Animating Newton's Cradle

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1 Aim(Question posed and why its interesting)

The purpose of this project is to animate Newton's Cradle from first principles. Newton's Cradle has been animated before however often via a ''physics-free" key-frame method or one reliant on a complicated physics engine. This project will demonstrate how strikingly real world behavior can be computed with simple, first-principles approach mathematical framework. Our elementary model will rightly predict how the system's behavior changes when multiple pendulums are raised. Additionally, it will provide insight on how a real cradle works and offer a possible explanation for some of the more curious behavior.

2 Method(Basic approach)

"Newton's Cradle can be modelled as multilple pendulums confined to swing along a single axis which undergo elastic collisions with one another. The pendulums rest such that there is a small amount of horizontal displacement between them."

2.1 Fundamental Framework

We will define the mathematics required for a generalised set of n independent pendulums, then compute their interactions in a cartesian coordinate system (x, y) which we will call the Cradle's Frame. For each $i \in \{1, 2, 3, ..., n\}$ let the ith pendulum have length ℓ, radius r_i and mass m_i. It will also need some displacement Δx_i along the x-axis, so that it 'almost touches' the others at rest. Each pendlum has a single degree of freedom along the axis (θ_i) in its own reference frame which obeys the differential equation of a pendulum;

$$\ddot{\theta}_i = -\frac{g}{\ell}\sin(\theta_i)$$

A transform to the Cradle's Frame can be computed by;

$$T(\theta_i, \Delta x_i) = (\ell\sin(\theta_i) + \Delta x_i, -\ell\cos(\theta_i))$$

We will say that a collision occurs if for any two pendulums i, j;

$$|T(\theta_i, \Delta x_i) - T(\theta_j, \Delta x_j)| = r_i + r_j$$

If we say that such a collision is elastic, from first principles we will arrive at;

$$\dot{\theta}_{i,\text{final}} = \frac{m_i - m_j}{m_i + m_j}\dot{\theta}_{i,\text{initial}} + \frac{2m_j}{m_i + m_j}\dot{\theta}_{j,\text{initial}} \quad \text{and} \quad \dot{\theta}_{j,\text{final}} = \frac{2m_i}{m_i + m_j}\dot{\theta}_{i,\text{initial}} + \frac{m_j - m_i}{m_i + m_j}\dot{\theta}_{j,\text{initial}}$$

We may also add the condition that each pendulum mass has the same density ρ such that m_i and r_i are related via $m_i = \frac{4\pi\rho}{3}r_i^3$. For the traditional Newton's Cradle with equal mass and radii pendlums the above collision formulae simplify. However, such a non traditional cradle will be explored.

3 Results(Notable results - the best or final output of code)

Click here to view the results discussed.

3.1 Traditional pendulum

One of the most striking results of this simulation is we have appeared to recreated the effect where the internal balls begin to move. Note how in this video of a real pendulum we see the same effect.

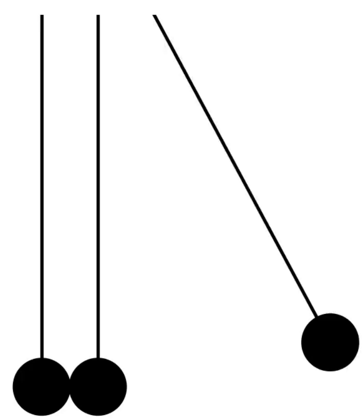

Figure 1: Grab

The explanation I can come up with for this originates from what we meant by the seperation between pendulums being "small."

"If the pendulums ever collide such that atleast one of them is non-perpendicular to the horizontal then there will be a runaway feedback loop where the internal ones begin to move."

A possible explanation for why a real pendulum exhibits this behavior is because the masses either touch on one another or have some non-zero seperation between them - in either case collisions will occur at an angle non-perpendicular to the horizontal and the aforementioned runaway feedback loop will occur.

With our simulated system we can place the pendulums closer together fairly easily, observe how this increases the time before the motion of the internal masses becomes obvious.

Original seperation between pendulums 1/100th of original seperation between pendulums

We could continue placing the masses closer and closer together, yet this comes at the cost of computation time. I thought this was a very interesting observation.

3.2 Non-equal masses

4 Conclusions(Answer to your question/Summary)