}{\sqrt{x^2 \pm a^2}} = \ln\left\lvert x + \sqrt{x^2 \pm a^2}\right\rvert + C$

2 常用积分技巧

2.1 换元法

1. 第一换元法:

$$\int f(\varphi(x))\varphi'(x)\mathrm{d}x = \int f(\varphi(x))\mathrm{d}\varphi(x) = F(x) + C.$$

2. 第二换元法:

$$\int f(x)\mathrm{d}x = \int f(\varphi(t))\mathrm{d}\varphi(t) = F(\varphi^{-1}(x)) + C.$$

3. 分部积分法:

$$\int u(x)v'(x)\mathrm{d}x = u(x)v(x) - \int u'(x)v(x)\mathrm{d}x.$$