三 角 函 数 公 式

三角函数公式

1 同角三角函数的关系

1.平方关系:

$$\sin^2\alpha+\cos^2\alpha=1,\tan^2\alpha+1=\sec^2,\cot^2\alpha+1=\csc^2\alpha.$$

2.倒数关系:

$$\sin\alpha\cdot\csc\alpha=1,\cos\alpha\cdot\sec\alpha=1,\tan\alpha\cdot\cot\alpha=1.$$

3.商数关系:

$$\frac{\sin\alpha}{\cos\alpha}=\tan\alpha,\frac{\cos\alpha}{\sin\alpha}=\cot\alpha.$$

2 诱导公式

1.

$$\sin(k\pi+\alpha)=\pm\sin\alpha;\quad\cos(k\pi+\alpha)=\pm\cos\alpha;$$

$$\tan(k\pi+\alpha)=\pm\tan\alpha;\quad\cot(k\pi+\alpha)=\pm\cot\alpha.$$

2.

$$\sin\left(\left(k+\frac{1}{2}\right)\pi+\alpha\right)=\pm\cos\alpha;\quad\cos\left(\left(k+\frac{1}{2}\right)\pi+\alpha\right)=\pm\sin\alpha;$$

$$\tan\left(\left(k+\frac{1}{2}\right)\pi+\alpha\right)=\pm\cot\alpha;\quad\cot\left(\left(k+\frac{1}{2}\right)\pi+\alpha\right)=\pm\tan\alpha;$$

口诀:奇变偶不变,符号看象限.

3 和差倍角公式

1.

$$\sin(\alpha\pm\beta)=\sin\alpha\cos\beta\pm\sin\beta\cos\alpha;\quad\cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\sin\alpha\sin\beta;$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}.$$

2.

$$\sin 2\alpha = 2\sin\alpha\cos\alpha; \quad \cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha;$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha}.$$

4 和(差)积互化公式

1.和差化积:

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}; \quad \sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2};$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}; \quad \cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}.$$

2.积化和差:

$$\sin\alpha\sin\beta = -\frac{1}{2}[\cos(\alpha+\beta) - \cos(\alpha-\beta)]; \quad \cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)].$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]; \quad \cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)].$$

5 万能置换公式

$$\sin\alpha = \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}; \quad \cos\alpha = \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}}; \quad \tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}.$$

6 三角形中的公式

1.正弦定理,余弦定理,射影定理.

2.

$$\sum_{cyc}\sin A = 4\prod_{cyc}\cos\frac{A}{2};$$

$$\sum_{cyc} \cos A = 1 + 4\prod_{cyc} \sin\frac{A}{2};$$

$$\sum_{cyc} \tan A = \prod_{cyc} \tan A;$$

$$\sum_{cyc} \sin^2 A = 2 + 2\prod_{cyc} \cos A;$$

$$\sum_{cyc} \left(\tan\frac{A}{2}\tan\frac{B}{2}\right) = 1.$$

7 三角形中的不等式

$$\prod_{cyc} \cos A \leq \frac{1}{8};$$

$$1 < \sum_{cyc} \cos A \leq \frac{3}{2};$$

$$\sum_{cyc} \sin A \leq \frac{3\sqrt{3}}{2}.$$

8 其他

1.

$$(\sin\alpha \pm \cos\alpha)^2 = 1 \pm \sin 2\alpha.$$

2.

$$\frac{1+\tan\alpha}{1-\tan\alpha} = \frac{\sin\alpha + \cos\alpha}{\cos\alpha - \sin\alpha} = \tan\left(\alpha + \frac{\pi}{4}\right).(\text{定义域略})$$

3.

$$\tan\alpha + \cot\alpha = \frac{2}{\sin 2\alpha};$$

$$\tan\alpha - \cot\alpha = -2\cot\alpha.(\text{定义域略})$$

4.

$$\sin(\alpha+\beta)\sin(\alpha-\beta) = \sin^2\alpha - \sin^2\beta = \cos^2\beta - \cos^2\alpha;$$

$$\cos(\alpha+\beta)\cos(\alpha-\beta) = \cos^2\alpha - \sin^2\beta.$$

5.

$$\sin 3\alpha = 4\sin(60^\circ - \alpha)\sin\alpha\sin(60^\circ + \alpha);$$

$$\cos 3\alpha = 4\cos(60^\circ - \alpha)\cos\alpha\cos(60^\circ + \alpha);$$

$$\tan 3\alpha = \tan(60^\circ - \alpha)\tan\alpha\tan(60^\circ + \alpha);$$