铁一中月考2物理多选最后一题的精确解. 设$v(t)$为速度与时间的依赖关系, 沿用题干中字母. 则答案有如下若干种情况:

对于实数x, 记

$$\operatorname{sign} x = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0. \end{cases}$$

记$\delta = \operatorname{sign}(mg - kv_0)$.

1. 若$F_0 > \mu mg$.

(1) $0 < v_0 < (\mu mg + F_0)/(\mu k)$. 则

$$v(t) = \frac{mg}{k} + \operatorname{sign}\left(\delta \ln\left(1 - \frac{\mu|kv_0 - mg|}{F_0}\right) + \frac{\mu k}{m}t\right) \cdot \frac{F_0}{\mu k}\left(1 - \exp\left(-\left|\delta \ln\left(1 - \frac{\mu|kv_0 - mg|}{F_0}\right) + \frac{\mu k}{m}t\right|\right)\right).$$

(2) $v_0 = (\mu mg + F_0)/(\mu k)$. 则$v(t) = v_0$.

(3) $v_0 > (\mu mg + F_0)/(\mu k)$. 则

$$v(t) = \frac{mg}{k} + \frac{F_0}{\mu k}\left(1 + \left(\frac{\mu k v_0 - \mu mg}{F_0} - 1\right)\exp\left(-\frac{\mu k}{m}t\right)\right).$$

2. 若$F_0 \le \mu mg$.

(1) $v_0 = (\mu mg \pm F_0)/(\mu k)$. 则$v = v_0$.

(2) $0 \le v_0 < (\mu mg - F_0)/(\mu k)$. 则

$$v(t) = \begin{cases} \dfrac{mg}{\mu} - \dfrac{F_0}{\mu k}\left(1 + \left(\dfrac{\mu(mg - kv_0)}{F_0} - 1\right)\exp\left(\dfrac{\mu k}{m}t\right)\right), & t < t_0 \\ 0, & t \ge t_0. \end{cases}$$

其中

$$t_0 = \frac{m}{\mu k}\ln\left(\frac{\mu mg - F_0}{\mu(mg - kv_0) - F_0}\right).$$

(3) $(\mu mg - F_0)/(\mu k) < v_0 < (\mu mg + F_0)/(\mu k)$. 则

$$v(t) = \frac{mg}{k} + \operatorname{sign}\left(\delta \ln\left(1 - \frac{\mu|kv_0 - mg|}{F_0}\right) + \frac{\mu k}{m}t\right) \cdot \frac{F_0}{\mu k}\left(1 - \exp\left(-\left|\delta \ln\left(1 - \frac{\mu|kv_0 - mg|}{F_0}\right) + \frac{\mu k}{m}t\right|\right)\right).$$

(4) $v_0 > (\mu mg + F_0)/(\mu k)$. 则

$$v(t) = \frac{mg}{k} + \frac{F_0}{\mu k}\left(1 + \left(\frac{\mu k v_0 - \mu mg}{F_0} - 1\right)\exp\left(-\frac{\mu k}{m}t\right)\right).$$