Equation of State and the Maximum Mass of Neutron Stars

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The relationship between the maximum neutron-star mass and observable parameters of the equation of state is explored. In particular, the roles of the nuclear incompressibility and the symmetry energy are considered. It is concluded that, for realistic symmetry energies, the compression modulus cannot, by itself, be severely limited by observed neutron-star masses. Several directions for further study are suggested.

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The maximum neutron-star mass ($M_{\max}$), subject only to the constraints of causality and general relativity, has been shown[1] to be $\sim 3M_\odot$ ($M_\odot \simeq 2\times 10^{33}$ g). Observed neutron-star masses generally lie well below this value. The measured[2] mass of 4U0900-40, $(1.85\pm 0.3)M_\odot$, may provide the lower limit for the maximum mass of the neutron star. The most accurate determinations[3] are for the two components of the binary pulsar PSR 1913+16, $(1.444\pm 0.01)M_\odot$ and $(1.384\pm 0.01)M_\odot$. Can the observed masses limit the equation of state (EOS) of dense matter? Can the parameters of symmetric nuclear matter,[4] which have the potential of being measured in other contexts, be constrained? The answers hinge on whether or not the bulk of the matter in maximum-mass neutron stars lies close to symmetric-matter saturation density, $n_0 \simeq 0.16$ fm^{-3}. If the bulk lies at much higher values the limits to symmetric-matter parameters cannot be effectively constrained. Here we examine the dependence of neutron-star structure on the EOS.

The importance of this sensitivity has bearing on other issues as well. The EOS so obtained must be consistent with those inferred from the analysis of giant resonances in laboratory nuclei.[5] They must also correspond to what is obtained in the studies of particle multiplicities and the matter, momentum, and energy flows in heavy-ion collisions.[6] Finally, there may be a significant effect of the EOS on supernova simulations as the strength of a supernova shock is correlated[7,8] with the behavior of the EOS near n_0.

Many approaches exist to determine the EOS through the many-body theory of interacting hadrons. These can be conveniently grouped into three types: (i) nonrelativistic potential models,[9,10] in which two-body forces determined from phase-shifts are employed and often supplemented by three nucleon interactions; (ii) field-theoretical models[11] with and without scalar self-interactions in which all calculations are inherently relativistic; and (iii) hybrid models[12-14] of these other approaches. As far as neutron-star structure is concerned, the only relevant quantity is the energy per baryon $E(n)$ expressed as a function of the baryon density n. From $E(n)$, the neutron-star's radius, red shift, moment of inertia, and binding energy, as functions of mass, can be determined.[15] In this Letter we concentrate on the neutron-star's maximum mass.

Neutron stars are in equilibrium with respect to weak interactions. The proton concentrations $x=Z/A$ in such matter depend most sensitively on the symmetry energy. Defining $E(x=\frac{1}{2})$ as the energy per particle of symmetric nuclear matter, various studies[10,16-18] have revealed that an excellent approximation for the specific energy of neutron-rich matter is

$$E(n,x)=E(n,x=\tfrac{1}{2})+S(n)(1-2x)^2, \qquad (1)$$

where $S(n)\equiv E(n,x=0)-E(n,x=\frac{1}{2})$ is the nuclear symmetry energy. We assume that Eq. (1) is valid over the entire range $0<x<\frac{1}{2}$ and for all n. The total energy per particle is obtained by adding the lepton energy $E_L(x)$, and in β equilibrium, one has $\partial[E(n,x)+E_L(x)]/\partial x=0$. The density dependence of S is uncertain and will determine how x varies with n in a neutron-star's interior. Many nuclear EOS's in current use[16,19] have S increasing with density near n_0, but then saturating at a few times n_0, and finally decreasing. On the other hand, potential contributions to S vary as n in recent relativistic Brueckner[13,18] and field-theoretic calculations.[14,17,20,21] In general, the maximum mass of a β-equilibrium configuration will be less than that of either a pure neutron-matter or a symmetric-matter configuration. The possible presence of hyperons, which will alter Eq. (1), is discussed later.

In the context of a mean-field model with scalar self-interactions, Glendenning[20] has argued that observed neutron-star masses set a relatively large lower limit to the compression modulus K_0. In particular, from the one-standard-deviation lower limit to the mass of U0900-40, $1.55M_\odot$, he derives that $K_0 > 225$ MeV. He states that saturation properties provide a clear constraint on K_0 because more than half the neutron-star's matter lies at densities less than $3n_0$. Other approaches have yielded a variety of results. For example, Prakash and Ainsworth[17] find results consistent with Glendenning, using a linear σ model. But the relativistic Brueckner-Hartree-Fock calculations of Müther, Prakash, and Ainsworth[18] in which $K_0 \simeq 200$ MeV have maximum masses of the order of $2.4M_\odot$. More recently, Wiringa, Fiks, and Fabrocini[10] have calculated a series of potential models with K_0 about 200 MeV and maximum masses in excess of $2M_\odot$. It is true that the latter two equations of state become acausal, but only at densities beyond that encountered in the maximum-mass case. These results suggest that the behavior of the high-density EOS is more important than the saturation parameters in the determination of neutron-star structure.

To investigate this relationship we consider a family of simple parametrizations of $E(n)$ as the structure depends only on this function. The energy function is constrained to reproduce known nuclear properties and to be causal at all densities. K_0 will serve as an input parameter. The parametrizations we use have only a modest microscopic foundation; however, they have the merit of being able to closely approximate more physically motivated calculations. Our objective is to find if K_0 is a useful diagnostic of the overall stiffness of the EOS, which ultimately sets the neutron-star's maximum mass.

It is common to employ local contact interactions to model the nuclear potential. Such forces lead to power-law density-dependent terms in $E(n)$. Repulsive contributions to $E(n)$ that vary faster than linear give rise to acausal behavior at high densities. To avoid this, we write the interaction energy per particle of symmetric matter as[22] $(A/2)u + Bu^\sigma/(1+B'u^{\sigma-1})$, where $u = n/n_0$ and A, B, B', and σ are constants. We note that the implied self-screening of repulsive interactions, while desirable, is not always guaranteed in potential-model calculations. The total energy is obtained by inclusion of the nucleon kinetic energies and the effect of finite-range forces between nucleons. For static nuclear matter,[23] a term of the form

$$Cu\int d^3p \, \frac{(4/h^3)\theta(p_F - p)}{1+(p/\Lambda)^2} \tag{2}$$

in the potential-energy density closely approximates momentum-dependent interactions of more realistic nuclear-matter calculations. In Eq. (2), p_F is the Fermi momentum and Λ is a finite-range force parameter. We use two such terms: one corresponding to a long-range attraction and the other to a short-range repulsion. The total energy per particle of symmetric nuclear matter is then

$$E(n,x=\tfrac{1}{2}) = \tfrac{3}{5}E_F^{(0)}u^{2/3} + \tfrac{1}{2}Au + \frac{Bu^\sigma}{1+B'u^{\sigma-1}} + 3\sum_{i=1,2} C_i \left(\frac{\Lambda_i}{p_F^{(0)}}\right)^3 \left(\frac{p_F}{\Lambda_i} - \tan^{-1}\frac{p_F}{\Lambda_i}\right). \tag{3}$$

Here $E_F^{(0)}$ is the Fermi energy at saturation. Equation (3) differs from that of Ref. 23 only in the extra finite-range term (that with C_2) which is needed to cover a wider range of input K_0 values. Similar parametrizations are known to reproduce properties of finite nuclei[5] and flow observables in heavy-ion reactions.[6]

To separate the kinetic and potential contributions to the symmetry energy, we write

$$S = (2^{2/3}-1)\tfrac{3}{5}E_F^{(0)}[u^{2/3} - F(u)] + S_0F(u),$$

where $F(u)$ defines the potential contributions to the symmetry energy and $F(1)\equiv 1$. We shall explore some simple forms of $F(u)$ which reproduce the symmetry energy of more realistic microscopic calculations. The parameters A, B, σ, C_1, and C_2 are determined with the constraints provided by the properties[4] of nuclear matter at saturation. For $\sigma < 1$, we use $B' = 0$, but employ small positive values of B' when $\sigma > 1$ to keep the equation of state causal for all densities. The finite-range parameters were assumed to be $\Lambda_1 = 1.5p_F^{(0)}$ and $\Lambda_2 = 3p_F^{(0)}$, where $p_F^{(0)}$ is the Fermi momentum of nuclear matter, but it can be shown that the values of the other parameters are insensitive to this choice. The parameter values are shown in Table I for three possible values of K_0.

The properties of maximum-mass neutron stars, obtained by the integration of the Tolman-Oppenheimer-Volkov equation (cf. Ref. 15), are displayed in Table II for the parameters shown in Table I and $S_0 = 30$ MeV. The choices for $F(u)$ mimic the results of the potential and hybrid models referred to earlier. Results for the maximum mass of a star with pure neutrons are shown in parenthesis. For a given parametrization, the maximum mass roughly scales[24,25] as $K_0^{1/2}$, and the stiffer the EOS the less important the symmetry energy.[17] Howev-

TABLE I. EOS parameters [Eq. (3)] determined with $E_0 = -16$ MeV, $n_0 = 0.16$ fm^{-3}, $m^* = 0.7m$, $U(n_0, p=0) = -76.34$ MeV, $\Lambda_1 = 1.5p_F^{(0)}$, and $\Lambda_2 = 3p_F^{(0)}$ for various values of the compression modulus K_0. All energies in MeV.

K_0	A	B	B'	σ	C_1	C_2
120	75.94	−30.88	0	0.498	−83.84	23.0
180	440.94	−213.41	0	0.927	−83.84	23.0
240	−46.65	39.54	0.3	1.663	−83.84	23.0

TABLE II. The mass (M_{max}), radius (R), central density (n_c), moment of inertia (I), gravitational binding energy (BE), and surface red shift (ϕ) of the maximum-mass neutron star are listed for different choices of the potential contribution to the symmetry energy $F(u)$ and compression modulus K_0. Results for the maximum mass with pure neutrons are shown in parenthesis.

$F(u)$	K_0 (MeV)	$M_{max}/M_\odot$	R (km)	n_c/n_0	I ($M_\odot$ km^2)	BE (10^{53} ergs)	ϕ
u	120	1.458(1.70)	9.114	10.841	43.83	3.431	0.776
	180	1.722(1.90)	9.879	8.680	66.73	4.733	0.696
	240	1.935(2.07)	10.57	7.269	90.93	5.890	0.677
$\frac{2u^2}{1+u}$	120	1.470(1.95)	9.895	9.631	49.36	3.260	0.749
	180	1.738(2.10)	10.318	8.166	70.82	4.572	0.708
	240	1.952(2.24)	10.933	6.953	95.32	5.734	0.687
$\sqrt{u}$	120	1.404(1.45)	8.435	12.28	37.75	3.476	0.712
	180	1.679(1.71)	9.324	9.46	60.07	4.812	0.683
	240	1.895(1.92)	10.112	7.740	83.83	5.954	0.667

er, even in the case $K_0=120$ MeV, relatively large neutron stars (with $M_{max}\sim 1.5M_\odot$) are possible. In addition, the more rapidly rising is the symmetry energy, the stiffer is the overall EOS. Changing the parameter S_0 to 35 MeV has an almost negligible effect on β-equilibrium stars, although the maximum mass of a pure neutron star is somewhat increased.

Figure 1 shows the fraction of the gravitational-mass exterior to a given radius versus the density at that radius for the maximum-mass configurations. Independently of the EOS employed here, most of the mass lies at densities higher than $3n_0$. Only for very stiff EOS's is this finding modified. Therefore, it is mostly the high-density properties that play an important role[17,18,25] in neutron-star structure.

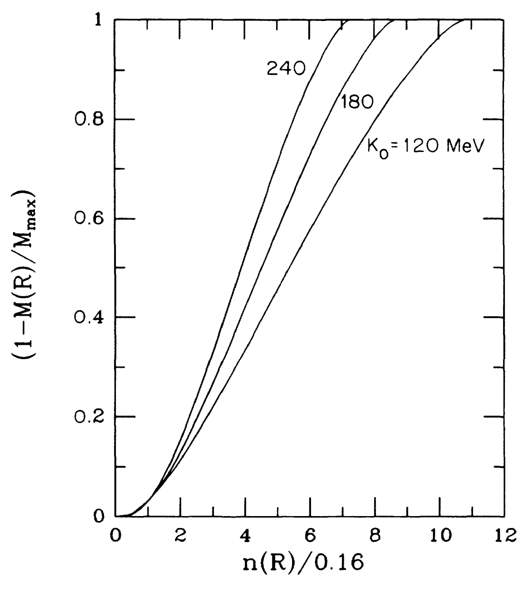

FIG. 1. The fraction of the total mass of maximum-mass neutron stars which lies at or below a given density. The potential contribution to the symmetry energy $F(u)=u$ is used, and each curve is labeled by the compression modulus K_0 in MeV.

We have demonstrated that observed neutron-star masses are consistent with K_0 smaller than 140 MeV, using a plausible EOS that is consistent with empirical nuclear matter properties and causality. Other equally plausible EOS's[17,20] have set a relatively larger lower limit to K_0, e.g., in Ref. 17 $M_{max}=(1.36,1.45,1.50)M_\odot$ were found for $K_0=(199,225,240)$ MeV and in Ref. 20 $M_{max}=(0.8,1.3,1.5,1.8)M_\odot$ were found for $K_0=(100, 200,240,285)$ MeV. In both these (field-theoretic) calculations the potential contribution to the symmetry energy varied linearly with density. The calculations of Ref. 20 included hyperons which substantially reduced M_{max}.[26] However, there are opposing points of view[20,27] regarding the importance and abundances of hyperons in dense neutron-star matter. The fact is almost nothing is known about hyperonic potentials from experiments. A further uncertainty, at the densities where hyperons might be abundant, is a possible transition to quark matter in which case the role of hyperons may be reduced. Our results imply that it is possible to construct an EOS at high densities, with acceptable saturation and causality properties, and including hyperons, which nevertheless has $M_{max}\gtrsim 1.5M_\odot$.

The above discussion shows that the relationship of M_{max} to K_0 is extremely model dependent and this is our main point. The compression modulus K_0 of symmetric matter by itself does not provide a good model-independent basis for contrasting the structure of neutron stars. This is because the high-density EOS is uncertain and there is no unique way to link the high-density EOS to properties around n_0. Evidently, star properties may not yet be firmly linked to equilibrium properties of symmetric nuclear matter.

Several directions for further study are suggested: (i) The symmetry energy, especially at high densities requires additional consideration. (ii) There is a need to carefully reexamine hyperonic interactions from both potential and field-theoretic approaches. (iii) Can observations of properties other than the mass constrain the EOS? The need for more detailed observations cannot be overemphasized. Further efforts in these directions may help to answer some open questions concerning the equation of state at high density.

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