RMP- Unscented Kalman Filter

Key words	EKF VS. UKF, UKF, Unscented Tranform, augmented state vector & covariance, incremental/local localization, scaling factor, sigma points
Status	note complete

Unscented Kalman Filter: highly non-linear models with even larger curvature

- idea:
  - even larger curvature: **no convergence with EKF (first-order Taylor series)**
  - local linearity assumption breaks down: if **higher order terms get significant**

  —> better approximation needed!!

- **Unscented Transformation**
  - Goal: **approximate a Gaussian probability distribution**
  - mean estimate and covariance are **accurate to second order** of Taylor expansion for any non-linear functions.
  - **Process:**
    1. select $2n_x + 1$ of **sigma points** χ which has sample mean $\bar{x}$ and sample covariance P_{xx},

$$
\begin{aligned}
\boldsymbol{\mathcal{X}}_0 &= \bar{\mathbf{x}} & W_0 &= \kappa/(n+\kappa) \\
\boldsymbol{\mathcal{X}}_i &= \bar{\mathbf{x}} + \left(\sqrt{(n+\kappa)\mathbf{P}_{xx}}\right)_i & W_i &= 1/2(n+\kappa) \\
\boldsymbol{\mathcal{X}}_{i+n} &= \bar{\mathbf{x}} - \left(\sqrt{(n+\kappa)\mathbf{P}_{xx}}\right)_i & W_{i+n} &= 1/2(n+\kappa)
\end{aligned}
$$

2. **transformation**: apply the **non-linear function (motion model)** to each sigma points to get transformed points $\bar{y}$ and P_{yy} —> $Y_i = g(\chi_i)$
3. The **estimated mean(weighted!) and covariance** are:

$$\bar{y} = \sum_{i=0}^{2n_x} W_i Y_i$$
$$P_y = \sum_{i=0}^{2n_x} W_i (Y_i - \bar{y})(Y_i - \bar{y})^T$$

—> **!! here $\bar{y} \neq f(\bar{x})$!!!** The peak is shifted !!

—> P_y not calculated from P_x: recalculated by the transformed sigma-points.

▼ scaling factor & Scaled Unscented Transformation

- combine sigma point selection and scaling into 1 step:

$$\lambda = \alpha^2(n_x + \kappa) - n_x$$

Unscented Kalman Filter (UKF)

- state variable: **augmented vector** with state, process noise and measurement noise

$$x_t^a = [x_t, v_t, n_t]^T$$

- covariance matrix: **augmented matrix**

$$P_t^a = \begin{bmatrix} P_t & 0 & 0 \\ 0 & Q_t & 0 \\ 0 & 0 & R_t \end{bmatrix}$$

▼ **Process**:

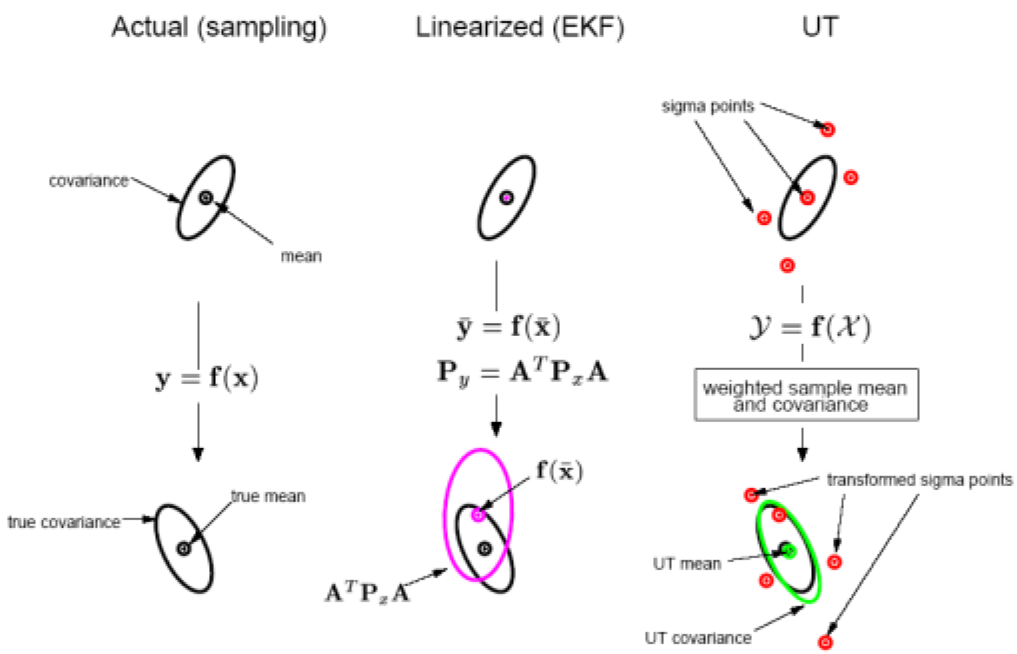
1. Initialization with augmented state variable x_0^a and covariance matrix P_0^a

2. compute sigma points χ^a_{t-1}
3. time update:
    a. **unscented transform**: project sigma points with **motion model** onto next timestamp
    b. get estimated mean and covariance using **weighted** transformed sigma points
4. measurement prediction:
    a. **unscented transform:** project transformed sigma points χ_t^- with **measurement model**
    b. get predicted measurement and covariance using **weighted transformed** sigma points
5. measurement update:
    a. compute Kalman Gain
    b. compute corrected state estimate and covariance

EKF VS. UKF:

- EKF:
    - calculates posterior mean and covariance **accurate to first order**
    - **computationally expensive** for **Jacobian** calculation
- UKF:
    - calculate posterior mean and covariance **accurate to second order —> better approximation than EKF**
    - **no Jacobian** calculation —> faster
    - doesn't require function to be smooth or differentiable
    - accept **arbitrary functions** —> just get sigma points and unscented transformation

- **fine tuning** possible: function with frequent changes — small κ, smooth function — large κ



- Filtering: **sequentially estimating** the states of a system as **a set of observations available** onine.
  - Solution: $p(X_k|Y_k) = p(x_k|Y_k)$ —> no need to keep track of all states.

▼ a general system: **non-linear, non-Gaussian**

- assumptions:
  - **states are markovian**: $p(x_k|x_{k-1}, x_{k-2}, \ldots, x_0) = p(x_k|x_{k-1})$