Spring 2019

Locomotion Concepts

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Week 2 overview

Overview

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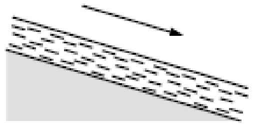
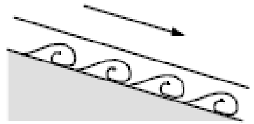
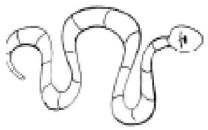
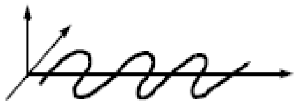
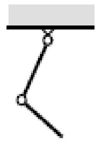
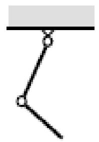
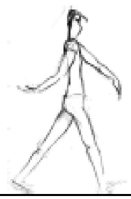
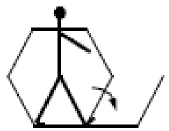
The goal of this week is to get an overview about legged robotic systems and rigid body kinematics of articulated systems. The material consists of three lecture segments, two short worked examples, and one problem set.

- **Legged Robotics:** The first segment provides an introduction to legged robots and a comparison to their natural counterparts. The concepts of static and dynamic stabilty will be covered and a brief outline about the underlying control concepts is given.
- **Basics of Kinematics:** The second segment starts with very basic kinematic tools such as rotations, translations, and homogeneous transformations of multi-body systems.
- **Application of Kinematics:** The third segment introduces the concept of generalized coordinates and Jacobians. This is followed by an explanation of inverse and differential kinematics and finally a discussion of some mobile-platform specific elements.
- **Worked Example 1:** In the first worked example, rotations and homogeneous transformations are applied to a planar 3-link robot arm to describe the robot kinematics.
- **Worked Example 2:** In the second worked example, a position control and trajectory following problem for the 3-link robot arm is analyzed using inverse and differential kinematics.

Locomotion Concepts: Principles Found in Nature

- On ground

Type of motion	Resistance to motion	Basic kinematics of motion
Flow in a Channel 	Hydrodynamic forces	Eddies 
Crawl 	Friction forces	 Longitudinal vibration
Sliding 	Friction forces	 Transverse vibration
Running 	Loss of kinetic energy	Oscillatory movement of a multi-link pendulum 
Jumping 	Loss of kinetic energy	Oscillatory movement of a multi-link pendulum 
Walking 	Gravitational forces	Rolling of a polygon (see figure 2.2) 

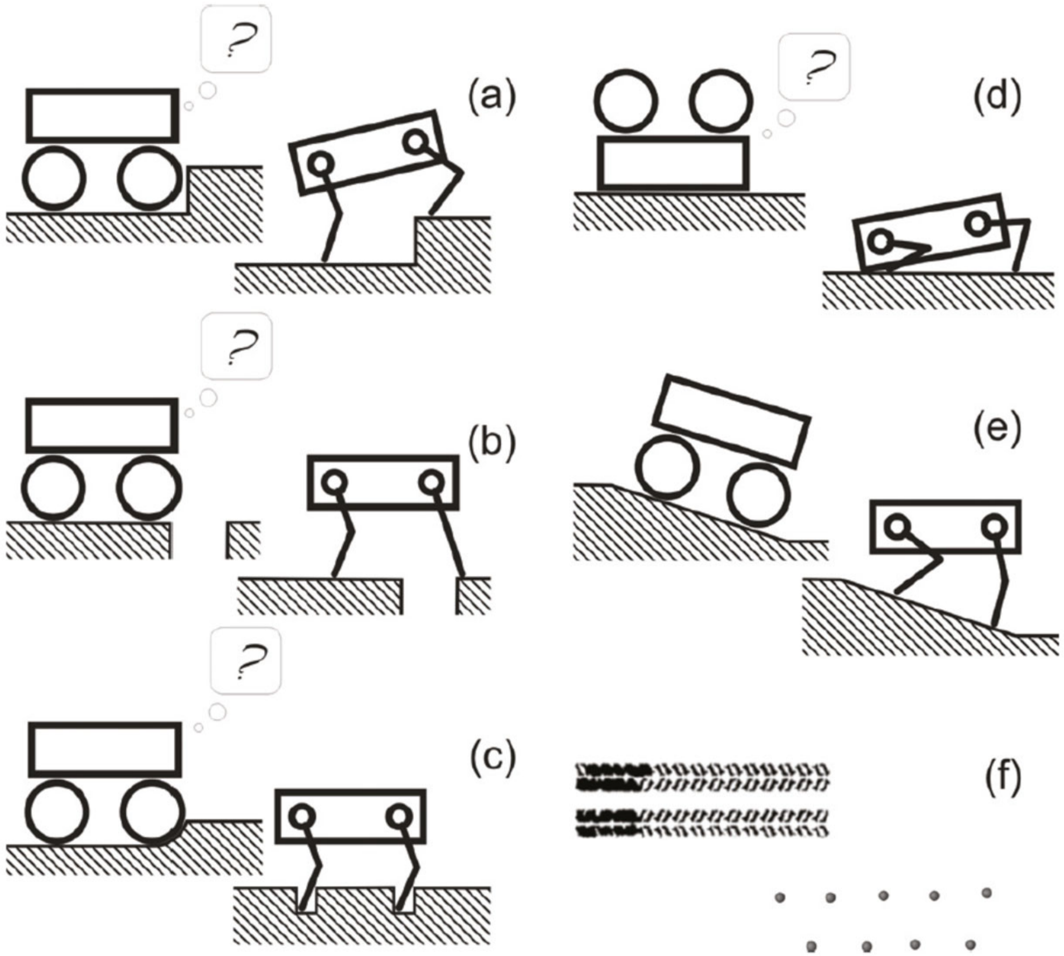
Locomotion Concepts

- Nature came up with a multitude of locomotion concepts
  - Adaptation to environmental characteristics
  - Adaptation to the perceived environment (e.g. size)
- Concepts found in nature
  - Difficult to imitate technically
  - Do not employ wheels
  - Sometimes imitate wheels (bipedal walking)
  - The smaller living creatures are, the more likely they fly
- Most technical systems today use wheels or caterpillars
  - Legged locomotion is still mostly a research topic

Characterization of locomotion concept

- Locomotion
  - physical interaction between the vehicle and its environment.
- Locomotion is concerned with interaction forces, and the mechanisms and actuators that generate them.
- The most important issues in locomotion are:
  - **stability**
    - number of contact points
    - center of gravity
    - static/dynamic stabilization
    - inclination of terrain
  - **characteristics of contact**
    - contact point or contact area
    - angle of contact
    - friction
  - **type of environment**
    - structure
    - medium (water, air, soft or hard ground)

Why legged robots?

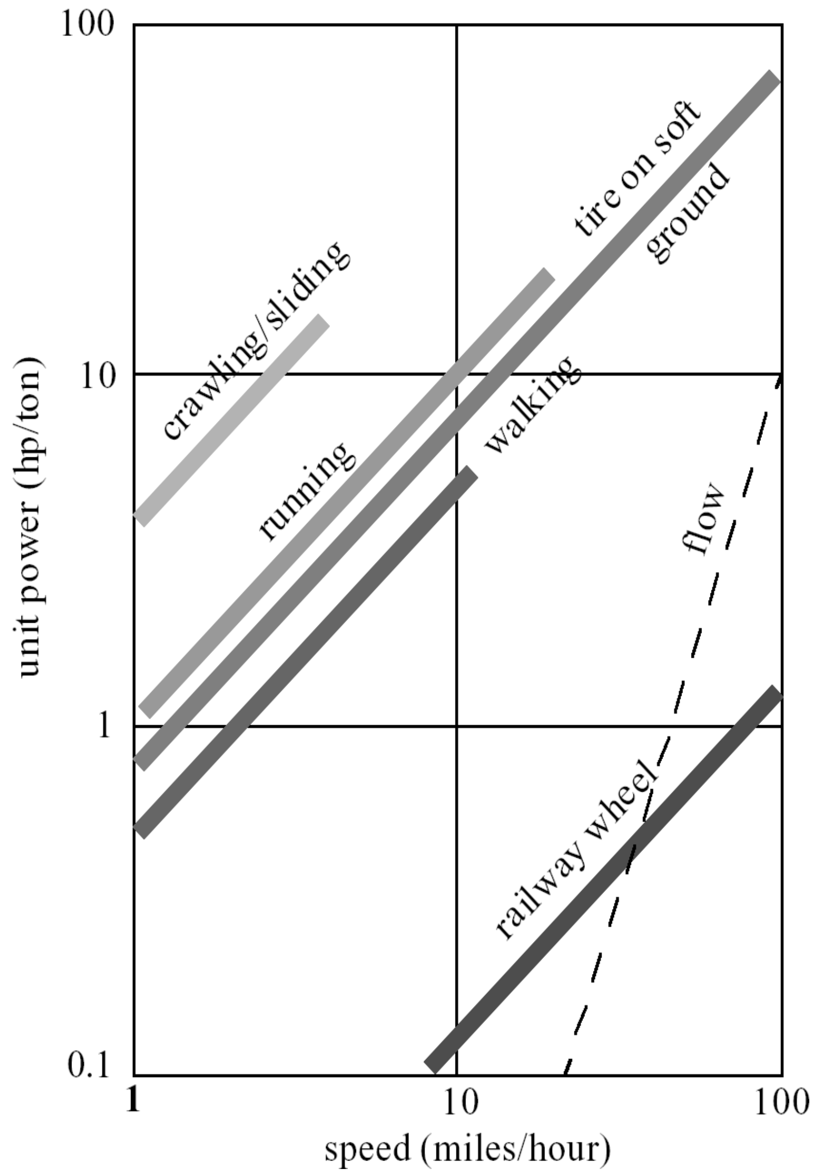


- Legged systems can overcome many obstacles that are not reachable by wheeled systems!

- But it is quite hard to achieve this since
  - many DOFs must be **controlled** in a coordinated way
  - the robot must **see** detailed elements of the terrain

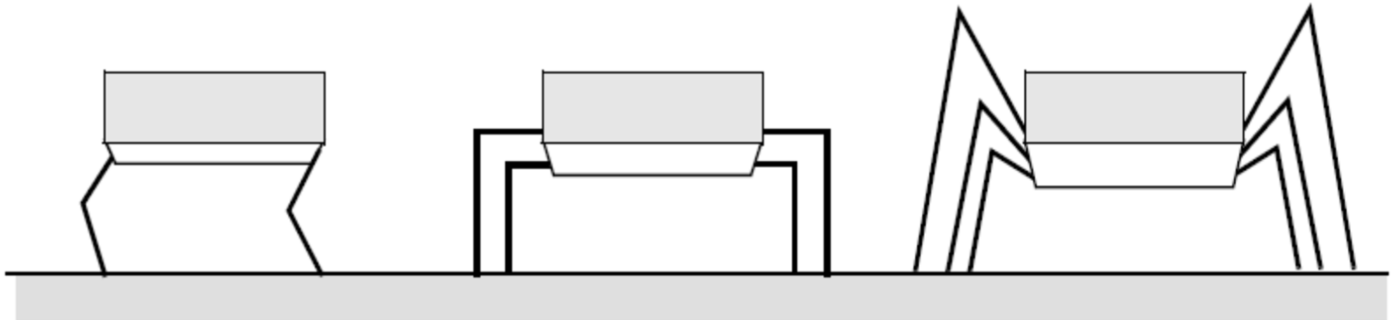
Walking or rolling?

- number of actuators
- structural complexity
- control expense
- energy efficient
  - terrain (flat ground, soft ground, climbing..)
- movement of the involved masses
  - walking / running includes up and down movement of COG
  - some extra losses



Mobile Robots with legs (walking machines)

- The fewer legs the more complicated becomes locomotion
  - Stability with point contact- at least three legs are required for static stability
  - Stability with surface contact – at least one leg is required
- During walking some (usually half) of the legs are lifted
  - thus loosing stability?
- For static walking at least 4 (or 6) legs are required
  - Animals usually move two legs at a time
  - Humans require more than a year to stand and then walk on two legs.



-

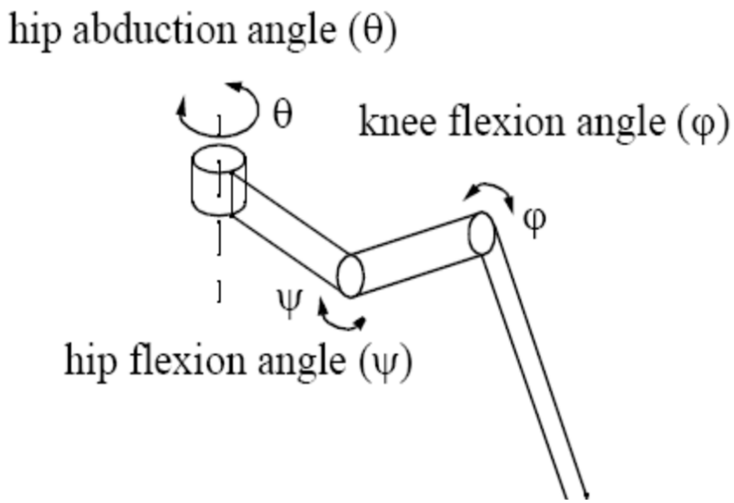
mammals
two or four legs

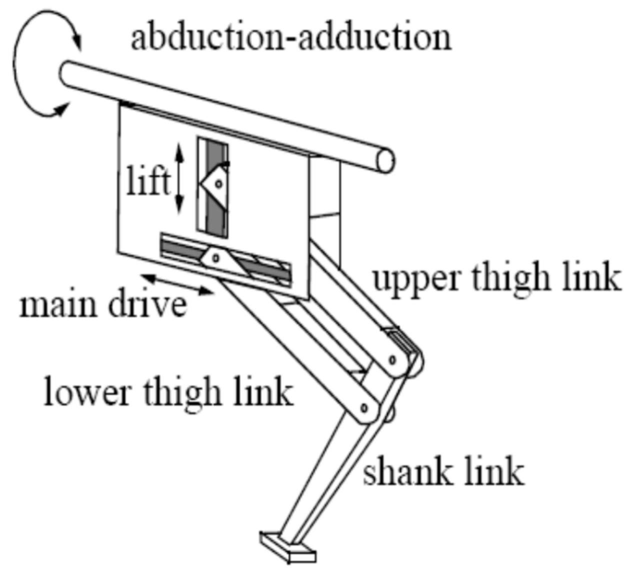
reptiles
four legs

insects
six legs

Number of Joints of Each Leg (DOF: degrees of freedom)

- A minimum of two DOF is required to move a leg forward
  - a *lift* and a *swing* motion.
  - Sliding-free motion in more than one direction not possible
- Three DOF for each leg in most cases (e.g. pictured below)
- 4th DOF for the ankle joint
  - might improve walking and stability
  - additional joint (DOF) increases the complexity of the design and especially of the locomotion control.





The number of distinct event sequences (gaits)

- The gait is characterized as the distinct sequence of ***lift and release events*** of the individual legs
  - it depends on the number of legs.
  - the number of possible events N for a walking machine with k legs is:

$$N = (2k-1)!$$

- For a biped walker (k=2) the number of possible events N is:

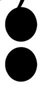
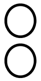
$$N = (2k-1)! = 3! = 3 \cdot 2 \cdot 1 = 6$$

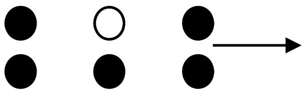
- For a robot with 6 legs (hexapod) N is already

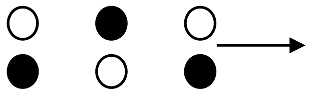
$$N = 11! = 39'916'800$$

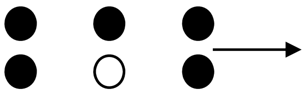
The number of distinct event sequences for biped:

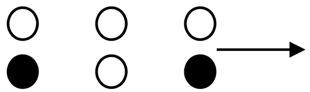
● *Leg down*
○ *Leg up*

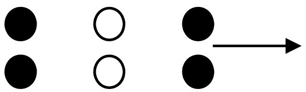
- With two legs (biped) one can have four different states
  - 1) Both legs down 
  - 2) Right leg down, left leg up 
  - 3) Right leg up, left leg down 
  - 4) Both leg up 
- A distinct event sequence can be considered as a change from one state to another and back.
- So we have the following $N = (2k-1)! = 6$ distinct event sequences (change of states) for a biped:

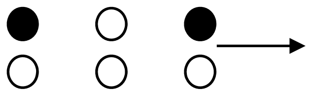
1 -> 2 -> 1  *turning on right leg*

2 -> 3 -> 2  *walking running*

1 -> 3 -> 1  *turning on left leg*

2 -> 4 -> 2  *hopping right leg*

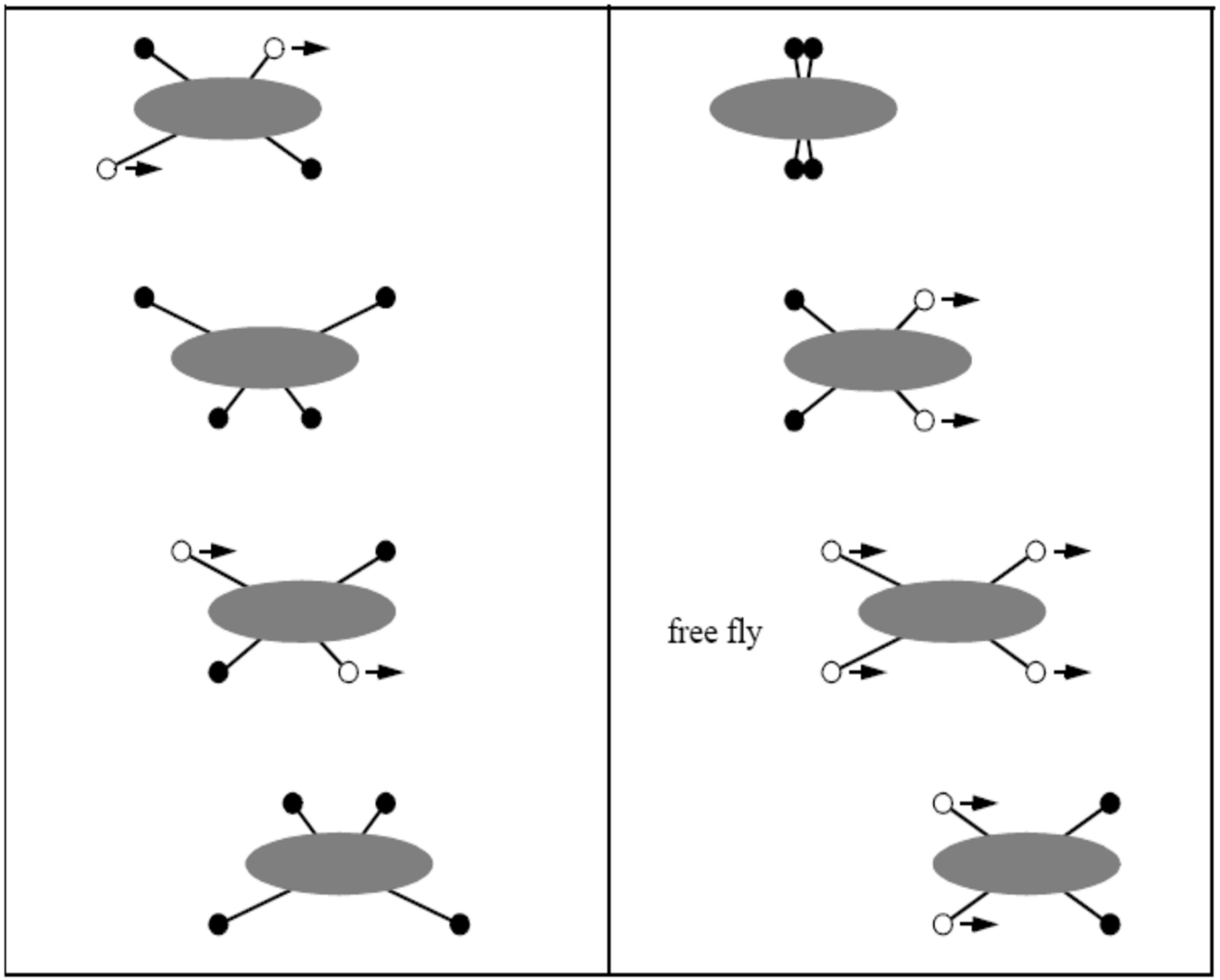
1 -> 4 -> 1  *hopping with two legs*

3 -> 4 -> 3  *hopping left leg*

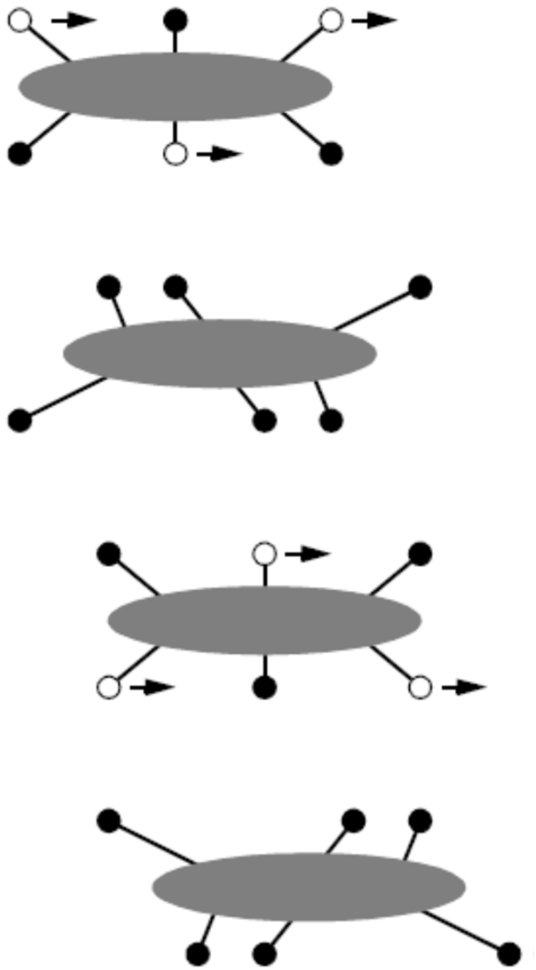
Most Obvious Natural Gaits with 4 Legs are Dynamic

- Changeover Walking

- Galloping

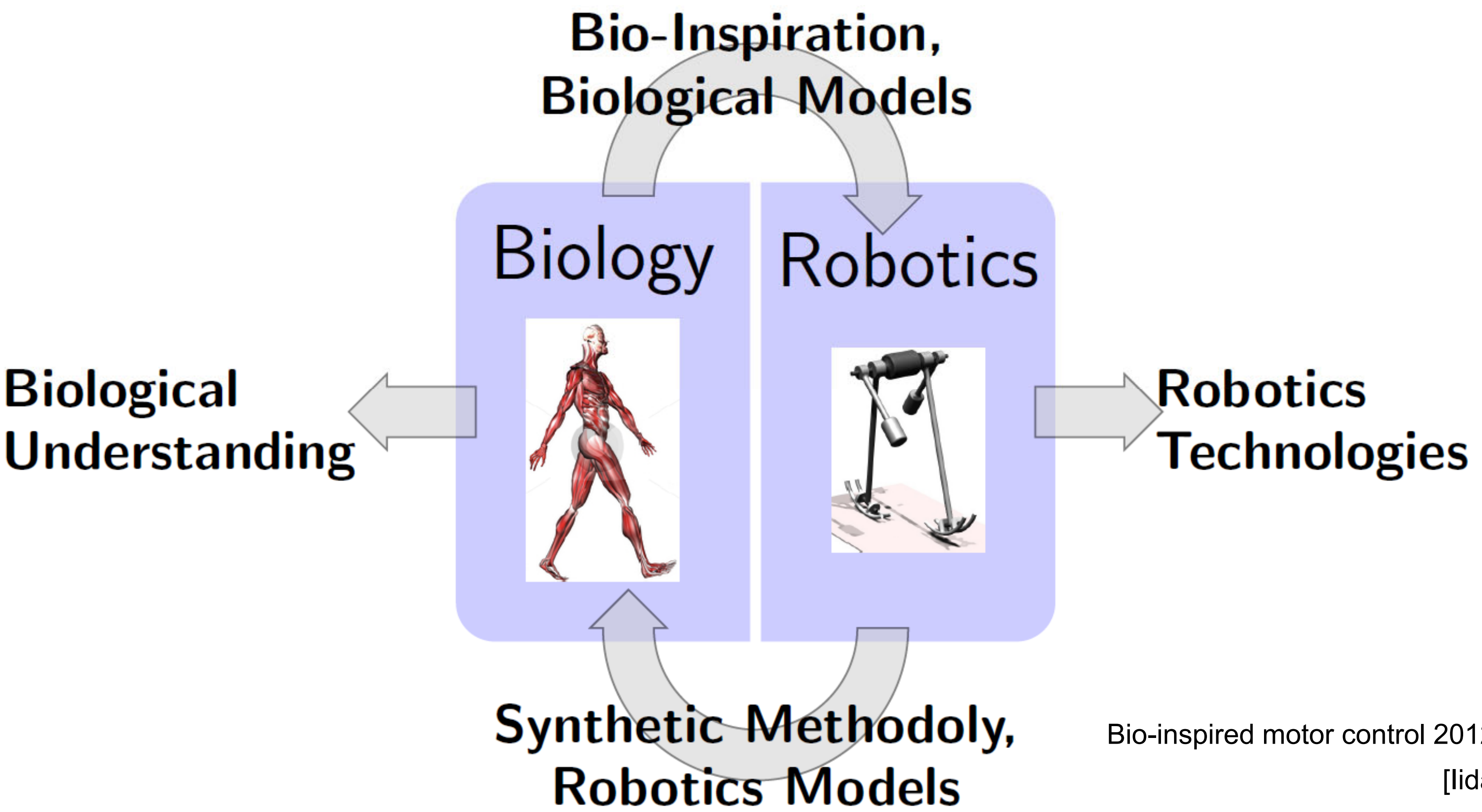


Most Obvious Gait with 6 Legs is Static



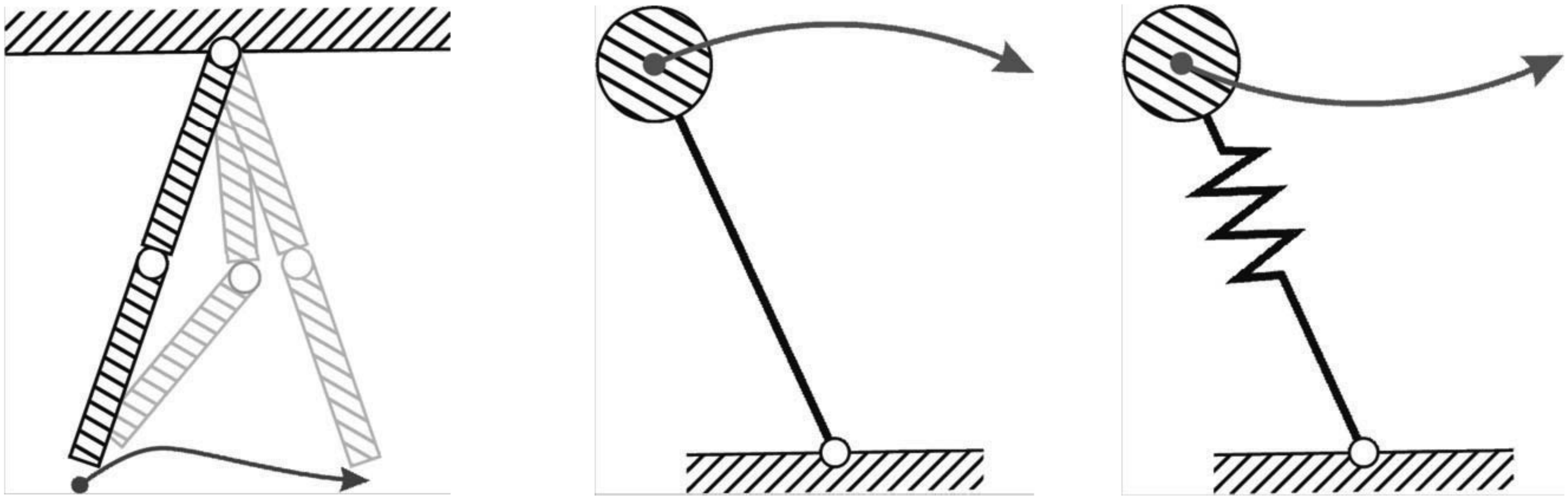
Muybridge 1878

Legged Robotics
Learning from Nature



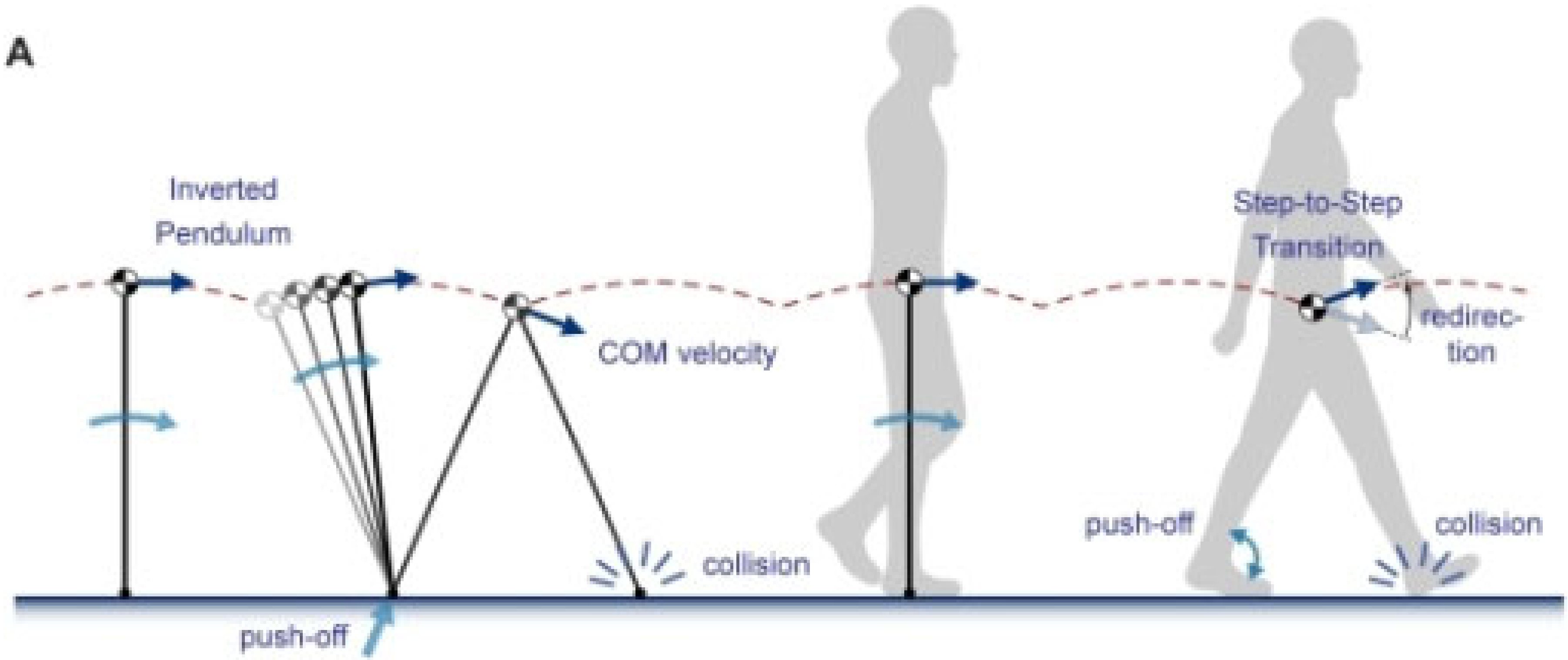
Bio-inspired motor control 2012
[Iida]

Fundamental principles behind locomtion

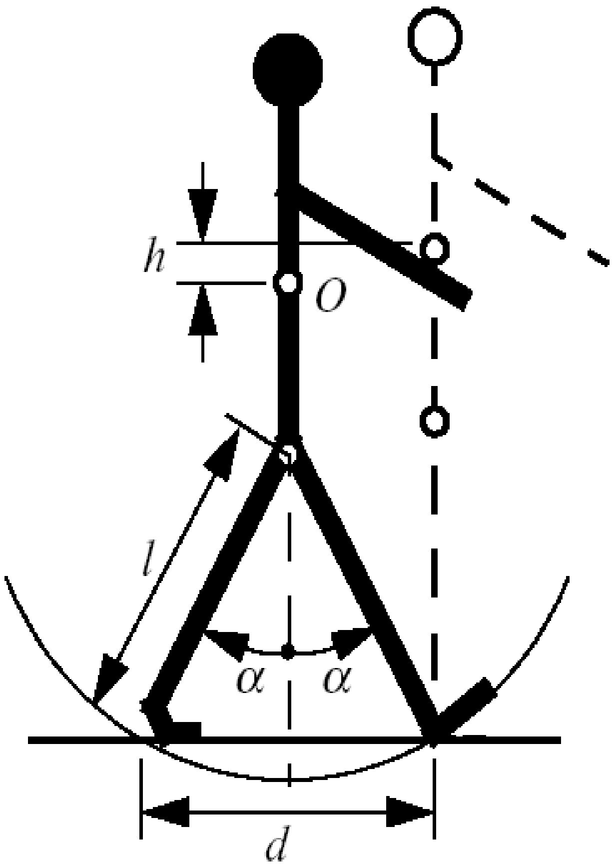


Static walking principles
Inverted Pendulum

- Static walking can be represented by inverted pendulum



Biped Walking



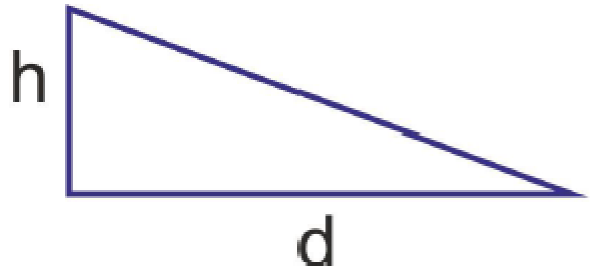
- Biped walking mechanism
  - not too far from real rolling
  - rolling of a polygon with side length equal to the length of the step
  - the smaller the step gets, the more the polygon tends to a circle (wheel)
- But…
  - rotating joint was not invented by nature
  - work against gravity is required
  - more detailed analysis follows later in this presentation

Static walking principles
Inverted Pendulum

- Static walking can be represented by inverted pendulum
- Exploit this in so-called passive dynamic walkers

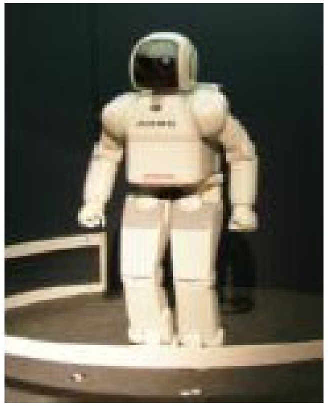
Energetically very efficient

$$COT = \frac{E_{used}}{m \cdot g \cdot d} = \frac{m \cdot g \cdot h}{m \cdot g \cdot d} = \frac{h}{d}$$



Efficiency Comparison

- Efficiency = c_{mt} = |mech. energy| / (weight x dist. traveled)



$c_{mt}^{est.} \approx 1.6$

Collins *et al.* 2005

$c_{mt} \approx 0.31$

$c_{mt} \approx 0.055$

Collins *et al.* 2005

C J. Braun, University of Edinburgh, UK

Case Study: Passive Dynamic Walker

- Forward falling combined with passive leg swing
- Storage of energy: potential ←→ kinetic in combination with low friction

C youtube material

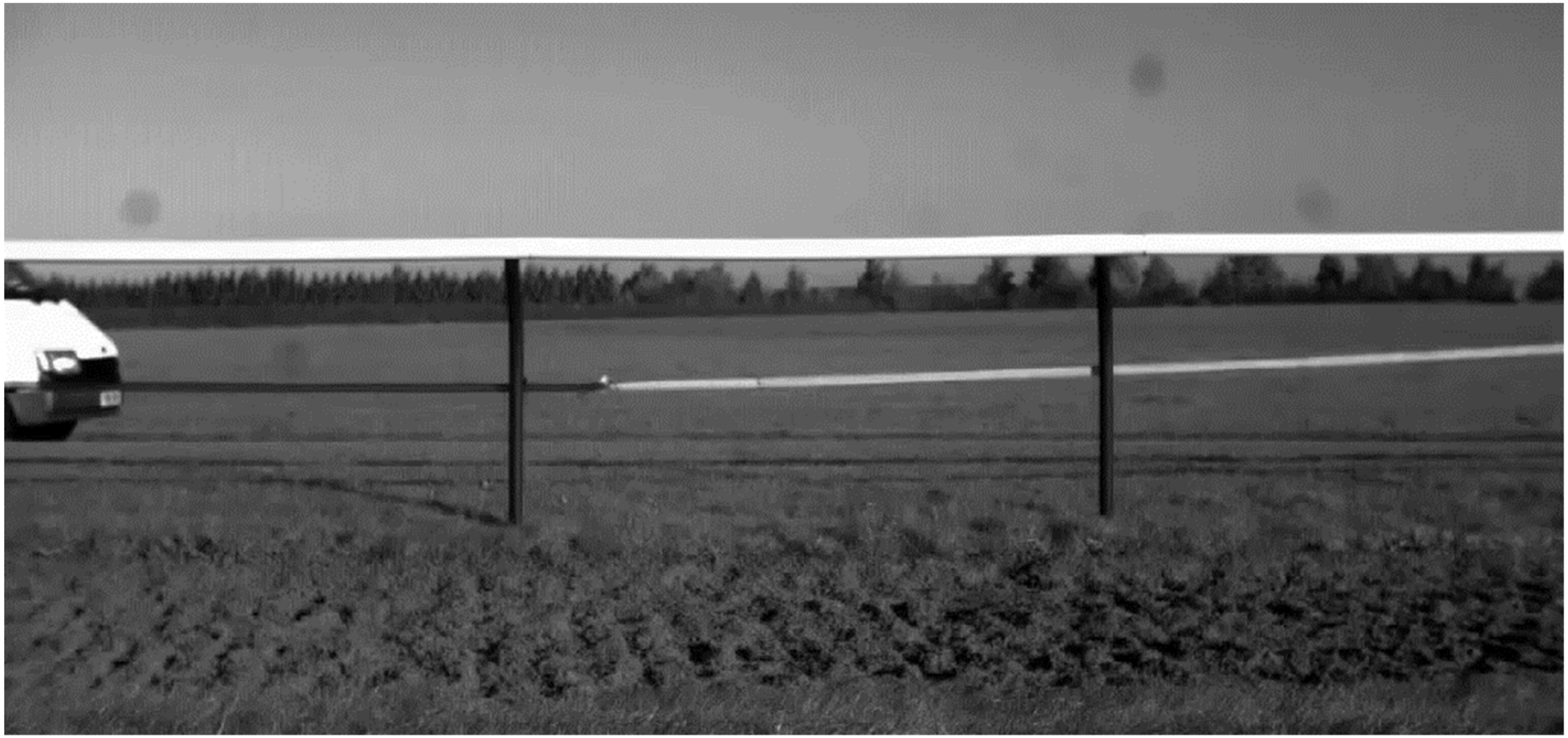
Static walking principles
Inverted Pendulum

- Static walking can be represented by inverted pendulum
- Exploit this in so-called passive dynamic walkers
- Add small actuation to walk on flat ground

Cornell Ranger

Total distance:	65.24 km
Total time:	30:49:02
Power:	16.0 W
COT:	0.28

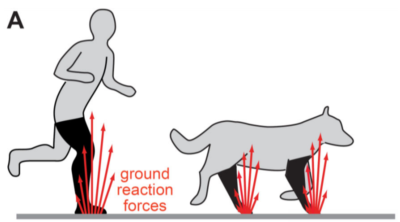
Towards Efficient Dynamic Walking: Optimizing Gaits



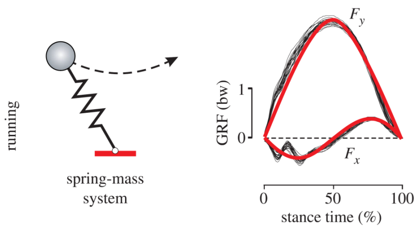
C Structure and motion laboratory
University of London

Dynamic Locomotion
SLIP model

- Biomechanical studies suggest SLIP models to describe complex running behaviors

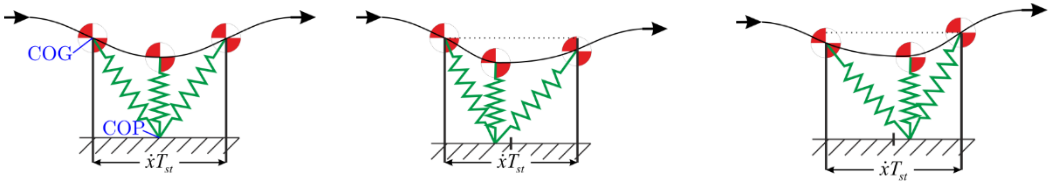


[Dickinson, Farley, Full 2000]



[Geyer 2006]

- Simple step-length rule to adjust the velocity



Constant speed
Accelerate
Decelerate

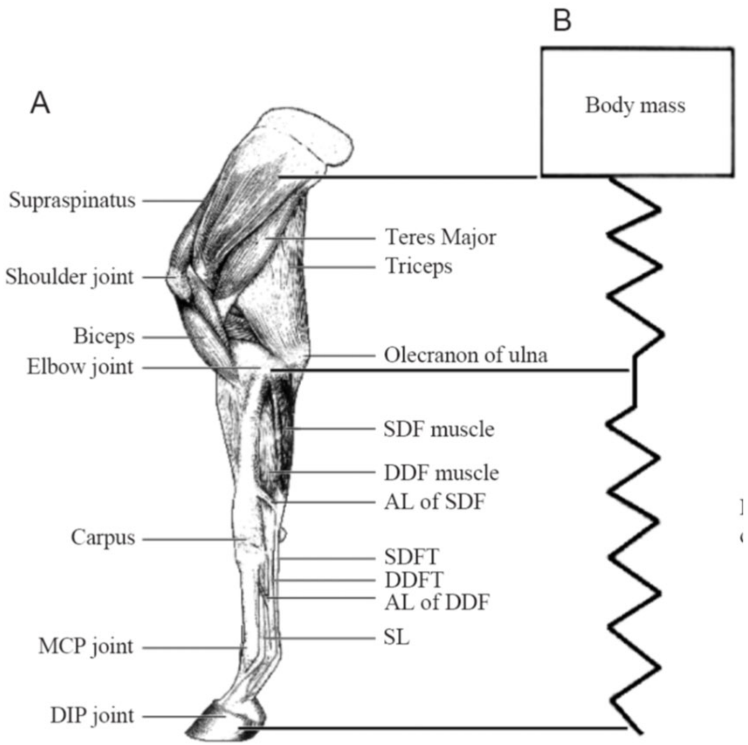
$$\mathbf{r}_F = \frac{1}{2}\dot{\mathbf{r}}_{HC,des}T_{st} + k_R^{FB}\left(\dot{\mathbf{r}}_{HC,des} - \dot{\mathbf{r}}_{HC}\right)\sqrt{h_{HC}}$$

[Raibert 1986]

Dynamic locomotion
Leg Structure

- Spring loaded inverted pendulum (SLIP)
  - ☑ are robust against collisions
  - ☑ can better handle uncertainties
  - ☑ can temporarily store energy
  - ☑ reduce peak power

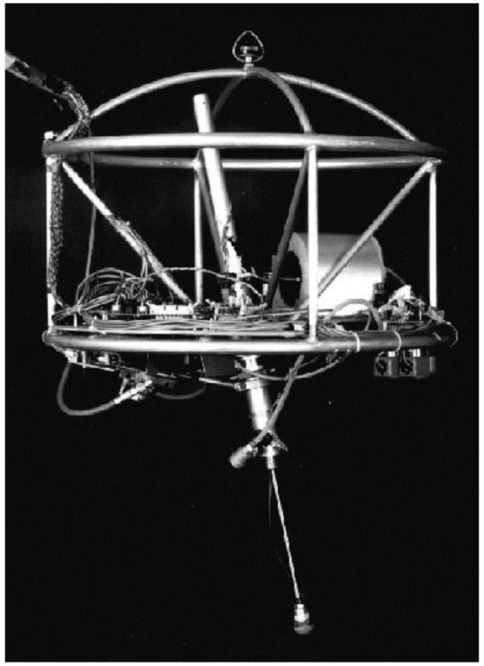
  [Alexander 1988,1990,2002,2003]



McGuigan & Wilson, 2003 – *J. Exp. Bio.*

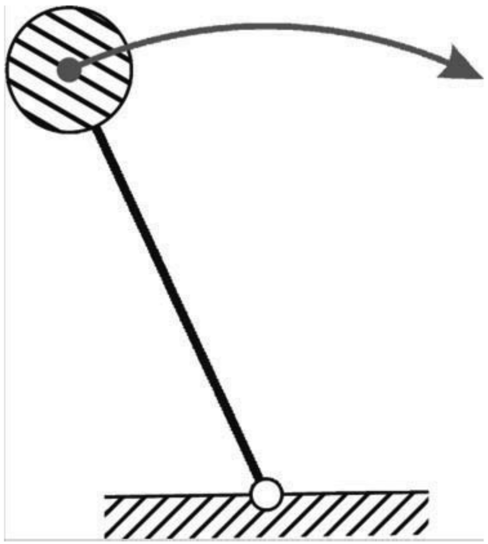
Dynamic Locomotion
SLIP principles in robotics

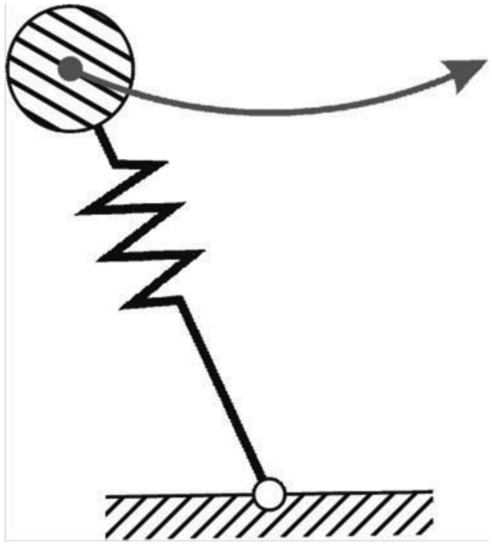
- Early Raibert hoppers (MIT leg lab) [1983]
  - Pneumatic piston
  - Hydraulic leg “angle” orientation



Understanding Locomotion
Summary

- Static locomotion (IP) ⇔ Dynamic Locomotion (SLIP)

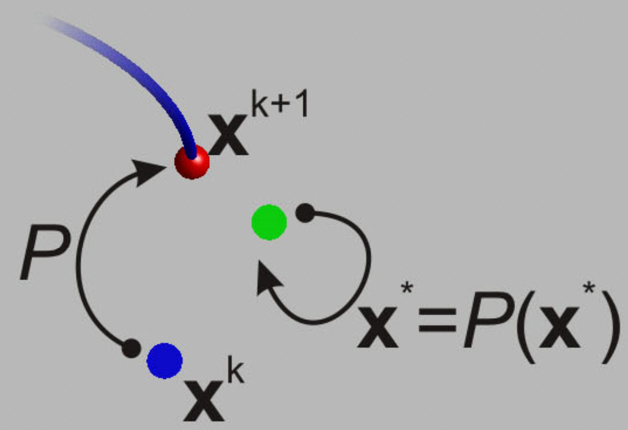


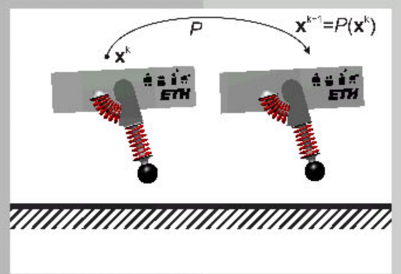


- How to determine stability?
- How to control (actively stabilize)?

Analyzing stability through limit cycles

- Poincaré Map $\mathbf{x}_{k+1} = P\left(\mathbf{x}_k\right)$
- Fix-Point $\mathbf{x}^* = P\left(\mathbf{x}^*\right)$
- Linearization of mapping $\Delta\mathbf{x}_{k+1} = \frac{\partial P}{\partial \mathbf{x}}\Delta\mathbf{x}_k = \mathbf{\Phi}\Delta\mathbf{x}_k$
- The system is stable iff: $\lambda_i\left(\mathbf{\Phi}\right) < 1$

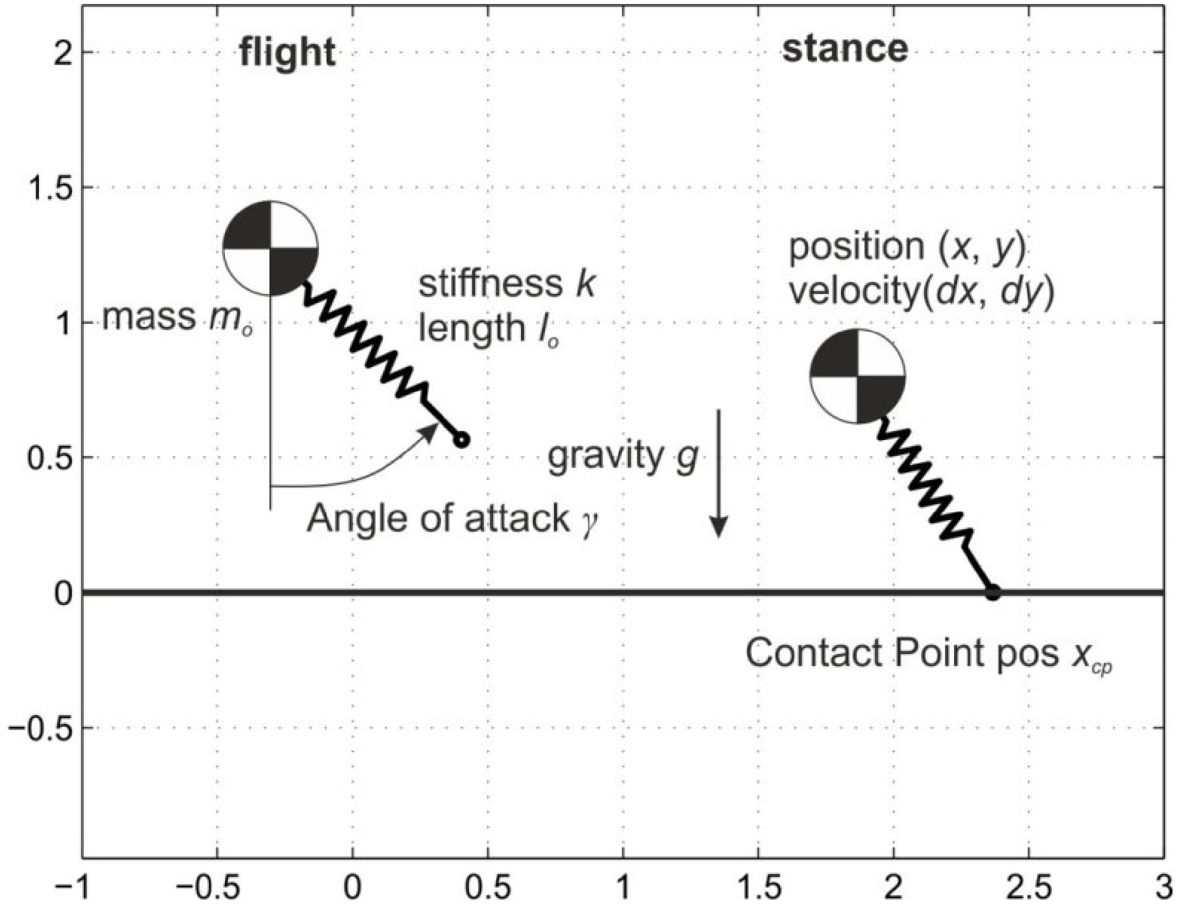




[C. David Remy, 2011]

Dynamic Locomotion
Stability Analysis

Discuss the edX limit-cycle movie



- Point mass: $\mathbf{q} = \begin{bmatrix} x \\ y \end{bmatrix}$
- Equation of Motion:
  - flight

$$\ddot{\mathbf{q}} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 \\ -g \end{bmatrix}$$

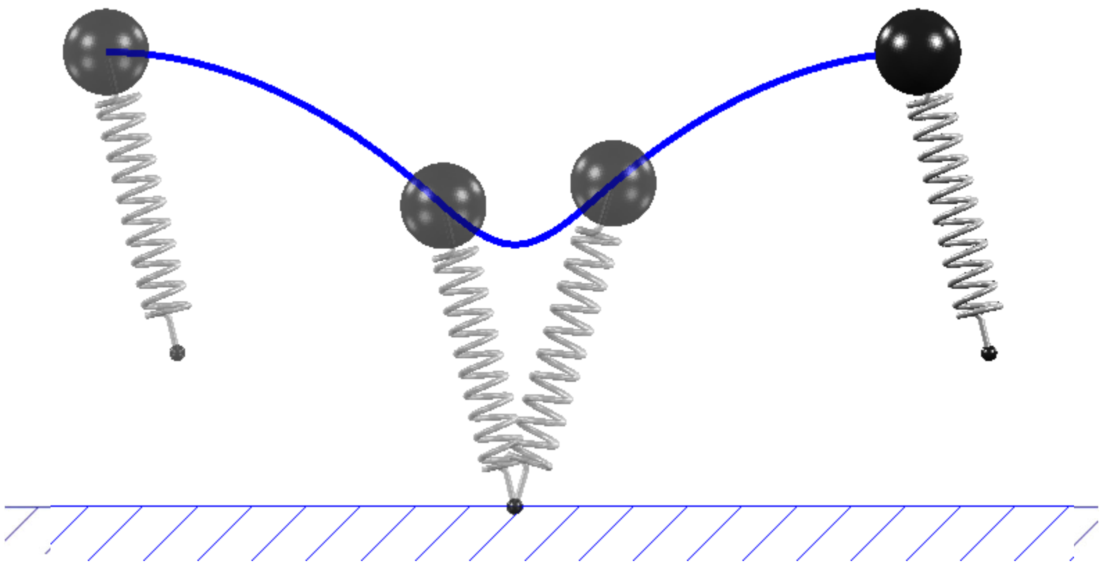
  - stance

$$\ddot{\mathbf{q}} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} \dfrac{-F^x_{spring}}{m_o} \\ \dfrac{F^y_{spring}}{m_o} - g \end{bmatrix}$$

$$\mathbf{F}_{spring} = f(\gamma, k, l)$$

Stability of Locomotion

Limit Cycle Analysis



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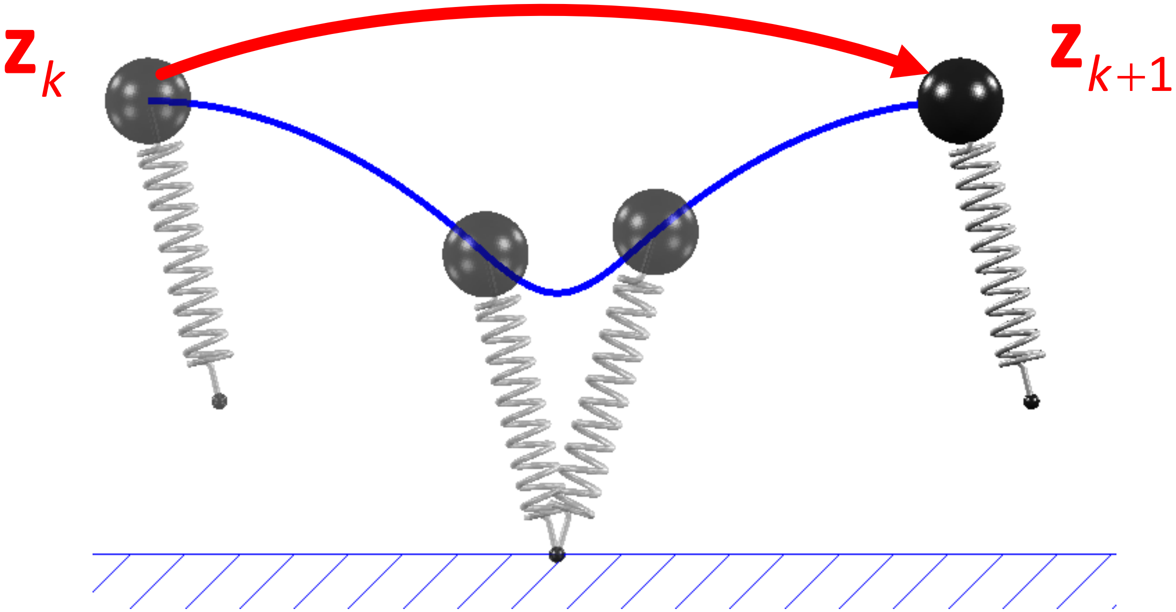
$$\mathbf{F}_{spring} = f\left(\gamma, k, l\right)$$

Stability of Locomotion
Limit Cycle Analysis



- Point mass: $\mathbf{q} = \begin{bmatrix} x \\ y \end{bmatrix}$
- System state

$$\mathbf{z} = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} \xrightarrow[\text{apex}]{\text{analysis at}} \mathbf{z} = \begin{bmatrix} y \\ \dot{x} \end{bmatrix}$$

- How does the state propagate from one apex to the next?

Stability of Locomotion
Limit Cycle Analysis

- Given the Poincaré mapping from one apex to the next

$$\mathbf{z}_{k+1} = P(\mathbf{z}_k)$$

  - P includes differential equations that cannot be analytically solved!

- Analyze periodic stability of a fixed point $\mathbf{z}^* = P(\mathbf{z}^*)$

- How can we do this? Linearization…

$$\mathbf{z}_{k+1} = \mathbf{z}^* + \Delta\mathbf{z}_{k+1} = P(\mathbf{z}^* + \Delta\mathbf{z}_k) = \underbrace{P(\mathbf{z}^*)}_{\mathbf{z}^*} + \left.\frac{\partial P}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}^*} \cdot \Delta\mathbf{z}_k + \underbrace{\cancel{\left.\frac{\partial^2 P}{\partial \mathbf{x}^2}\right|_{\mathbf{x}=\mathbf{x}^*} \cdot \Delta\mathbf{z}_k^2}}_{\approx 0} + \ldots$$

$$\Delta\mathbf{z}_{k+1} = \frac{\partial P}{\partial \mathbf{z}} \Delta\mathbf{z}_k = \mathbf{\Phi}\Delta\mathbf{z}_k$$

?

Stability of Locomotion

Numerical Limit Cycle Analysis

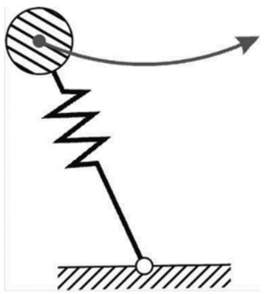
- Find the linearization of the Poincaré map around fix-point

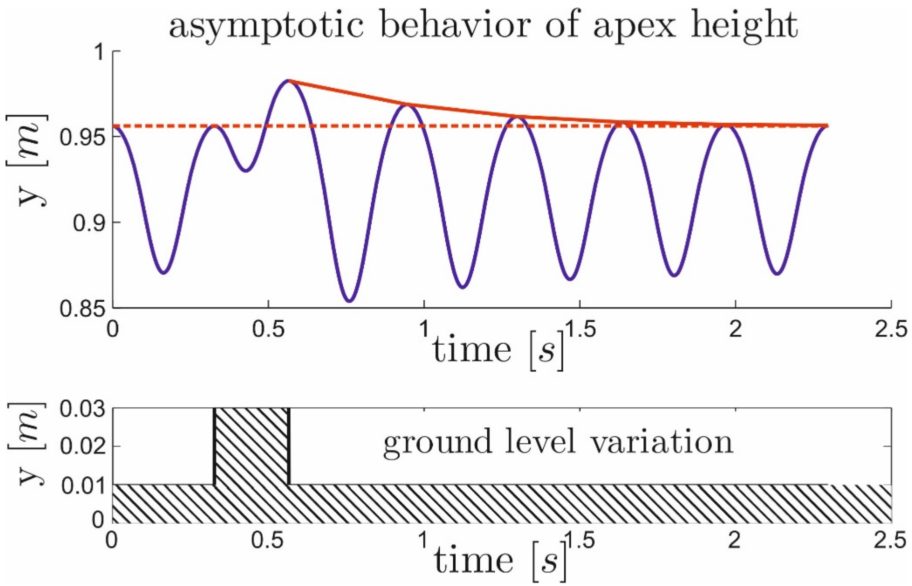
$$\begin{bmatrix} \Delta y_{k+1} \\ \Delta \dot{x}_{k+1} \end{bmatrix} = \left.\frac{\partial P}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}^*} \cdot \begin{bmatrix} \Delta y_k \\ \Delta \dot{x}_k \end{bmatrix}$$

  - Choose $\Delta \mathbf{z}_k = \begin{bmatrix} \Delta y_k \\ \Delta \dot{x}_k \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} h$, with a small h
  - Simulate until the next apex, starting from $\begin{bmatrix} y_{k+1} \\ \dot{x}_{k+1} \end{bmatrix} = P\left(\mathbf{z}^* + \begin{bmatrix} 1 \\ 0 \end{bmatrix} h \right)$
  - Calculate $\left.\frac{\partial P}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}^*} = \begin{bmatrix} \frac{y_{k+1} - y^*}{h} & * \\ \frac{\dot{x}_{k+1} - \dot{x}^*}{h} & * \end{bmatrix}$
  - Do the same thing for $\Delta \mathbf{z}_k = \begin{bmatrix} \Delta y_k \\ \Delta \dot{x}_k \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} h$

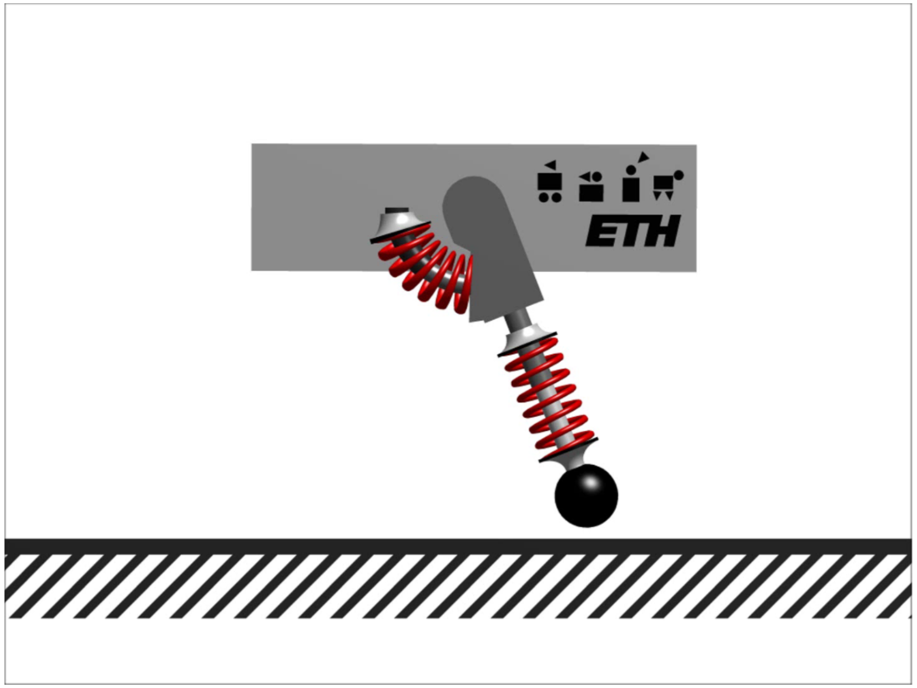
Stability of Locomotion
Numerical Limit Cycle Analysis

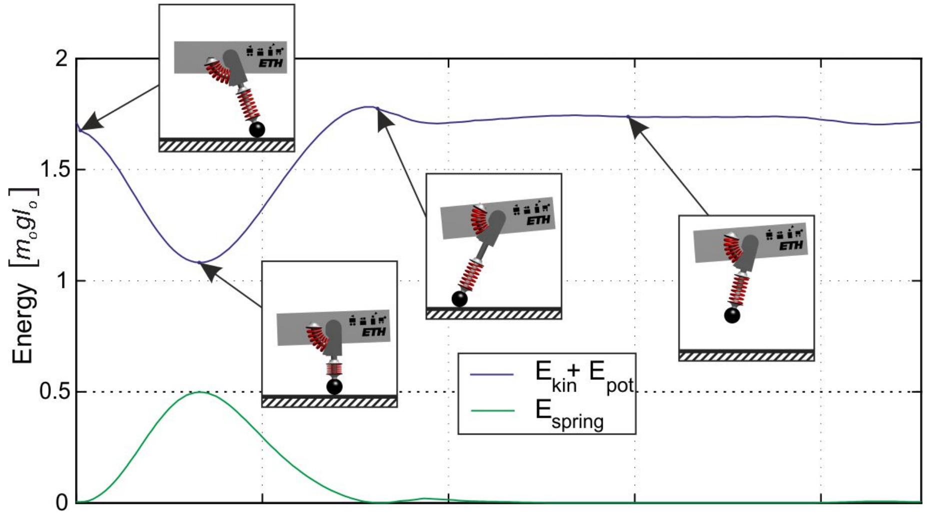
- The previous evaluation results in a 2x2 matrix $\left.\dfrac{\partial P}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}^*}$
- Eigenvalue analysis:
  - System is energy conservative: $\lambda_1 = 1$
  - System can be
    - stable $|\lambda_2| < 1$
    - unstable: $|\lambda_2| > 1$





Efficient Walking and Running | series elastic actuation

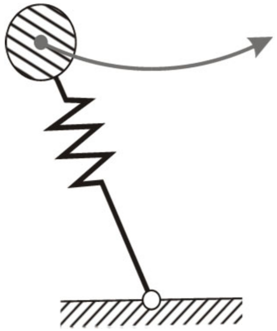


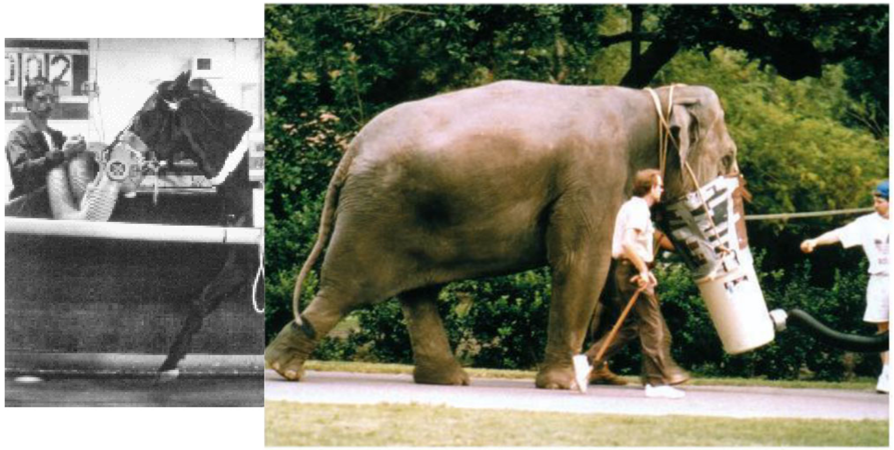


https://www.youtube.com/watch?v=6igNZiVtbxU

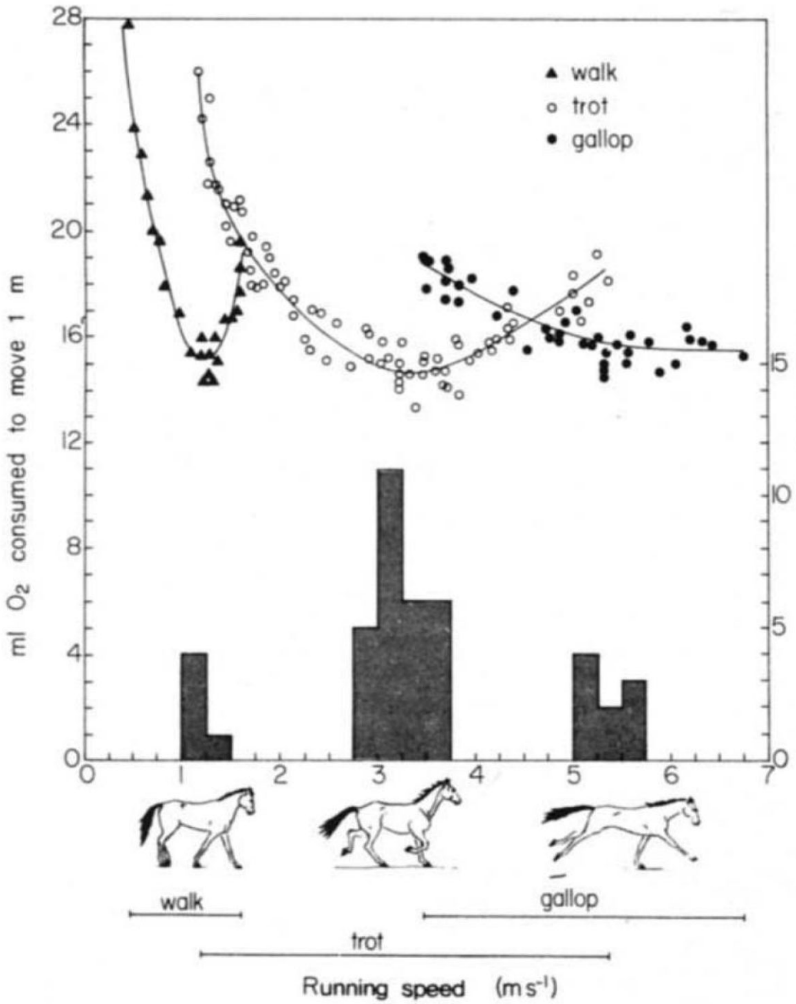
Towards Efficient Dynamic Walking: Optimizing Gaits

- Nature optimizes its gaits
- Storage of “elastic” energy
- To allow locomotion at varying frequencies and speeds, different gaits have to utilize these elements differently





Langman et al., J Exp. Bio. 1995



Hoyt & Taylor, 1981 - *Nature*

Week 2 overview

Overview

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The goal of this week is to get an overview about legged robotic systems and rigid body kinematics of articulated systems. The material consists of three lecture segments, two short worked examples, and one problem set.

- **Legged Robotics:** The first segment provides an introduction to legged robots and a comparison to their natural counterparts. The concepts of static and dynamic stabilty will be covered and a brief outline about the underlying control concepts is given.
- **Basics of Kinematics:** The second segment starts with very basic kinematic tools such as rotations, translations, and homogeneous transformations of multi-body systems.
- **Application of Kinematics:** The third segment introduces the concept of generalized coordinates and Jacobians. This is followed by an explanation of inverse and differential kinematics and finally a discussion of some mobile-platform specific elements.
- **Worked Example 1:** In the first worked example, rotations and homogeneous transformations are applied to a planar 3-link robot arm to describe the robot kinematics.
- **Worked Example 2:** In the second worked example, a position control and trajectory following problem for the 3-link robot arm is analyzed using inverse and differential kinematics.