Animating Newton's Cradle

Lachlan Macartney

1 Aim(Question posed and why its interesting)

The purpose of this project is to animate Newton's Cradle from first princples. Newton's Cradle has been animated before however often via a ''physics-free" key-frame method or one reliant on a complicated physics engine. This project will demonstrate how strikingly real world behavior can be computed with simple, first-principles approach mathematical framework. Our elementary model will rightly predict how the system's behavior changes when multiple pendulums are raised. Additionally, it will provide insight on how a real cradle works and offer a possible explanation for some of the more curious behavior.

2 Method(Basic approach)

"Newton's Cradle can be modelled as mutilple pendulums confined to swing along a single axis which undergo elastic collisions with one another. The pendulums rest such that there is a small amount of horizontal displacement between them."

2.1 Fundamental Framework

We will define the mathematics required for a generalised set of n independent pendulums, then compute their interactions in a cartesian coordinate system (x, y) which we will call the Cradle's Frame. For each $i \in \{1, 2, 3, ..., n\}$ let the ith pendulum have length ℓ, radius r_i and mass m_i. It will also need some displacement Δx_i along the x-axis, so that it 'almost touches' the others at rest. Each pendlum has a single degree of freedom along the axis (θ_i) in its own reference frame which obeys the differential equation of a pendulum;

$$\ddot{\theta}_i = -\frac{g}{\ell}\sin(\theta_i)$$

A transform to the Cradle's Frame can be computed by;

$$T(\theta_i, \Delta x_i) = (\ell \sin(\theta_i) + \Delta x_i, -\ell \cos(\theta_i))$$

We will say that a collision occurs if for any two pendulums i, j;

$$|T(\theta_i, \Delta x_i) - T(\theta_j, \Delta x_j)| = r_i + r_j$$

If we say that such a collision is elastic, from first principles we will arrive at;

$$\dot{\theta}_{i,\text{final}} = \frac{m_i - m_j}{m_i + m_j}\dot{\theta}_{i,\text{initial}} + \frac{2m_j}{m_i + m_j}\dot{\theta}_{j,\text{initial}} \quad \text{and} \quad \dot{\theta}_{j,\text{final}} = \frac{2m_i}{m_i + m_j}\dot{\theta}_{i,\text{initial}} + \frac{m_j - m_i}{m_i + m_j}\dot{\theta}_{j,\text{initial}}$$

We may also add the condition that each pendulum mass has the same density ρ such that m_i and r_i are related via $m_i = \frac{4\pi\rho}{3} r_i^3$. For the traditional Newton's Cradle with equal mass and radii pendlums the above collision formulae simplify. However, such a non traditional cradle will be explored.

3 Results(Notable results - the best or final output of code)

3.1 Traditional pendulum

3.2 Non-equal masses

4 Conclusions(Answer to your question/Summary)