Chapter 7

Wavelets and Multiresolution Processing

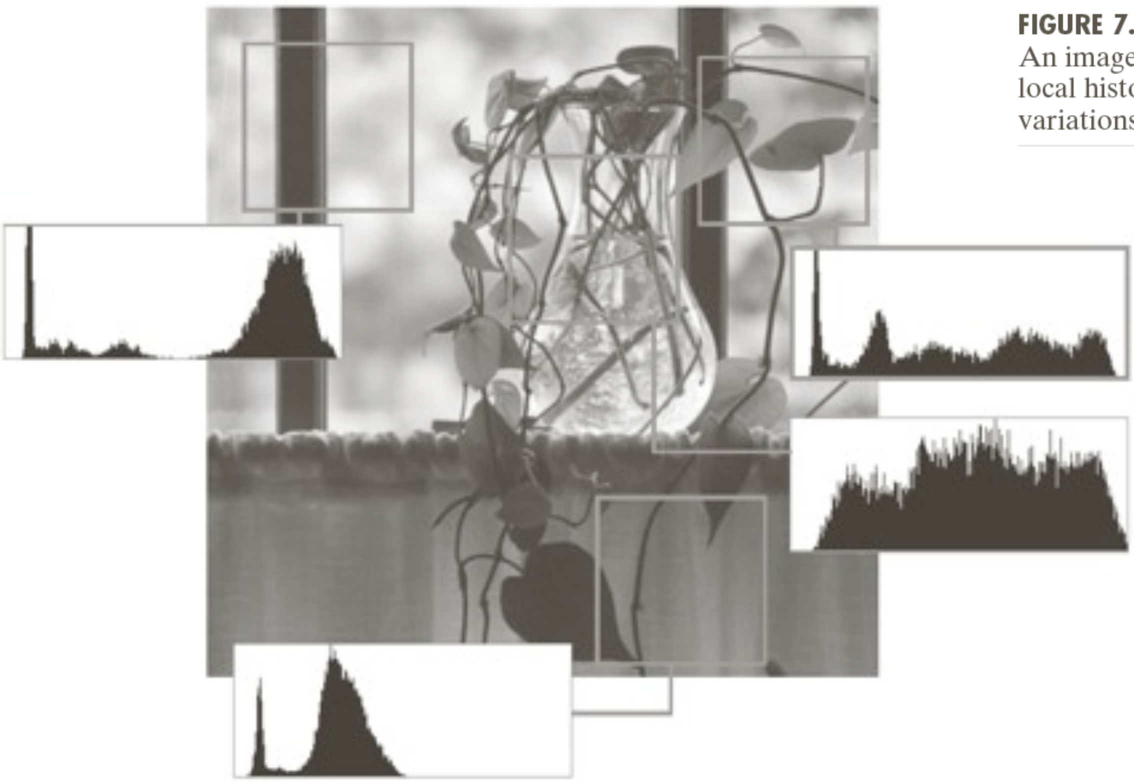
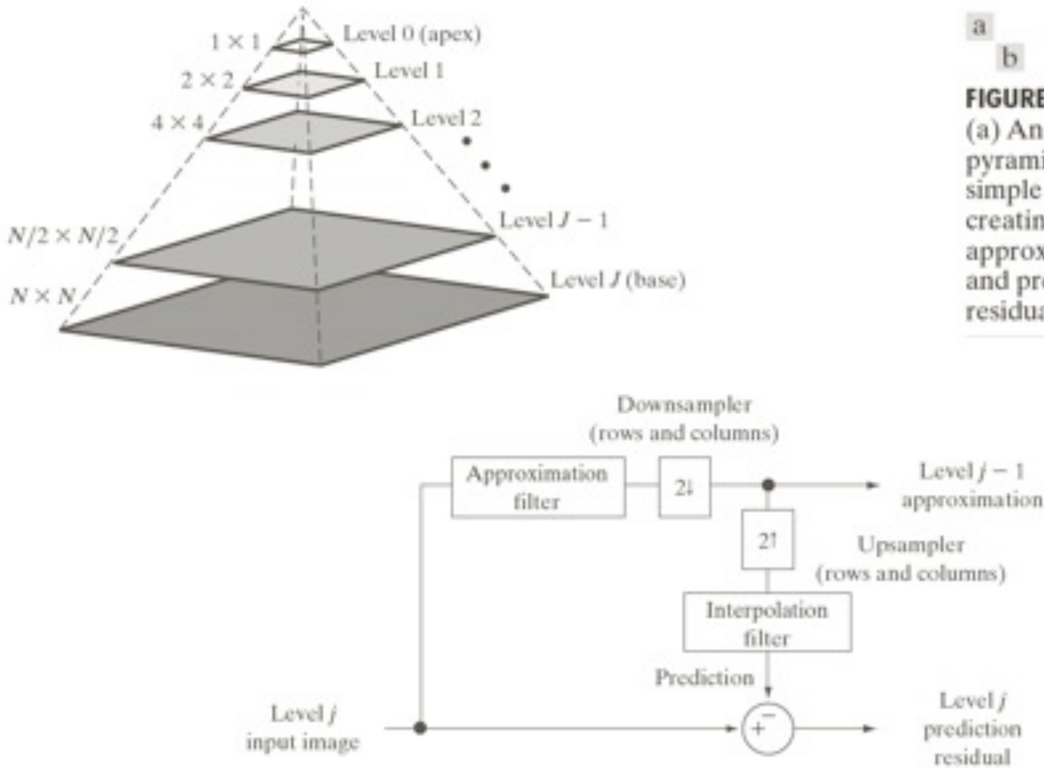
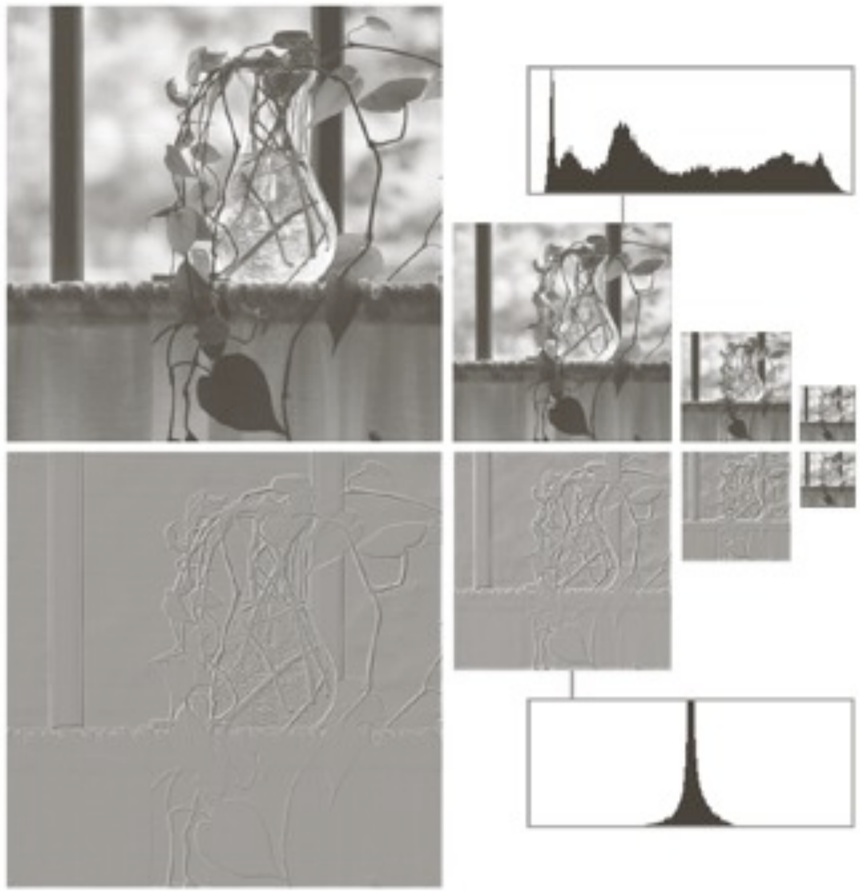


FIGURE 7.1 An image and its local histogram variations.



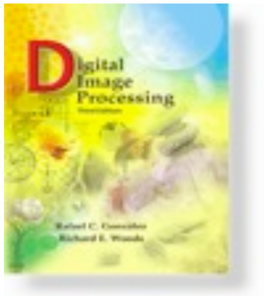
a b

FIGURE 7.2 (a) An image pyramid. (b) A simple system for creating approximation and prediction residual pyramids.



a
b

FIGURE 7.3 Two image pyramids and their histograms: (a) an approximation pyramid; (b) a prediction residual pyramid.



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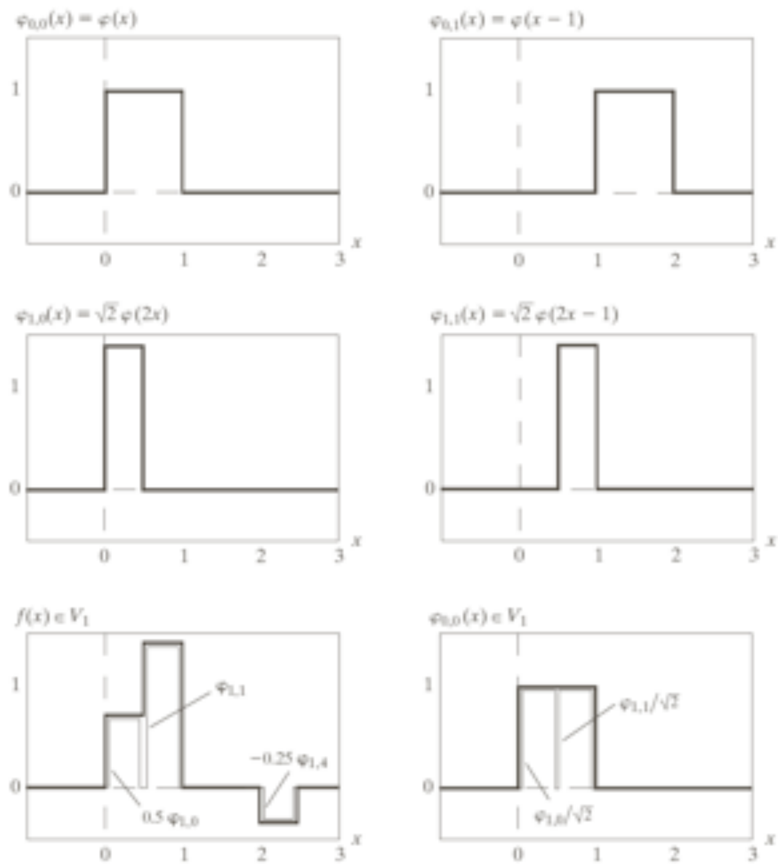


a b
c d
e f

FIGURE 7.11 Some Haar scaling functions.

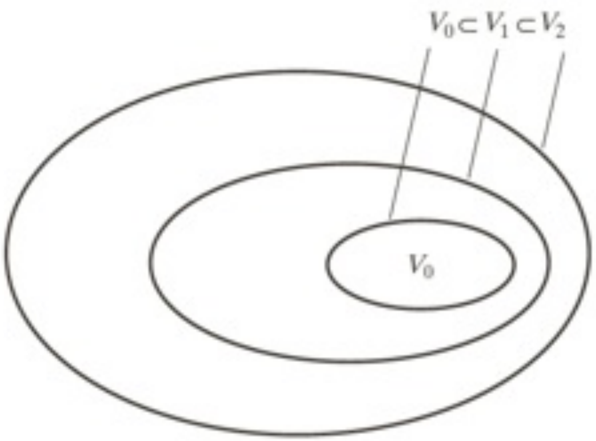


FIGURE 7.12 The nested function spaces spanned by a scaling function.

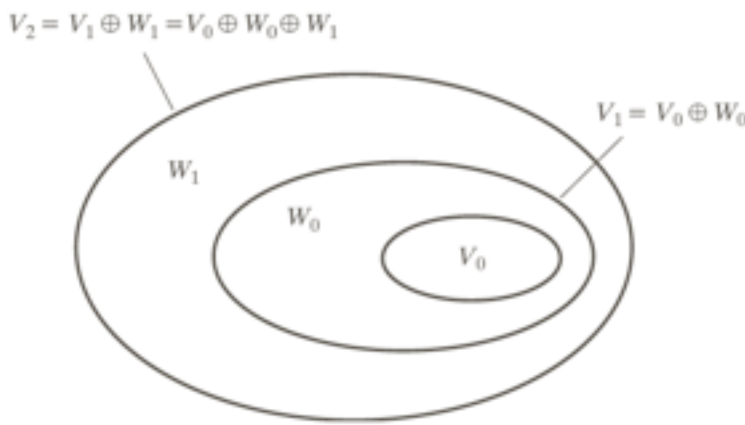
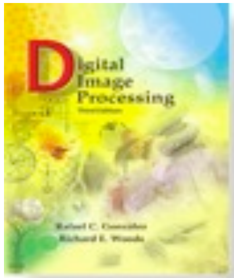


FIGURE 7.13 The relationship between scaling and wavelet function spaces.



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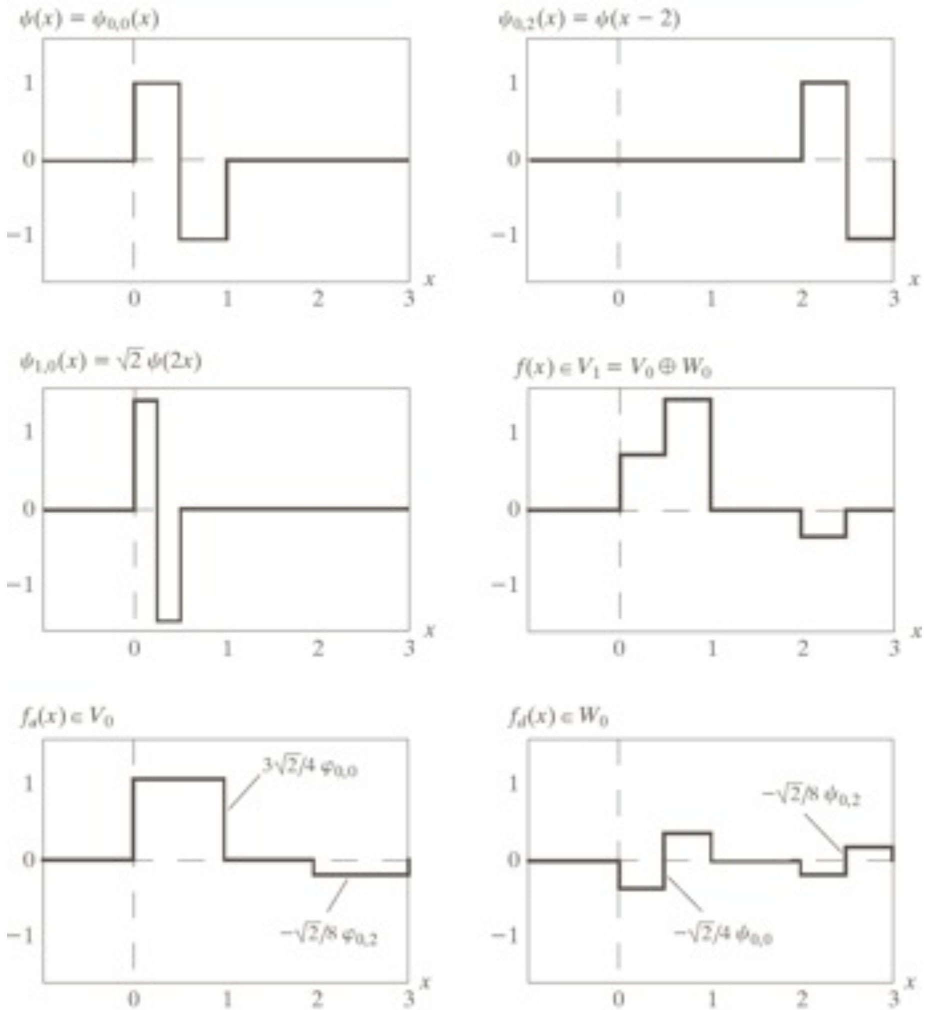

a b
c d
e f

FIGURE 7.14 Haar wavelet functions in W_0 and W_1.

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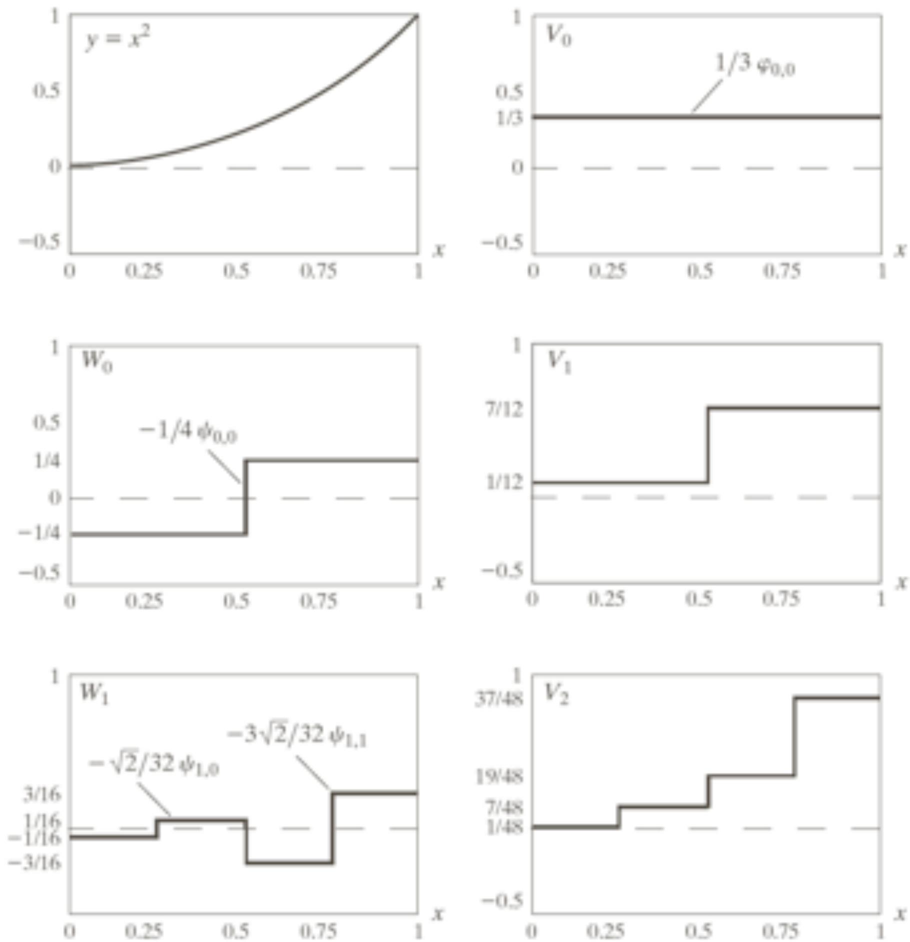
Wavelets and Multiresolution Processing



a b
c d
e f

FIGURE 7.15 A wavelet series expansion of $y = x^2$ using Haar wavelets.

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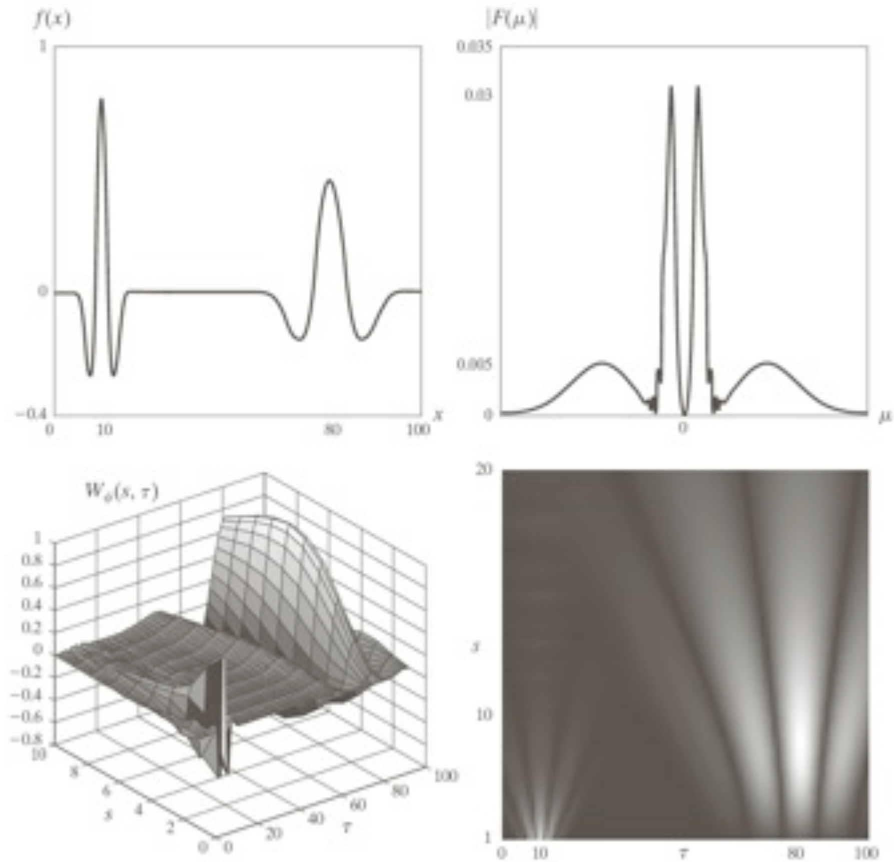
Wavelets and Multiresolution Processing



a b
c d

FIGURE 7.16 The continuous wavelet transform (c and d) and Fourier spectrum (b) of a continuous 1-D function (a).

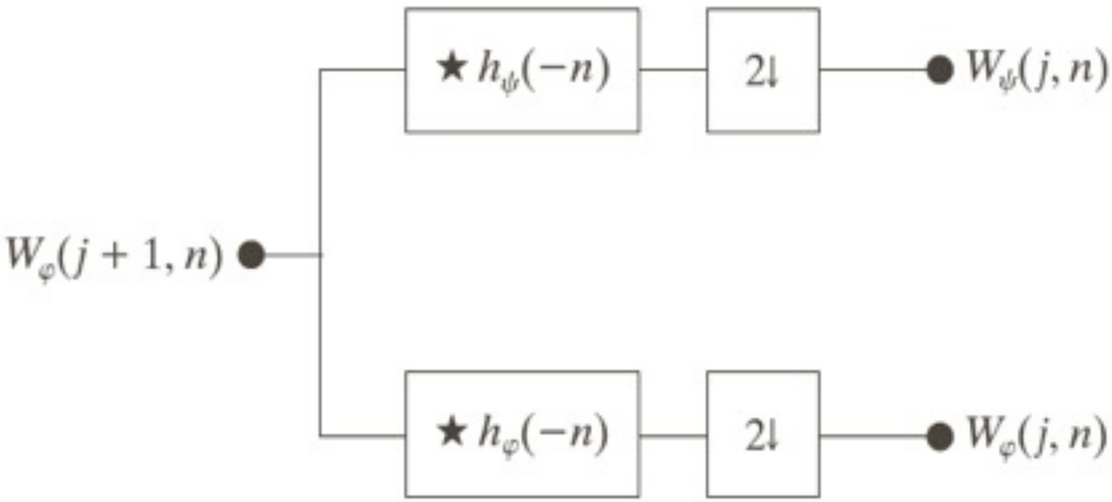
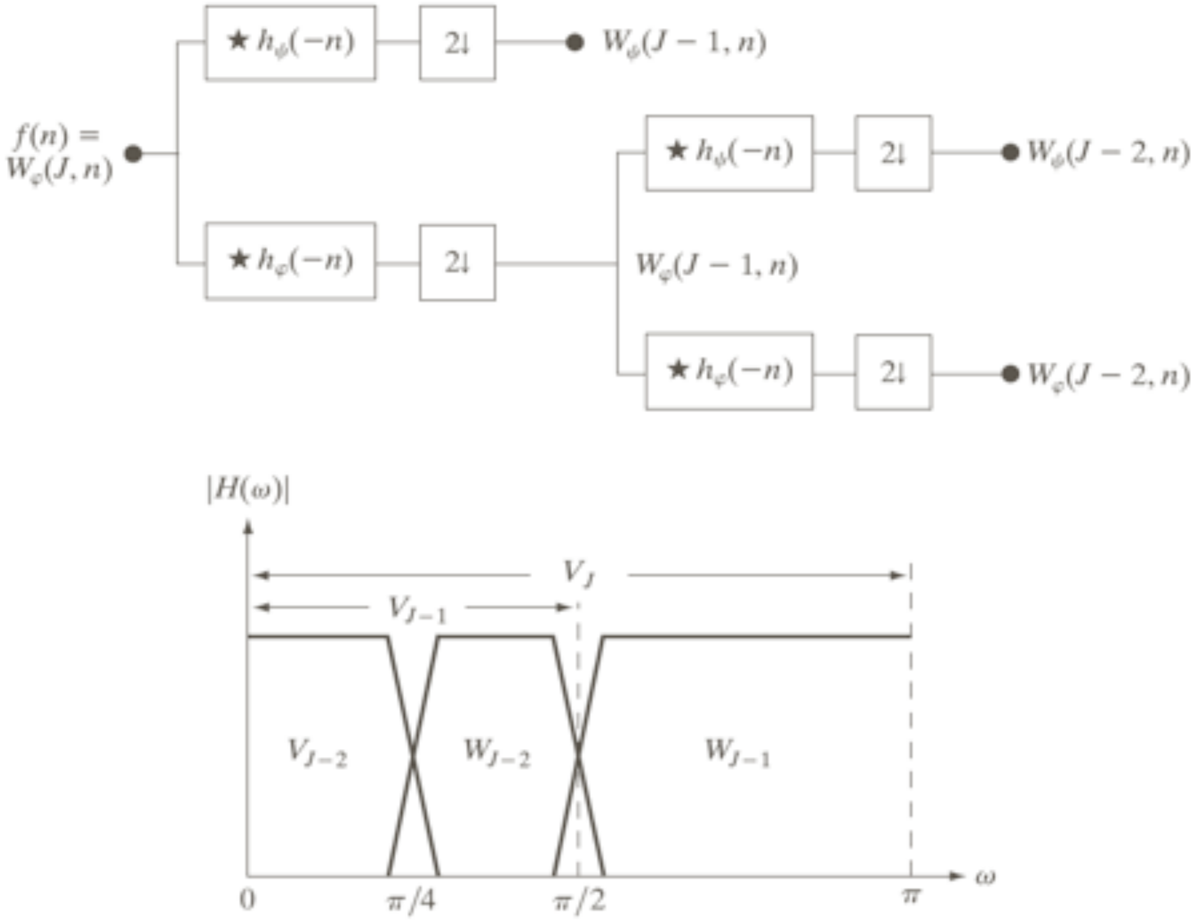


FIGURE 7.17 An FWT analysis bank.

Analysis



a
b

FIGURE 7.18 (a) A two-stage or two-scale FWT analysis bank and (b) its frequency splitting characteristics.

n	$h_\varphi(n)$
0	$1/\sqrt{2}$
1	$1/\sqrt{2}$

TABLE 7.2 Orthonormal Haar filter coefficients for $h_\varphi(n)$.

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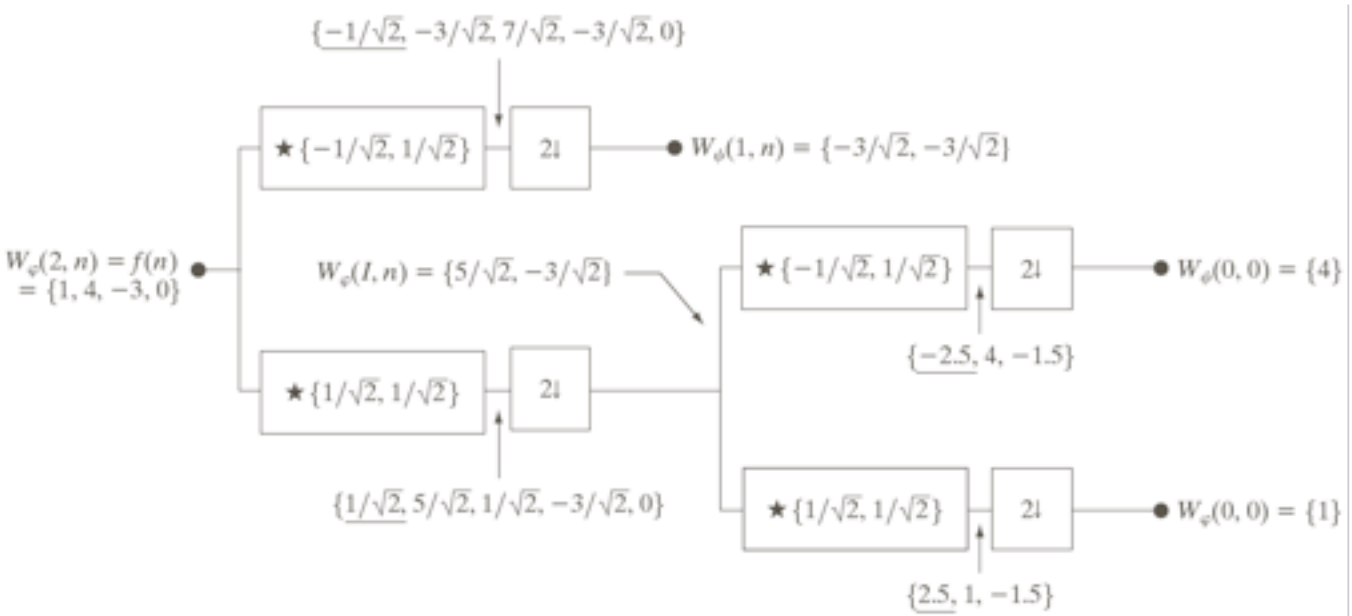


FIGURE 7.19 Computing a two-scale fast wavelet transform of sequence {1, 4, −3, 0} using Haar scaling and wavelet vectors.

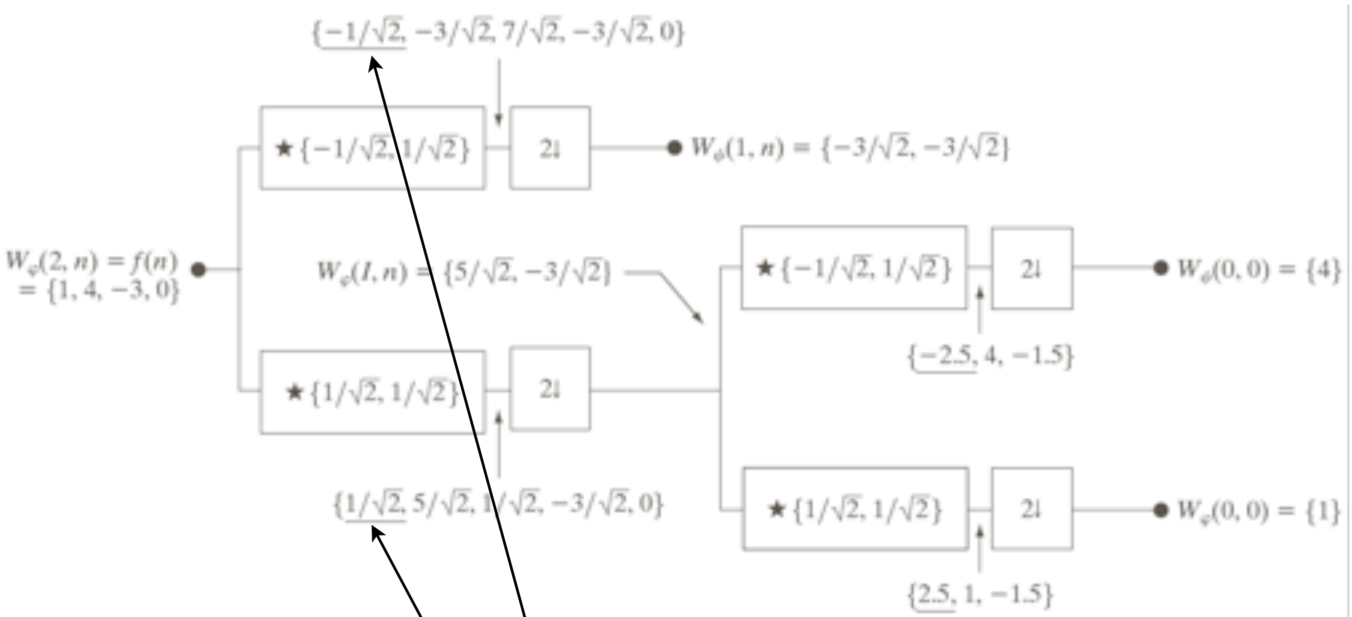


FIGURE 7.19 Computing a two-scale fast wavelet transform of sequence $\{1, 4, -3, 0\}$ using Haar scaling and wavelet vectors.

These coefficients correspond to neg. n

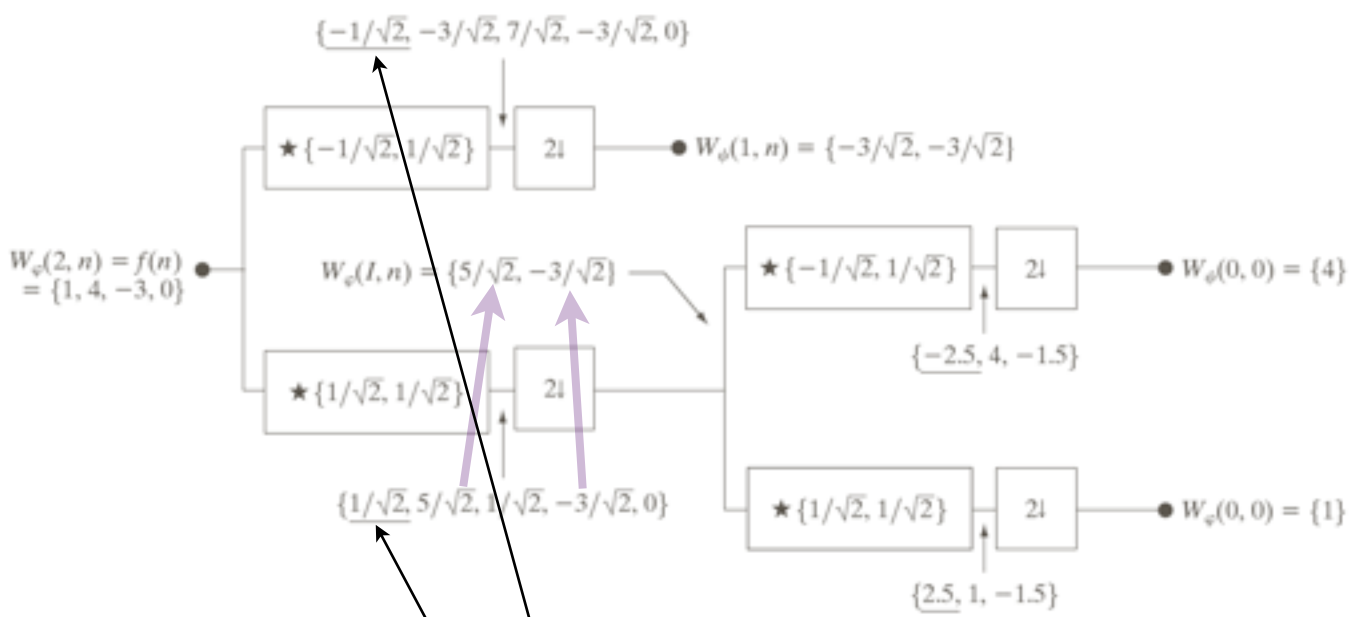


FIGURE 7.19 Computing a two-scale fast wavelet transform of sequence $\{1, 4, -3, 0\}$ using Haar scaling and wavelet vectors.

- Downsample by taking every second element

These coefficients correspond to neg. *n*

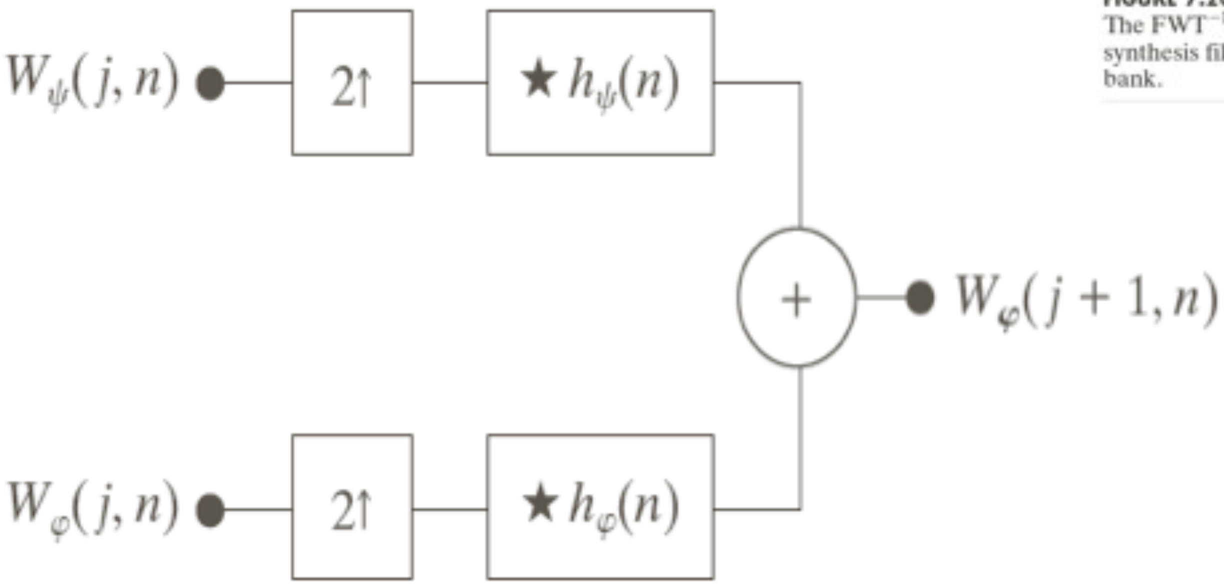


FIGURE 7.20 The FWT^{-1} synthesis filter bank.

Synthesis

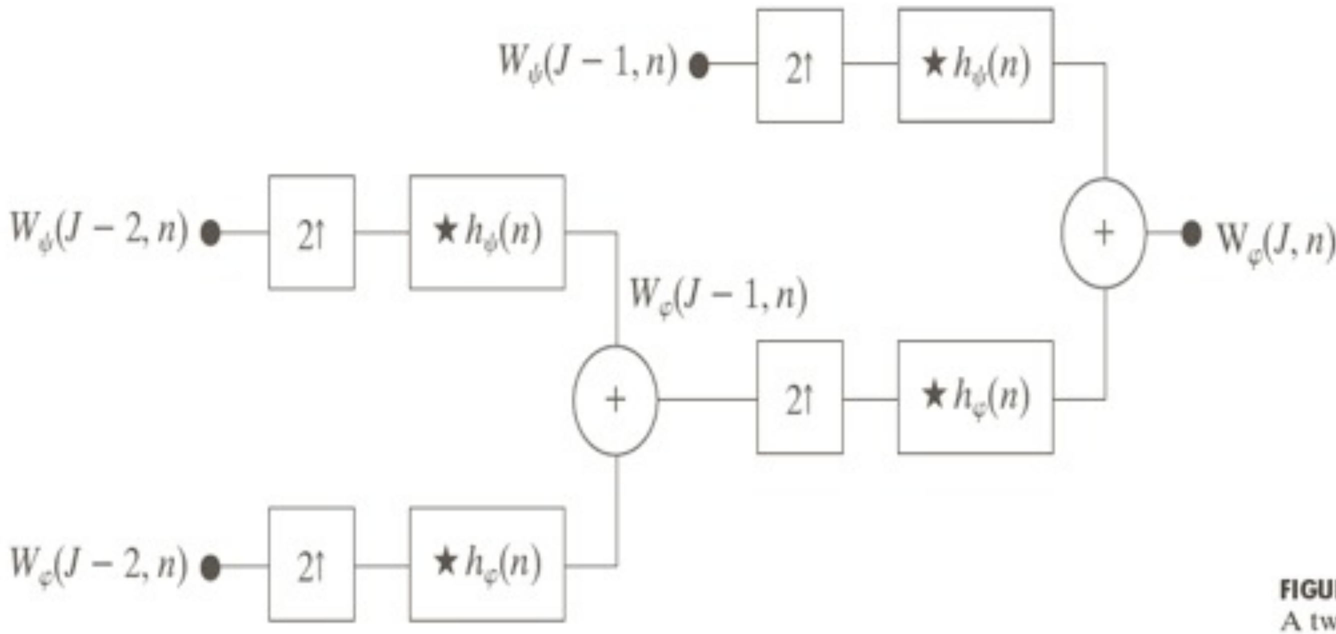


FIGURE 7.21
A two-stage or two-scale FWT^{-1} synthesis bank.

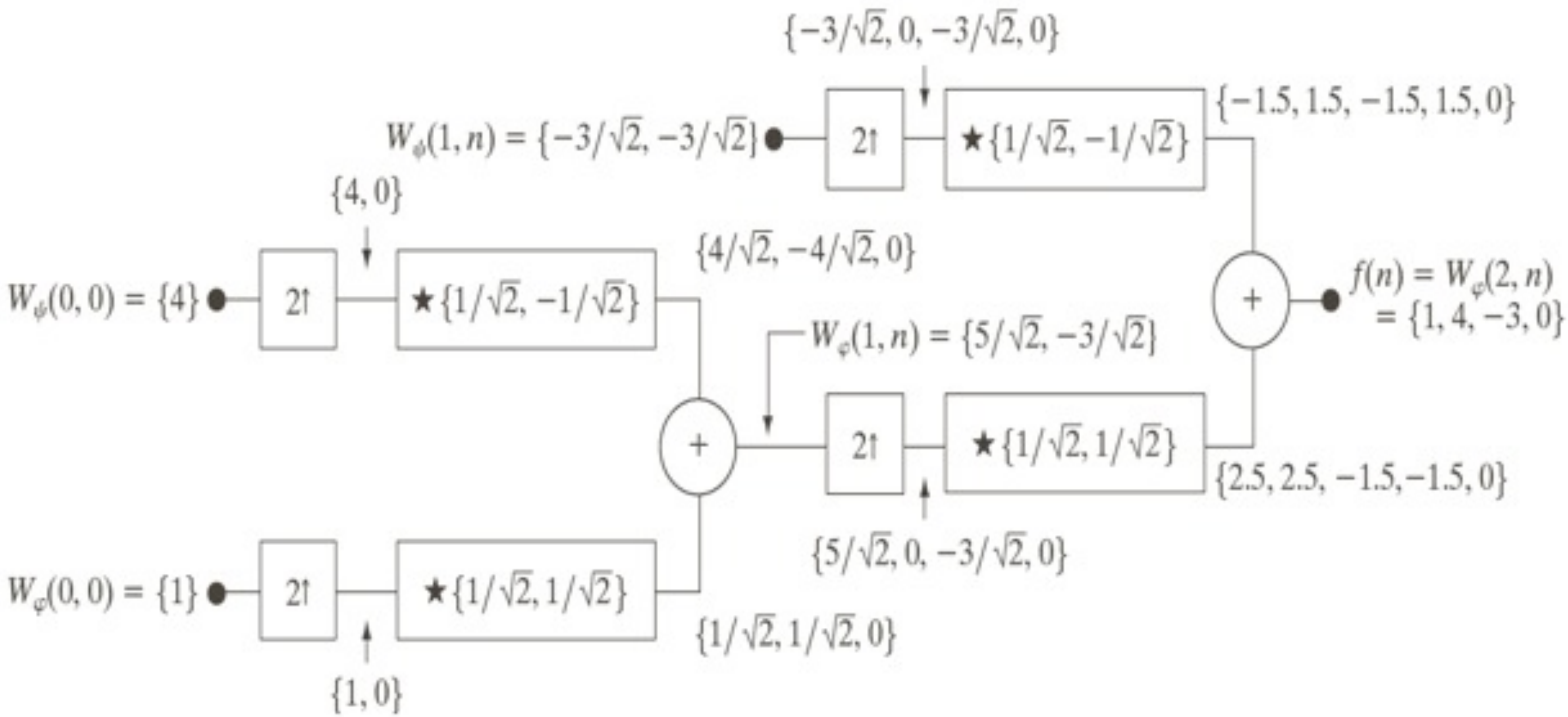


FIGURE 7.22 Computing a two-scale inverse fast wavelet transform of sequence $\{1, 4, -1.5\sqrt{2}, -1.5\sqrt{2}\}$ with Haar scaling and wavelet functions.

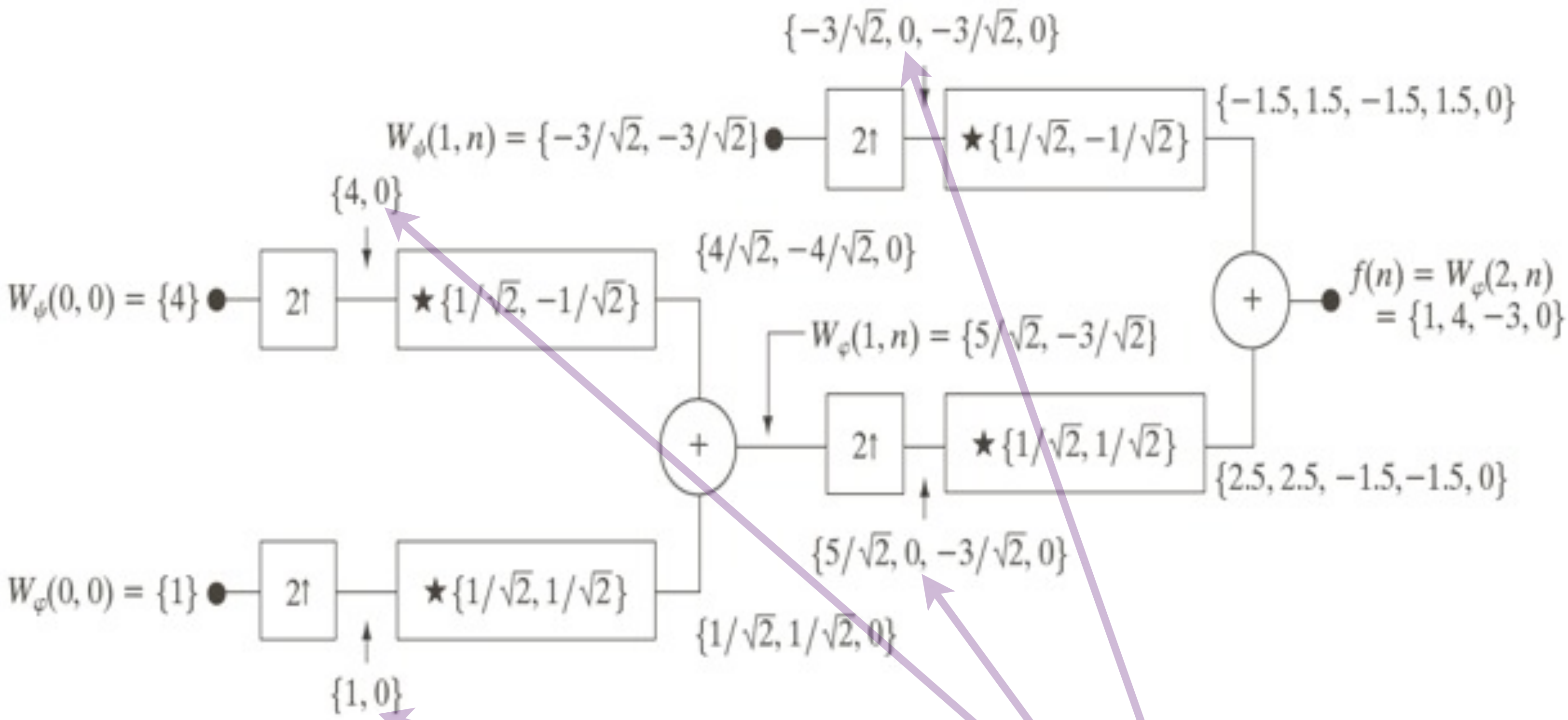
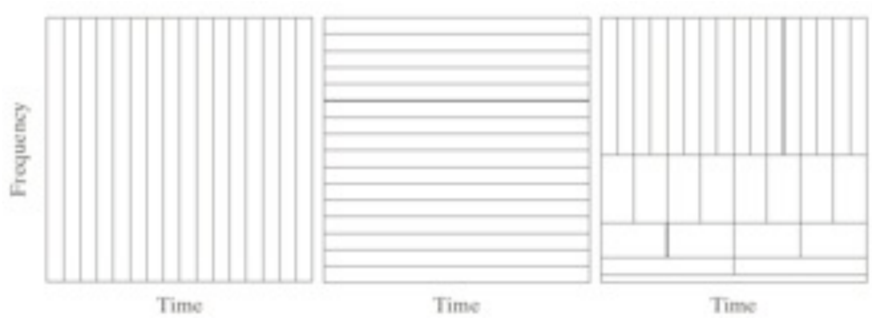


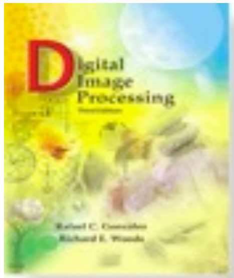
FIGURE 7.22 Computing a two-scale inverse fast wavelet transform of sequence $\{1, 4, -1.5\sqrt{2}, -1.5\sqrt{2}\}$ with Haar scaling and wavelet functions.

Upsampling by adding a 0 every 2nd element

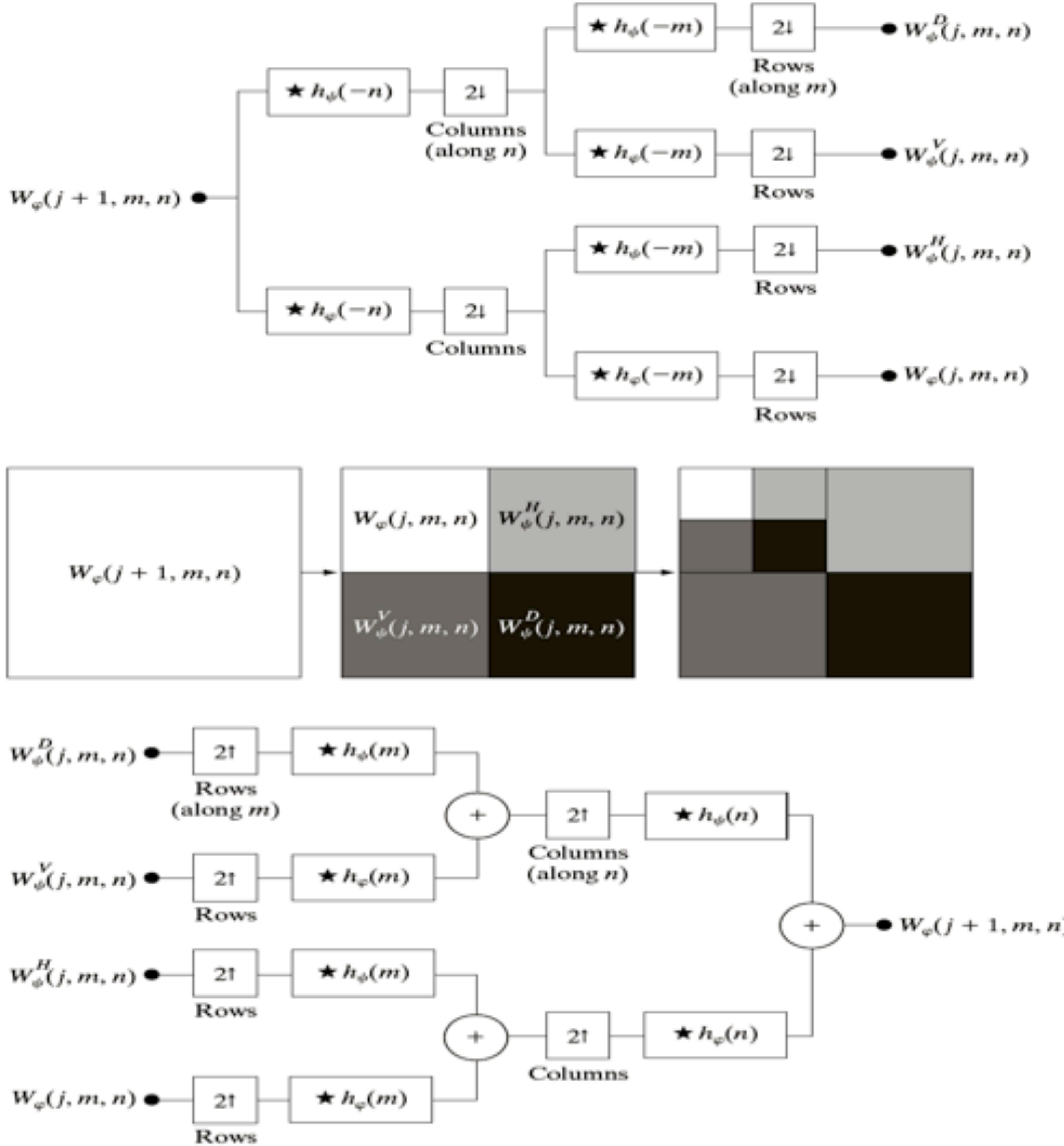


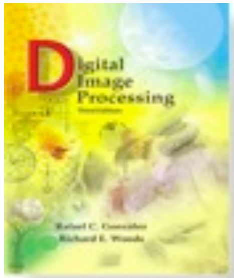
a b c

FIGURE 7.23 Time-frequency tilings for the basis functions associated with (a) sampled data, (b) the FFT, and (c) the FWT. Note that the horizontal strips of equal height rectangles in (c) represent FWT scales.


Digital Image Processing

2D scheme


$W_\varphi(j+1, m, n)$
$\star\, h_\psi(-n)$
$2\downarrow$
Columns (along n)
$\star\, h_\psi(-m)$
$2\downarrow$
Rows (along m)
$W_\psi^D(j, m, n)$
$\star\, h_\varphi(-m)$
$2\downarrow$
Rows
$W_\psi^V(j, m, n)$
$\star\, h_\varphi(-n)$
$2\downarrow$
Columns
$\star\, h_\psi(-m)$
$2\downarrow$
Rows
$W_\psi^H(j, m, n)$
$\star\, h_\varphi(-m)$
$2\downarrow$
Rows
$W_\varphi(j, m, n)$
$W_\varphi(j+1, m, n)$
$W_\varphi(j, m, n)$
$W_\psi^H(j, m, n)$
$W_\psi^V(j, m, n)$
$W_\psi^D(j, m, n)$
$W_\psi^D(j, m, n)$
$2\uparrow$
Rows (along m)
$\star\, h_\psi(m)$
$W_\psi^V(j, m, n)$
$2\uparrow$
Rows
$\star\, h_\varphi(m)$
$+$
$2\uparrow$
Columns (along n)
$\star\, h_\psi(n)$
$W_\psi^H(j, m, n)$
$2\uparrow$
Rows
$\star\, h_\psi(m)$
$W_\varphi(j, m, n)$
$2\uparrow$
Rows
$\star\, h_\varphi(m)$
$+$
$2\uparrow$
Columns
$\star\, h_\varphi(n)$
$+$
$W_\varphi(j+1, m, n)$

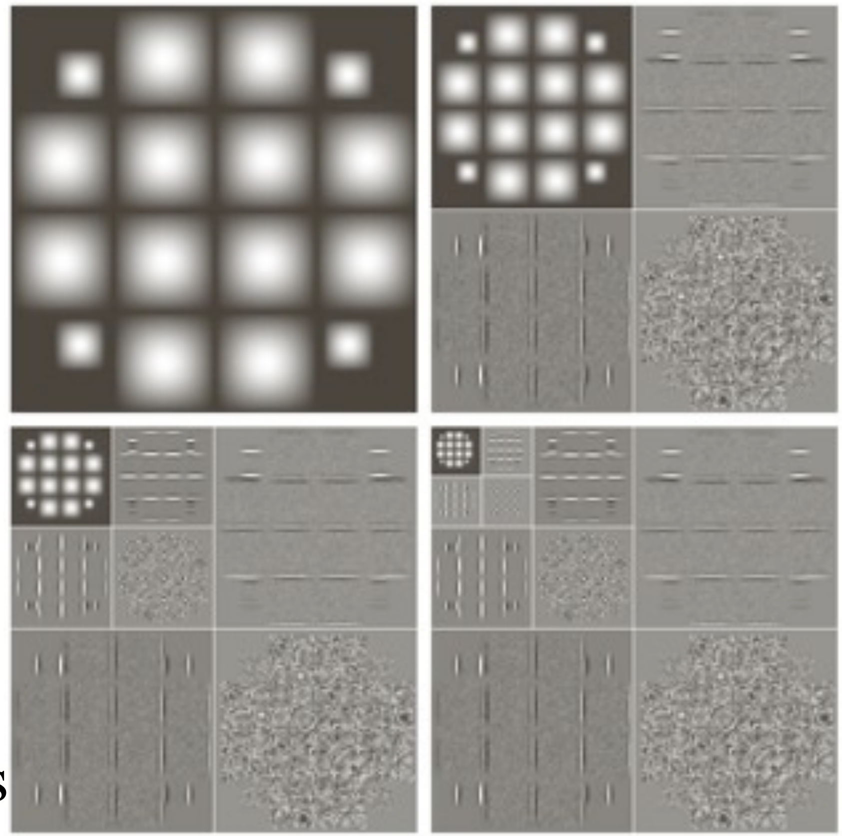


Original

1 scale

2 scales

3 scales



Note that the wavelet components have typically 0 mean value (encode the small variations in the details of the image)

Wavelets - so what now?

As we have done image manipulation (filtering, smoothing, in spatial domain, frequency domain, etc.) it is possible to apply the same tools in the wavelets formalism.

1. Apply DWT
2. Perform filtering/operation
3. Apply IDWT

The advantage of wavelets wrt to FFT is the cost (O(N) wavelets, O (NlogN) for FFT)

Example:

Picking up edge information by modifying the approximation part

Modifying also the horizontal detail information

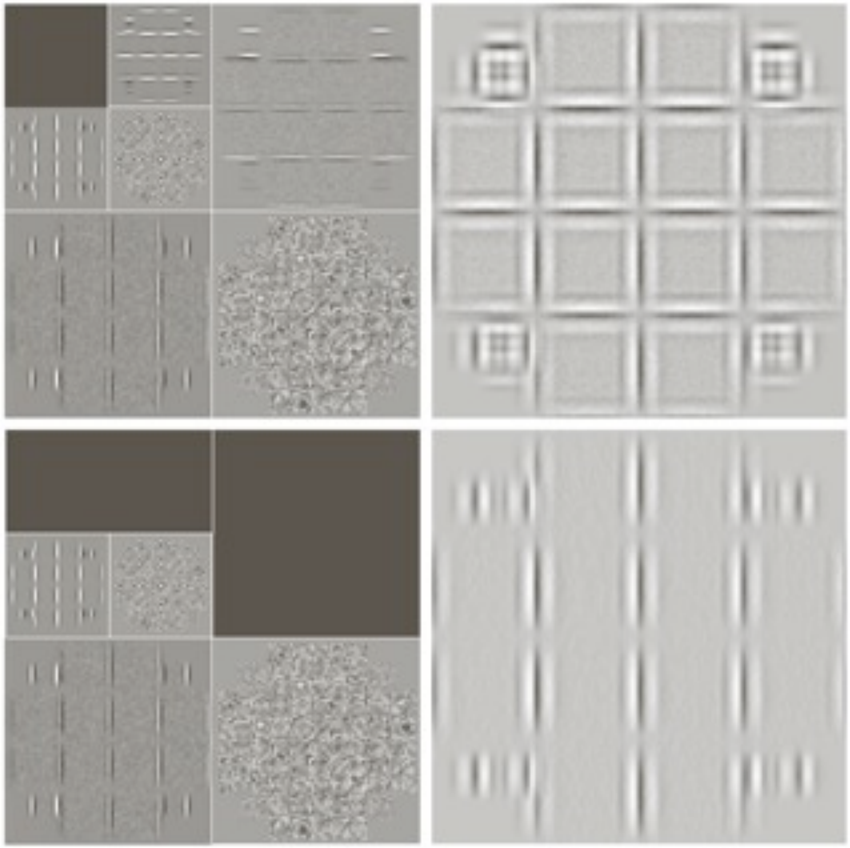
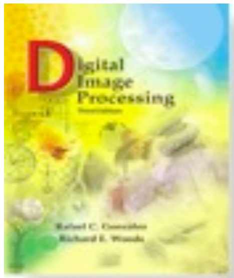
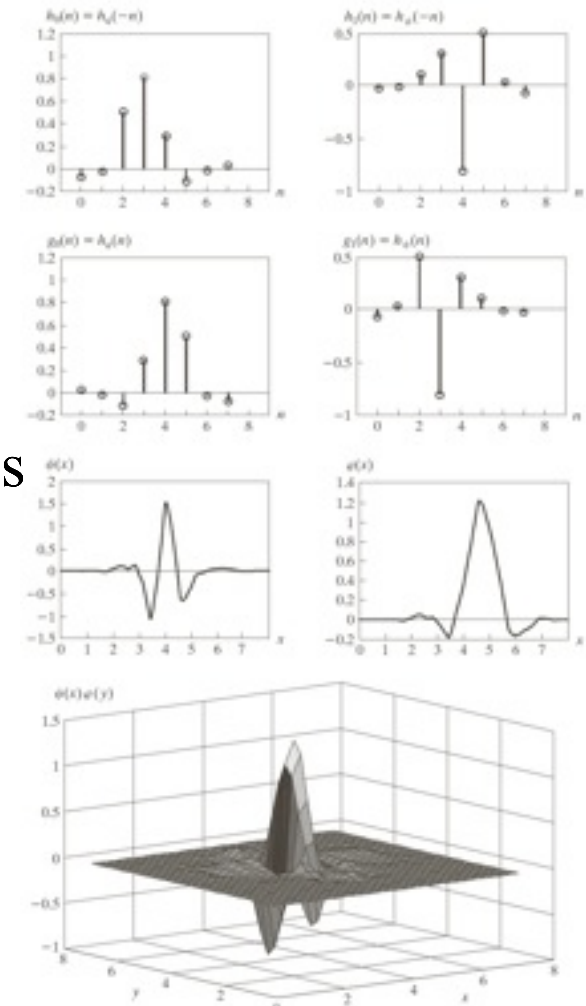


FIGURE 7.27 Modifying a DWT for edge detection: (a) and (c) two-scale decompositions with selected coefficients deleted; (b) and (d) the corresponding reconstructions.



Haar wavelets converge slowly.

Other families of wavelets have better convergence properties



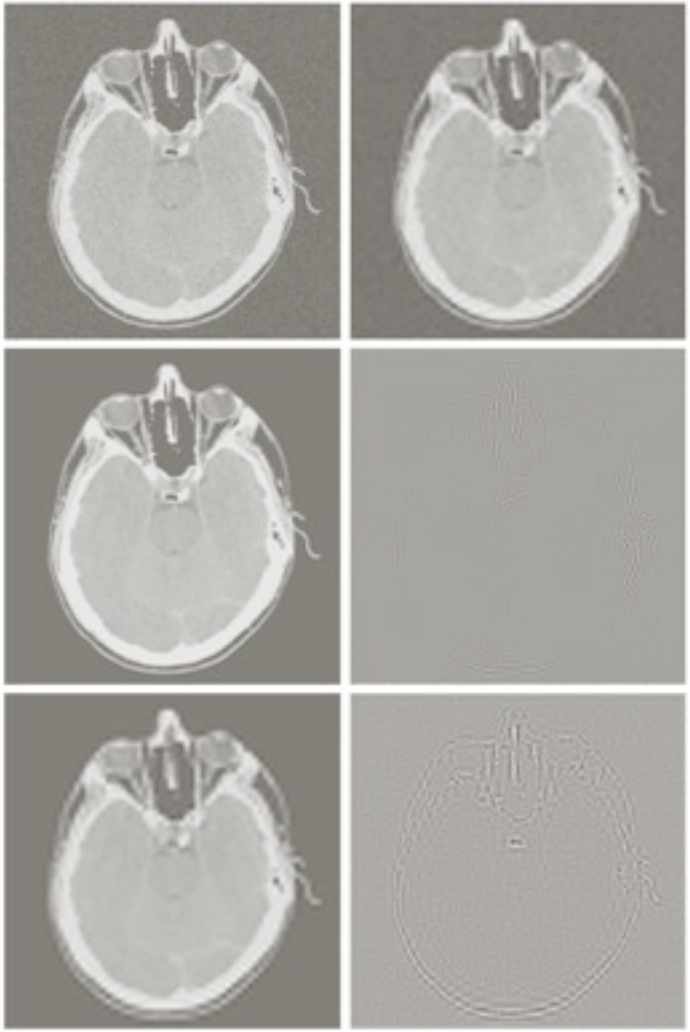
a b
c d
e f
g

FIGURE 7.26 Fourth-order symlets: (a)–(b) decomposition filters; (c)–(d) reconstruction filters; (e) the one-dimensional wavelet; (f) the one-dimensional scaling function; and (g) one of three two-dimensional wavelets, $\psi^V(x, y)$. See Table 7.3 for the values of $h_\varphi(n)$ for $0 \le n \le 7$.

n	$h_\varphi(n)$
0	0.0322
1	−0.0126
2	−0.0992
3	0.2979
4	0.8037
5	0.4976
6	−0.0296
7	−0.0758

TABLE 7.3 Orthonormal fourth-order symlet filter coefficients for $h_\varphi(n)$. (Daubechies [1992].)

Example of noise removal:
here it is set to zero the coefficients of the wavelets (detail coefficients), while keeping the approximation coefficients



a b
c d
e f

FIGURE 7.28 Modifying a DWT for noise removal: (a) a noisy CT of a human head; (b), (c) and (e) various reconstructions after thresholding the detail coefficients; (d) and (f) the information removed during the reconstruction of (c) and (e). (Original image courtesy Vanderbilt University Medical Center.)