Problem Set 1, Answers

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Problem 1. A language L is in $\Sigma_2^{\mathbf{P}}$ iff there is a polynomial time TM M such that:

$$x \in L \iff \exists u_1 \forall u_2 M(x, u_1, u_2) \tag{1}$$

where the u_i are polynomial size and we treat M as returning true or false. This can also be written as:

$$x \in L \iff \exists u_1 \neg(\exists u_2(\neg M(x, u_1, u_2)) \tag{2}$$

The answer to $\exists u_2(\neg M(x, u_1, u_2))$ can be found in a single call to an **NP** oracle, so $\Sigma_2^{\mathbf{P}} \in \mathbf{NP}^{\mathbf{NP}}$.

The other direction is similar. TODO: prove the other direction as well