

$$y' = 2y^{\frac{1}{2}} + 2y \quad | : y^{\frac{1}{2}} \quad \text{Bernoulli equation}$$

$$\frac{y'}{y^{\frac{1}{2}}} = 2 + 2y^{\frac{1}{2}}$$

$$\text{Substitution: } z = y^{\frac{1}{2}}; \quad z' = \frac{1}{2} y^{-\frac{1}{2}} y'$$

$$z' = z + 1$$

$$z' - z = 1$$

$$\ln z = x + C$$

$$z = e^x \cdot C(x)$$

$$e^x C'(x) + e^x C(x) - e^x C(x) = 1$$

$$e^x C'(x) = 1$$

$$C(x) = -e^{-x} + C$$

$$z = -1 + Ce^x$$

$$y = (-1 + Ce^x)^2$$

$$y(0) = 1; \quad 1 = (-1 + C)^2$$

$$C = 2$$

$$C = 0 - \text{trivial solution}$$

IVP Solution:

$$\underline{y = (1 - 2e^x)^2}$$