4.1 Vertical and Horizontal Translations

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Vertical and Horizontal Translations

Learning Intention

To be able to sketch graphs that have been translated from a parent function.

Success Criteria:

I can:

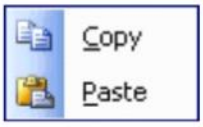
- ❑ Shift the graph of a function vertically.
- ❑ Shift the graph of a function horizontally.
- ❑ Combine horizontal and vertical translations.

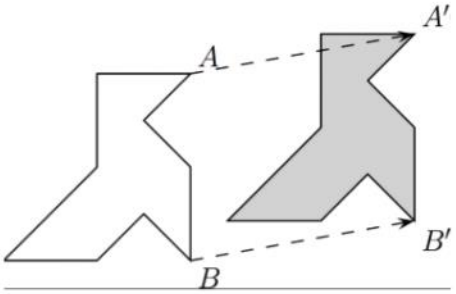
Textbook Reference

4A

Translating a function is:

shifting it vertically or horizontally to produce a further graph (function / relation). This changes the domain and/or range.





Shifting up and down

$y = x^2$

$y = x^2 + 2$; $y = x^2 - 3$

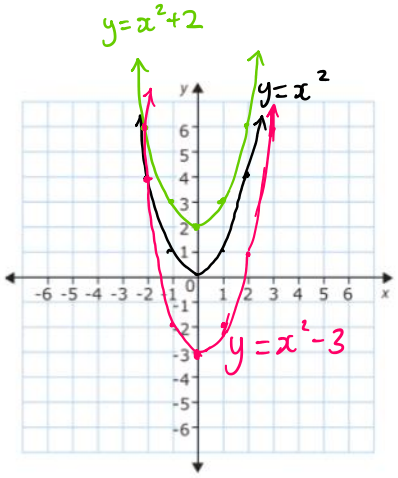
Table of values

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$x^2 + 2$	11	6	3	2	3	6	11
$x^2 - 3$	6	1	-2	-3	-2	1	6

+2 -3

→ To shift a graph h units up, add h to $f(x)$, the new function is $f(x) + h$

→ To shift a graph h units down, subtract h from $f(x)$, the new function is $f(x) - h$



Shifting left and right

$y = x^2$

$y = (x + 3)^2$; $y = (x - 2)^2$

Table of values

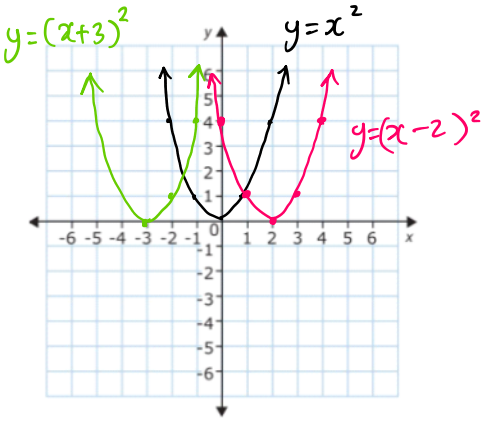
x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9
$(x+3)^2$	0	1	4	9	16	25	36
$(x-2)^2$	25	16	9	4	1	0	1

- To shift a graph h units to the _____, replace x by $x - h$.

If the graph is a function $f(x)$, the new function is _____.

- To shift a graph h units to the _____, replace x by $x + h$.

The new function is _____.



Some familiar functions

- **Quadratic** ..... $y = x^2$ .....
- **Cubic** ..... $y = x^3$ .....
- **Reciprocal** ..... $y = \frac{1}{x}$ ..... $(y = \frac{k}{x})$ — x : denominator
- **Exponential** ..... $y = 2^x$ ..... $(y = a^x)$ — x is in the exponent
- **Circle** ..... $x^2 + y^2 = r^2$ ..... — Both x & y are squared.

Combining horizontal and vertical translations

Example 1 *(no sketching)*

How is the graph of $y = (x+2)^3 - 1$ obtained using translations?

$y = x^3$ —(Horizontal Translation: 2 units to the left)→ $y = (x+2)^3$ —(Vertical Translation: 1 unit down)→ $y = (x+2)^3 - 1$

Example 2 *(no sketching)*

How is the graph of $y = \frac{4}{x-1} + 2$ obtained using translations?

$y = \frac{4}{x}$ —(Horizontal Translation: 1 unit to the right)→ $y = \frac{4}{x-1}$ —(Vertical Translation: 2 units up)→ $y = \frac{4}{x-1} + 2$

Think Pair Share

Does the order of the translations matter?

Example 3 – Translating hyperbolas

Sketch the graph of $\mathbf{y = \frac{1}{x}}$. Then sketch the graph of $y = \frac{1}{x+1}$, and state the asymptotes of each graph.

VA
$x = 0$

Asymptotes of $y = \frac{1}{x}$:

Vertical Asymptote : $x = 0$

Horizontal Asymptote : $y = 0$

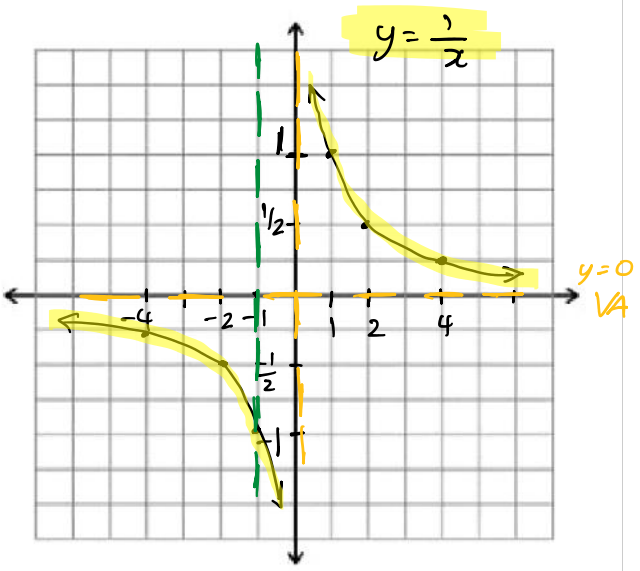
• Table of values:

x	-4	-2	-1	0	1	2	4
$y = \frac{1}{x}$	$\frac{-1}{4}$	$\frac{-1}{2}$	-1		1	$\frac{1}{2}$	$\frac{1}{4}$
$y = \frac{1}{x+1}$	$\frac{-1}{3}$	-1		1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{5}$

Asymptotes of $y = \frac{1}{x+1}$:

VA $x = -1$

HA $y = 0$



Example 4 – Translating circles

a. Complete the squares in x and in y of the relation $x^2 + y^2 - 6x + 8y = 0$.

b. Identify the circle with centre the origin that can be translated to it, and state the translations.

c. Sketch both circles on the same diagram, and explain why each circle passes through the centre of the other circle.

$x^2 + y^2 = 5^2$

$(x-3)^2 + (y+4)^2 = 25$

a) $x^2 - 6x + y^2 + 8y = 0$

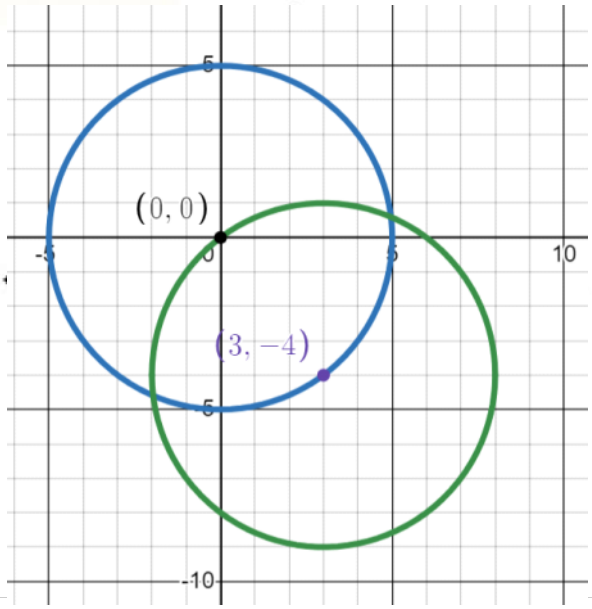
$x^2 - 6x + 9 + y^2 + 8y + 16 = 9 + 16$

$(x-3)^2 + (y+4)^2 = 25$

b)

c) 3 to the right → horizontal

4 down → vertical



Example 5

The graph of $y = 2^x$ is shifted down 2 units.

a. Write down the equation of the shifted graph.

b. Construct tables of values, and sketch the two graphs.

c. State the asymptotes of the two graphs.

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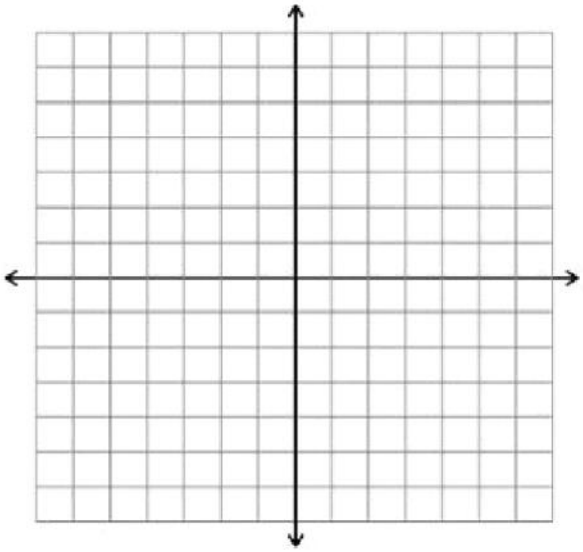

A Summary

SHIFTING (OR TRANSLATING) RIGHT AND LEFT

- To shift a graph h units to the *right*, replace x by $x - h$.
- Alternatively, if the graph is a function, the new function rule is $y = f(x - h)$.

SHIFTING (OR TRANSLATING) UP AND DOWN

- To shift a graph k units *upwards*, replace y by $y - k$.
- Alternatively, if the graph is a function, the new function rule is $y = f(x) + k$.

THE COMPLETED SQUARE AND THE VERTEX OF A PARABOLA

The completed square form of a quadratic

$$y = a(x - h)^2 + k \quad \text{or} \quad y - k = a(x - h)^2$$

displays its graph as the parabola $y = ax^2$ shifted right h units and up k units.