Determinant simplification

So knowing these properties of the determinant, what operations can we do on the columns of a matrix? We can do 3 things:

1. We can switch any two columns. This will have the effect of negating the determinant.
2. We can pull any constant times a column out of the determinant: i.e. $\det(\mathbf{a}_1, 2\mathbf{a}_2) = 2\det(\mathbf{a}_1, \mathbf{a}_2)$.
3. We can add a scalar multiple of one of the columns to another column. This will not change the value of the determinant at all. That is $\det(\mathbf{a}_1, \mathbf{a}_2) = \det(\mathbf{a}_1, \mathbf{a}_2 + k\mathbf{a}_1)$.

Trigonometric simplification

$$sin(\theta_1 \pm \theta_2) = sin(\theta_1)cos(\theta_2) \pm cos(\theta_1)sin(\theta_2) \quad (1)$$

$$cos(\theta_1 \pm \theta_2) = cos(\theta_1)cos(\theta_2) \mp sin(\theta_1)sin(\theta_2) \quad (2)$$

Newton-Euler inward and outward loop

Outward iterations: $i : 0 \to 5$

$$
\begin{aligned}
{}^{i+1}\omega_{i+1} &= {}^{i+1}_{i}R\,{}^{i}\omega_i + \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1},\\
{}^{i+1}\dot{\omega}_{i+1} &= {}^{i+1}_{i}R\,{}^{i}\dot{\omega}_i + {}^{i+1}_{i}R\,{}^{i}\omega_i \times \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1} + \ddot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1},\\
{}^{i+1}\dot{v}_{i+1} &= {}^{i+1}_{i}R({}^{i}\dot{\omega}_i \times {}^{i}P_{i+1} + {}^{i}\omega_i \times ({}^{i}\omega_i \times {}^{i}P_{i+1}) + {}^{i}\dot{v}_i),\\
{}^{i+1}\dot{v}_{C_{i+1}} &= {}^{i+1}\dot{\omega}_{i+1} \times {}^{i+1}P_{C_{i+1}}\\
&\quad + {}^{i+1}\omega_{i+1} \times ({}^{i+1}\omega_{i+1} \times {}^{i+1}P_{C_{i+1}}) + {}^{i+1}\dot{v}_{i+1},\\
{}^{i+1}F_{i+1} &= m_{i+1}\,{}^{i+1}\dot{v}_{C_{i+1}},\\
{}^{i+1}N_{i+1} &= {}^{C_{i+1}}I_{i+1}\,{}^{i+1}\dot{\omega}_{i+1} + {}^{i+1}\omega_{i+1} \times {}^{C_{i+1}}I_{i+1}\,{}^{i+1}\omega_{i+1}.
\end{aligned}
$$

Inward iterations: $i : 6 \to 1$

$$
\begin{aligned}
{}^{i}f_i &= {}^{i}_{i+1}R\,{}^{i+1}f_{i+1} + {}^{i}F_i,\\
{}^{i}n_i &= {}^{i}N_i + {}^{i}_{i+1}R\,{}^{i+1}n_{i+1} + {}^{i}P_{C_i} \times {}^{i}F_i\\
&\quad + {}^{i}P_{i+1} \times {}^{i}_{i+1}R\,{}^{i+1}f_{i+1},\\
\tau_i &= {}^{i}n_i^{T}\,{}^{i}\hat{Z}_i.
\end{aligned}
$$

Transformation matrix from DH-Table

$$ {}^{i-1}_{i}\mathbf{T} = \mathbf{R}_x(\alpha_{i-1}) \cdot \mathbf{D}_x(a_{i-1}) \cdot \mathbf{R}_z(\theta_i) \cdot \mathbf{D}_z(d_i) \qquad (1) $$

Calculate the products in this direction: first do ans = Rz*Dz, then Dx*ans, etc. etc.

Remember that this is the way to use the matrix: ${}^{i-1}p = {}^{i-1}_{i}T \; * \; {}^{i}p$

Order for transformations (from 0 to n)

$$ {}^{0}_{n}\mathbf{T} = \prod_{i=1}^{n} {}^{i-1}_{i}\mathbf{T} = {}^{0}_{1}\mathbf{T} \cdot {}^{1}_{2}\mathbf{T} \cdot {}^{2}_{3}\mathbf{T} \cdot \ldots \qquad (1) $$

Jacobian

Is always going to be like this:

- First three rows from derivatives of $p_{(\theta)}$ with respect to all n thetas
- Last three rows are the directions of the n-frame Z axis with respect to the 0-frame Z axis

So the Jacobian will ALWAYS be a 6xN matrix, where N is the number of joints in the robot

Jacobian and velocities

$$ \dot{\vec{x}} = \bar{J} \cdot \dot{\vec{\theta}} \qquad (1) $$

Velocity Formulas for Jacobian in n-frame computation

$$ {}^{i+1}\omega_{i+1} = {}^{i+1}_{i}R \cdot {}^{i}\omega_i + \dot{\Theta}_{i+1} \cdot {}^{i+1}\hat{Z}_{i+1} \qquad (1) $$
$$ {}^{i+1}v_{i+1} = {}^{i+1}_{i}R({}^{i}v_i + {}^{i}\omega_i \times {}^{i}P_{i+1}) \qquad (2) $$

Careful with rotation matrices, as you can see they must be first inverted!

$$ {}^{n}J\dot{\Theta} = \begin{pmatrix} {}^{n}v_n \\ {}^{n}\omega_n \end{pmatrix}, $$

MVG

$$ \tau = M(\Theta)\ddot{\Theta} + V(\Theta, \dot{\Theta}) + G(\Theta) $$

where M is a n x n matrix, whereas V and G are 1 x n vectors. M si deriva raccogliendo tutti gli elementi contenenti la derivate seconda di theta 1, 2, ecc. Invece V contiene tutti gli elementi con la derivata prima di theta 1, 2, ecc. G ha tutto ciò che resta.

Lagrange

Formula for kinetic energy at joint i:

$$k_i = \frac{1}{2} m_i v_{C_i}^{\mathrm{T}} \cdot v_{C_i} + \frac{1}{2} {}^{i}\omega_i^{\mathrm{T}} \cdot {}^{C_i}I_i \cdot {}^{i}\omega_i$$

Formula for potential energy at joint i:

$$u_i = -m_i \cdot {}^{0}g^{\mathrm{T}} \cdot {}^{0}P_{C_i} + u_{\mathrm{ref}_i}$$

Lagrange equation:

$$\tau_i = \frac{d}{dt}\frac{\partial k}{\partial \dot{\Theta}_i} - \frac{\partial k}{\partial \Theta_i} + \frac{\partial u}{\partial \Theta_i}$$