Problem: Show that, if n is a positive integer, then

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Proof: The proof will be by induction. Let $P(n)$ be the statement

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$

Since $\sum_{i=1}^{1} i^2 = 1 = \frac{1\cdot 2\cdot 3}{6}$, $P(1)$ is true. Assume, by induction, that $P(k)$ is true for some $k > 0$. Then, by the associative law of addition and the inductive assumption,

$$\sum_{i=1}^{k+1} i^2 = \left(\sum_{i=1}^{k} i^2\right) + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2.$$

By finding a common denominator and simplifying,

$$\frac{k(k+1)(2k+1)}{6} + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + \frac{6(k+1)^2}{6} = \frac{(k+1)[(2k^2+k)+(6k+6)]}{6}$$

Since $(2k^2+k)+(6k+2) = 2k^2+7k+6 = (k+2)(2k+3) = [(k+1)+1][2(k+1)+1]$, we have that

$$\frac{k(k+1)(2k+1)}{6} + (k+1)^2 = \frac{(k+1)[(k+1)+1][2(k+1)+1]}{6}.$$

That is, $P(k+1)$ is true. Therefore, since $P(1)$ is true and for $k > 0$, the truth of $P(k)$ implies the truth of $P(k+1)$, the Principle of Mathematical Induction implies that $P(n)$ is true for all $n > 0$. Q.E.D.