The 4-Component Models for Slurry Pipeline Friction and Pump Solids Effect

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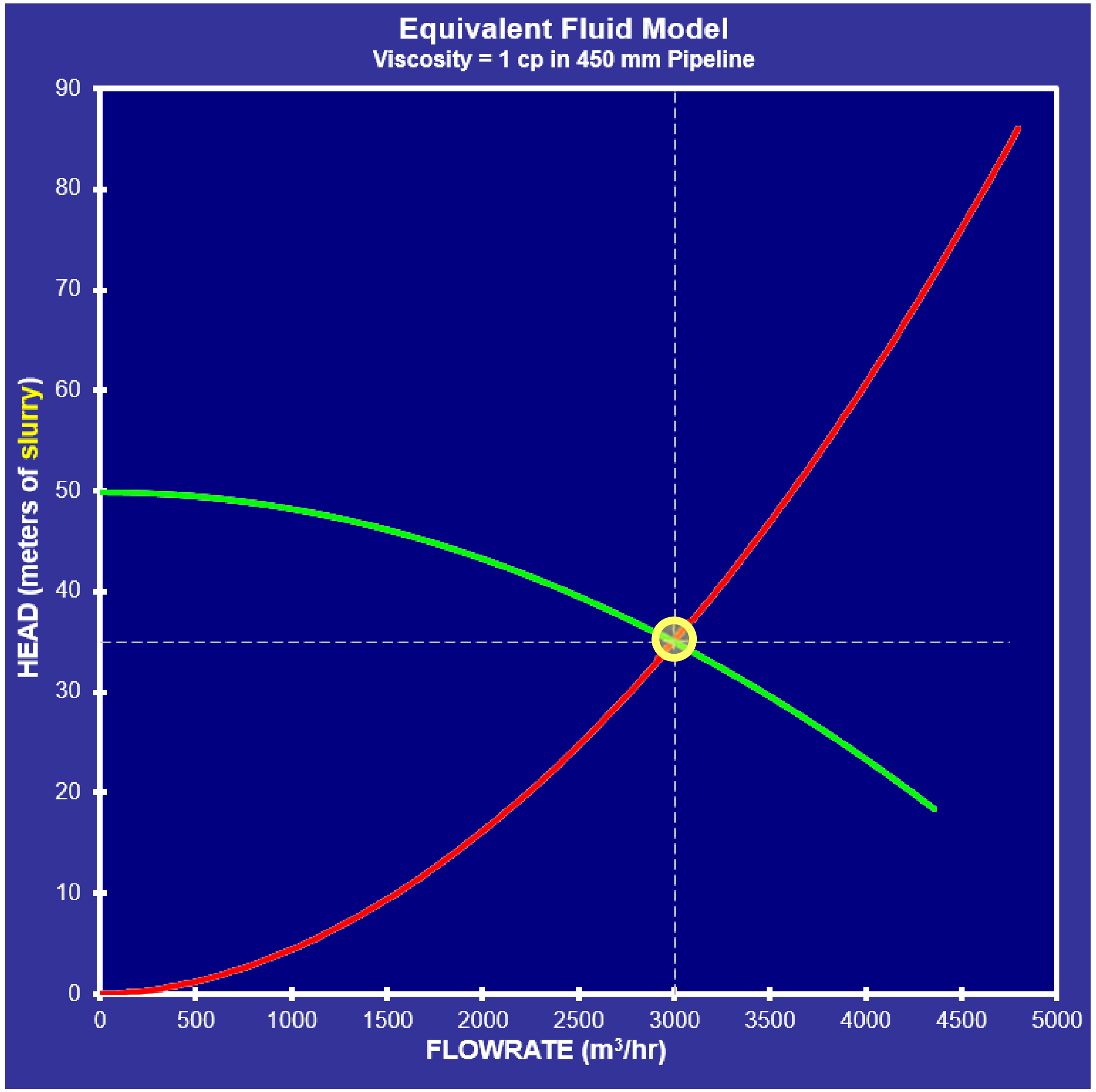
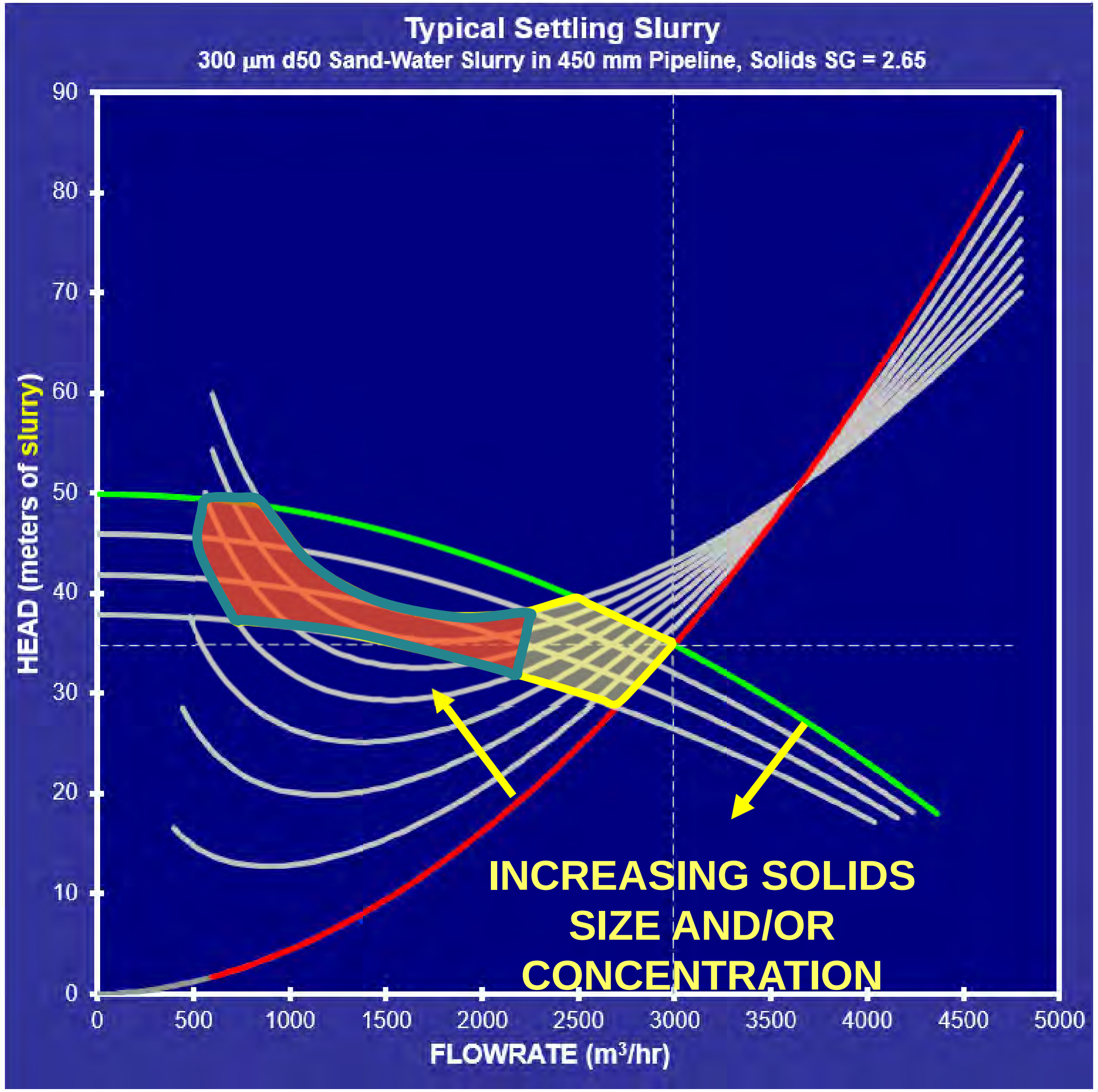
Why do settling slurry flow models matter?



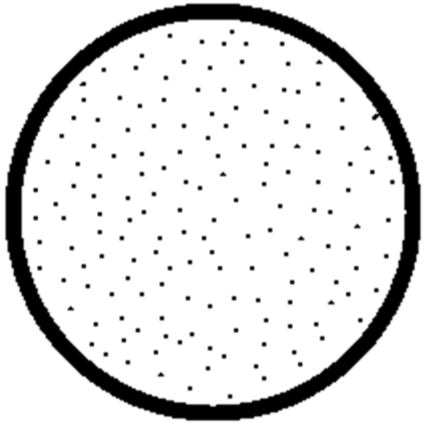
- When pumping simple liquids, centrifugal pumps interact with their piping systems in a pretty straightforward way.
- The piping system resistance curve (red) is generally upward sloping.
- The pump performance curve (green) is generally downward sloping.
- Their combined action leads to a stable operating condition at the point where the two curves intersect.
- In many cases, the stability of this condition can be maintained with little oversight.

Why do settling slurry flow models matter?



- When pumping slurries, additional variables must be considered, especially changes in the concentration and size of the solid particles.
- Both the system resistance and the pump performance may be strong functions of these changing properties.
- Such changes may occur on a regular and somewhat uncontrolled basis, due to natural changes in the source of solids and/or variability in the process itself.
- The range of possible intersections between pump and system curves may now span a large area.
- In some cases, certain combinations may be unstable or not even feasible, leading to unexpected swings in flowrate and the potential for system blockage.
- Accurate models are needed to predict both the pump and pipeline response to slurry properties, so these conditions may be analyzed, and safe operating procedures developed.

Idealized Settling Slurry Flow Regimes



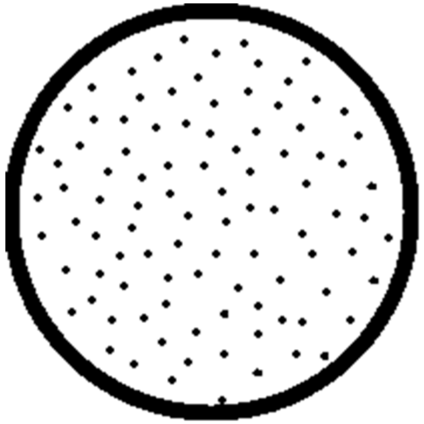
"Carrier Fluid"

particles < 40μm

Solids assumed to "combine" with the liquid.

Solids may affect the mixture viscosity.

A standard "pure-fluid" model may be used.



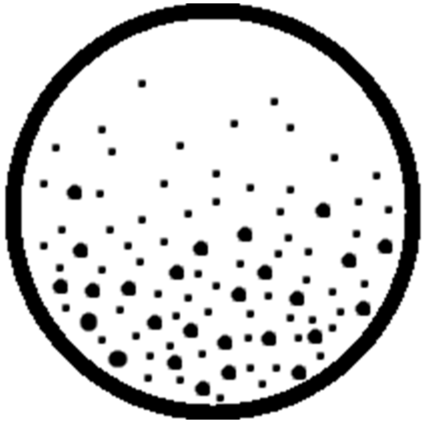
"Pseudo-homogeneous"

40 μm < particles < 200 μm

Solids fully supported by turbulence.

Solids do not affect the mixture viscosity.

Treated as an "equivalent fluid".



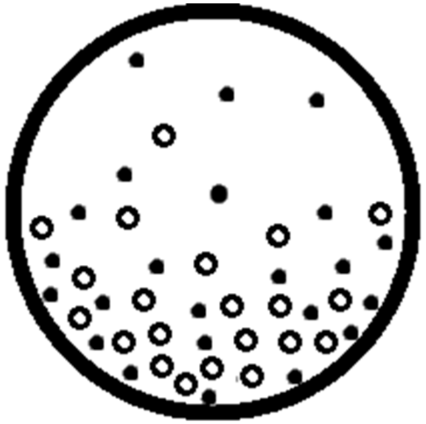
"Heterogeneous"

200 μm < particles < 0.015D

Solids supported by a combination of turbulence and mechanical contact with the pipe wall.

A partly-stratified model of slurry flow is required.

(D = pipe diameter)



"Fully Stratified"

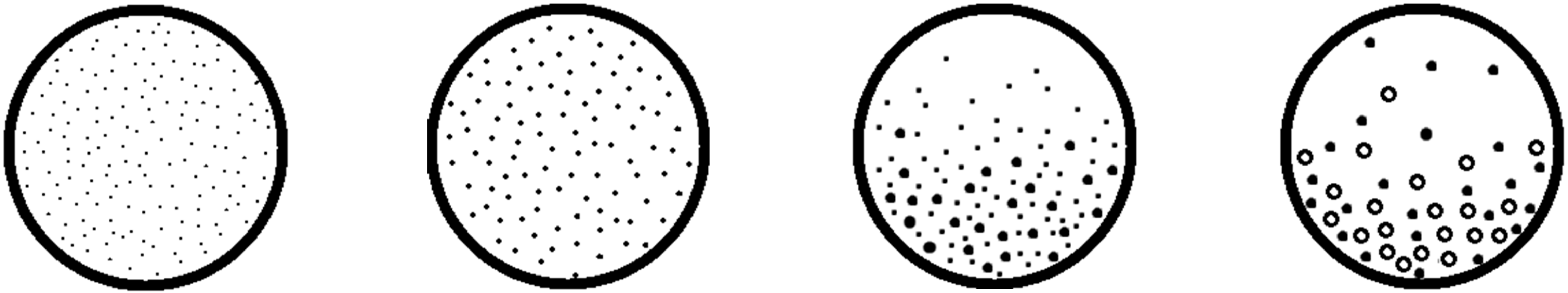
0.015D < particles

Solids supported mainly by mechanical contact with the pipe wall.

A sliding bed friction model of slurry flow is required.

(D = pipe diameter)

Idealized Settling Slurry Flow Regimes

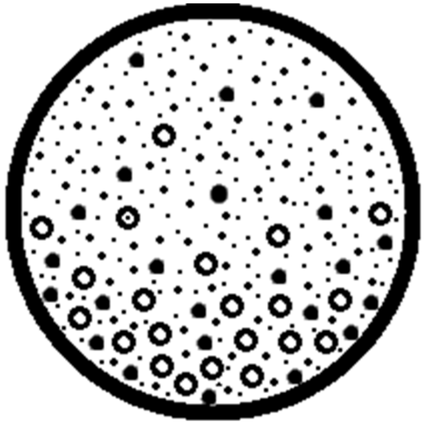


“Carrier Fluid”	“Pseudo-homogeneous”	“Heterogeneous”	“Fully Stratified”
particles < 40μm	40 μm < particles < 200 μm	200 μm < particles < 0.015D	0.015D < particles

Modelling challenges:

- Effective models exist for each regime, however, in combining them, each coarser fraction must consider the supporting effects of the finer fractions below it.
- Quality data across all regimes in industrial sized pipelines with accurate PSD sampling is scarce.

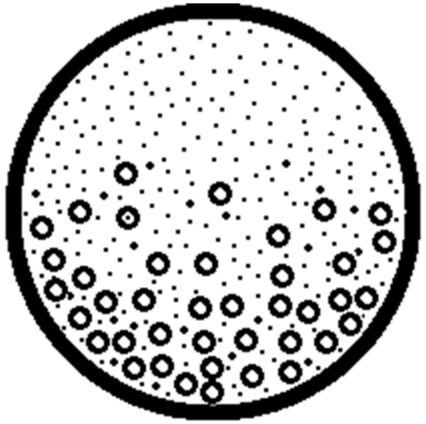
Practical Settling Slurry Flow Examples



"Broad size distribution"

Particles range from less than 40μm to greater than 0.015D

Often generated by mineral processing operations (crushing, grinding, etc.)



"Bi-modal size distribution"

Particles grouped into two different size ranges

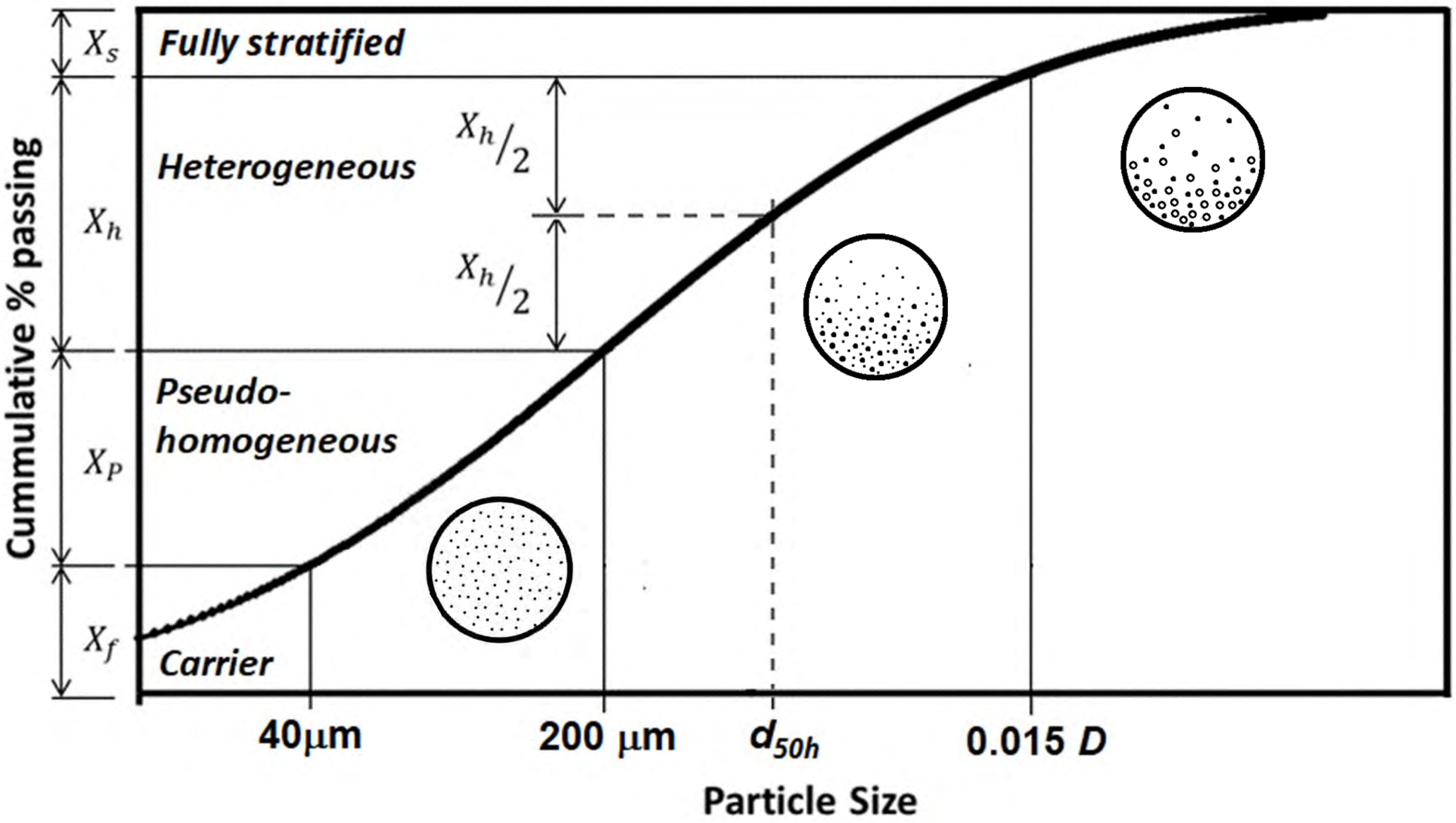
Often generated by hydrotransport operations (Oil Sands, Phosphate, Industrial minerals, Dredging)

- Of special interest are broad and bi-modal particle size distributions.
- These are not accurately described by any model developed for only one basic flow regime, or by models which describe the overall distribution of solids with only one or two average particle sizes.

The 4-Component Model

The four volume fractions or "components"

$$X_f + X_p + X_h + X_s = 1$$

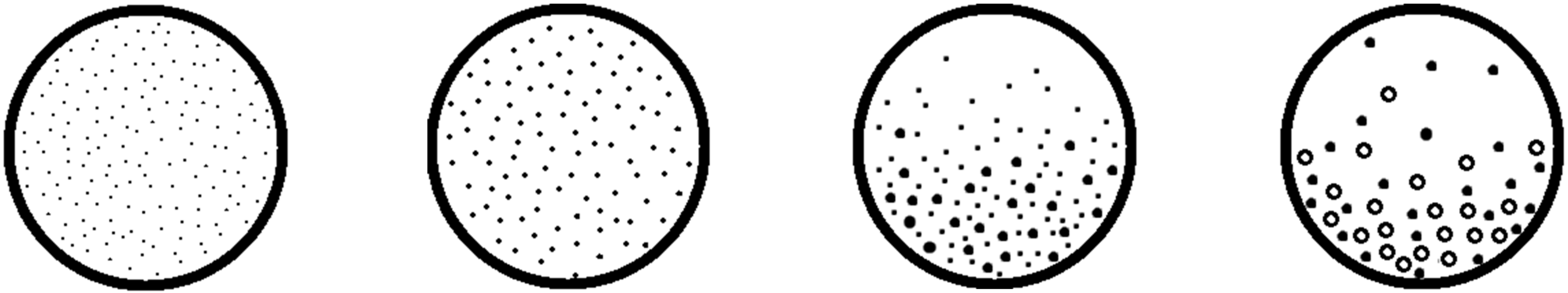


1. The slurry solids are divided into 4 groups or volume "fractions" based on the 4 basic flow regimes.
2. An established model for pipe friction or pump solids effect is applied to each fraction.
3. The fractional results are combined to provide the total effect.

Model calculation strategy

1. The contribution of each component is calculated using a settling flow model applicable to that particle size range.
2. The contributions are weighted according to the concentration of that component present in the mixture.
3. In applying each model, the solids for that component are treated as running in a “fluid mixture” whose density includes all of the previous, finer components.
    - *This captures that portion of the interaction between components due to mixture density effects.*
4. For particle-particle interaction effects between components, additional “cross correlating” empirical parameters are needed. These are weighted according to the relative velocity of the flow.
    - *More weight at lower velocity (i.e. with stratified flow).*
    - *Less weight at higher velocity (i.e. with well mixed flow).*

Empirical "Cross-Correlation" Parameters



"Carrier Fluid"	"Pseudo-homogeneous"	"Heterogeneous"	"Fully Stratified"
particles < 40μm	40 μm < particles < 200 μm	200 μm < particles < 0.015D	0.015D < particles

"Carrier Fluid." support of "Pseudo-Homog." fraction

"Pseudo-Homog." support of "Heterogeneous" fraction

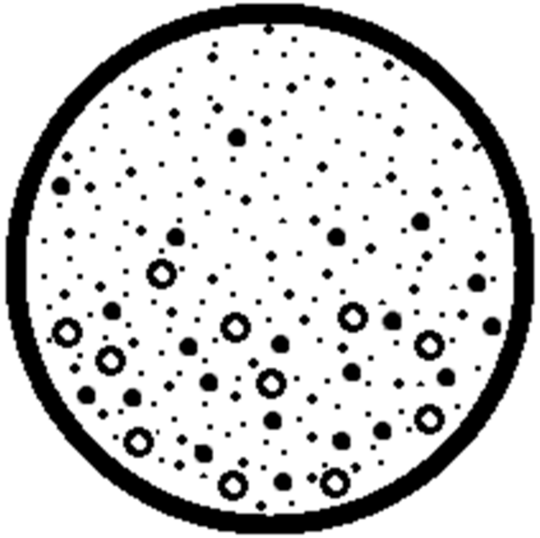
"Heterogeneous" support of "Fully Stratified" fraction

"Carrier Fluid" support of "Heterogeneous" fraction

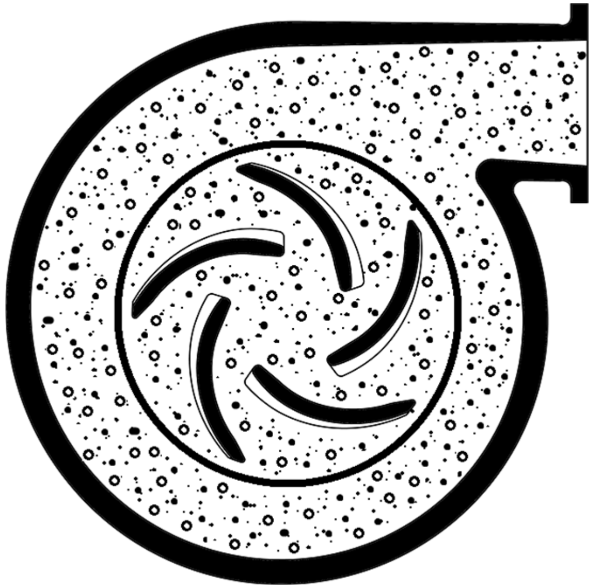
"Pseudo-Homogeneous" support of "Fully Stratified" fraction

"Carrier Fluid" support of "Fully Stratified" fraction

Advantages of the 4-Component Models



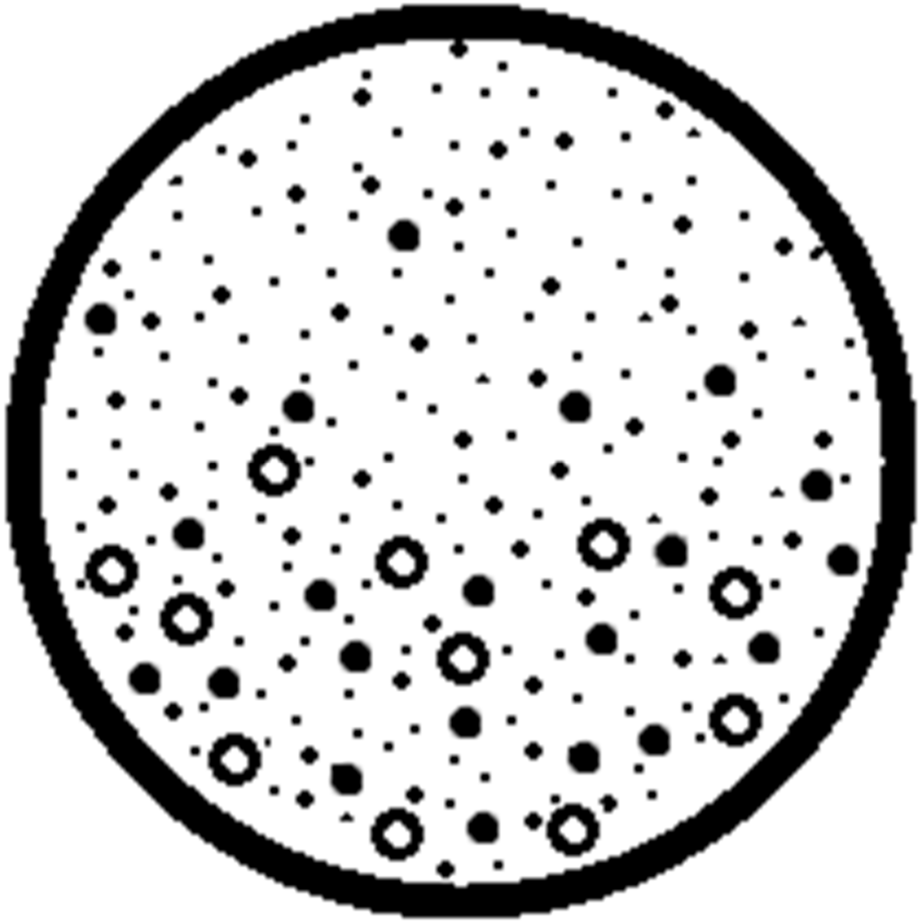
Pipeline Friction Loss



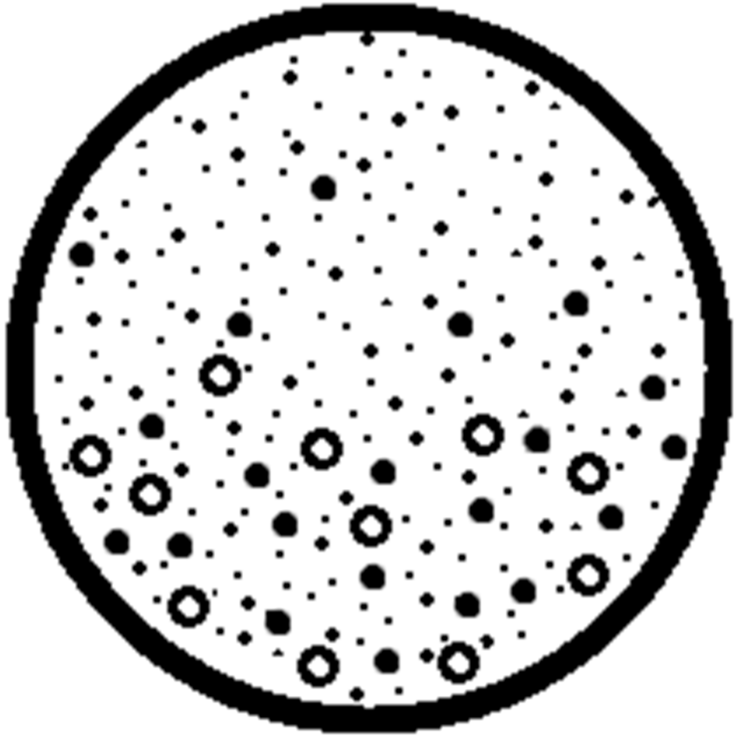
Pump Solids Effect

- Based on well established sub-models for each fraction.
- Can handle broad or bi-modal slurries … but collapse to established models as the particle size distribution narrows.
- Default calibrations are available for common settling slurries … or custom calibrations can be derived from test data.
- “Open source” formulae (included in this presentation).
- Also available in the GIW SLYSEL pump and pipeline calculation program.

The 4-Component Model for Pipeline Friction Loss



The 4-Component Model for Pipeline Friction Loss



The 4-component slurry

The pressure gradient (friction loss) for the complete mixture is determined as the sum the “Carrier Fluid” pressure gradient, plus the additional contributions (excess gradients) from each successively coarser fraction.

$$i_m = i_f + \Delta i_p + \Delta i_h + \Delta i_s$$

$$j_m = i_m / S_m$$

Where:

i = pressure gradient in meters of ***water*** head per meter of pipe.

i_m = total mixture pressure gradient.

i_f = pressure gradient of the “Carrier Fluid”.

Δi_p = excess pressure gradient of the “Pseudo-homogeneous” fraction.

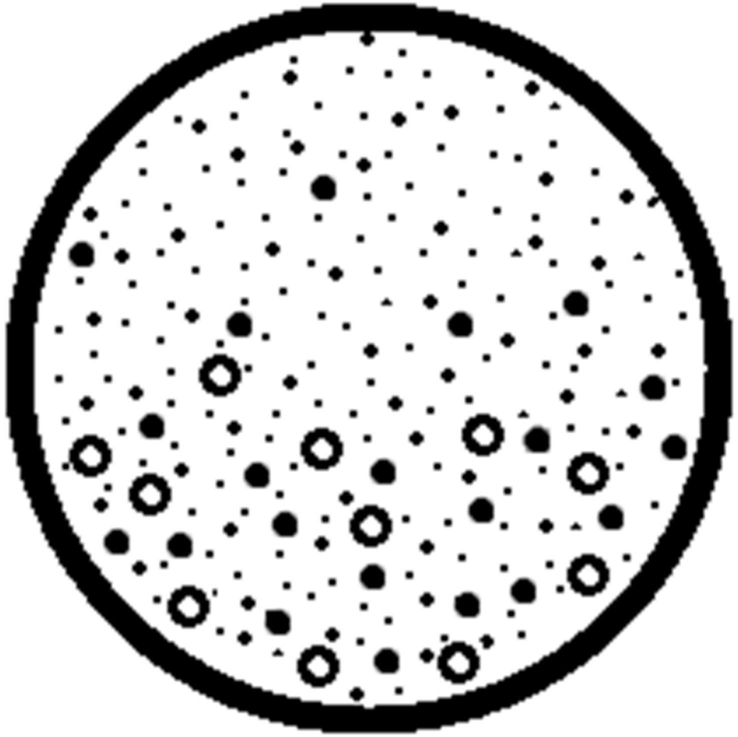
Δi_h = excess pressure gradient of the “Heterogeneous” fraction.

Δi_s = excess pressure gradient of the “Fully Stratified” fraction.

j_m = hydraulic gradient in meters of ***slurry*** head per meter of pipe.

S_m = specific gravity of the total slurry mixture (density relative to water).

Specific gravity terms



The specific gravity (S) is the ratio of a given density to the density of standard water, which for simplicity is taken as water at 5ºC, having a density of 1000 kg/m³.

$S = \rho / \rho_w$, where ρ_w = 1000 kg/m³

The cumulative specific gravities are important terms for the 4-component models. These are the mixture densities as each successive solids fraction is added to the mix.

S_w = SG of water at 5ºC = 1.0

S_l = SG of liquid (without solids)

S_s = SG of solids

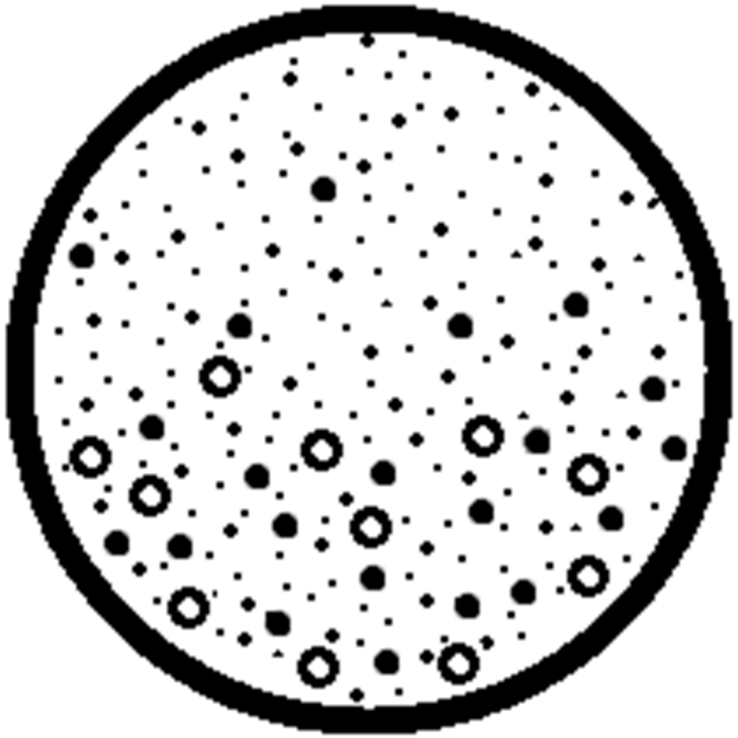
S_f = SG of liquid + X_f solids

S_{fp} = SG of liquid + X_f and X_p solids

S_{fph} = SG of liquid + X_f, X_p and X_h solids

S_m = SG of liquid + X_f, X_p, X_h and X_s solids
= SG of total mixture

Specific gravity terms



$$S_f = S_l + \frac{X_f \cdot C_v \cdot (S_s - S_l)}{1 - C_v \cdot (1 - X_f)}$$

$$S_{fp} = S_l + \frac{(X_f + X_p) \cdot C_v \cdot (S_s - S_l)}{1 - C_v \cdot (1 - X_f - X_p)}$$

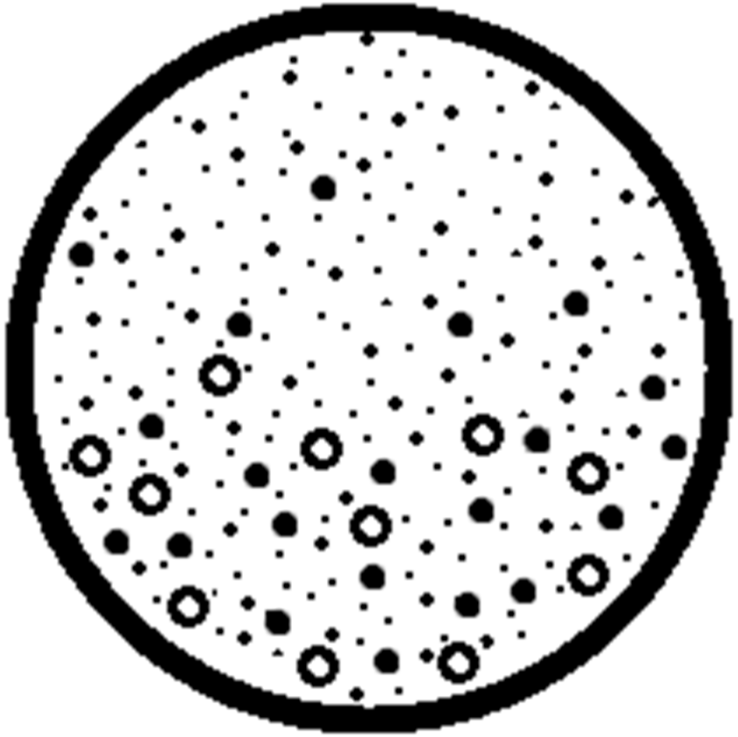
$$S_{fph} = S_l + \frac{(X_f + X_p + X_h) \cdot C_v \cdot (S_s - S_l)}{1 - C_v \cdot (1 - X_f - X_p - X_h)}$$

$$S_m = S_l + C_v \cdot (S_s - S_l)$$

Where:

C_v = total delivered volumetric concentration of solids (all fractions).

Concentration terms



The component volumetric concentrations are also important terms for the 4-component models.

These represent the volumetric concentration of each independent fraction within a mixture made of the liquid, the fraction itself, and all finer fractions.

C_{vf} = volume concentration of X_f solids within the mixture of liquid + X_f particles.

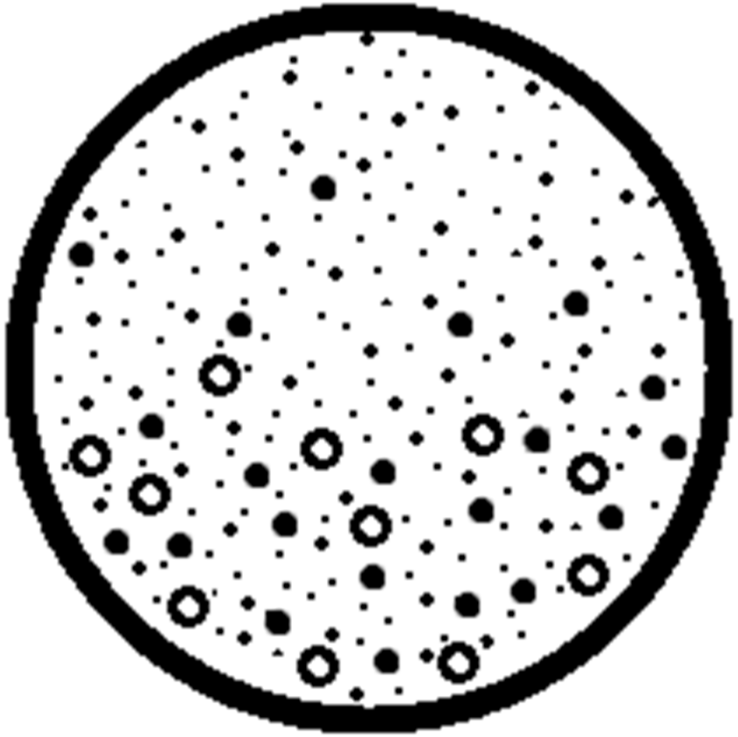
C_{vp} = volume concentration of X_p solids within the mixture of liquid + X_p and X_f particles.

C_{vh} = volume concentration of X_h solids within the mixture of liquid + X_h , X_p and X_f particles.

C_{vs} = volume concentration of X_s solids within the total mixture.

NOTE: In each case, the volume of the <u>coarser</u> fractions is <u>excluded</u>.

Concentration terms



$$C_{vf} = \frac{X_f \cdot C_v}{1 - C_v \cdot \left(1 - X_f\right)}$$

$$C_{vp} = \frac{X_p \cdot C_v}{1 - C_v \cdot \left(1 - X_f - X_p\right)}$$

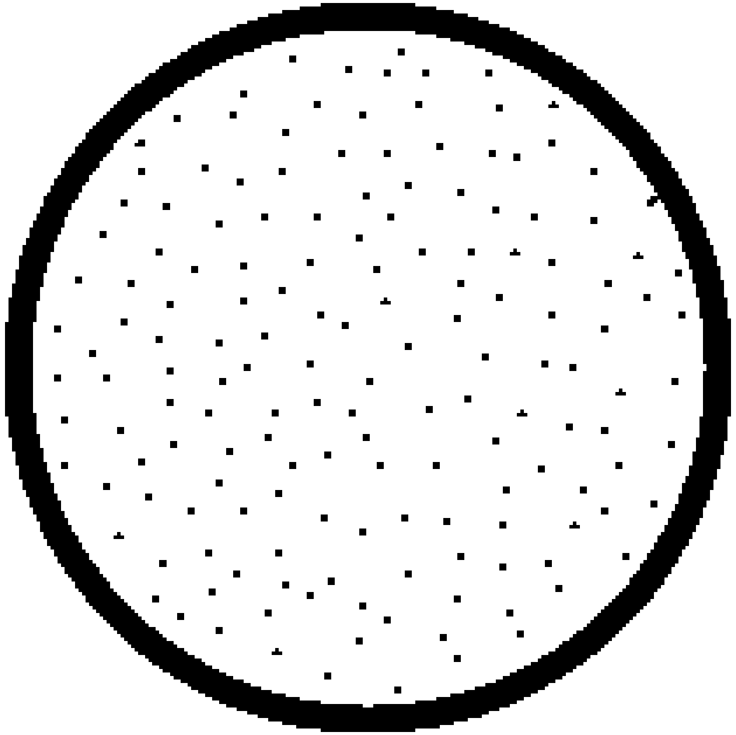
$$C_{vh} = \frac{X_h \cdot C_v}{1 - C_v \cdot \left(1 - X_f - X_p - X_h\right)}$$

$$C_{vs} = X_s \cdot C_v$$

Where:

C_v = total delivered volumetric concentration of solids (all fractions).

The “Carrier Fluid” fraction



“Carrier Fluid” fraction

X_f

$d < 40\mu m$

The pressure gradient for the “Carrier Fluid” is calculated according to the Darcy-Weisbach friction factor relationship.

The “Carrier Fluid” density and viscosity are used when calculating the Reynolds number to determine the friction factor (f_f).

$$i_f = S_f \cdot f_f \cdot \frac{V_m^2}{2 \cdot g \cdot D}$$

Where:

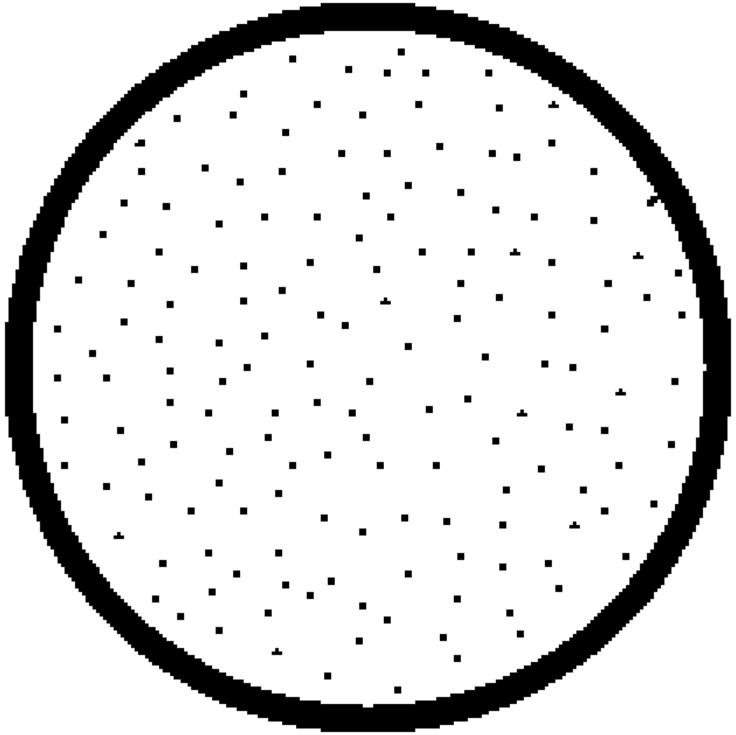
f_f = Darcy-Weisbach friction factor.

V_m = average slurry mixture velocity in the pipe (m/s)

D = pipe inner diameter (m)

g = acceleration of gravity (typically 9.81 m/s^2)

The “Carrier Fluid” fraction - viscosity correction



“Carrier Fluid” fraction

$\boldsymbol{X_f}$

d < 40μm

The effect of the particles on the viscosity of the “Carrier Fluid” mixture should be included.

This may be determined by a number of methods. Laboratory tests with sand-water mixtures found the method by Gillies et al. (1999) to be most accurate:

$$\frac{\mu_f}{\mu_l} = 1 + 2.5 \cdot C_{vf} + 10 \cdot C_{vf}^2 + 0.0019 \cdot e^{20 \cdot C_{vf}}$$

where:

μ_f = dynamic viscosity of the “Carrier Fluid” (liquid + fines).

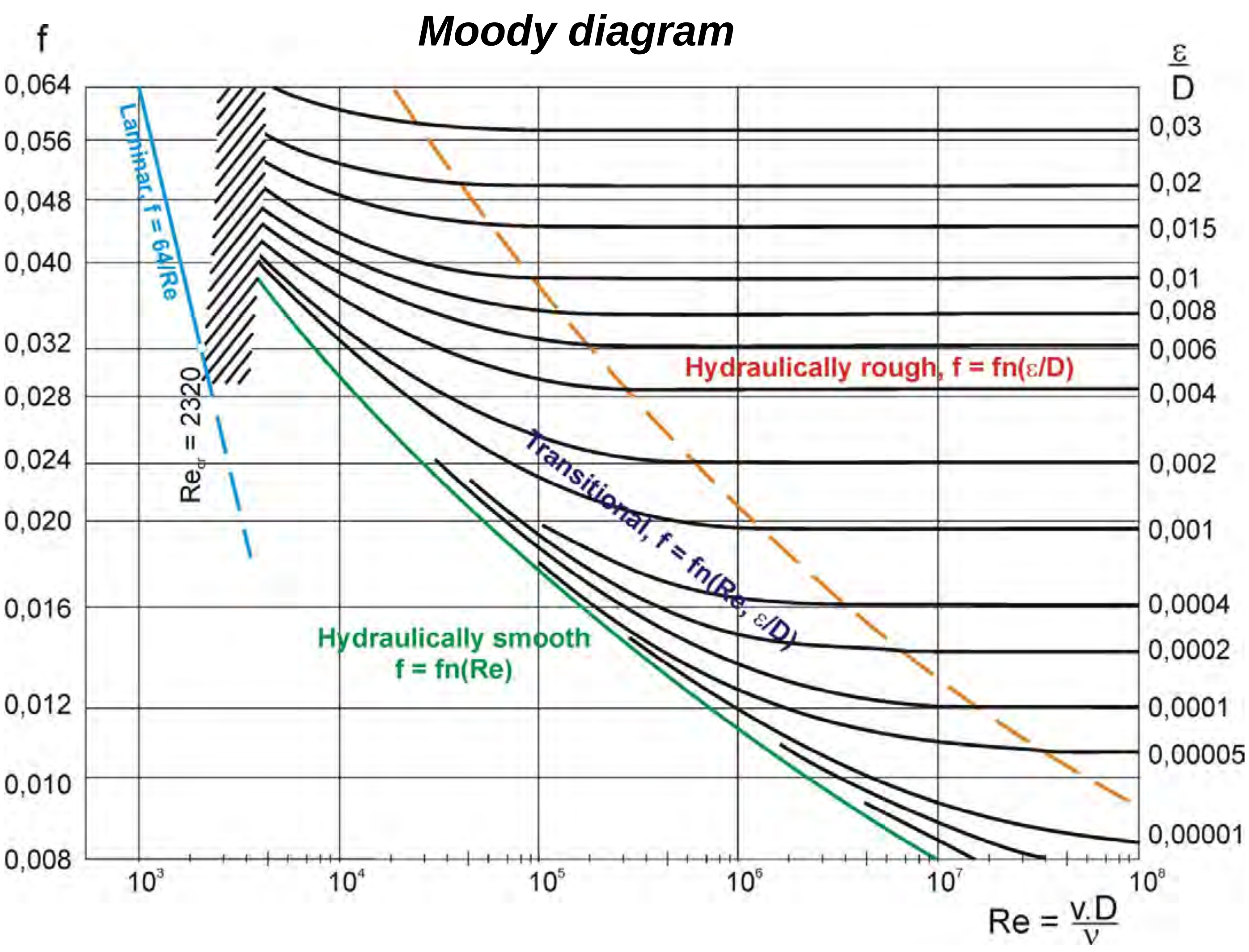
μ_l = dynamic viscosity of the liquid (without solids).

For pure water (with T in °C):

$$\mu(\text{kg/m-s}) \simeq (1.002\text{E-3}) \cdot 10^{\left(\frac{1.1709(20-T)-0.001827(T-20)^2}{T+89.93}\right)}$$

$$\rho(\text{kg/m}^3) \simeq 1000\left(1 - \frac{(T-3.9863)^2 (T+288.9414)}{508929.2(T+68.12963)}\right)$$

Calculating the Darcy-Weisbach friction factor



The Darcy-Weisbach friction factor (f) is a function of the pipeline Reynolds number (Re) and the ratio of the pipe wall roughness to the pipeline diameter (ε / D).

It can be read from the Moody Diagram at left.

Most slurry pipes are worn smooth by the action of the slurry solids and a typical value for the pipe wall roughness is ε = 0.000015 m.

The pipeline Reynolds Number is:

$$\mathrm{Re} = \frac{V_m D}{\nu_f}$$

Where:

$$\nu_f = \mu_f / \rho_f \ \ (\mathrm{m}^2/\mathrm{s})$$

Calculating the Darcy-Weisbach friction factor

For turbulent pipe flows ($Re > 10^4$) The Darcy-Weisbach friction factor (f) can be calculated using the **Colebrook-White** equation. This equation is implicit, and must be solved by iteration:

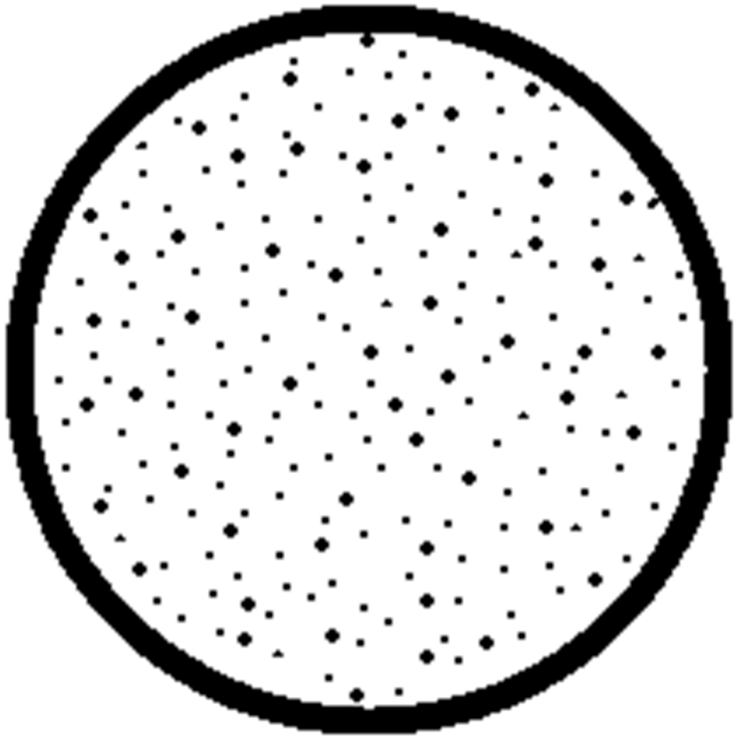
$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{Re\sqrt{f}}\right)$$

For most industrial pipelines using water as the carrier fluid, the friction factor falls in the "transitional" range between $10^5 < Re < 10^7$ and the more convenient **Swamee-Jain** equation, which can be solved directly, may be used:

$$f = \frac{0.25}{\left[\log\left(\frac{\varepsilon}{3.7D} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$$

More information on alternative Darcy-Weisbach friction factor formulae can be found on Wikipedia:
https://en.wikipedia.org/wiki/Darcy_friction_factor_formulae

The “Pseudo-homogeneous” fraction



“Pseudo-homogeneous” fraction

$$X_p$$

$$40\mu m < d < 200\ \mu m$$

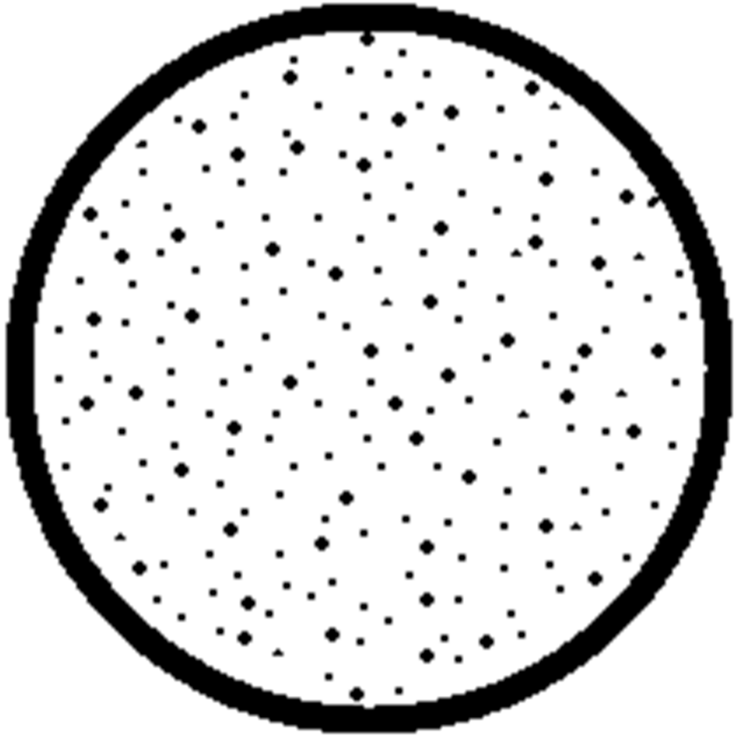
The excess pressure gradient contribution for the “Pseudo-homogeneous” component is based on an “equivalent fluid” treatment, where the particles affect the density, but not the viscosity of the mixture:

$$\Delta i_p = i_p - i_f = A'' \cdot C_{vp} \cdot \left(S_s - S_f\right) \cdot i_f / S_f$$

which can be algebraically reduced to:

$$\Delta i_p = A'' \cdot \left(S_{fp} - S_f\right) \cdot i_f / S_f$$

The “Pseudo-homogeneous” fraction



“Pseudo-homogeneous” fraction

X_p

$40\mu m < d < 200\ \mu m$

The empirical parameter A'' accounts for near wall, hydrodynamic lift effects which may be experienced by the “pseudo-homogeneous” particles.

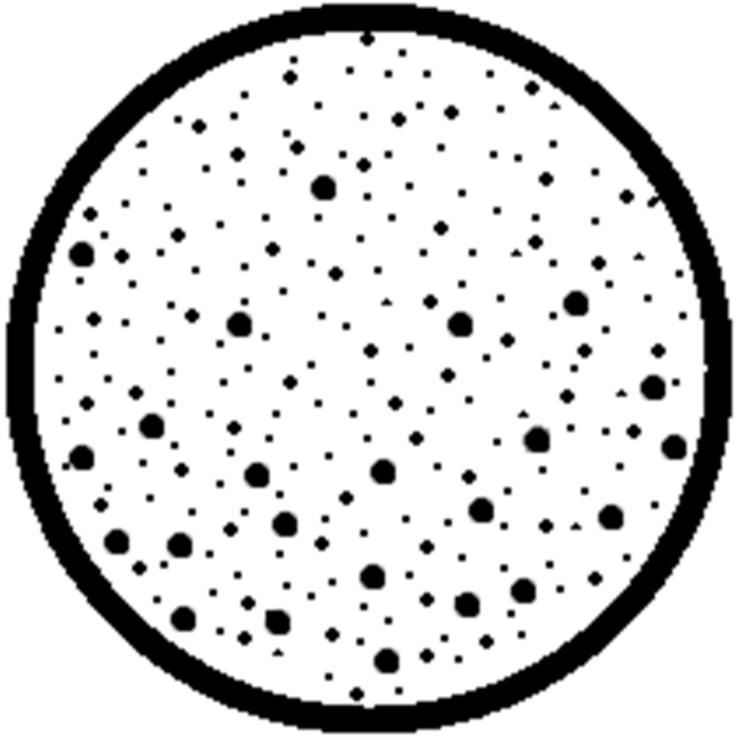
These reduce the pressure gradient from the theoretical value obtained from the “equivalent fluid” model alone.

$$A'' = 1 - \left(A_f'' \cdot X_f + A_p'' \cdot X_p \right)$$

Based on laboratory testing described below, the following default values are recommended for the correlation constants:

$$A''_f = 1.0, \qquad A''_p = 0.5$$

The “Heterogeneous” fraction



“Heterogeneous” fraction

$\boldsymbol{X_h}$

200μm < d < 0.015 D

The excess pressure gradient contribution for the “Heterogeneous” component is based on the Wilson V_{50} model:

$$\Delta i_h = C'' \cdot \frac{\mu_s}{2} \cdot C_{vh} \cdot \left(S_s - S_{fp}\right) \cdot \left(\frac{V_{50,h}}{V_m}\right)^M$$

which can be reduced to:

$$\Delta i_h = C'' \cdot \frac{\mu_s}{2} \cdot \left(S_{fph} - S_{fp}\right) \cdot \left(\frac{V_{50,h}}{V_m}\right)^M$$

where:

$$V_{50,h} = 44.1 \cdot \frac{d_{50h}^{0.35}}{\nu_r^{0.25}} \cdot \left(\frac{S_s - S_{fp}}{1.65}\right) \quad \text{and} \quad \nu_r = \nu_f / \nu_w = \mu_f / \left(\mu_w S_f\right)$$

μ_s = sliding friction coef. between particles and pipe wall, generally from 0.4 for softer, rounded particles to 0.5 for harder, angular particles.

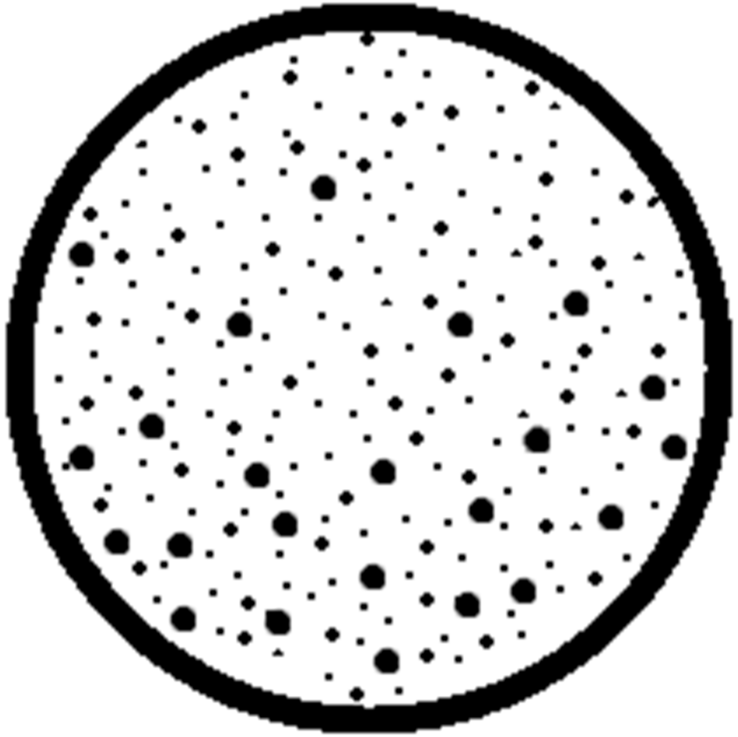
d_{50h} = d_{50} of the X_h fraction solids in (m).

M = 1.0 when using the four component model.

C'' = the empirical parameter for effects of inter-granular support from smaller sized fractions.

μ_w = viscosity of water @ 20 °C = 0.0010 (kg/m-s)

The "Heterogeneous" fraction



"Heterogeneous" fraction

X_h

$200 \mu m < d < 0.015\ D$

The empirical parameter for particle-particle interactions between components is based on the volumes of the finer X_f and X_p fractions.

It is weighted according to the slurry mixture velocity, becoming stronger at lower velocity where the flow is more stratified and weaker at high velocity where the flow is more well mixed.

$$C'' = 1 - \left(C''_f \cdot X_f + C''_p \cdot X_p\right) \cdot \left(\frac{V_{100,s} - V_m}{V_{100,s} - V_{sm,h}}\right)^{n_C}$$

Where:

$$C'' = 1.0 \text{ for any } V_m > V_{100,s}$$

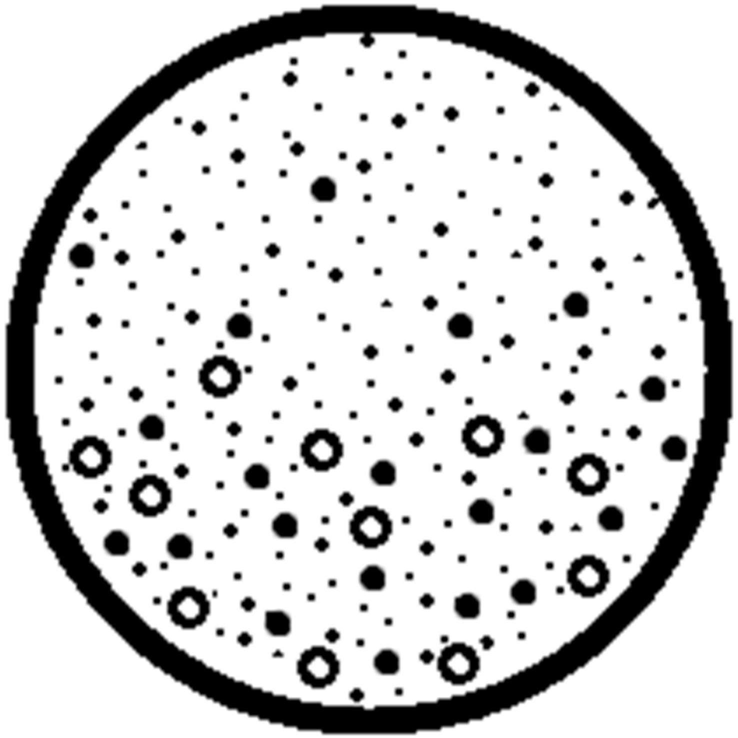
$V_{sm,h}$ = the maximum velocity at limit of stationary deposition for d_{50h} sized particles.

$V_{100,s}$ = the pseudo-homogeneous velocity for 0.015D sized particles.

Based on laboratory testing described below, the following default values are recommended for the correlation constants:

$$C''_f = 1.0, \qquad C''_p = 0.5, \qquad n_C = 0.5$$

The “Fully Stratified” fraction



“Fully Stratified” fraction

$\boldsymbol{X_s}$

$\boldsymbol{0.015D < d}$

The excess pressure gradient contribution for the “Fully Stratified” component is based on the model proposed by Wilson and Addie, (1995):

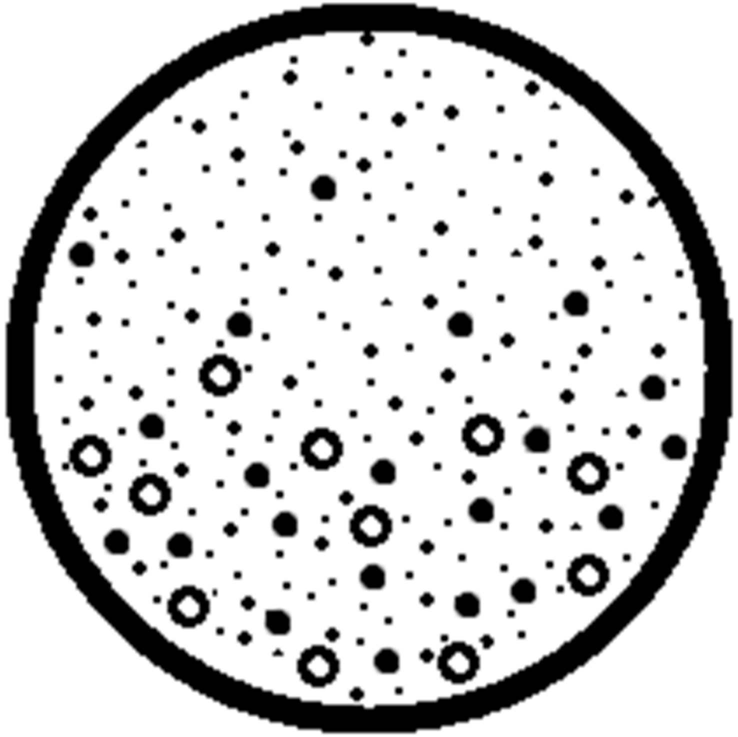
$$\Delta i_s = B'' \cdot 2\mu_s \cdot C_{vs} \cdot \left(S_s - S_{fph}\right) \cdot \left(\frac{V_{sm,s}}{V_m}\right)^{0.25}$$

Where:

$V_{sm,s}$ = the maximum velocity at limit of stationary deposition for 0.015D sized particles.

B'' = the empirical parameter for effects of inter-granular support from smaller sized fractions.

The “Fully Stratified” fraction



“Fully Stratified” fraction

$\boldsymbol{X_s}$

0.015D < d

The empirical parameter for particle-particle interactions between components is based on the volumes of the finer X_f , X_p and X_h fractions.

It is weighted according to the slurry mixture velocity, becoming stronger at lower velocity where the flow is more stratified and weaker at high velocity where the flow is more well mixed.

$$B'' = 1 - \left(B_f'' \cdot X_f + B_p'' \cdot X_p + B_h'' \cdot X_h\right) \cdot \left(\frac{V_{100,s} - V_m}{V_{100,s} - V_{sm,s}}\right)^{n_B}$$

Where:

$$B'' = 1.0 \text{ for any } V_m > V_{100,s}$$

Based on laboratory testing described below, the following default values are recommended for the correlation constants:

$$B''_f = 1.0, \quad B''_p = 1.0, \quad B''_h = 0.5, \quad n_B = 0.5$$

Determination of V_{sm} (the deposition velocity)

Values for the **maximum velocity at the limit of stationary deposition** (V_{sm}) are required to evaluate the heterogeneous and stratified contributions to the pressure gradient.

Various methods exist for determination of these values. Close approximations can be made for particles sizes > 200 μm using the following equations based on Wilson's deposition nomograph.

$$V_{sm} = MIN\left(V_{sm,\mathrm{nom}}, V_{sm,\mathrm{max}}\right)$$

where:

$$V_{sm,\mathrm{nom}} = 8.8\left(\mu_s \frac{S_s - S_f}{0.66 \cdot S_f}\right)^{0.55} \frac{D^{0.7} d^{1.75}}{d^2 + 0.11 D^{0.7}} \quad \text{and} \quad V_{sm,\mathrm{max}} = \left(\frac{0.018}{f_f}\right)^{0.13} \sqrt{2gD\left(\frac{S_s}{S_f} - 1\right)}$$

$V_{sm,\mathrm{nom}}$ (m/s) = a correlation for the nominal value of V_{sm}, based on Wilson's nomograph (1979).

$V_{sm,\mathrm{max}}$ (m/s) = an alternative maximum value for V_{sm}, based on Wilson's 1992 two-layer model analysis.

d = the particle diameter in (mm) = d_{50h} for $V_{sm,h}$, or d_s for $V_{sm,s}$

D = the pipe diameter in (m)

f_f = the Darcy-Weisbach friction factor for the carrier fluid (μ_f , S_f) in pipe diameter D at velocity V_{sm}.
Since f_f is a function of V_{sm}, the $V_{sm,\mathrm{max}}$ equation must be solved iteratively.
[Tip: In turbulent flow, f_f is a weak function of velocity and $V_{sm,\mathrm{max}}$ is a weak function of f_f. For preliminary calculations, $V_{sm,\mathrm{max}}$ may be approximated without iteration, if an approximate value for f_f is known.]

g, μ_s, S_s, S_f are as previously defined.

Determination of $V_{100,s}$ (the pseudo-homogeneous velocity)

A value for the **pseudo-homogeneous velocity of the 0.15D sized particle** ($V_{100,s}$) is required to evaluate the stratified contributions to the pressure gradient.

Various methods exist for determination of this value. Close approximations can be made for industrial sized pipes using the following equation based on Newit's analysis (1955).

$$V_{100,s} = \sqrt[3]{1800 \cdot g \cdot D \cdot v_{t,s}}$$

where:

$V_{100,s}$ (m/s) = the pseudo-homogeneous velocity for the 0.15D sized particle in pipe diameter D.

D = the pipe diameter in (m)

$v_{t,s}$ (m/s) = the terminal settling velocity for the d_s sized particle. Various methods exist for determining v_t. For water based slurries, semi-angular particles, and pipe sizes from 0.075 to 1.5 m, $v_{t,s}$ can be estimated using the "Newton's law" relationship with a shape factor correction (ξ) as follows:

$$v_{t,s} = 1.73\xi\sqrt{gd_s\left(S_s - S_f\right)} \quad \text{where} \quad \xi = 0.4d_s^{-0.04}$$

where:

d_s = the 0.015D particle diameter in (m)

g, S_s are as previously defined.

Determination of $V_{100,s}$ (the pseudo-homogeneous velocity)

An alternate method for calculation of the **pseudo-homogeneous velocity of the 0.15*D* sized particle** ($V_{100,s}$) is based on determining the flow velocity at which the pipe system curves for heterogeneous and pseudo-homogeneous flow of the 0.015*D* sized particle intersect.

$$V_{100,s} = \sqrt[3]{\frac{\mu_s}{A''\cdot f_{f,100,s}} \cdot g \cdot D \cdot V_{50,s}}$$

where:

$$V_{50,s} = 44.1 \cdot \frac{d_s^{0.35}}{\nu_r^{0.25}} \cdot \left(\frac{S_s - S_{fp}}{1.65} \right)$$

D = the pipe diameter in (m)

d_s = the 0.015*D* particle diameter in (m)

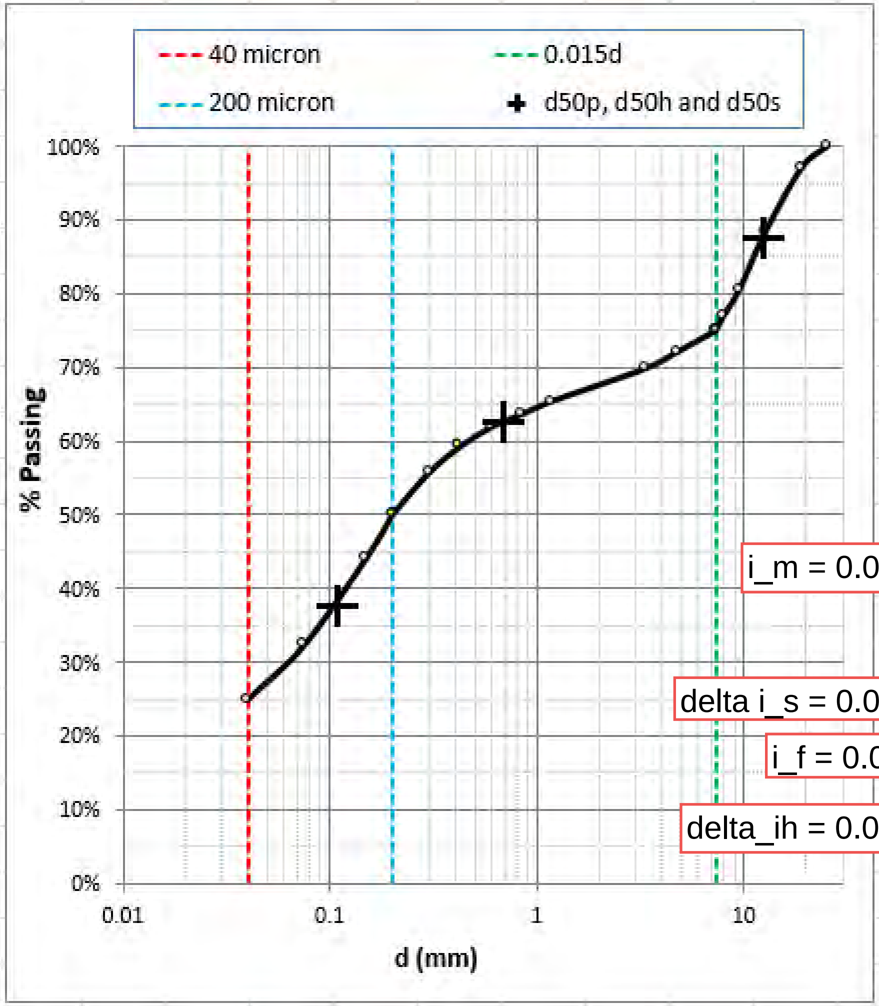
$f_{f,100,s}$ = the Darcy-Weisbach friction factor for the carrier fluid (μ_f , S_f) in pipe diameter D at velocity $V_{100,s}$. Since $f_{f,100,s}$ is a function of $V_{100,s}$, the equation must be solved iteratively.

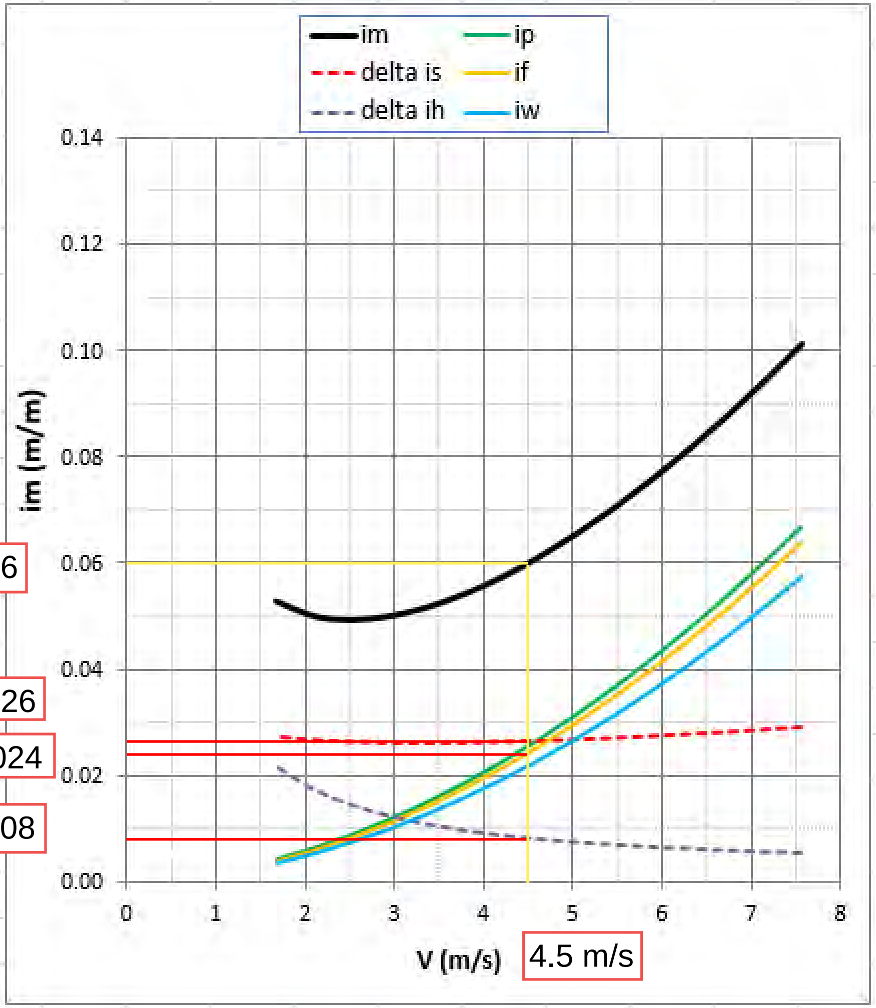
[Tip: In turbulent flow, f_f is a weak function of velocity and $V_{100,s}$ is a weak function of f_f. For preliminary calculations, $V_{100,s}$ may be approximated without iteration, if an approximate value for f_f is known.]

g, μ_s, S_s, S_{fp} and A'' are all as previously defined.

Examples

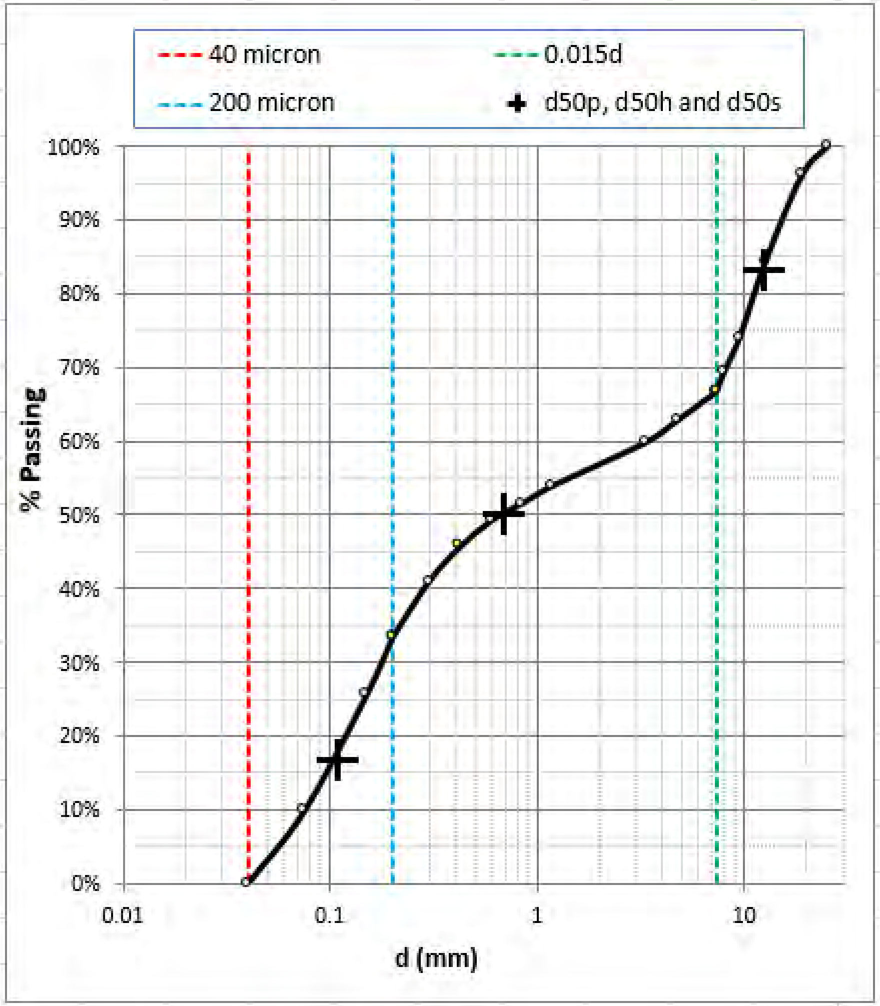
$D =$	0.489	m
$\varepsilon =$	2.0E-06	m
$\mu_s =$	0.50	
$S_m =$	1.328	
$S_s =$	2.650	
$C_v =$	20.0%	
$T =$	10.0	^{o}C
liquid =	H_2O	
$X_f =$	0.25	
$X_p =$	0.25	
$X_h =$	0.25	
$X_s =$	0.25	
$d_{50p} =$	0.11	mm
$d_{50h} =$	0.68	mm
$d_s =$	7.33	mm
$d_{50s} =$	12.4	mm

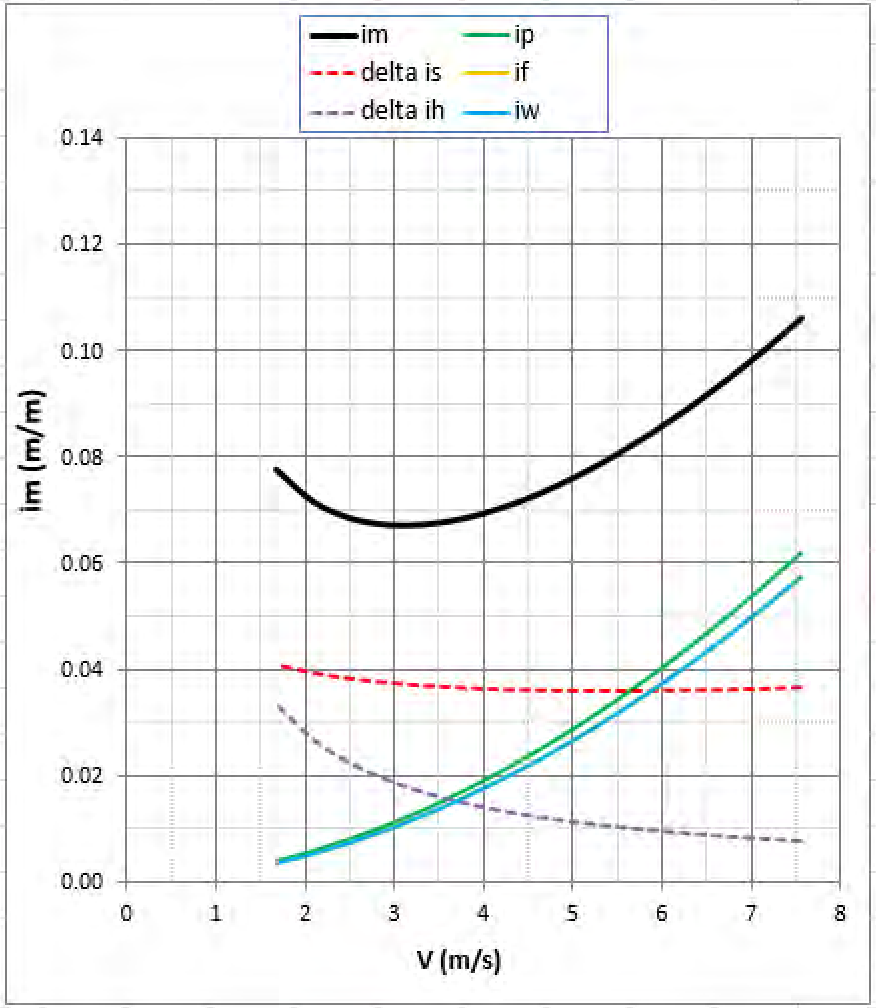




Examples

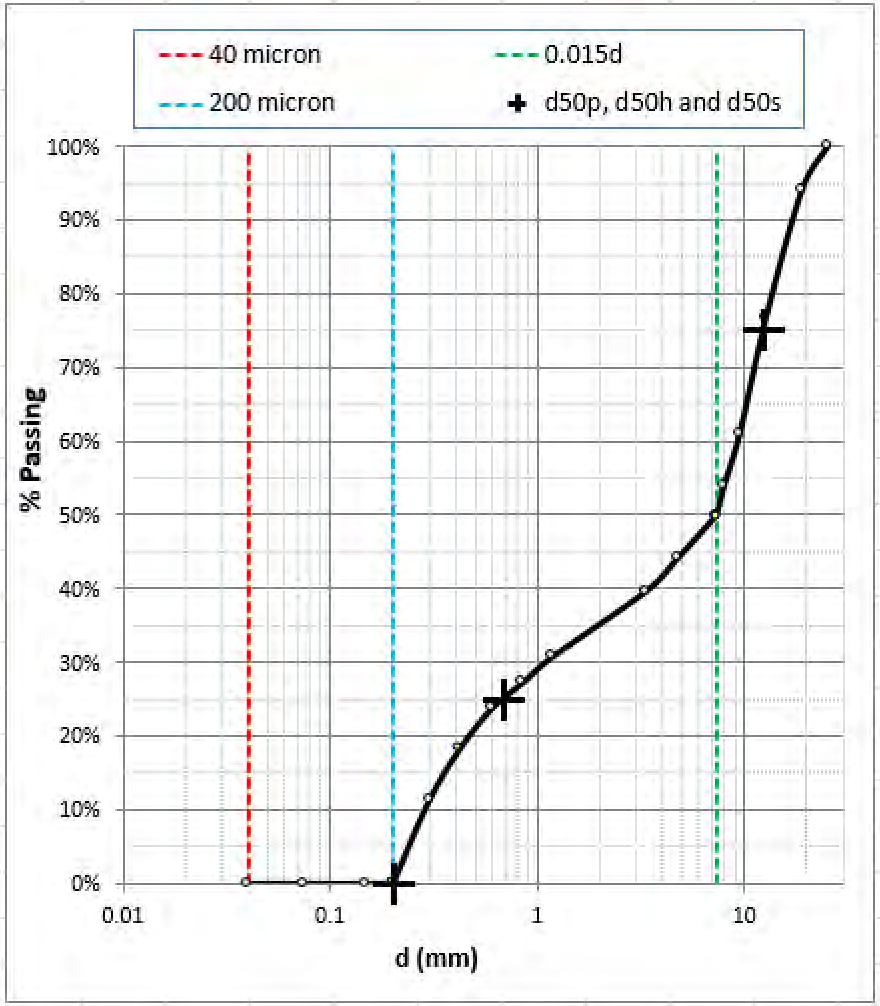
D =	0.489	m
ε =	2.0E-06	m
μ_s =	0.50	
S_m =	1.246	
S_s =	2.650	
C_v =	15.0%	
T =	10.0	°C
liquid =	H_2O	
X_f =	0.00	
X_p =	0.33	
X_h =	0.33	
X_s =	0.33	
d_{50p} =	0.11	mm
d_{50h} =	0.68	mm
d_s =	7.33	mm
d_{50s} =	12.4	mm

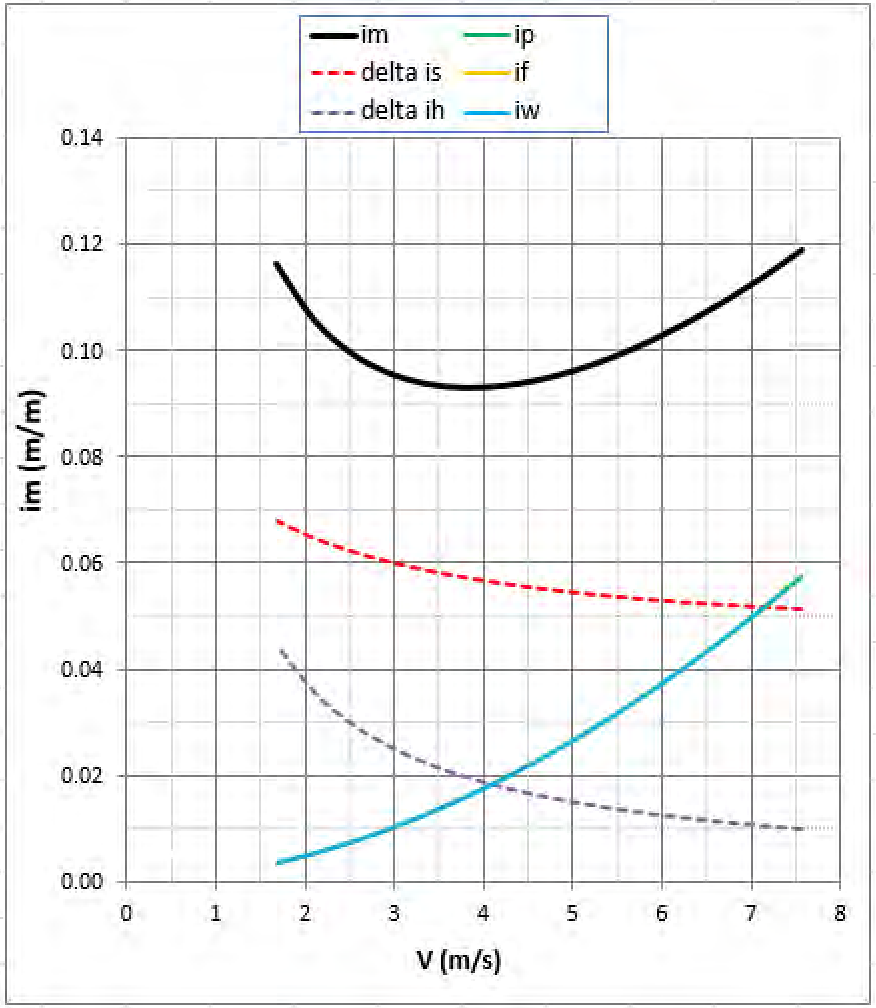




Examples

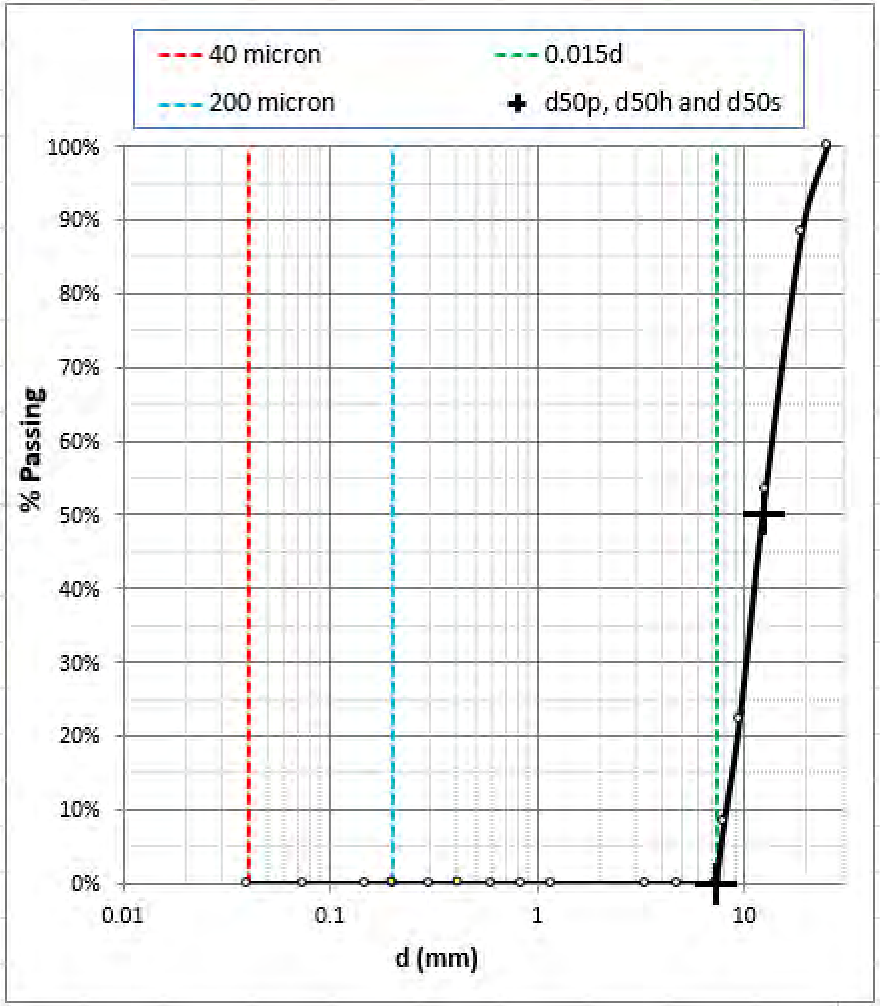
D =	0.489	m
ε =	2.0E-06	m
μ_s =	0.50	
S_m =	1.163	
S_s =	2.650	
C_v =	10.0%	
T =	10.0	°C
liquid =	H_2O	
X_f =	0.00	
X_p =	0.00	
X_h =	0.50	
X_s =	0.50	
d_{50p} =		mm
d_{50h} =	0.69	mm
d_s =	7.33	mm
d_{50s} =	12.4	mm

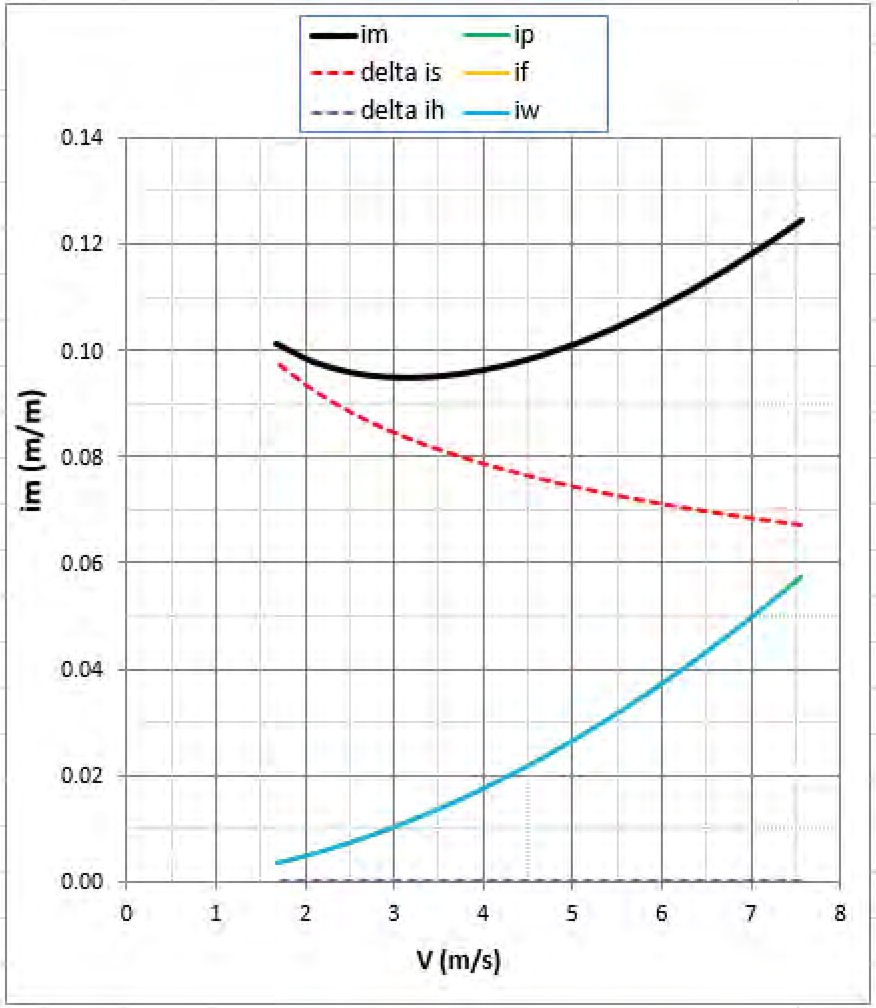




Examples

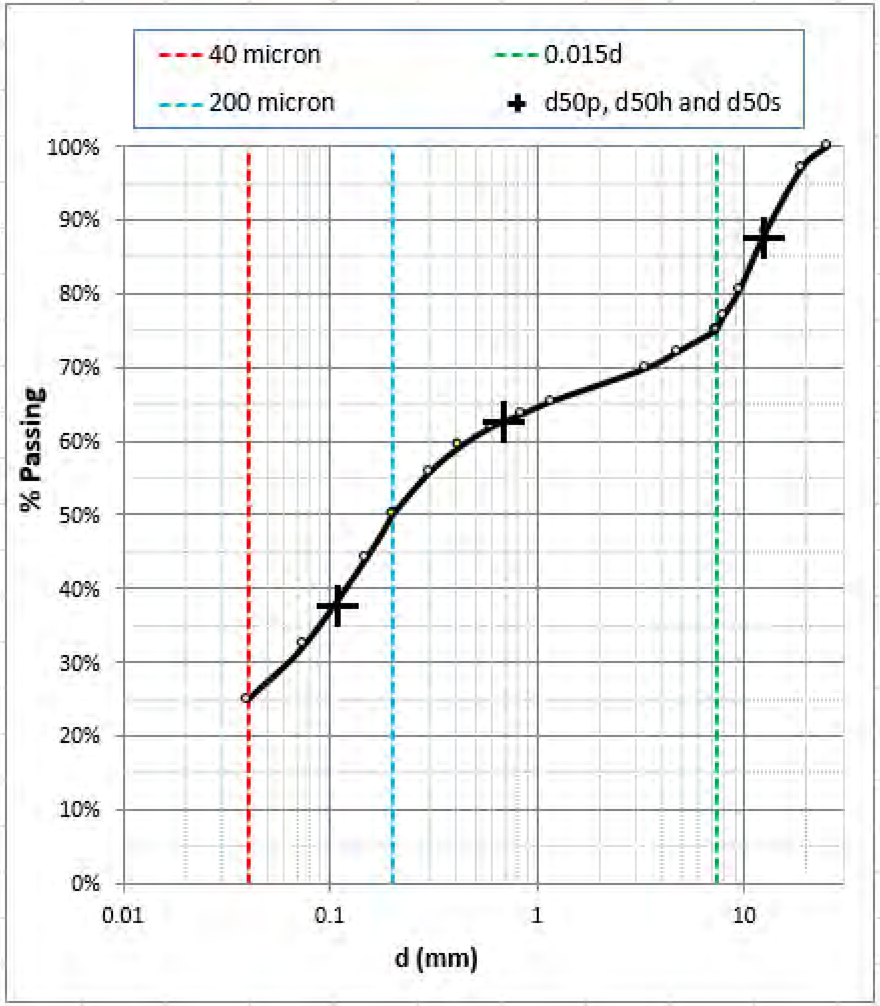
D =	0.489	m
ε =	2.0E-06	m
μ_s =	0.50	
S_m =	1.081	
S_s =	2.650	
C_v =	5.0%	
T =	10.0	°C
liquid =	H_2O	
X_f =	0.00	
X_p =	0.00	
X_h =	0.00	
X_s =	1.00	
d_{50p} =		mm
d_{50h} =		mm
d_s =	7.33	mm
d_{50s} =	12.4	mm

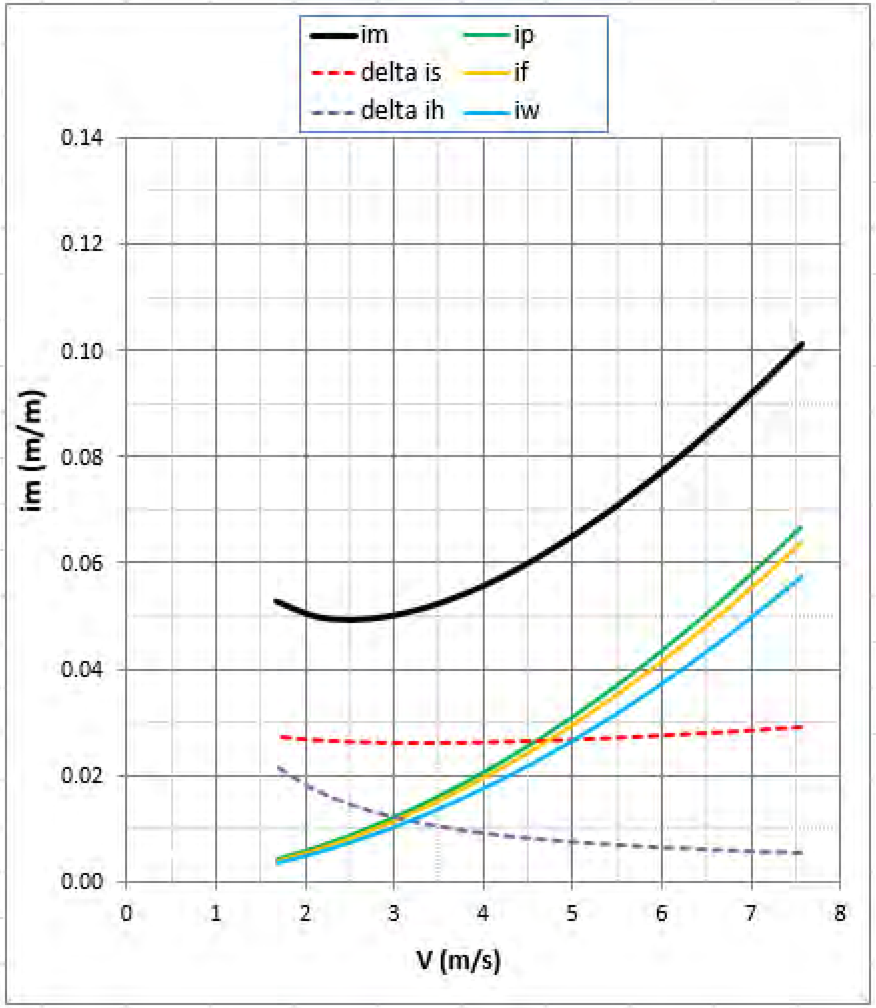




Examples

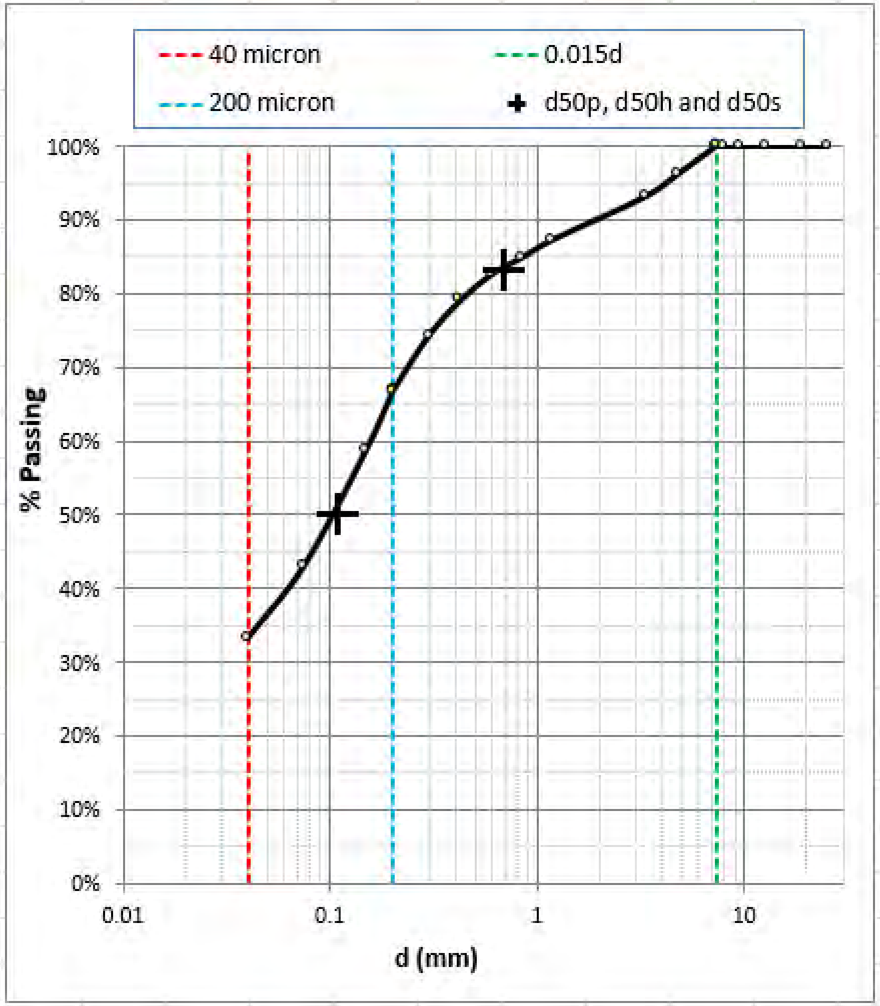
D =	0.489	m
ε =	2.0E-06	m
μ_s =	0.50	
S_m =	1.328	
S_s =	2.650	
C_v =	20.0%	
T =	10.0	°C
liquid =	H_2O	
X_f =	0.25	
X_p =	0.25	
X_h =	0.25	
X_s =	0.25	
d_{50p} =	0.11	mm
d_{50h} =	0.68	mm
d_s =	7.33	mm
d_{50s} =	12.4	mm

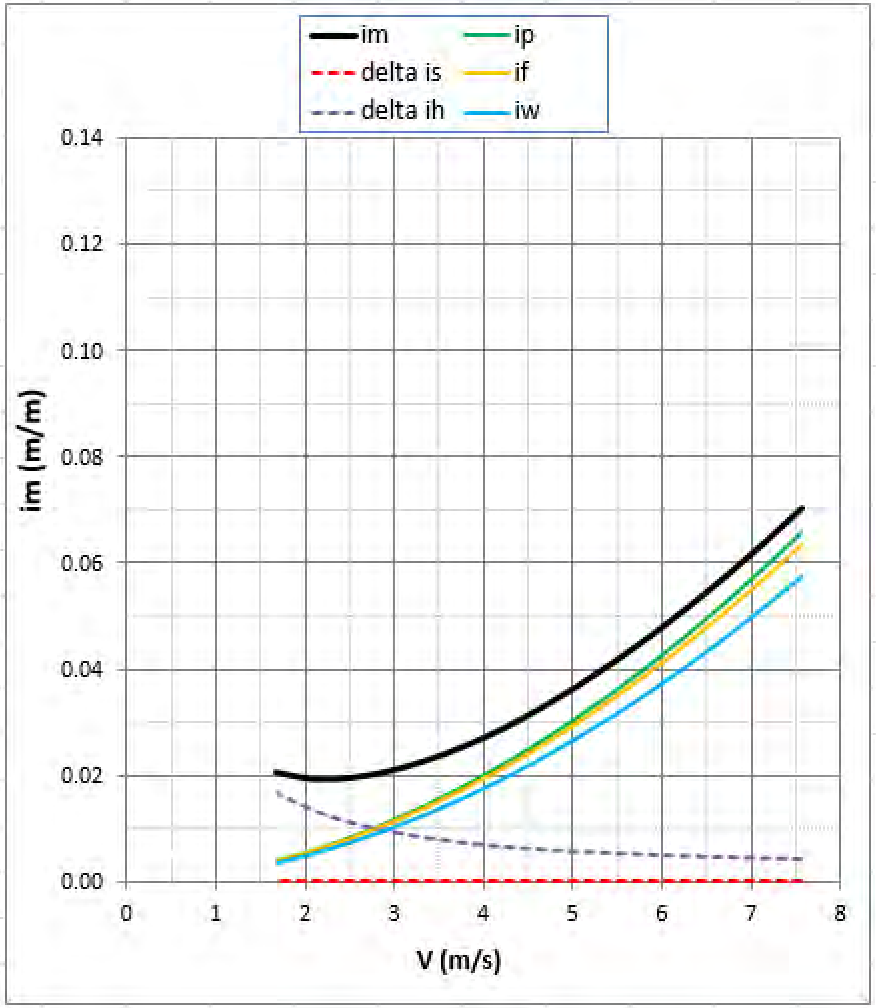




Examples

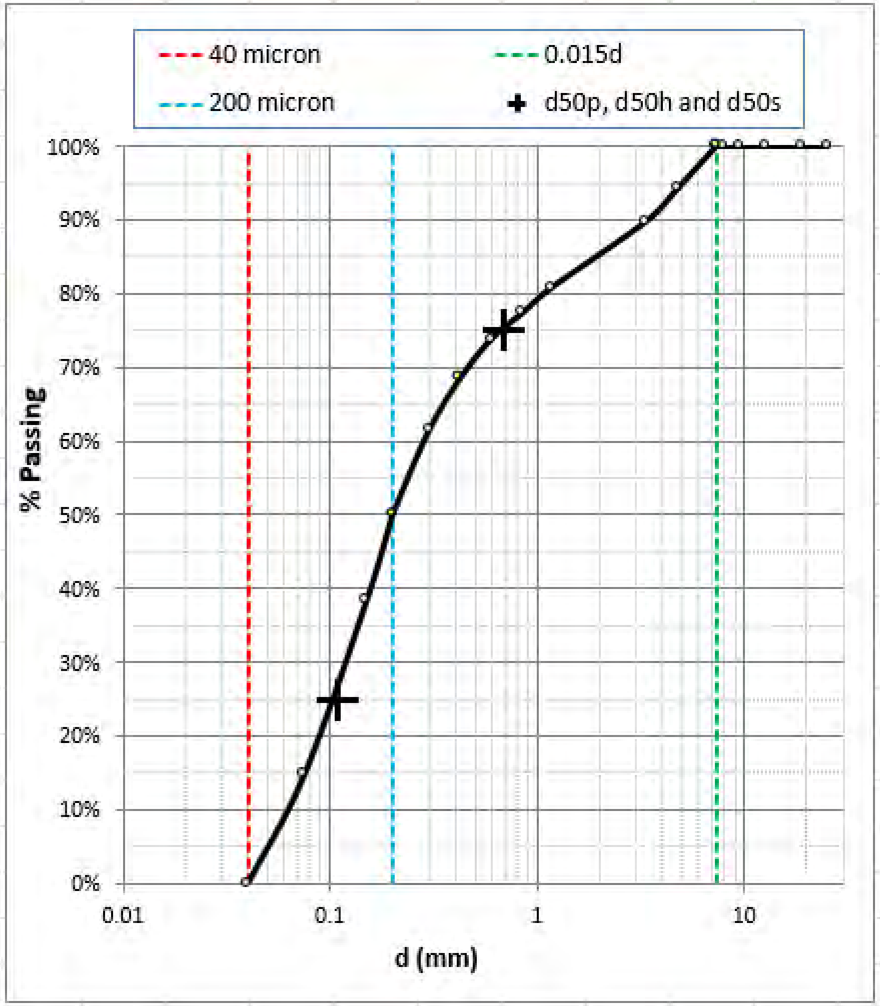
D =	0.489	m
ε =	2.0E-06	m
μ_s =	0.50	
S_m =	1.246	
S_s =	2.650	
C_v =	15.0%	
T =	10.0	°C
liquid =	H_2O	
X_f =	0.33	
X_p =	0.33	
X_h =	0.33	
X_s =	0.00	
d_{50p} =	0.11	mm
d_{50h} =	0.68	mm
d_s =	7.33	mm
d_{50s} =		mm

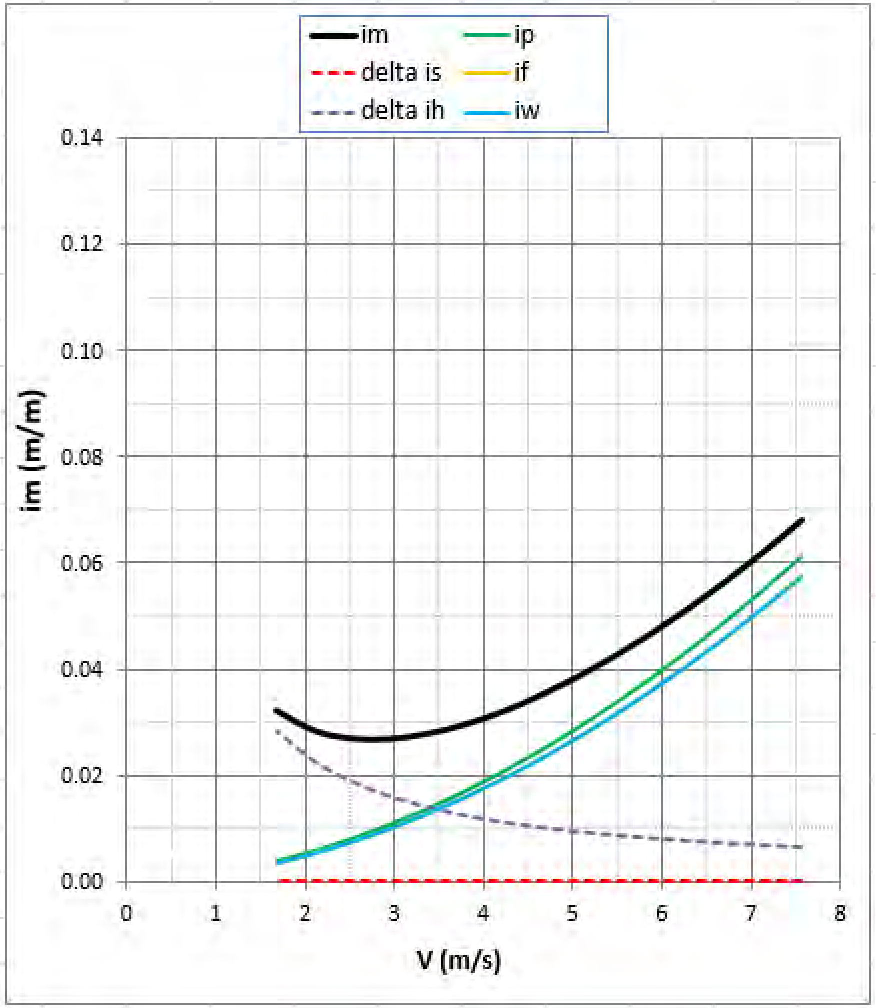




Examples

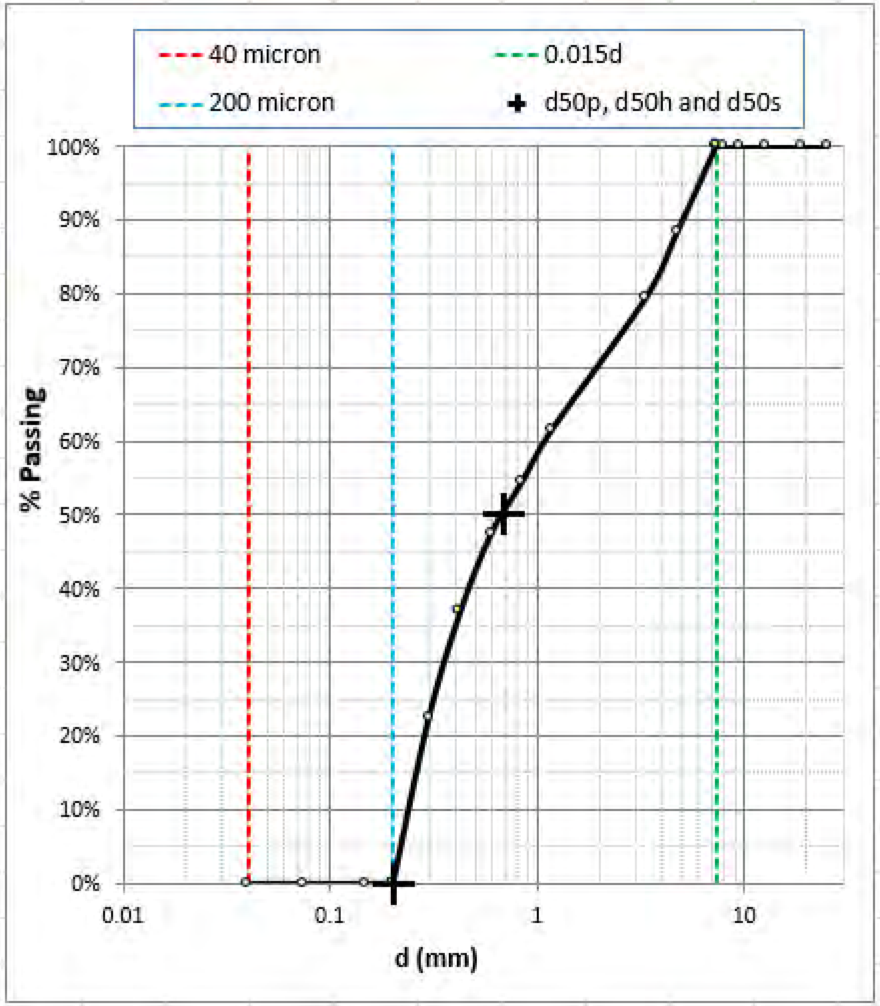
D =	0.489	m
ε =	2.0E-06	m
μ_s =	0.50	
S_m =	1.163	
S_s =	2.650	
C_v =	10.0%	
T =	10.0	°C
liquid =	H_2O	
X_f =	0.00	
X_p =	0.50	
X_h =	0.50	
X_s =	0.00	
d_{50p} =	0.11	mm
d_{50h} =	0.68	mm
d_s =	7.33	mm
d_{50s} =		mm

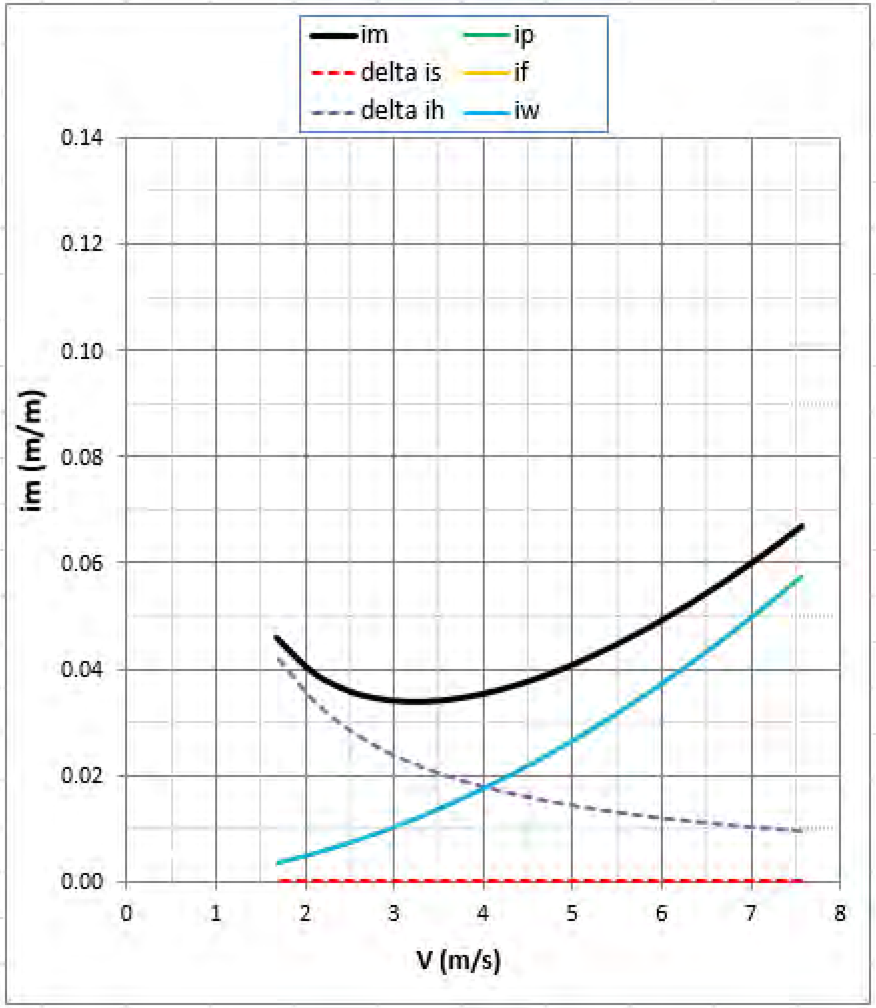




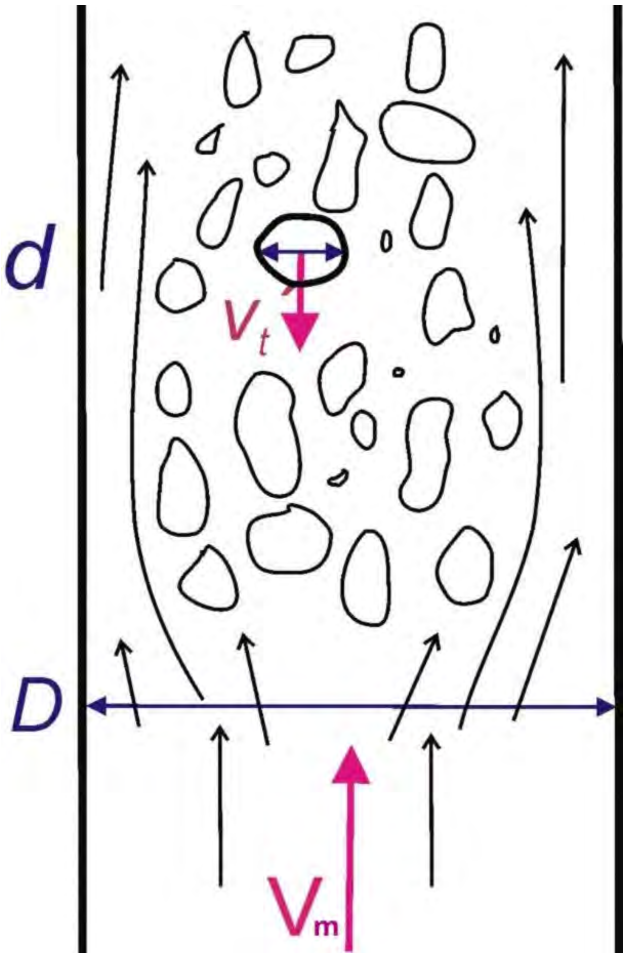
Examples

D =	0.489	m
ε =	2.0E-06	m
μ_s =	0.50	
S_m =	1.081	
S_s =	2.650	
C_v =	5.0%	
T =	10.0	°C
liquid =	H_2O	
X_f =	0.00	
X_p =	0.00	
X_h =	1.00	
X_s =	0.00	
d_{50p} =		mm
d_{50h} =	0.69	mm
d_s =	7.33	mm
d_{50s} =		mm





Special topic: Vertical conveying



For vertical conveying, bed friction due to stratification of solids against the pipe wall does not exist, and the heterogeneous and stratified solids contributions must be modified.

A certain minimum velocity should also be maintained to control the slip of particles within the slurry. Excessive slip will increase the in-situ concentration, the pressure gradient and the chances of an unsteady or collapsing flow.

It is recommended that the minimum vertical transport velocity be set = 2x the terminal settling velocity of the largest particle treated as a sphere (with ξ=1):

$$V_{m,\min} = 2v_{ts,(\text{largest particle})} \simeq 1.73\sqrt{gd_{\max}\left(S_s - S_f\right)}$$

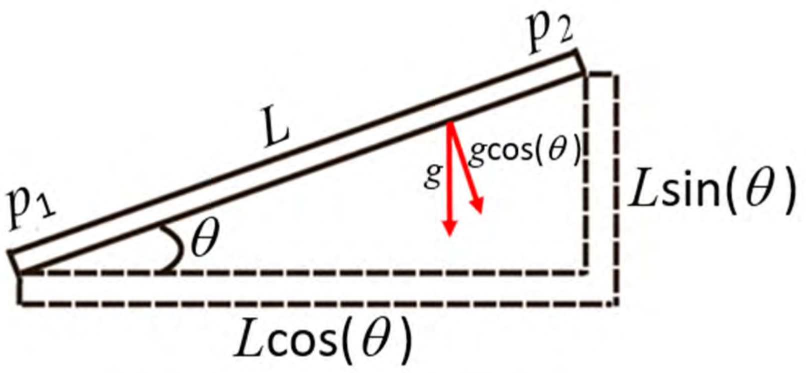
Under these conditions, particle slip is minimized and X_h and X_s solids can be treated as part of the pseudo-homogeneous load. To apply the 4-component model, modify X_p, X_h and X_s accordingly:

$$X_{p,vert} = X_p + X_h + X_s$$

$$X_{h,vert} = X_{s,vert} = 0$$

Note: Only the friction loss is thus calculated. The static lift must be added to this.

Special topic: Inclined conveying



For inclined conveying, the contributions for the stratified components X_h and X_s should be corrected for the angle of inclination.

First, the deposition velocities should be corrected, as they can be up to 50% higher than those for horizontal flow. The new values are then used in calculating the B'' and C'' parameters.

The following method based on Wilson and Tse may be used:

$$V_{sm,\theta} = V_{sm,horiz} + \Delta_D \sqrt{2g\left(S_s / S_f - 1\right)D}$$

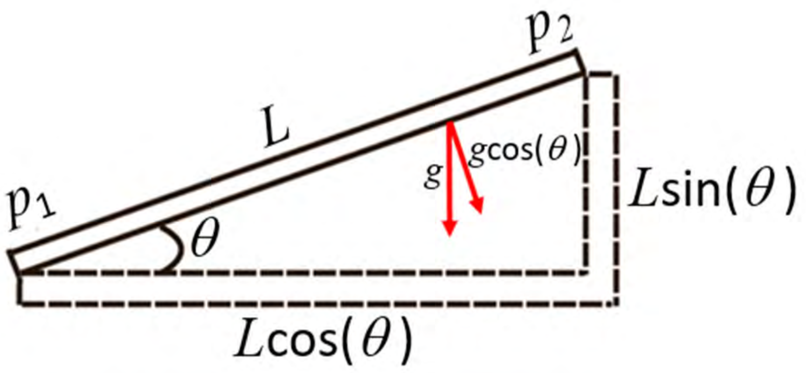
where:

$$\Delta_D = 0.75\theta - \frac{0.50\left(0.6366\theta\right)^2}{1-0.6366\theta} \quad \text{for } \theta > 0$$

$$\Delta_D = 0.75\theta - \frac{0.02\left(2.29\theta\right)^2}{1-2.29\theta} \quad \text{for } \theta < 0$$

with θ expressed in radians $\{\ \text{radians} = \text{degrees} \cdot \left(\pi / 180\right)\ \}$

Special topic: Inclined conveying



The pressure gradient contributions for the X_h and X_s factions are then calculated (using the new B'' and C'' parameters), and further corrected for inclination angle according to the Worster-Denny formula:

$$i_m = i_f + \Delta i_p + (\Delta i_h + \Delta i_s) \cdot \cos\theta$$

Note that this includes only the friction loss component of the pressure gradient. The static lift must be added to this.

This method predicts a reduced pressure gradient (for friction only), however, the in-situ concentration may be higher than expected due to increased slip between particles and fluid. For a conservative calculation in a short line, or at low angles, the $\cos\theta$ term may be omitted.

Also remember that the increase in the deposition velocity V_{sm} must be considered when setting the operating flowrate. This may result in higher required operating velocities and corresponding higher friction losses than in a purely horizontal line.

Pipeline Friction Model Limitations

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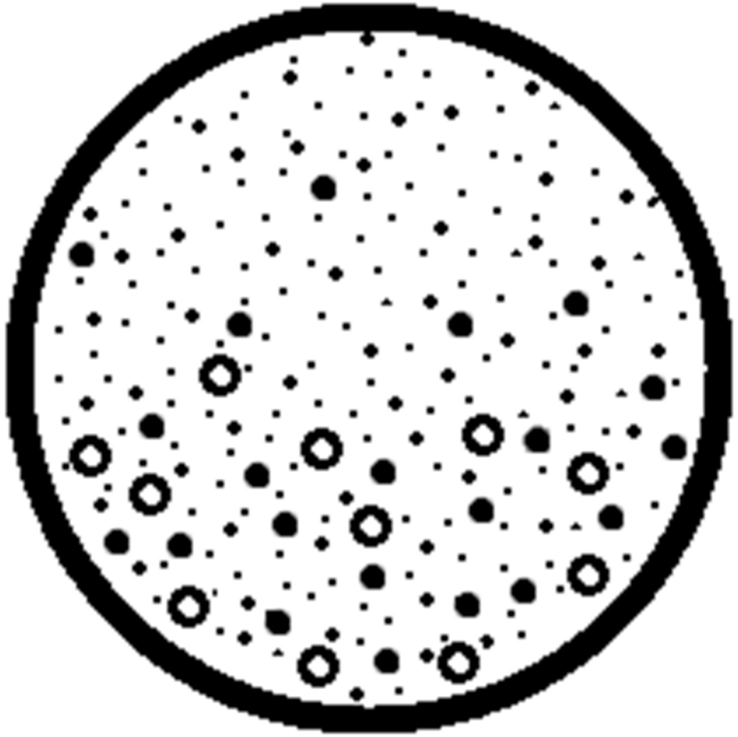
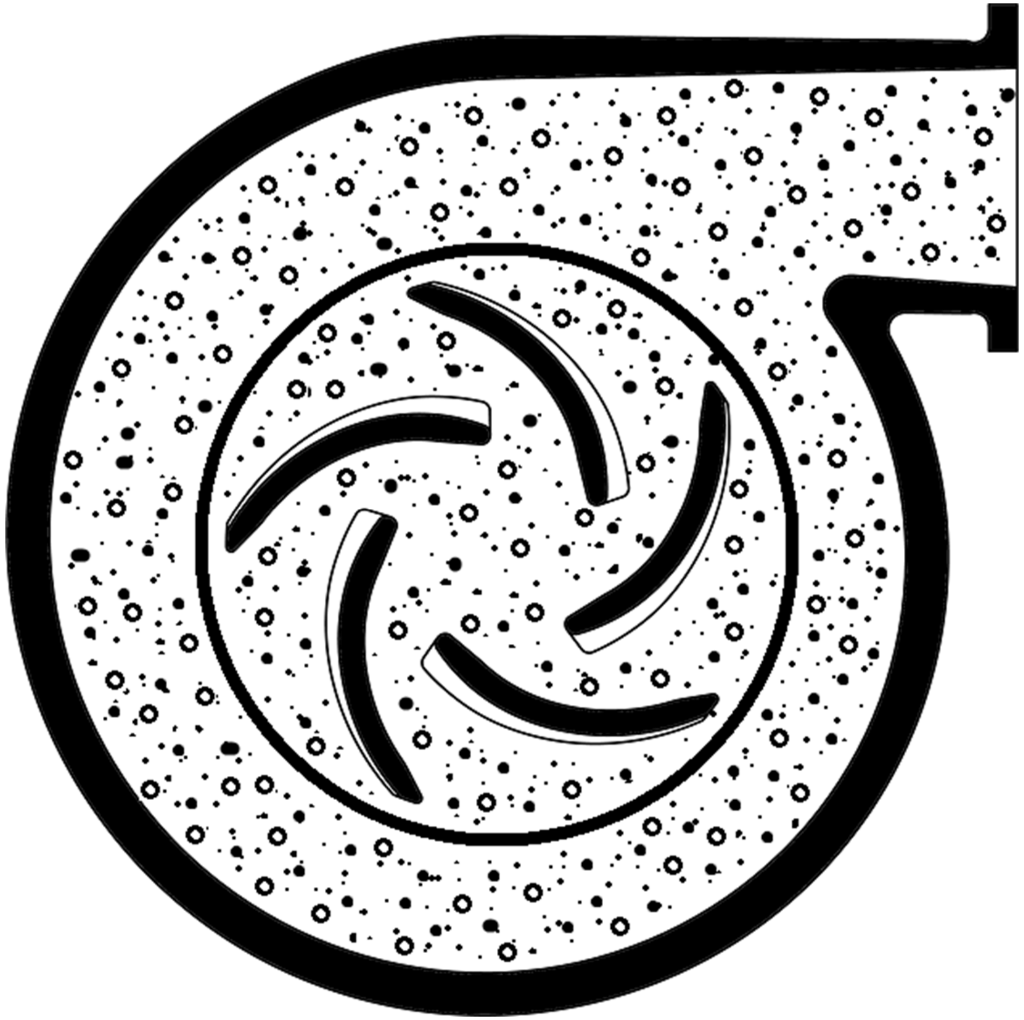


The 4-component slurry

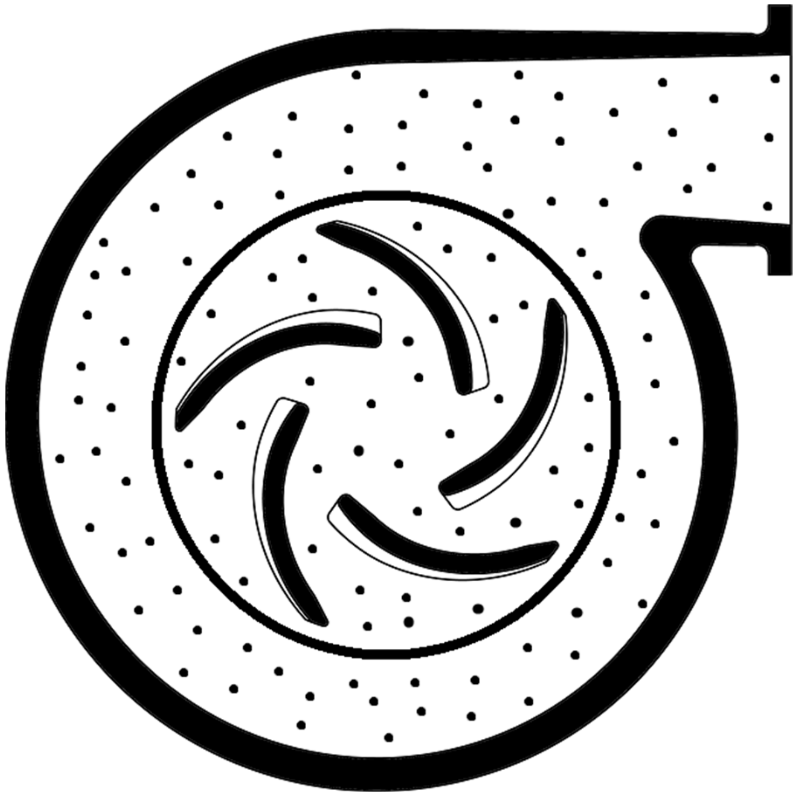
The model as described above is not valid for:

- Pipeline velocities below the limit of stationary deposition. At this velocity, solids begin to deposit in the pipe, effectively changing the shape, size and roughness of the conduit.
    - o *Reasonable approximations of the pressure gradient in a partially filled pipe can be made by using a pipe diameter equal to the hydraulic diameter ($4A/P$) of the partially filled pipe.*
    - o *However, depending on the slurry composition, the flow may become unstable at these low velocities.*
- Cases where the liquid or “Carrier Fluid” exhibit significant non-Newtonian properties.
- There may also be an upper limit on the Newtonian viscosity of the liquid.
    - o *Successful results with liquid viscosities up to 8 cP have been obtained, but the upper limit beyond that is not yet known.*

Centrifugal Pump Solids Effect



Centrifugal Pump Solids Effect



The mono-sized Sellgren - HI model

The baseline model for settling slurry performance effects on centrifugal pumps is the mono-sized particle formulation described by Sellgren (2017) and incorporated into the Hydraulic Institute Standard for Centrifugal Slurry Pumps (ANSI/HI 12.1-12.6-2016):

$$r_h = S_1(1.11/D_2)^{0.9} \cdot (d_{50})^{S2} \cdot [(S_s - 1)/1.65]^{0.65} \cdot (C_v/0.15) \cdot (1-X)^2$$

Where:

r_h = Percent head reduction factor (e.g. 1.0 = 1% derate)

D_2 = Pump impeller outer diameter (m)

d_{50} = 50% weight passing solids diameter (mm) based on screen sieving.

S_S = Solids specific gravity.

C_v = Volumetric concentration of solids.

X = Fraction of solids smaller than 0.075 mm.

S_1 = an empirical function of D_2 ranging from 4.04 to 6.5.

S_2 = 0.4*$(d_{50})^{-0.25}$

Centrifugal Pump Solids Effect

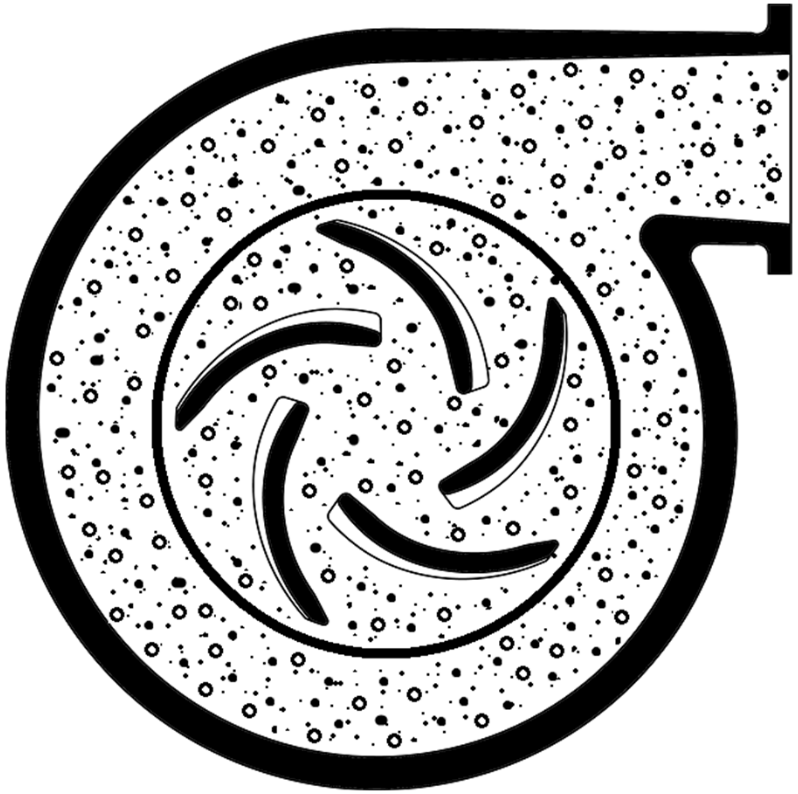
per ANSI/Hydraulic Institute Standard 12.1-12.6-2016

Head reduction factor	1 Pump size effect	2 Particle size effect	3 SG of Solids effect	4 Volumetric concentration effect	5 < 75 μm fines effect

$$r_h = S_1(1.11/D_2)^{0.9} \cdot (d_{50})^{S2} \cdot [(S_s - 1)/1.65]^{0.65} \cdot (C_v/0.15) \cdot (1-X)^2$$

1. An empirical correlation for the effect of pump size relative to a baseline impeller diameter of 1.11 m.
2. The effect of particle size. This term is proportional to the inverse root of the drag coefficient, $C_d^{-0.5}$, for a d_{50} sized particle in water.
3. The effect of solids specific gravity relative to silica based solids (S_s=2.65).
4. The effect of volumetric solids concentration relative to a baseline of 15% (0.15).
5. The effect of fine particles, defined as those with diameter < 0.075 mm.

The 4-Component Model for Pump Solids Effect



The 4-component slurry model

For the 4-component pump solids effect model, we take a similar approach as with the pipeline, calculating the additive effect of each coarser component.

A "root sum of squares" formulation is used to balance the viscous effects of the "Carrier Fluid", against the drag effects of the larger solids. At high viscosity, the "Carrier Fluid" contribution will dominate, whereas at low viscosity the remaining solids terms are most important.

$$r_h = \sqrt{\left(r_{h,f}\right)^2 + \left(r_{h,p} + r_{h,h} + r_{h,s}\right)^2}$$

Where:

r_h = total percent head reduction factor (1.0 = 1% derate)

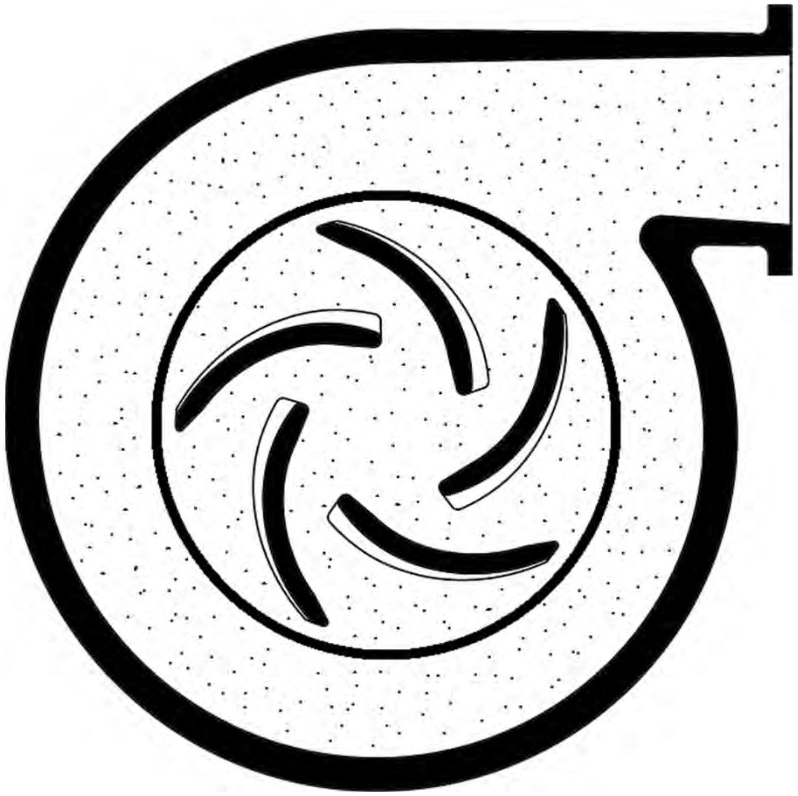
$r_{h,f}$ = contribution of the "Carrier Fluid" viscosity.

$r_{h,p}$ = contribution of the "Pseudo-homogeneous" fraction.

$r_{h,h}$ = contribution of the "Heterogeneous" fraction.

$r_{h,s}$ = contribution of the "Fully Stratified" fraction.

The “Carrier Fluid” fraction



“Carrier Fluid” fraction

X_f

$d < 40\mu m$

The velocities and inertial effects in a centrifugal pump are much greater than in a pipeline. Therefore, small changes in “Carrier Fluid” viscosity do not have a noticeable effect on the head produced.

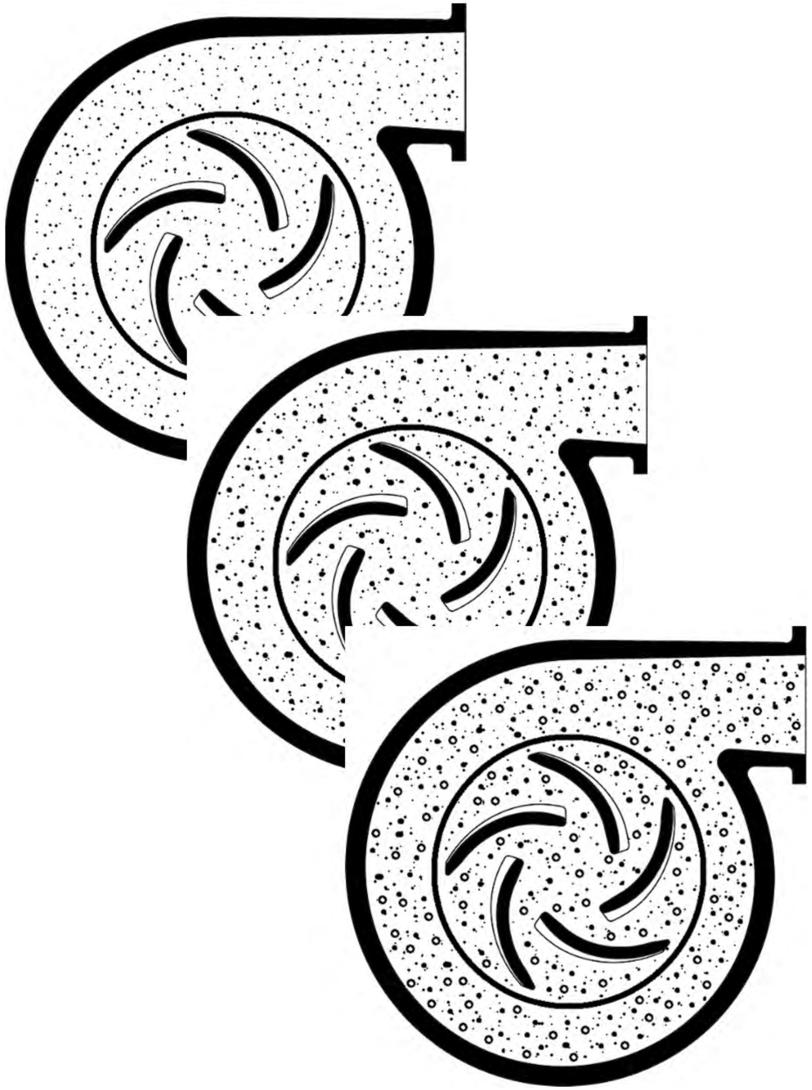
In general, if the apparent viscosity of the “Carrier Fluid” is less than 20 cP (0.02 kg/m-s), it can be assumed that:

$$r_{h,f} = 0$$

For viscosities in excess of 20 cP, the Hydraulic Institute Standard ANSI/HI 9.7.6 for the Effects of Liquid Viscosity on Pump Performance should be used.

Note that the derate provided by this viscosity correction standard varies with flowrate, and is not a fixed value. Therefore, the total r_h derate equation, as shown on the previous slide, must be evaluated on a point-by-point basis along the pump curve.

The remaining fractions: X_p, X_h and X_s



For the remaining fractions, a modified version of the HI equation is used, where the particle size, mixture density and concentration of each individual fraction are considered.

As before, the solids in each fraction are treated as running in a “fluid” or “mixture” with density including all of the previous finer components.

This equation takes the general form:

$$r_{h,x} = N'' \cdot 8(D_{REF}/D_2)^{S1} \cdot (d_{50x})^{S2} \cdot (S_s - S_x)/1.65 \cdot (X_x \cdot C_v/0.15)$$

- *The highlighted terms are selected to represent each fraction, as shown on the next slide.*
- *Note that the “fines effect” term from the original HI equation is omitted, since the effects of each finer component are implicitly included in the 4-component model.*

The remaining fractions: X_p, X_h and X_s

Head reduction factor		Pump size	Particle size	SG of Solids vs. "mixture"	Fraction concentration

$$r_{h,p} = A'' \cdot 8(D_{REF}/D_2)^{S1} \cdot (d_{50p})^{S2} \cdot (S_s\text{-}S_f)/1.65 \cdot (X_p \cdot C_v/0.15)$$

$$r_{h,h} = C'' \cdot 8(D_{REF}/D_2)^{S1} \cdot (d_{50h})^{S2} \cdot (S_s\text{-}S_{fp})/1.65 \cdot (X_h \cdot C_v/0.15)$$

$$r_{h,s} = B'' \cdot 8(D_{REF}/D_2)^{S1} \cdot (d_{50s})^{S2} \cdot (S_s\text{-}S_{fph})/1.65 \cdot (X_s \cdot C_v/0.15)$$

Where:

A'', B'', C'' = 4-Component Empirical Parameters from pipeline model based on pump discharge diameter

D_{REF} = 1.0 m

$S1$ = $0.5 \cdot (D_2/D_{REF})$

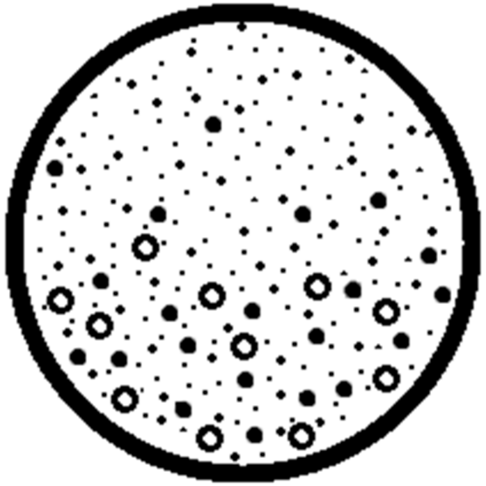
$S2$ = 0.4

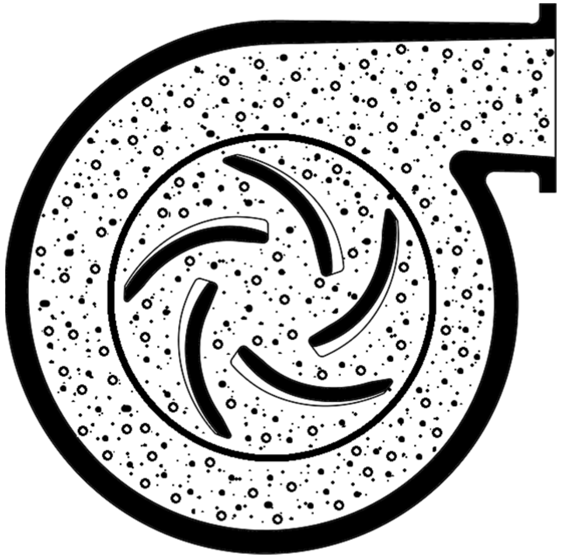
d_{50p} = The average particle size of the "Pseudo-homogeneous" fraction in (mm).

d_{50h} = The average particle size of the "Heterogeneous" fraction in (mm).

d_{50s} = The average particle size of the "Fully Stratified" fraction in (mm).

Modification for high density solids





Limited testing has been carried out on solids with a specific gravity substantially higher than 2.65. However, the results of these tests indicate a shift of the particle size boundary between the X_f and X_p fractions to a lower value provides a better prediction of the measured results.

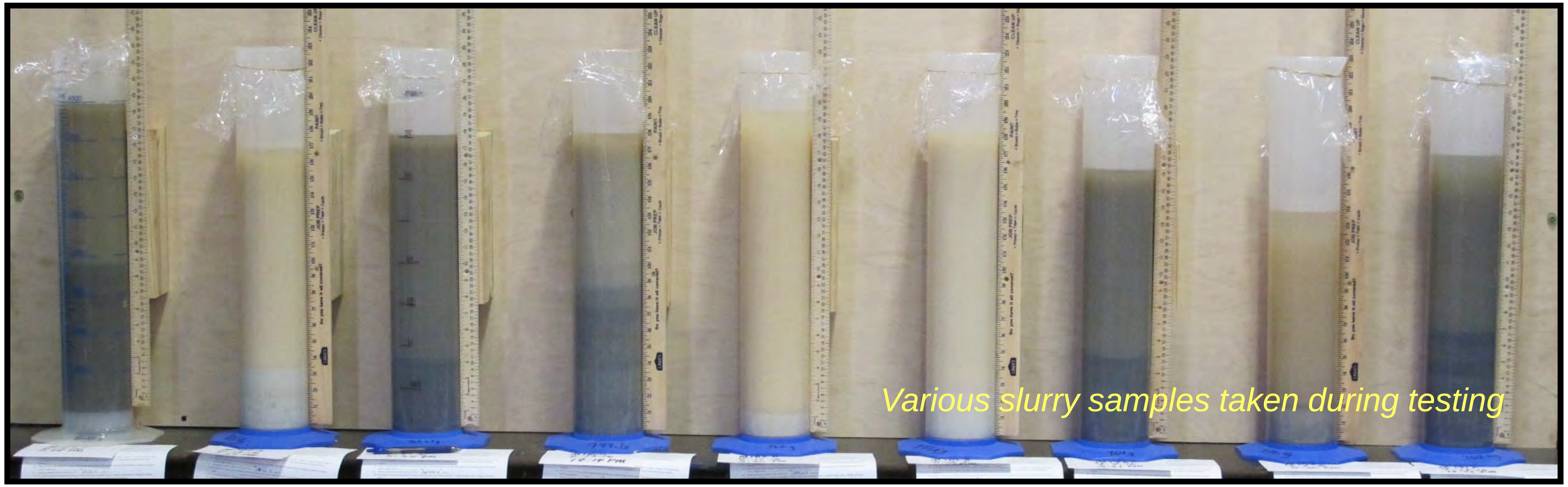
The following correlation has been proposed:

$$X_f \text{ boundary size} = 40\mu\text{m} \cdot \left(\frac{2.65}{S_s} \right)$$

where:

S_s = the solids specific gravity.

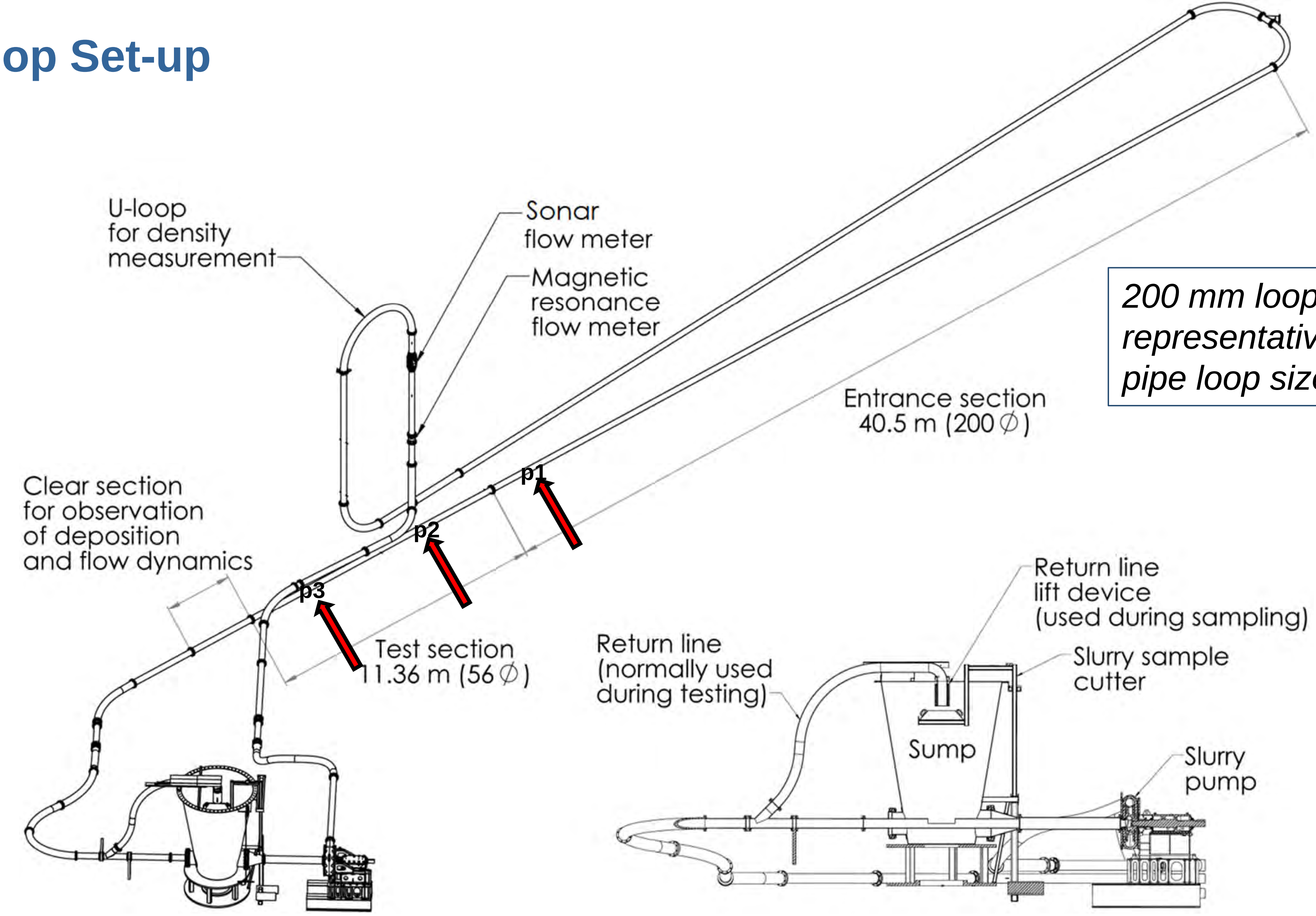
The Test Program



In order to provide a comprehensive data set for validation and calibration of the models, pipe loop test have been carried out over a range of particle size distributions, solids concentrations, pipeline diameters and pump sizes:

- 28 tests in a 200 mm pipe loop with 2.65 SG solids and GIW 8x10 LSA-32 pump with 806.5 mm impeller.
- 12 tests in a 100 mm pipe loop with 2.65 SG solids and GIW 3x4 LCC-12 pump with 310 mm impeller.
- 19 tests in a 100 mm pipe loop with 4.75 SG solids and GIW 4x6 LCC-16 pump with 395 mm impeller.
- 24 tests in a 500 mm pipe loop with 2.65 SG solids and GIW 24x24 TBC 57 pump with 1435 mm impeller.

Pipe Loop Set-up



200 mm loop shown - representative of other pipe loop sizes

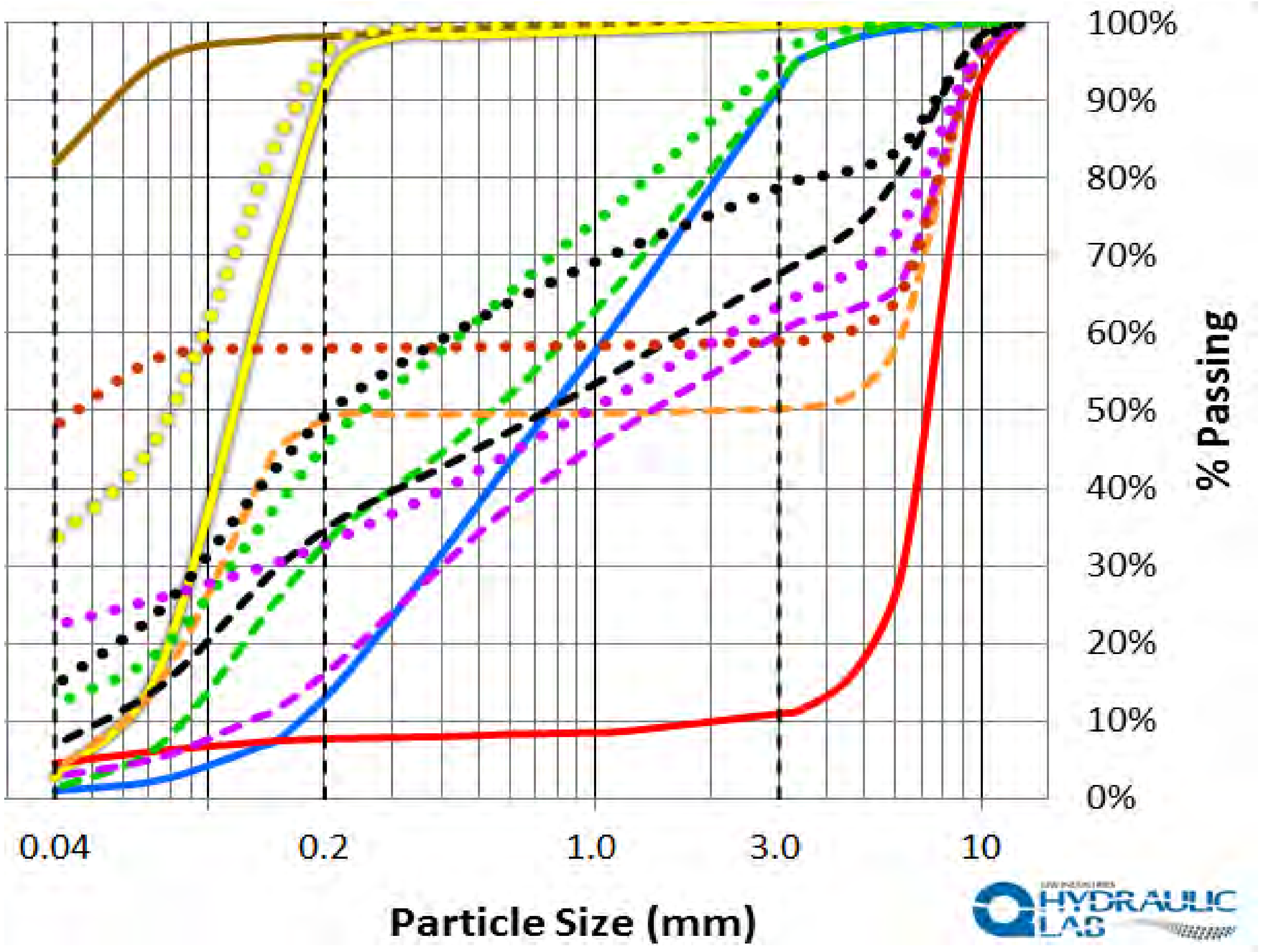
Pump and Sump for 200mm (8”) Pipe Loop

Pump and Sump for 500mm (20”) Pipe Loop

500mm (20”) Pipe Loop

Solids Size Distributions (PSDs) for 200 mm tests

KSB MINING



X_f silica "rock flour"

X_p silica sand

X_h granite

X_s granite

$X_f + X_p$

$X_p + X_h$

$X_f + X_p + X_h$

$X_h + X_s$

$X_f + X_h + X_s$

$X_p + X_h + X_s$

$X_f + X_p + X_h + X_s$

$X_p + X_s$ bi-modals

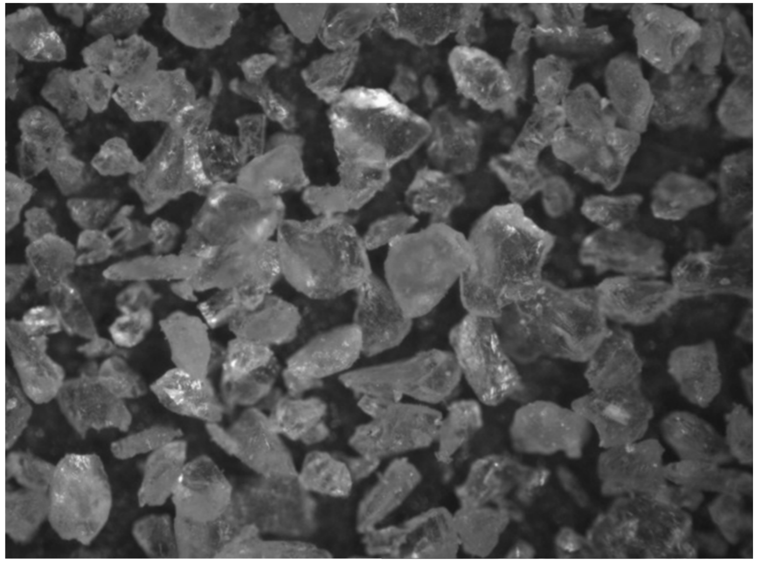
$X_f + X_s$ bi-modals

Solids Size Distributions (PSDs) for 200 mm tests

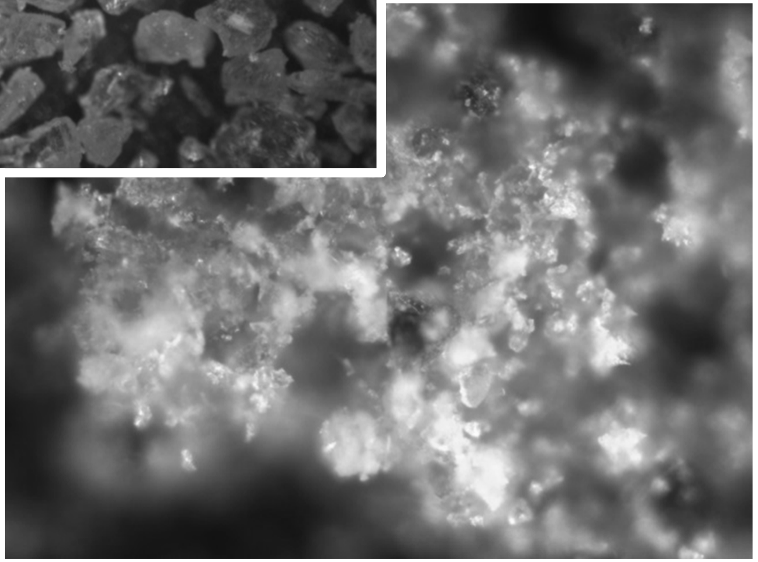
KSB MINING

X_s fraction

X_p fraction magnified



X_h fraction with magnified inset



X_f fraction magnified

Test Program – Matrix of tests

28 primary tests in the 200 mm loop, 12 of these repeated in the 100 mm loop.

Table 1. Test matrix showing target percentages for volumetric concentration.
(All tests were carried out in 200 mm loop, highlighted tests repeated in 100 mm.)

Test	1	2	3	4a	4b	5a	5b	6a	6b	7a	7b	7c	7d	8a	8b	8c	8d	8e	9a	9b	10a	10b	11a	11b	12	13	14	15
Xf										5	5	5	5	10	10	10	10	10										
Xp						5	5	5	5		5	5	5					5			10	10	10	10		15		
Xh				10	10			10	10			10	10				20	20	20	20			20	20	30			
Xs	5	10	15		5		5		5				5		5	10	10	10		10		10		10		15	20	30
Total	5	10	15	10	15	5	10	15	20	5	10	20	25	10	15	20	40	45	20	30	10	20	30	40	30	30	20	30

Table 2. Test matrix showing actual measured fraction content and volumetric concentration from the 200 mm loop tests.

Test	1	2	3	4a	4b	5a	5b	6a	6b	7a	7b	7c	7d	8a	8b	8c	8d	8e	9a	9b	10a	10b	11a	11b	12	13	14	15
Xf	.05	.02	.03	.01	.03	.03	.04	.01	.03	.82	.33	.12	.15	.82	.48	.34	.22	.24	.01	.06	.02	.03	.01	.07	.02	.07	.04	.06
Xp	.03	.01	.02	.05	.08	.66	.35	.17	.19	.15	.51	.23	.20	.17	.10	.05	.06	.15	.06	.07	.77	.38	.17	.16	.05	.33	.02	.01
Xh	.04	.02	.03	.86	.50	.31	.15	.76	.45	.03	.15	.61	.45	.01	.01	.02	.35	.30	.85	.49	.20	.09	.74	.45	.86	.10	.06	.09
Xs	.89	.96	.92	.08	.40	.00	.46	.06	.32	.00	.00	.04	.21	.00	.41	.59	.36	.31	.08	.39	.01	.50	.07	.32	.07	.50	.89	.84
Cv	5%	11%	15%	12%	16%	5%	10%	16%	21%	4%	8%	20%	24%	8%	14%	18%	34%	38%	22%	29%	10%	21%	31%	35%	25%	30%	20%	30%

Sample taking

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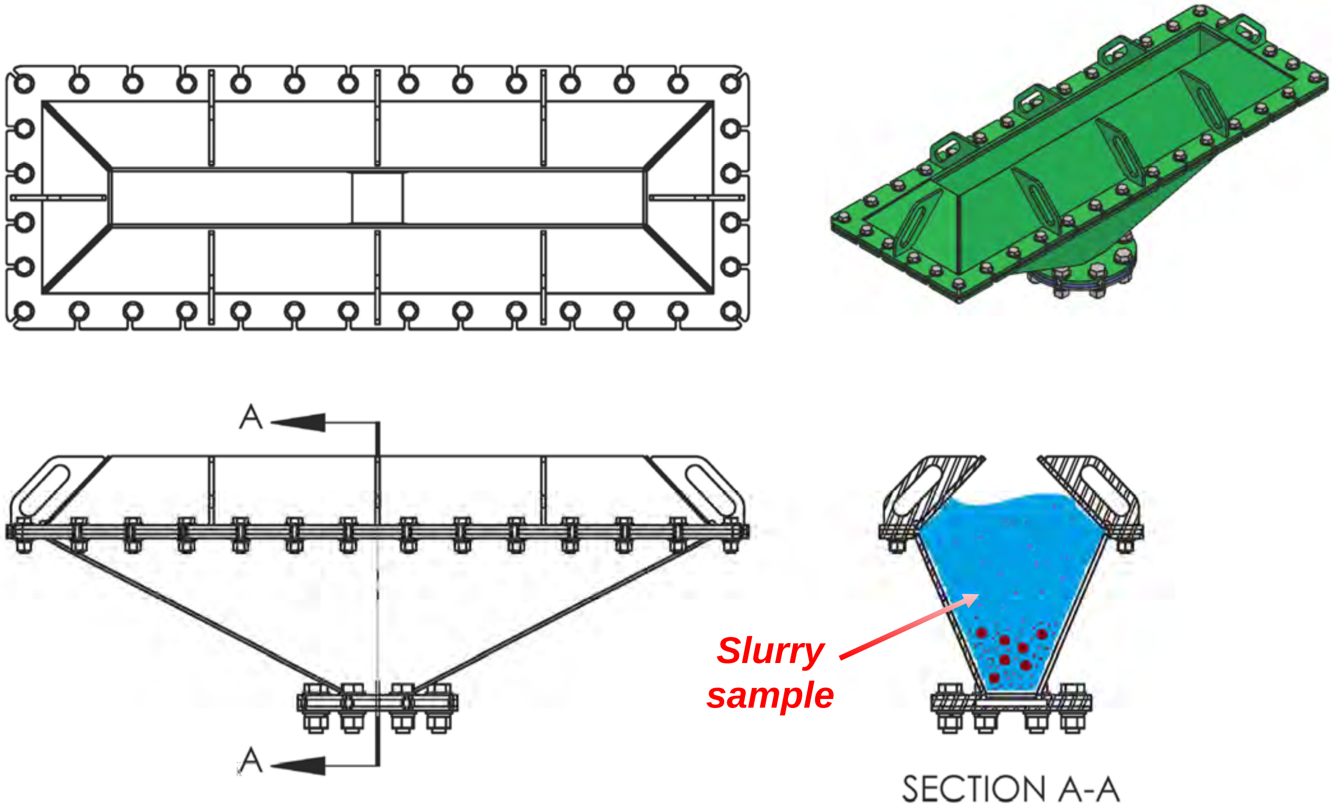
- Retractable discharge pipe lifted above surface of slurry.
- Sample cutter passes through entire flow stream.
- Sample taken at slurry velocity about midway between deposition and pseudo-homogeneous flow.

Pipe system	Solids SG	Sample cutter opening (mm x mm)	Sample cutter velocity (m/s)	Slurry velocity (m/s)	Ave. sample volume (litres)
500 mm (20 inch)	2.65	76 x 760	2	7.5	22
200 mm (8 inch)	2.65	27 x 600	1	5	8
100 mm (4 inch)	2.65	30 x 310	0.45	4	3.5
100 mm (4 inch)	4.75	20 x 310	0.5	4.5	2

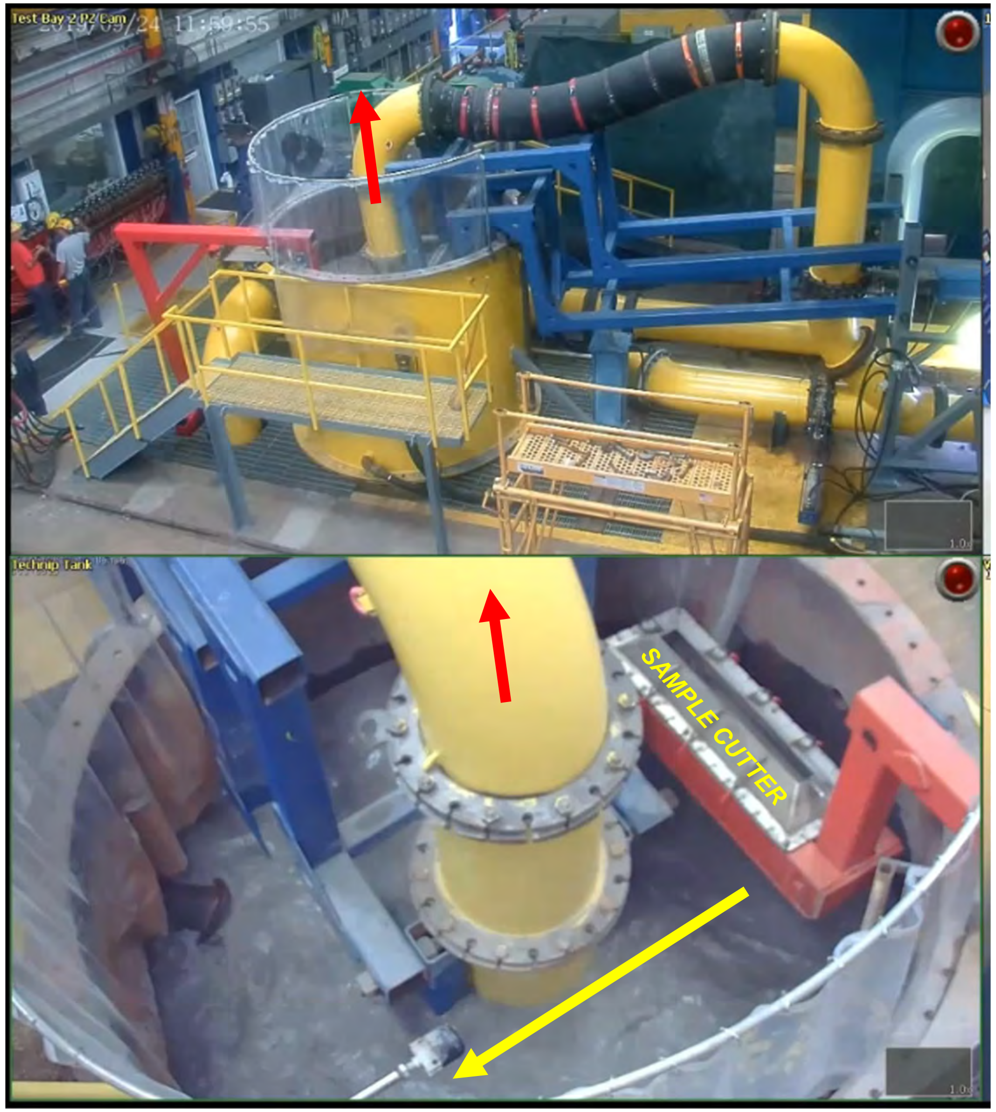
500 mm (20”) Pipe Loop Sample Cutter

Sample cutter positioned directly underneath the raised return pipe.



Typical sample cutter geometry

500 mm (20”) Sample Taking



In video at left, sample cutter is ready to cut a sample. The procedure is as follows:

1. The return pipe (yellow elbow) is raised (red arrow) about one meter above the level of the liquid.
2. The sample cutter (silver box with trapezoidal cross section), attached to the swing arm (orange beam), passes through the entire flow from right to left (yellow arrow).
3. The return pipe is quickly lowered back into the sump and below the liquid level, to reduce air entrainment.
4. The sample cutter is removed and emptied, and the sample processed to obtain the particle size distribution.

Emptying the 500 mm sample cutter

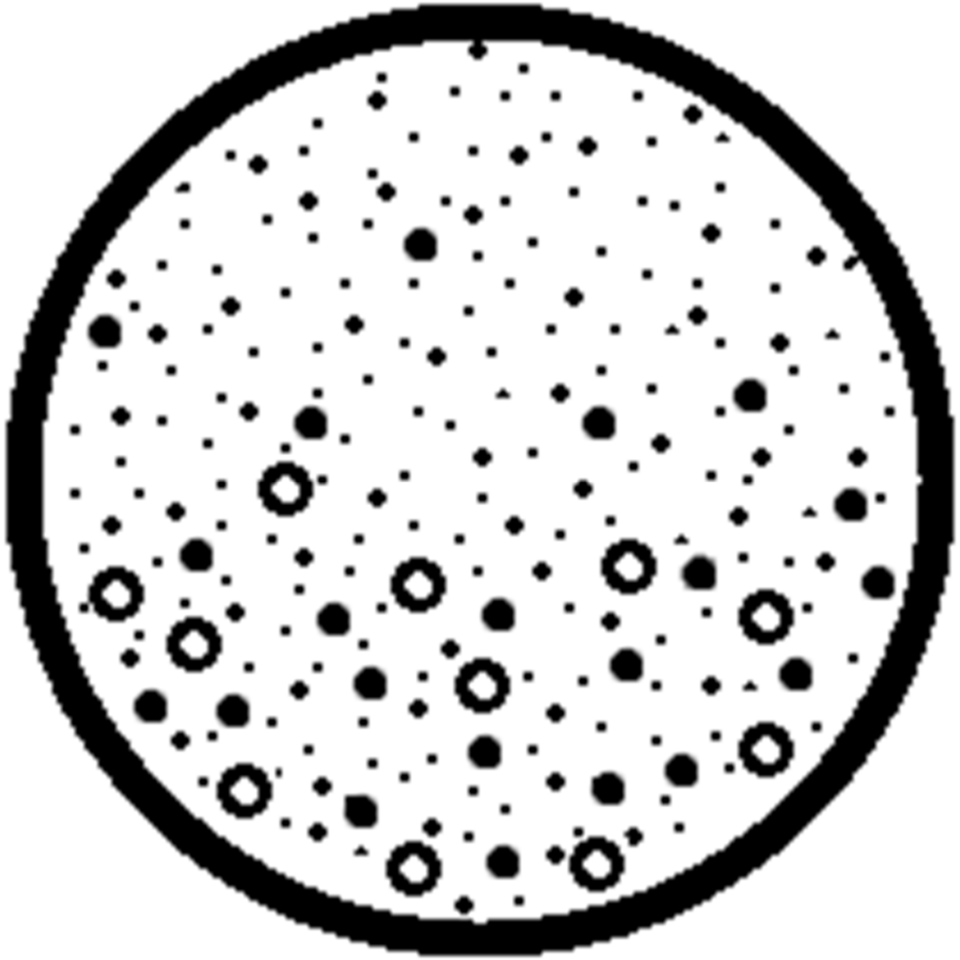
500 mm (20”) Sample Taking

Top view of sump and sample taking mechanism

Sample taking

- PSDs processed dry.
- First cut to 150 µm.
- Additional sieving of fines down to 40 µm.
- Special care taken to avoid fines adhering to coarse during sieving process.
- Accurate determination of fines is important!

Pipeline Friction Loss Results – 200 mm (8”) loop



Empirical Parameters – 200 mm loop

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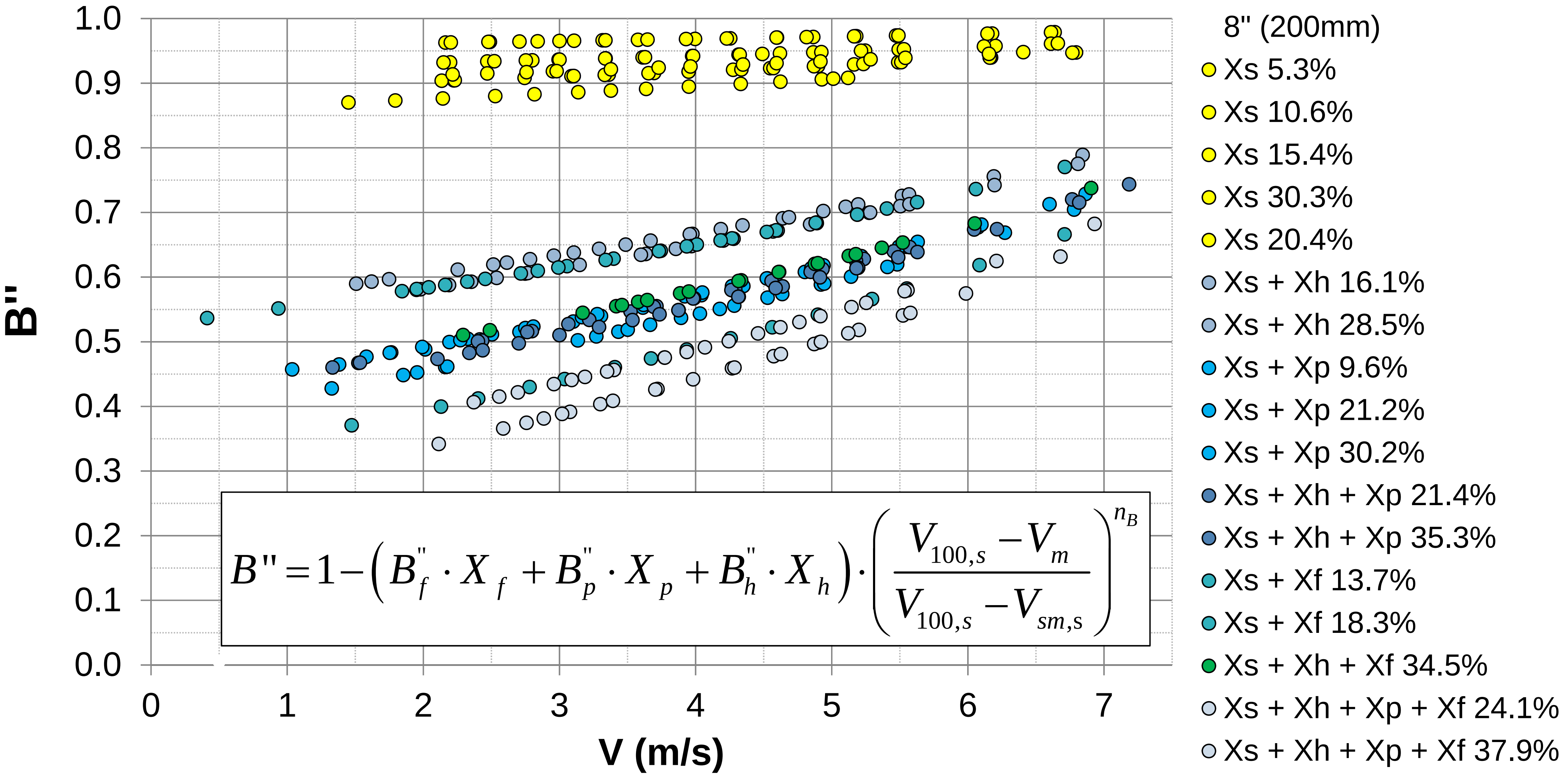
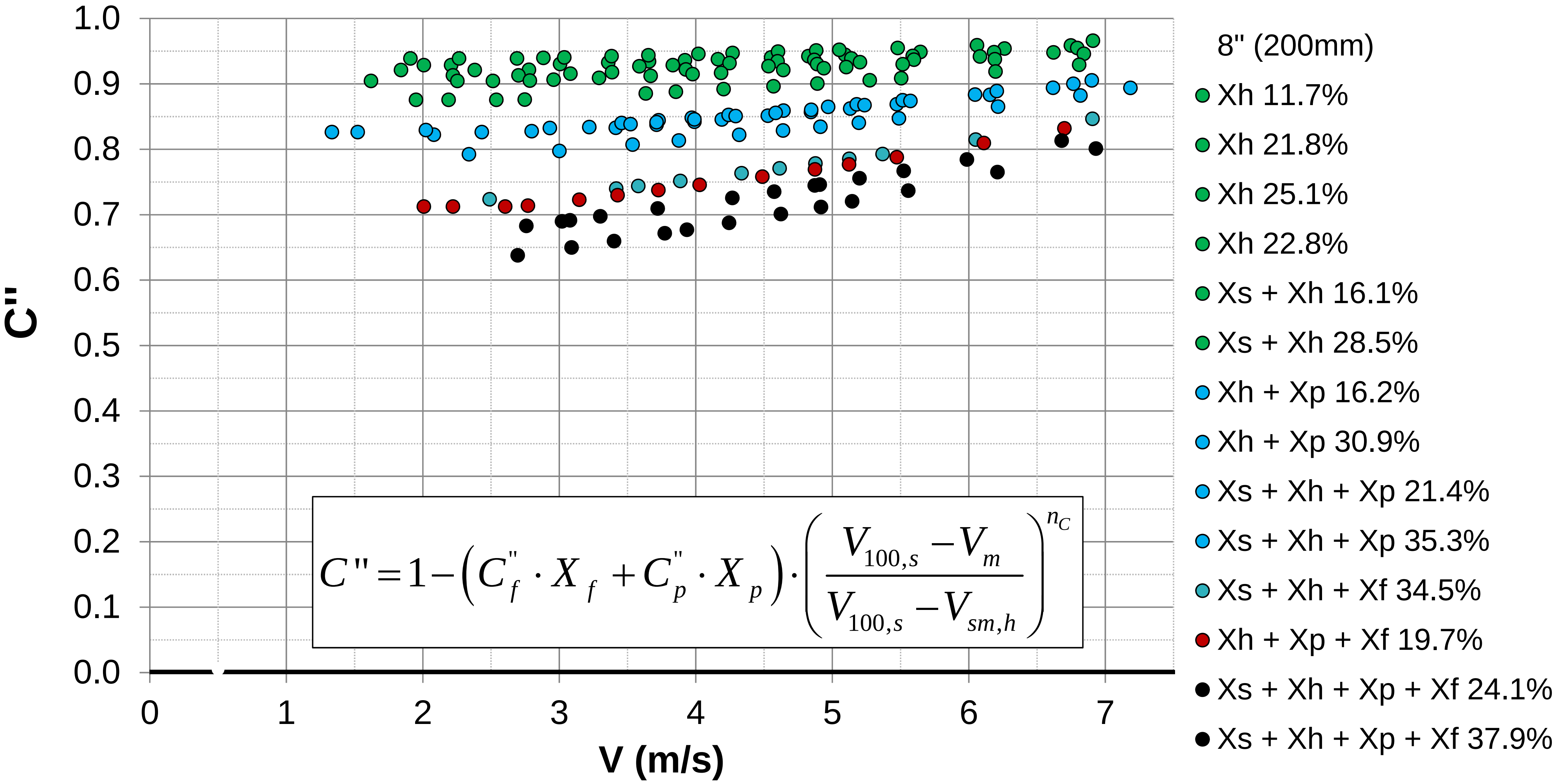


$$B'' = 1 - \left(B''_f \cdot X_f + B''_p \cdot X_p + B''_h \cdot X_h\right) \cdot \left(\frac{V_{100,s} - V_m}{V_{100,s} - V_{sm,s}}\right)^{n_B}$$

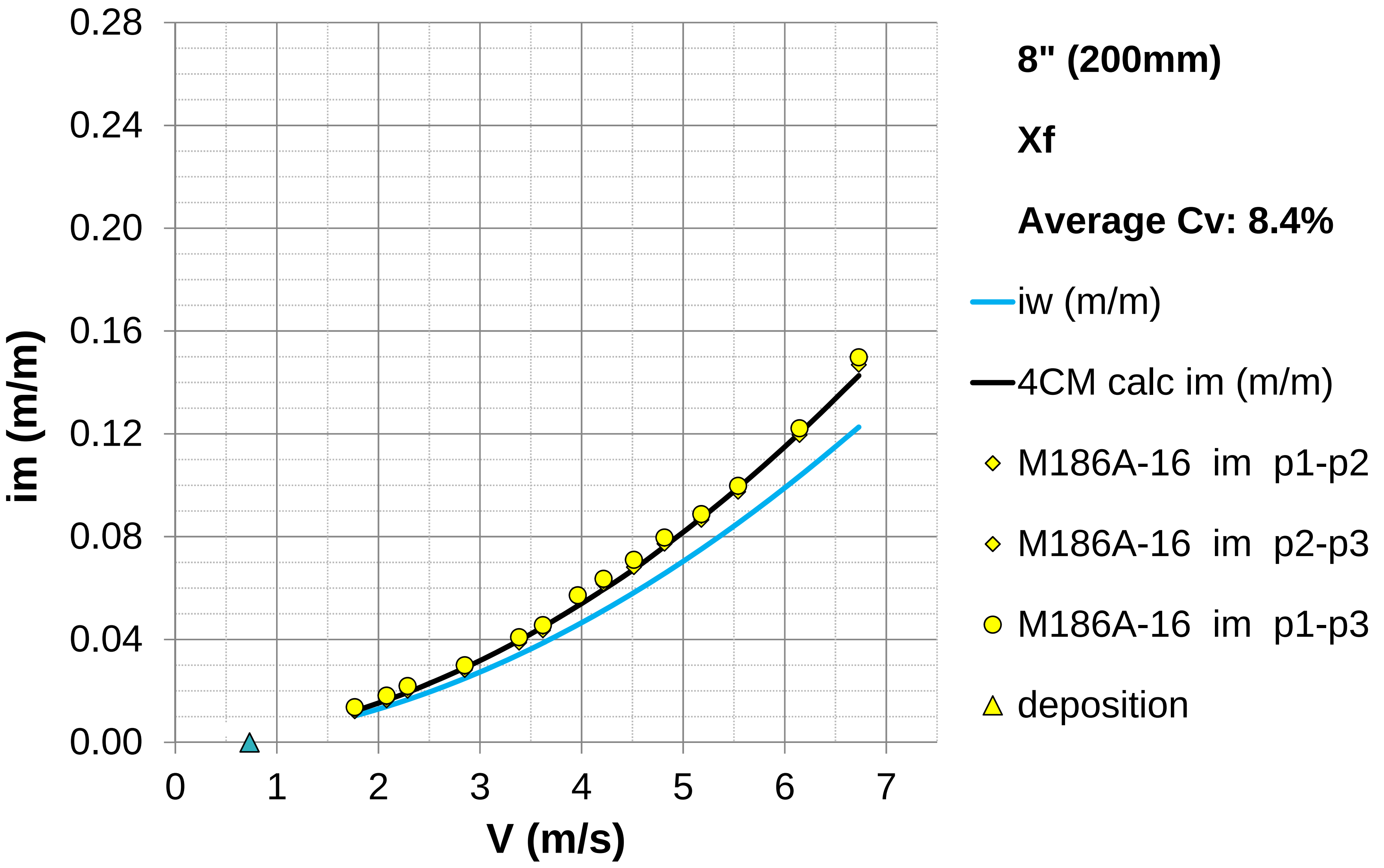
Empirical Parameters – 200 mm loop

KSB MINING



$$C'' = 1 - \left(C_f'' \cdot X_f + C_p'' \cdot X_p \right) \cdot \left(\frac{V_{100,s} - V_m}{V_{100,s} - V_{sm,h}} \right)^{n_C}$$

Individual results, 200 mm loop



Fine

C_v : 8.4%

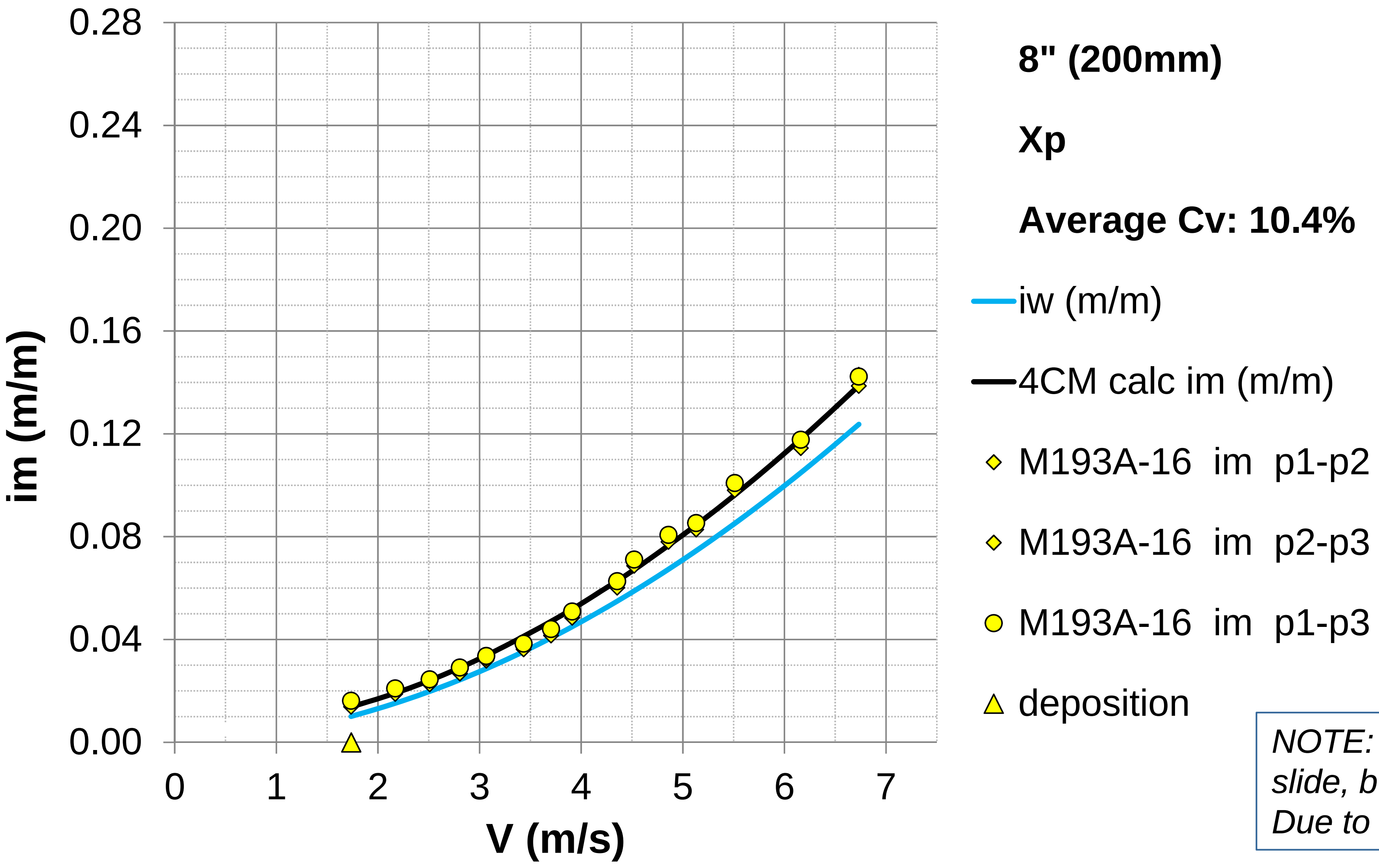
X_f : 0.823

X_p : 0.167

X_h : 0.008

X_s : 0.003

Individual results, 200 mm loop

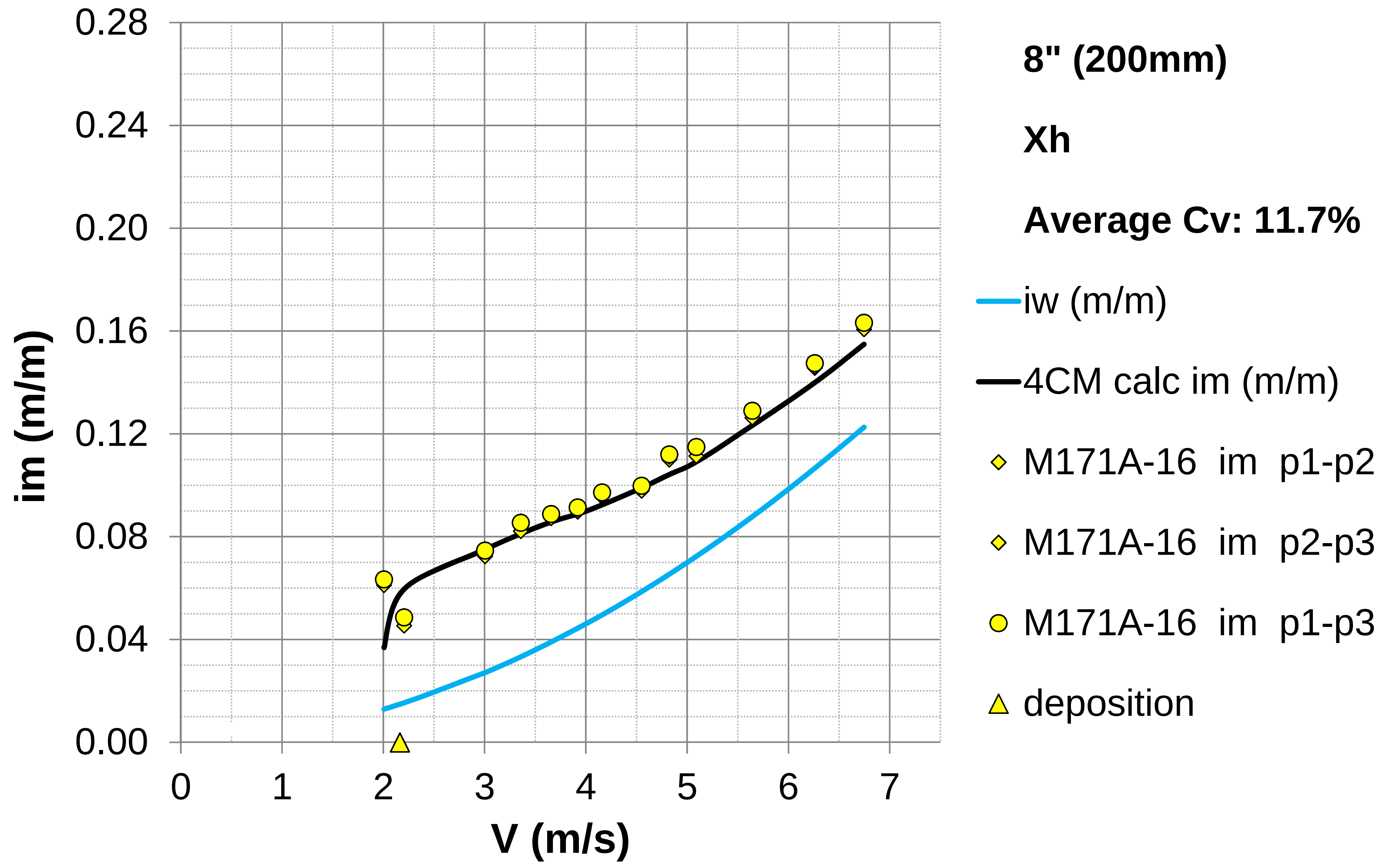


Pseudo-homogenous

C_v :	**10.4%**
X_f :	**0.021**
X_p :	**0.771**
X_h :	**0.201**
X_s :	**0.007**

NOTE: Slightly higher C_v than previous slide, but slightly lower friction losses. Due to effect of near wall lift.

Individual results, 200 mm loop



Heterogeneous

C_v : **11.7%**

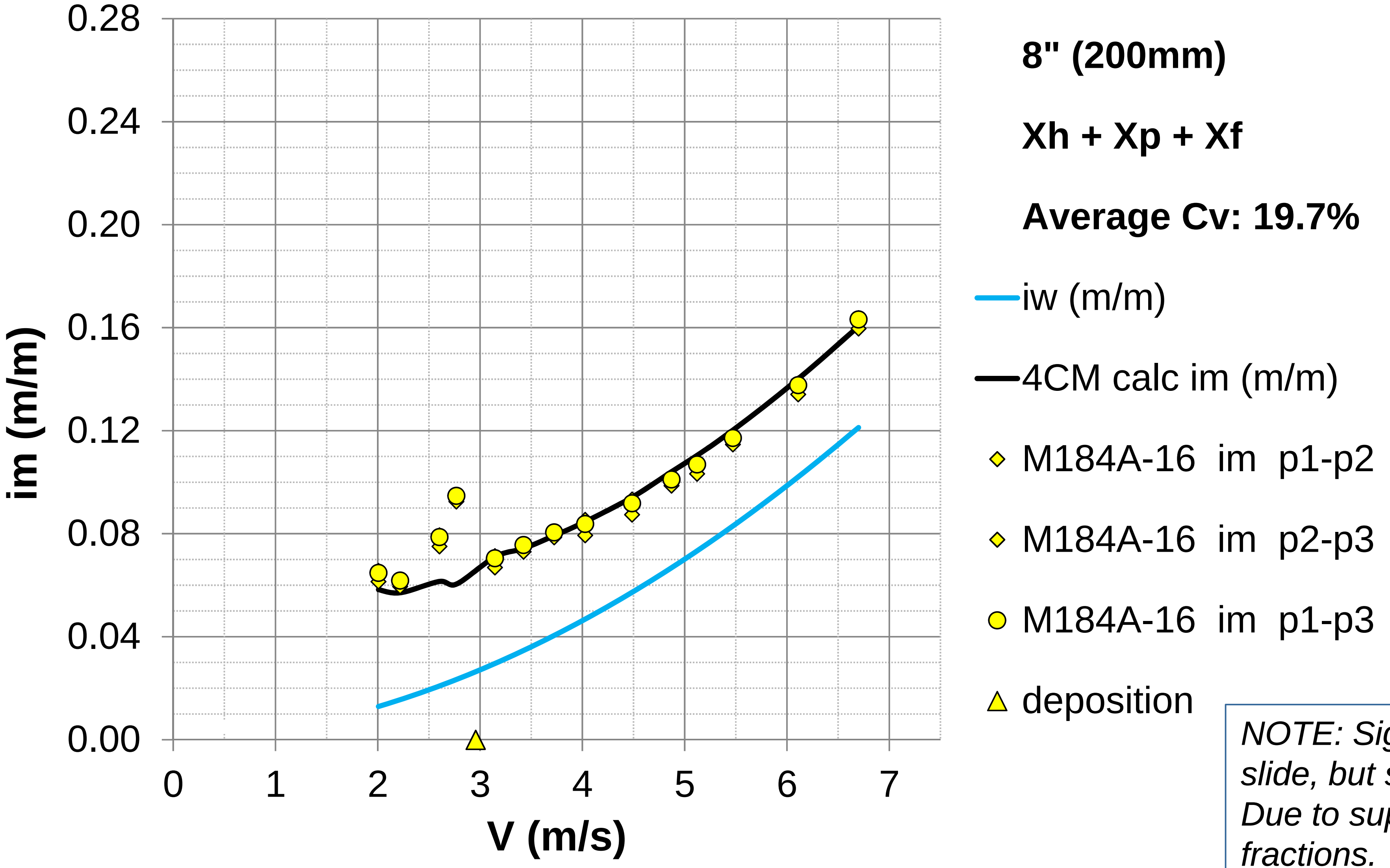
X_f : **0.010**

X_p : **0.058**

X_h : **0.852** *(C_v=10%)*

X_s : **0.078**

Individual results, 200 mm loop



Medium-Fine mix

C_v : **19.7%**

X_f : **0.121**

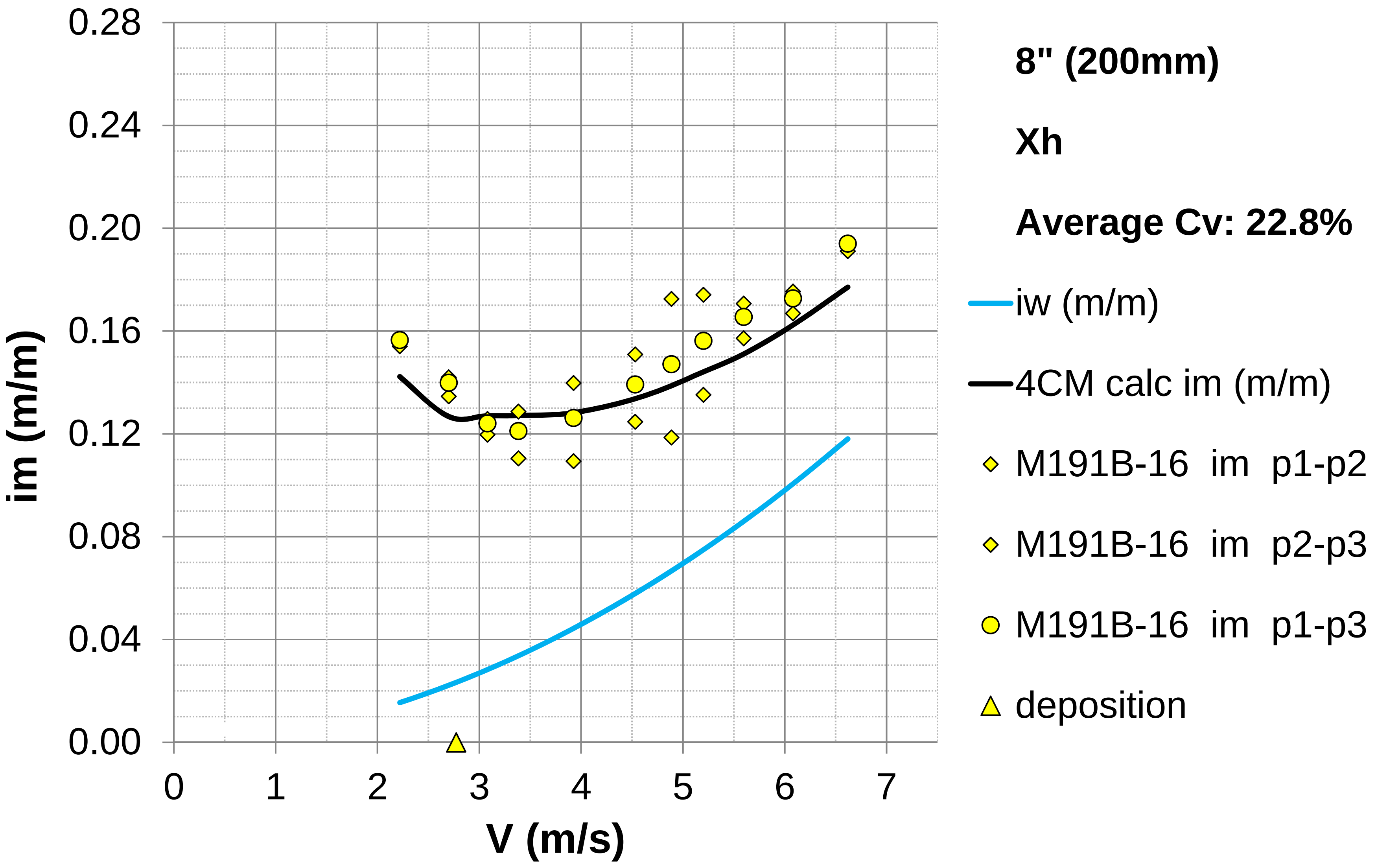
X_p : **0.244**

X_h : **0.591** *(C_v=11.6%)*

X_s : **0.045**

NOTE: Significantly higher C_v than previous slide, but slightly lower friction losses. Due to supporting effects of Xf and Xp fractions.

Individual results, 200 mm loop



Heterogeneous

C_v : 22.8%

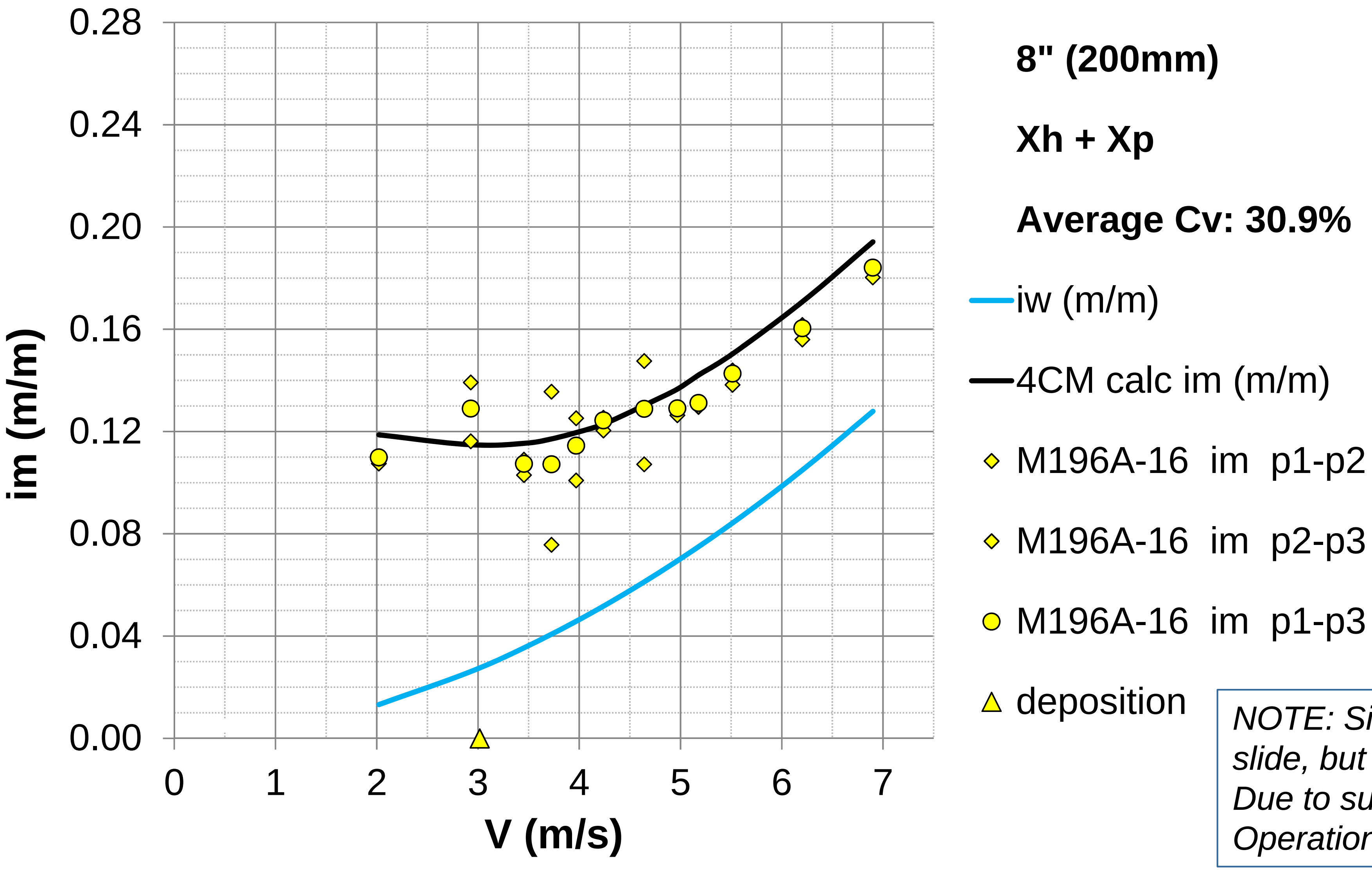
X_f : 0.025

X_p : 0.073

X_h : 0.812 (C_v=18.5%)

X_s : 0.085

Individual results, 200 mm loop



Medium mix

C_v : **30.9%**

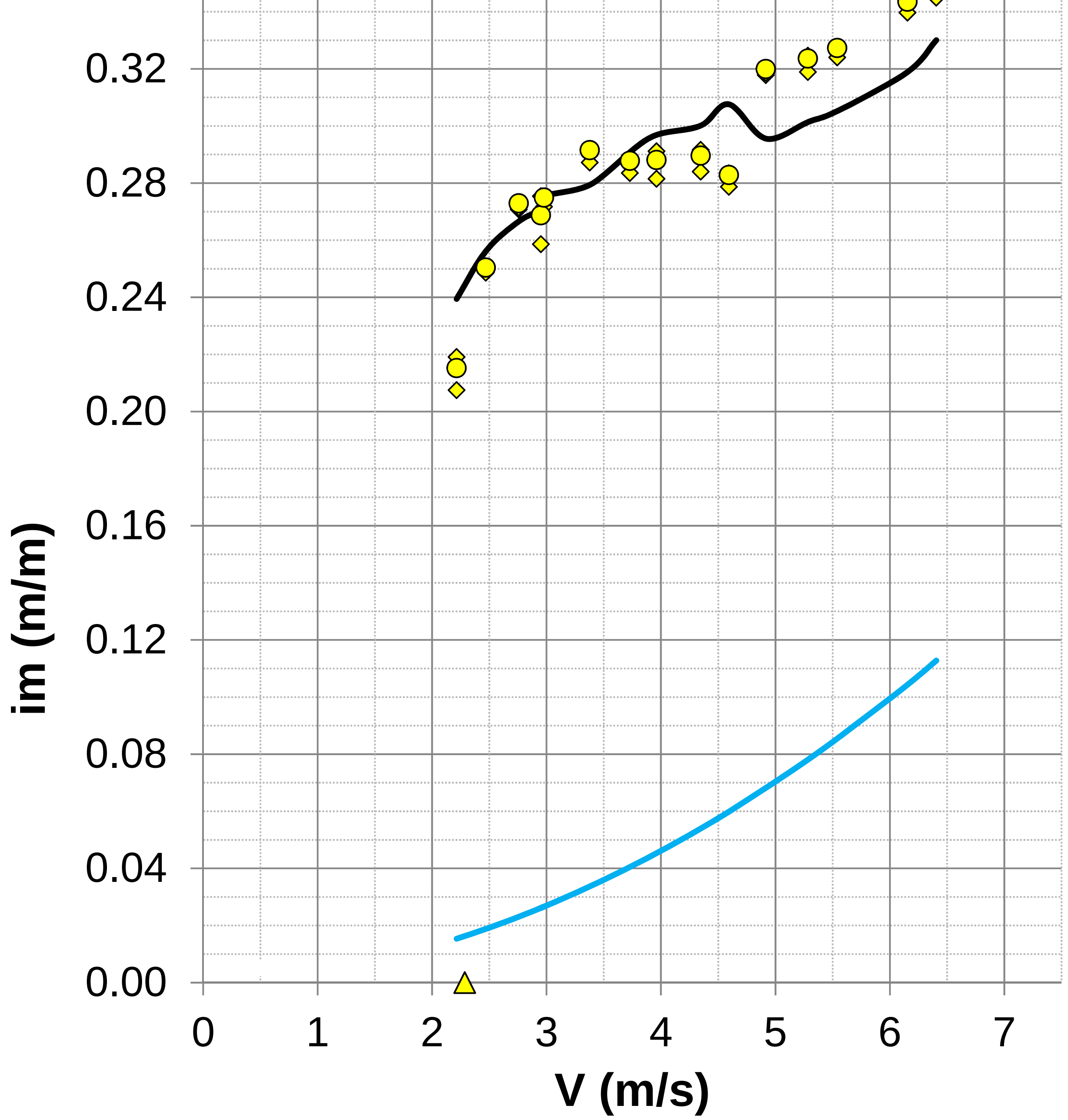
X_f : **0.013**

X_p : **0.232**

X_h : **0.680** *(C_v=21%)*

X_s : **0.075**

NOTE: Significantly higher C_v than previous slide, but slightly lower friction losses. Due to supporting effects of Xp fractions. Operation less stable.



8" (200mm)

Xs

Average Cv: 20.4%

iw (m/m)

4CM calc im (m/m)

M198 -16 im p1-p2

M198 -16 im p2-p3

M198 -16 im p1-p3

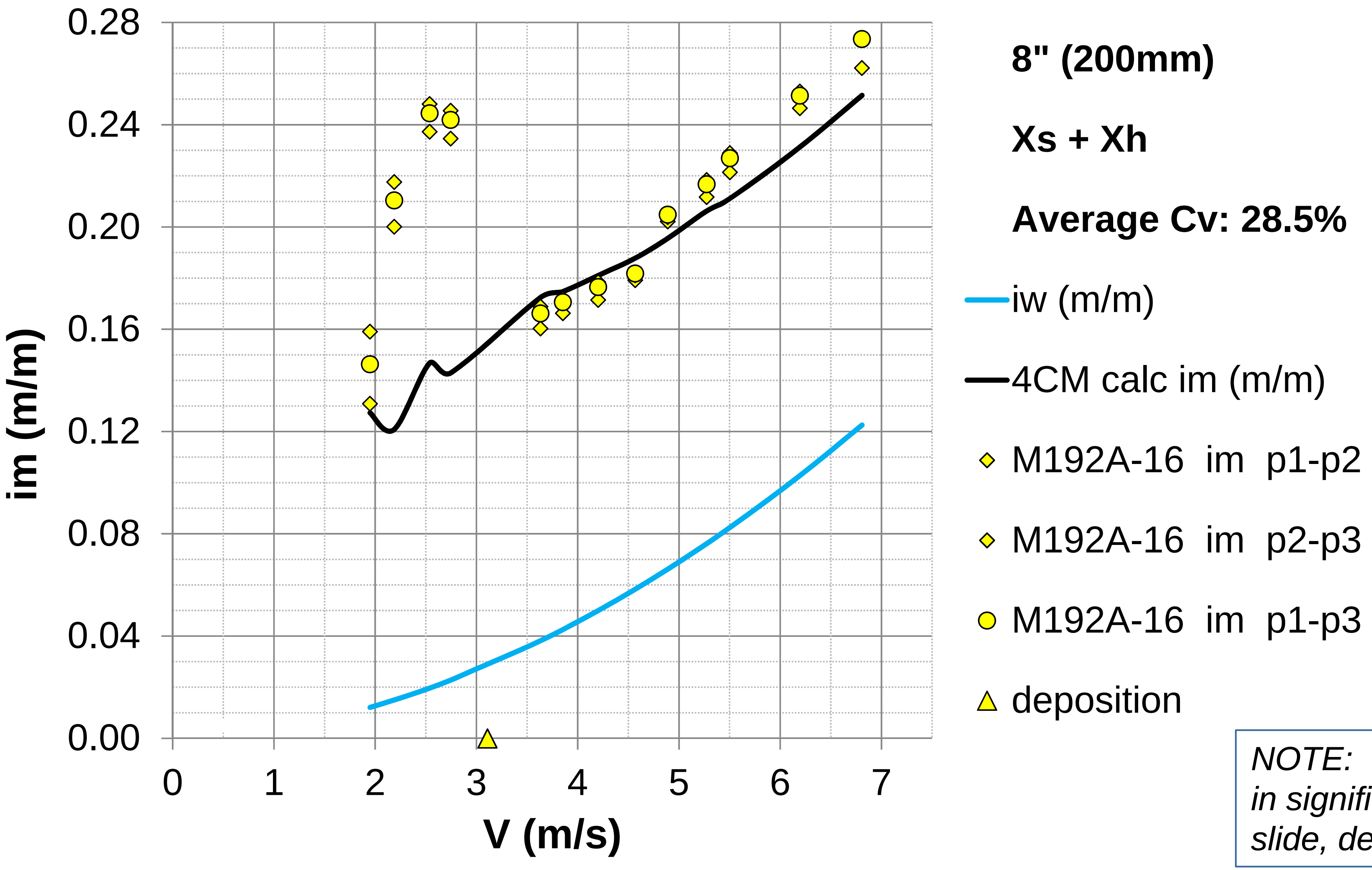
deposition

Fully Stratified

C_v :	**20.4%**
X_f :	**0.037**
X_p :	**0.016**
X_h :	**0.061**
X_s :	**0.886** *(C_v=18%)*

NOTE: Stratified only solids result if very high friction losses.

Individual results, 200 mm loop



Coarse mix

C_v : **28.5%**

X_f : **0.056**

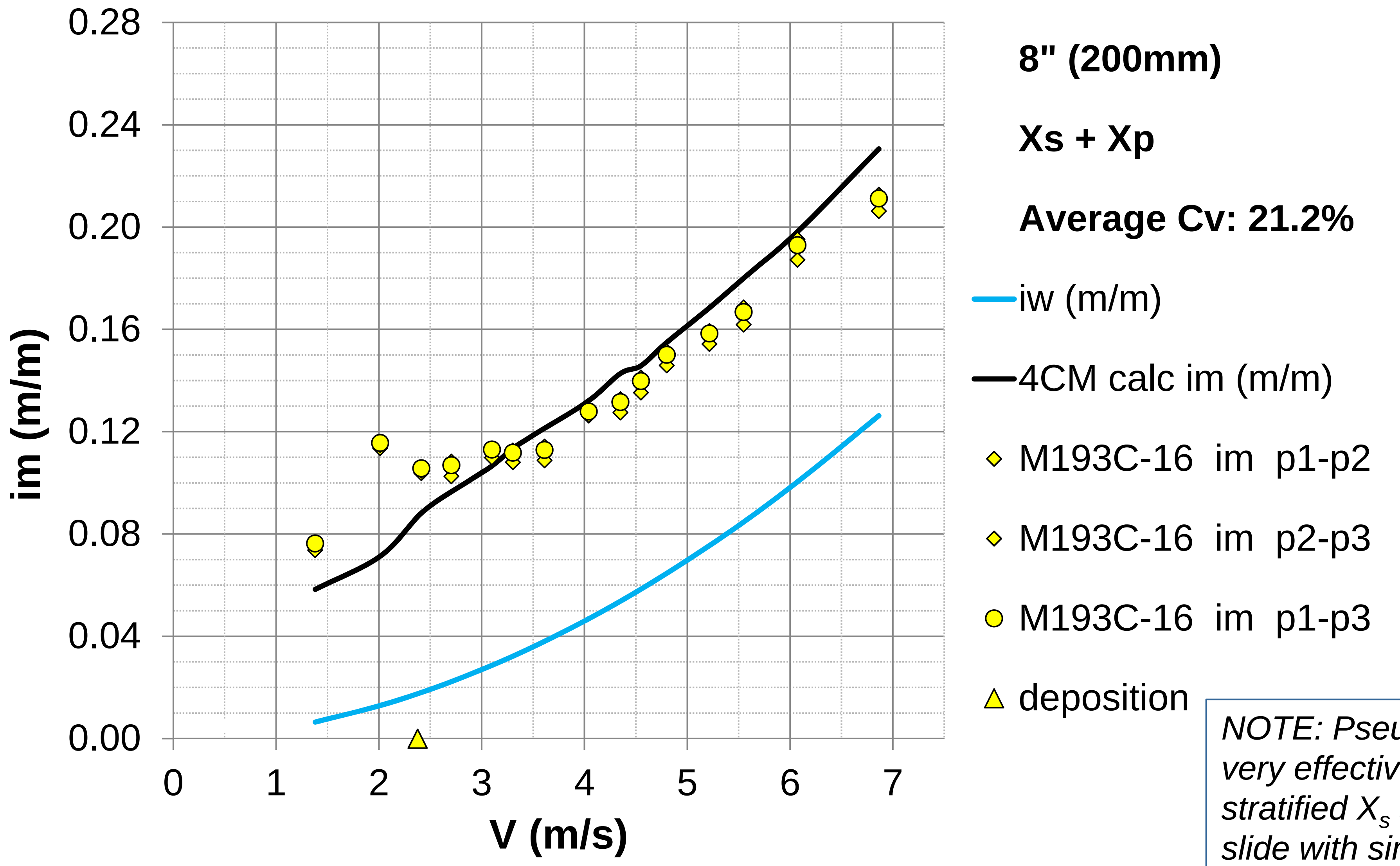
X_p : **0.087**

X_h : **0.472** *(C_v=13.5%)*

X_s : **0.385** *(C_v=11%)*

NOTE: Shift from Xs to Xh solids results in significantly lower losses than previous slide, despite higher total conventration

Individual results, 200 mm loop



Bi-modal

C_v : 21.2%

X_f : 0.035

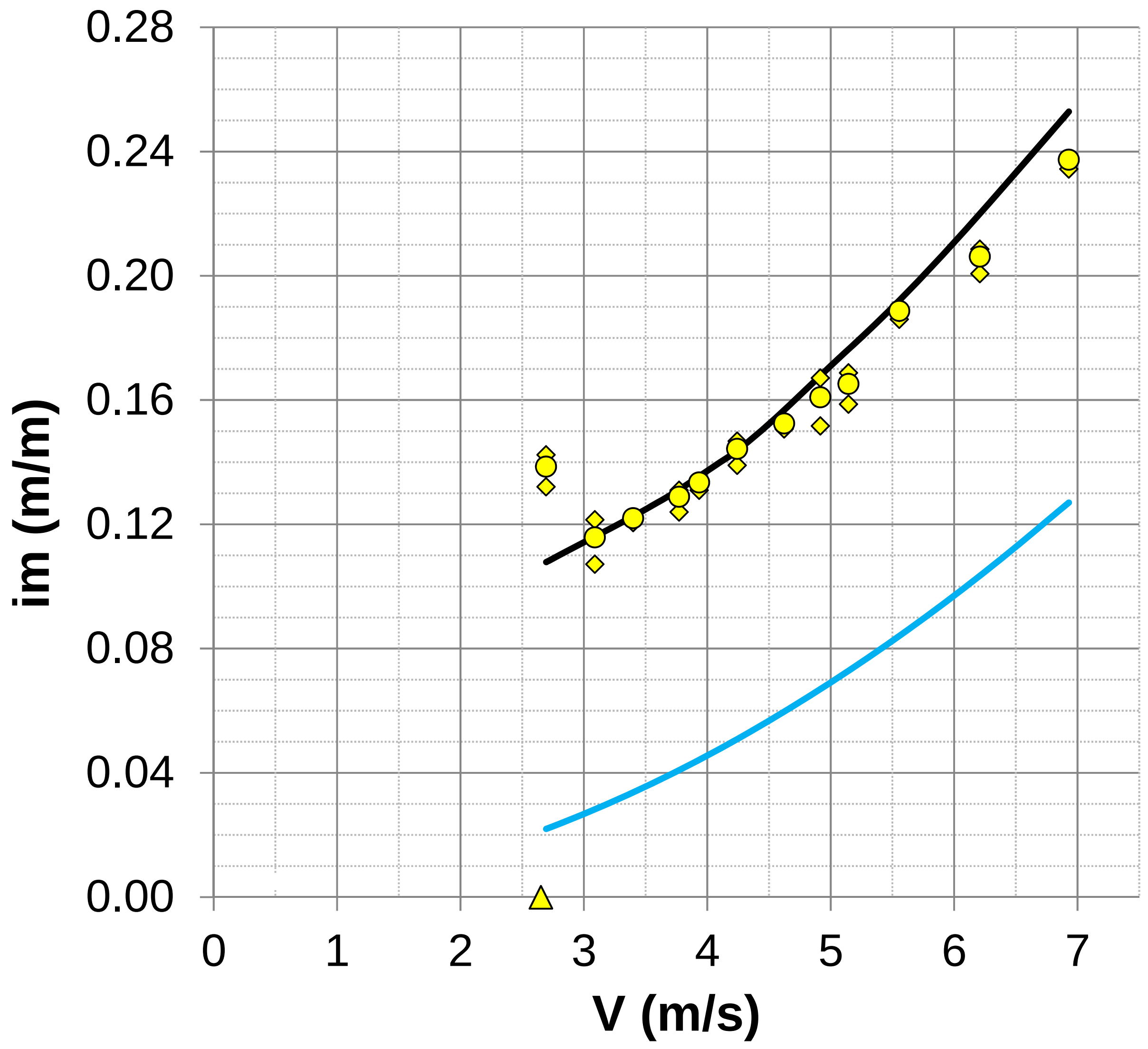
X_p : 0.451

X_h : 0.016

X_s : 0.498 *(C_v=10.5%)*

NOTE: Pseudo-homogeneous X_p solids are very effective at supporting the fully stratified X_s solids. (Compare with previous slide with similar X_s concentration.)

Individual results, 200 mm loop



8" (200mm)

Xs + Xh + Xp + Xf

Average Cv: 37.9%

iw (m/m)

4CM calc im (m/m)

M189A-16 im p1-p2

M189A-16 im p2-p3

M189A-16 im p1-p3

deposition

Broad, high Cv

C_v : **37.9%**

X_f : **0.239**

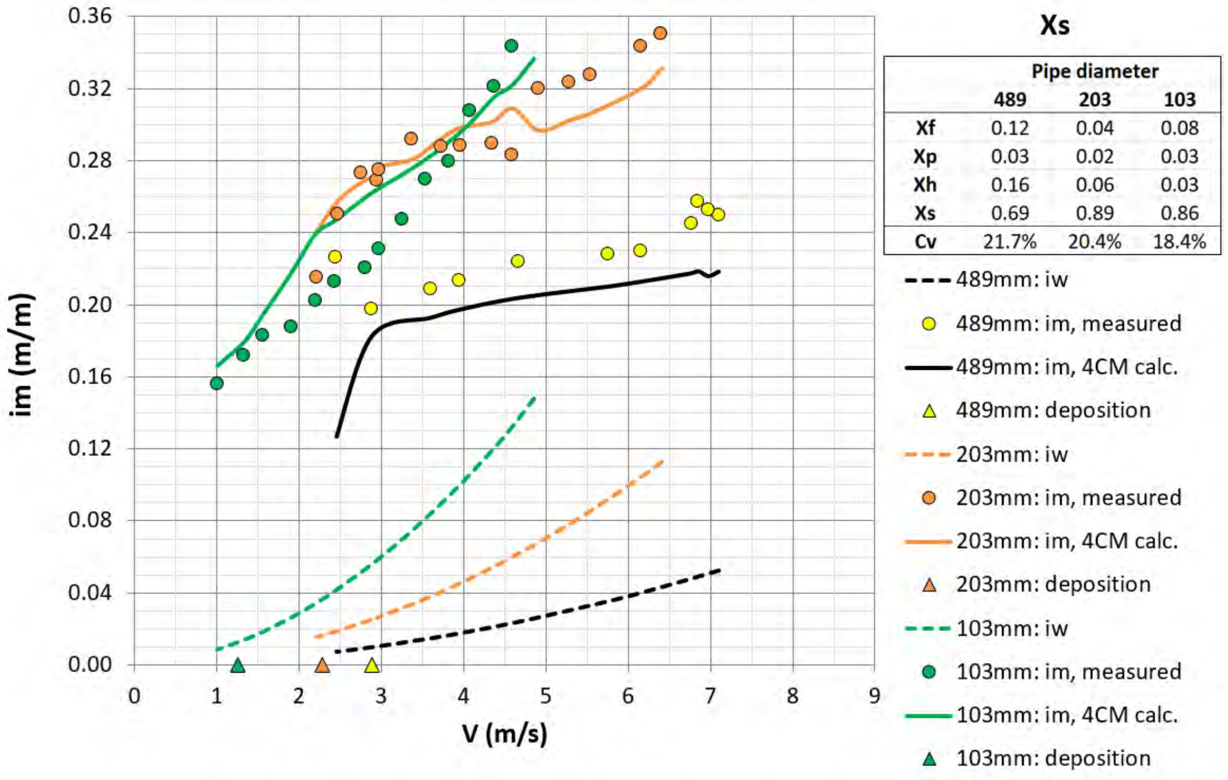
X_p : **0.193**

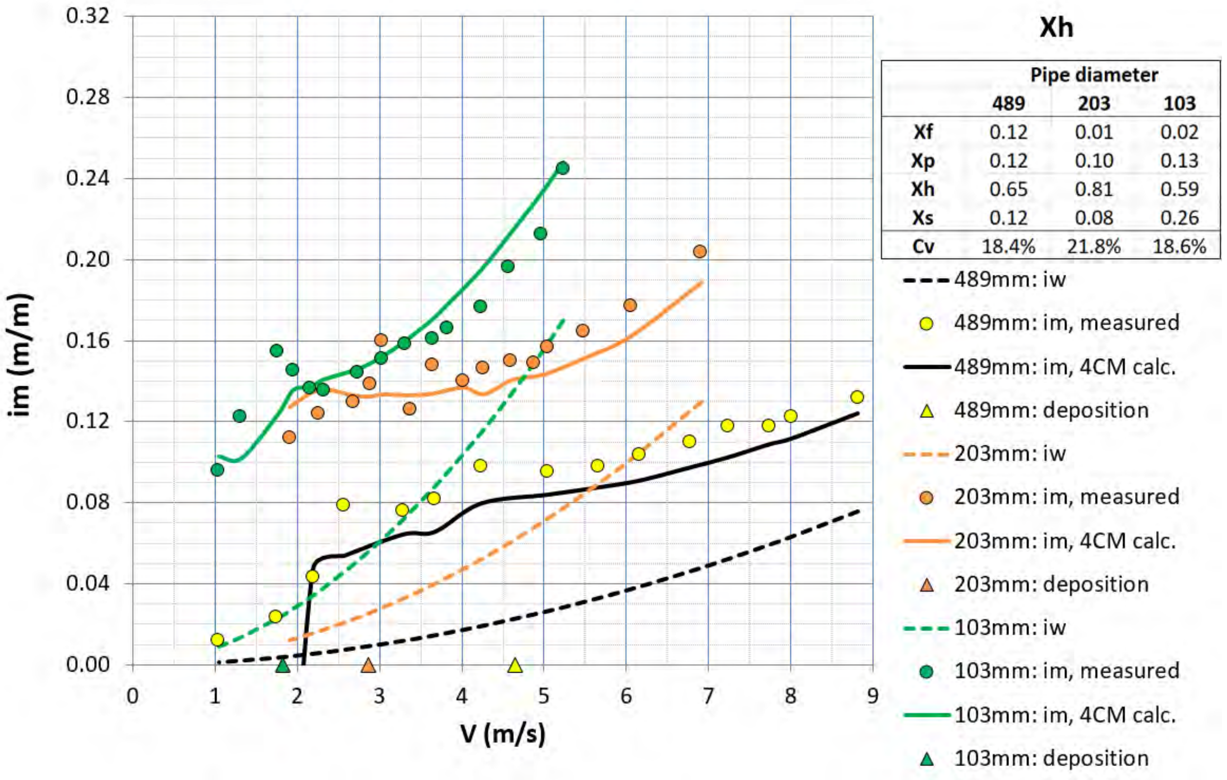
X_h : **0.260** *(C_v=9.9%)*

X_s : **0.308** *(C_v=11.7%)*

NOTE: Broad particle size distributions can exhibit relatively low losses, even at high concentration.

Individual results, 100, 200 and 500 mm loops





These figures show slurries where the single X_s or X_h components are dominant, resulting in relatively narrow particle size distributions. Volumetric concentrations are near 20%. Note the very visible effect of particle size and pipe size on the measured pressure gradient.

Individual results, 100, 200 and 500 mm loops

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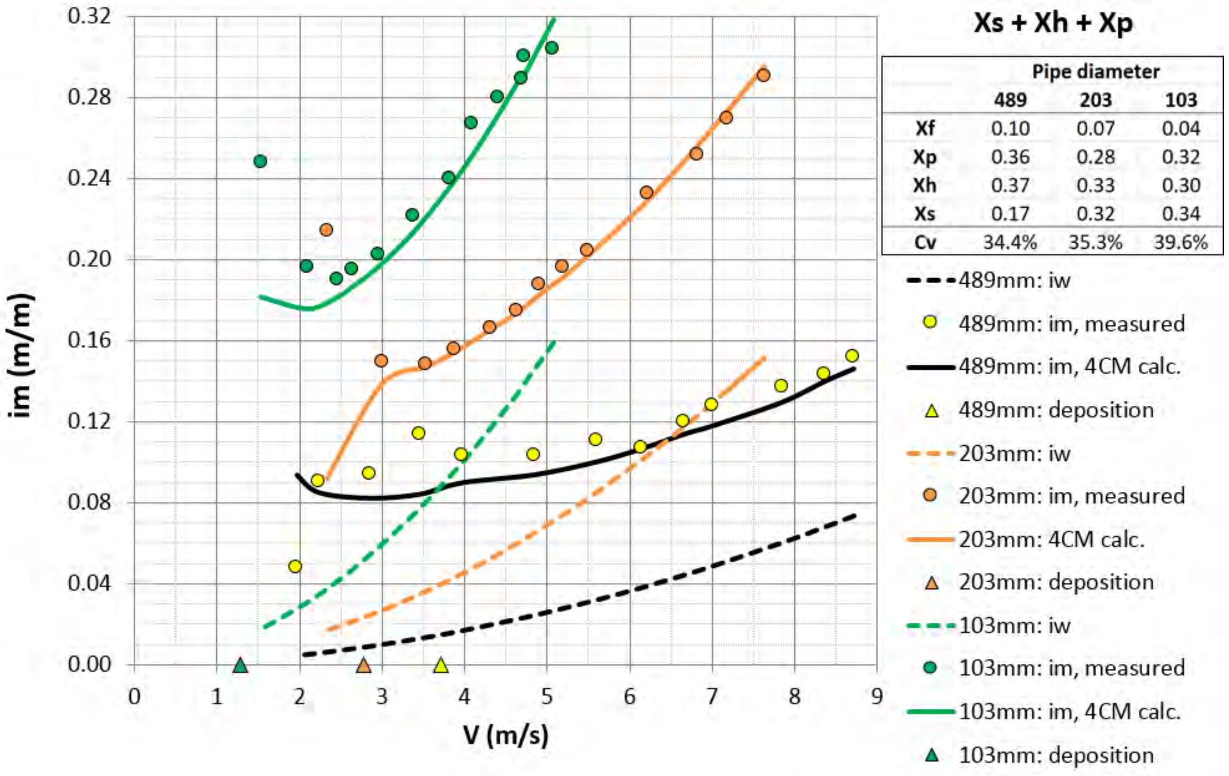
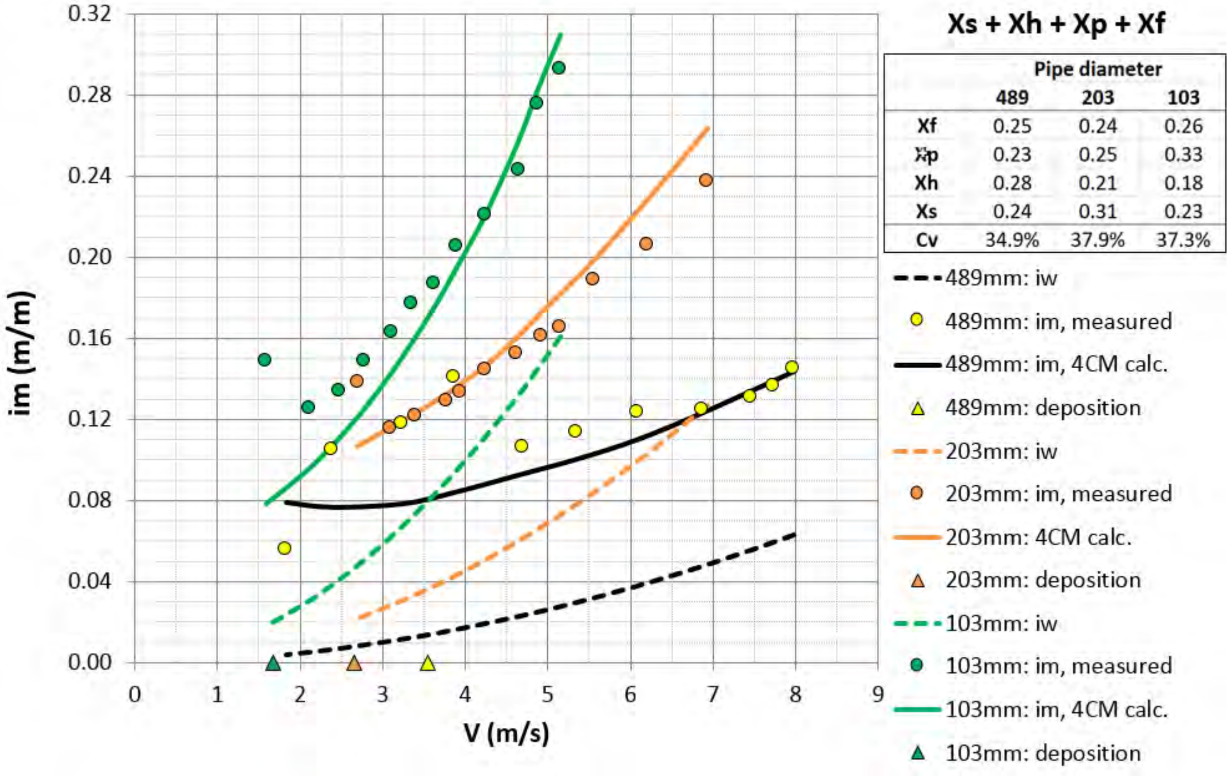
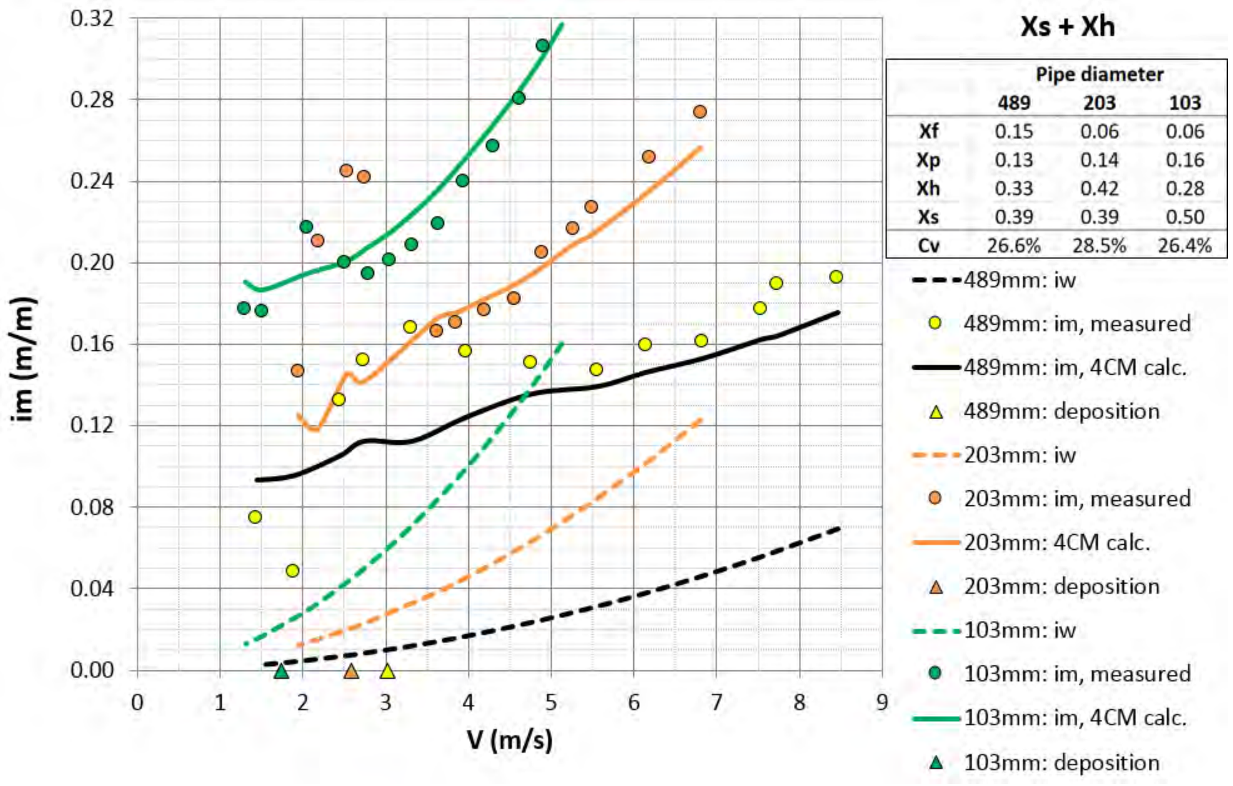
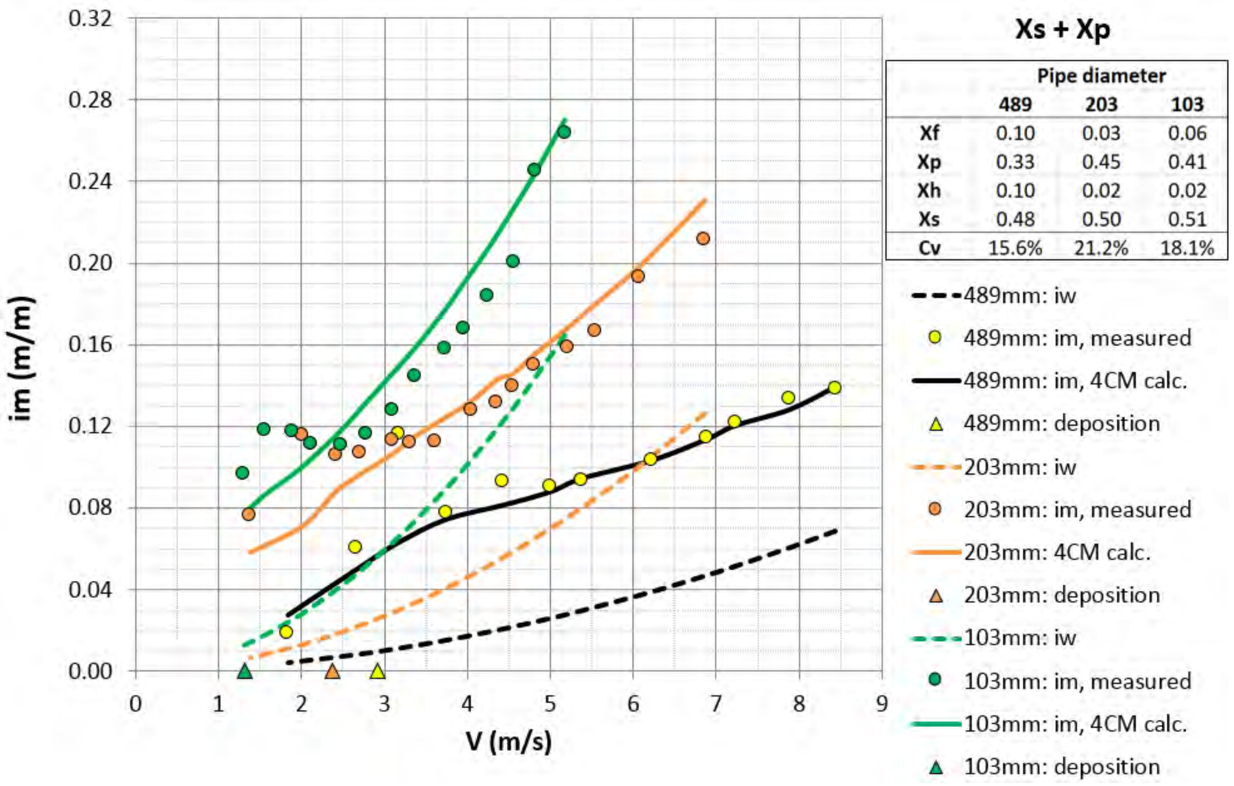




These figures show mixtures of three and four components respectively at volumetric concentrations near 35%. These tests represent concentrated slurries with broad particle size distributions. Note that the pressure gradients shown in these figures are comparable to the lower concentration, narrow PSD distributions seen in the previous slide.

Individual results, 100, 200 and 500 mm loops

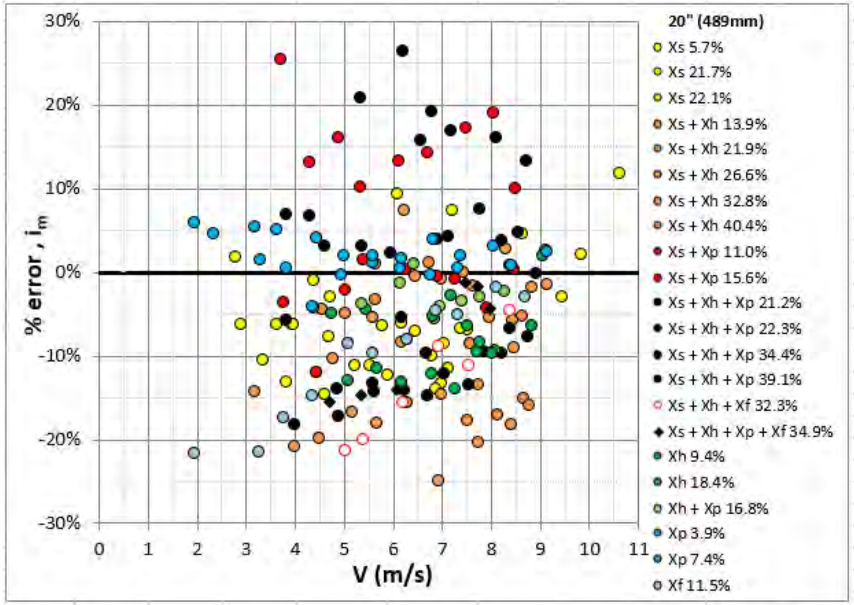


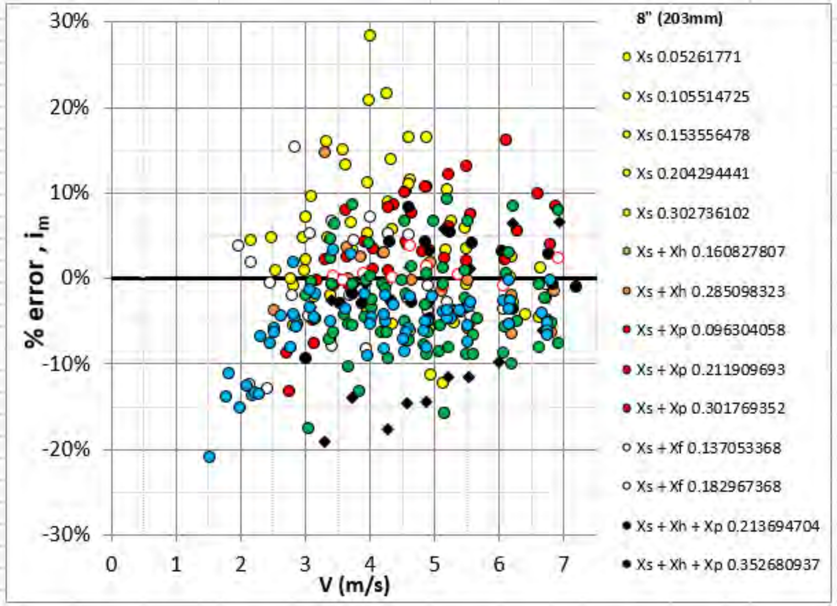


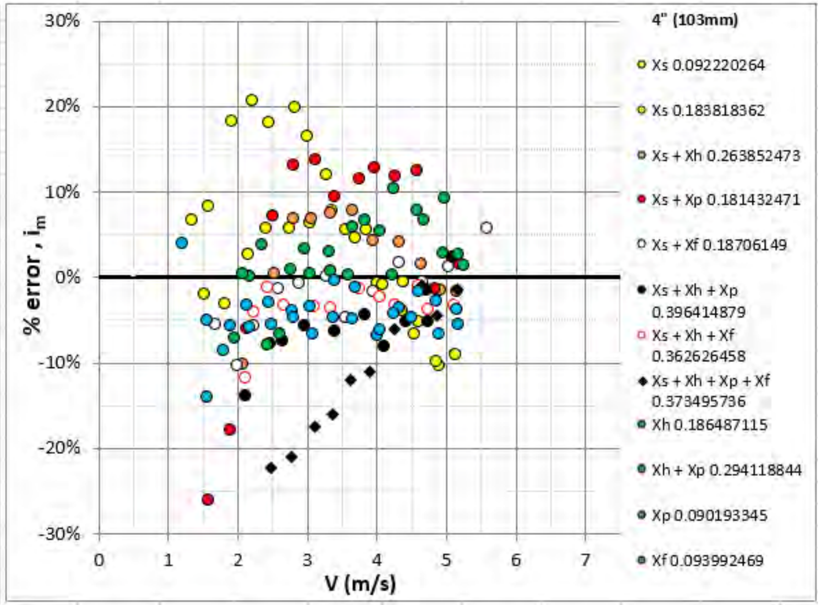
These figures show stratified X_s solids combined with heterogeneous X_h and pseudo-homogeneous X_p solids respectively.

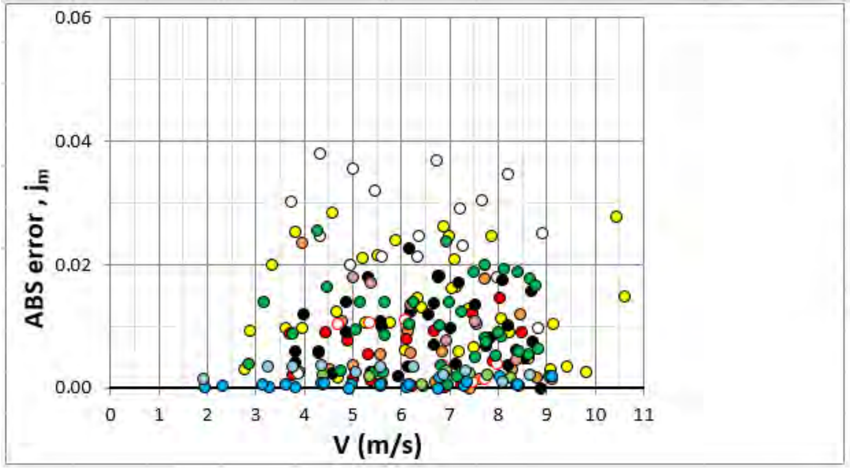
Note that the $X_s + X_p$ mixtures represent bimodal PSDs, having a relatively small fraction of X_h surrounded by larger fractions of X_s and X_p.

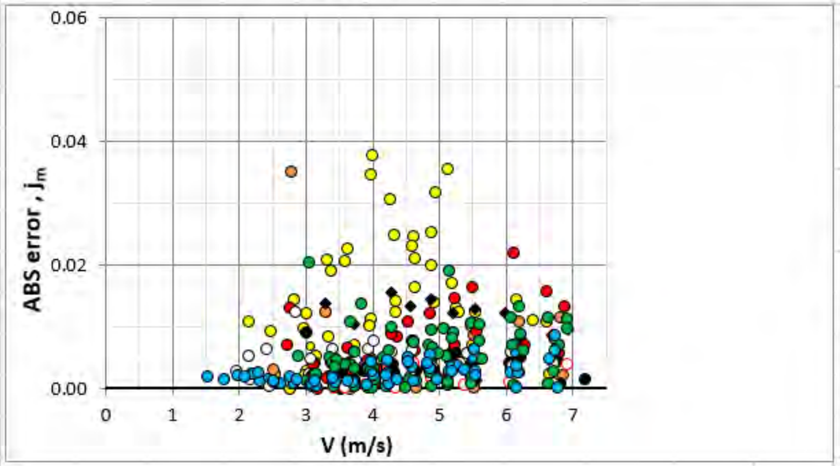
Error summary – 4CM Pipeline pressure gradient

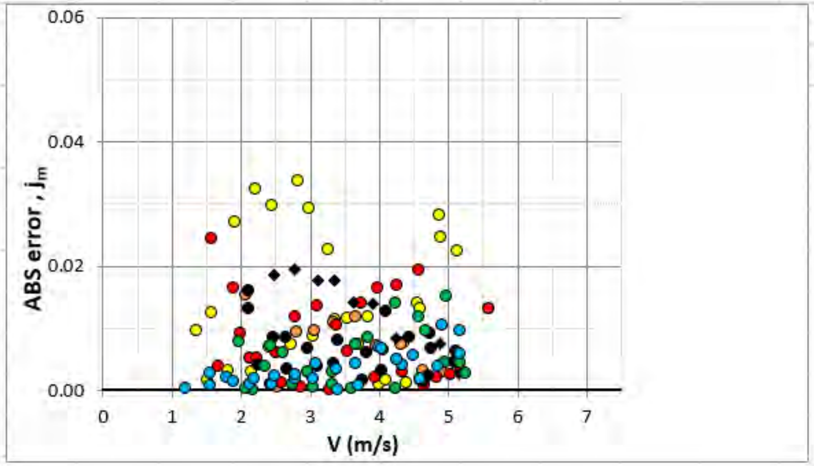










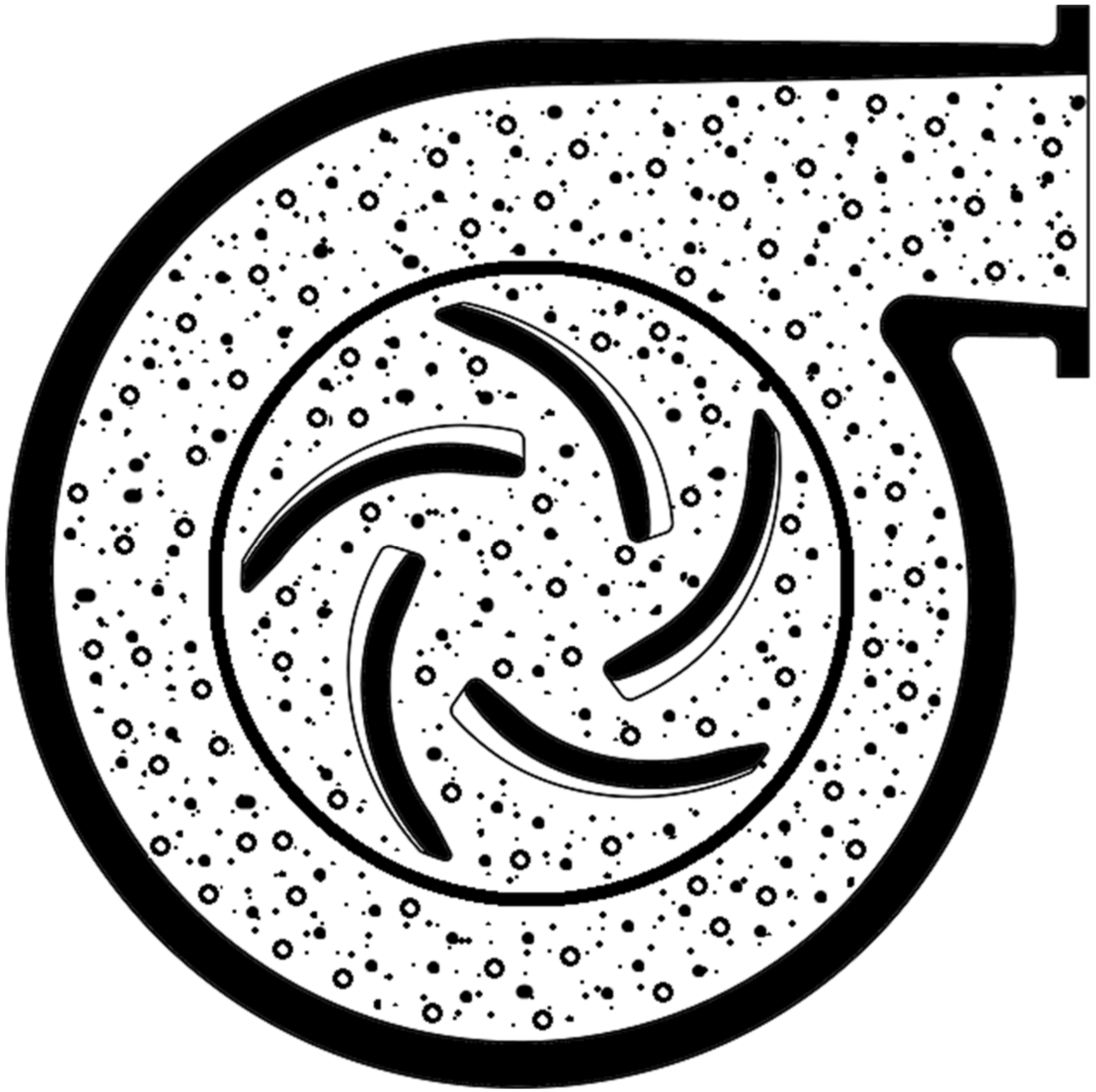


500 mm (20”)

200 mm (8”)

100 mm (4”)

Pump Solids Effect Results – 200 mm loop



Tested pump speeds

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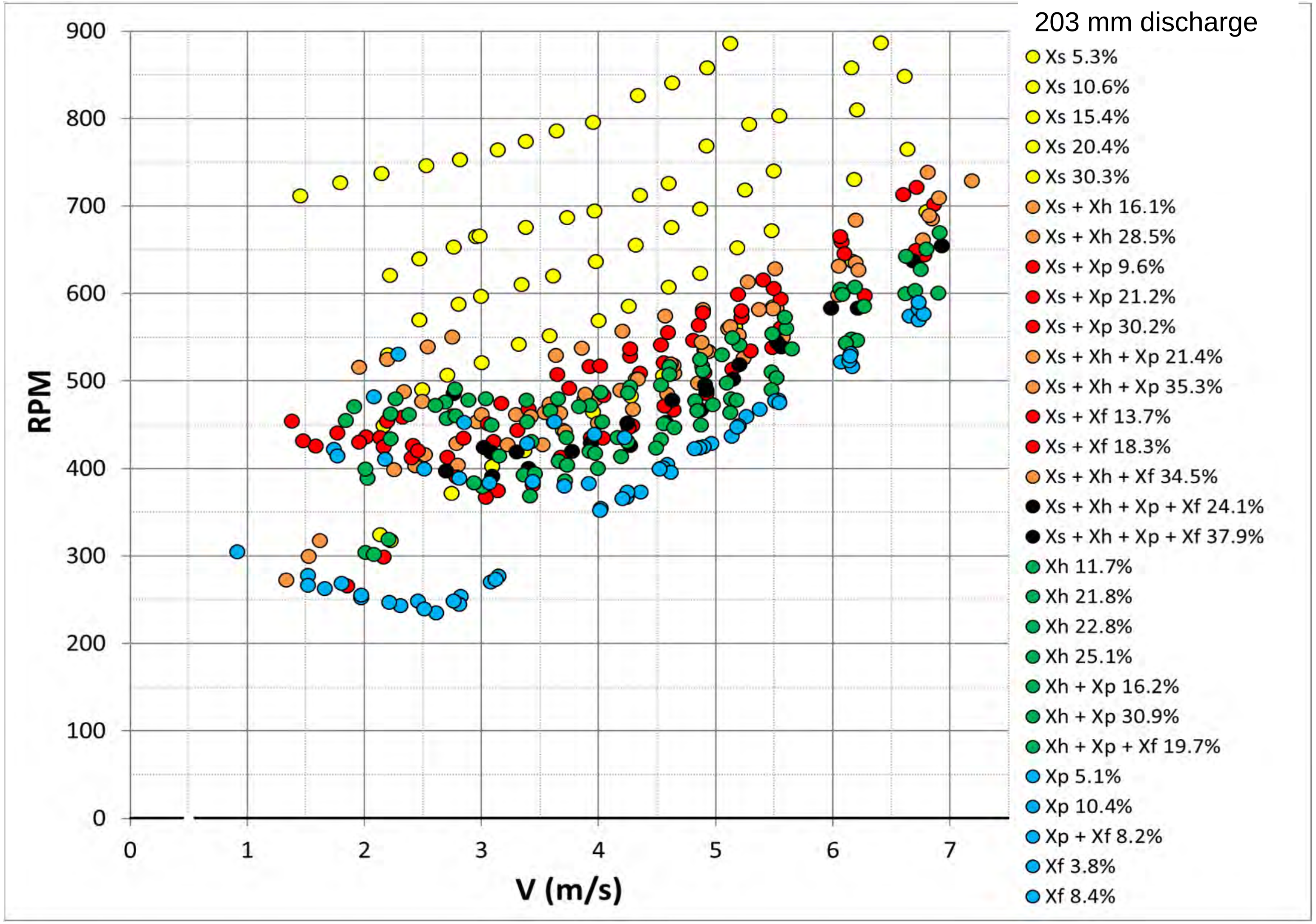
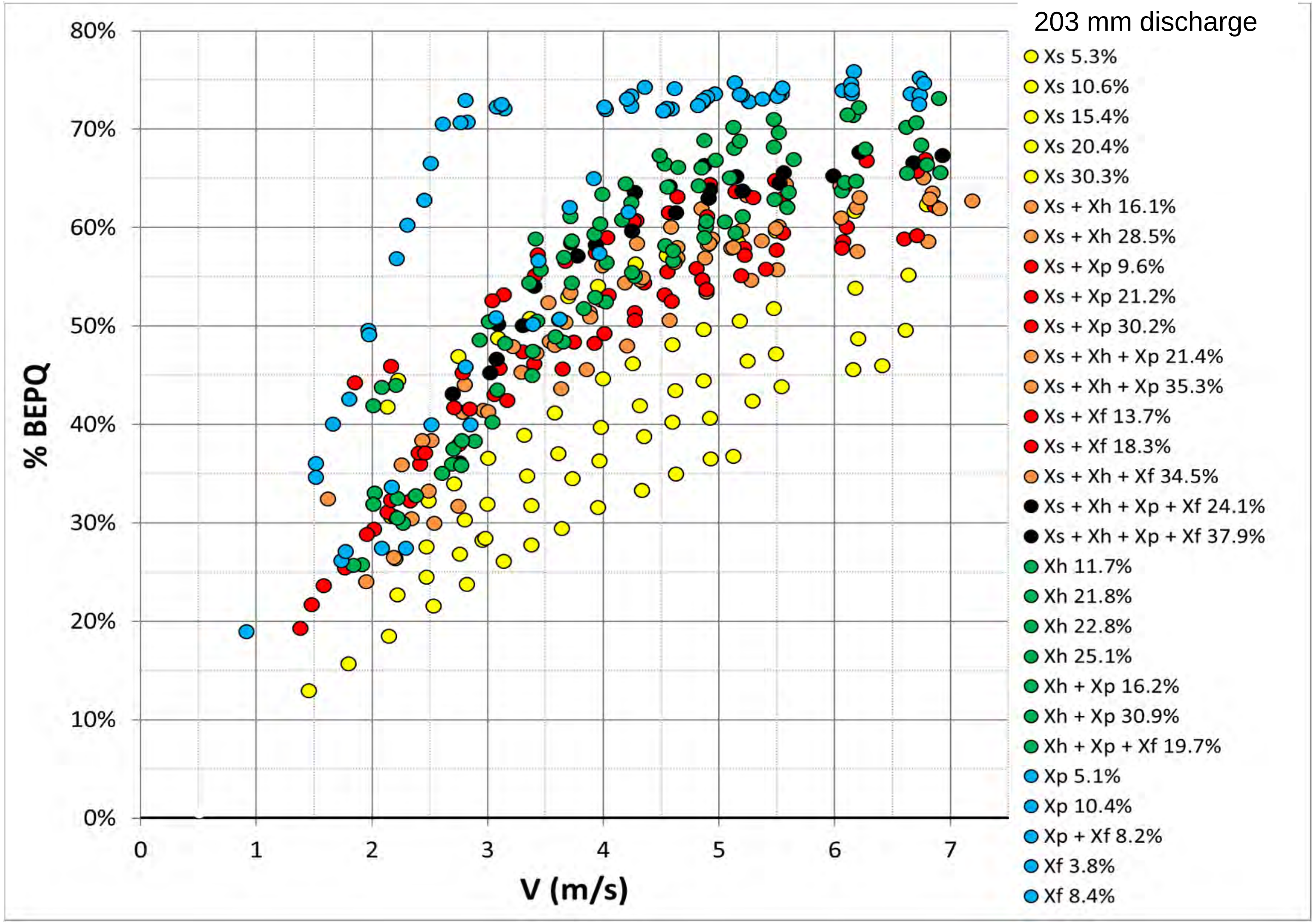
8x10 LSA 32
806.5 mm impeller
254 mm suction
203 mm discharge



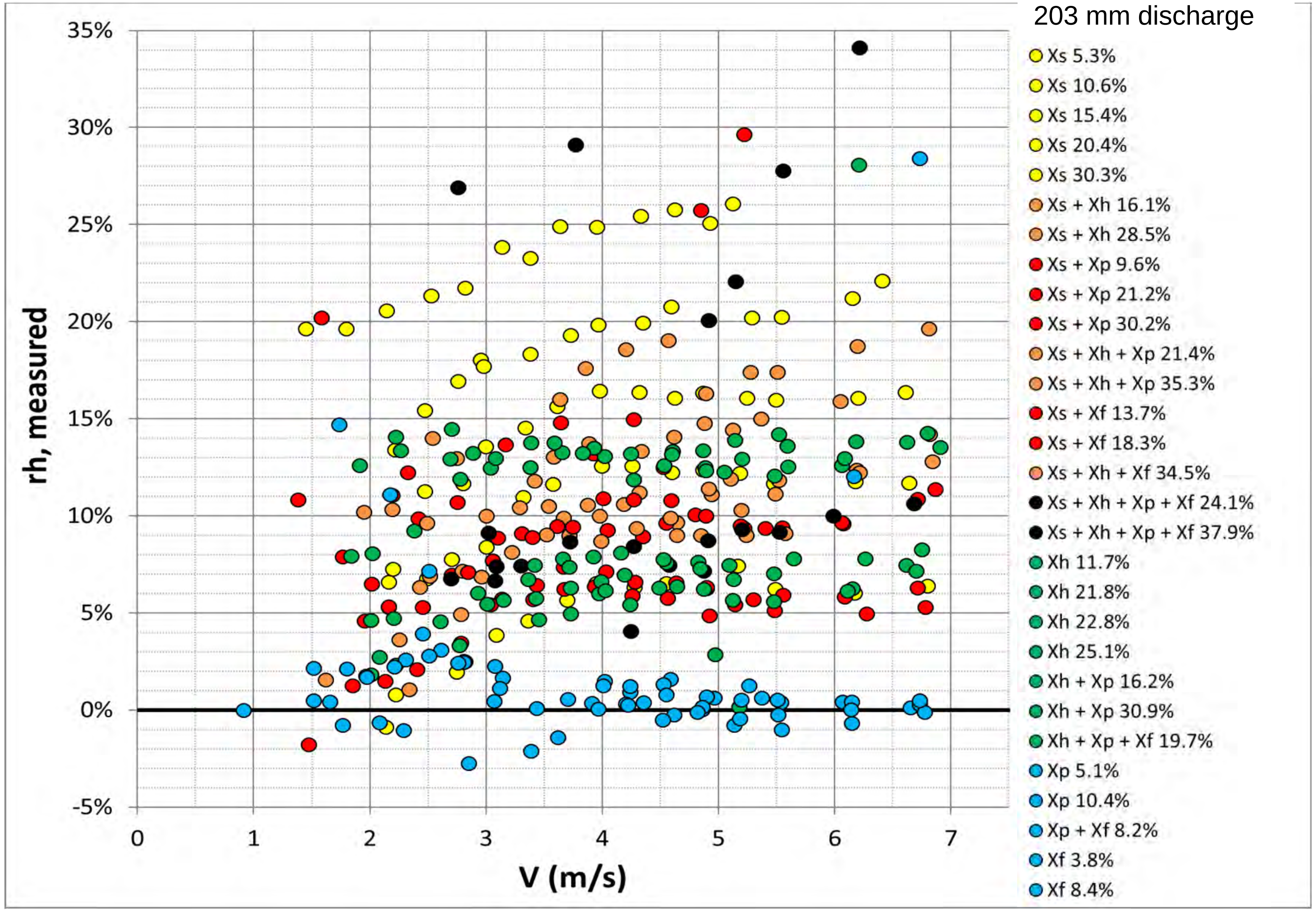
Tested %BEPQ *(best efficiency flowrate)*

8x10 LSA 32
806.5 mm impeller
254 mm suction
203 mm discharge



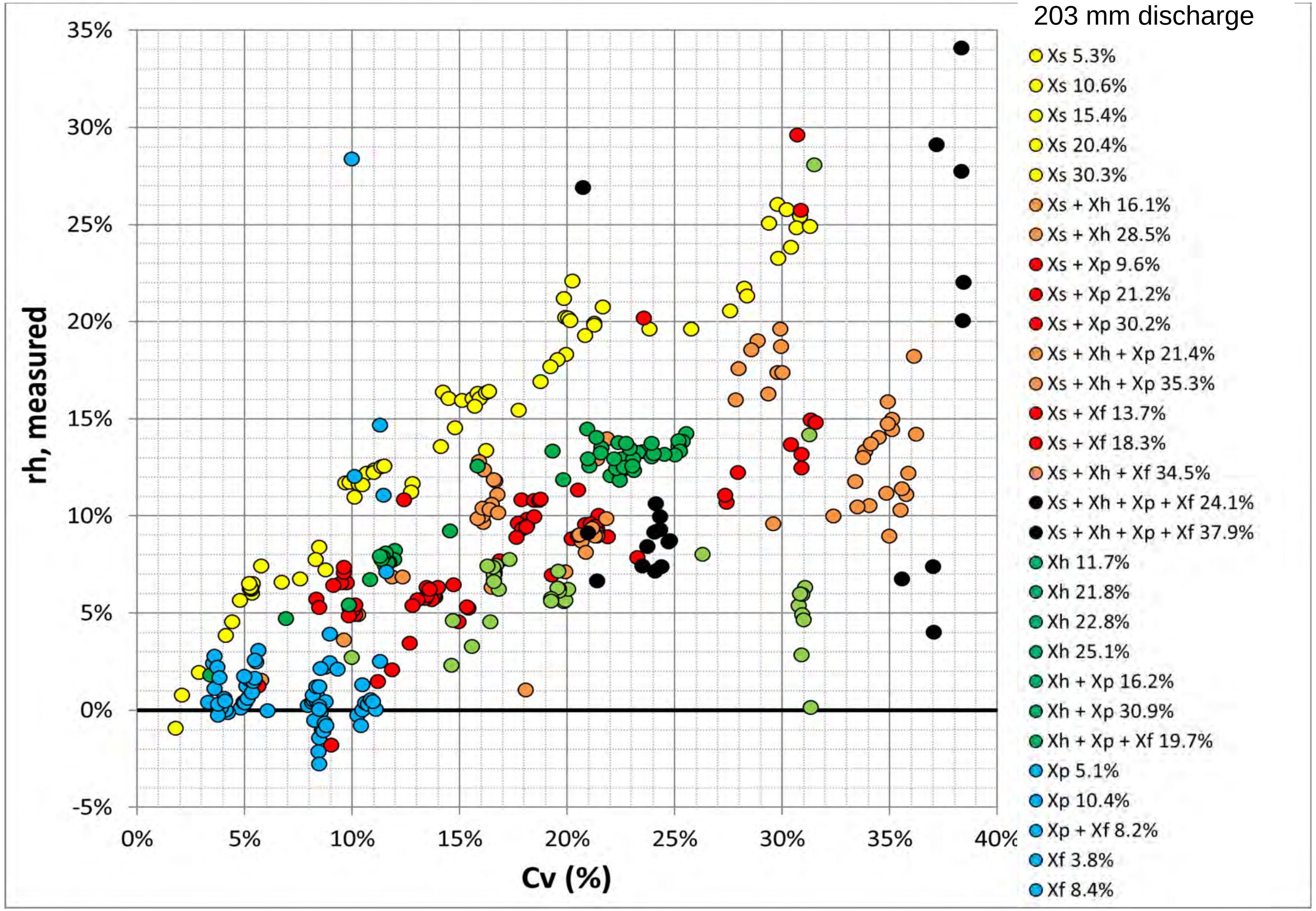
Summary of data – by flow velocity

8x10 LSA 32
806.5 mm impeller
254 mm suction
203 mm discharge



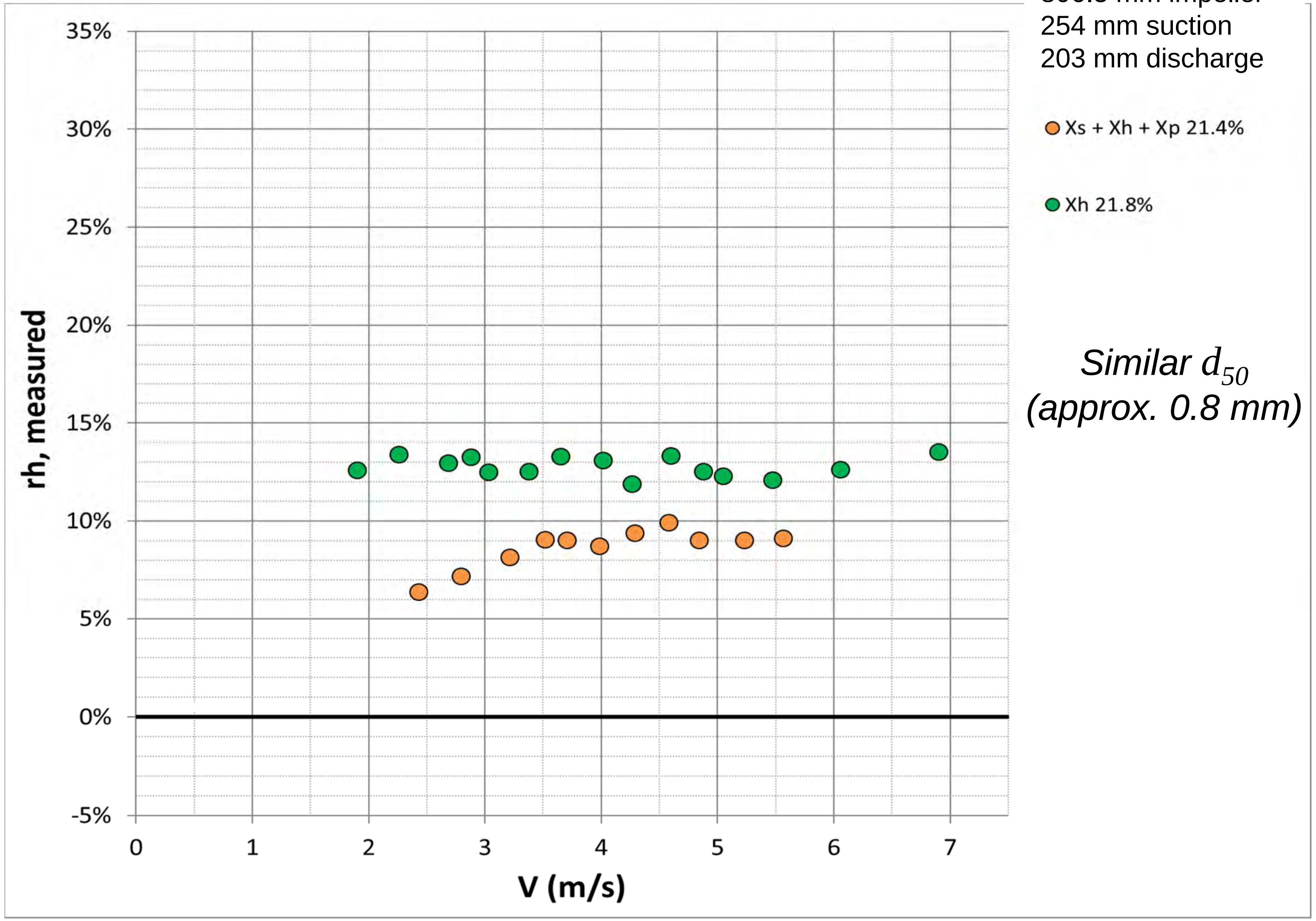
Summary of data – by total C_v

8x10 LSA 32
806.5 mm impeller
254 mm suction
203 mm discharge



Broad vs. narrow PSD (d_{50} and C_v constant)

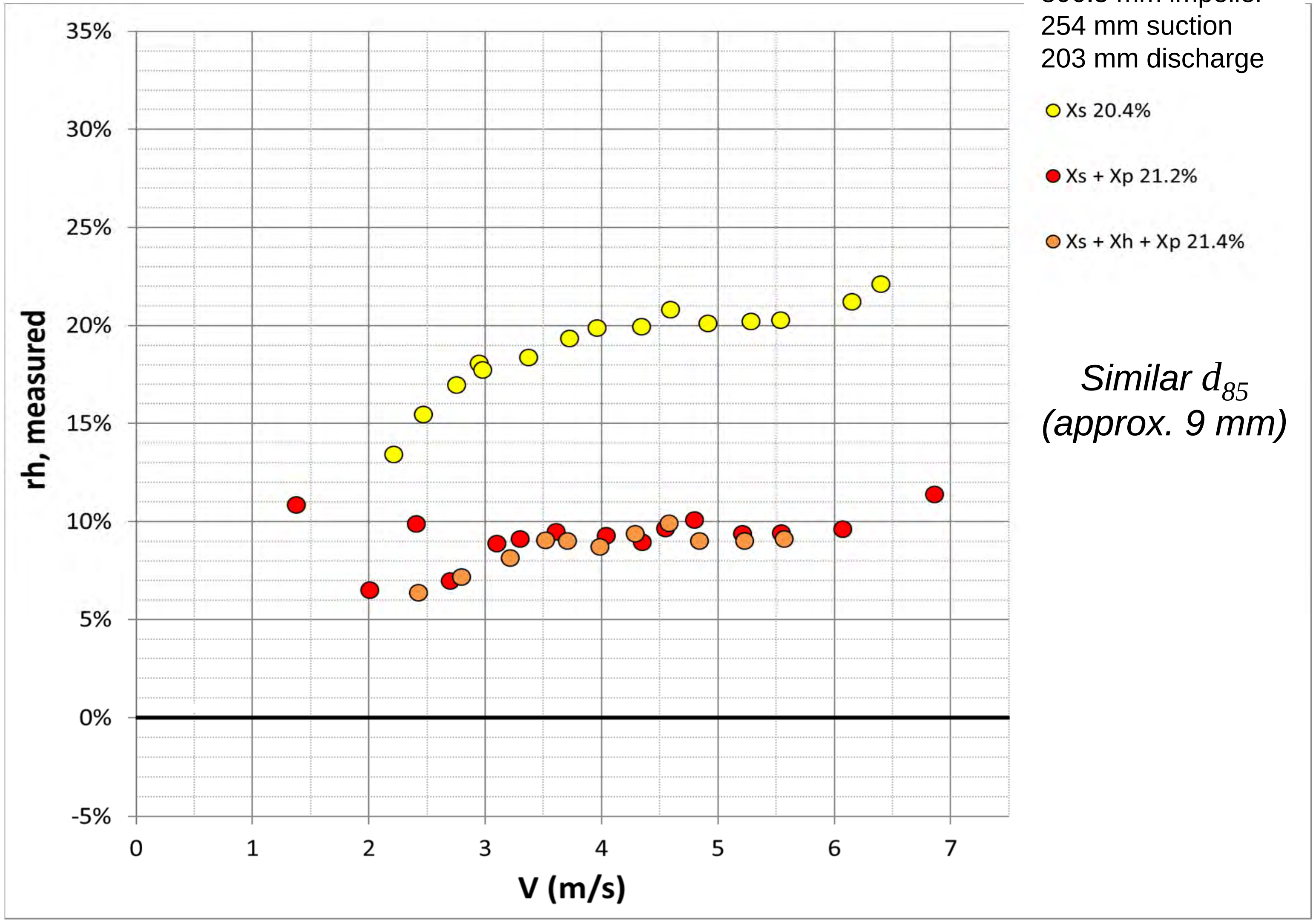
8x10 LSA 32
806.5 mm impeller
254 mm suction
203 mm discharge



Similar d_{50} (approx. 0.8 mm)

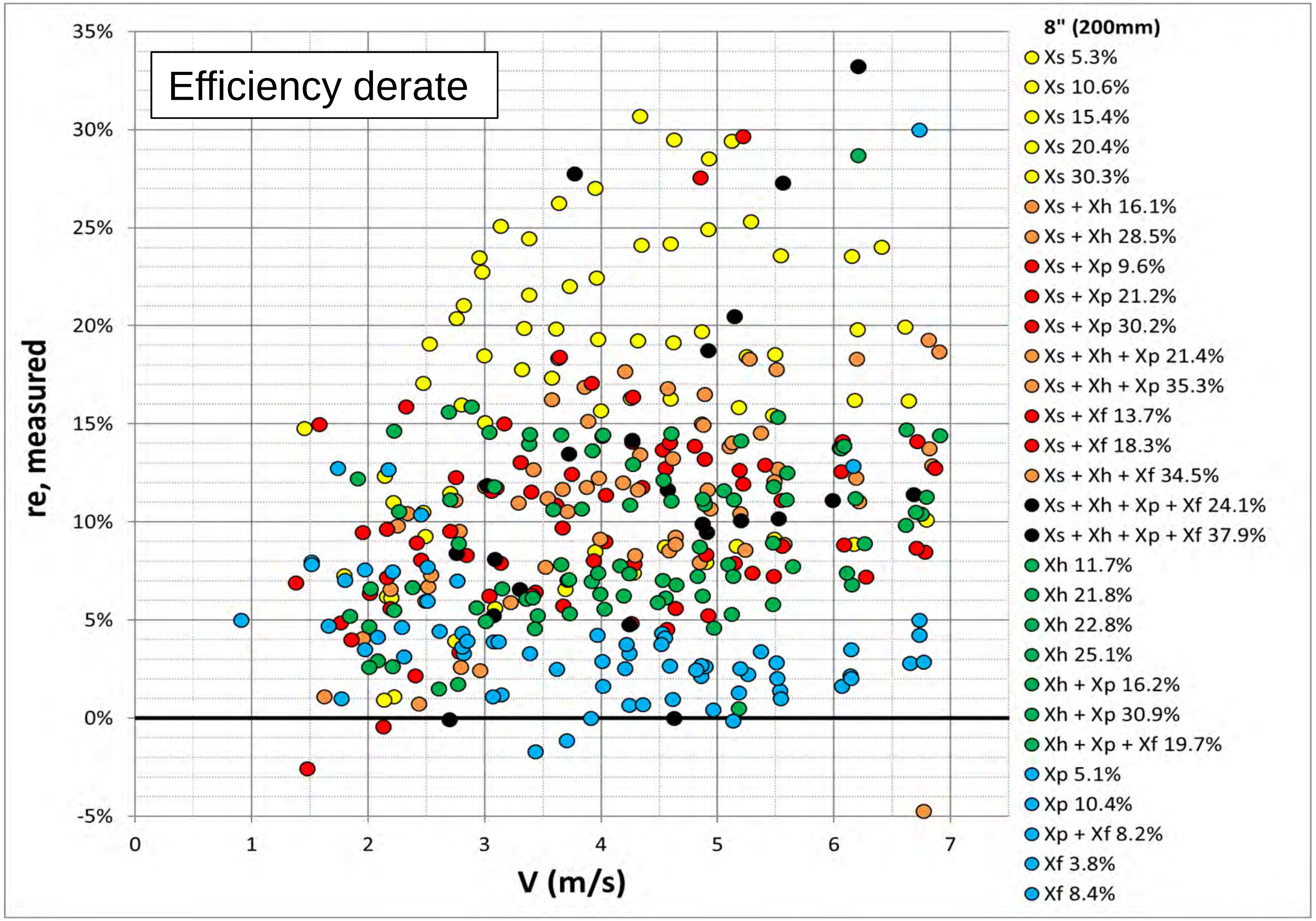
Broad vs. narrow PSD (d_{85} and C_v constant)

8x10 LSA 32
806.5 mm impeller
254 mm suction
203 mm discharge



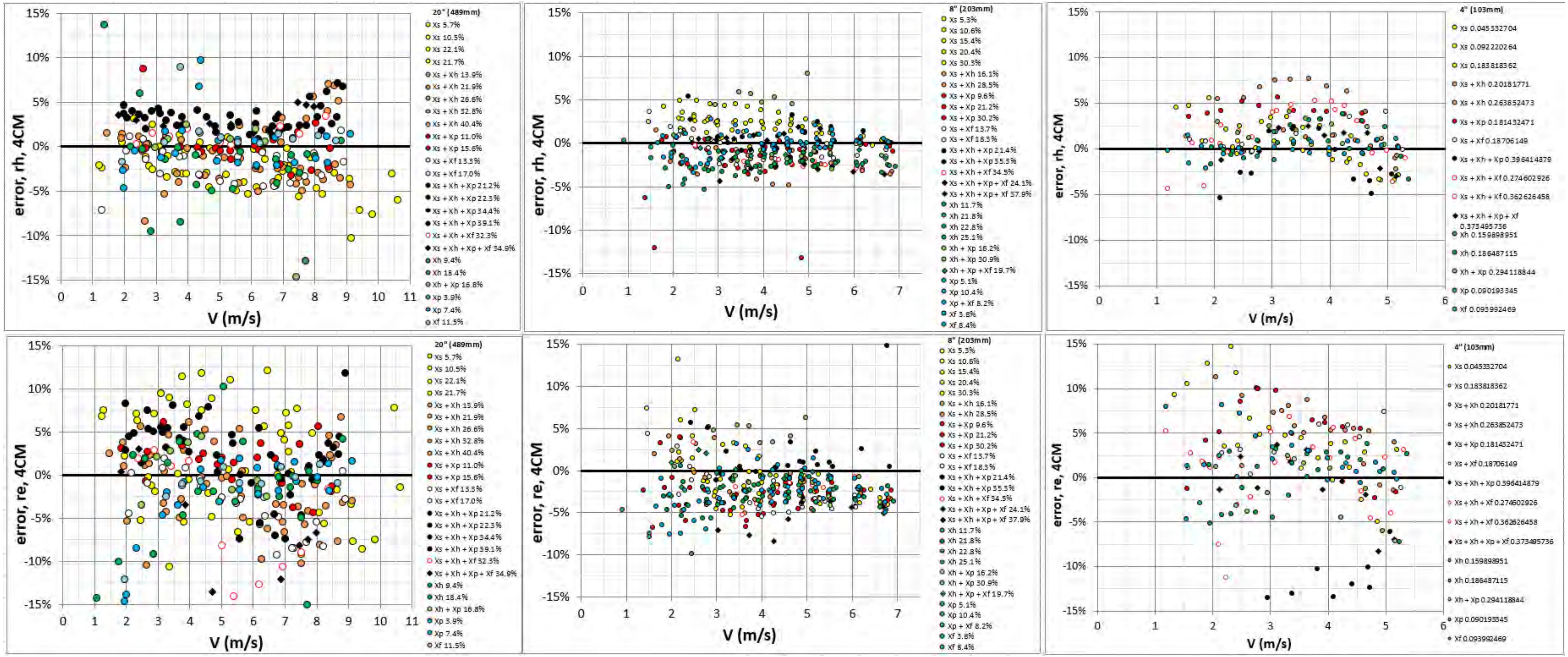
Similar d_{85}
(approx. 9 mm)

Pump Solids Effect on Efficiency



- The solids effect on efficiency (r_e) is more difficult to predict that the effect on head.
- In general, the standard assumption that $r_e = r_h$ provides the best overall fit to the data when the “Carrier Fluid” viscosity is low.
- Fortunately, the head effect is of greatest importance for system design, while efficiency is largely concerned with the sizing of the pump driver.
- For highly concentrated slurries, or those with significant viscous and/or non-Newtonian effects, efficiency losses can be greater.
- When in doubt, consult with your slurry pump expert.

Error summary – 4CM Pump solids effect



500 mm (20")　　***200 mm (8")***　　***100 mm (4")***

Acknowledgements

This presentation is based on a number of works by the author and his colleagues:

Dr. Anders Sellgren, Professor Emeritus, Luleå University, Sweden

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4-Component model references

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