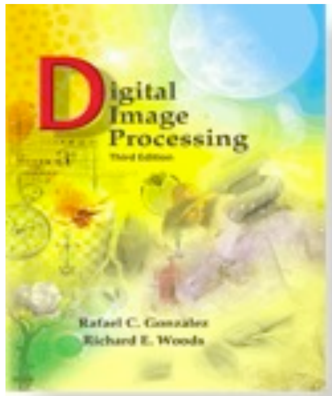


Last time: Laplacian mask

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\nabla^2 f \approx f(x-1,y) + f(x+1,y) + f(x,y-1) + f(x,y+1) - 4f(x,y)$$

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

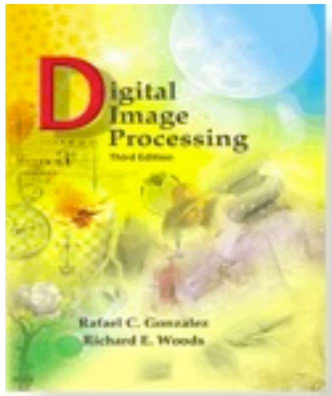
0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

Sharpening:

$$g(x,y) = f(x,y) + c[\nabla^2 f(x,y)]$$

Use c <0 for schemes with negative central weight, c>0 for those with positive central weight.

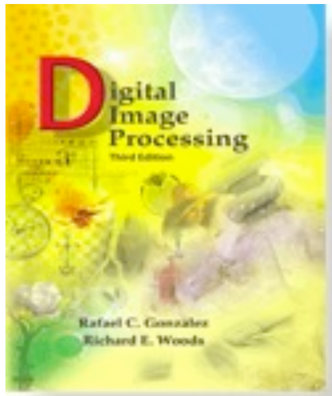


The gradient operator

The gradient is the vector of the partial derivatives of a scalar function

$$\nabla f = \mathrm{grad}(f) = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \end{bmatrix}$$

It points in the direction where *f* changes the most



There are (at least) 2 main ways to use the gradient:

1. Through its magnitude (2-norm as a vector)

$$M(x,y) = \|\nabla f\| = \sqrt{f_x^2 + f_y^2}$$

2. Through the 1-norm

$$M(x,y) = |f_x| + |f_y|$$

Both formulas are *nonlinear*.

The first formula is *isotropic* (=rotationally invariant) the second is not.

NB. The component of the gradient vector are not rotationally invariant, however the magnitude of the gradient is.

This is because if Q is a orthogonal matrix, then $\|Q\mathbf{v}\| = \|\mathbf{v}\|$

$$\sqrt{f_x^2 + f_y^2}$$

$$\frac{\partial f}{\partial x} \approx f(x+1) - f(x),$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Consider the central pixel f_5:

$$f_x \approx f_8 - f_5 \qquad f_y \approx f_6 - f_5$$

$$M(x, y) = \sqrt{(f_8 - f_5)^2 + (f_6 - f_5)^2}$$

and a similar formula is obtained for the other use of the gradient

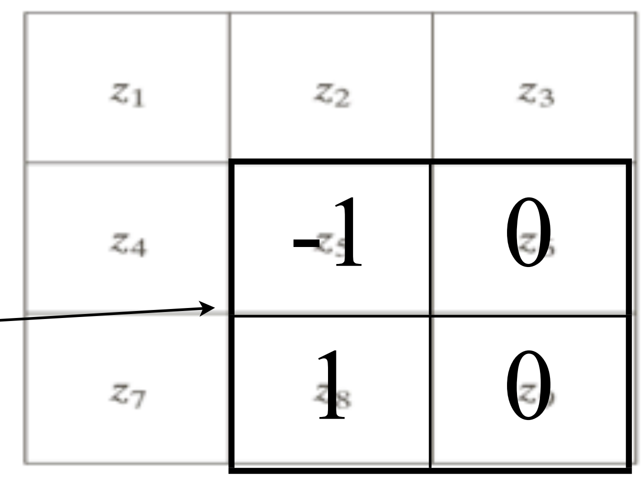
$$M(x, y) = |f_8 - f_5| + |f_6 - f_5|$$

$$\sqrt{f_x^2 + f_y^2}$$

$$\frac{\partial f}{\partial x} \approx f(x+1) - f(x),$$

Consider the central pixel f_5:

$$f_x \approx f_8 - f_5 \qquad f_y \approx f_6 - f_5$$



$$M(x, y) = \sqrt{(f_8 - f_5)^2 + (f_6 - f_5)^2}$$

and a similar formula is obtained for the other use of the gradient

$$M(x, y) = |f_8 - f_5| + |f_6 - f_5|$$

$$\sqrt{f_x^2 + f_y^2}$$

$$\frac{\partial f}{\partial x} \approx f(x+1) - f(x),$$

Consider the central pixel f_5:

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

$$f_x \approx f_8 - f_5 \qquad f_y \approx f_6 - f_5$$

$$M(x, y) = \sqrt{(f_8 - f_5)^2 + (f_6 - f_5)^2}$$

and a similar formula is obtained for the other use of the gradient

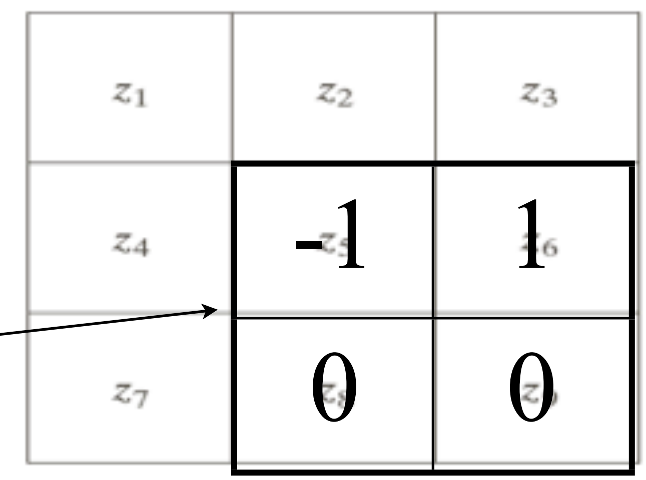
$$M(x, y) = |f_8 - f_5| + |f_6 - f_5|$$

$$\sqrt{f_x^2 + f_y^2}$$

$$\frac{\partial f}{\partial x} \approx f(x+1) - f(x),$$

Consider the central pixel f_5:

$$f_x \approx f_8 - f_5 \qquad f_y \approx f_6 - f_5$$



$$M(x,y) = \sqrt{(f_8 - f_5)^2 + (f_6 - f_5)^2}$$

and a similar formula is obtained for the other use of the gradient

$$M(x,y) = |f_8 - f_5| + |f_6 - f_5|$$

There exist also *cross-gradient* formulas, (Roberts)

$$f_x \approx f_9 - f_5 \qquad f_y \approx f_8 - f_6$$

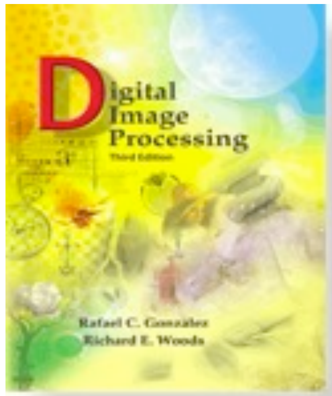
z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

which can be implemented by masks and correlation/convolution.

−1	0
0	1

0	−1
1	0

However, these masks have even dimension and that's unpractical because they do not have a symmetry center, hence it is not clear which weight corresponds to what.



We wish to use discretizations using an odd number of pixels (with an obvious centre of symmetry)

$$f_x \approx (f_7 + 2f_8 + f_9) - (f_1 + 2f_2 + f_3)$$

$$f_y \approx (f_3 + 2f_6 + f_9) - (f_1 + 2f_4 + f_7)$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Now, the partial derivatives can be computed by linear filters, the nonlinearity comes then from the final assembly of *M(x,y)*

−1	−2	−1
0	0	0
1	2	1

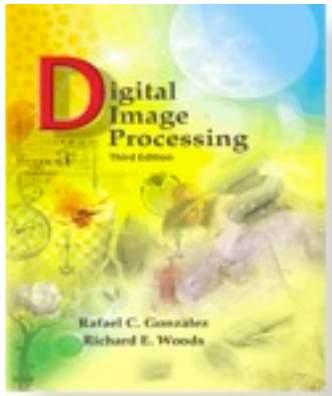
−1	0	1
−2	0	2
−1	0	1

Sobel operators

We have seen several methods for smoothing and sharpening and enhancing:

- histogram equalization/matching
- linear filters
- nonlinear filters

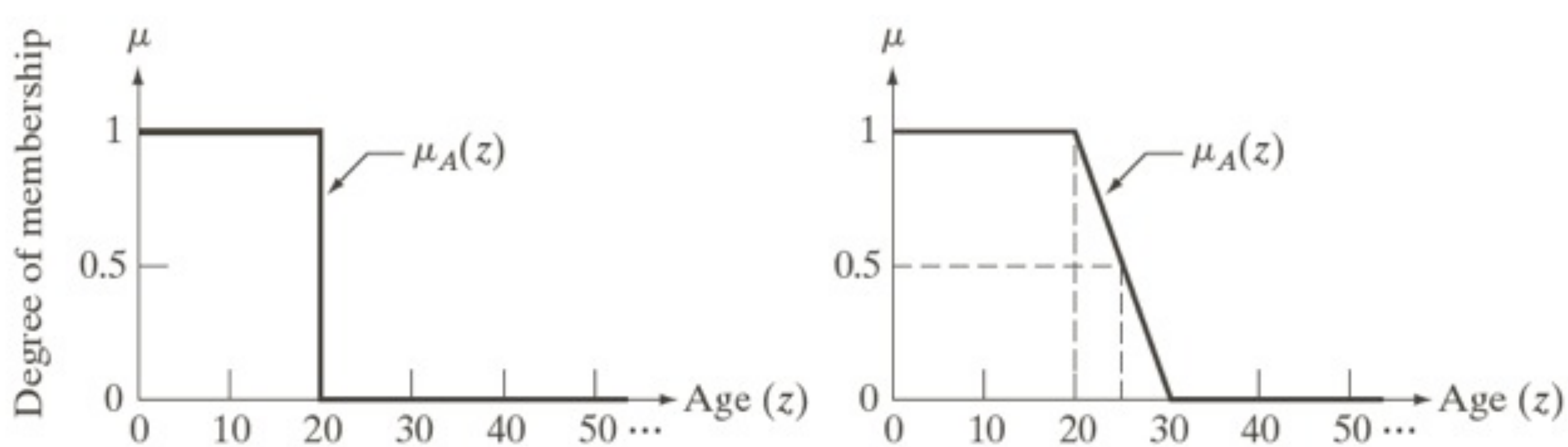
In practical applications, it is often necessary one or more techniques, depending on what we wish to do with the image.



Fuzzy sets and fuzzy techniques

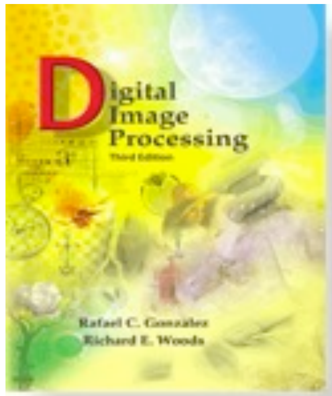
Classical set theory has a strong definition of membership: either an element is member or not (1 or 0)

For fuzzy sets, one needs to define a membership function that assigns to each element a value between 0 and 1



Property of the set A: "z is in A if z is young"

Left: Example of hard membership function
Right: Example of soft (fuzzy) membership function.



A fuzzy set is thus characterized by

$$A = \{z, \mu_A(z) | z \in Z\}$$

Z is called *universe of discourse*
$0 \leq \mu_A(z) \leq 1$ is the *membership* function (degree of membership of z in A)

$\mu_A(z) = 1 :$ z is a full member of A

$\mu_A(z) = 0 :$ z is not a member of A

Intermediate values correspond to partial membership

Thus, fuzzy sets are completely determined by the universe Z and the membership function -> we can operate on fuzzy sets using tools from algebra.

Empy set: $\mu_A(z) = 0 \quad \text{in } Z$

Equality: $A = B$ iff $\mu_A(z) = \mu_B(z)$ for all $z \in Z$

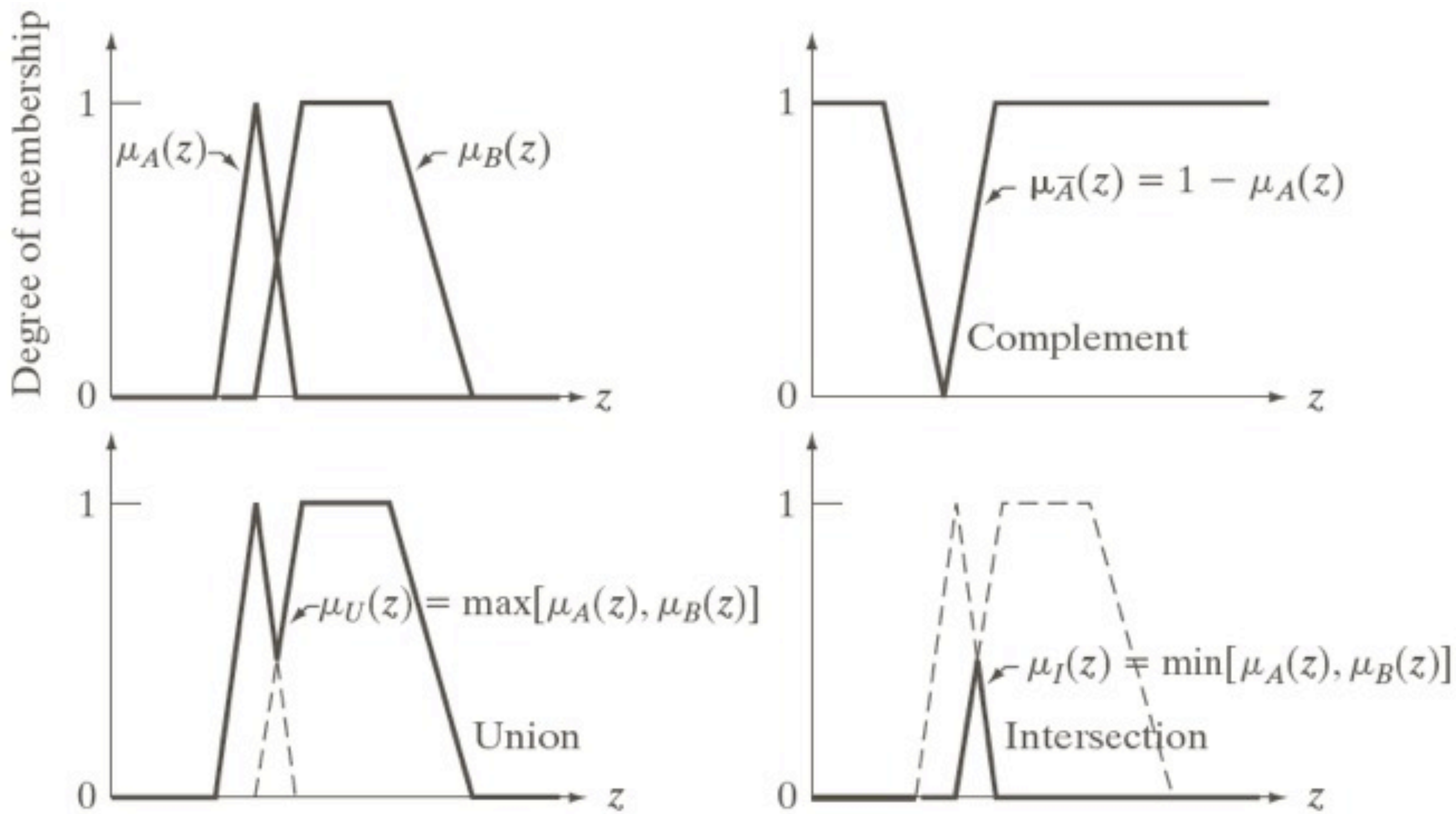
Complement: $\bar{A}$, $\mu_{\bar{A}}(z) = 1 - \mu_A(z)$

Subset: A is a subset of B if $\mu_A(z) \leq \mu_B(z)$ for all z

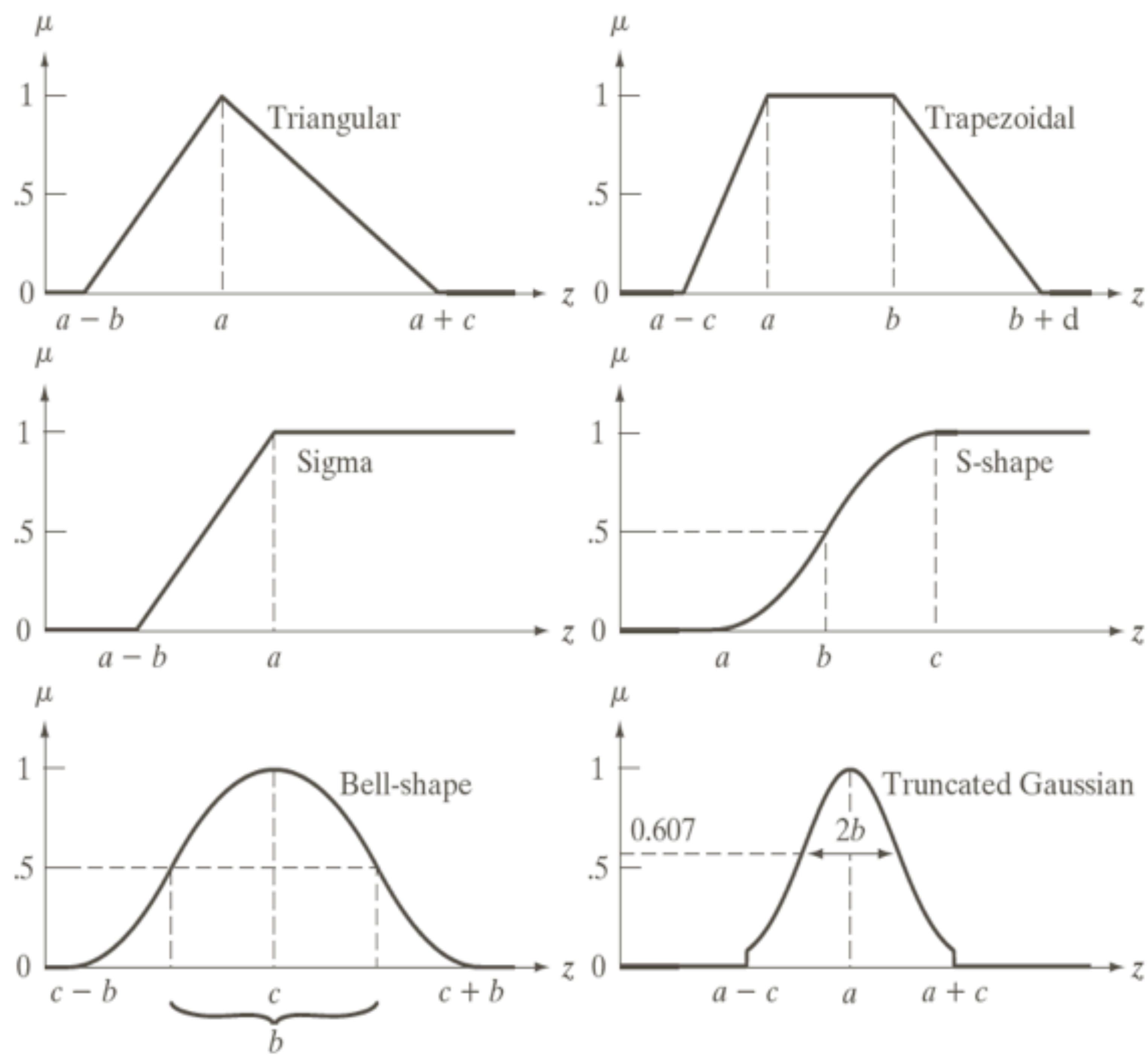
Union: $U = A \cup B$, has membership function $\mu_U(z) = \max\{\mu_A(z), \mu_B(z)\}$

Intersection: $V = A \cap B$, has membership function $\mu_V(z) = \min\{\mu_A(z), \mu_B(z)\}$

Chapter 3
Intensity Transformations & Spatial Filtering



Some common membership functions are:



Principal steps of fuzzy logic:

1. Fuzzification of the input
2. Apply fuzzy logical operations
3. Apply implications
4. Apply aggregation

NB. The output of fuzzy operations is still a fuzzy set, while we need a non-fuzzy intensity transformation

5. Defuzzify (computing the center of gravity)

$$v_0 = \frac{\sum_{v=0}^{K} vQ(v)}{\sum_{v=0}^{K} Q(v)}$$

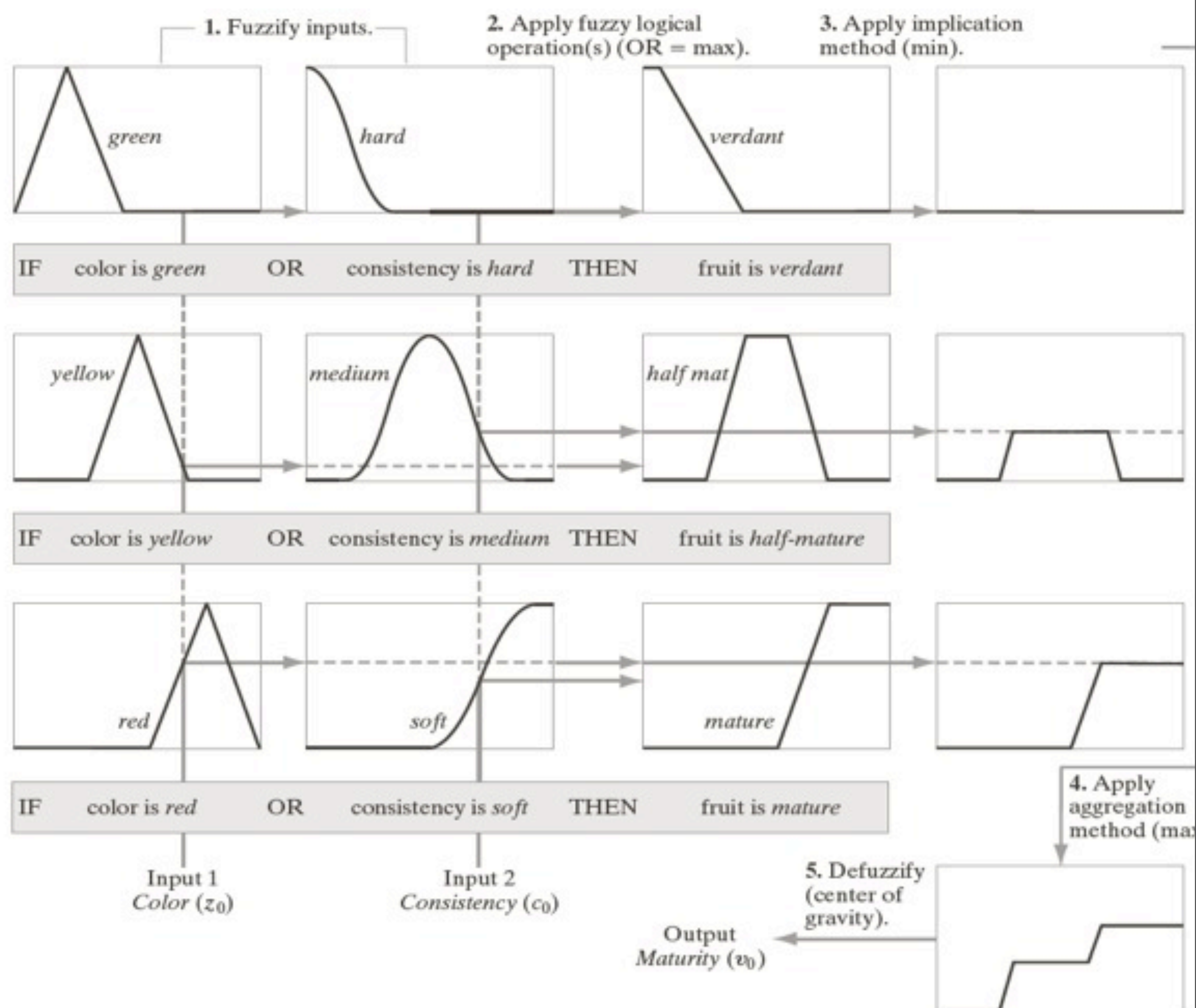
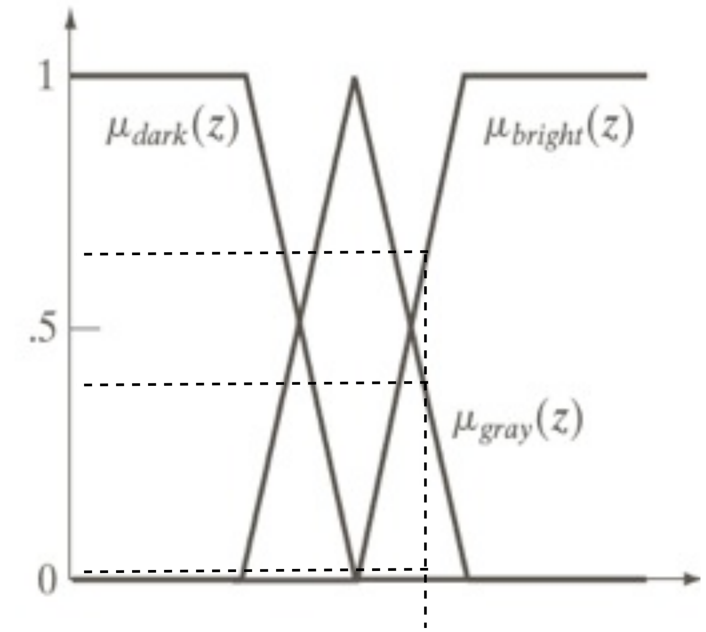


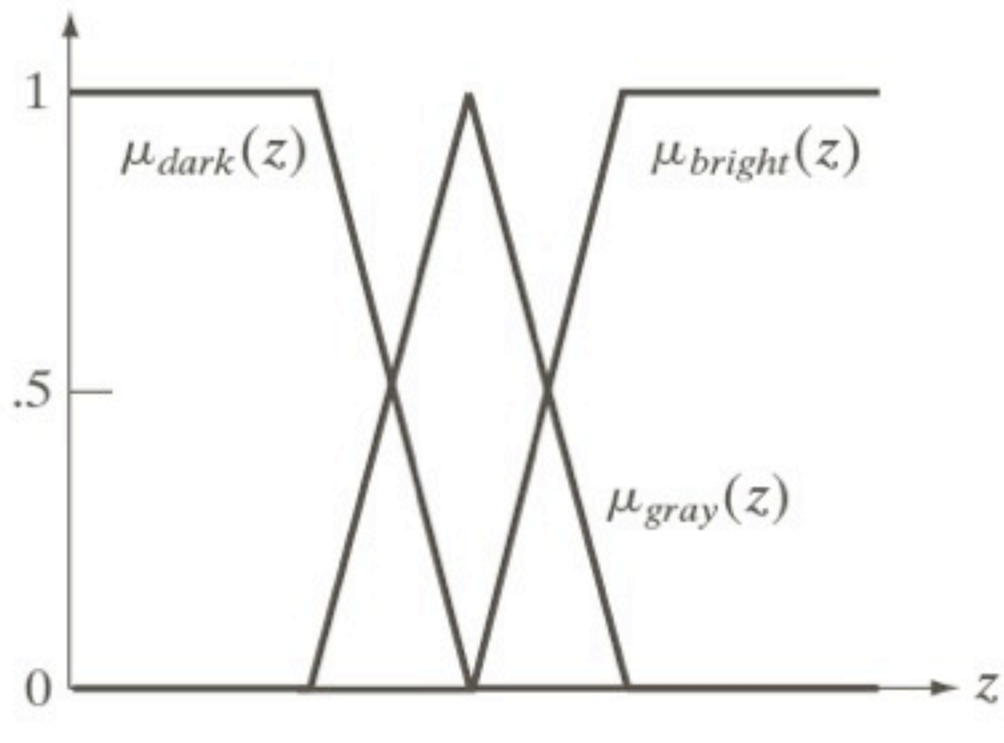
FIGURE 3.52 Example illustrating the five basic steps used typically to implement a fuzzy, rule-based syster (1) fuzzification, (2) logical operations (only OR was used in this example), (3) implicatio (4) aggregation, and (5) defuzzification.

Fuzzy sets for intensity transformations

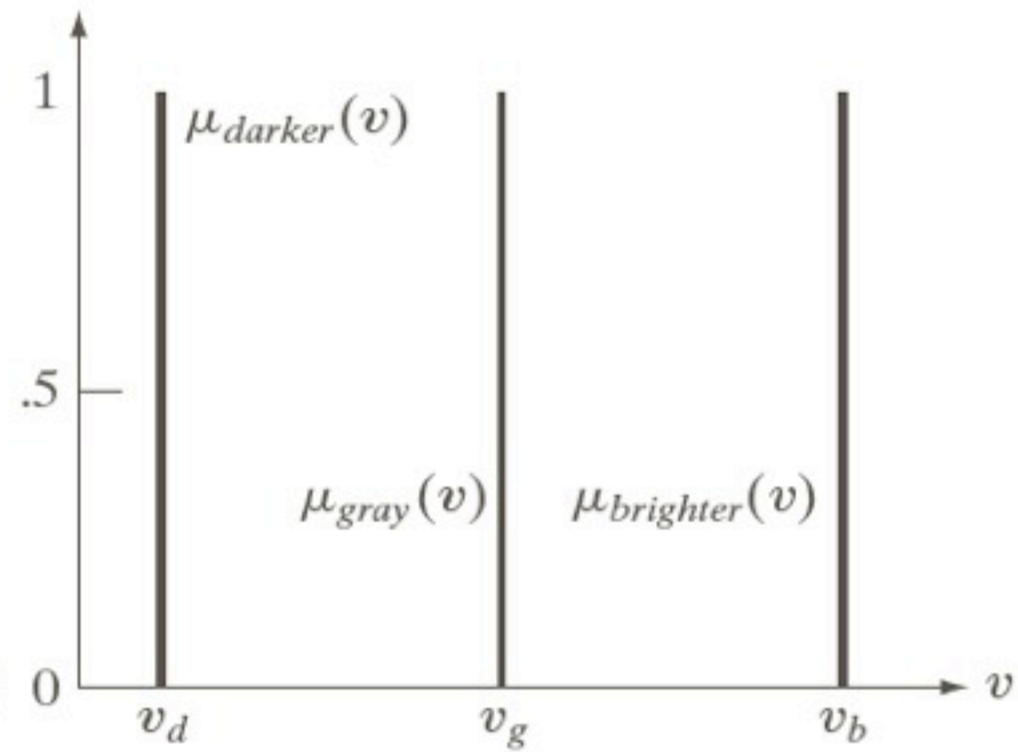
To operate on fuzzy sets, we have to define the problem-specific knowledge in terms of IF-THEN rules (implications or inference)



Example:
IF a pixel is dark THEN make it darker
IF a pixel is gray THEN make it gray
IF a pixel is bright THEN make it brighter

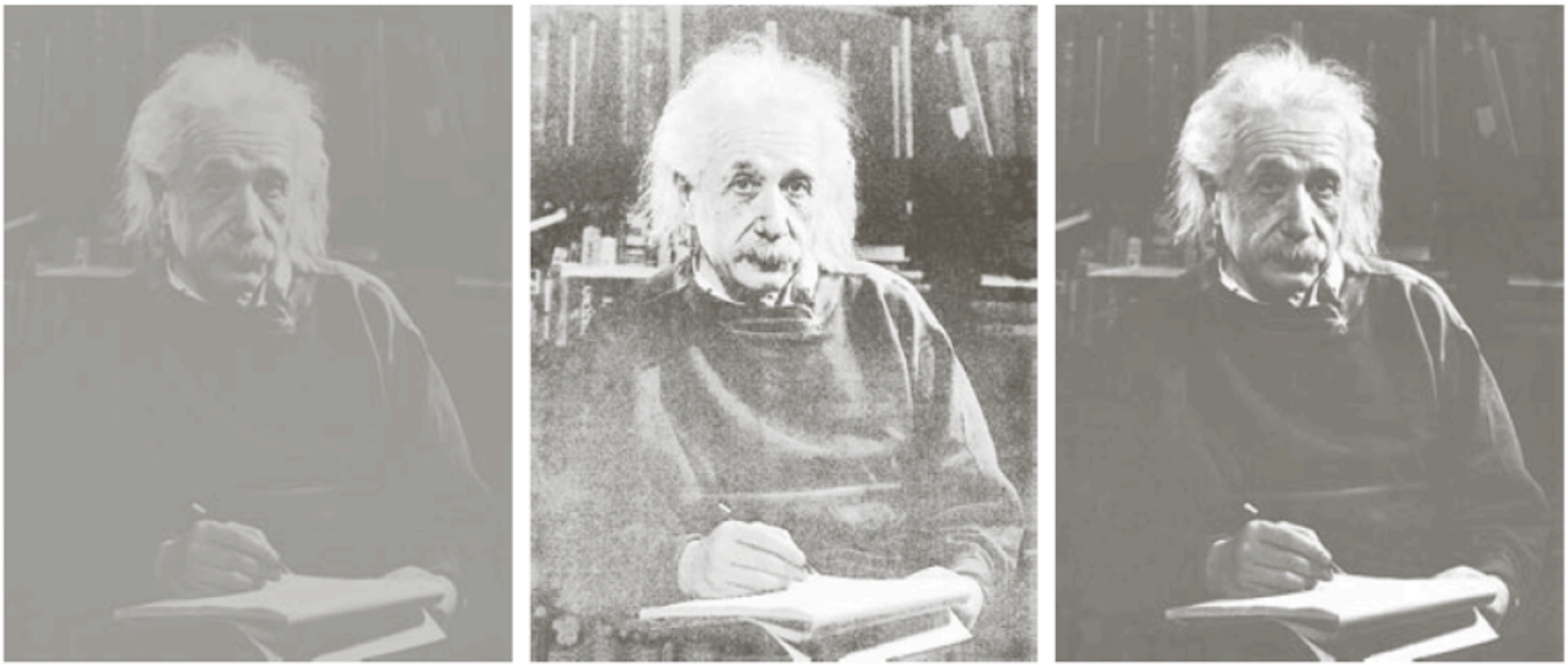


Input



Simplified output (singletons)

$$v_0 = \frac{\mu_{dark}(z_0)v_d + \mu_{gray}(z_0)v_g + \mu_{bright}(z_0)v_b}{\mu_{dark}(z_0) + \mu_{gray}(z_0) + \mu_{bright}(z_0)}$$



a b c

FIGURE 3.54 (a) Low-contrast image. (b) Result of histogram equalization. (c) Result of using fuzzy, rule-based contrast enhancement.

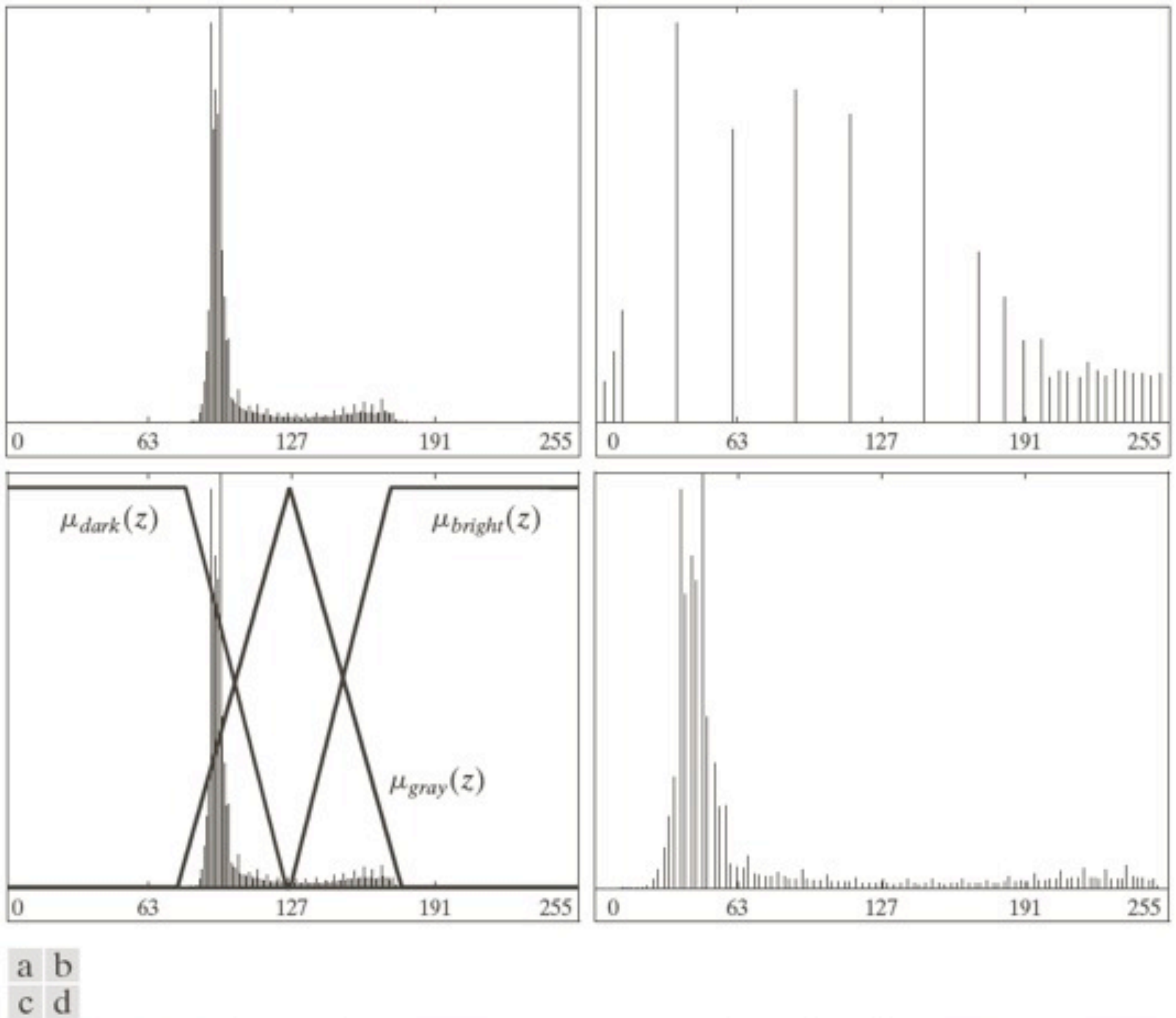
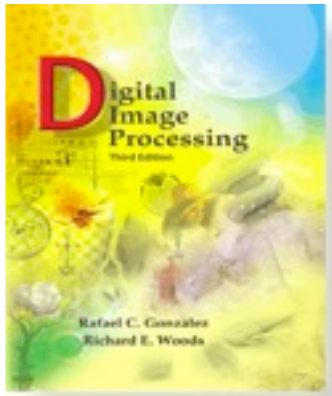


FIGURE 3.55 (a) and (b) Histograms of Figs. 3.54(a) and (b). (c) Input membership functions superimposed on (a). (d) Histogram of Fig. 3.54(c).

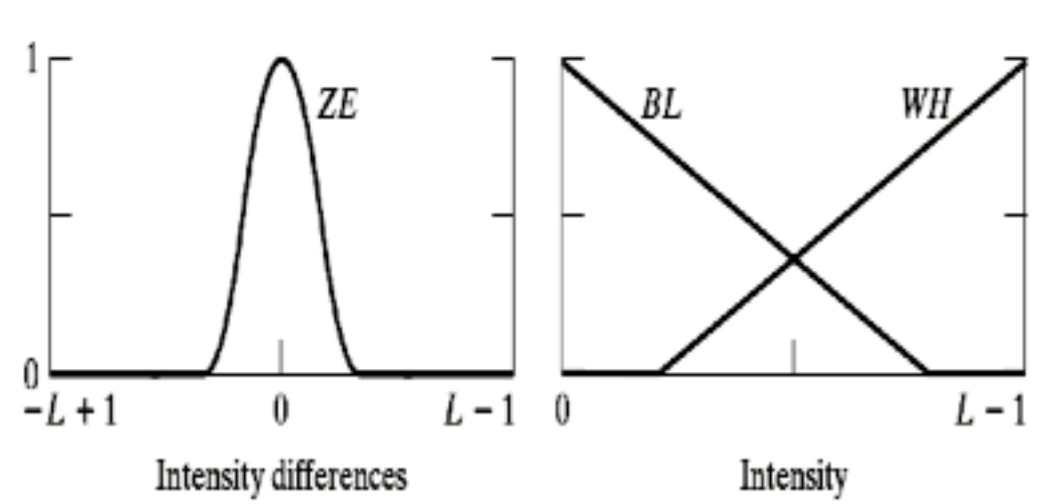


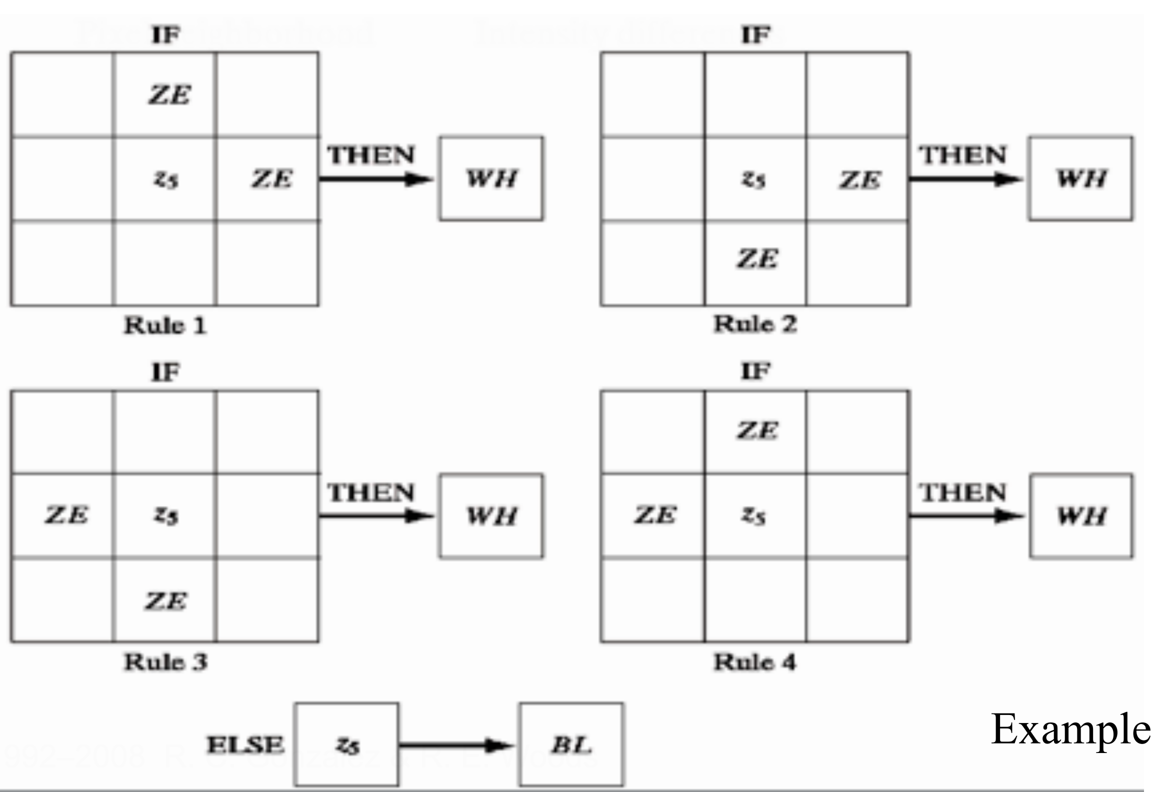
Fuzzy sets for spatial filtering.

The basic approach is to define neighborhood properties that capture the essence of the image.

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

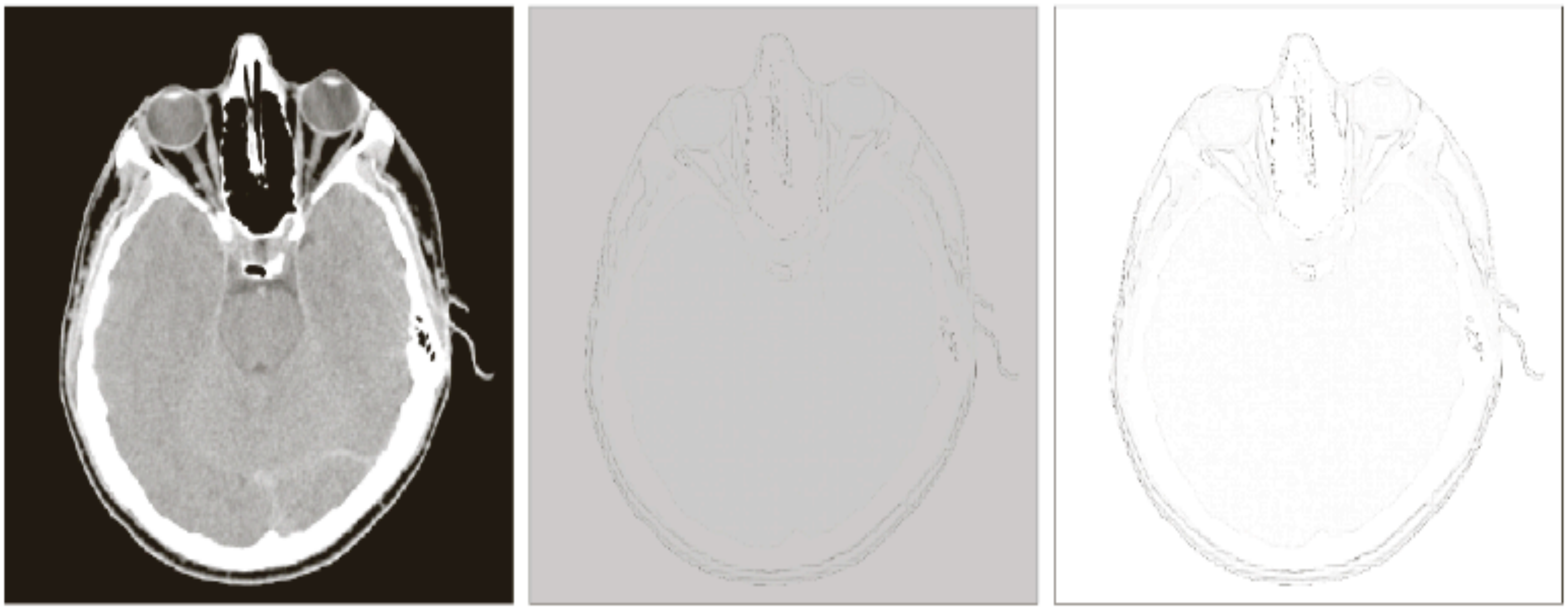
d_1	d_2	d_3
d_4	0	d_6
d_7	d_8	d_9





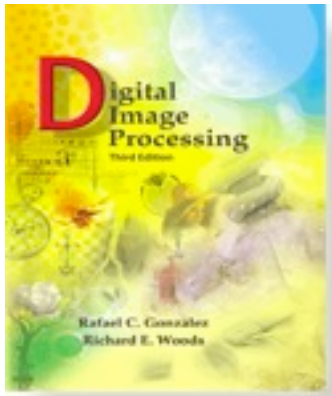
Uniform region: if neighborhood pixels have "zero" intensity difference.

Example of inference rule using 4-neighborhood



a b c

FIGURE 3.59 (a) CT scan of a human head. (b) Result of fuzzy spatial filtering using the membership functions in Fig. 3.57 and the rules in Fig. 3.58. (c) Result after intensity scaling. The thin black picture borders in (b) and (c) were added for clarity; they are not part of the data. (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)



Fuzzy techniques are computationally intensive because the rules must be applied to each pixel.

The use of singletons (simplified output) makes it possible to reduce the complexity.

A practical approach is to use fuzzy set techniques to determine the histogram of a well balanced image, then use faster methods like histogram matching to obtain similar results.