RMP- Kalman Filter

Key words	Kalman Gain, appications & graphs, incremental/local localization, linear motion & measurement model, measurement correction, observability matrix, prediction, sum of squares error optimal, uncertain data, zero-mean Gaussian noise
Status	note complete

Kalman Filter: Fusion of Uncertain Data

- assumptions until now:
  - **obstacle**s**: absolute accurate knowledge**
  - **robot positions**: **absolute accurate knowledge**
- real world:
  - **obstacles**: boundaries have **uncertainty** due to **sensor measurement error**
  - **robot position**: **not absolute, uncertainty in true position** (variance in elipse)
  - by sending the robot to a **new position**: might have **completely different uncertainty** in robot pose & obstacles

Kalman Filter:

▼ Idea & assumptions:

- an **estimate for parameter x** and **estimate for uncertainty of x** (variance covariance matrix) —> $(\vec{x}_t, \tilde{P}_t)$
- **more trust** on measurements with **lower variance (more accurate)**
- motion model & measurement model: **linear models**
- error characteristics: **zero-mean Gaussian noise** N$(0, \sigma^2)$

—> **optimal: minimizes sum of squares error**

▼ **Prediction (Time Update)**

1. prediction $\hat{x}^-_{k+1}$ based on **linear motion model**
2. project the covariance P_k forward to P^-_{k+1}

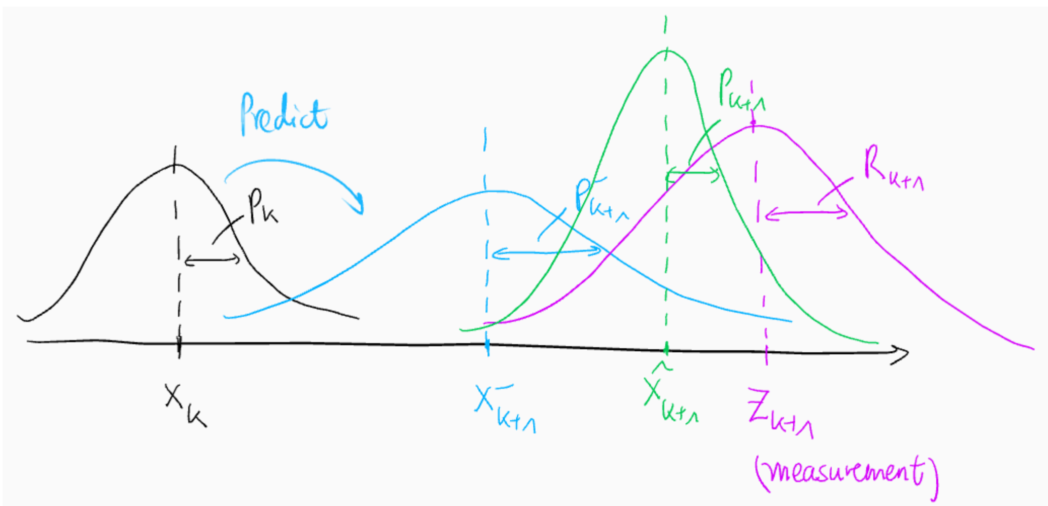
—> **a priori estimate** $(\hat{x}^-_{k+1}, P^-_{k+1})$

▼ **Correction (Measurement Update)**

1. compute Kalman Gain K_{k+1}
2. update prediction $\hat{x}^-_{k+1}$ with measurements z_{k+1} to $\hat{x}_{k+1}$
3. update error covariance to P_{k+1}

—> **a posteriori estimate** ($\hat{x}_{k+1}, P_{k+1}$)

- one-dimensional case:



▼ **linear** motion model and measurement model

- motion model:
  - process noise variance Q
  - state vector x_k: not readable, with uncertainty
  - control vector u_k: **readable,** another source of measurement (odometry, IMU, wheels)

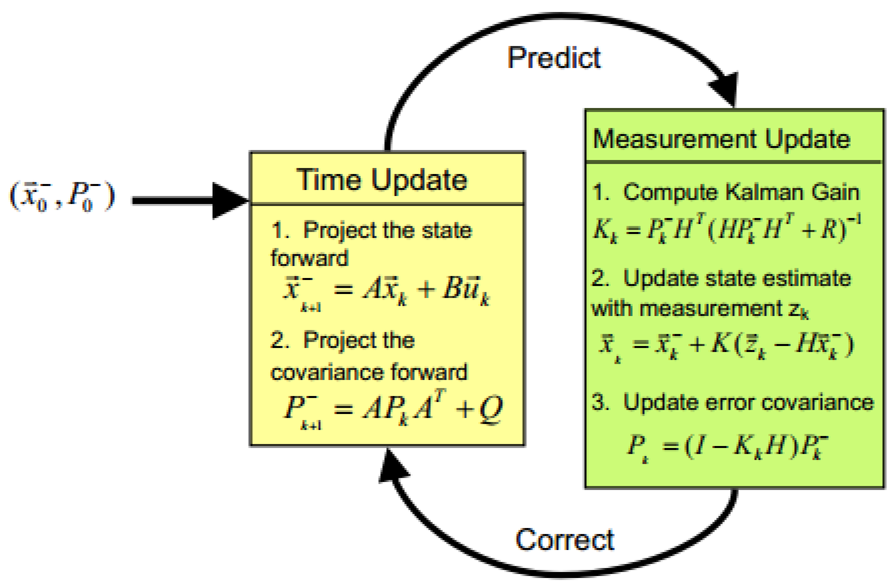
$$x_{k+1} = A \cdot x_k + B \cdot u_k + w_k$$

- measurement model:
  - sensor noise variance R

$$z_{k+1} = H \cdot x_{k+1} + v_{k+1}$$

▼ **Kalman Gain K**:

- **K = 0 —>** $P^-_{k+1} = 0$, **100% trust on prediction —>** $\hat{x}_{k+1} = \hat{x}^-_{k+1}$
- **K = 1 (1D case) or K =** $\frac{1}{H}$ —> R = 0, **100% trust on measurement —>** $\hat{x}_{k+1} = z_{k+1}$



▼ state representation: selection of motion model

- position p
- position p + velocity v
- position p + velocity v + acceleration a

—> all can be **estimated as state**, or some are **directly readable as control vector**

▼ measurement: what to measure? all? some? —> **Observability Matrix**

- **observability of a system:**
  - a system with a **n-dimensional state vector** is **observable,** if the observability matrix O has $\mathbf{rank}(\mathbf{O}) = \mathbf{n}$

$$O = \begin{bmatrix} H \\ H \cdot A \\ H \cdot A^2 \\ \vdots \\ H \cdot A^{n-1} \end{bmatrix}$$

—> if system not observable, wrong measured parameters —>system diverges

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Kalman Filter Applications:

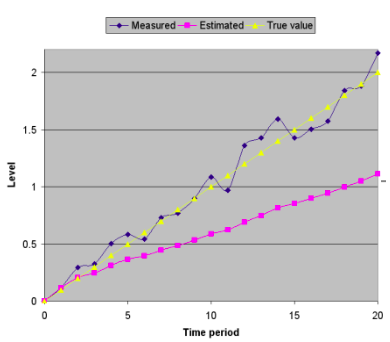
Problem: a lag between true and estimation

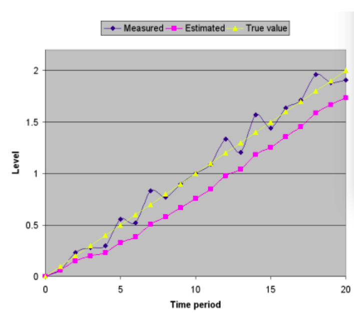
- wrong model
- reliability of the motion model: Q

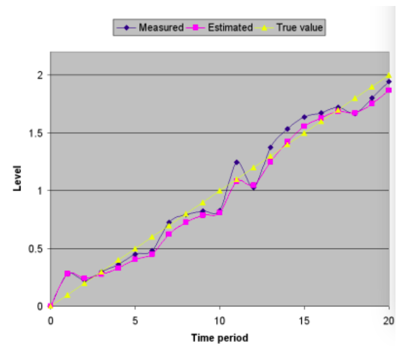
Solution:

- relax the model with **larger process noise q**
- a better model

increase q:







change model:

- if the initialization is bad, KF takes the first measurement as a good initialization
- if the model is non-linear —> it won't converge to true