Differential Equations: Numerical Methods

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Github link:
https://github.com/lackadaisicalcynic/Differential-Equations

Exact solution of the given differential equation

$y' = 2y^{\frac{1}{2}} + 2y \quad | : y^{\frac{1}{2}}$ Bernoulli equation

$\frac{y'}{y^{\frac{1}{2}}} = 2 + 2y^{\frac{1}{2}}$

Substitution: $z = y^{\frac{1}{2}}$; $z' = \frac{1}{2} y^{-\frac{1}{2}} y'$

$z' = z + 1$

$z' - z = 0$

$\ln z = x + C$

$z = e^x \cdot C(x)$

$e^x C'(x) + e^x C(x) - e^x C(x) = 1$

$e^x C'(x) = 1$

$C(x) = -e^{-x} + C$

$z = -1 + Ce^x$

$y = (-1 + Ce^x)^2$

$y(0) = 1; \quad 1 = (-1 + C)^2$

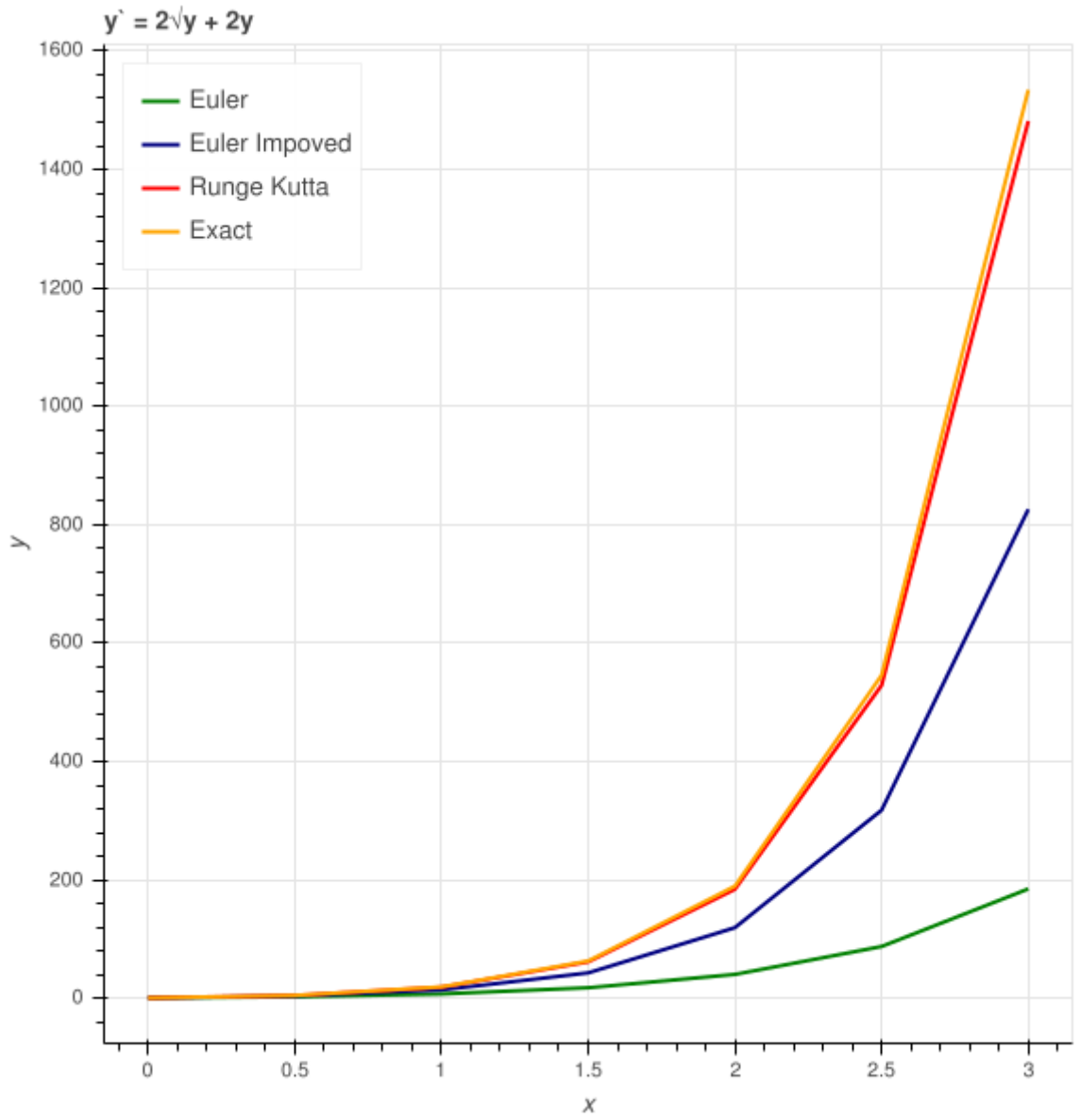
$\begin{cases} C = 2 \\ C = 0 - \text{trivial solution} \end{cases}$

IVP Solution:

$y = (1 - 2e^x)^2$

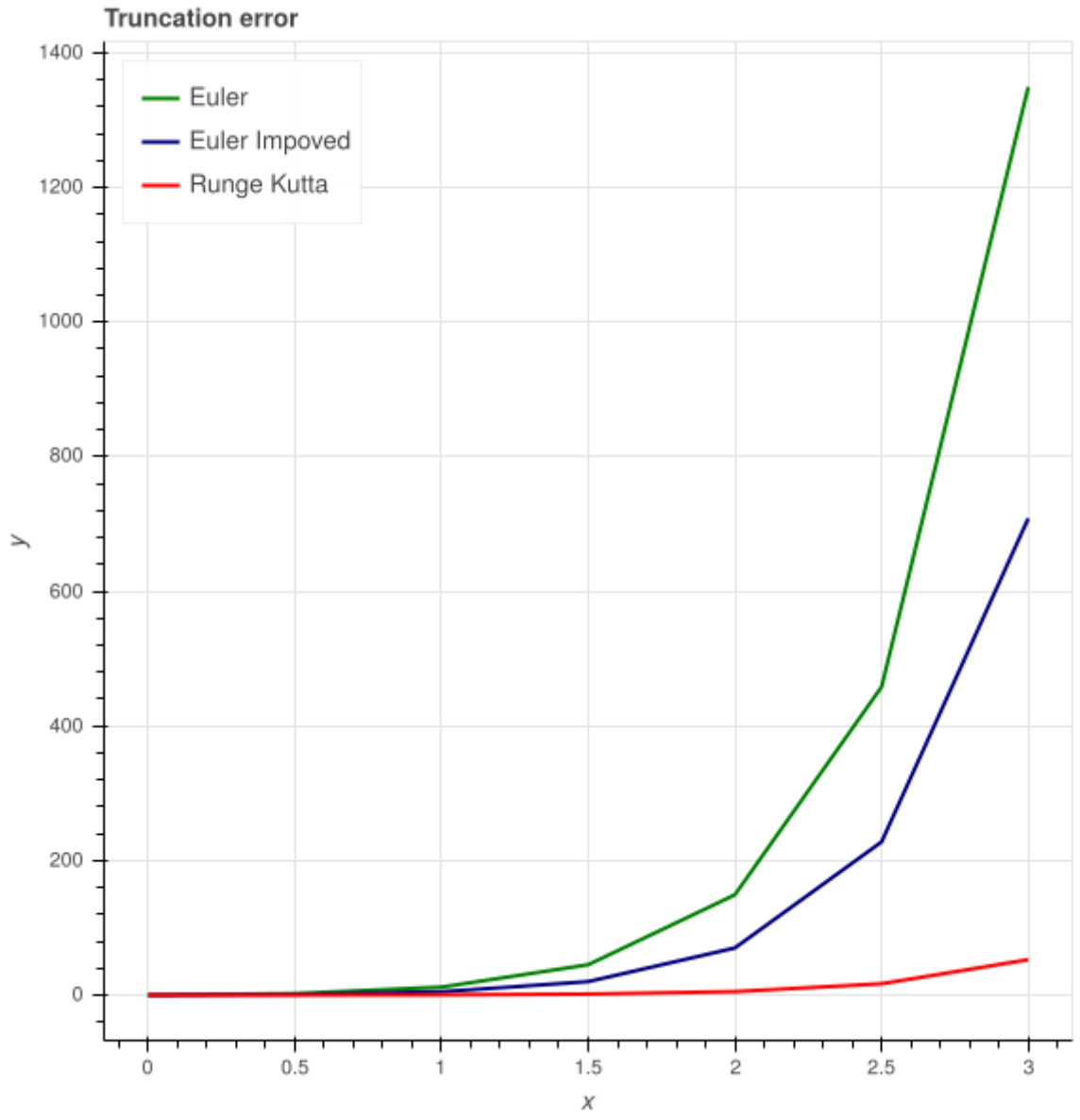
No points of discontinuity, x belongs to R, y > 0.

Sample graph to consider numerical methods comparing to each other and to the exact solution



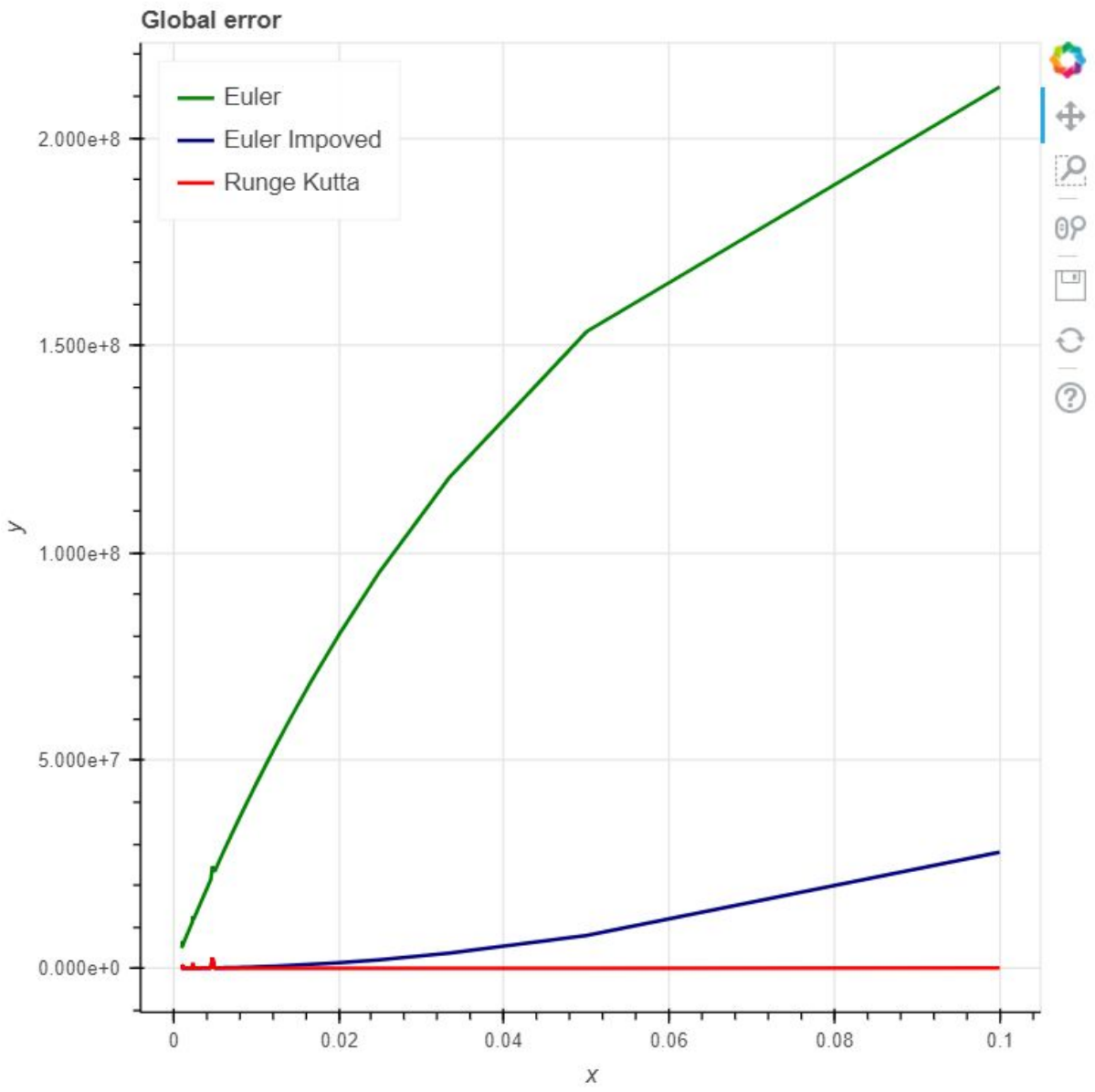
Step = 0.5, Interval = [0:3]

Local truncation error sample graph:



Step h=0.5, Interval = [0:3]

Global truncation error graph:



Interval of function [0:9], OX: step for numerical method [0.001 : 0.1], OY: Global truncation error for corresponding value of step.