

Differential Equations: Numerical Methods

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Github link:

<https://github.com/lackadaisicalcynic/Differential-Equations>

Exact solution of the given differential equation

$$y' = 2y^{\frac{1}{2}} + 2y \quad | : y^{\frac{1}{2}} \quad \text{Bernoulli equation}$$

$$\frac{y'}{y^{\frac{1}{2}}} = 2 + 2y^{\frac{1}{2}}$$

$$\text{Substitution: } z = y^{\frac{1}{2}}; \quad z' = \frac{1}{2} y^{-\frac{1}{2}} y'$$

$$z' = z + 1$$

$$z' - z = 1$$

$$\ln z = x + C$$

$$z = e^x \cdot C(x)$$

$$e^x C'(x) + e^x C(x) - e^x C(x) = 1$$

$$e^x C'(x) = 1$$

$$C(x) = -e^{-x} + C$$

$$z = -1 + Ce^x$$

$$y = (-1 + Ce^x)^2$$

$$y(0) = 1; \quad 1 = (-1 + C)^2$$

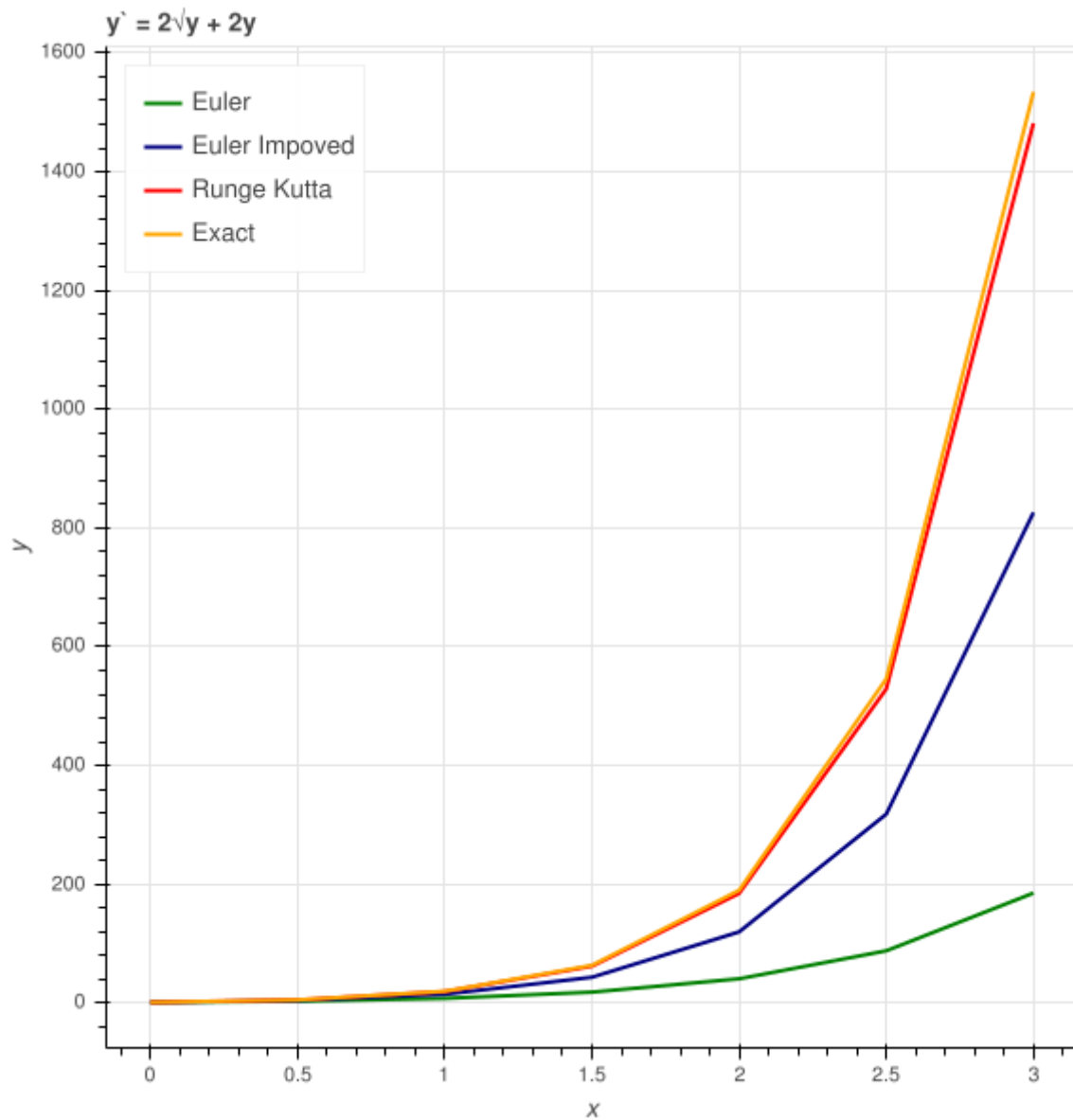
IVP Solution:

$$C = 2$$

$C = 0$ - trivial solution

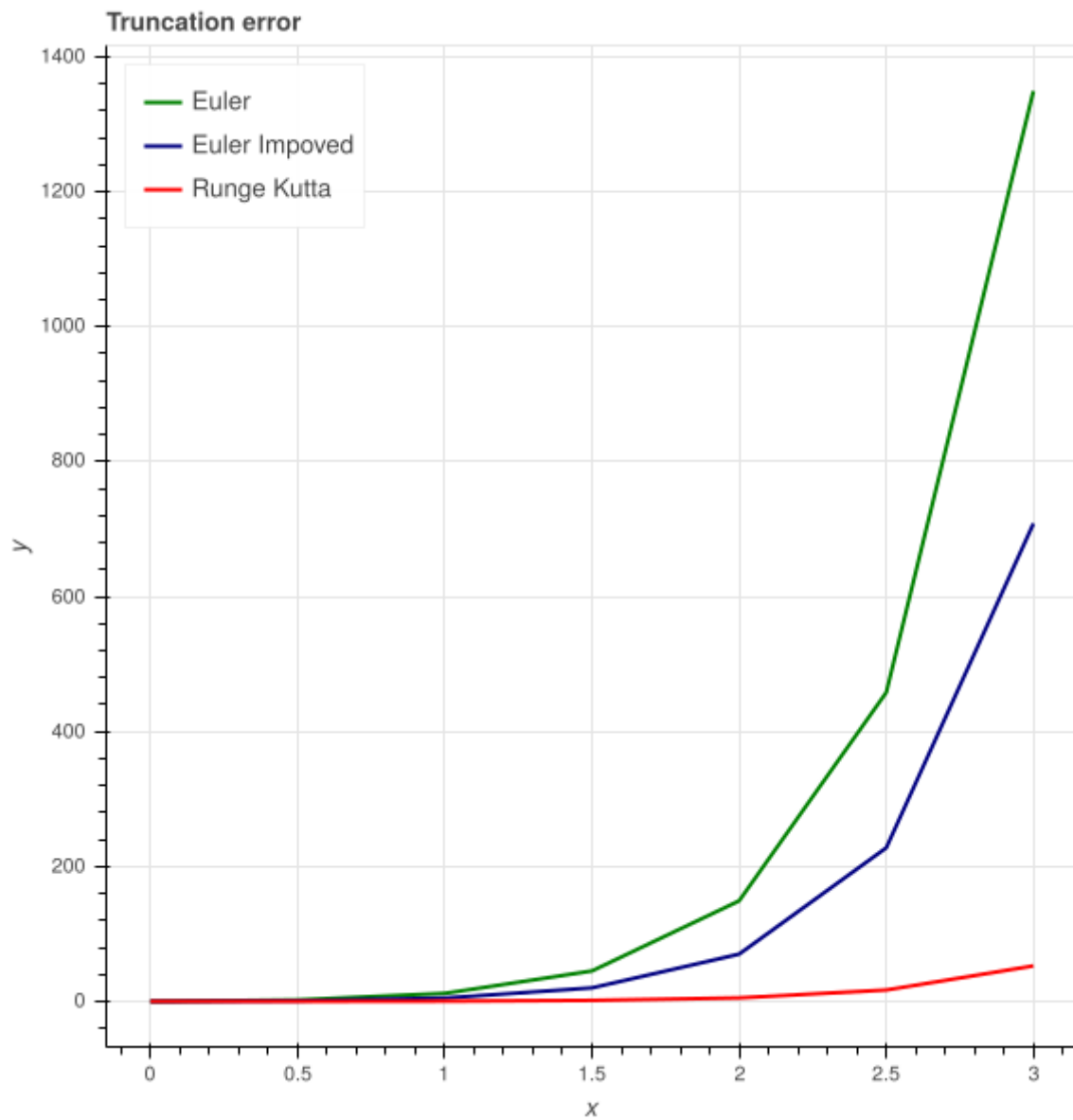
$$y = (1 - 2e^x)^2$$

Sample graph to consider numerical methods comparing to each other and to the exact solution



Step = 0.5, Interval = [0; 3]

Local truncation error sample graph:



Step $h=0.5$, Interval = $[0; 3]$