



Perception III: Fundamentals of Image Processing (incl. image features)

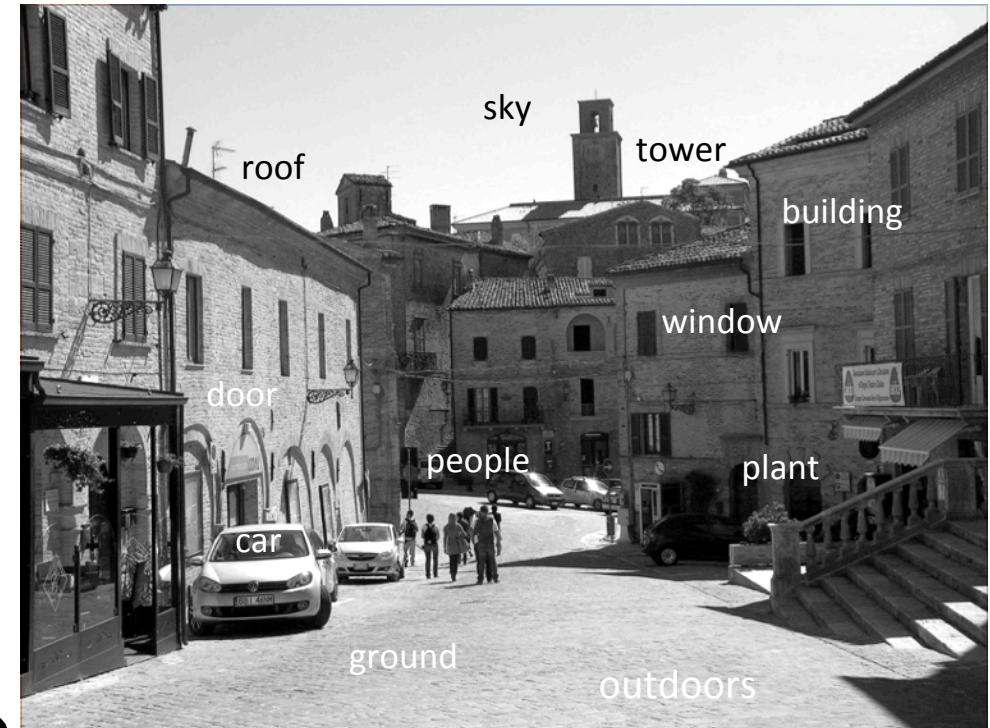
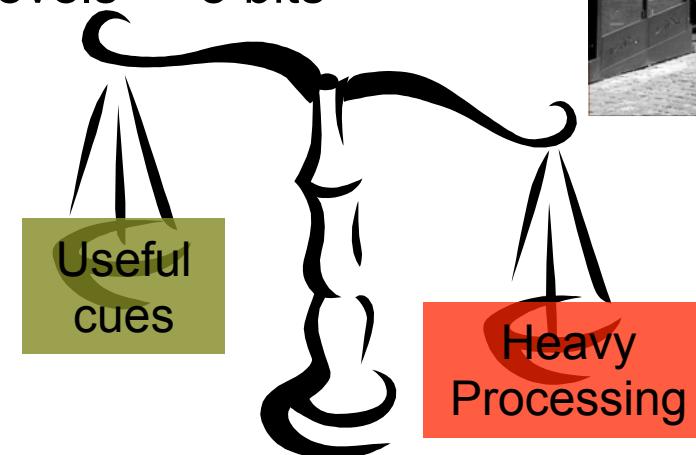
Autonomous Mobile Robots

Margarita Chli

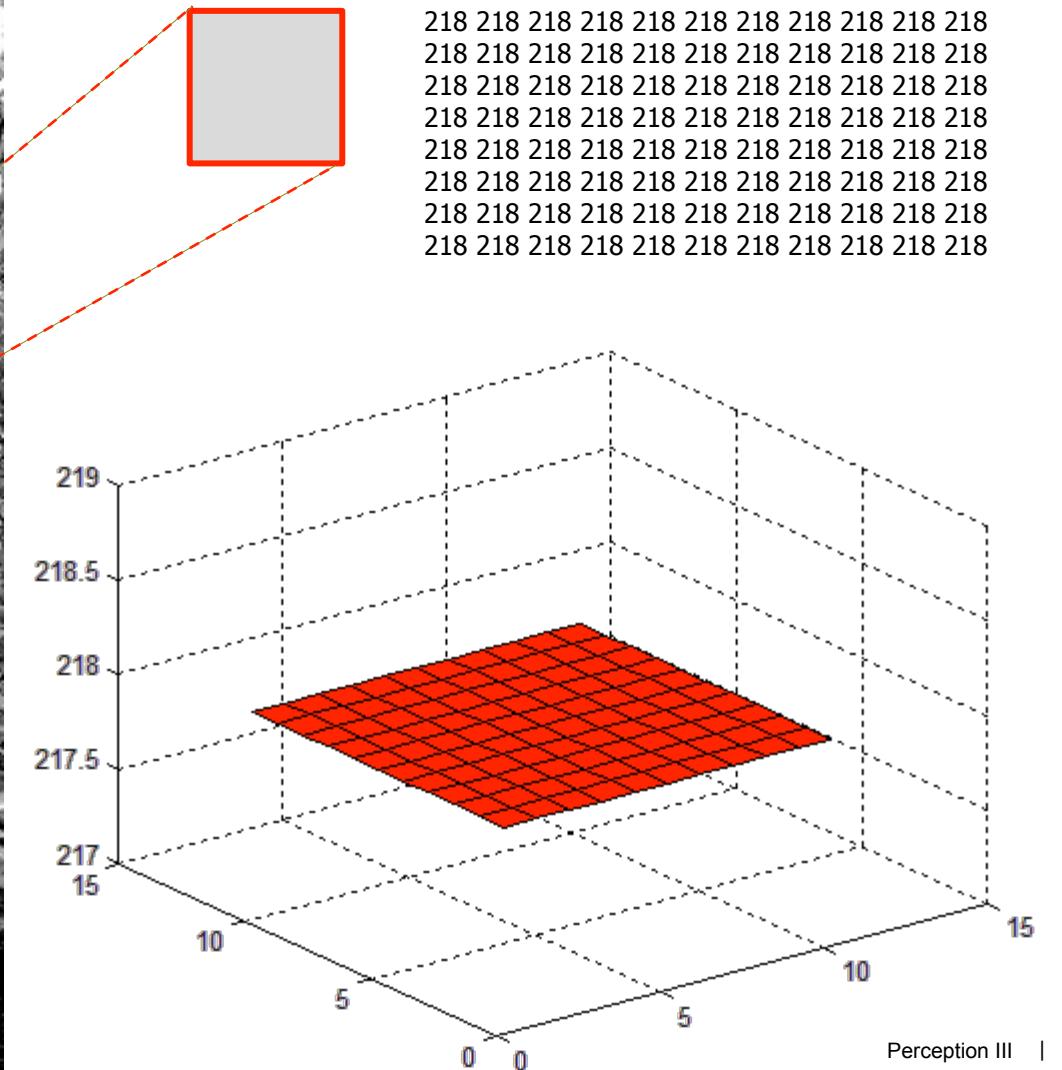
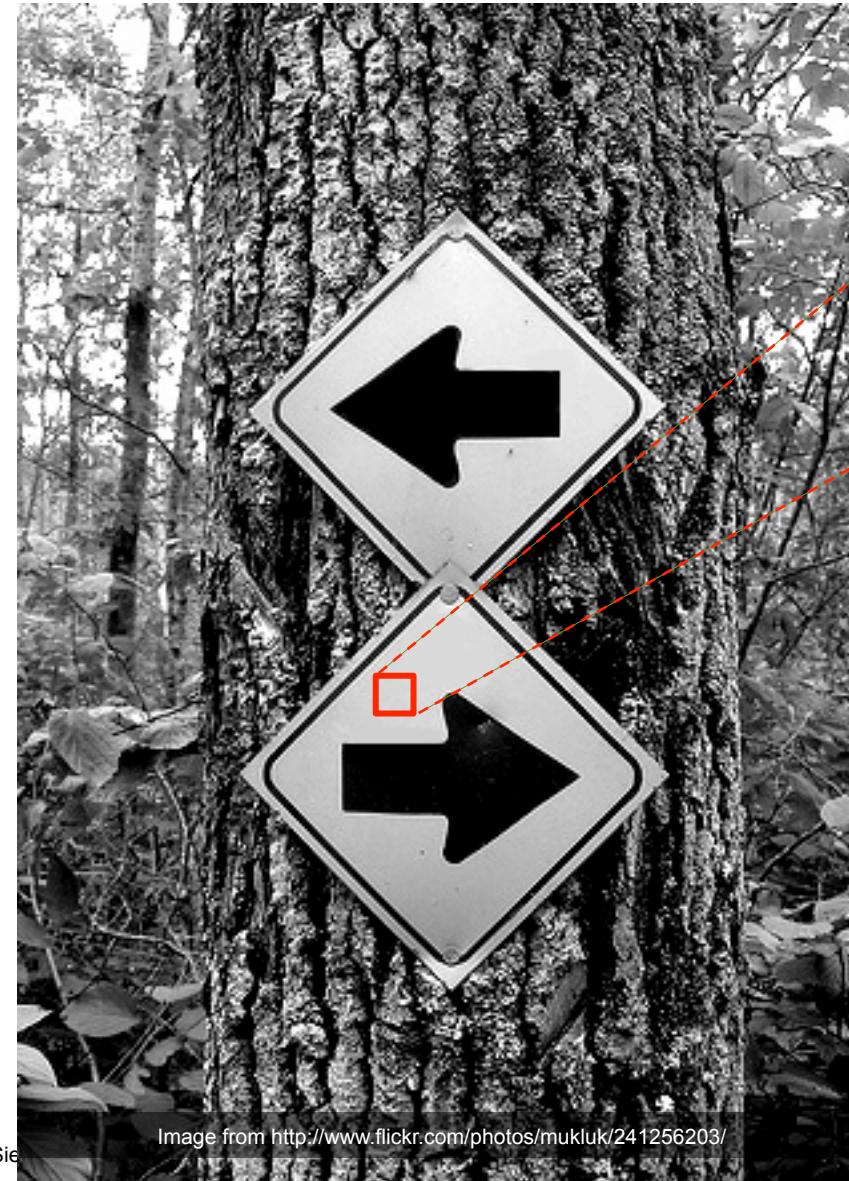
Roland Siegwart and Nick Lawrance

Image Intensities & Data Reduction

- Images capture a lot of information
- Monochrome image \Rightarrow matrix of intensity values
- Typical sizes:
 - 320 x 240 (QVGA)
 - 640 x 480 (VGA)
 - 1280 x 720 (HD)
- Intensities sampled to 256 grey levels \Rightarrow 8 bits



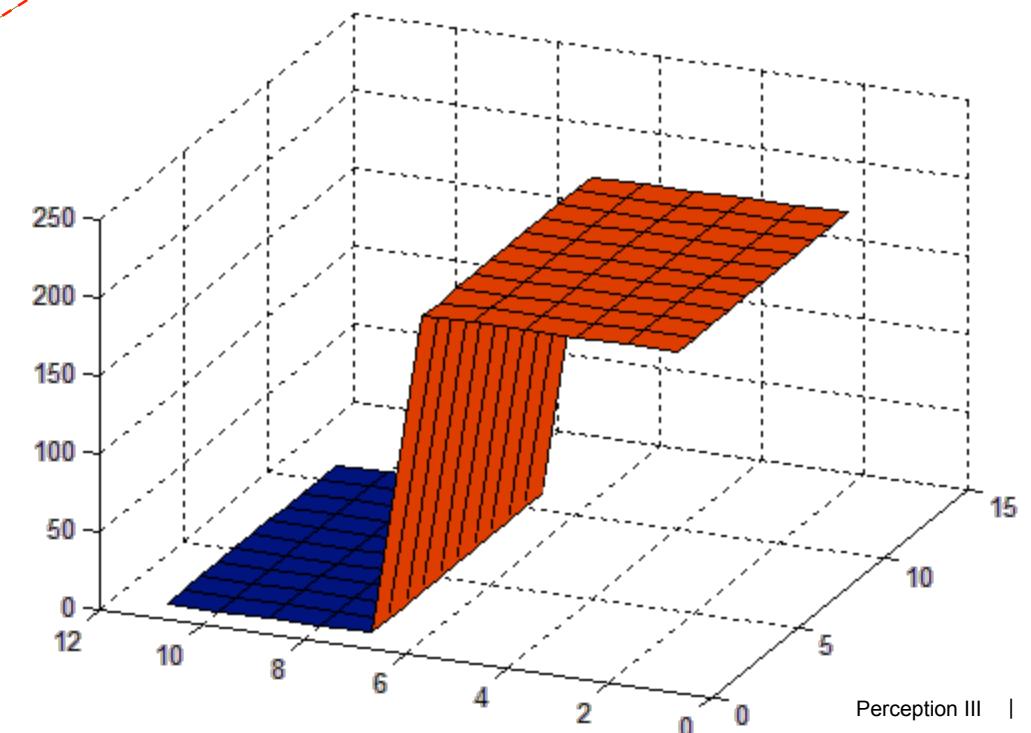
What is useful, what is redundant?



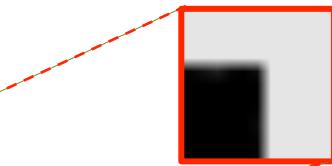
What is useful, what is redundant?



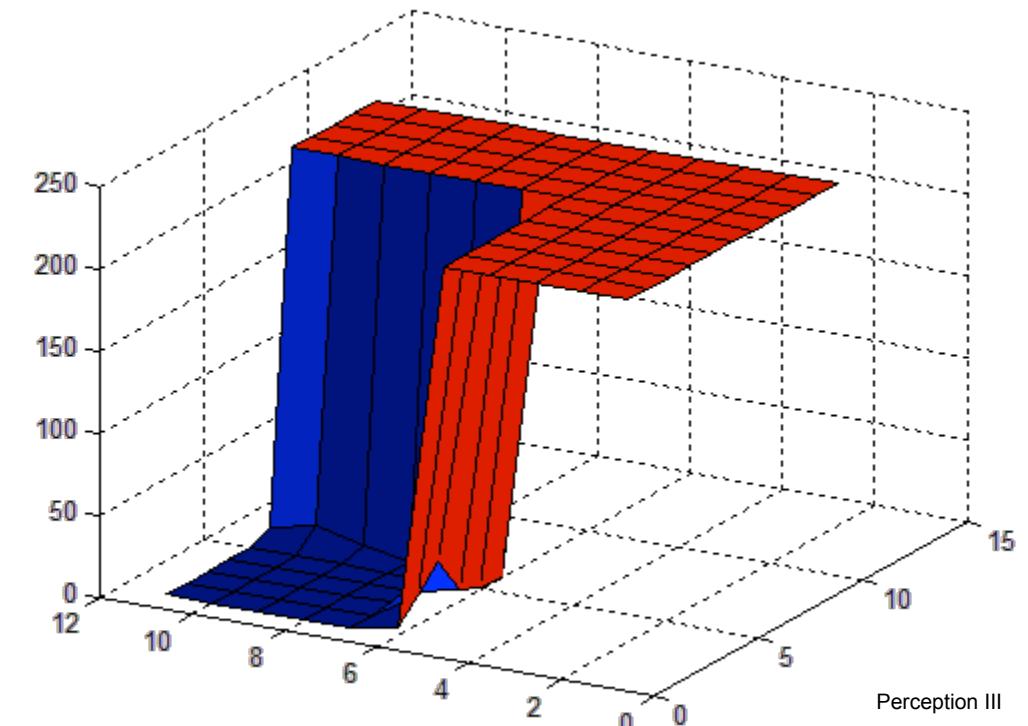
208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208
208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208
208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208
208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208
208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208
208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208
208	207	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208	208
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0



What is useful, what is redundant?



229	229	229	229	229	229	229	229	229	229	229	229	229
229	229	229	229	229	229	229	229	229	229	229	229	229
229	229	229	229	229	229	229	229	229	229	229	229	229
229	229	229	229	229	229	229	229	229	229	229	229	229
229	229	229	229	229	229	229	229	229	229	229	229	229
229	229	229	229	229	230	229	229	229	229	229	229	229
5	17	31	7	1	0	229	229	229	229	229	229	229
0	0	1	0	0	0	229	229	229	229	229	229	229
0	0	0	0	0	0	229	229	229	229	229	229	229
0	0	0	0	1	4	229	229	229	229	229	229	229
0	0	0	0	0	11	229	229	229	229	229	229	229
0	0	0	0	0	5	229	229	229	229	229	229	229



Today's Outline

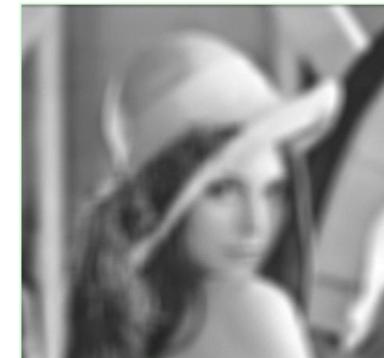
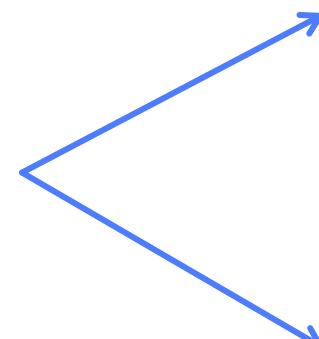
- Sections 4.3 – 4.5 of the book
- Image filtering
 - Correlation
 - Convolution
- Edge / Corner extraction
- Point Features
 - Harris corners
 - SIFT features
 - + some more recent image features from the state of the art

Optional Reading:

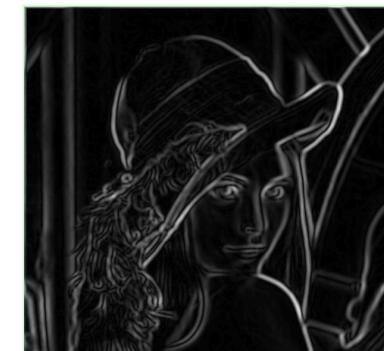
- Harris Corner Detector: C. Harris and M. Stephens. "A combined corner and edge detector." *Alvey vision conference*, 1988. [[paper](#)]
- Shi-Tomasi features: J. Shi and C. Tomasi. "Good features to track." *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)* 1994. [[paper](#)]
- SIFT features: D. G. Lowe. "Distinctive image features from scale-invariant keypoints." *International Journal of Computer Vision (IJCV)*, 2004. [[paper](#)][[demo code](#)]
- FAST corner detector: E. Rosten, R. Porter, and T. Drummond. "Faster and better: A machine learning approach to corner detection." *IEEE Transactions on Pattern Analysis and Machine Intelligence (PAMI)*, 2010. [[paper](#)]
- BRIEF descriptor: M. Calonder, V. Lepetit, C. Strecha, P. Fua. "Brief: Binary robust independent elementary features." *European Conference on Computer Vision (ECCV)*, 2010. [[paper](#)]
- BRISK features: S. Leutenegger, M. Chli, and R. Y. Siegwart. "BRISK: Binary robust invariant scalable keypoints." *International Conference on Computer Vision (ICCV)*, 2011. [[paper](#)]
- Open source implementation of some of these methods (and others) in [OpenCV](#).

Image filtering

- **filtering:** accept / reject certain components
- example: a low-pass filter allows low frequencies \Rightarrow blurring (smoothing) effect on an image – used to reduce image noise
- Smoothing can be achieved not only with **frequency filters**, but also with **spatial filters**.



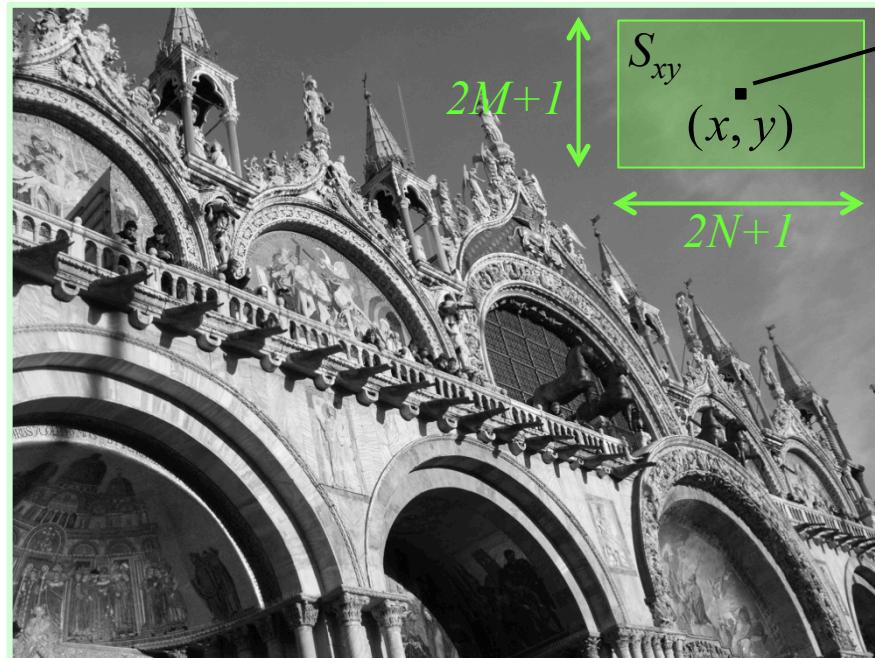
Low-pass filtering:
retains low-frequency components
(smoothing)



High-pass filtering:
retains high-frequency components (edge detection)

Image filtering | spatial filters

- S_{xy} : neighborhood of pixels around the point (x,y) in an image I
- Spatial filtering operates on S_{xy} to generate a new value for the corresponding pixel at output image J

Image I Filtered Image $J = F(I)$

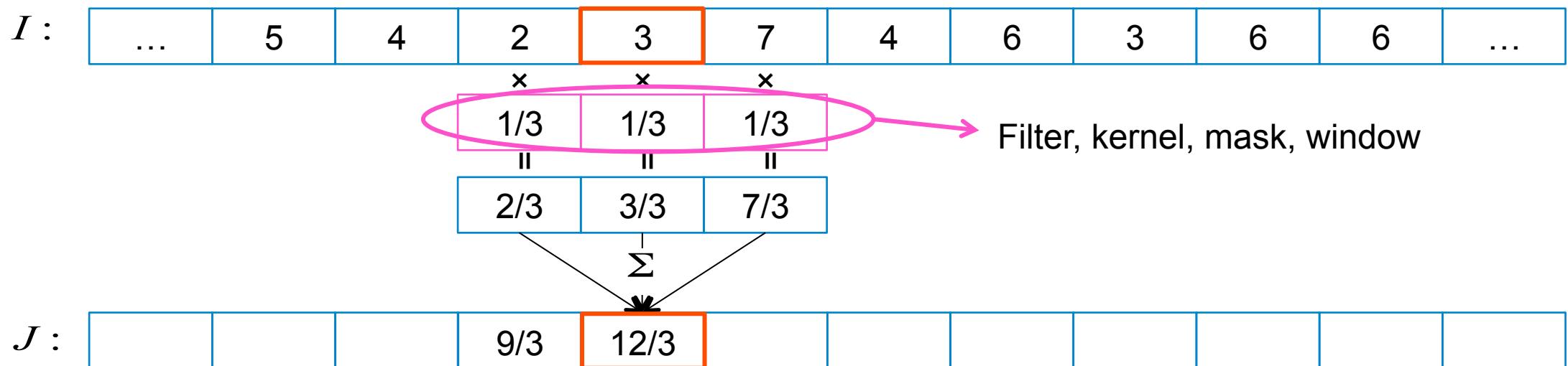
- For example, an averaging filter is: $J(x,y) = \frac{\sum_{(u,v) \in S_{xy}} I(u,v)}{(2M+1)(2N+1)}$

Image filtering | linear, shift-invariant filters

- **Linear:** every pixel is replaced by a linear combination of its neighbours
- **Shift-invariant:** the same operation is performed on every point on the image
- Why filter?
 - Noise reduction, image enhancement, feature extraction, ...
- Basic & very useful filtering operations:
 - Correlation
 - Convolution
- Brief study of these filters in the simplest case of 1D images (i.e. a row of pixels) & their extension to 2D

Image filtering | correlation

- An averaging filter

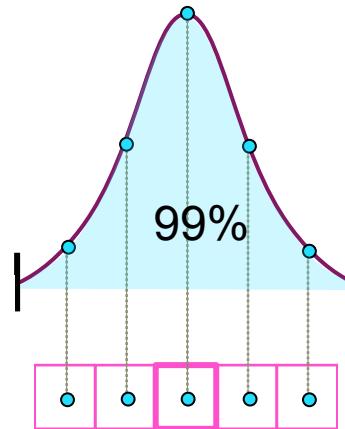


- Formally, Correlation is $J(x) = F \circ I(x) = \sum_{i \in [-N, N]} F(i)I(x + i)$

- In this smoothing example $F(i) = \begin{cases} 1/3, & i \in [-1, 1] \\ 0, & i \notin [-1, 1] \end{cases}$

Image filtering | constructing filter from a continuous fn

- Common practice for image smoothing:
use a Gaussian



$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$\mu = 0$$

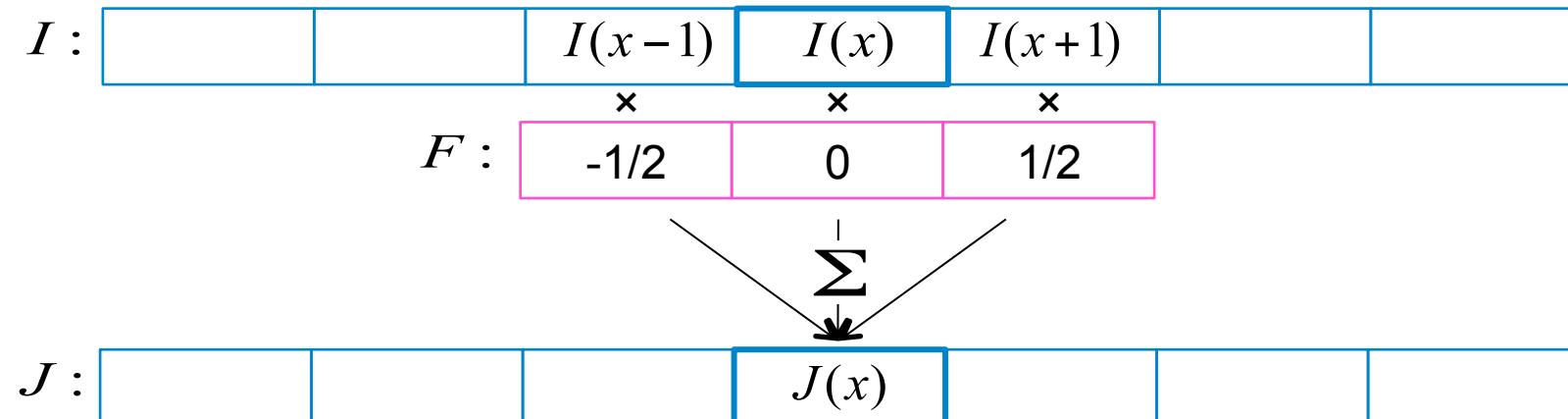
σ : controls the amount of smoothing

Normalize filter so that values always add up to 1

- Near-by pixels have a bigger influence on the averaged value rather than more distant ones

Image filtering | taking derivatives with correlation

- **Derivative** of an image:
quantifies how quickly intensities change
(along the direction of the derivative)
- Approximate a derivative operator:



$$J(x) = \frac{I(x+1) - I(x-1)}{2}$$

Image filtering | matching using correlation

- Find locations in an image that are similar to a **template**

Correspondence Search | the problem

- **goal:** identify image regions / patches in the left & right images, corresponding to the same scene structure
 - Typical **similarity measures:** Normalized Cross-Correlation (NCC) , Sum of Squared Differences (SSD), Sum of Absolute Differences (SAD), ...
 - **Exhaustive** image search can be computationally very expensive!
Can we search for correspondences more efficiently?

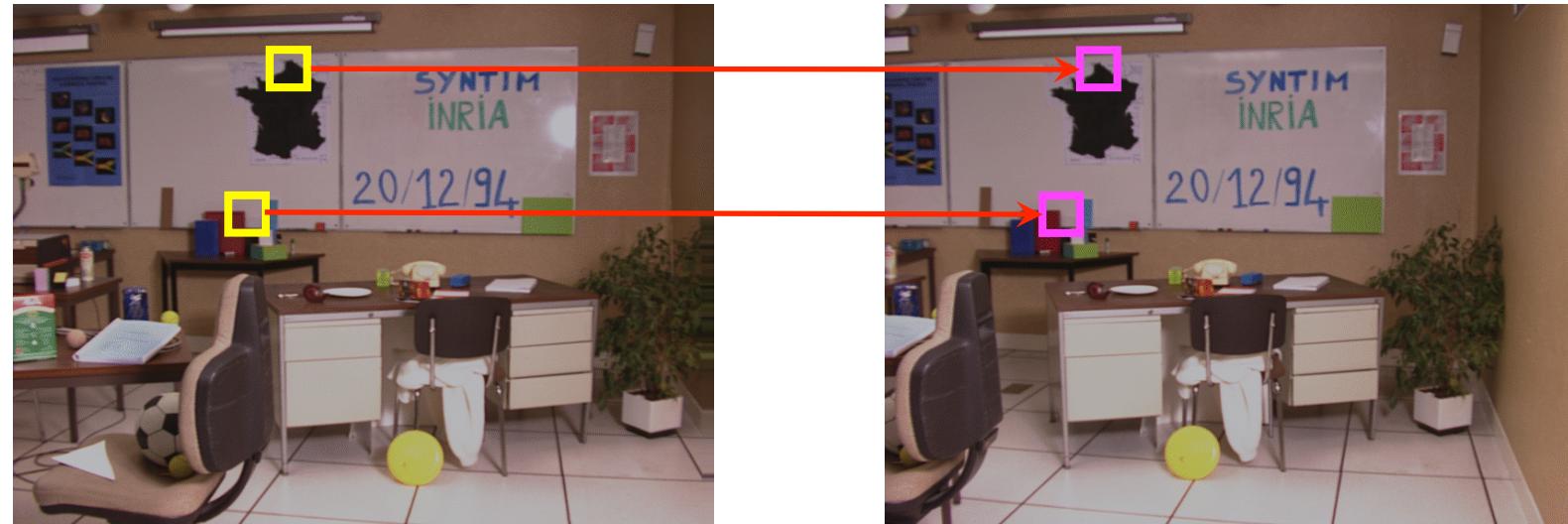


Image filtering | matching using correlation

- Find locations in an image that are similar to a **template**
- Filter = template  ⇒ test it against all image locations



- Similarity measure: Sum of Squared Differences (**SSD**) – minimize

$$\sum_{i=-N}^N (F(i) - I(x+i))^2$$



Image filtering | matching using correlation

- Find locations in an image that are similar to a **template**
- Filter = template  ⇒ test it against all image locations



- Similarity measure: Sum of Squared Differences (**SSD**) – minimize

$$\sum_{i=-N}^N (F(i) - I(x+i))^2 = \sum_{i=-N}^N (F(i))^2 + \sum_{i=-N}^N (I(x+i))^2 - 2 \sum_{i=-N}^N (F(i)I(x+i))$$

Correlation



- Similarity measure: Correlation? – maximize



Image filtering | NCC: Normalized Cross Correlation

- Find locations in an image that are similar to a **template**
- Filter = template  ⇒ test it against all image locations



- Correlation value is affected by the magnitude of intensities
- Similarity measure: Normalized Cross Correlation (**NCC**) – maximize

$$\frac{\sum_{i=-N}^{i=N} (F(i)I(x+i))}{\sqrt{\sum_{i=-N}^{i=N} (F(i))^2} \sqrt{\sum_{i=-N}^{i=N} (I(x+i))^2}}$$



Image filtering | ZNCC: Zero-mean NCC

- Find locations in an image that are similar to a **template**
- Filter = template  ⇒ test it against all image locations



- Correlation value is affected by the magnitude of intensities
- Similarity measure: Zero-mean Normalized Cross Correlation (**ZNCC**) – maximize

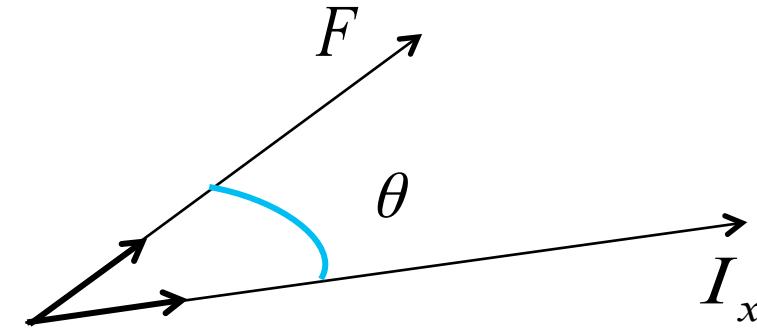
$$\frac{\sum_{i=-N}^{i=N} (F(i) - \mu_F)(I(x+i) - \mu_{I_x})}{\sqrt{\sum_{i=-N}^{i=N} (F(i) - \mu_F)^2} \sqrt{\sum_{i=-N}^{i=N} (I(x+i) - \mu_{I_x})^2}} , \text{ where}$$

$$\begin{cases} \mu_F = \frac{\sum_{i=-N}^N F(i)}{2N+1} \\ \mu_{I_x} = \frac{\sum_{i=-N}^N I(x+i)}{2N+1} \end{cases}$$

Image filtering | correlation as a dot product

- Considering the filter F and the portion of the image I_x as vectors \Rightarrow their correlation is:

$$\langle F, I_x \rangle = \|F\| \|I_x\| \cos \theta$$



- In **NCC** and **ZNCC** we consider the unit vectors of F and I_x , hence we measure their similarity based on the angle θ

Image filtering | correlation in 2D

$$F \circ I(x, y) = \sum_{j \in [-M, M]} \sum_{i \in [-N, N]} F(i, j)I(x + i, y + j)$$

- Example:
Constant averaging filter

$$F = \begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix}$$

- If $\text{size}(F) = (2N + 1)^2$ i.e. this is a square filter
- 2D Correlation \Rightarrow no. multiplications per pixel $= (2N + 1)^2$
no. additions per pixel $= (2N + 1)^2 - 1$

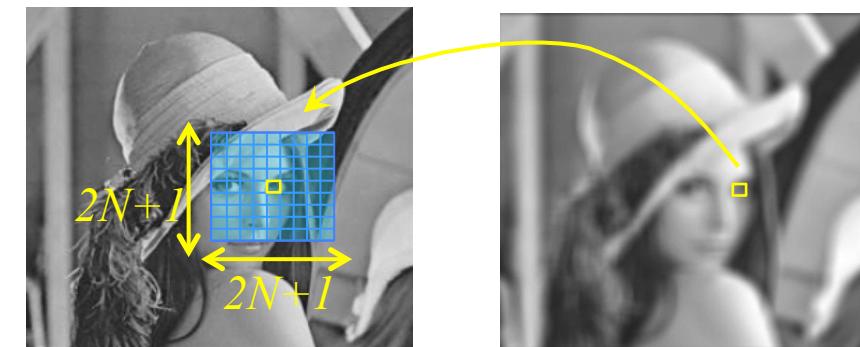


Image filtering | correlation in 2D

$$F \circ I(x, y) = \sum_{j \in [-M, M]} \sum_{i \in [-N, N]} F(i, j)I(x + i, y + j)$$

- Example:
Constant averaging filter

$$F = \begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

“separable” filter

- If $\text{size}(F) = (2N + 1)^2$ i.e. this is a square filter
 - 2D Correlation \Rightarrow no. multiplications per pixel $= (2N + 1)^2$
no. additions per pixel $= (2N + 1)^2 - 1$
 - 2×1 D Correlation \Rightarrow no. multiplications per pixel $= 2(2N + 1)$
no. additions per pixel $= 4N$

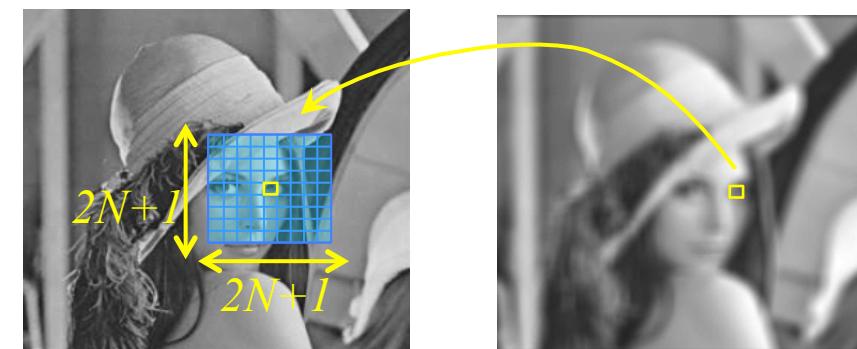


Image filtering | correlation in 2D

$$F \circ I(x, y) = \sum_{j \in [-M, M]} \sum_{i \in [-N, N]} F(i, j)I(x + i, y + j)$$

- Example:
Constant averaging filter

$$F = \begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

“separable” filter

- If $\text{size}(F) = (2N + 1)^2$ i.e. this is a square filter

- 2D Correlation \Rightarrow no. multiplications per pixel $= (2N + 1)^2$
no. additions per pixel $= (2N + 1)^2 - 1$
- 2×1 D Correlation \Rightarrow no. multiplications per pixel $= 2(2N + 1)$
no. additions per pixel $= 4N$
- 2×1 D Correlation \Rightarrow no. multiplications per pixel $= 1$
(with const. factor) no. additions per pixel $= 4N$

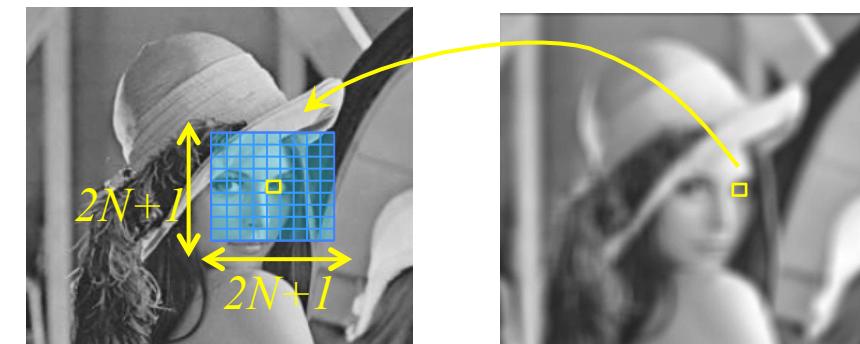


Image filtering | 2D gaussian smoothing

- A general, 2D Gaussian $G(x, y) = \frac{1}{2\pi|S|^{1/2}} e^{-\frac{1}{2}\begin{pmatrix} x \\ y \end{pmatrix} S^{-1} \begin{pmatrix} x & y \end{pmatrix}}$
- We usually want to smooth by the same amount in both x and y directions $S = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix}$
- So this simplifies to:

$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{y^2}{2\sigma^2}}$$

$g_\sigma(x)$ $g_\sigma(y)$

- Another separable filter

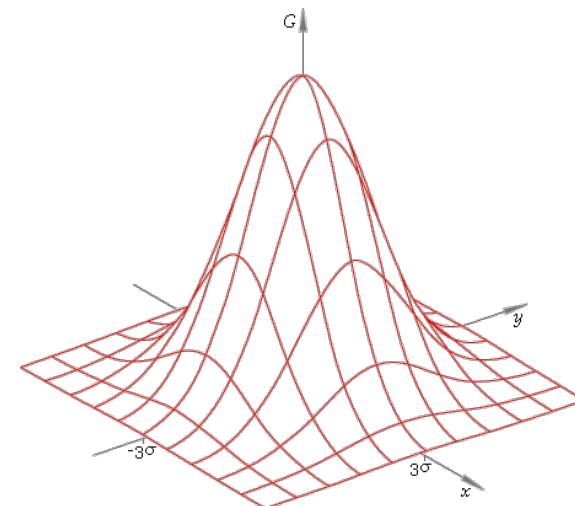


Image filtering | convolution

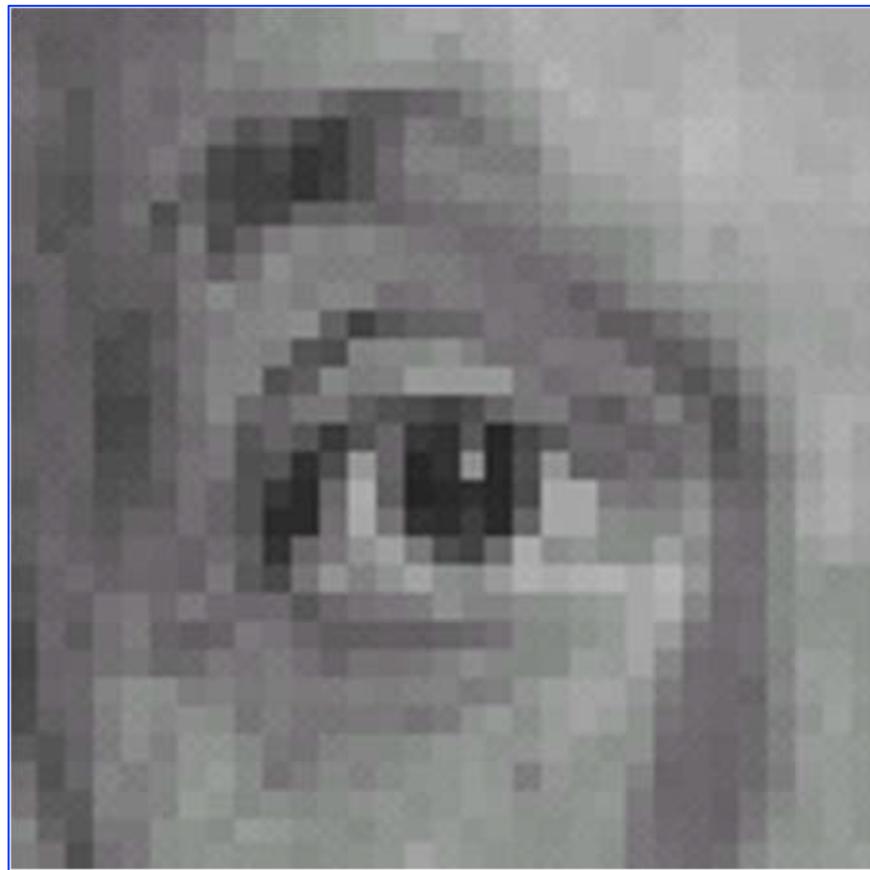
- Convolution is **equivalent** to Correlation with a flipped filter before correlating
- **CONVOLUTION:** $J(x) = F * I(x) = \sum_{i \in [-N, N]} F(i)I(x-i)$
- **CORRELATION:** $J(x) = F \circ I(x) = \sum_{i \in [-N, N]} F(i)I(x+i)$
- Likewise, in 2D we flip the filter both horizontally & vertically

$$J(x, y) = F * I(x, y) = \sum_{j \in [-M, M]} \sum_{i \in [-N, N]} F(i, j)I(x-i, y-j)$$

- Key difference between correlation and convolution is that **convolution is associative**:
$$F * (G * I) = (F * G) * I$$
- Very useful!
- Example: smooth an image & take its derivative \Rightarrow convolve the Derivative filter with the Gaussian Filter & convolve the resulting filter with the Image

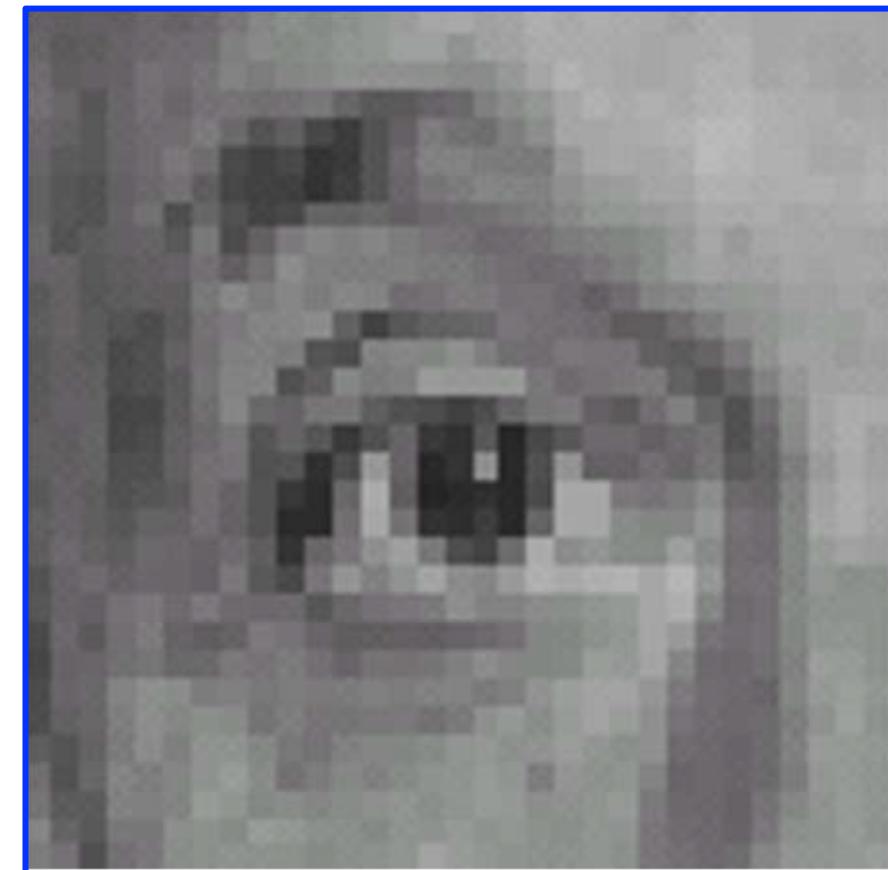
So if $F = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$
 $F' = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$
Then, $F * I(x) = F' \circ I(x)$

Image filtering | examples



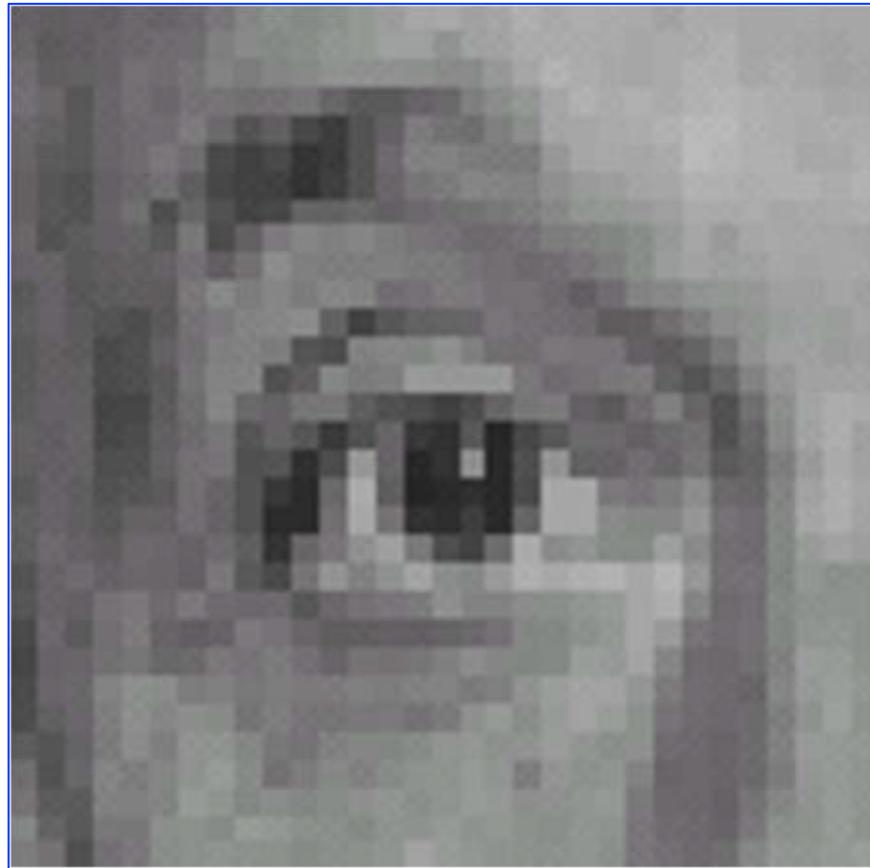
original image

$$\begin{matrix} * & \begin{matrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{matrix} & = \\ & \begin{matrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{matrix} & \end{matrix}$$



filtered (no change)

Image filtering | examples



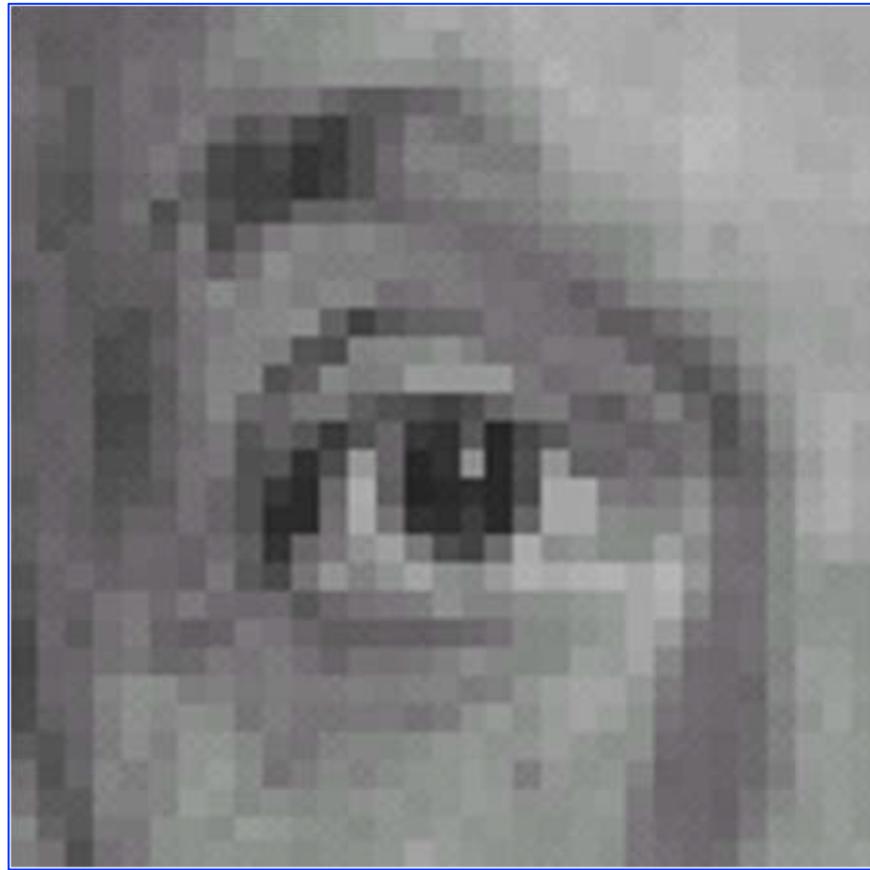
original image

$$\begin{matrix} * & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{matrix} & = \\ & \begin{matrix} & & \\ & & \\ & & \end{matrix} & \end{matrix}$$



filtered (shifted left by 1 pixel)

Image filtering | examples

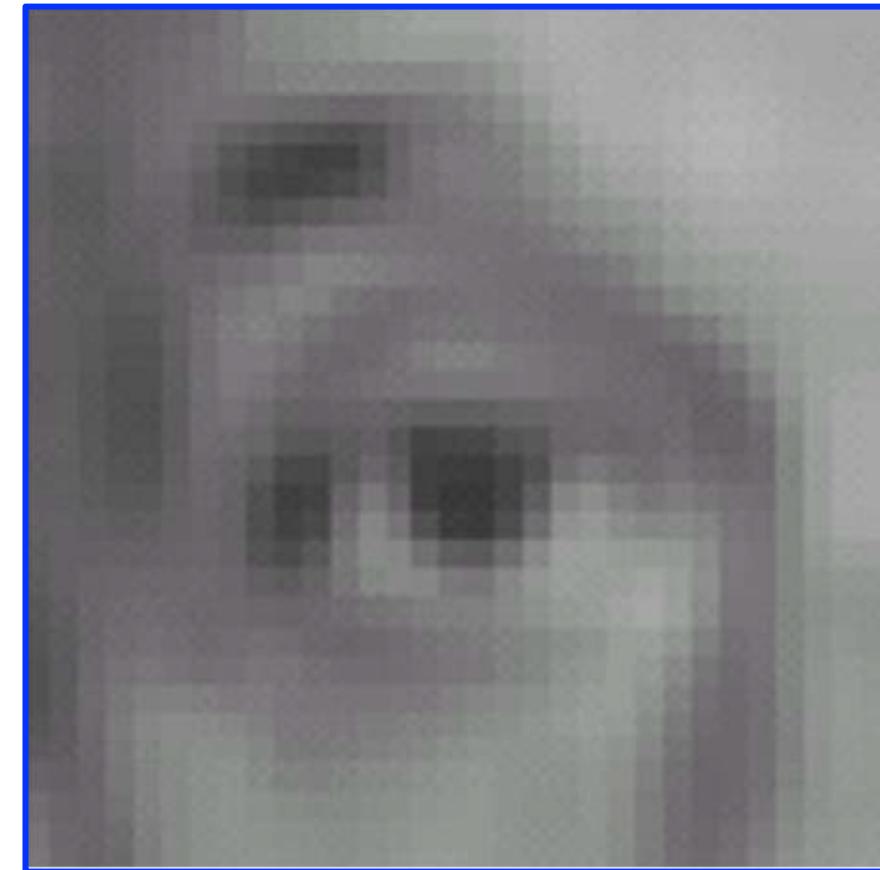


original image

*

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

=



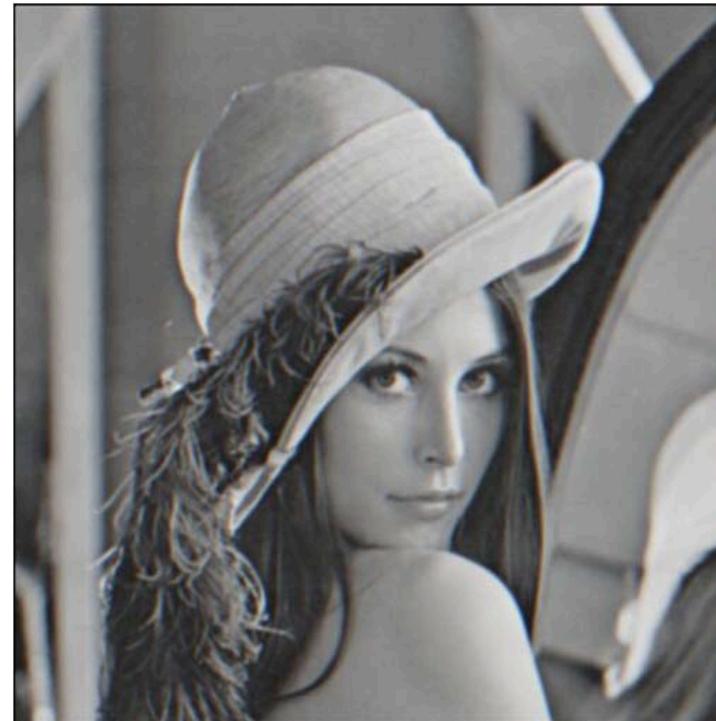
filtered (blurred with a box filter)

Image filtering | examples

- What does blurring take away?



original image



smoothed (5x5)



detail

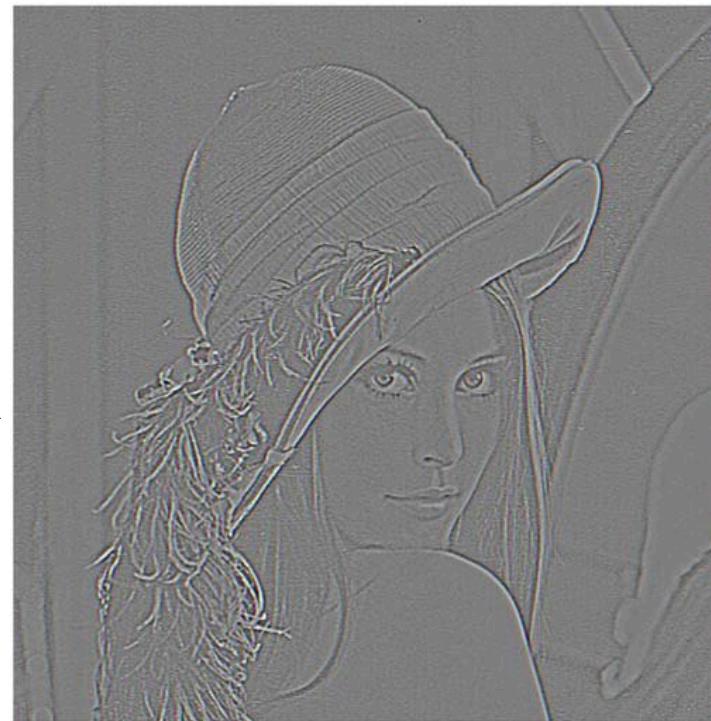
Image filtering | examples

- Let's add it back:



original image

+ a



detail

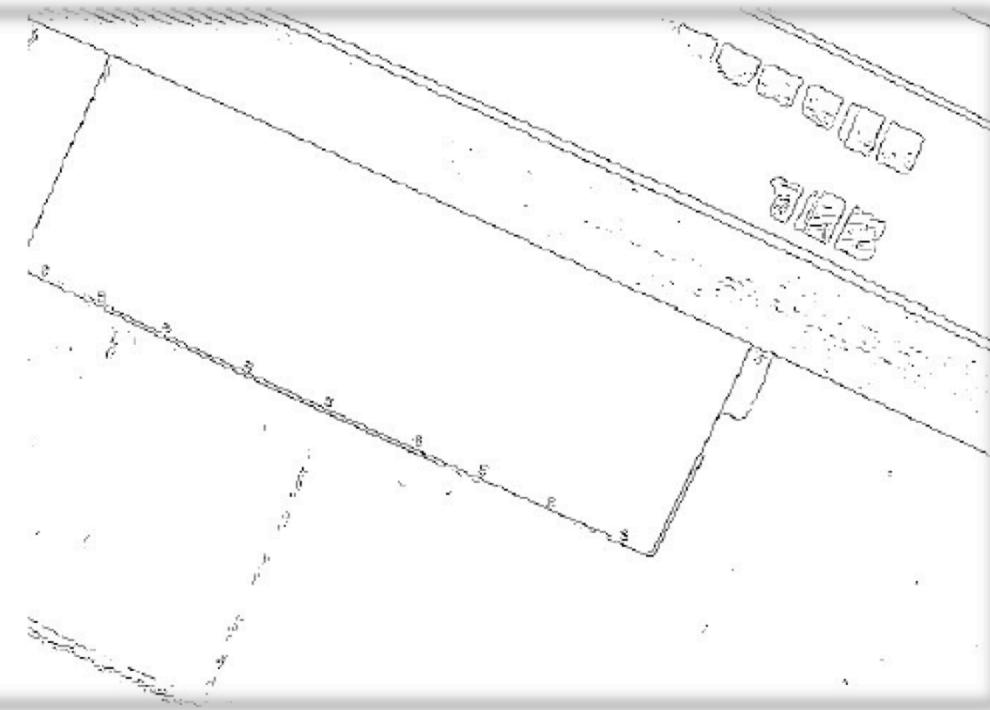
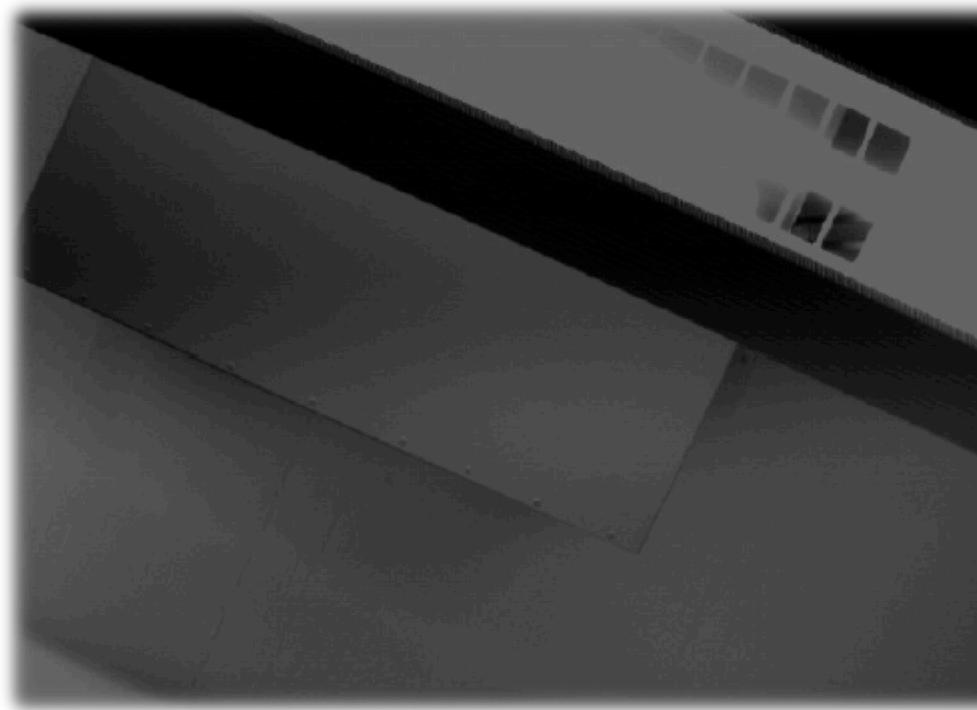
=



sharpened

Edge detection

- Ultimate goal of edge detection: an idealized line drawing.
- Edge contours in the image correspond to important scene contours.



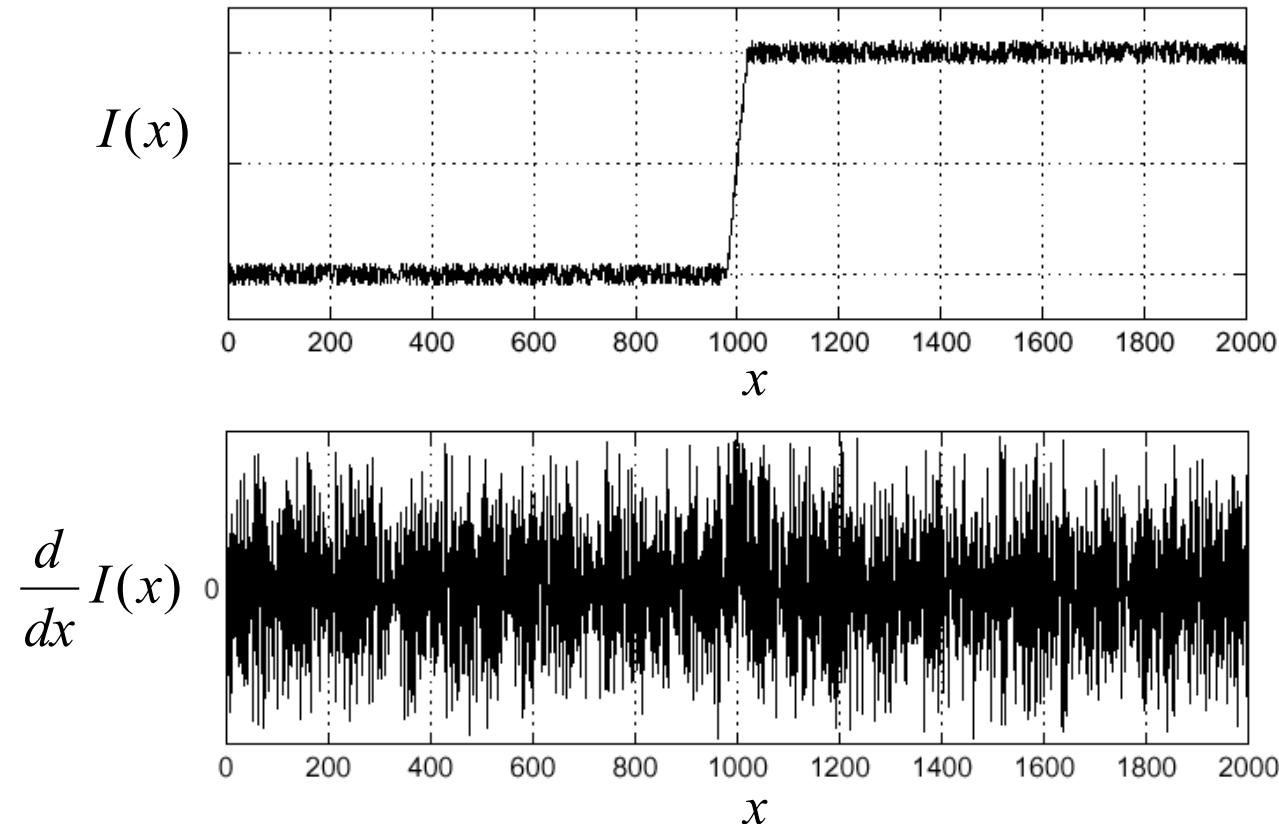
Edge detection | edge = intensity discontinuity in 1 direction

- Edges correspond to sharp changes of intensity
- **How to detect an edge?**
 - Change is measured by 1st order derivative in 1D
 - Big intensity change \Rightarrow magnitude of derivative is large
 - Or 2nd order derivative is zero.

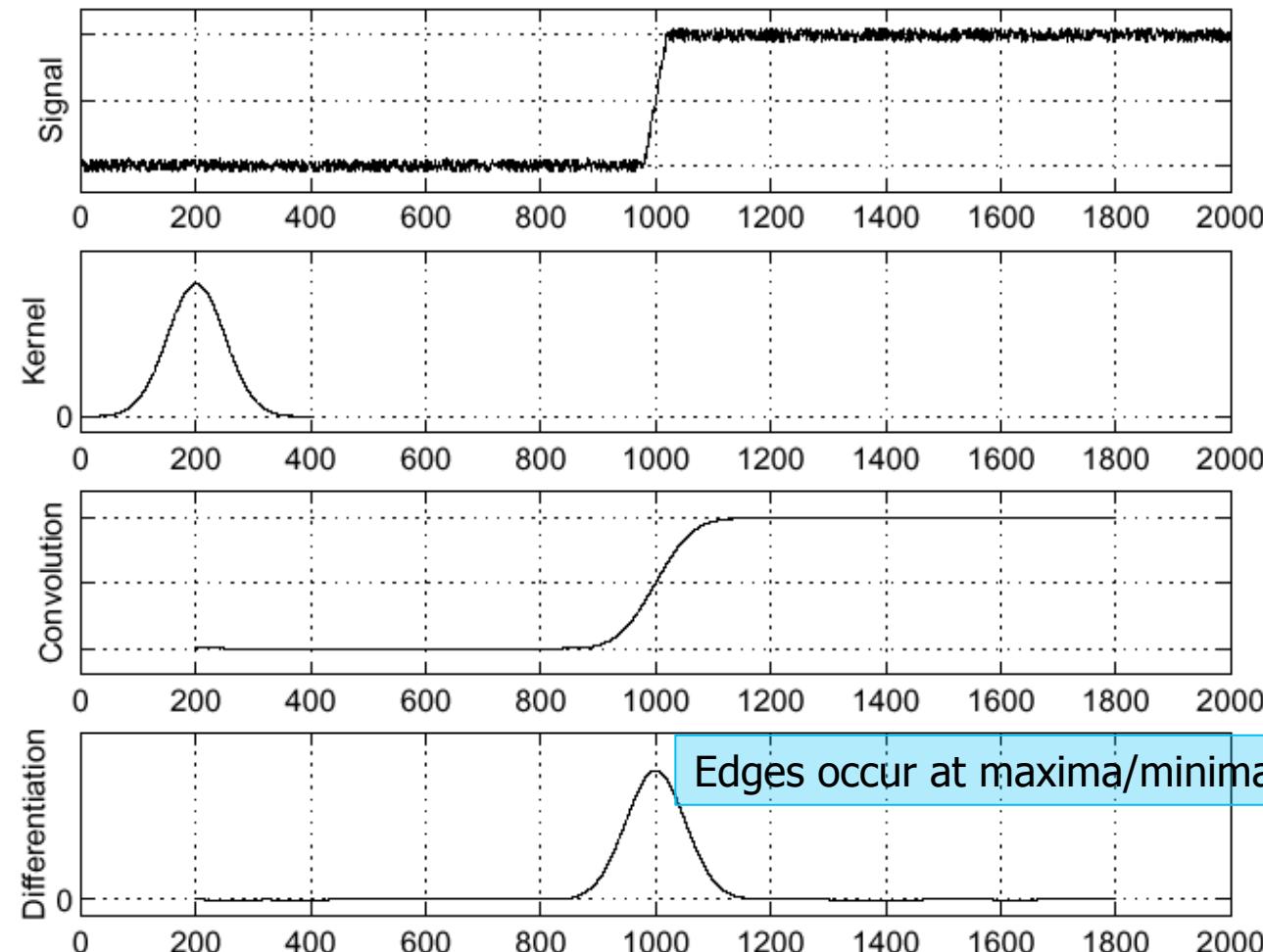


1D edge detection |

- Consider a single row or column of the image, where image intensity shows an obvious change



1D edge detection | solution: smooth first



$$I(x)$$

$$g_\sigma(x)$$

$$s(x) = I(x) * g_\sigma(x)$$

$$s'(x) = \frac{d}{dx}(s(x))$$

Edges occur at maxima/minima of $s'(x)$

- Where is the edge?

1D edge detection | use properties of convolution

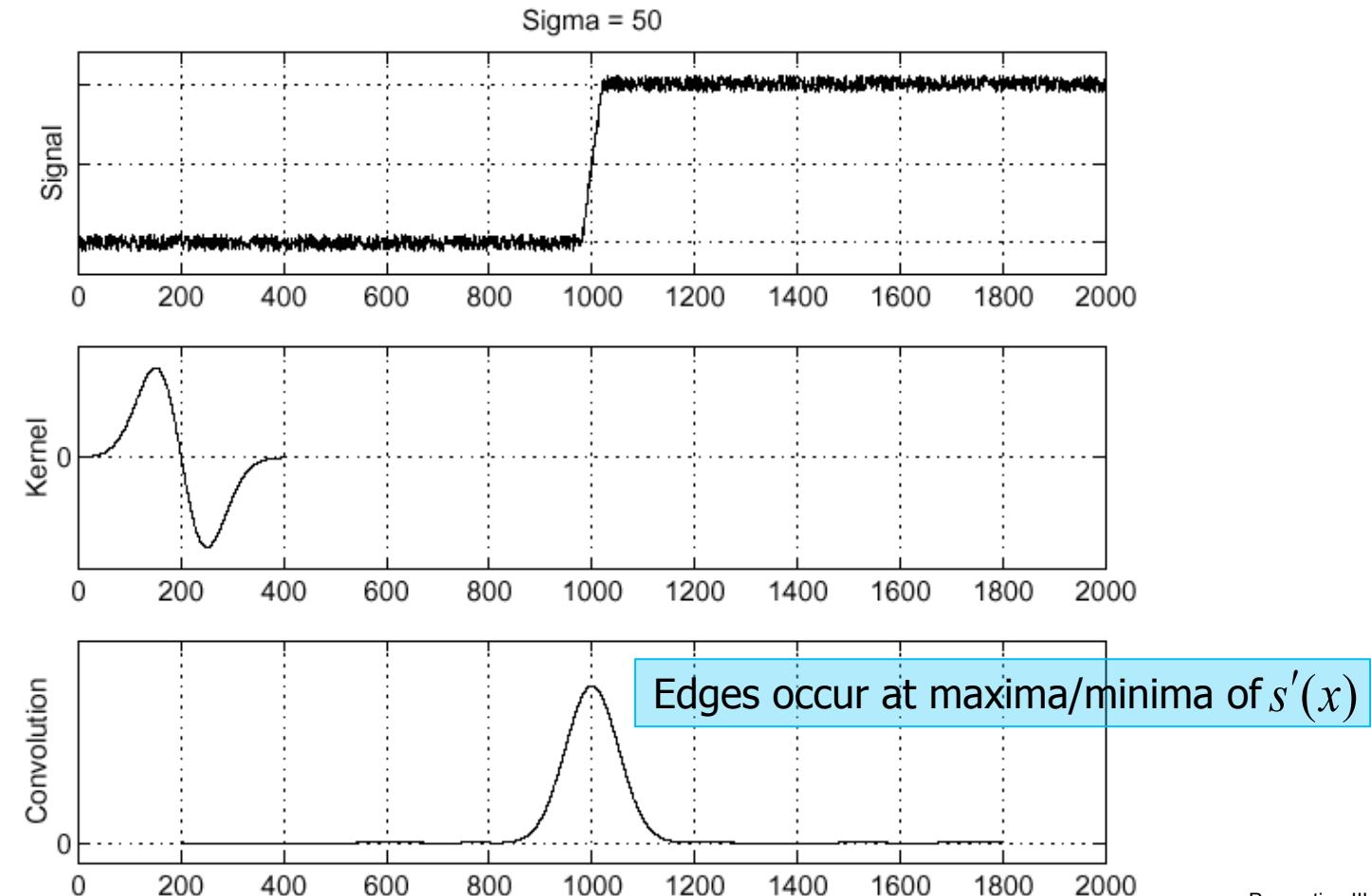
- $s'(x) = \frac{d}{dx} (g_\sigma(x) * I(x)) = g'_\sigma(x) * I(x)$

- This saves us one operation:

 $I(x)$

$$g'_\sigma(x) = \frac{d}{dx} g_\sigma(x)$$

$$s'(x) = g'_\sigma(x) * I(x)$$



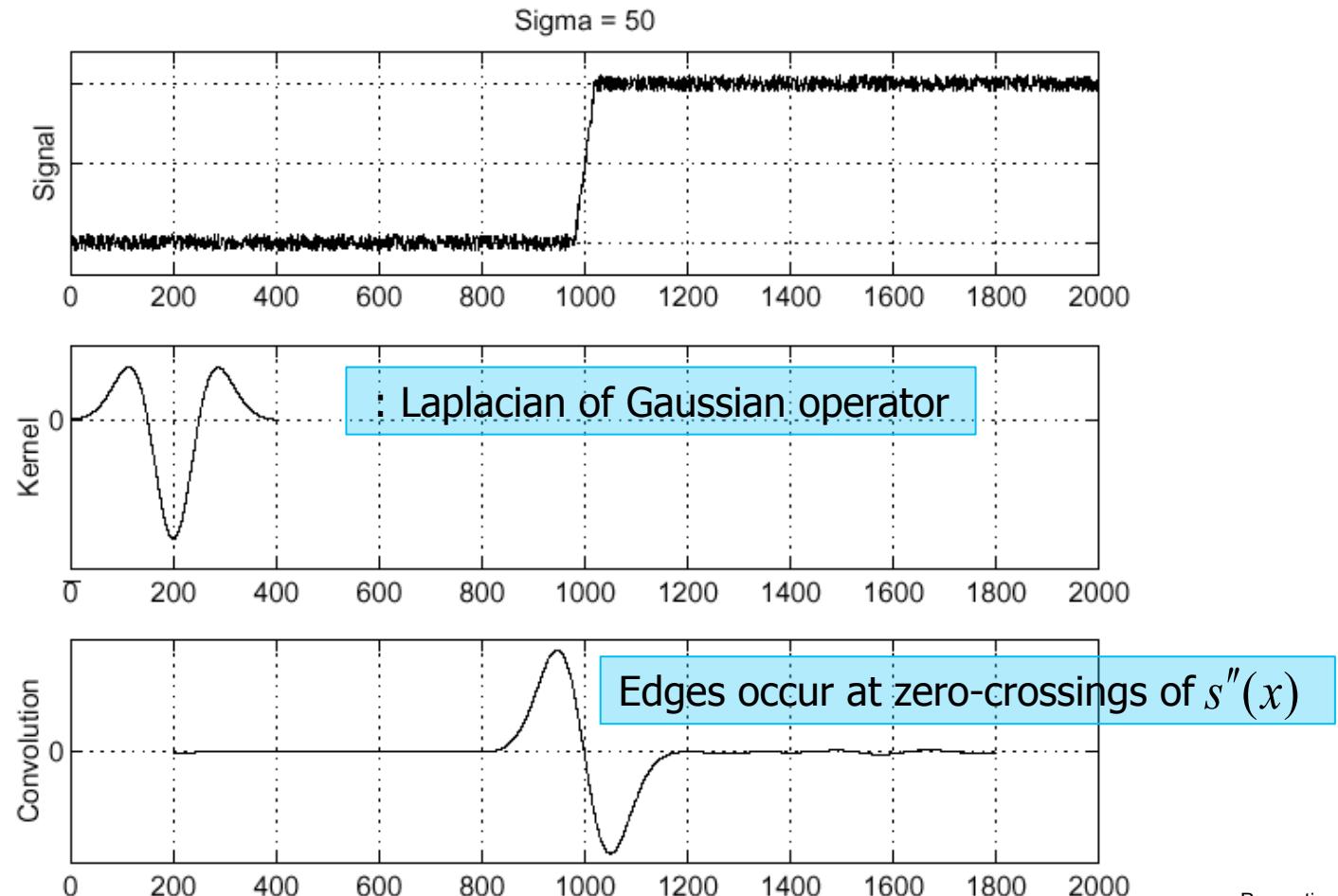
1D edge detection | zero-crossings

- Locations of Maxima/minima in $s'(x)$ are equivalent to zero-crossings in $s''(x)$

 $I(x)$

$$g''_\sigma(x) = \frac{d^2}{dx^2} g_\sigma(x)$$

$$s''(x) = g''_\sigma(x) * I(x)$$



2D edge detection

- Find gradient of smoothed image in both directions

$$\nabla S = \nabla(G_\sigma * I) = \begin{bmatrix} \frac{\partial(G_\sigma * I)}{\partial x} \\ \frac{\partial(G_\sigma * I)}{\partial y} \end{bmatrix} = \begin{bmatrix} \frac{\partial G_\sigma}{\partial x} * I \\ \frac{\partial G_\sigma}{\partial y} * I \end{bmatrix} = \begin{bmatrix} g'_\sigma(x)g_\sigma(y)*I \\ g_\sigma(x)g'_\sigma(y)*I \end{bmatrix}$$

Usually use a separable filter such that:
 $G_\sigma(x,y) = g_\sigma(x)g_\sigma(y)$

- Discard pixels with $|\nabla S|$ (i.e. edge strength) below a certain threshold
- Non-maxima suppression:** identify local maxima of $|\nabla S|$
⇒ detected edges

2D edge detection | example

$$\nabla S = \nabla(G_\sigma * I) = \begin{bmatrix} \frac{\partial(G_\sigma * I)}{\partial x} \\ \frac{\partial(G_\sigma * I)}{\partial y} \end{bmatrix}$$



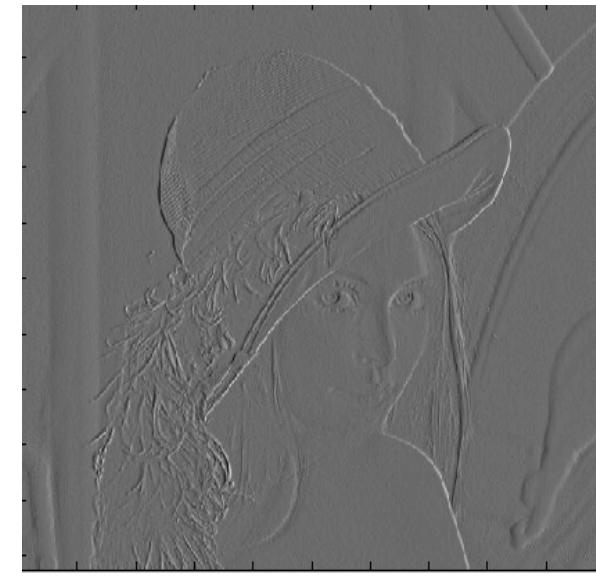
I : original image (Lena)

2D edge detection | example using the Canny edge detector

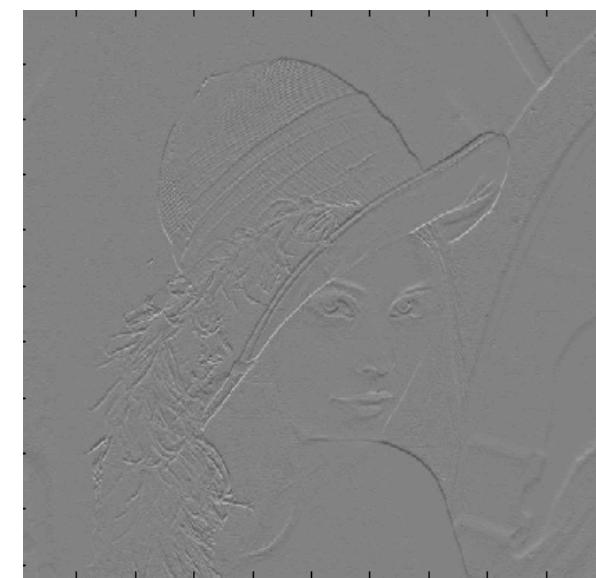


$$|\nabla S| = \sqrt{S_x^2 + S_y^2}$$

$|\nabla S|$: Edge strength



$$S_x = \frac{\partial(G_\sigma * I)}{\partial x}$$



$$\nabla S = \nabla(G_\sigma * I)$$

$$S_y = \frac{\partial(G_\sigma * I)}{\partial y}$$

2D edge detection | example using the Canny edge detector



Thresholding $|\nabla S|$

2D edge detection | example using the Canny edge detector



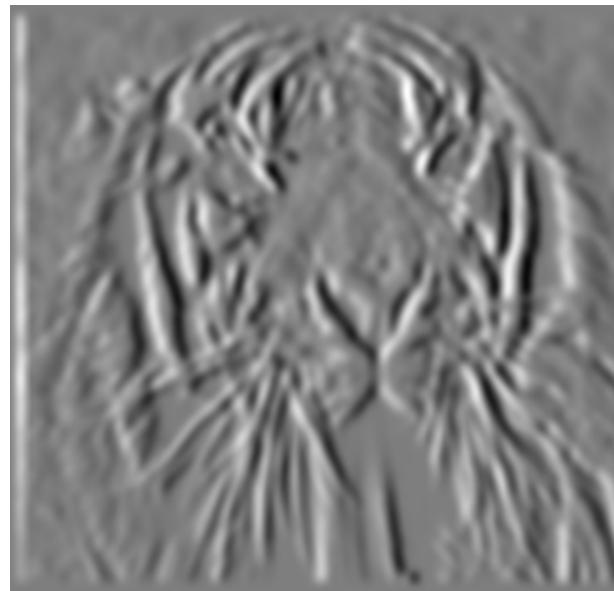
Thinning: non-maximal suppression
⇒ **edge image**

2D edge detection | partial derivatives of an image



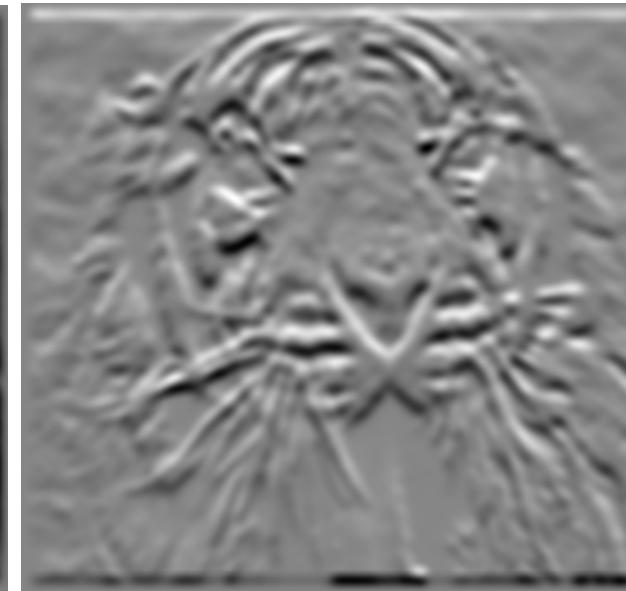
$$S_x = \frac{\partial(G_\sigma * I)}{\partial x}$$

$$F_x = \begin{array}{|c|c|} \hline -1 & 1 \\ \hline \end{array}$$



$$S_y = \frac{\partial(G_\sigma * I)}{\partial y}$$

$$F_y = \begin{array}{|c|c|} \hline -1 \\ \hline 1 \\ \hline \end{array}$$



$$|\nabla S| = \sqrt{S_x^2 + S_y^2}$$



2D edge detection | other approx. of derivative filters

- Prewitt:

$$F_x = \begin{matrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{matrix}$$

$$F_y = \begin{matrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{matrix}$$

- Sobel:

$$F_x = \begin{matrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{matrix}$$

$$F_y = \begin{matrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{matrix}$$

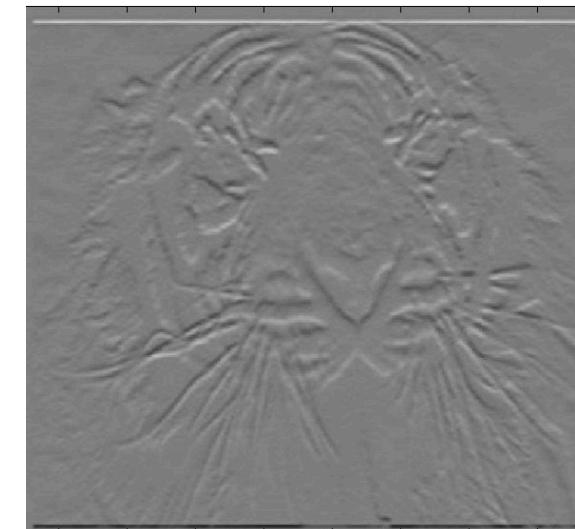
- Roberts:

$$F_x = \begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix}$$

$$F_y = \begin{matrix} 1 & 0 \\ 0 & -1 \end{matrix}$$

Sample Matlab code

```
>> im = imread('lion.jpg')
>> Fy = fspecial('sobel');
>> outim = imfilter(double(im), Fy);
>> imagesc(outim);
>> colormap gray;
```



2D edge detection | derivative of gaussian filter

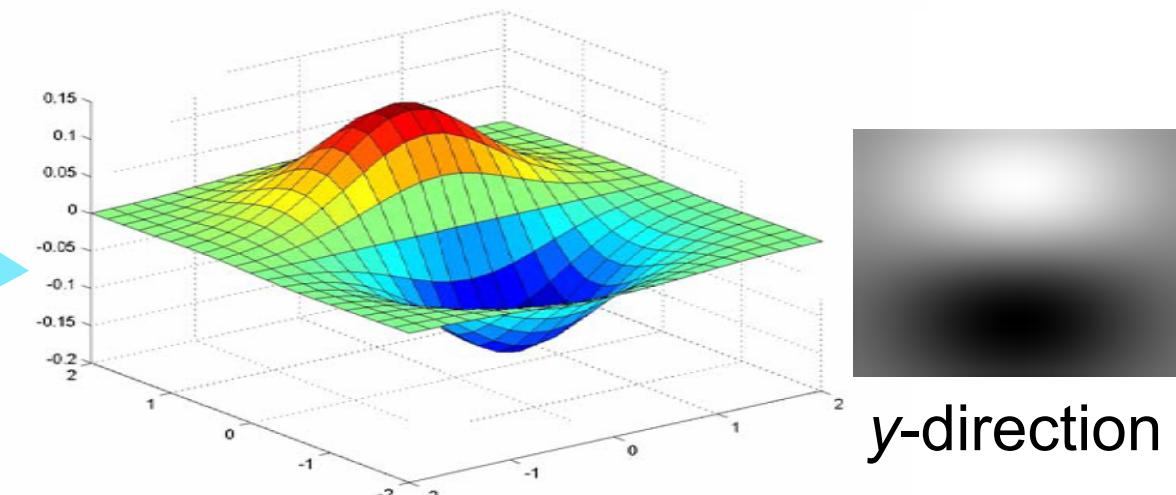
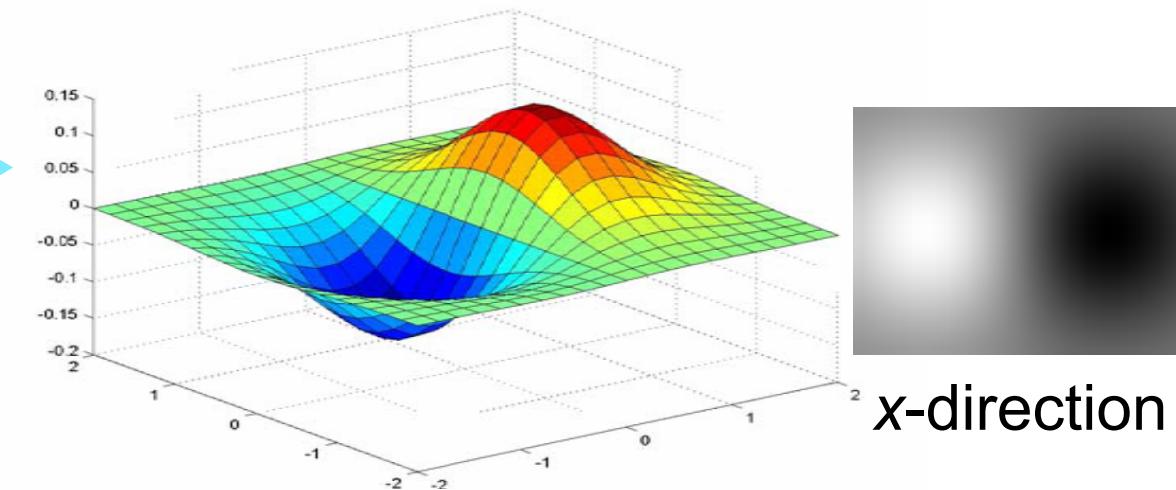
$$\nabla S = \nabla * G * I = \begin{pmatrix} (F_x * G) * I \\ (F_y * G) * I \end{pmatrix}$$

$$F_x = \begin{pmatrix} 1 & -1 \end{pmatrix}$$

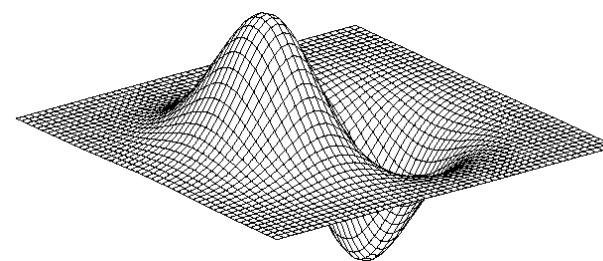
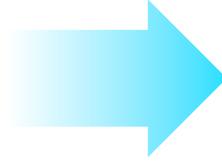
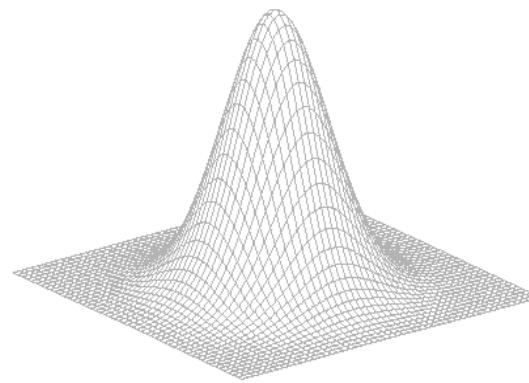
$$F_y = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

↓

$$\begin{pmatrix} 0.0030 & 0.0133 & 0.0219 & 0.0133 & 0.0030 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0219 & 0.0983 & 0.1621 & 0.0983 & 0.0219 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0030 & 0.0133 & 0.0219 & 0.0133 & 0.0030 \end{pmatrix}$$



2D edge detection | popular edge detection filters



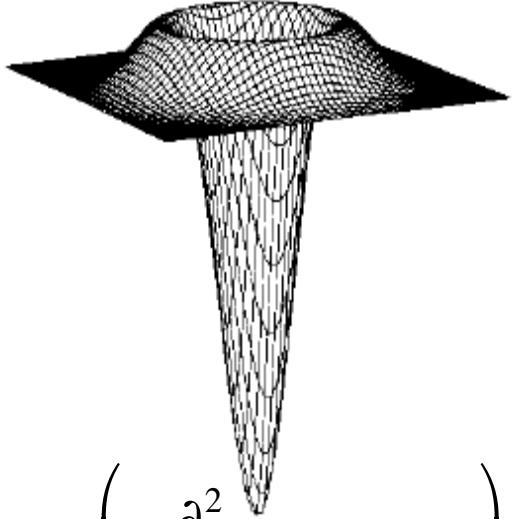
derivative of Gaussian

Gaussian

$$G_\sigma(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$

$$\nabla G_\sigma(u, v) = \begin{pmatrix} \frac{\partial}{\partial u} G_\sigma(u, v) \\ \frac{\partial}{\partial v} G_\sigma(u, v) \end{pmatrix}$$

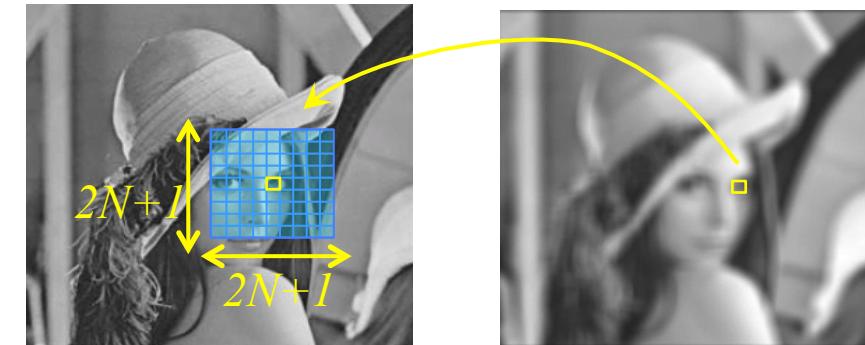
Laplacian of Gaussian


$$\nabla^2 G_\sigma(u, v) = \begin{pmatrix} \frac{\partial^2}{\partial u^2} G_\sigma(u, v) \\ \frac{\partial^2}{\partial v^2} G_\sigma(u, v) \end{pmatrix}$$

Key points on smoothing + derivative masks

Smoothing masks

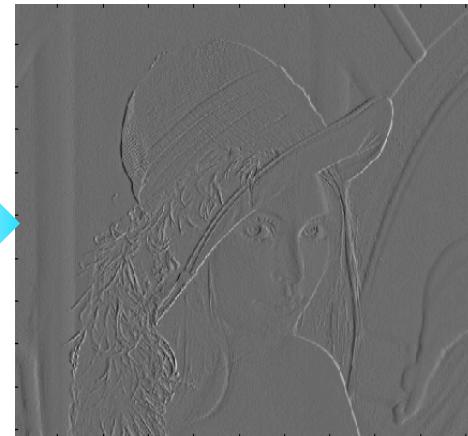
- Values positive
- Always **sum to 1** → constant regions same as input
- Amount of **smoothing proportional to mask size**



Derivative masks

- Opposite signs used to get high response in regions of high contrast
- Always **sum to 0** → no response in constant regions
- High absolute value at points of high contrast

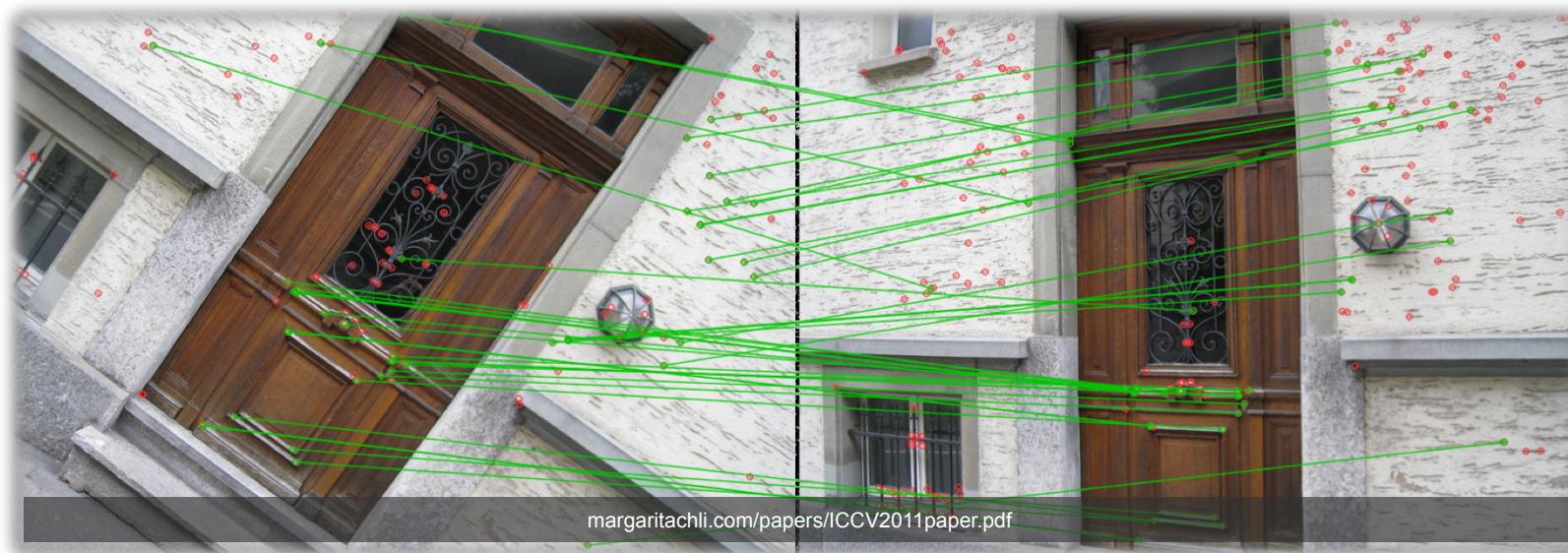
$$F_x = \begin{pmatrix} 1 & -1 \end{pmatrix}$$



Point Features

Image Feature Extraction:

- Edges
- Points: **Shi-Tomasi & Harris corners**
SIFT features
- and more recent algorithms from the state of the art...



Point features | applications

Point features are widely used in:

- Robot navigation
- Object recognition
- 3D reconstruction
- Motion tracking
- Indexing and database retrieval ⇒ Google Images
- ...
- Image stitching: this panorama was generated using **AUTOSTITCH** (freeware)



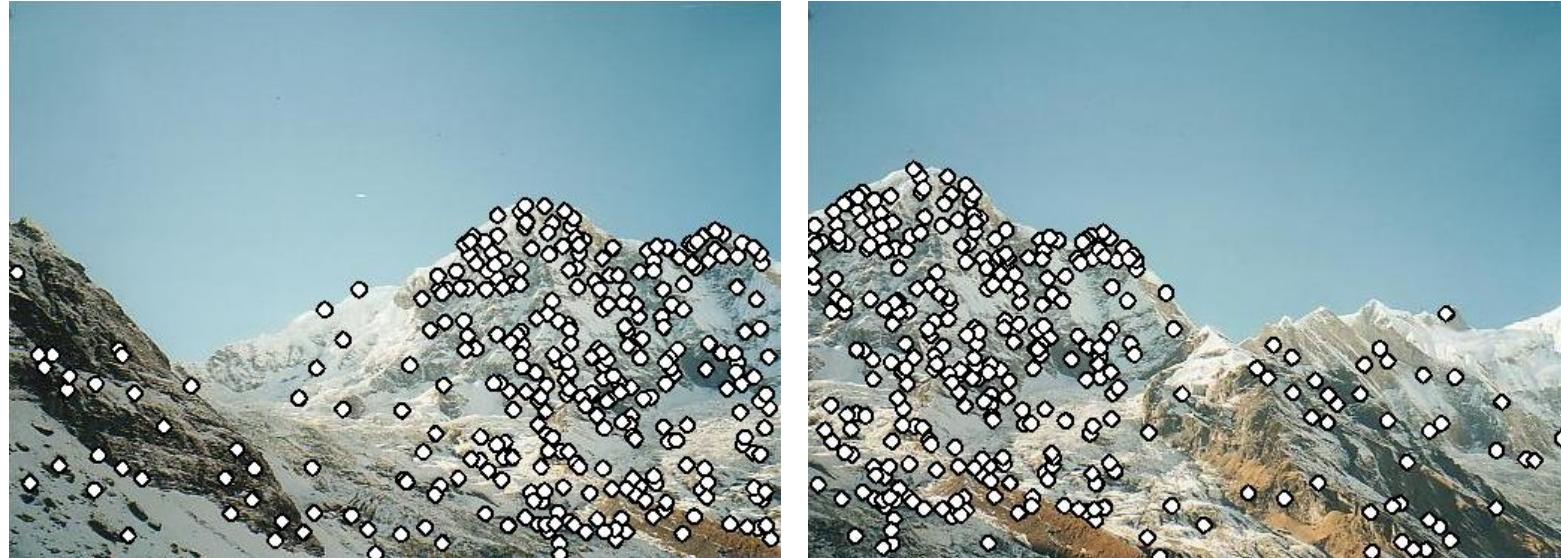
Point features | how to build a panorama?

- We need to match (align) images



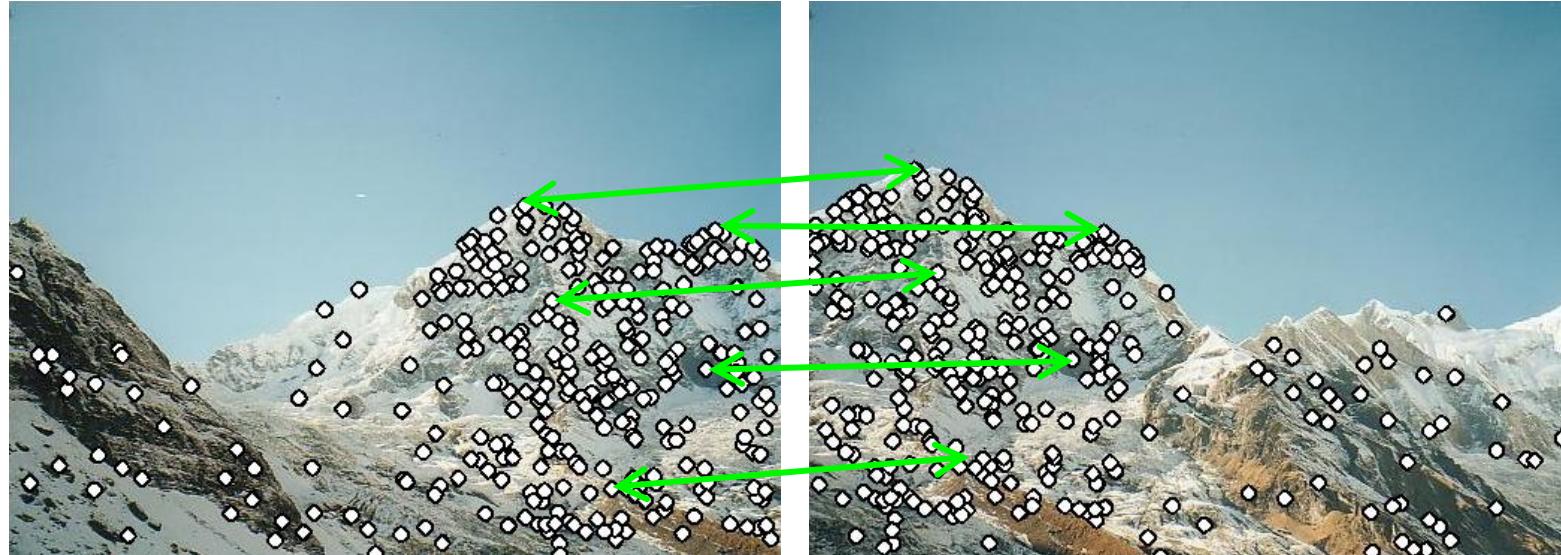
Point features | how to build a panorama?

- Detect feature points in both images



Point features | how to build a panorama?

- Detect feature points in both images
- Find corresponding pairs



Point features | how to build a panorama?

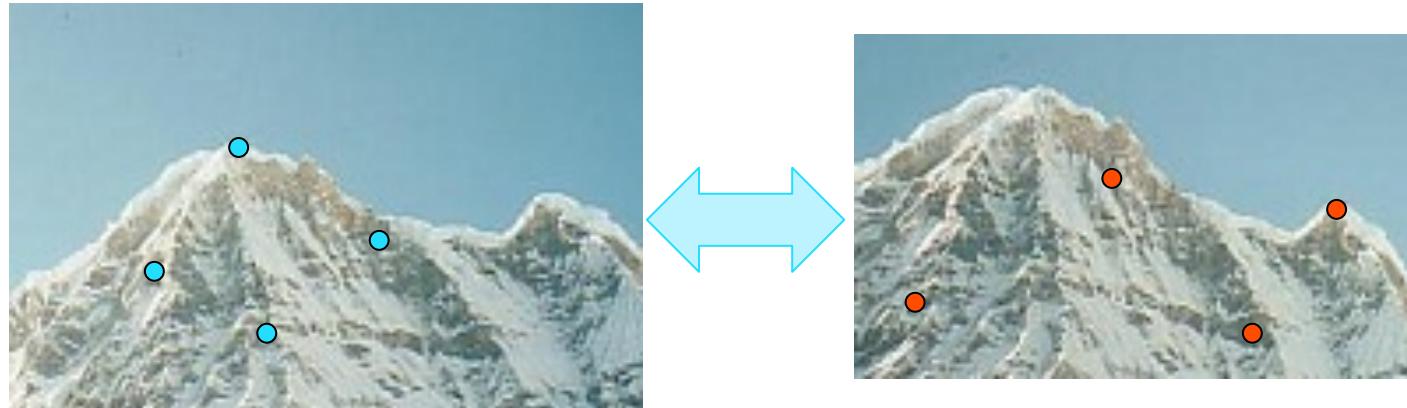
- Detect feature points in both images
- Find corresponding pairs
- Use these pairs to align images



Point features | feature extraction

Problem 1:

- Detect the **same** points **independently** in both images, if they are in the field of view



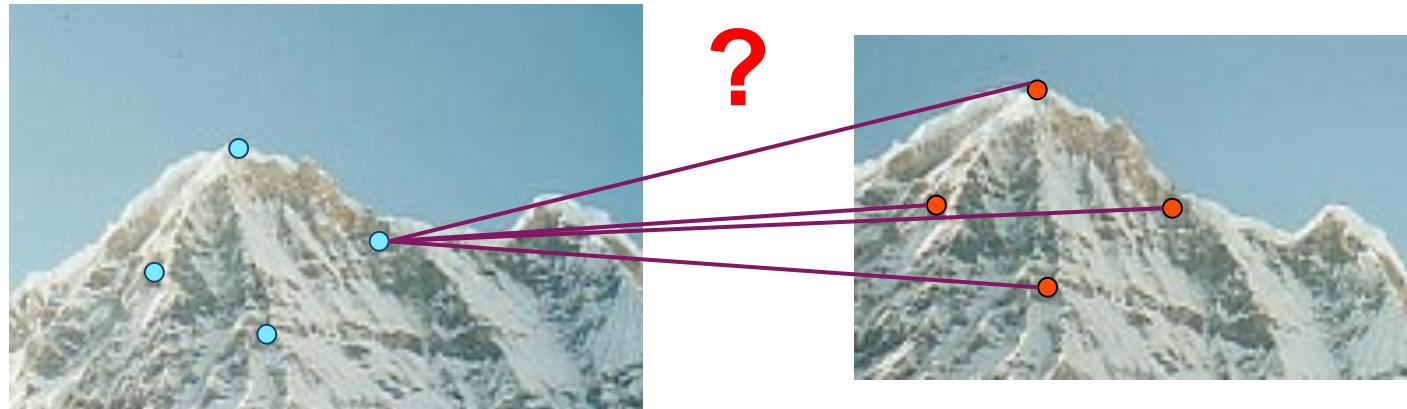
no chance to match!

We need a **repeatable** feature detector

Point features | feature matching

Problem 2:

- For each point, identify its correct correspondence in the other image(s)



We need a **reliable** and **distinctive** feature descriptor

Point features | what is a distinctive feature?

- Some patches can be localized or matched with higher accuracy than others

Image 1

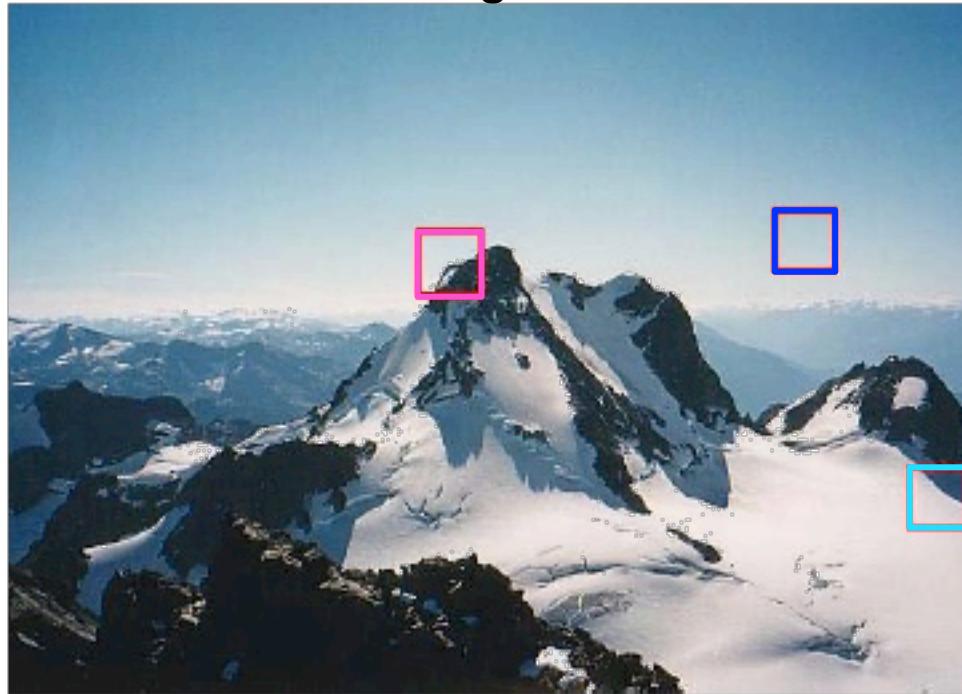
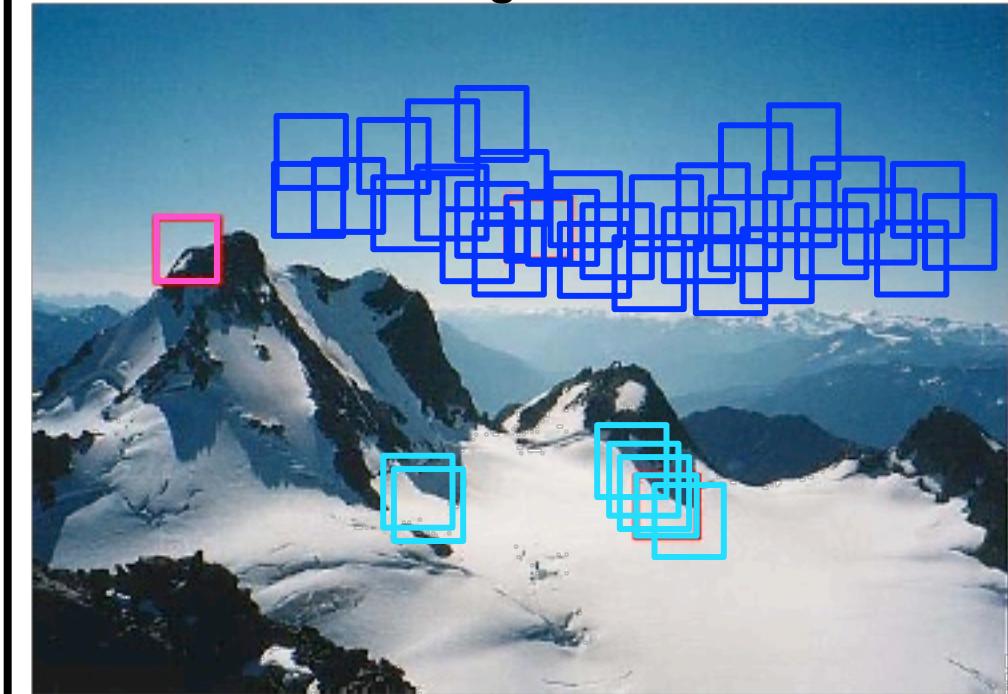


Image 2



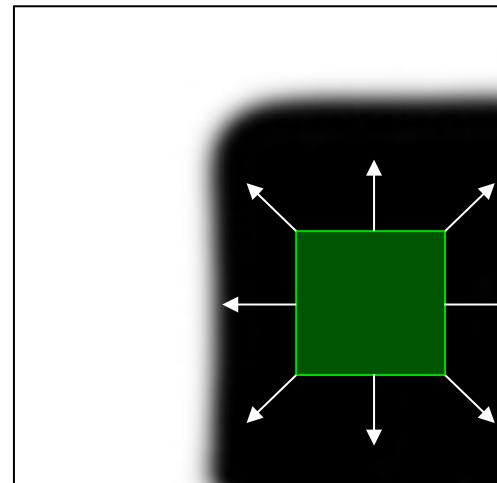
What is useful, what is redundant?



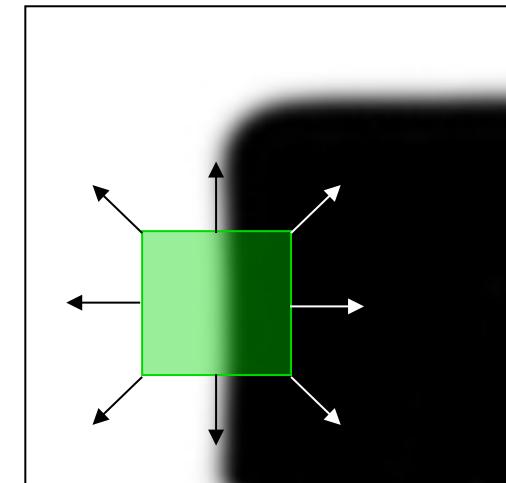
- Images capture a lot of information
 - What is useful, what is redundant?
 - Select features that:
 - Are distinctive
 - Do not vary much in appearance
 - Can be detected & matched very fast
- ➔ use point features: e.g. **corners**

Corner detection | how do we identify corners?

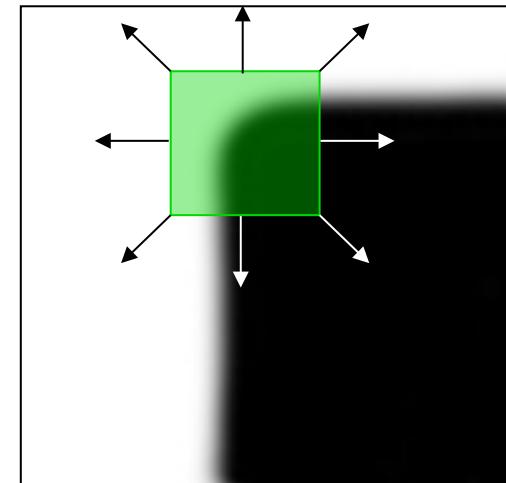
Property: Shifting a window in **any direction** should give a **large change** of intensity in at least 2 directions



“flat” region:
no intensity change



“edge”:
no change along the edge
direction

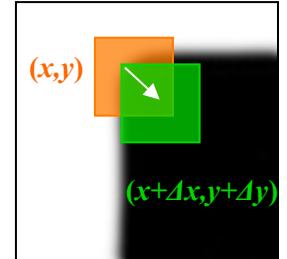


“corner”:
significant change in at
least 2 directions

Corner detection | how do we implement this?

- Two image patches of size P one centered at (x, y) and one centered at $(x + \Delta x, y + \Delta y)$
- The Sum of Squared Differences between them is:

$$SSD(\Delta x, \Delta y) = \sum_{x,y \in P} (I(x, y) - I(x + \Delta x, y + \Delta y))^2$$



- Let $I_x = \frac{\partial I(x, y)}{\partial x}$ and $I_y = \frac{\partial I(x, y)}{\partial y}$. Approximating with a **1st order Taylor expansion**:

$$I(x + \Delta x, y + \Delta y) \approx I(x, y) + I_x(x, y)\Delta x + I_y(x, y)\Delta y$$

- This produces the approximation

$$SSD(\Delta x, \Delta y) \approx \sum_{x,y \in P} (I_x(x, y)\Delta x + I_y(x, y)\Delta y)^2$$

Corner detection | how do we implement this?



zürich

Corner detection | how do we implement this?

- Two image patches of size P one centered at (x, y) and one centered at $(x + \Delta x, y + \Delta y)$
- The Sum of Squared Differences between them is:

$$SSD(\Delta x, \Delta y) = \sum_{x,y \in P} (I(x, y) - I(x + \Delta x, y + \Delta y))^2$$

- Let $I_x = \frac{\partial I(x, y)}{\partial x}$ and $I_y = \frac{\partial I(x, y)}{\partial y}$. Approximating with a **1st order Taylor expansion**:

$$I(x + \Delta x, y + \Delta y) \approx I(x, y) + I_x(x, y)\Delta x + I_y(x, y)\Delta y$$

- This produces the approximation

$$SSD(\Delta x, \Delta y) \approx \sum_{x,y \in P} (I_x(x, y)\Delta x + I_y(x, y)\Delta y)^2$$

$$SSD(\Delta x, \Delta y) \approx \sum_{x,y \in P} (I_x(x, y)\Delta x + I_y(x, y)\Delta y)^2$$

- This can be written in a matrix form as

$$\sum_{x,y \in P} \begin{bmatrix} \Delta x & \Delta y \end{bmatrix} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

$$\Rightarrow SSD(\Delta x, \Delta y) \approx \begin{bmatrix} \Delta x & \Delta y \end{bmatrix} M \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

$$M = \sum_{x,y \in P} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

“Second moment matrix”

Corner detection | interpreting matrix M

- Since M is symmetric $\Rightarrow M = \sum_{x,y \in P} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$
- The Harris (and the “Shi-Tomasi”) detector analyses the **eigenvalues**, λ_1 and λ_2 , to decide if we are in presence of a corner \Rightarrow i.e. look for large intensity changes in at least 2 directions

Corner detection | eigen decomposition

- The **eigenvectors** ν and **eigenvalues** λ of a square matrix A satisfy:

$$A\nu = \lambda\nu$$

- Then ν is an eigenvector of A and λ is the corresponding eigenvalue.
- The eigenvalues are found by solving: $\det(A - \lambda I) = 0$

Corner detection | eigen decomposition

- The **eigenvectors** ν and **eigenvalues** λ of a square matrix A satisfy:

$$A\nu = \lambda\nu$$

- Then ν is an eigenvector of A and λ is the corresponding eigenvalue.
- The eigenvalues are found by solving: $\det(A - \lambda I) = 0$

- In this case, $A = M$ is a 2×2 matrix, so:

$$\det \begin{bmatrix} m_{11} - \lambda & m_{12} \\ m_{21} & m_{22} - \lambda \end{bmatrix} = 0$$

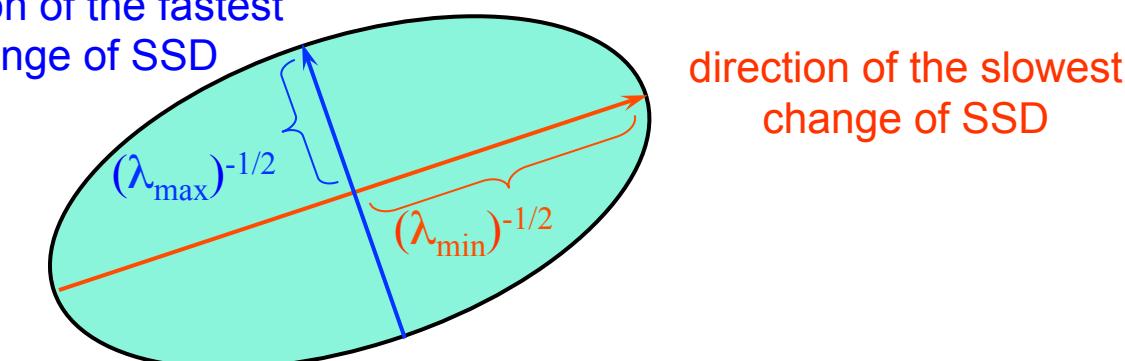
$$\lambda_{1,2} = \frac{1}{2} \left[(m_{11} + m_{22}) \pm \sqrt{4m_{12}m_{21} + (m_{11} - m_{22})^2} \right] = 0$$

- For each λ we can find its corresponding ν (forming the columns in R) by

$$\begin{bmatrix} m_{11} - \lambda & m_{12} \\ m_{21} & m_{22} - \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

Corner detection | interpreting matrix M

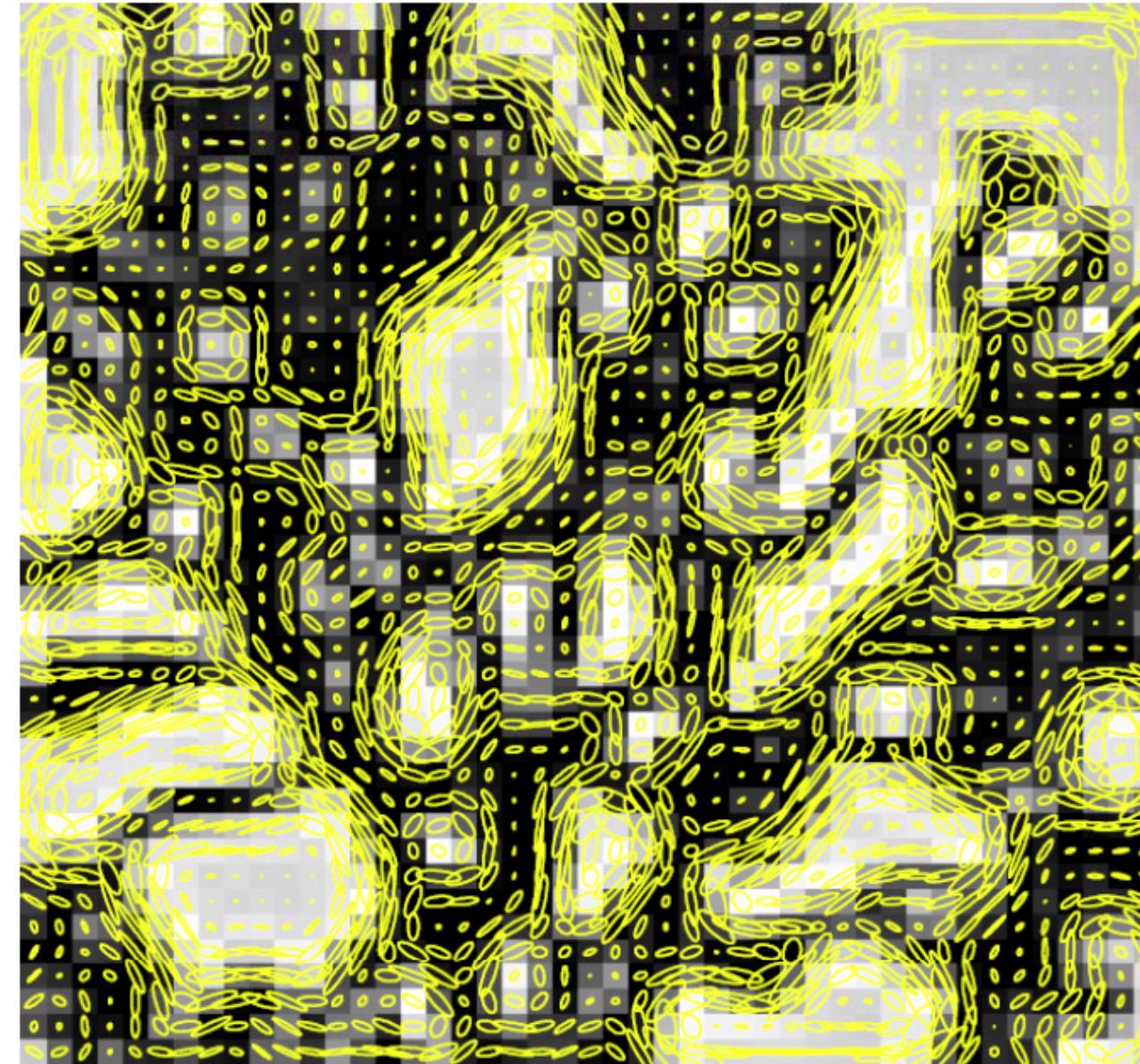
- Since M is symmetric $\Rightarrow M = \sum_{x,y \in P} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$
- The Harris (and the “Shi-Tomasi”) detector analyses the **eigenvalues**, λ_1 and λ_2 , to decide if we are in presence of a corner \Rightarrow i.e. look for large intensity changes in at least 2 directions
- We can visualize $[\Delta x \quad \Delta y] M \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \text{const}$ as an **ellipse** with axis-lengths determined by λ_1 and λ_2 and the axes’ orientations determined by R (i.e. the **eigenvectors** of M)
- The eigenvectors identify the orthogonal directions of largest and smallest changes of SSD



Corner detection | visualization of 2nd moment matrices



Corner detection | visualization of 2nd moment matrices



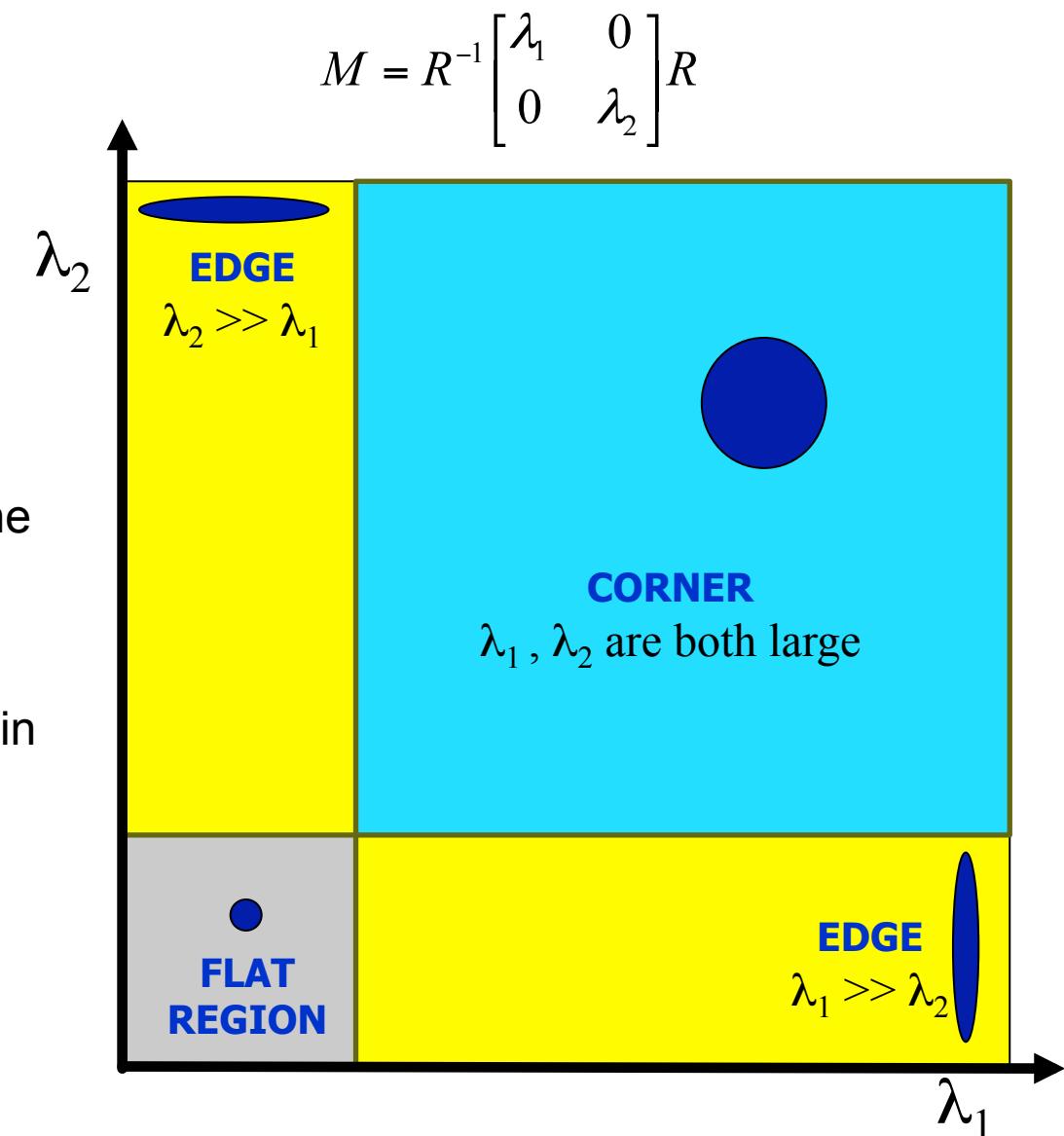
Corner detection | interpreting the eigenvalues

Does patch P describe a corner or not?

- **No structure:** $\lambda_1 \approx \lambda_2 \approx 0$
SSD is almost constant in all directions, so it's a **flat** region
- **1D structure:** $\lambda_1 \gg \lambda_2$ is large (or vice versa)
SSD has a large variation only in one direction, which is the one perpendicular to the **edge**.
- **2D structure:** λ_1, λ_2 are both large
SSD has large variations in all directions and then we are in presence of a **corner**.
- **Shi-Tomasi [1] cornerness criterion:**

$$C_{SHI-TOMASI} = \min(\lambda_1, \lambda_2) > \text{thresh.}$$

[1] J. Shi and C. Tomasi. "Good Features to Track.". CVPR 1994



Corner detection | corner response function

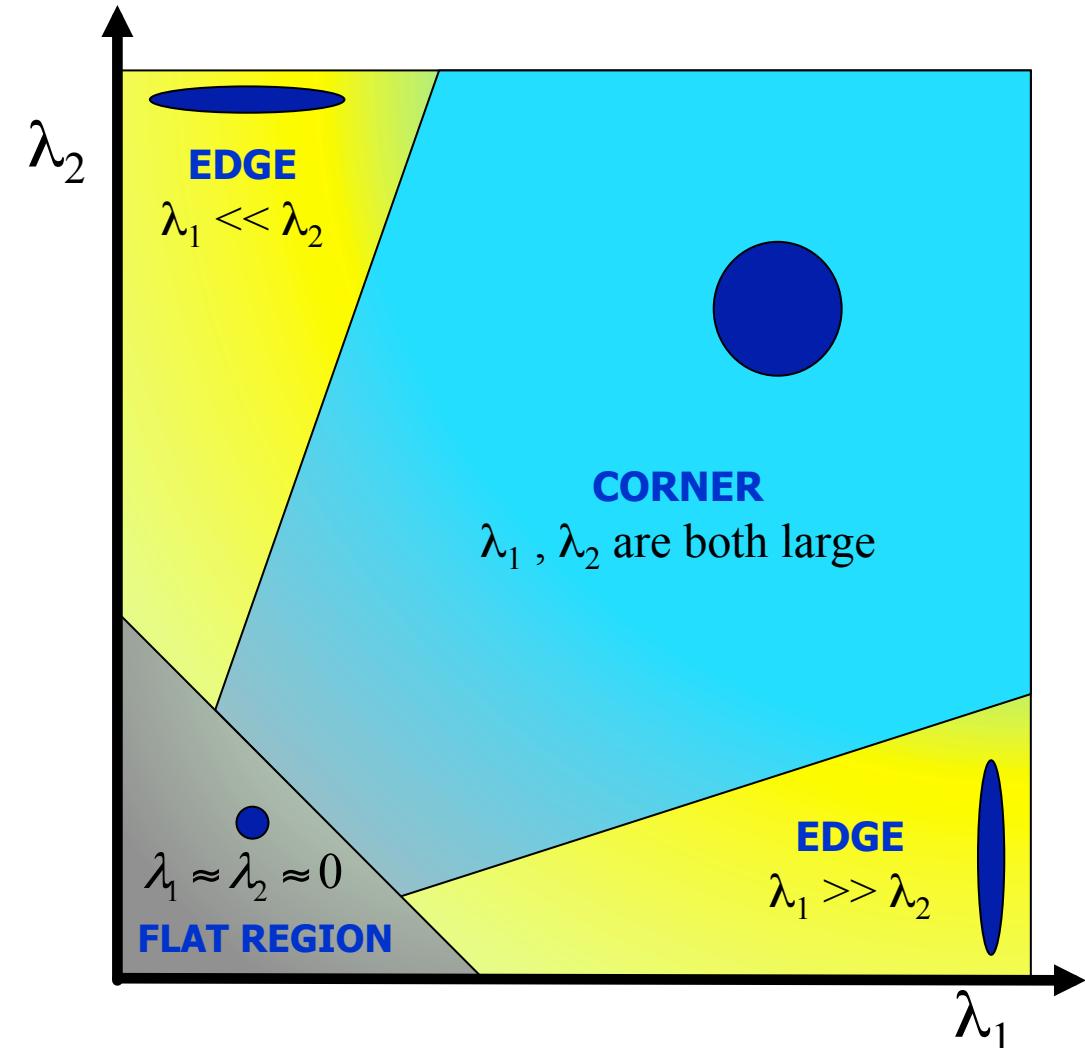
- Computing λ_1 and λ_2 is expensive
 ⇒ Harris & Stephens suggested using a “**cornerness function**” instead:

$$C_{HARRIS} = \lambda_1 \lambda_2 - \kappa(\lambda_1 + \lambda_2)^2 = \det(M) - \kappa \cdot \text{trace}^2(M)$$

where $\kappa = \text{const.} \in [0.04, 0.15]$

- Harris** cornerness criterion [2]
- Last step of Harris corner detector: extract local maxima of the cornerness function

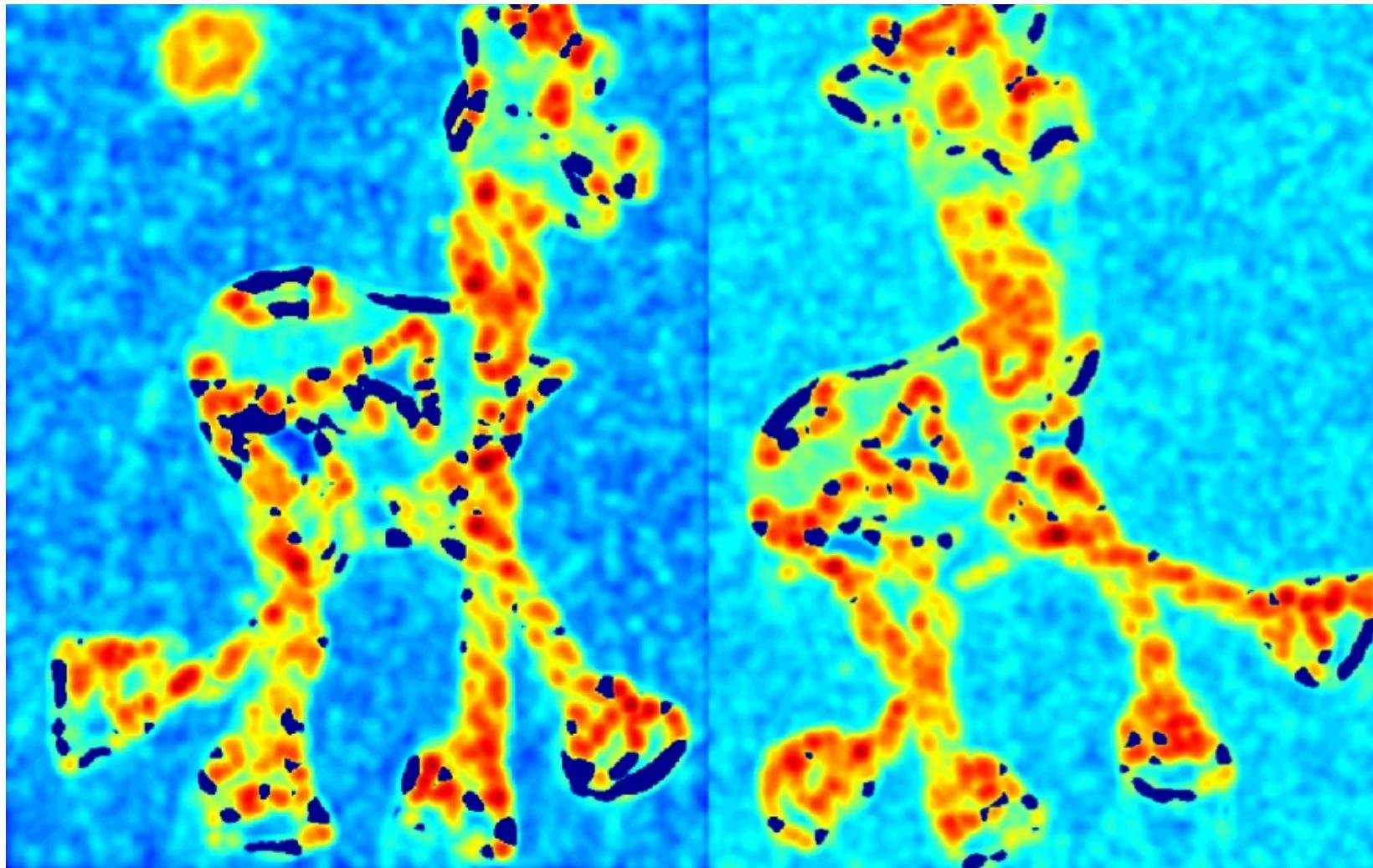
[2] C.Harris and M.Stephens. ["A Combined Corner and Edge Detector."](#),
Proceedings of the Alvey Vision Conference, 1988



Harris corners | workflow



Harris corners | workflow



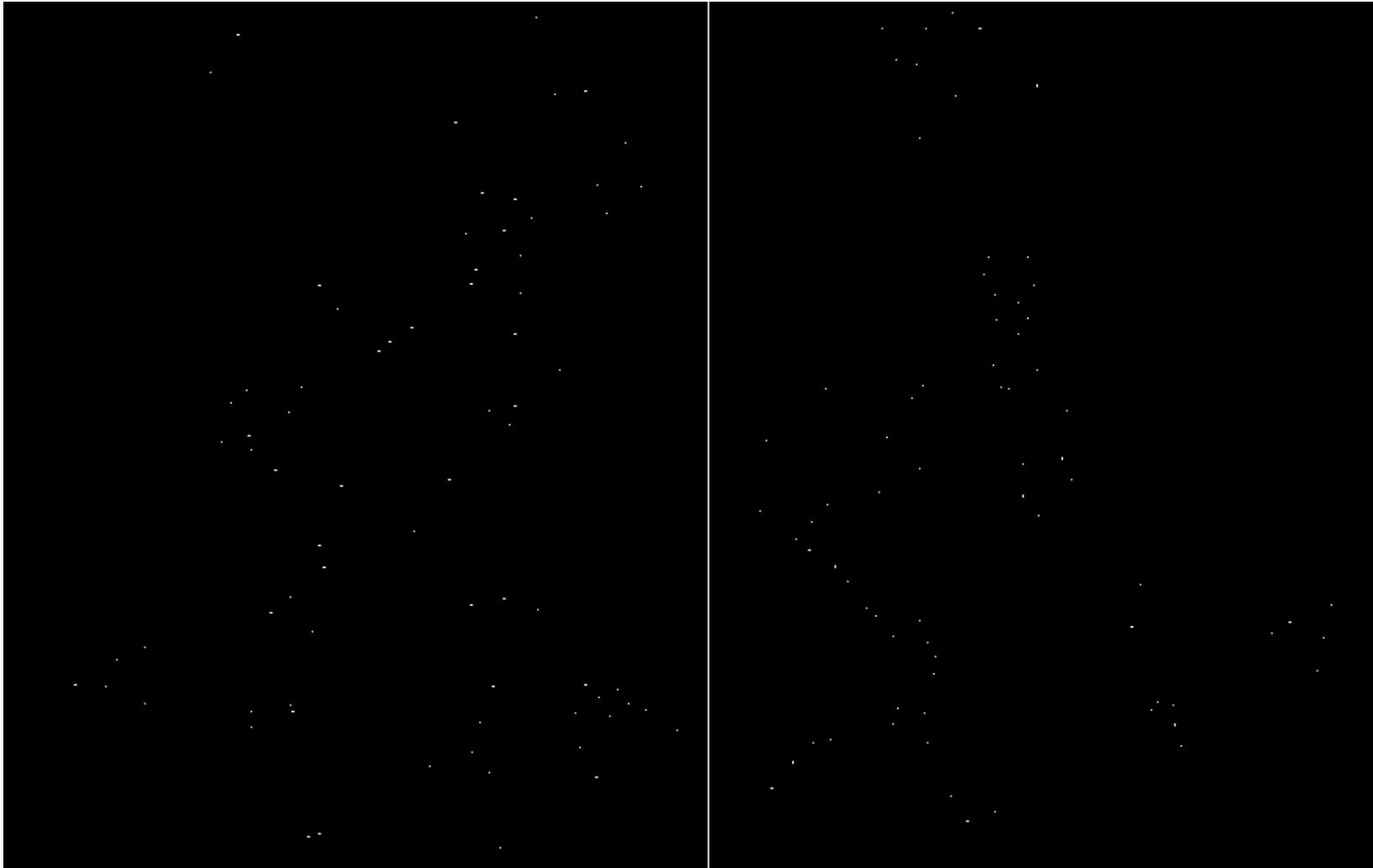
- Compute corner response C

Harris corners | workflow



- Find points with large corner response: $C > \text{threshold}$

Harris corners | workflow



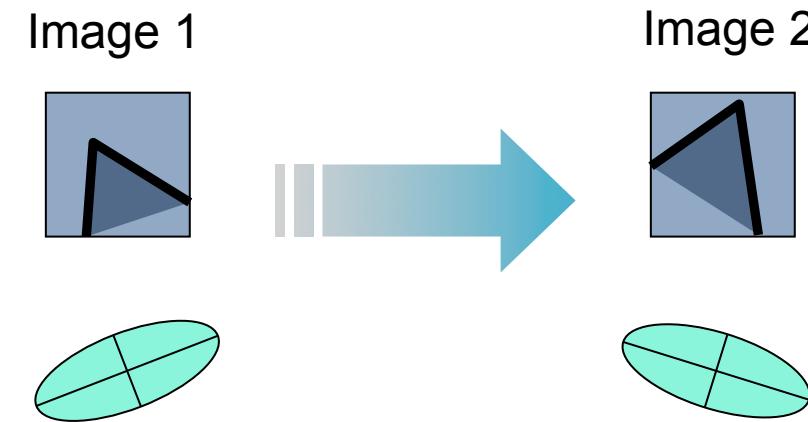
- Take only the points of local maxima of thresholded C

Harris corners | workflow



Harris corners | properties

- Rotation invariance?

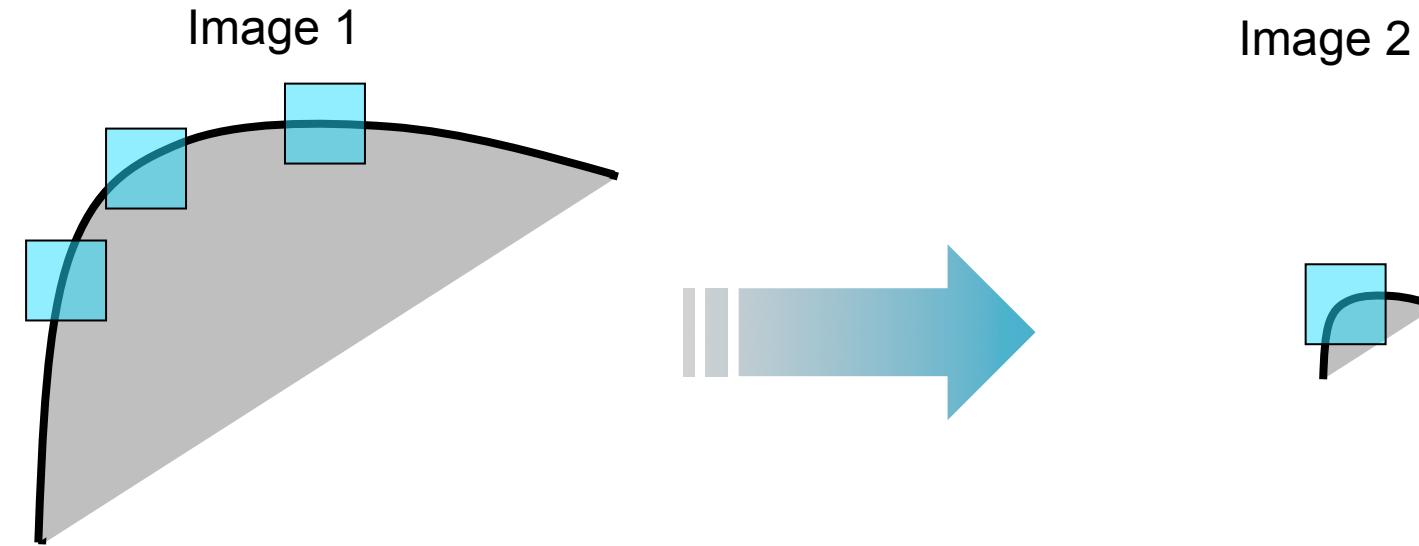


Ellipse rotates, but its shape (i.e. eigenvalues) remains the same

Harris corners are **invariant to image rotation**

Harris corners | properties

- Scale invariance?



All points will be
classified as **edges**

Corner!

Harris corners are **not invariant to scale change**

Harris corners | summary

- Harris detector: probably the most widely used and known corner detector



Invariance to:

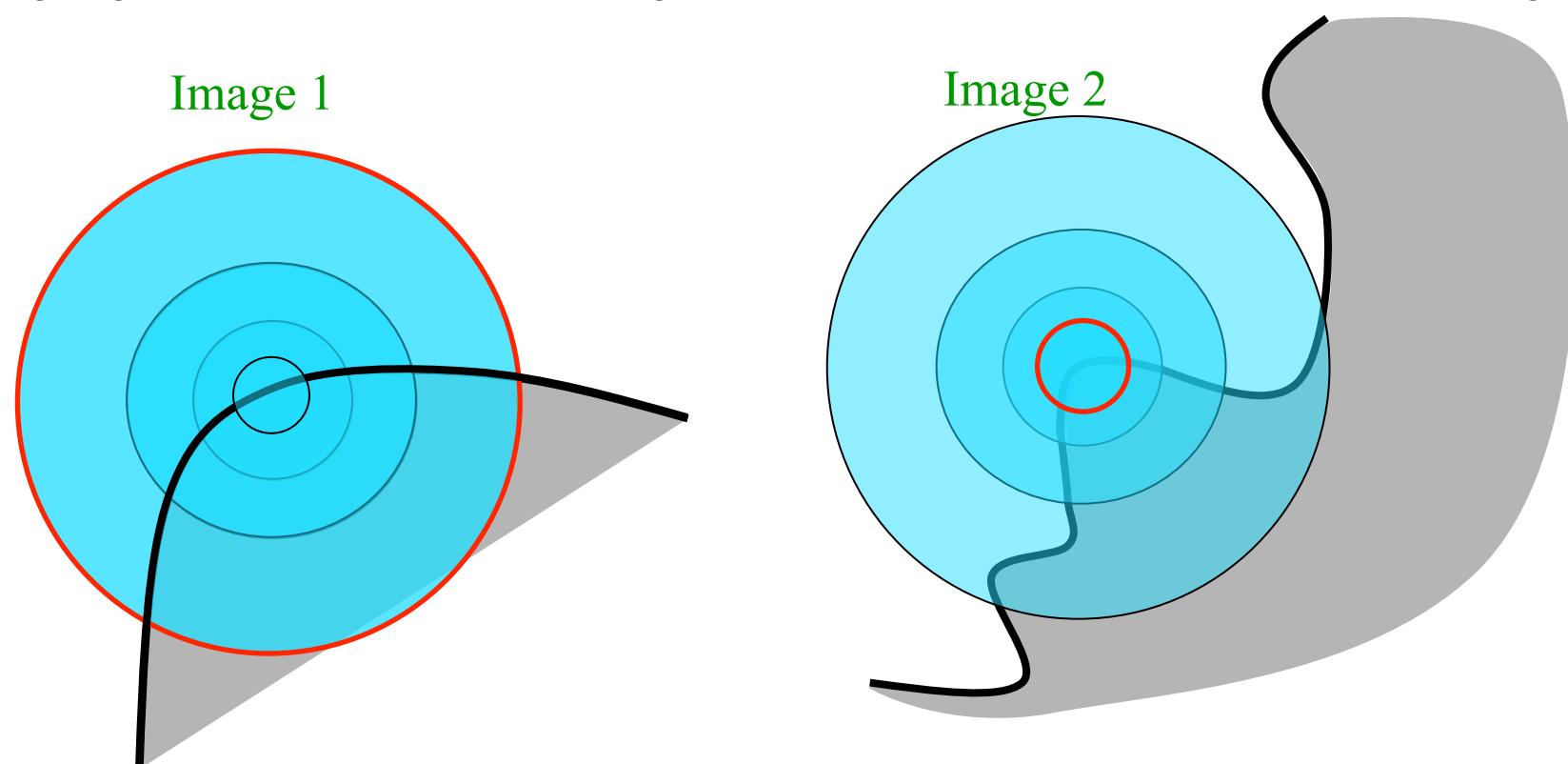
- Rotation (in-plane)
- Linear intensity changes
- note: to make the matching invariant to these we need suitable descriptor & matching

NO invariance to:

- Scale changes
- Geometric affine changes: an image transformation, which distorts the neighborhood of the corner, can distort its ‘cornerness’ response

Scale-invariant feature detection

- Consider regions (e.g. discs) of different sizes around a point
- **Aim:** corresponding regions look the same in image space, when the appropriate scale-change is applied

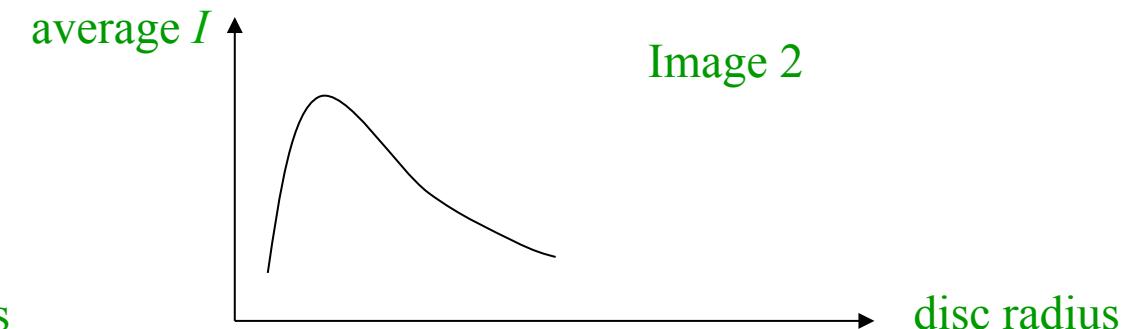
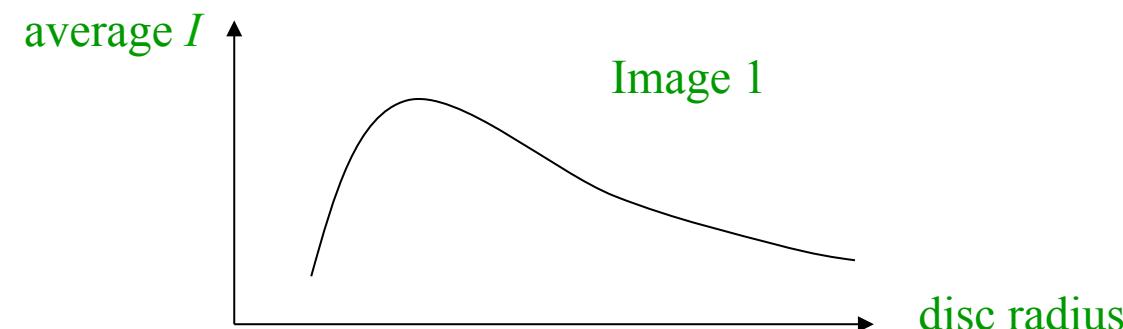
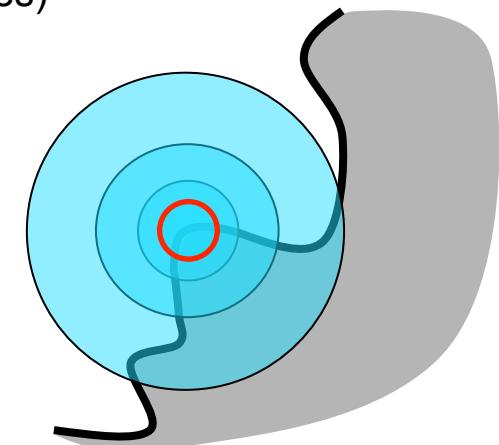
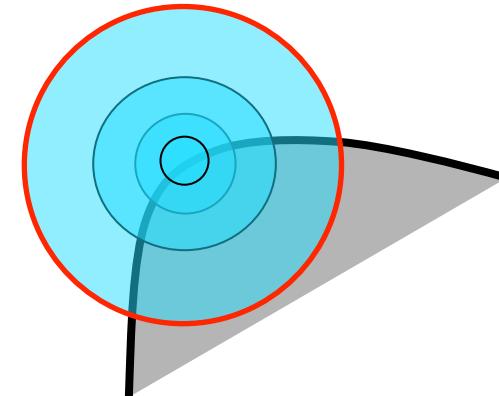


- Choose corresponding regions (discs) **independently** in each image

Scale-invariant feature detection

- Approach: design a function to apply on the region (disc) , which is “scale invariant”
(i.e. remains constant for corresponding regions, even if they are at different scales)

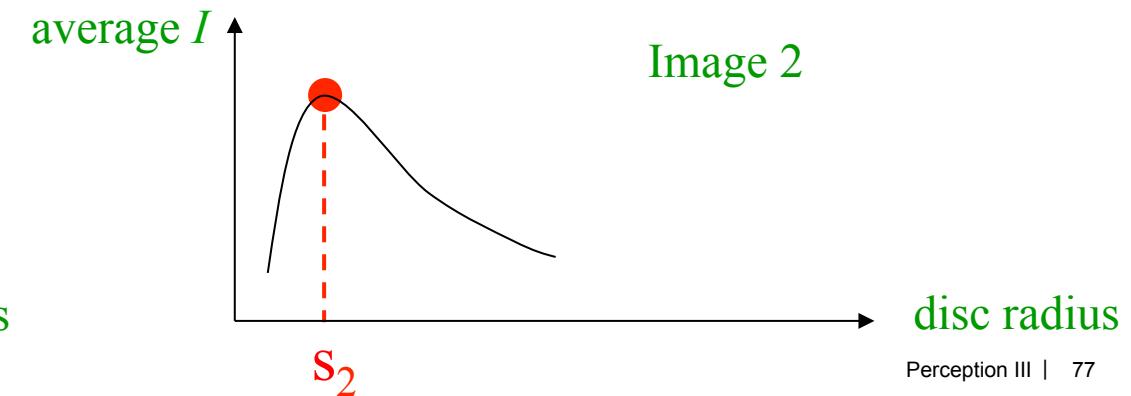
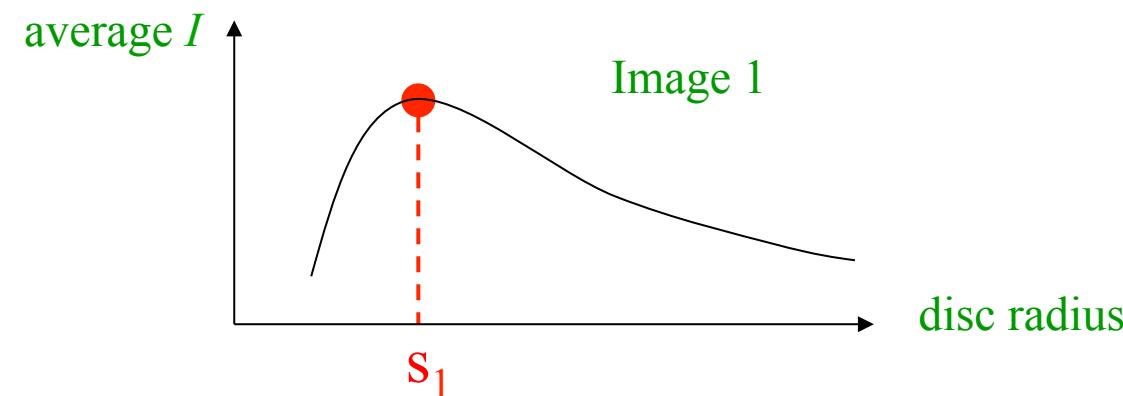
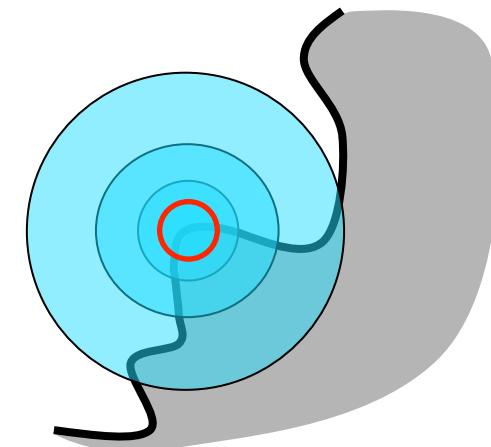
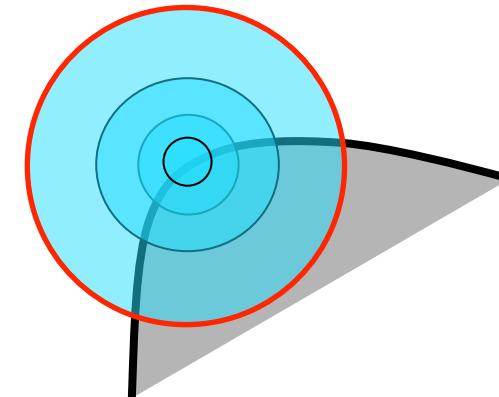
example: **average image intensity** over corresponding regions (at different scales) should remain constant



Scale-invariant feature detection

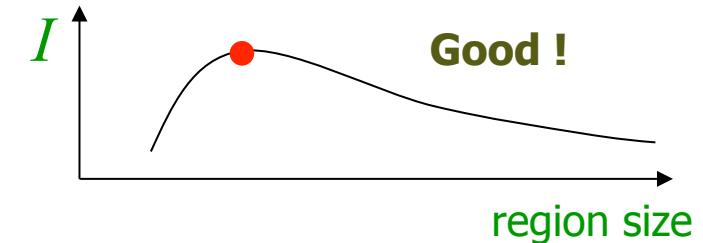
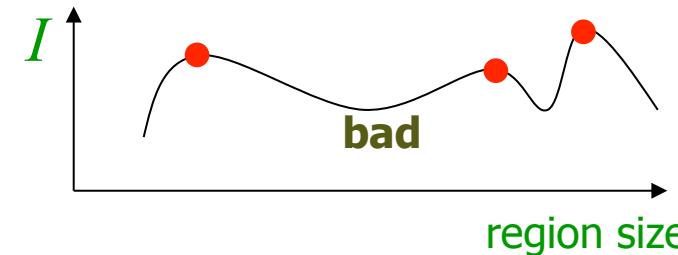
- Identify the local maximum in each response \Rightarrow these occur at corresponding region sizes

The corresponding scale-invariant region size is found in each image *independently*!



Scale-invariant feature detection

- A “good” function for scale detection has one clear, sharp peak



- Sharp, local intensity changes in an image, are good regions to monitor for identifying relative scale in usual images.
⇒ look for blobs or corners (i.e. sharp intensity discontinuities)

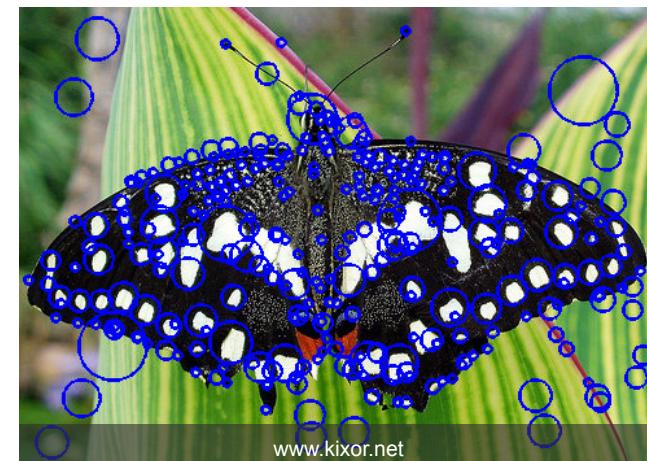
Scale-invariant feature detection | LoG scale detector

- Functions of determining scale: convolve image with kernel to identify sharp intensity discontinuities

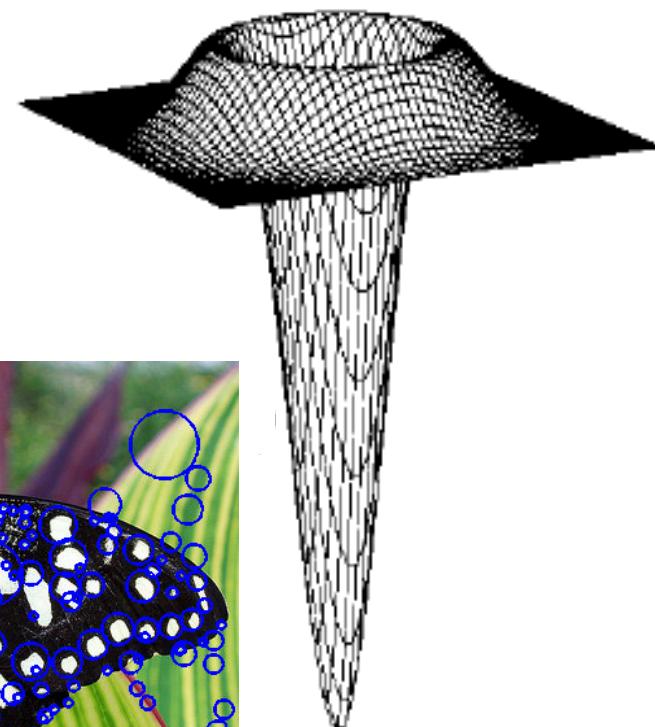
$$f = \text{Kernel} * \text{Image}$$

- Detected scale corresponds to **local maxima or minima** of the convolved image region

$$LoG = \nabla^2 G(x, y) = \frac{\partial^2 G(x, y)}{\partial x^2} + \frac{\partial^2 G(x, y)}{\partial y^2}$$

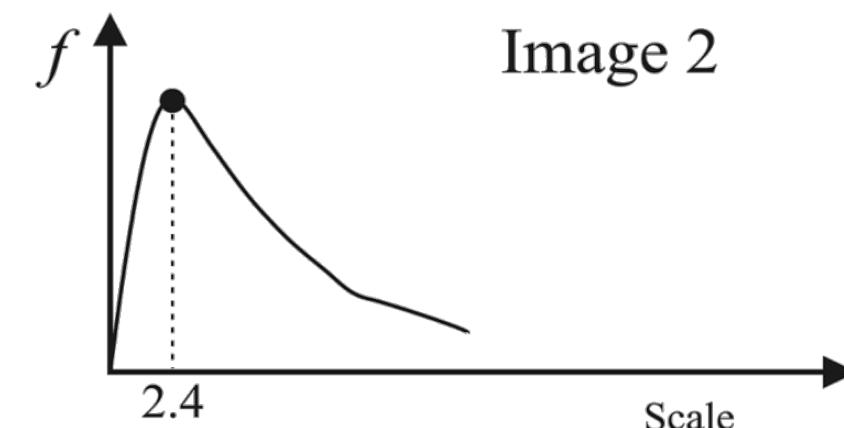
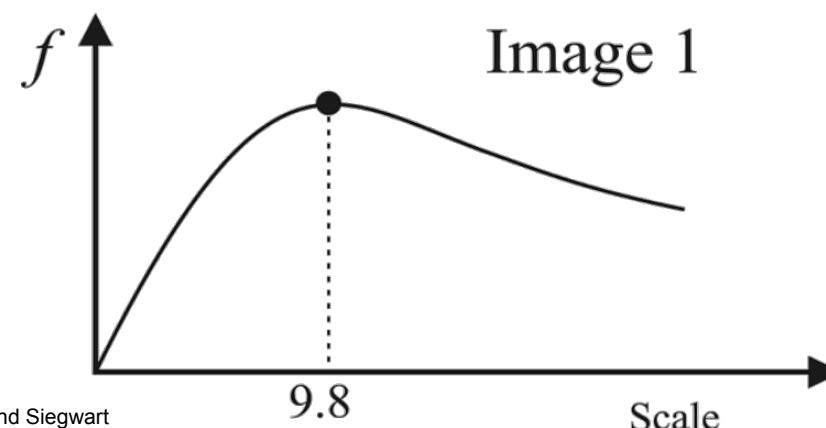


Laplacian of Gaussian



Scale-invariant feature detection | LoG scale detector

- Response of LoG for corresponding regions:



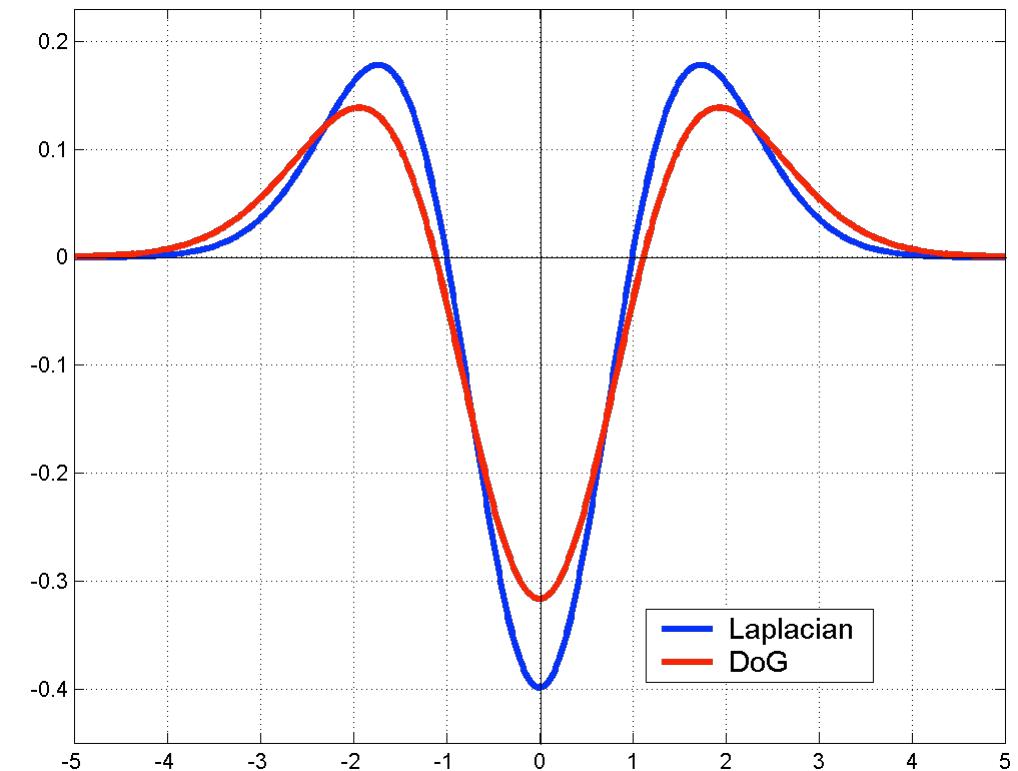
Scale-invariant feature detection | DoG scale detector

- Approximation to the LoG kernel for efficiency:

Difference of Gaussians (DoG) kernel:

$$DoG = G_{k\sigma}(x, y) - G_\sigma(x, y)$$

- Used in the **SIFT** feature detector [Lowe et al., IJCV 2004]
- The **SURF** feature detector [Bay et al, CVIU 2008] implements the DoG kernel using a linear combination of rectangular functions

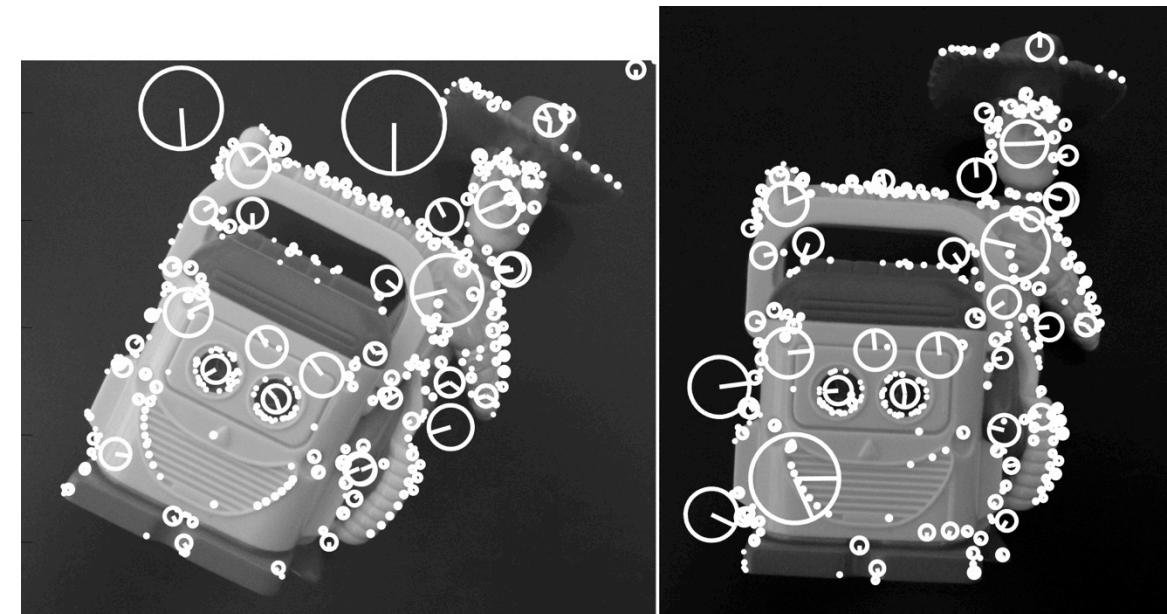


SIFT features [Lowe et al., IJCV 2004]

- **SIFT** = Scale Invariant Feature Transform
an approach for detecting and describing regions of interest in an image
- SIFT detector uses **DoG kernel**, SIFT descriptor is based on **gradient orientations**

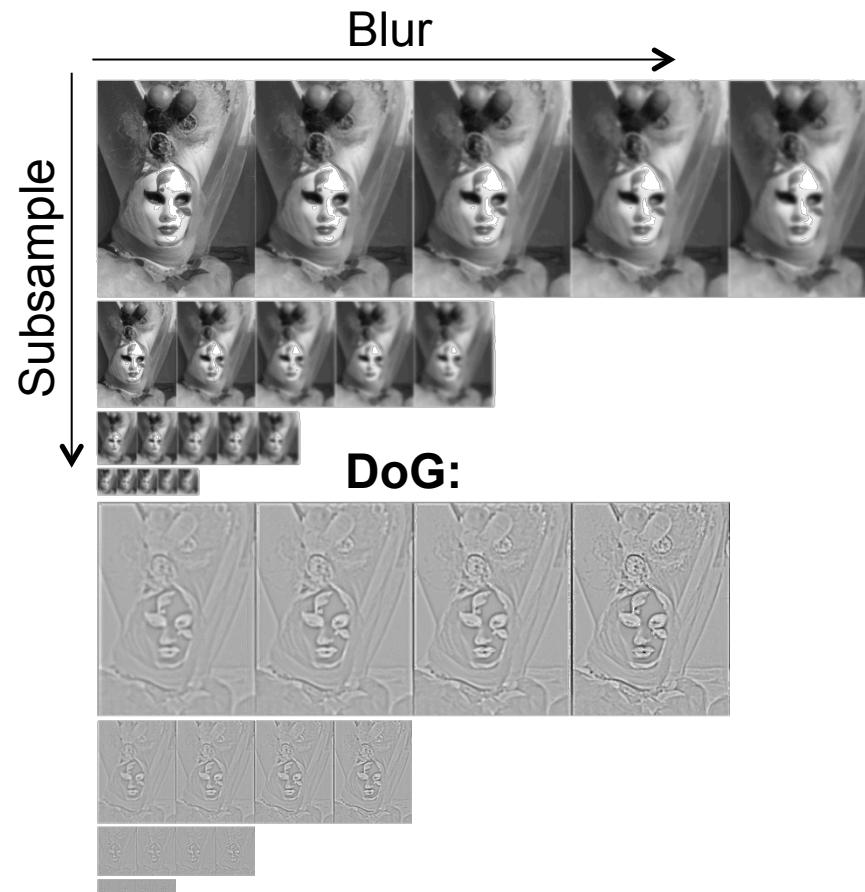
Main SIFT stages:

1. Extract keypoints + scale
2. Assign keypoint orientation
3. Generate keypoint descriptor

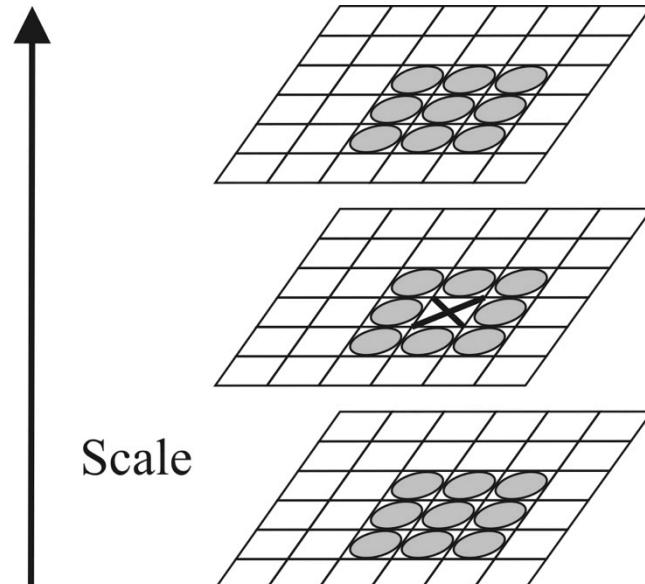


SIFT features | detector (keypoint location + scale)

1. Scale-space pyramid: subsample and blur original image
2. Difference of Gaussians (DoG) pyramid: subtract successive smoothed images



3. Keypoints: local extrema in the DoG pyramid



SIFT features | keypoint orientation assignment

Define “orientation” of keypoint to achieve **rotation invariance**

- Sample intensities around the keypoint
- Compute a histogram of orientations of intensity gradients
- Peaks in histogram: dominant orientations
- **Keypoint orientation = histogram peak**
- If there are multiple candidate peaks, construct a different keypoint for each such orientation

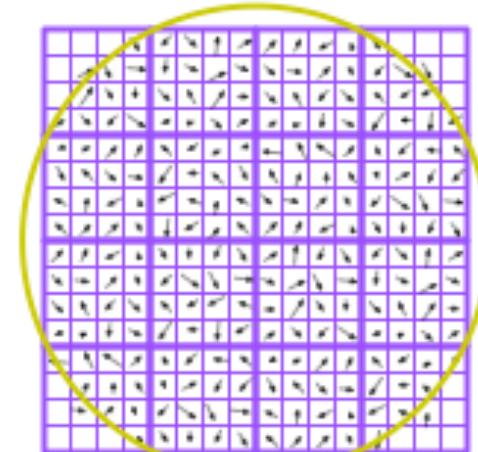
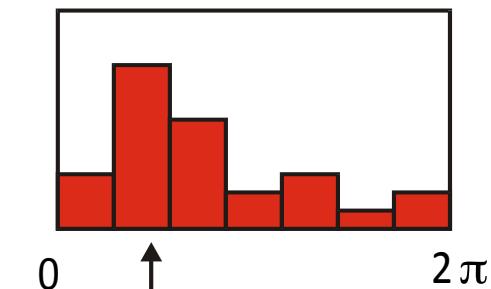
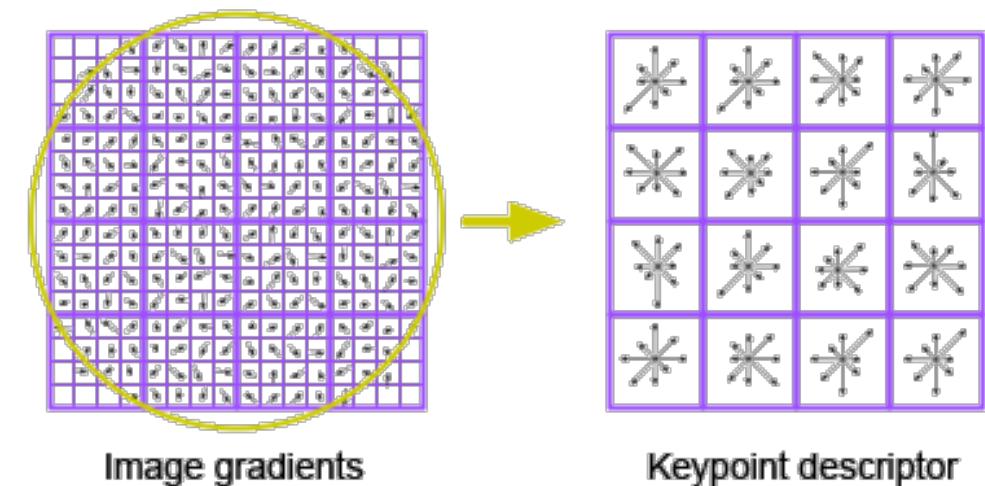


Image gradients

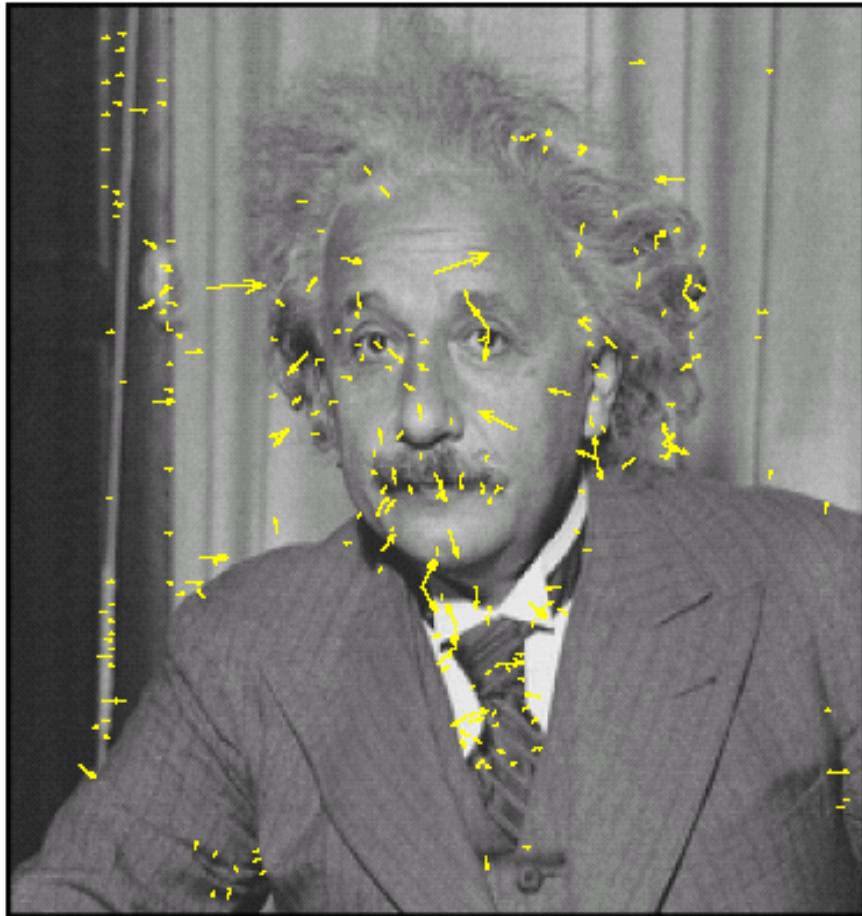


SIFT features | descriptor

- **Descriptor** : “identity card” of keypoint
- Simplest descriptor: matrix of intensity values around a keypoint (image patch)
- Ideally, a descriptor should be
 - highly distinctive +
 - tolerant/invariant to common image transformations
- **SIFT descriptor**: 128-element vector
- Describe all gradient orientations **relative to the keypoint orientation**
- Divide keypoint neighborhood in **4×4** regions and compute orientation histograms along **8** directions
- SIFT descriptor: concatenation of all **4×4×8 (=128)** values
- Descriptor Matching: L_2 -distance (i.e. SSD) between these descriptor vectors



SIFT features



SIFT keypoints with detected orientation & scale

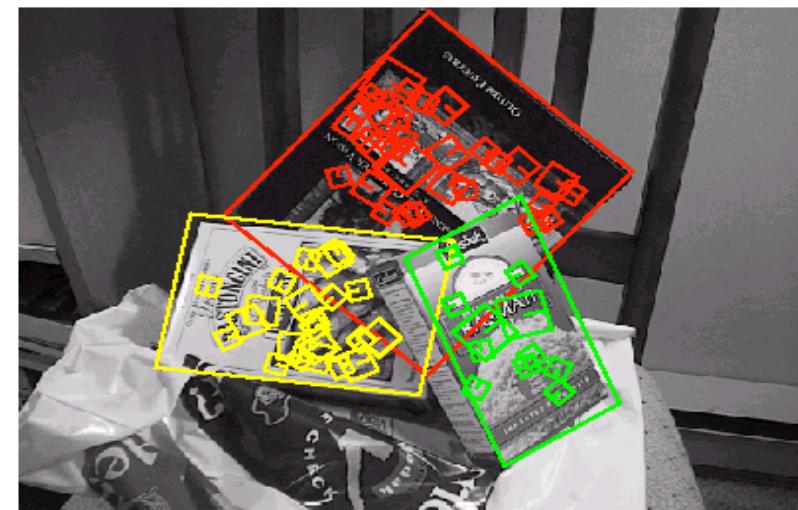
- SIFT features are reasonably **invariant** to changes in:
rotation, scaling, illumination
- Very powerful in capturing + describing **distinctive** structure, but also **computationally demanding**

SIFT features | code and demos

- **SIFT feature detector code:**
 - for Matlab & C code to run with compiled binaries
 - for Win and Linux (freeware)

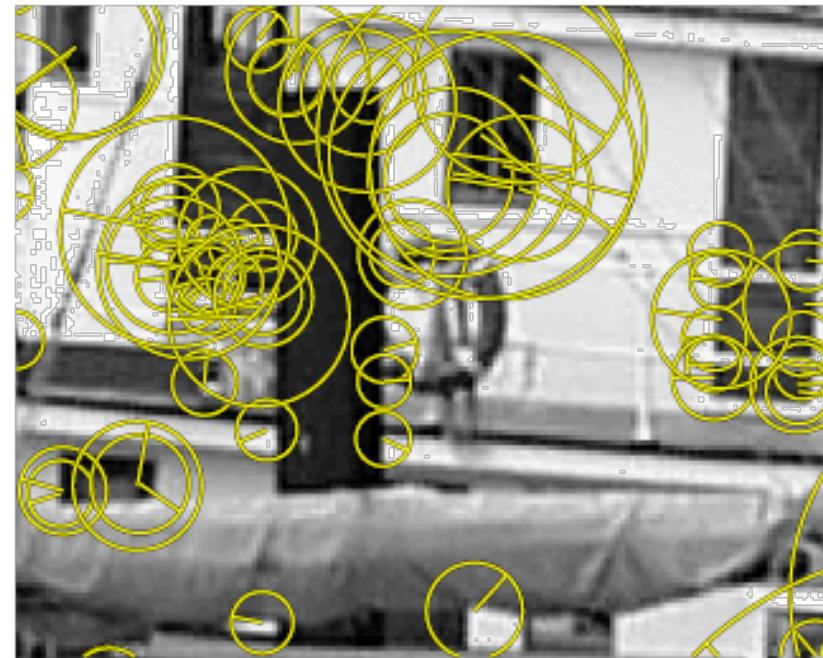
<http://www.cs.ubc.ca/~lowe/keypoints/>
- Make your own panorama with **AUTOSTITCH** (freeware):

<http://matthewwalunbrown.com/autostitch/autostitch.html>



More recent features from SOTA

- ...suitable for Robotics applications



(a) Boat image 1



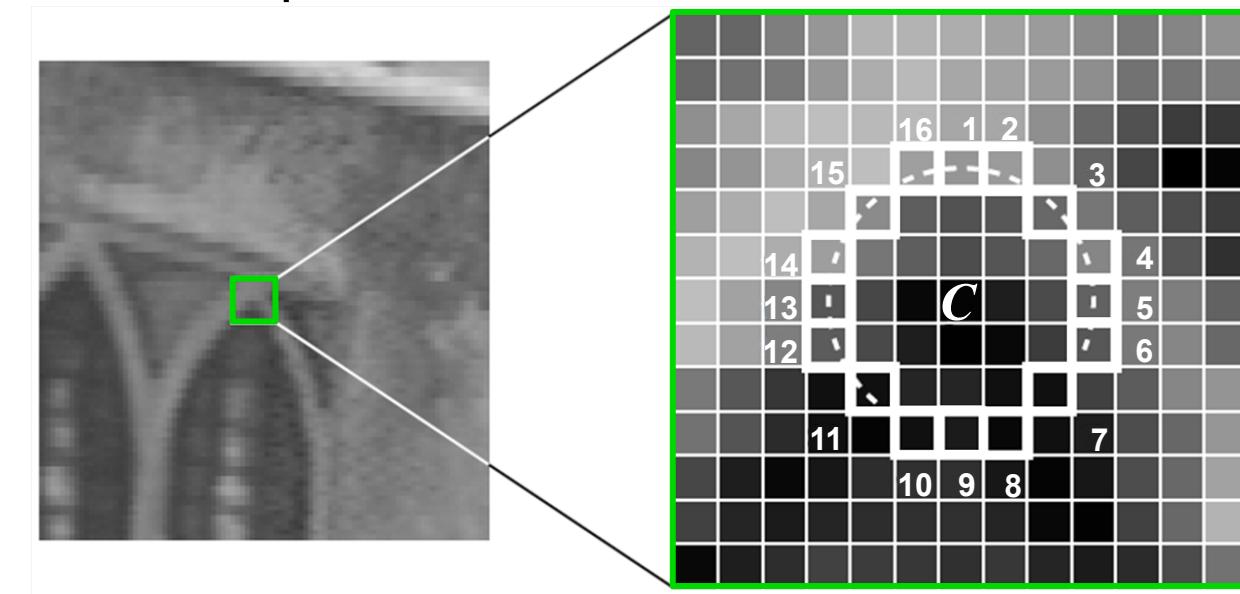
(b) Boat image 2

FAST corner detector

[Rosten, Porter and Drummond, PAMI 2010]

- **FAST:** Features from Accelerated Segment Test
- Studies intensity of pixels on circle around candidate pixel C

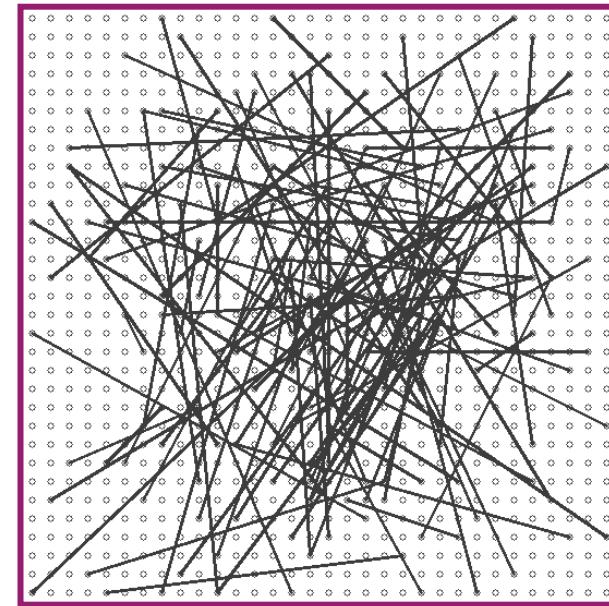
- C is a FAST corner if a set of N contiguous pixels on circle are:
 - all brighter than `intensity_of(C)+threshold`,
 - or
 - all darker than `intensity_of(C)+threshold`



- Typical FAST mask: test for **12** contiguous pixels in a **16-pixel** circle
- **Very fast detector** – in the order of 100 Mega-pixels/second

BRIEF descriptor [Calonder, Lepetit, Strecha and Fua, ECCV 2010]

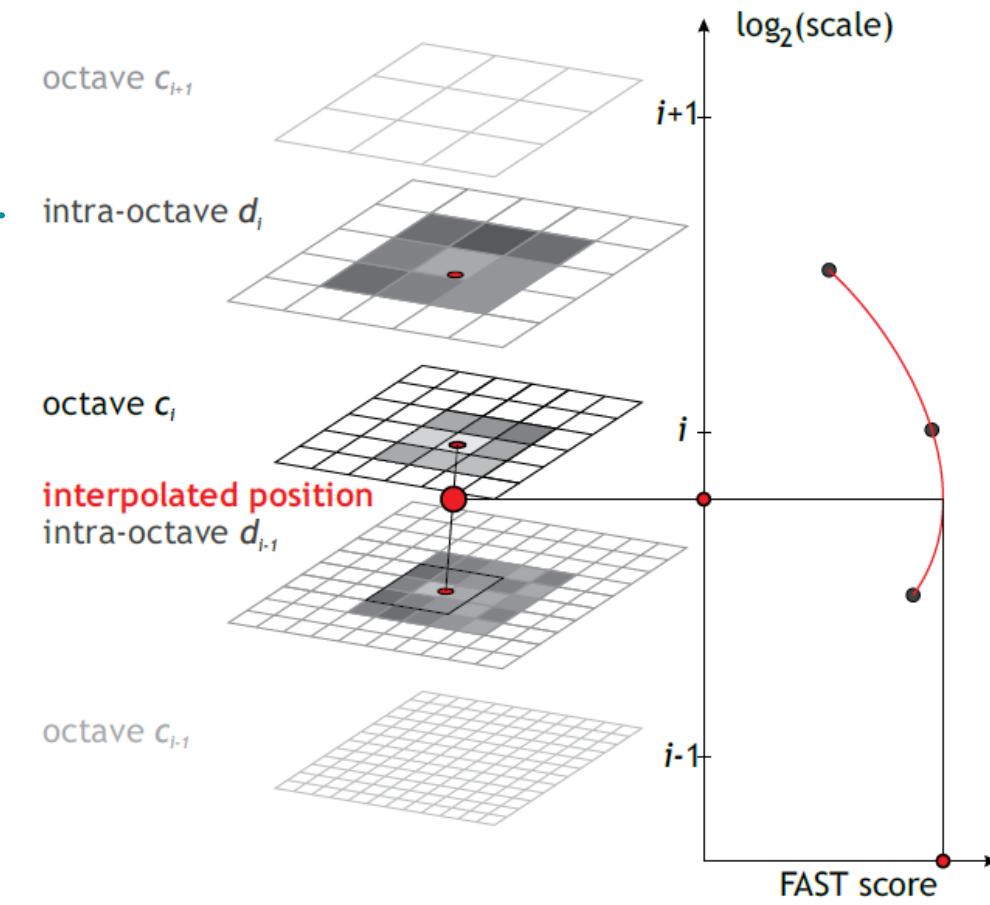
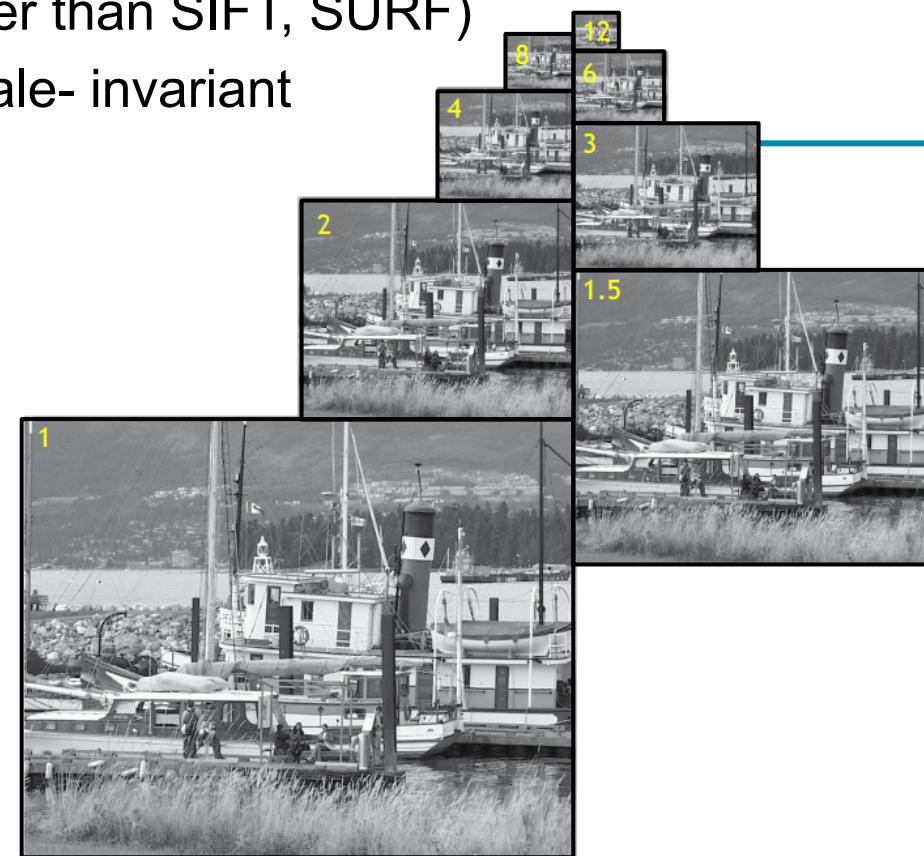
- **BRIEF** : Binary Robust Independent Elementary Features
- Goal: high speed (in description and matching)
- **Binary** descriptor formation:
 - Smooth image
 - **for each** detected keypoint (e.g. FAST),
 - sample** all intensity pairs (I_1, I_2) (typically 256 pairs) according to pattern around the keypoint
 - for each** intensity pair p
 - **if** $I_1 < I_2$ **then set** bit p of descriptor to 1
 - **else set** bit p of descriptor to 0
 - Not scale/rotation invariant (extensions exist...)
 - Allows **very fast** Hamming Distance matching: counting the no. different bits in the descriptors tested



Pattern for intensity pair samples – generated randomly

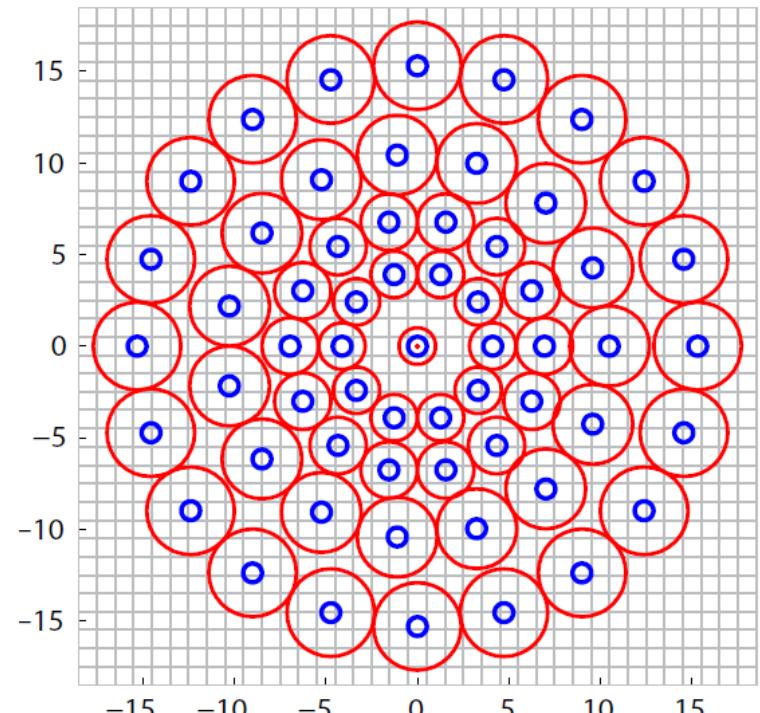
BRISK features [Leutenegger, Chli, Siegwart, ICCV 2011] | detector

- **BRISK: Binary Robust Invariant Scalable Keypoints**
- Detects corners in scale-space based on Harris detection
- High-speed (faster than SIFT, SURF)
- Rotation- and scale- invariant



BRISK features | descriptor

- **Binary**, formed by pairwise intensity comparisons (like BRIEF)
- **Pattern** defines intensity comparisons in the keypoint neighborhood
- **Red circles**: size of the smoothing kernel applied
- **Blue circles**: smoothed pixel value used
- Compare short- and long-distance pairs for orientation assignment & descriptor formation



BRISK sampling pattern

BRISK feature | in action

- Precision – Recall: comparable to SIFT & SURF
- Much faster than SIFT & SURF (10x faster)
- Rotation- and scale- invariant
- Slower than BRIEF, but scale- and rotation-invariant
- BRISK, FAST, BRIEF + many more point detectors and descriptors are publicly available in the [OpenCV library](#)

