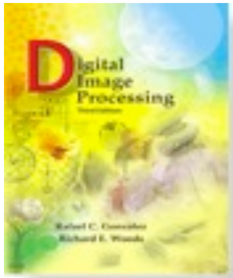


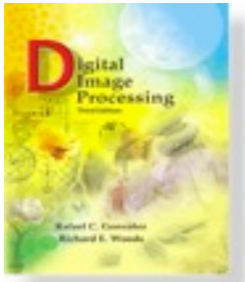
Last time:

Histogram: is a discrete function that counts how many pixels have a given intensity value.

$$h(r_k) = n_k$$

Number of pixels that have such intensity value

k-th intensity value



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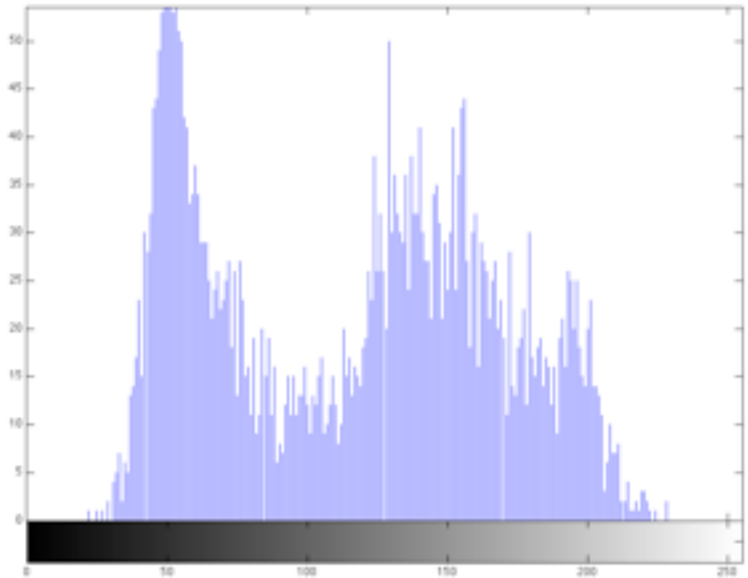
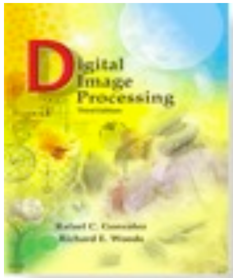
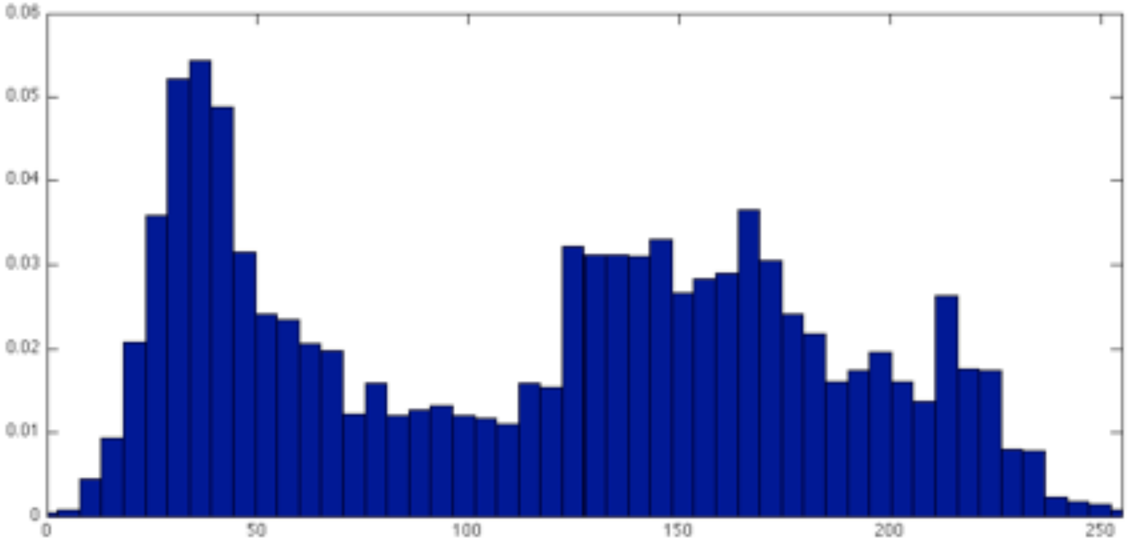




It is typical to scale n_k by MN, the number of pixels. The corresponding number represents the probability of a pixel having the given intensity.



$$p(r_k) = \frac{n_k}{MN} \qquad \sum_k p(r_k) = 1$$

$$\sum_k n_k = MN$$

Probability that a random pixels has intensity r_k.

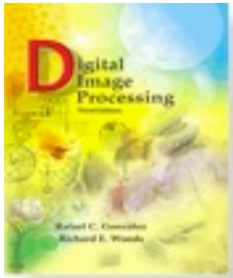
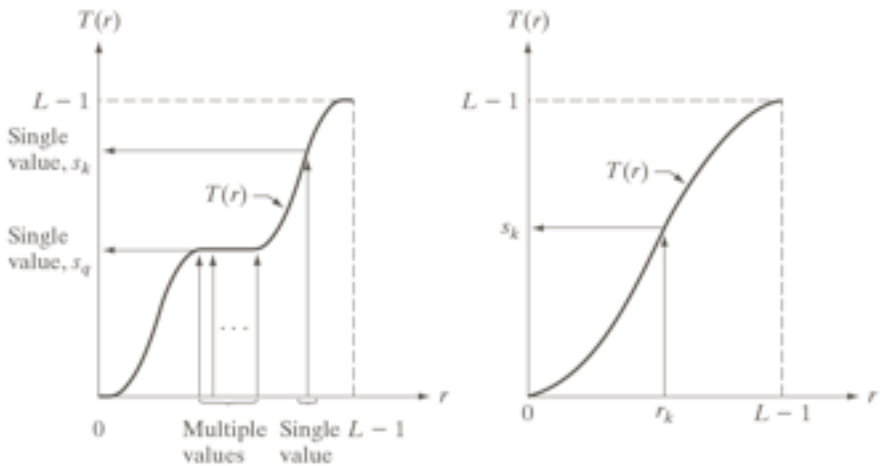


Image enhancement by *histogram equalization*

Consider an intensity transformation $s = T(r)$ (r input intensity level, s output intensity level). Let us assume that

- T is monotonically increasing
  that is $T(r_1) \leq T(r_2)$ for $r_1 \leq r_2$
  (strictly increasing if T^{-1} is required)
- T maps $[0, L-1]$ to $[0, L-1]$



We can identify the intensity level to a *random variable* in $[0, L-1]$. Then the function $p = h/MN$ is the probability density function of such random variable.

Denote by

- p_r the probability distribution of the original image (r-variable) and
- p_s the probability distribution of the image with intensity $s = T(r)$.

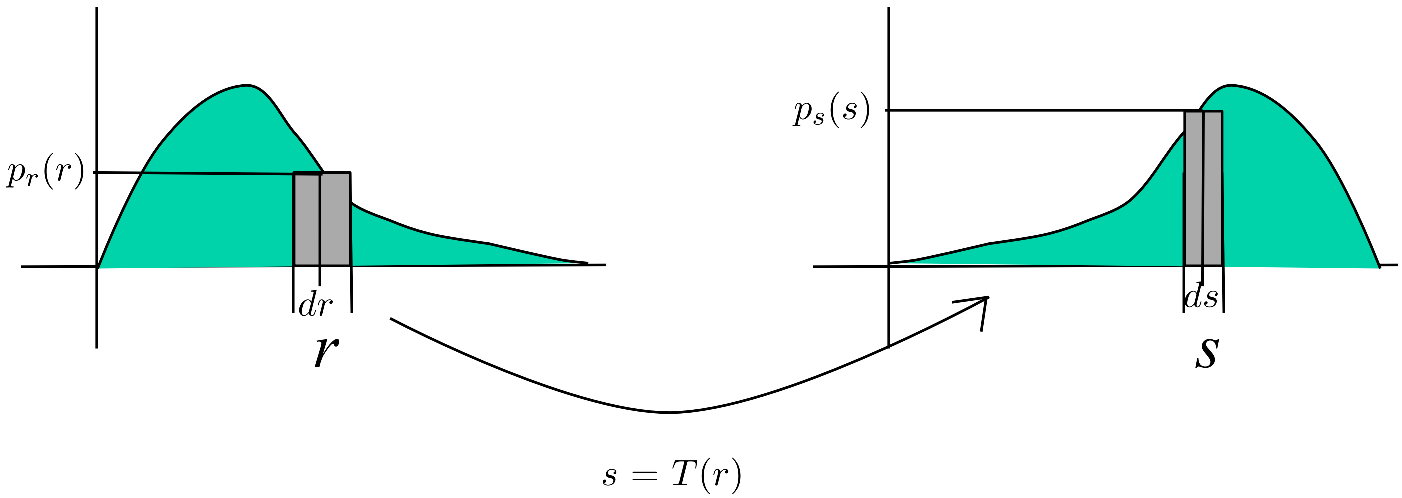
These two distributions will generally be different.

Thus, given the *change of variables* $s = T(r)$, the relation between p_s and p_r is given by the formula

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|.$$

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right|.$$

Where does it come from?



(Recall, probability distributions are positive)

The gray regions must have the same area (we are just relabeling the variables)

$$p_r(r)dr = p_s(s)ds$$

(take the absolute value is for the case when T is decreasing)

$$p_s(s) = p_r(r) \left| \frac{dr}{ds} \right| \qquad \qquad p_s(s) = p_r(r) \frac{1}{|T'(r)|}$$

(Alternative formulation)

Example. Consider the transformation

$$s = T(r) = (L-1) \int_0^r p_r(w) dw.$$

The transformation is strictly increasing because p_r is a positive function.

$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right| \qquad \Longleftrightarrow \qquad p_s(s) = p_r(r)\frac{1}{|T'(r)|}$$

(Alternative formulation)

Example. Consider the transformation

$$s = T(r) = (L-1)\int_0^r p_r(w)dw.$$

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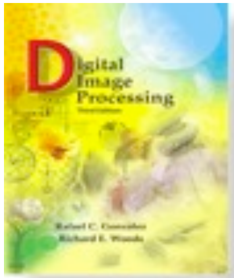
Cumulative probability



$$p_s(s) = p_r(r)\left|\frac{dr}{ds}\right| \qquad \longleftrightarrow \qquad p_s(s) = p_r(r)\frac{1}{|T'(r)|}$$

(Alternative formulation)

Example. Consider the transformation

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw.$$

The transformation is strictly increasing because p_r is a positive function.

Scaling factor so that *T(L-1)* = *L-1*

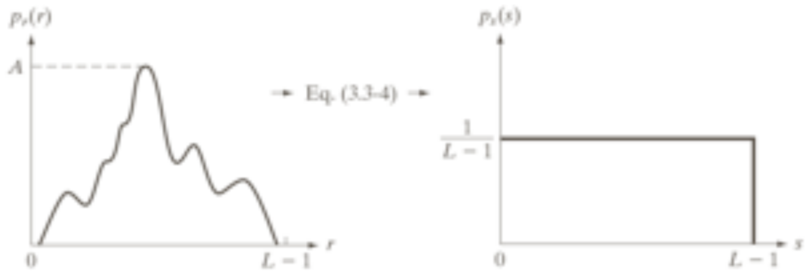
Cumulative probability

What is $p_s(s)$?

Let’s use the formula $p_s(s) = p_r(r)\dfrac{1}{|T'(r)|}$

We derive $T(r)$ with respect to r

$$T'(r) = \frac{d}{dr}T(r) = (L-1)p_r(r)$$



a b

FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

Hence

$$p_s(s) = p_r(r)\frac{1}{|T'(r)|} = p_r(r)\frac{1}{(L-1)p_r(r)} = \frac{1}{L-1}$$

The uniform distribution!

For discrete density functions, the equivalent of

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw.$$

is obtained by replacing the integral by a sum,

$$\begin{aligned} s_k &= T(r_k) = (L-1)\sum_{j=0}^{\kappa} p_r(r_j) \\ &= \frac{L-1}{MN}\sum_{j=0}^{k} n_j \qquad k = 0, 1, \ldots, L-1. \end{aligned}$$

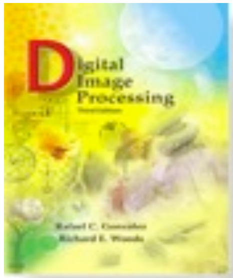
This transformation is called *histogram equalization/linearization*.

Example:

$$
\begin{aligned}
s_k &= T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) \\
&= \frac{L-1}{MN}\sum_{j=0}^{k} n_j \qquad k = 0, 1, \ldots, L-1.
\end{aligned}
$$

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

TABLE 3.1 Intensity distribution and histogram values for a 3-bit, 64×64 digital image.



Example:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$= \frac{L-1}{MN}\sum_{j=0}^{k} n_j \qquad k = 0, 1, \ldots, L-1.$$

r_k	n_k	$p_r(r_k) = n_k/MN$
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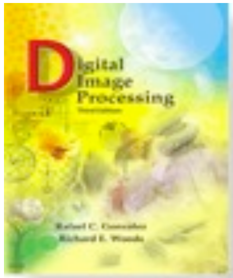
TABLE 3.1 Intensity distribution and histogram values for a 3-bit, 64×64 digital image.

$$s_0 = (L-1)p_r(r_0) = 7 \cdot 0.19 = 1.33$$

$$s_1 = 7 \cdot (0.19 + 0.25) = 3.08$$

$$s_3 = 7 \cdot (0.19 + 0.25 + 0.21) = 4.55$$



Example:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$= \frac{L-1}{MN}\sum_{j=0}^{k} n_j \qquad k = 0, 1, \ldots, L-1.$$

r_k	n_k	$p_r(r_k) = n_k/MN$
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$$s_0 = (L-1)p_r(r_0) = 7 \cdot 0.19 = 1.33$$

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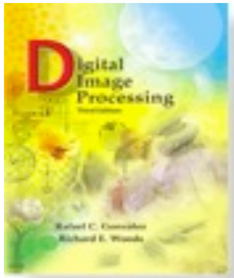
$$s_3 = 7 \cdot (0.19 + 0.25 + 0.21) = 4.55$$

$$s_0 = 1$$

$$s_1 = 3$$

$$s_2 = 5$$

and so on...



Example:

$$
\begin{aligned}
s_k &= T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) \\
&= \frac{L-1}{MN}\sum_{j=0}^{k} n_j \qquad k = 0, 1, \ldots, L-1.
\end{aligned}
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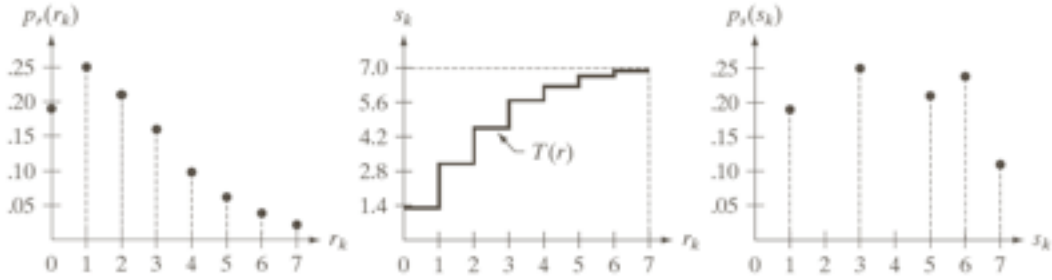
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

TABLE 3.1 Intensity distribution and histogram values for a 3-bit, 64×64 digital image.

$$
\begin{aligned}
s_0 &= (L-1)p_r(r_0) = 7 \cdot 0.19 = 1.33 \\
s_1 &= 7 \cdot (0.19 + 0.25) = 3.08 \\
s_3 &= 7 \cdot (0.19 + 0.25 + 0.21) = 4.55
\end{aligned}
\quad \Longrightarrow \quad
\begin{aligned}
s_0 &= 1 \\
s_1 &= 3 \\
s_2 &= 5
\end{aligned}
$$

and so on...

Since the s_k is an intensity transformation, it must be an integer value. This rounding causes the resulting histogram not necessarily to to result into a uniform histogram.

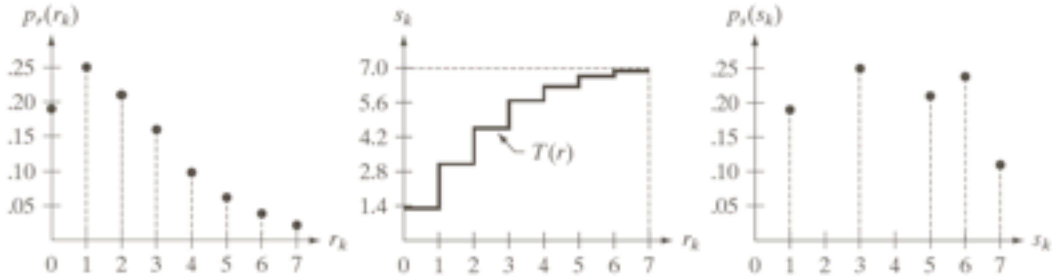


a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Though the resulting histogram is not uniform, the final result is that the intensity values are spread out (span a wider range).

This results in an image that has a enhanced contrast.



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Though the resulting histogram is not uniform, the final result is that the intensity values are spread out (span a wider range).

This results in an image that has a enhanced contrast.

NB. Note that the resulting image might reduce the actual number of intensity levels (because of discretization).

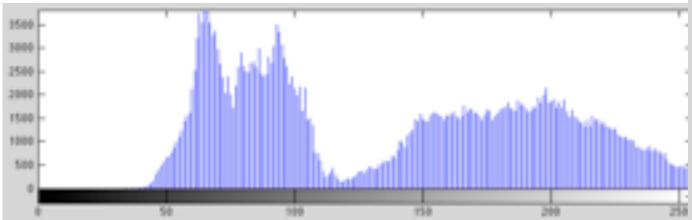
Seljord lake
(Norwegian equivalent
of Loch Ness)

Picture taken with a cell
phone camera (low contrast)

Seljord lake
(Norwegian equivalent
of Loch Ness)

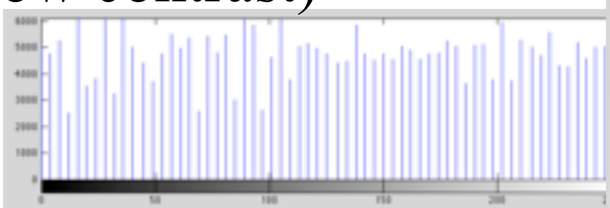
Picture taken with a cell
phone camera (low contrast)

Before and after
histogram equalization



Seljord lake (Norwegian equivalent of Loch Ness)

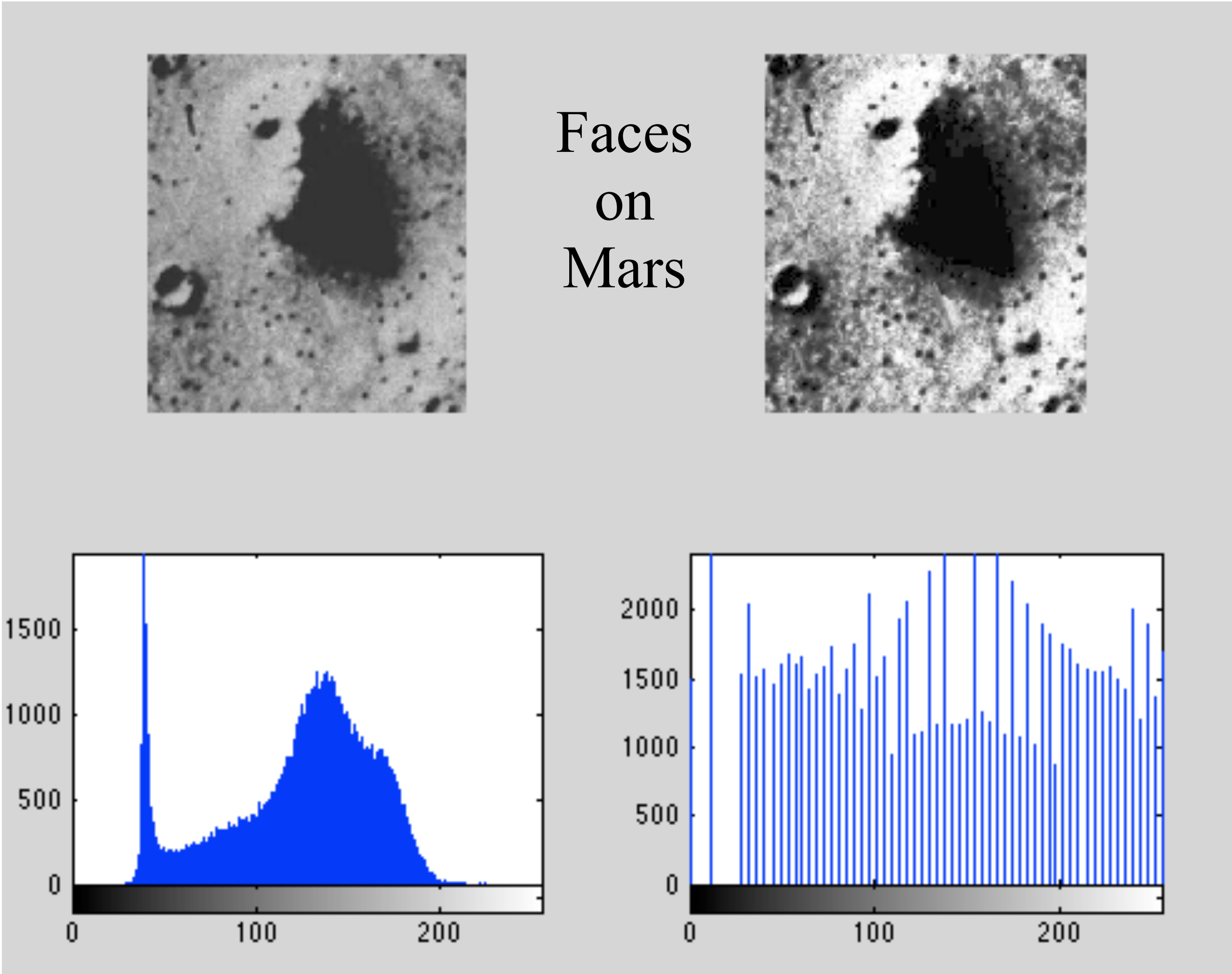
Picture taken with a cell phone camera (low contrast)



Before and after histogram equalization

Matlab command for histogram equalization:

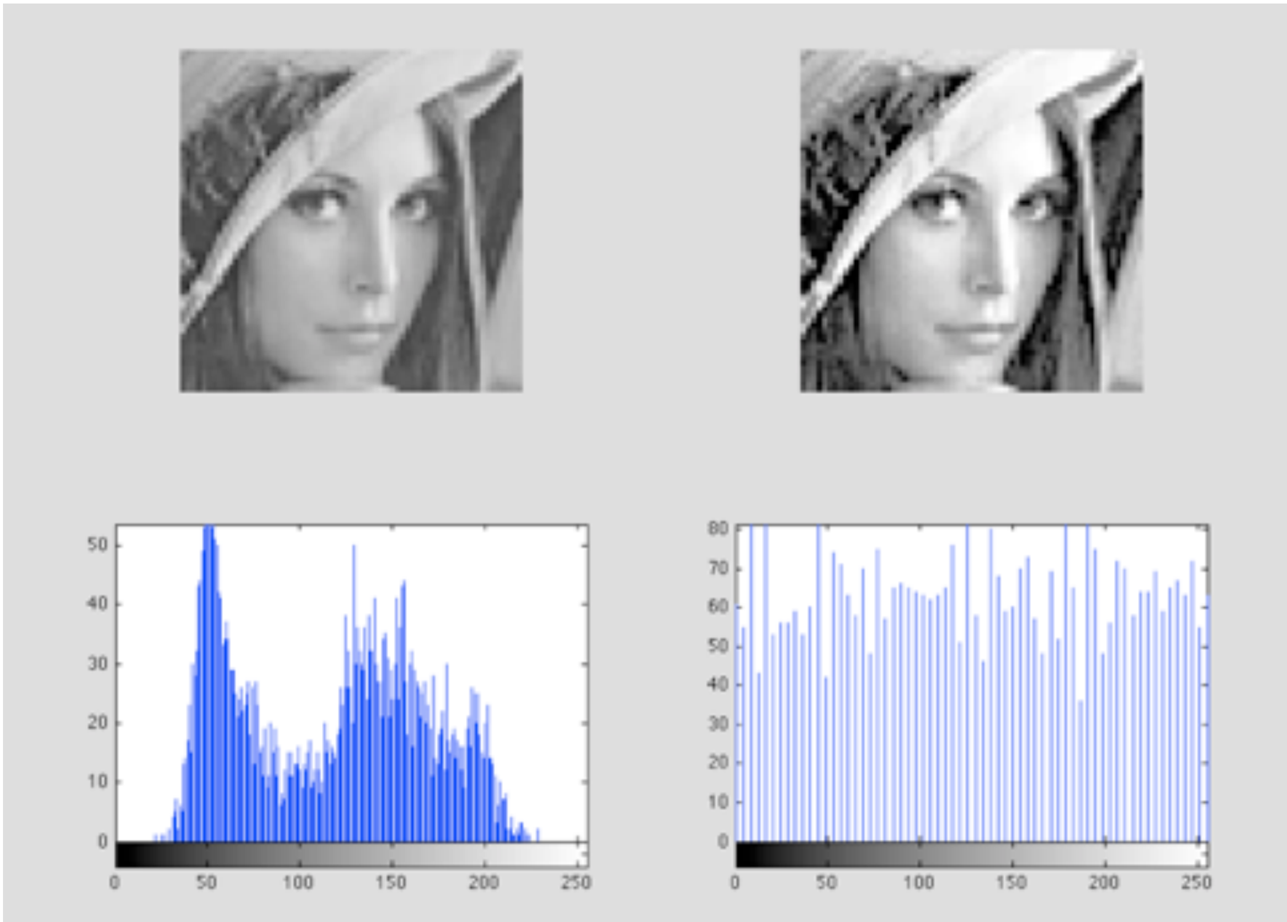
`histeq`

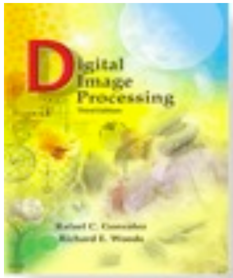


Not in all cases contrast enhancement gives a visual enhancement of the picture

(Enhancement is a visual property, hence partly subjective).

For landscapes, it works generally
well. For portraits, depends





dark image

light image

low contrast

high contrast

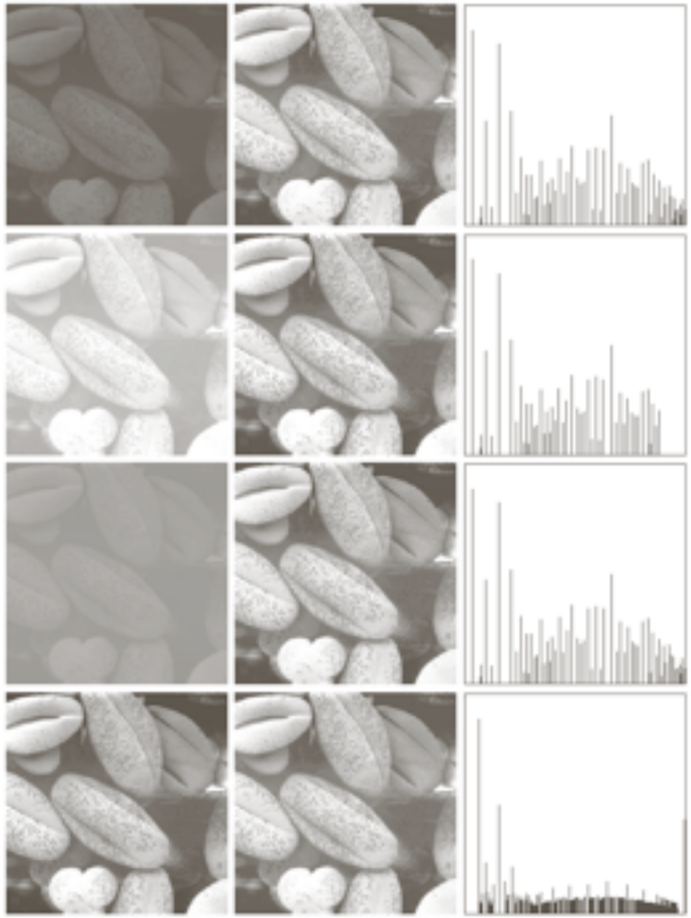


FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

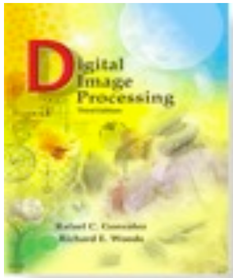
Histogram matching

As we have seen, the histogram equalization does not necessarily "enhance" equally well all the images.

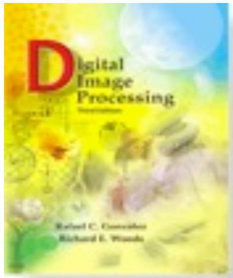
Example: the face

Assume that we have found the "winning formula", that is, a histogram that works well for enhancing a given type of images.

We might wish to apply an intensity transformation to match the prescribed histogram (probability density distribution).



$$p_r(r) \longrightarrow p_z(z)$$



equidistribution

$$p_r(r) \longrightarrow p_z(z)$$

$$\begin{tikzcd} & \text{equidistribution} & \\ p_r(r) \arrow[ur, "T(r)"] \arrow[rr] & & p_z(z) \end{tikzcd}$$

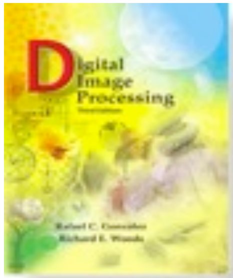
$$
\begin{tikzcd}
& \text{equidistribution} & \\
p_r(r) \arrow[ur, "T(r)"] \arrow[rr] & & p_z(z) \arrow[ul, "G(z)"']
\end{tikzcd}
$$

$$
\begin{tikzcd}
& \text{equidistribution} \arrow[dr, shift left, "G^{-1}(s)"] & \\
p_r(r) \arrow[ur, "T(r)"] \arrow[rr] & & p_z(z) \arrow[ul, shift left, "G(z)"]
\end{tikzcd}
$$

$$\begin{tikzcd}[column sep=small]
& \text{equidistribution} \arrow[dr, shift left, "G^{-1}(s)"] & \\
p_r(r) \arrow[ur, "T(r)"] \arrow[rr] & & p_z(z) \arrow[ul, shift left, "G(z)"]
\end{tikzcd}$$

$$z = G^{-1}(s) = G^{-1}(T(r))$$

Therefore, to match the histograms we have to compute the equidistribution of the probability density of first image and then the inverse using the equidistribution of the probability density of the given histogram.



$$
\begin{tikzcd}
& \text{equidistribution} \arrow[dr, shift left, "G^{-1}(s)"] & \\
p_r(r) \arrow[ur, "T(r)"] \arrow[rr] & & p_z(z) \arrow[ul, shift left, "G(z)"]
\end{tikzcd}
$$

$$z = G^{-1}(s) = G^{-1}(T(r))$$

In practice, this requires the computation of the equalization of the first image and then G^{-1}

For discrete probabilities, it is ``exactly'' the same.

Example:

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

z_q	Specified $p_z(z_q)$
$z_0 = 0$	0.00
$z_1 = 1$	0.00
$z_2 = 2$	0.00
$z_3 = 3$	0.15
$z_4 = 4$	0.20
$z_5 = 5$	0.30
$z_6 = 6$	0.20
$z_7 = 7$	0.15

$$\begin{aligned} s_k &= T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) \\ &= \frac{L-1}{MN}\sum_{j=0}^{k} n_j \qquad k = 0, 1, \ldots, L-1. \end{aligned}$$

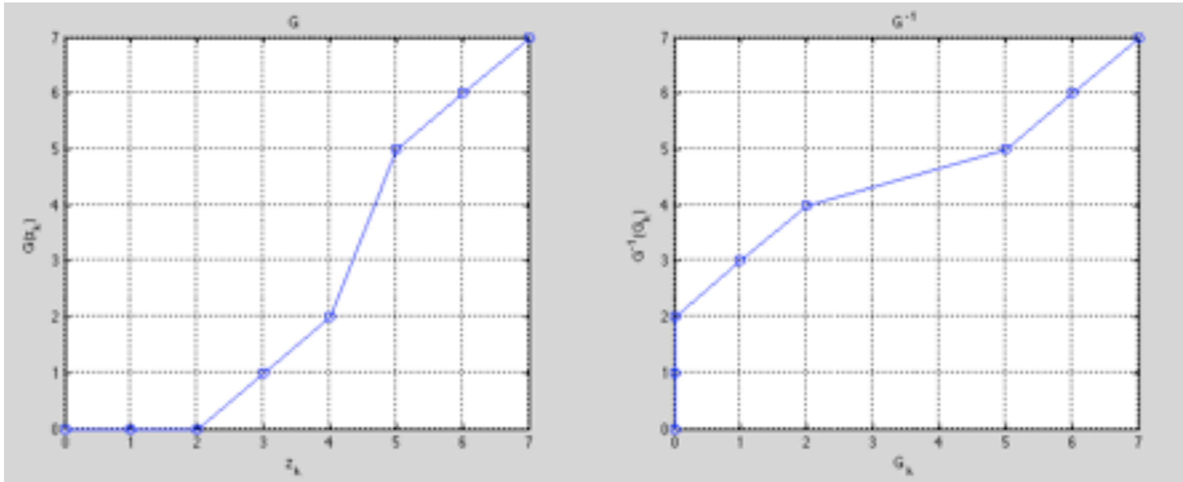
$$\begin{aligned} s_0 &= 1 \\ s_1 &= 3 \\ s_2 &= 5 \\ s_3 &= 6 \\ s_4 &= 6 \\ s_5 &= 7 \\ s_6 &= 7 \\ s_7 &= 7 \end{aligned}$$

$$\begin{aligned} G(z_0) &= 0(0) \\ G(z_1) &= 0(0) \\ G(z_2) &= 0(0) \\ G(z_3) &= 1(1.05) \\ G(z_4) &= 2(2.45) \\ G(z_5) &= 5(4.55) \\ G(z_6) &= 6(5.95) \\ G(z_7) &= 7(7) \end{aligned}$$

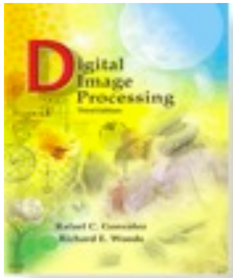
*(L-1)*cum.prob.*
*=7*0.15=1.05*

*(L-1)*cum.prob.*
=7(0.15+0.2)*
=2.45

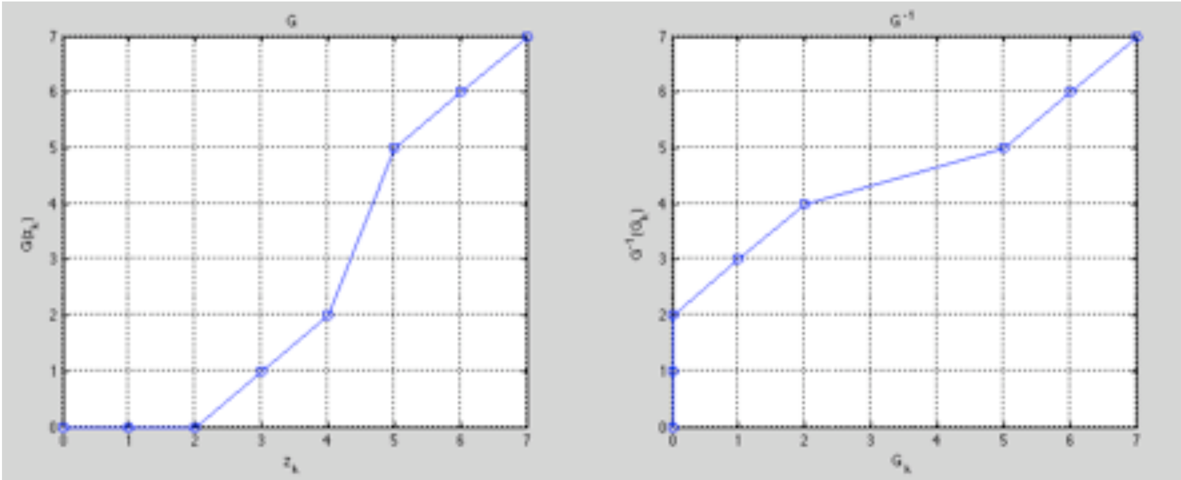
$$
\begin{aligned}
G(z_0) &= 0(0)\\
G(z_1) &= 0(0)\\
G(z_2) &= 0(0)\\
G(z_3) &= 1(1.05)\\
G(z_4) &= 2(2.45)\\
G(z_5) &= 5(4.55)\\
G(z_6) &= 6(5.95)\\
G(z_7) &= 7(7)
\end{aligned}
$$



Now we can compute the inverse of G for the range of interest.



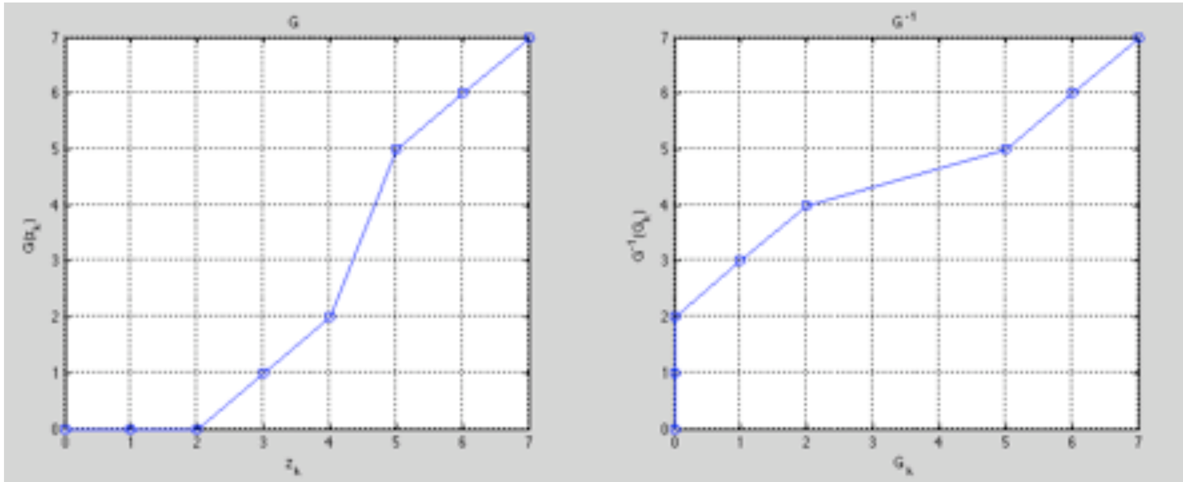
$$
\begin{aligned}
G(z_0) &= 0(0) \\
G(z_1) &= 0(0) \\
G(z_2) &= 0(0) \\
G(z_3) &= 1(1.05) \\
G(z_4) &= 2(2.45) \\
G(z_5) &= 5(4.55) \\
G(z_6) &= 6(5.95) \\
G(z_7) &= 7(7)
\end{aligned}
$$



Now we can compute the inverse of G for the range of interest.

$$
\begin{aligned}
s_0 &= 1 \\
s_1 &= 3 \\
s_2 &= 5 \\
s_3 &= 6 \\
s_4 &= 6 \\
s_5 &= 7 \\
s_6 &= 7 \\
s_7 &= 7
\end{aligned}
$$

$$
\begin{aligned}
G(z_0) &= 0(0)\\
G(z_1) &= 0(0)\\
G(z_2) &= 0(0)\\
G(z_3) &= 1(1.05)\\
G(z_4) &= 2(2.45)\\
G(z_5) &= 5(4.55)\\
G(z_6) &= 6(5.95)\\
G(z_7) &= 7(7)
\end{aligned}
$$

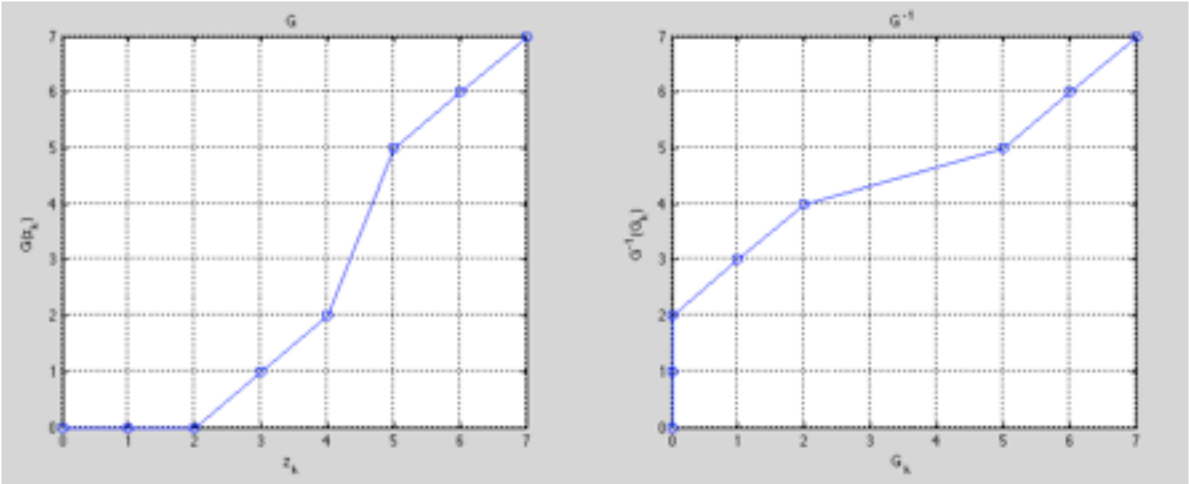


Now we can compute the inverse of G for the range of interest.

$$
\begin{aligned}
s_0 &= 1\\
s_1 &= 3\\
s_2 &= 5\\
s_3 &= 6\\
s_4 &= 6\\
s_5 &= 7\\
s_6 &= 7\\
s_7 &= 7
\end{aligned}
$$

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

$$
\begin{aligned}
G(z_0) &= 0(0)\\
G(z_1) &= 0(0)\\
G(z_2) &= 0(0)\\
G(z_3) &= 1(1.05)\\
G(z_4) &= 2(2.45)\\
G(z_5) &= 5(4.55)\\
G(z_6) &= 6(5.95)\\
G(z_7) &= 7(7)
\end{aligned}
$$



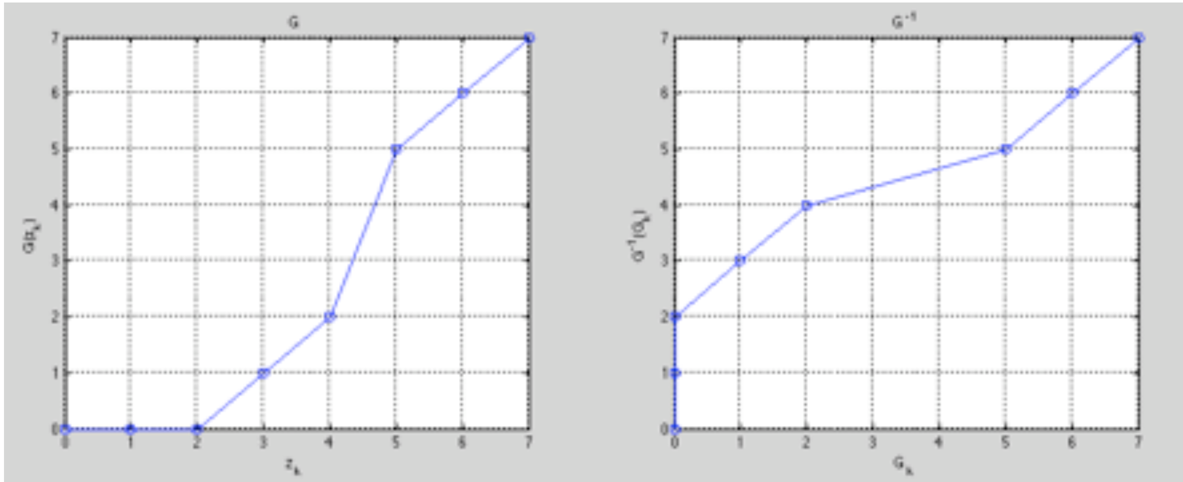
Now we can compute the inverse of G for the range of interest.

$$
\begin{aligned}
r_0 &= 0\\
r_1 &= 1\\
r_2 &= 2\\
r_3 &= 3\\
r_4 &= 4\\
r_5 &= 5\\
r_6 &= 6\\
r_7 &= 7
\end{aligned}
$$

$$
\begin{aligned}
s_0 &= 1\\
s_1 &= 3\\
s_2 &= 5\\
s_3 &= 6\\
s_4 &= 6\\
s_5 &= 7\\
s_6 &= 7\\
s_7 &= 7
\end{aligned}
$$

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

$$
\begin{aligned}
G(z_0) &= 0(0)\\
G(z_1) &= 0(0)\\
G(z_2) &= 0(0)\\
G(z_3) &= 1(1.05)\\
G(z_4) &= 2(2.45)\\
G(z_5) &= 5(4.55)\\
G(z_6) &= 6(5.95)\\
G(z_7) &= 7(7)
\end{aligned}
$$



Now we can compute the inverse of G for the range of interest.

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\begin{aligned}
r_0 &= 0 & s_0 &= 1\\
r_1 &= 1 & s_1 &= 3\\
r_2 &= 2 & s_2 &= 5\\
r_3 &= 3 & s_3 &= 6\\
r_4 &= 4 & s_4 &= 6\\
r_5 &= 5 & s_5 &= 7\\
r_6 &= 6 & s_6 &= 7\\
r_7 &= 7 & s_7 &= 7
\end{aligned}
$$

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

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\end{aligned}
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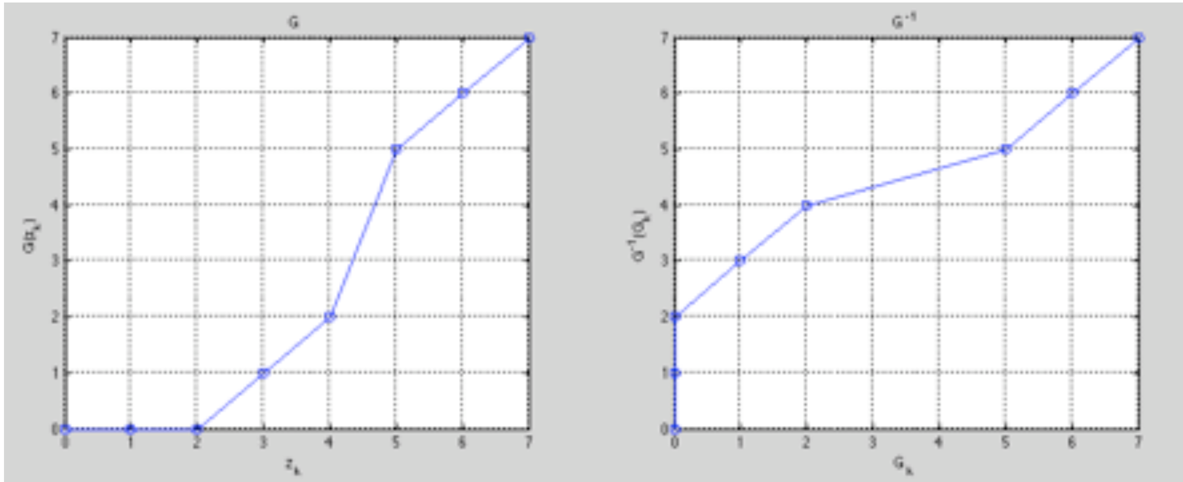

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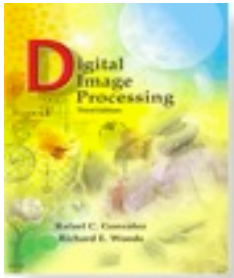
r_3 &= 3 & s_3 &= 6\\
r_4 &= 4 & s_4 &= 6\\
r_5 &= 5 & s_5 &= 7\\
r_6 &= 6 & s_6 &= 7\\
r_7 &= 7 & s_7 &= 7
\end{aligned}
$$

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7



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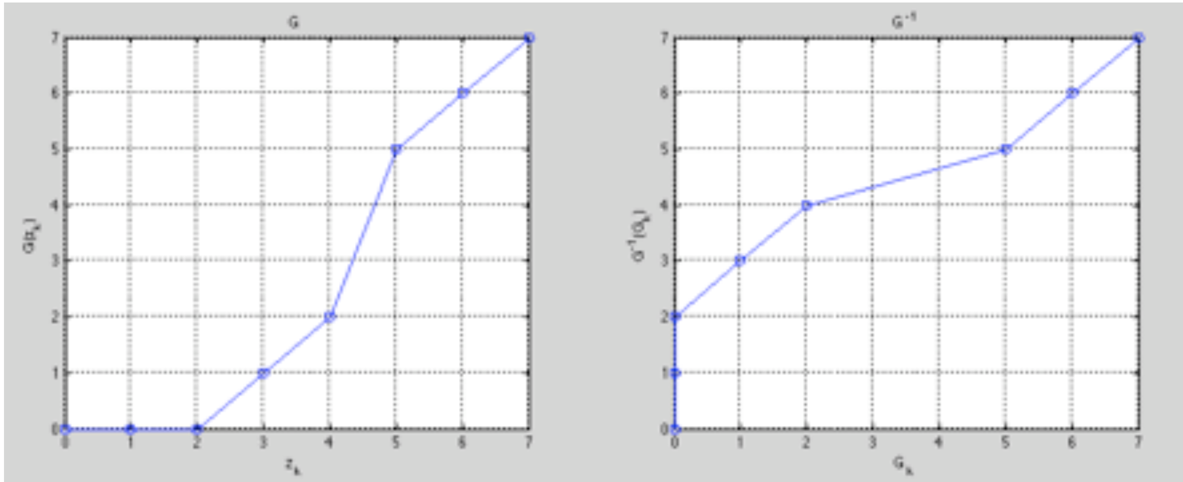
G(z_6) &= 6(5.95)\\
G(z_7) &= 7(7)
\end{aligned}
$$



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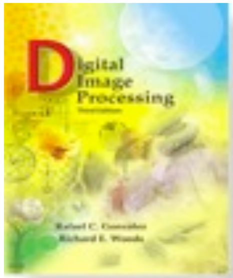
r_6 &= 6 & s_6 &= 7\\
r_7 &= 7 & s_7 &= 7
\end{aligned}
$$

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7



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\begin{aligned}
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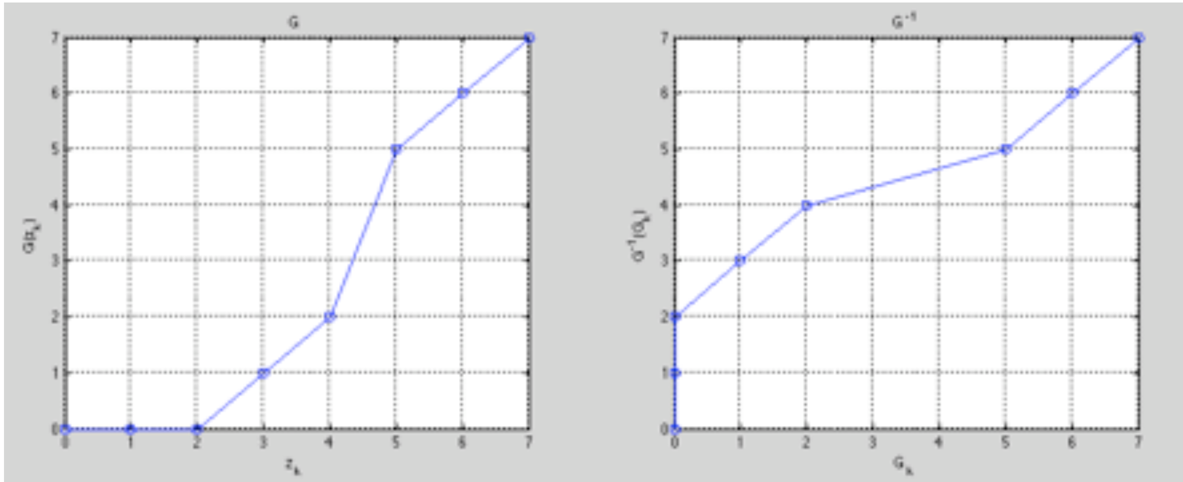
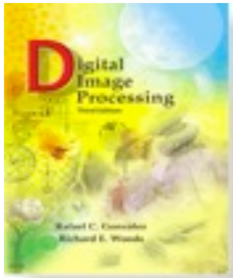
\end{aligned}
$$



Now we can compute the inverse of G for the range of interest.

$$
\begin{aligned}
r_0 &= 0 \rightarrow s_0 = 1\\
r_1 &= 1 \rightarrow s_1 = 3\\
r_2 &= 2 \rightarrow s_2 = 5\\
r_3 &= 3 \rightarrow s_3 = 6\\
r_4 &= 4 \quad s_4 = 6\\
r_5 &= 5 \quad s_5 = 7\\
r_6 &= 6 \quad s_6 = 7\\
r_7 &= 7 \quad s_7 = 7
\end{aligned}
$$

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7



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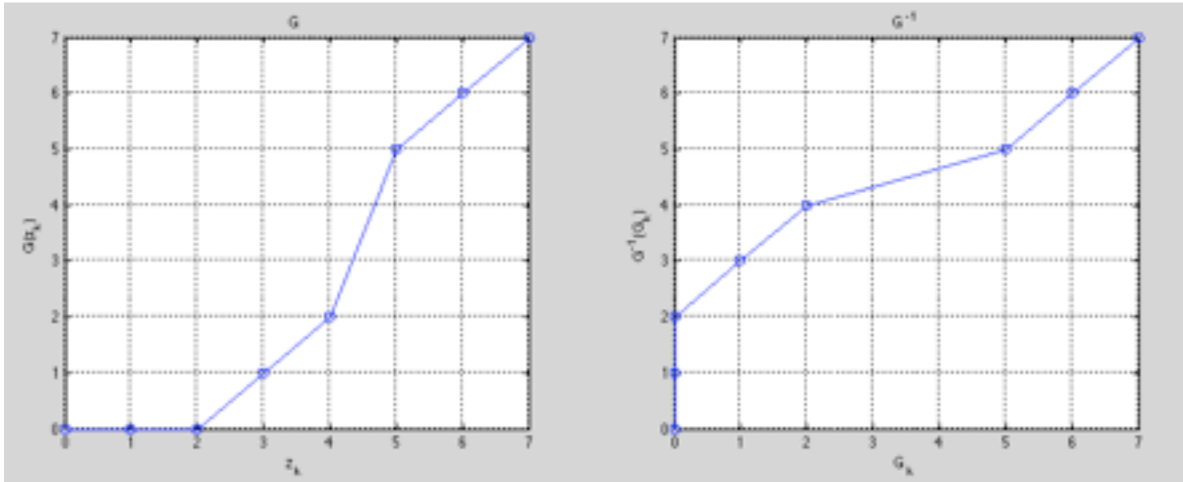
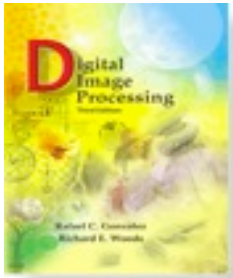
G(z_5) &= 5(4.55)\\
G(z_6) &= 6(5.95)\\
G(z_7) &= 7(7)
\end{aligned}
$$



Now we can compute the inverse of G for the range of interest.

$r_0 = 0$	→	$s_0 = 1$
$r_1 = 1$	→	$s_1 = 3$
$r_2 = 2$	→	$s_2 = 5$
$r_3 = 3$	→	$s_3 = 6$
$r_4 = 4$	→	$s_4 = 6$
$r_5 = 5$		$s_5 = 7$
$r_6 = 6$		$s_6 = 7$
$r_7 = 7$		$s_7 = 7$

s_k	→	z_q
1	→	3
3	→	4
5	→	5
6	→	6
7	→	7



Chapter 3
Intensity Transformations & Spatial Filtering

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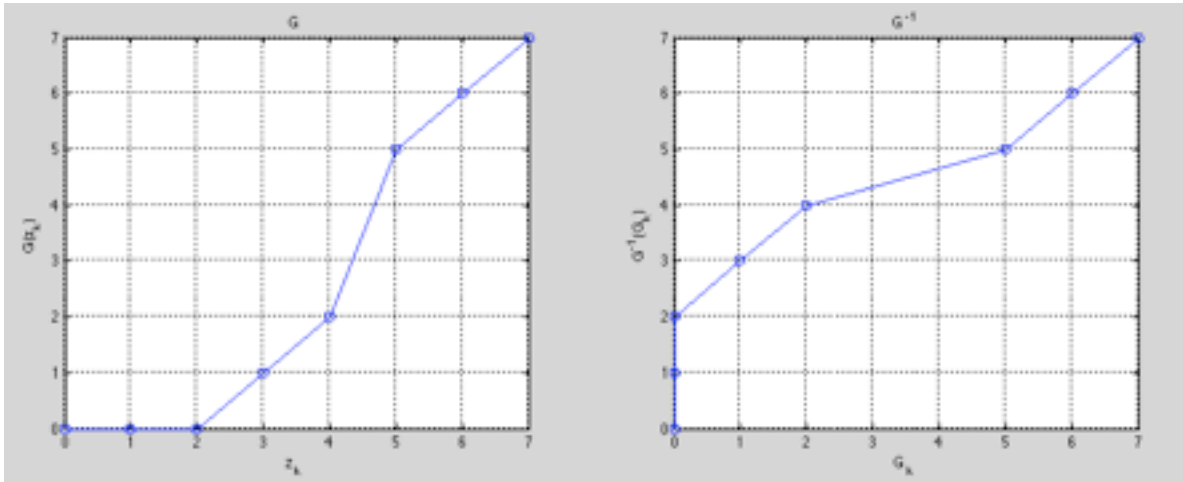
G(z_4) &= 2(2.45)\\
G(z_5) &= 5(4.55)\\
G(z_6) &= 6(5.95)\\
G(z_7) &= 7(7)
\end{aligned}
$$



Now we can compute the inverse of G for the range of interest.

$$
\begin{aligned}
r_0 = 0 &\rightarrow s_0 = 1 \rightarrow 3\\
r_1 = 1 &\rightarrow s_1 = 3 \rightarrow 4\\
r_2 = 2 &\rightarrow s_2 = 5 \rightarrow 5\\
r_3 = 3 &\rightarrow s_3 = 6 \rightarrow 6\\
r_4 = 4 &\rightarrow s_4 = 6 \rightarrow 6\\
r_5 = 5 &\rightarrow s_5 = 7 \rightarrow 7\\
r_6 = 6 & \quad s_6 = 7\\
r_7 = 7 & \quad s_7 = 7
\end{aligned}
$$

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

$$
\begin{aligned}
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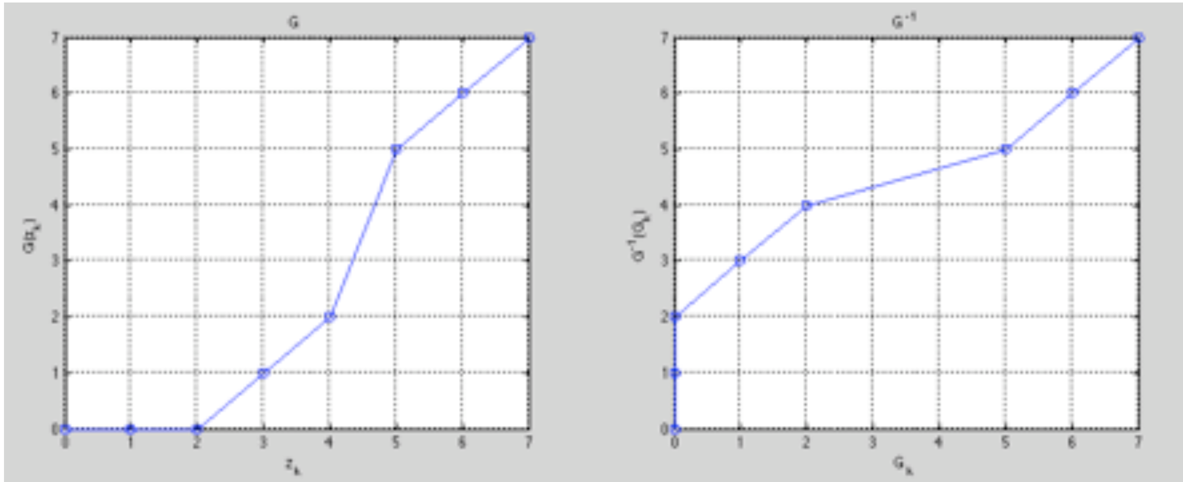
G(z_4) &= 2(2.45)\\
G(z_5) &= 5(4.55)\\
G(z_6) &= 6(5.95)\\
G(z_7) &= 7(7)
\end{aligned}
$$



Now we can compute the inverse of G for the range of interest.

$$
\begin{aligned}
r_0 &= 0 \rightarrow s_0 = 1\\
r_1 &= 1 \rightarrow s_1 = 3\\
r_2 &= 2 \rightarrow s_2 = 5\\
r_3 &= 3 \rightarrow s_3 = 6\\
r_4 &= 4 \rightarrow s_4 = 6\\
r_5 &= 5 \rightarrow s_5 = 7\\
r_6 &= 6 \rightarrow s_6 = 7\\
r_7 &= 7 \quad\;\; s_7 = 7
\end{aligned}
$$

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
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$$
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G(z_6) &= 6(5.95)\\
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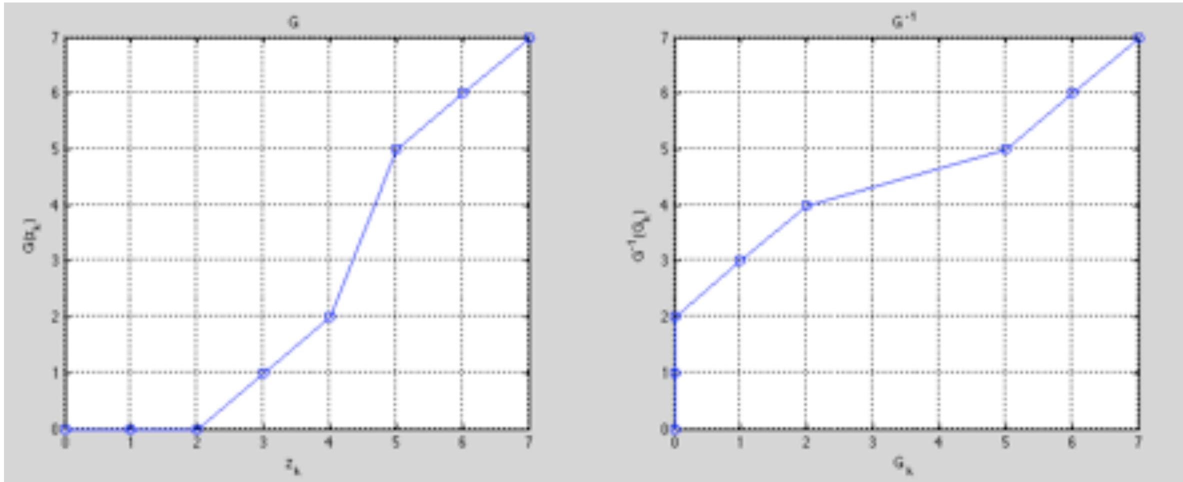
\end{aligned}
$$



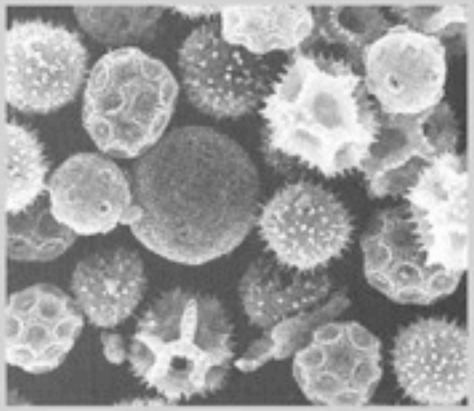
Now we can compute the inverse of *G* for the range of interest.

$$
\begin{aligned}
r_0 &= 0 \rightarrow s_0 = 1\\
r_1 &= 1 \rightarrow s_1 = 3\\
r_2 &= 2 \rightarrow s_2 = 5\\
r_3 &= 3 \rightarrow s_3 = 6\\
r_4 &= 4 \rightarrow s_4 = 6\\
r_5 &= 5 \rightarrow s_5 = 7\\
r_6 &= 6 \rightarrow s_6 = 7\\
r_7 &= 7 \rightarrow s_7 = 7
\end{aligned}
$$

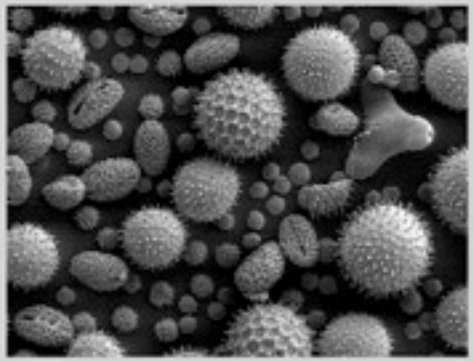
s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

Since we are working with integer number, because of rounding the computed transformation does not give an *exact* histogram match.

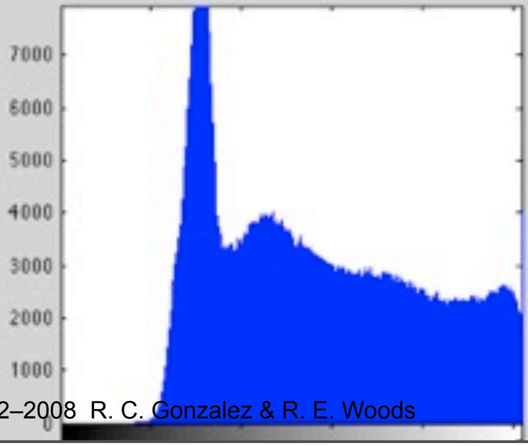
However, there will be a tendency to move the intensity values to have a similar match.

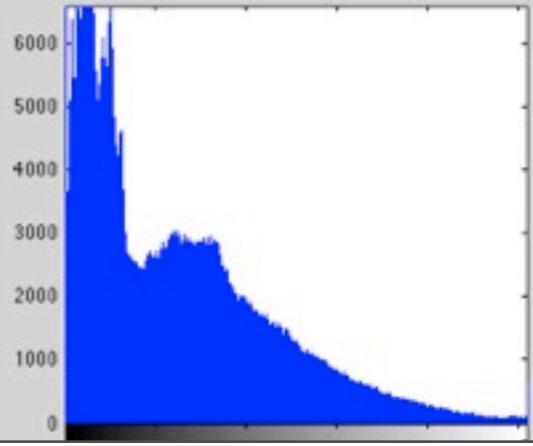


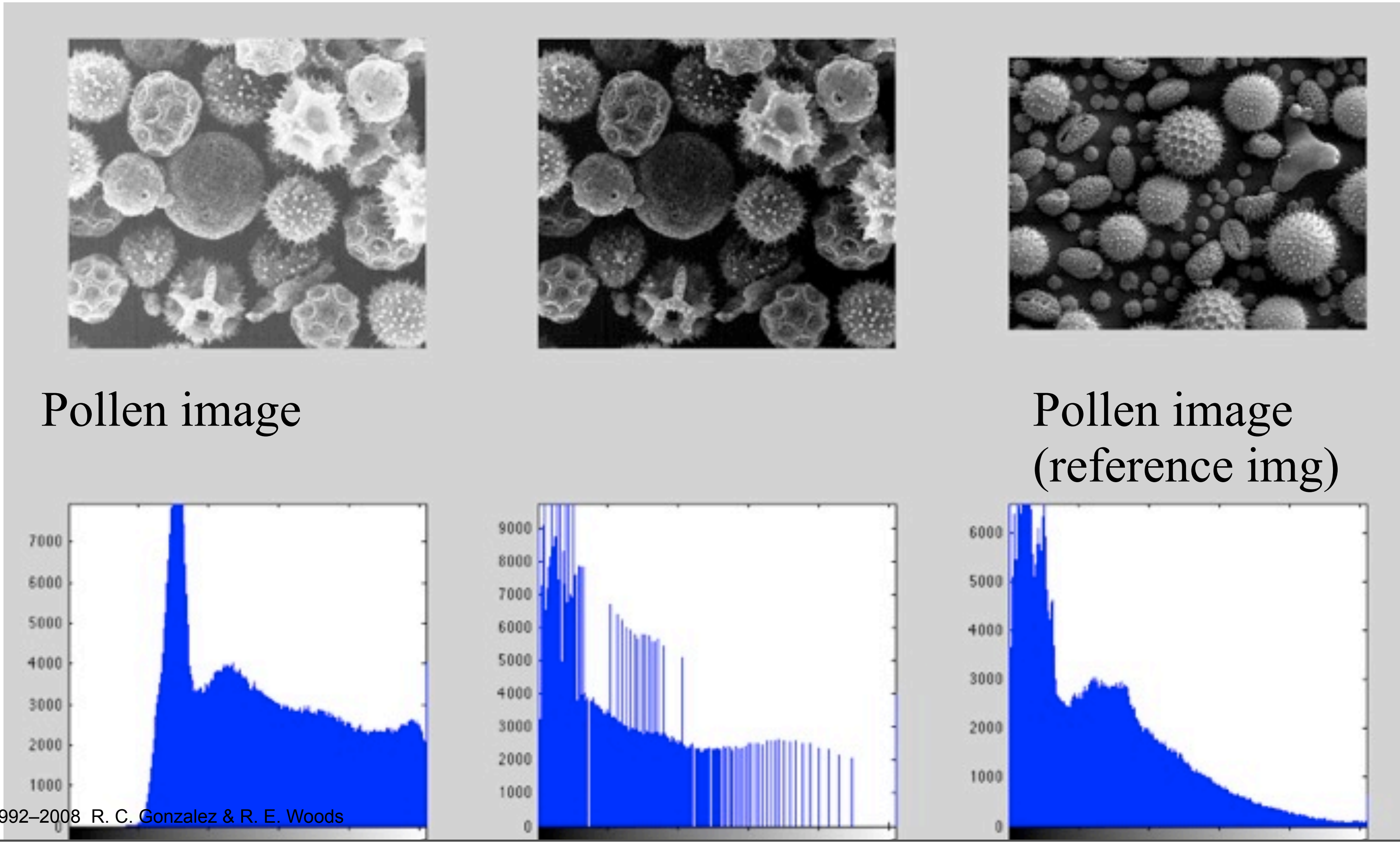
Pollen image

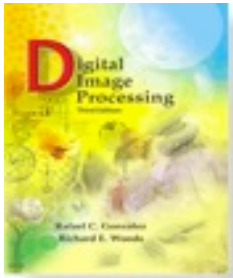


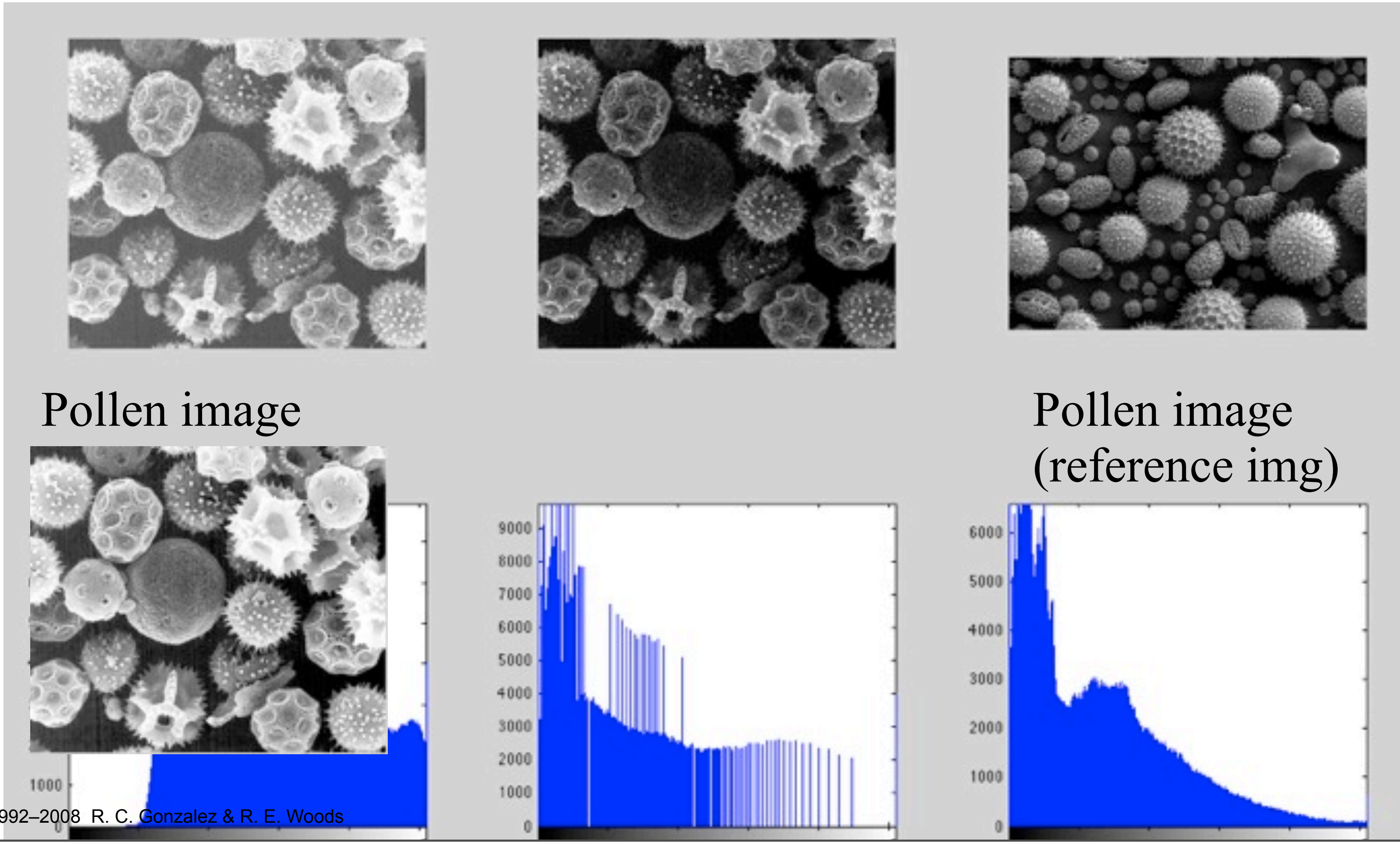
Pollen image
(reference img)






Pollen image
Pollen image
(reference img)
7000
6000
5000
4000
3000
2000
1000
0
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
6000
5000
4000
3000
2000
1000
0





Pollen image
Pollen image
(reference img)
1000
0
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
6000
5000
4000
3000
2000
1000
0

How to do the histogram matching in Matlab?

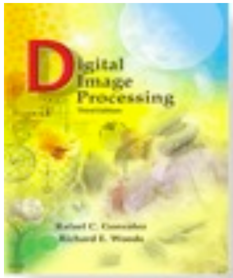
`J=histeq(I)` : histogram equalization of the image I

`J=histeq(I, HIST)` : creates J from I by intensity transformation so that the histogram of J and HIST match.

The histogram transformations that we have seen up to this point are *global*, in the sense that they act on the whole image.

In some applications, one might wish to apply different transformations to different regions of the image

Local histogram processing



Further image statistics (normalized quantities):

Definitions:

Average intensity:

$$m = \sum_{i=1}^{L-1} r_i p(r_i)$$

(also called *mean* intensity)

nth moment:

$$\mu_n = \sum_{i=1}^{L-1} (r_i - m)^n p(r_i)$$

Second moment:

$$\mu_2 = \sum_{i=1}^{L-1} (r_i - m)^2 p(r_i)$$

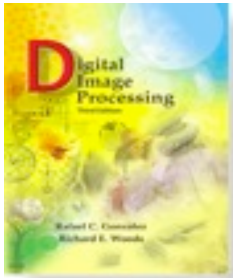
(this is the intensity variance, and equals σ^2)

The mean m (average) and the variance σ^2 can be computed directly from the image data,

$$m = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y)$$

$$\sigma^2 = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} (f(x, y) - m)^2$$

It is an easy exercise to see that the values computed in this manner are the same obtained with the probability density function.



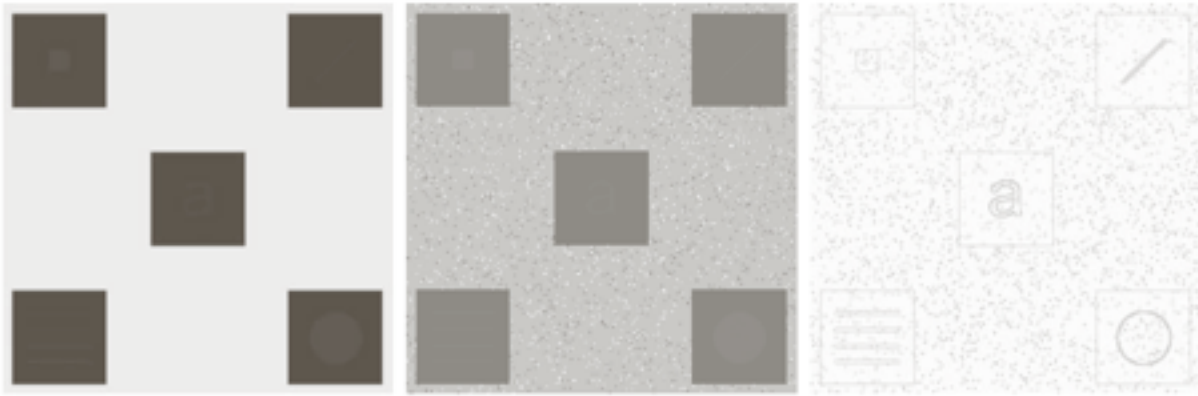
The previous definitions of mean, average, moments, can easily be adapted to a *subregion* of an image (instead of the full one).

$$m_{S_{xy}} = \sum_{i=1}^{L-1} r_i p_{S_{xy}}(r_i) = \frac{1}{|S_{xy}|} \sum_{(s,t)\in S_{xy}} f(s,t)$$

$$\sigma^2 = \sum_{i=1}^{L-1} (r_i - m_{S_{x,y}})^2 p_{S_{xy}}(r_i) = \frac{1}{|S_{xy}|} \sum_{(s,t)\in S_{xy}} (f(s,t) - m_{S_{xy}})^2$$

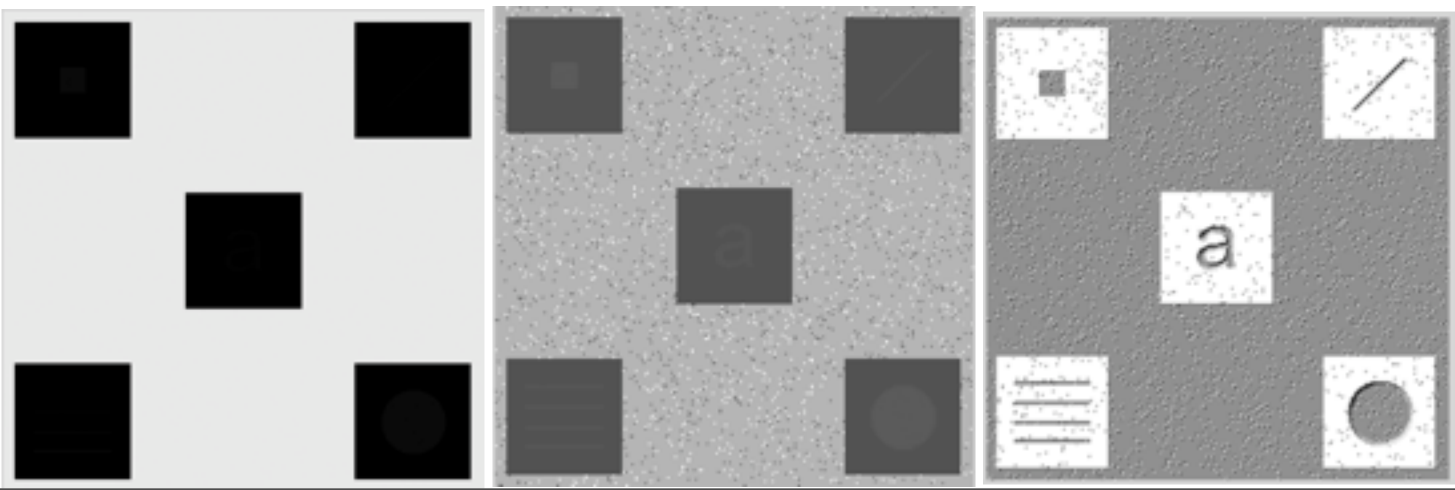
Example: we have an image, we are satisfied from parts of it (foregreound), want to enhance other parts (background)

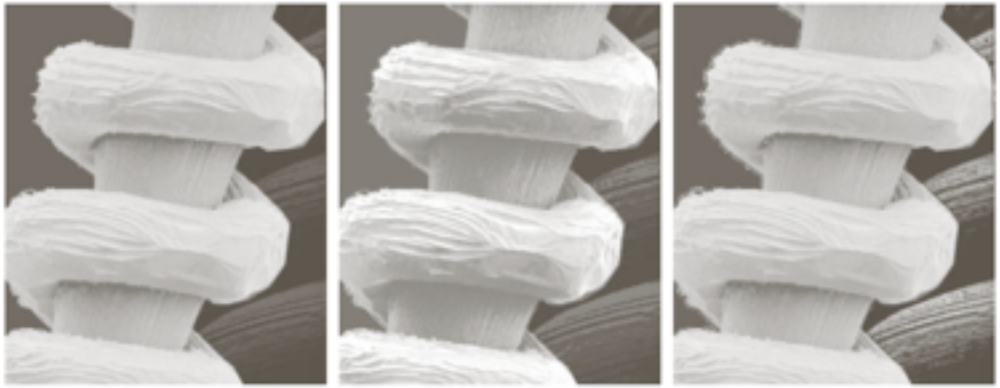
Chapter 3
Intensity Transformations & Spatial Filtering



a b c

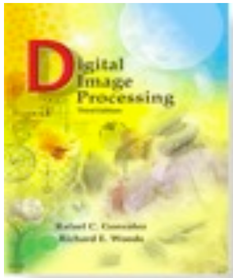
FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3 × 3.





a b c

FIGURE 3.27 (a) SEM image of a tungsten filament magnified approximately 130×. (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

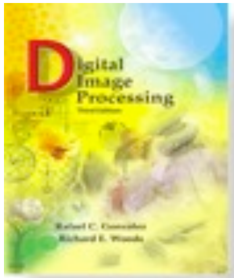


Fundaments of spatial filtering

Filtering = sieving, removing unwanted components

Image filtering adapts and expands techniques already used in signal processing.

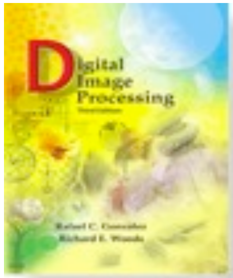
Filtering is accomplished directly on the image using *masks* (*kernels*, *templates*, *windows*)



We have already seen an example of spatial filtering

(blurring by 9 pixels averaging (D8))

This is a special example of *linear filter* as the result (response) is obtained as a linear combination of the input image pixel values.

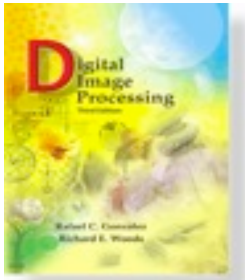


We have already seen an example of spatial filtering

(blurring by 9 pixels averaging (D8))

This is a special example of *linear filter* as the result (response) is obtained as a linear combination of the input image pixel values.

Example of a blurred image by a 3x3 mask



A generic D_8 *linear* filter will have the following expression

$$
\begin{aligned}
g(x,y) \quad = \quad & w(-1,-1)f(x-1,y-1) + w(-1,0)f(x-1,0) + \cdots \\
& + w(0,0)f(x,y) + \cdots + w(1,1)f(x+1,y+1)
\end{aligned}
$$

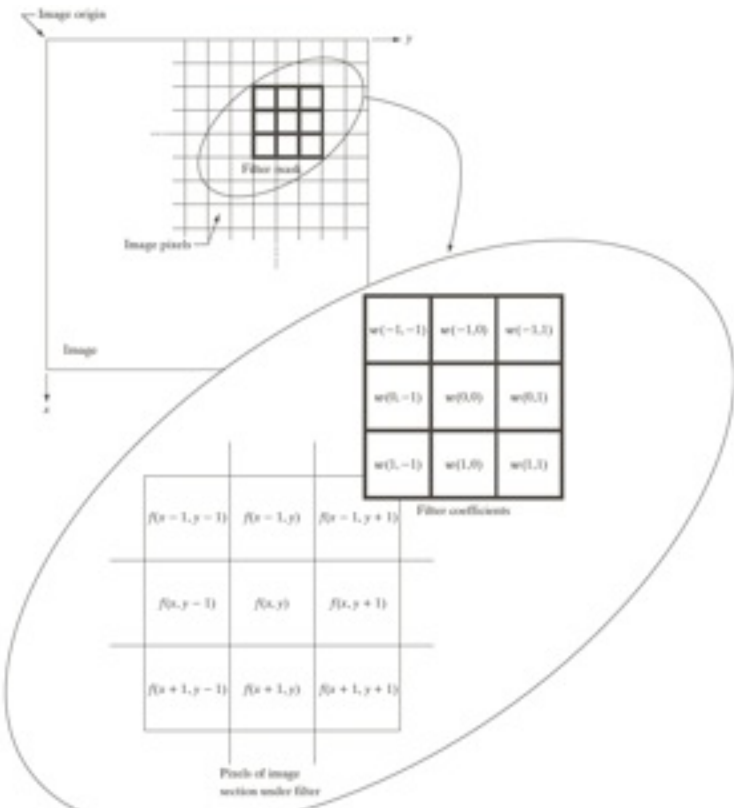
or, for a generic mask using mn pixels,

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

where

$$m = 2a+1, \qquad n = 2b+1$$

,



2 important filter concepts: Spatial correlation and convolution

Correlation: is the process of moving a filter mask over an image and computing the sum of the (pointwise) product.

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

Convolution: mechanically the same (except for the mask, rotated by 180 degrees).

$$g(x,y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$