

COMENIUS UNIVERSITY, BRATISLAVA
FACULTY OF MATHEMATICS, PHYSICS AND INFORMATICS

Use of SAT Solvers in Cryptanalysis
MASTER'S THESIS

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I hereby declare that I have written this thesis by myself, only with the help of the resources listed in the bibliography, under the careful supervision of my thesis supervisor.

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Abstrakt

SAT solvery sú univerzálny nástroj pre hľadanie riešení boolovských problémov splniteľnosti. V minulosti boli používané aj na kryptografické problémy, ako napríklad hľadanie vzoru hašovacích funkcií alebo kľúča prúdových šifier. Avšak tieto prístupy a riešenia nie je možné ľahko upraviť a opätovne využiť. Taktiež vyžadujú značnú prácu pri tvorbe programov, ktoré vytvárajú požadované SAT inštancie.

Na odstránenie týchto problémov sme v našej práci vytvorili knižnicu pre jednoduché modelovanie SAT inštancií. Sústredili sme sa špeciálne na rôzne problémy súvisiace s kryptografickými hašovacími funkciami, ale vytvorená knižnica je dostatočne všeobecná a je ju možné použiť aj na iné účely.

S pomocou tejto knižnice sme vytvorili modely pre niekoľko kryptografických hašovacích funkcií. Ďalej sme testovali niekoľko SAT solverov, optimalizácií a heuristík a porovnávali sme ich efektivitu. Okrem iného sme využívali logický minimalizér *Espresso* na redukciu veľkosti inštancií, a tiež vlastné poradie vetvenia pri ohodnocovaní premenných s pomocou upraveného SAT solveru.

KEÚČOVÉ SLOVÁ: SAT, kryptografia, hašovacie funkcie, heuristiky

Abstract

SAT solvers are a universal tool for finding solutions to boolean satisfiability problems. In the past they have also been used for cryptographic problems, such as finding preimages for hash functions or obtaining the key for stream ciphers. However these solutions are not easily reusable or modifiable and require significant effort to produce the programs that create the SAT instances.

To avoid these issues we create a modeling library that allows simple creation of SAT instances. While we focus specifically on various problems related to cryptographic hash functions, the library is generic enough that it can be used for other purposes as well.

Using this library we create models for several cryptographic hash functions. Various SAT solvers, optimizations and heuristics are evaluated on these models to compare their performance. These include the use of the *Espresso* logic minimizer to reduce the instance size, forcing custom variable branching order with help of modified SAT solvers and others.

KEYWORDS: SAT, cryptography, hash functions, heuristics

Contents

Introduction	1
1 Background	2
1.1 SAT solvers	2
1.1.1 Implementation	2
1.1.2 Interfacing with SAT solvers	4
1.2 Cryptographic problems	5
1.2.1 Hash functions	5
1.3 Encoding	8
1.3.1 Boolean circuits	8
1.3.2 Tseitin transformation	9
1.3.3 Arithmetic gates	9
1.4 Related work	10
2 Modeling library	11
2.1 Goals	11
2.2 Our approach	12
2.2.1 Operator overloading	12
2.2.2 Boolean circuit creation	12
2.2.3 Instance generation	13
2.2.4 Solving and working with solution	13
2.3 Using the library	13
2.3.1 Simple example	14
2.3.2 SHA-1 example	15
2.3.3 Comparison to existing work	16
2.3.4 The N-Queens problem	18
2.4 Hash function toolkit	20
3 Optimizations	21
3.1 Merging operators	21
3.2 Expression encoding	22
3.3 Branching order	23

3.3.1	Implementation	23
4	Experiments	25
4.1	Methodology	25
4.2	Merging operators	27
4.3	Expression encoding evaluation	27
4.3.1	SHA-1 analysis	28
4.3.2	SHA-3 analysis	28
4.3.3	Discussion	29
4.4	Branching order evaluation	30
4.4.1	SHA-1 analysis	30
4.4.2	SHA-3 analysis	31
4.4.3	Discussion	32
	Conclusion	33
	A Library	34
	B HashToolkit	35
	Bibliography	36

Introduction

SAT solvers are programs for finding solutions to instances of boolean satisfiability problems, which consist of boolean variables and clauses. Using backtracking and various heuristics these programs explore the space of all possible truth assignments of the variables and look for one which leads to all clauses being satisfied. Since the SAT language is *NP*-complete, SAT solvers can be used to model and solve any problem which itself is in the *NP* class.

This is not only a theoretical observation – SAT solvers are commonly used in a wide variety of areas including formal program verification, model checking or finding solutions to difficult problems such as routing connections in microchips.

Cryptographic *hash functions* are functions that are easily computable but which are computationally hard to invert. The process of finding an input to hash function that produces a desired output is called a *preimage attack*.

There are several works that use the power of SAT solvers to solve instances modeling such preimage attacks. These instances were created by special handcrafted programs which generated the required variables and clauses based on some input parameters.

However this approach requires the creation of such programs, a process which is both time consuming and error-prone. Furthermore, once such a program is created for one hash function, it is not easily modifiable for a different one and much of the work needs to be repeated.

We propose a solution based on *operator overloading* and *dynamic typing* (features of the *Python* programming language) which allows modeling problems for SAT with minimal work. We create a library for this purpose that can be used together with off-the-shelf implementations of hash functions available online. As we demonstrate, only minimal changes to such implementation are required to turn them into programs capable of generating SAT instances.

While this automatic and transparent process is very convenient, it gives us less control over the generated instances which might lead to inefficiencies. To help reduce them we implement some optimizations and also evaluate whether they are even required.

CHAPTER 1

Background

1.1 SAT solvers

SAT solvers (shortened from *satisfiability*) are programs which take a boolean satisfiability problem and find a solution or solutions to this problem. The problem, which we will call an *instance*, consists of a number of variables and boolean clauses composed of these variables. The solution, if it exists, is an assignment of truth values to the variables such that the clauses are satisfied. If no such assignment exists the instance is called *unsatisfiable*.

Most modern SAT solvers expect input in *conjunctive normal form* in which all clauses are disjunctions of literals (a literal is either a variable or a negation of a variable) and the instance is a conjunction of such clauses. Thus all clauses must be true, and in each clause at least one literal must be true in order for the instance to be satisfied. As the name suggests this is a normal form, which means every boolean expression can be converted to an equivalent one that satisfies these requirements. Thus we will from now on only consider such instances without loss of generality.

By modeling a decision problem as a SAT instance we can take advantage of the advanced optimizations and heuristics employed by modern SAT solvers. Examples of such uses in practice include formal program verification [5, 6], model checking [4, 24] or even routing of connections in microchips [29, 30].

1.1.1 Implementation

Since SAT is an *NP*-complete language [7] and assuming $P \neq NP$ we can not in general find solutions in deterministic polynomial time. A simple exponential time algorithm is to try every possible boolean truth assignment. To remember which assignments have been tried and which have not we can implement this algorithm as a recursive backtracking, where for every input variable we try in turn both possible values. This will only require linear memory to implement in terms of number of variables.

While there are no known algorithms that perform asymptotically better in the worst case, we can still improve on this simple exponential time algorithm significantly by avoiding paths that are guaranteed to lead to a *conflict* – a clause in which all literals have been assigned to false.

DPLL algorithm

First such improvement is the *DPLL algorithm* (Davis-Putnam-Logemann-Loveland, [8, 9]) which enhances the naïve backtracking algorithm by first applying two rules at each step of the recursion:

Unit propagation

We will call a clause a *unit* if it only contains one unassigned literal. If the clause is already satisfied we can ignore it during further search. Otherwise it can only be satisfied by assigning the necessary value to the remaining literal. This means there will be no branching for this literal and thus the search time will be reduced by avoiding the obviously wrong choice.

Pure literal elimination

We will call a literal *pure* if its variable only occurs in the formula with a single polarity (that is, always negated or always non-negated). All pure literals can be assigned the required value to make all clauses containing them true and these clauses can then also be ignored during further search.

After these rules are applied we continue as before by picking an unassigned variable and trying both possible truth value assignments recursively. These two simple rules will help to reduce the search space the algorithm has to look through and thus reduce the total running time.

Conflict-Driven Clause Learning

It can happen that after some partial assignment of variables there is already a conflict and it is not possible to extend this assignment to a satisfying one. However, the DPLL algorithm will continue to explore the entire search subspace. With *conflict-driven clause learning* (CDCL, [20, 40]) we can avoid this by the use of *implication graph* and *learnt clauses*.

The *implication graph* is a directed graph in which vertices are variables with their assignment. These will be marked as either *decision* or *forced*.

At first we start with an empty implication graph and execute the DPLL algorithm. Every time we make an arbitrary decision (the recursive step in the algorithm) we will add a decision vertex representing this variable and the assignment we have chosen into the graph. Every time either of the two rules of the DPLL algorithm forces some

assignment of a variable we will add a forced vertex into the graph, with edges from every vertex that is a part of this forced choice.

For example, with clause $(x \vee y \vee z)$ and with partial assignment $x = 0, y = 0$ the unit propagation rule will force the assignment of $z = 1$. We will add this as a forced vertex, with edges from both $x = 0$ and $y = 0$.

When we add an vertex to the graph for some variable but this variable is already present with the opposite assignment we have found a conflict. Now comes the part of CDCL which speeds up this search – we will find all the decision vertices (variables) that are responsible for this conflict. Let us call them x_1, \dots, x_k with assignments v_1, \dots, v_k . From this we can build a *learnt clause* L :

$$\begin{aligned} (x_1 = v_1 \wedge \dots \wedge x_k = v_k) &\Rightarrow \perp \\ \perp &\Rightarrow \overline{(x_1 = v_1 \wedge \dots \wedge x_k = v_k)} \\ \top &\Rightarrow (x_1 = \overline{v_1} \vee \dots \vee x_k = \overline{v_k}) \\ L &:= (x_1^{(\overline{v_1})} \vee \dots \vee x_k^{(\overline{v_k})}) \end{aligned}$$

The notation $x^{(v)}$ is used to represent x if $v = 1$ and \bar{x} if $v = 0$. Symbols \top and \perp represent tautology and contradiction, respectively.

We can now add this learnt clause L into the list of clauses we have to satisfy, since not satisfying L is guaranteed to lead to a conflict. In the next step of the backtracking algorithm we will not return only a single level up from the recursion (that is, removing the assigning of just a single variable) but all the way back until we reach the first point where one of the conflicting variables was assigned a value.

This method is used in all modern SAT solvers and improves their performance significantly over the DPLL algorithm [40].

1.1.2 Interfacing with SAT solvers

Since our work depends on providing a suitable input to SAT solvers we will briefly describe the widely supported *DIMACS CNF* format¹ which we use. Later in section 3.3.1 we will describe augmenting this format for optimization purposes.

The file begins with a line in form

p *cnf vars clauses*

where *vars* and *clauses* are the number of variables and clauses in this instance, respectively.

The following lines are in the form

$v_1 \ v_2 \ \dots \ v_i \ 0$

¹As in the case of *JPEG*, the acronym refers to the institute that created the format – the *Center for Discrete Mathematics and Theoretical Computer Science*.

and each represents a single clause consisting of a disjunction of literals v_1 through v_i . Each literal v_i is an integer between 1 and $vars$ for a variable in positive polarity or an integer between -1 and $-vars$ for negative polarity.

The output of a SAT solver for a satisfiable instance is a list of $vars$ numbers. For every $1 \leq v_i \leq vars$ either v_i or $-v_i$ is present in the list, indicating the polarity of the i -th variable in some satisfying assignment.

1.2 Cryptographic problems

We make use of SAT solvers to find solutions to instances which represent a cryptographic primitive. The variables represent the inputs, outputs and the internal state of the primitive. The clauses describe the behavior of the primitive as a relation between the variables. Solutions to such instances are pairs of inputs and corresponding outputs.

By placing additional restrictions on the possible solutions (by introducing more clauses) we can find a truth assignment that gives us the desired inputs and outputs to the cryptographic primitive. With a stream cipher this can mean finding the secret key (input) by restricting the output to observed keystream. With hash functions this can mean reversing the output for a *preimage* attack or finding two different inputs with the same output (a *collision*).

In our work we will focus specifically on hash functions, which we describe in more detail in the following section.

1.2.1 Hash functions

A *hash function* is designed to reduce a long input *message* into a shorter, fixed-length *digest* (short from *message digest*, also called *hash*). The computation in this direction is often designed to be fast and efficient. However, a *cryptographic* hash function should further have the property that it is infeasible to compute it in the opposite direction – that is, given a digest to find a message.²

By infeasible (or hard) we mean that the computation should take time exponential to some parameters of the hash function and its input, and thus is not practical for sufficient input sizes.

More precisely, hash function h is a function $h : X \rightarrow Y$ where X is the (potentially unbounded) set of input messages and $h(x) \in Y = \{0, 1\}^n$ is an n -bit message digest. For a more formal discussion of hash functions (which is beyond the scope of our work) we refer the reader to [36].

The properties which we require these functions to have are:

²Not *the* message, since there might be multiple input messages that have the same digest.

Preimage resistance: Given a digest d it is hard to find a message m such that $h(m) = d$.

Second preimage resistance: Given a message m_1 it is hard to find a message $m_2 \neq m_1$ such that $h(m_1) = h(m_2)$.

Collision resistance: It is hard to find messages m_1, m_2 such that $h(m_1) = h(m_2)$ and $m_1 \neq m_2$.

Most modern hash functions have multiple rounds in which the state is modified using a *round function*. Starting from the initial state (usually some constant) a part of the input message (with some form of padding) is processed and a new state produced. This is repeated until the whole input has been processed and the final state is then used to produce the digest.

Reduced hash functions and partial attacks

The expected time to find a preimage for actual hash functions as used in practical cryptography is usually so large we couldn't obtain a solution in a reasonable time. For this reason we will be working with *reduced* instances of hash functions – modifications that are similar but easier to attack. To create such reduced instances we can simply lower the number of rounds used in the computation.

Similarly, finding a full preimage where all the output digest bits are fixed to a certain value could take a long time. Instead we will focus on *partial* preimage attacks, where only a certain number of output digest bits are fixed.

The SHA hash functions

We will briefly describe two hash functions on which we focus in our work and to whose internals we will be referring later.

SHA-1, short for *Secure Hash Algorithm* [31] is a widely used hash function based on the *Merkle-Damgård* construction [26] with output digest size of 160 bits. The internal state consists of five 32-bit words h_0, \dots, h_4 which are first initialized to some constant values.

After padding³ the message to a length that is multiple of 512 bits, it is processed in *chunks* of that same size. Each chunk is broken into 16 32-bit words W_0, \dots, W_{15} . From those 64 additional words W_{16}, \dots, W_{79} are created by *extending* the chunk:

$$W_i = (W_{i-3} \oplus W_{i-8} \oplus W_{i-14} \oplus W_{i-16}) \lll 1 \quad (16 \leq i < 80) \quad (1.1)$$

where \lll means cyclic left shift on 32-bit words and \oplus means bitwise exclusive or.

These words are then processed in 80 rounds, during which a set of five 32-bit *round words* A, B, C, D and E are updated. These are first initialized to the current values

³Details of the padding are not important for our purposes and can be found in the specification.

of h_0 through h_4 to make the result of every chunk depend on all previous chunks as well.

In each round i , first the values f and k are computed based on the round number. Then T is computed as

$$T = (A \lll 5) + f + E + k + W_i$$

where addition is performed modulo 2^{32} .

Finally, the five round words are updated as follows

$$E \leftarrow D \quad D \leftarrow C \quad C \leftarrow B \lll 30 \quad B \leftarrow A \quad A \leftarrow T$$

The value of k is one of four constants, depending on the value of $\lfloor \frac{i}{20} \rfloor$ (that is, one value for rounds 0 to 19, another for rounds 20 to 39 and so on). Similarly, f is computed by one of four *round functions*:

$$\begin{aligned} f_0 &= Ch(B, C, D) = (B \wedge C) \oplus (\bar{B} \wedge D) \\ f_1 &= f_3 = B \oplus C \oplus D \\ f_2 &= Maj(B, C, D) = (B \wedge C) \oplus (B \wedge D) \oplus (C \wedge D) \\ f &= f_{\lfloor \frac{i}{20} \rfloor} \end{aligned}$$

After the last round the values of the round words are added to the current state (modulo 2^{32}). Once all chunks have been processed this way the resulting state is the output digest.

SHA-3 is a recently standardized [11] hash function family based on the *Keccak* family [3] that won the selection competition against fifty other candidates. It is based on a *sponge* construction which allows the output digest size to be variable.

We will specifically focus on the variant *SHA-3-512* which has 1600-bit state updated in 24 rounds (divided into five steps called θ , ρ , π , χ and ι) and produces a 512-bit digest. The state is represented as a 5×5 matrix S of 64-bit words.

We will describe the steps θ and χ in little more detail since we will be referring to them later in section 4.3.2. In the θ step an auxiliary array C is filled as follows:

$$C_i = S_{i,0} \oplus S_{i,1} \oplus S_{i,2} \oplus S_{i,3} \oplus S_{i,4} \quad (0 \leq i < 5)$$

This is then used, along with two additional arrays B and D to compute the new values of the state S in the χ step:

$$A_{i,j} = B_{i,j} \oplus (\overline{B_{i+1,j}} \wedge B_{i+2,j}) \quad (0 \leq i, j < 5)$$

The most important difference compared to SHA-1 is that not the whole state is used for output. Instead it is divided into two parts: the 576-bit *bitrate* and 1024-bit *capacity*, and only the bitrate part is used for the output while the capacity part stays hidden.

The sponge construction then works in two phases – *absorbing* and *squeezing*. During absorbing, chunks of the padded input message are xored with the state and the 24 rounds are performed until the message is fully processed. Afterwards the bitrate part of the state is used for the output digest. If it is not long enough another series of 24 rounds is performed on the whole state and the resulting bitrate part is appended to the digest and so on until the desired length is reached.

This flexible design allows arbitrary output digest sizes for specific applications. It also makes reversing the process harder, since the entire output state is not known.

1.3 Encoding

The most important step when using SAT solvers for any problem is the encoding of the model to a suitable instance. In case of cryptographic problems this previously (see section 1.4) required significant effort of either generating the SAT instance by hand, or by first translating the model to a more suitable form and then using existing tools to generate the instance.

In our work we created a library [33] which automates large parts of this process. This makes it easier, faster and less error-prone. To achieve this we made use of *boolean circuit* representation to translate almost unmodified implementations of cryptographic primitives to SAT instances transparently.

1.3.1 Boolean circuits

The cryptographic primitive we want to encode into a SAT instance can be thought of as a *boolean circuit* – a *DAG* (directed acyclic graph) in which vertices are boolean operators (*gates*, such as the *AND gate*, *XOR gate* and so on). The edges represent flow of values from one gate to another. *Inputs* are special vertices that have no incoming edges. *Outputs* are also special vertices that have exactly one incoming edge and no outgoing edges.

Given this representation the simplest way to encode this as a CNF formula would be to take each output vertex and recursively expand it in the following way: At first we start with an output vertex v encoded as (v) . This will have one incoming edge from a gate vertex g with inputs x_1, \dots, x_k . We will encode this as $g(x_1, \dots, x_k)$ where g is the appropriate boolean operation of the gate – for example, if the output vertex is connected to an *AND* gate this would be $(x_1 \wedge \dots \wedge x_k)$.

Now we repeat this process recursively, expanding each gate node until the encoding only refers to the input vertices. The resulting encoding has to be converted into conjunctive normal form and can then be solved with any SAT solver.

However, this approach produces very large output formula with many clauses. Since we are recursively expanding each vertex until we reach the input vertices, large subgraphs which are referenced (connected by an outgoing edge) multiple times will needlessly be repeated in the output encoding. The total length of the formula then can be exponential in the size of the input circuit.

1.3.2 Tseitin transformation

To reduce the number of clauses of the resulting encoding we can use the *Tseitin transformation* [42]. Instead of generating a large number of clauses when expanding the circuit we will add a new variable for each vertex.

Each gate vertex can then be encoded as a boolean function with constant number of clauses using only variables corresponding to gates (or inputs) that are connected via an incoming edge. The following table shows how to encode the most common boolean gates with two inputs A and B . Variable C refers to the new variable representing this gate in the encoding.

<i>AND</i>	$A \wedge B$	$(C \vee \bar{A} \vee \bar{B}) \wedge (\bar{C} \vee A) \wedge (\bar{C} \vee B)$
<i>OR</i>	$A \vee B$	$(\bar{C} \vee A \vee B) \wedge (C \vee \bar{A}) \wedge (C \vee \bar{B})$
<i>XOR</i>	$A \oplus B$	$(\bar{C} \vee \bar{A} \vee \bar{B}) \wedge (\bar{C} \vee A \vee B) \wedge$ $(C \vee \bar{A} \vee B) \wedge (C \vee A \vee \bar{B})$

Other binary gates such as *NAND* can be encoded similarly. In case of multiple inputs we can either extend this encoding or replace the n -ary gates with multiple binary ones.

This encoding produces formula with linear number of variables and linear length with respect to the size of the boolean circuit, if we limit it to unary and binary gates.

1.3.3 Arithmetic gates

Most cryptographic algorithms make heavy use of arithmetic operations, such as modular addition. These usually work on variables of some fixed size, for example 32-bit integers.

The n -bit variable can be represented using n binary variables in the SAT instance. However, these operations can not be performed on individual bits like in the case of boolean operators. We need to introduce additional helper variables that will represent the carry bits during the addition operation.

This can be thought of as taking the boolean circuit for a binary adder with carry (either ripple-carry or lookahead-carry), composed of several full-adder circuits. These can then be encoded using the Tseitin transformation.

However, for simplicity we extend our model of boolean circuits to support modular addition nodes directly. When encoding these nodes to CNF we use the additional carry variables and output the clauses required to model a binary adder.

1.4 Related work

The idea of using SAT solvers for cryptographic problems was first introduced in [23]. The authors designed an encoding of the *DES* (*Data Encryption Standard*) symmetric cipher and created a program to generate instances. However the work is very specific to *DES* and modifying the tool for a different cipher would require significant changes.

The first attempt to simplify this process can be found in [18] where *operator overloading* (see section 2.2.1) in the *C++* language is used. Our library also makes use of this feature with the additional advantage that we use a *dynamic* programming language (*Python*) and therefore require smaller changes to existing code. Additionally their work uses only simple Tseitin transform without additional optimizations, and the code for the tool is not available.

Another work which automates the generation of instances makes use of the *Verilog* hardware description language⁴ [28]. After writing the cryptographic primitive in this language a free, but proprietary, compiler is used to generate equations which are then turned to a CNF instance. Since *Verilog* is quite a niche language most cryptographic primitives do not have implementations available. Additionally the toolkit itself is also not publicly available.

An in-depth analysis of *SHA-1* preimage attacks was done in [32]. The instances were generated using a custom, hand-built 1000-line *C++* program. The experiments investigated the speed of various SAT solvers, effects of preprocessing and simplifying the instance and various heuristics.

⁴HDLs describe the behavior of logic circuits. They are used for programming FPGAs or designing ASICs.

CHAPTER 2

Modeling library

The works mentioned in previous chapter created their models for various cryptographic problems mostly by hand. While the results obtained are interesting, they are hard to reproduce by others. Also this approach does not help to solve similar problems (like using a different hash function instead), as a new model would have to be created from scratch.

To address these issues we provide an easy to use and reusable library for modeling SAT instances. While the library can be used for modeling any problem we specifically focus on making modeling cryptographic problems as simple as possible. In this chapter we will state our goals for this library, describe its design and inner functionality. We will also show examples of its use.

2.1 Goals

The main goals of our library are as follows:

Existing implementation reuse: In order to simplify the modeling of cryptographic primitives as much as possible we want to allow reuse of existing implementations. Most commonly used primitives – such as hash functions, block and stream ciphers and others – have widely available implementations in all popular programming languages.

The library should therefore allow using these implementations with only minor changes. In addition to saving time this also makes the modeling less error-prone as we can build upon a well tested implementation.

Output abstraction: The library should take care of generating the output in proper format for some SAT solver. With solvers that support advanced features, such as *XOR* clauses, it should be possible to take advantage of them.

Model parsing: After successfully solving the instance with a SAT solver we obtain a model in form of a satisfying variable assignment. The library should be able to load this model and map the truth assignment back to variables defined by the user. This makes it easy to extract for example the colliding messages out of the model.

2.2 Our approach

To achieve the goals stated in previous section we take advantage of a technique called *operator overloading*. This is a feature present in many programming languages. For our library we have decided to use the *Python* language which also supports it. Python has the additional advantage that it is very popular and therefore has implementations of virtually all commonly used cryptographic primitives.

2.2.1 Operator overloading

As the name suggests, operator overloading allows us to overload (override) existing behavior of operators in some programming language. While the feature is only *syntactic sugar* (which means it does not allow us to do anything more than would be possible without it; it just simplifies the syntax) it not only greatly increases the readability of the code, but also allows us to reuse existing implementations as we will show.

The reason this feature is useful in our library is that we can change the type of some variables in existing code without having to change anything else. For example, we can take an implementation of some hash function and change the type of all variables from the built-in integers to our new data type. Without operator overloading this could not be possible, since operators such as addition or bit shift would not be defined for our new type by the language. With operator overloading however we can provide these definitions ourselves.

Our library provides a new data type *BitVector* that supports all the operations as the built-in integer type. Since Python uses *dynamic typing* it is sufficient to change the types of the constants used by the cryptographic primitive implementation. All other variables are a result of operations on these constants and will therefore have the proper type.

2.2.2 Boolean circuit creation

The difference between the built-in integer type and *BitVector* is that while the integer variable only holds one given value, our type instead stores how its value can be obtained from other variables.

More specifically, each time an operation is applied to one or more operands (variables of type *BitVector* or constants) the result is another instance of type *BitVector* which stores these operands. That means that the output of some cryptographic primitive is not a single value but instead a boolean circuit representation. The circuit will form a directed acyclic graph.

2.2.3 Instance generation

Once we have the boolean circuit for a model we can then take this representation and output a SAT instance using the Tseitin transformation described in section 1.3.2.

The order of clauses in the output is irrelevant, however since the circuit forms a DAG we can process it in topological order. For every node (representing an operator applied to one or more operands) we first assign numbers to all required variables. This is because the *DIMACS* input format (described in section 1.1.2) used by most SAT solvers uses integers to refer to variables.

In most cases we have one variable for every bit of the vector. However, an addition node needs additional carry variables. On the other hand, for a negation node we don't need to introduce additional variables, as we can simply use the variables already assigned to the operand with reversed polarity. Similarly for cyclic bit shift we can reuse existing variables but with different ordering.

Once all variables have been assigned integer values we pass through all the nodes again. This time we generate the clauses which model the operator behavior. The number of clauses required depends on the operator and the bit width of the *BitVector* node.

2.2.4 Solving and working with solution

After the model has been turned into an instance we can run a SAT solver on this generated list of clauses and wait for it to terminate. If the instance is satisfiable the solver will find a satisfying truth assignment and output it as truth values for all the variables. The library will load, parse and store this data for later use.

We can then easily query any of the *BitVector* variables for its value. The mapping from variables to integer labels will be used to find the appropriate assignment and to reconstruct the value of the node.

2.3 Using the library

We will demonstrate the use of our library on two examples. The first one is very simple and shows the entire process from start to finish. The second example shows a real-world use case – modifying existing *SHA-1* implementation to use our library for

CHAPTER 3

Optimizations

Modern SAT solvers use many advanced optimizations and heuristics to improve the solving time. However these are usually designed for general problem instances. With additional knowledge about a specific problem we can try to come up with better heuristics which lead to decrease of solving time.

In this chapter we will describe several optimizations that we implemented and evaluated. Some of these involve a simple change in output instance generation, others involve changes to the SAT solvers themselves.

3.1 Merging operators

Since all operators in the *Python* programming language are either unary or binary so are all the nodes in the created boolean circuit. Writing an expressions such as $X = A \ \& \ B \ \& \ C \ \& \ D$ will result in the boolean circuit shown in figure 3.1(a) since the *and* operator is left-to-right associative.

A simple recursive algorithm can be used to walk over the boolean circuit in reverse topological order and merge such structures into a single n -ary node. At each step, if one or both of the operands of some boolean operator node are nodes of the same type, they can be merged together to a single node. After repeating this procedure for the whole tree we obtain the result shown in figure 3.1(b).

We implemented such optimization in our library and modified the clause generation code to support arbitrary arities. This is a simple optimization that can reduce the number of variables and clauses required to encode the given model as a SAT instance. Next we describe a more advanced method that can optimize not just sequences of identical operators but arbitrary expressions.

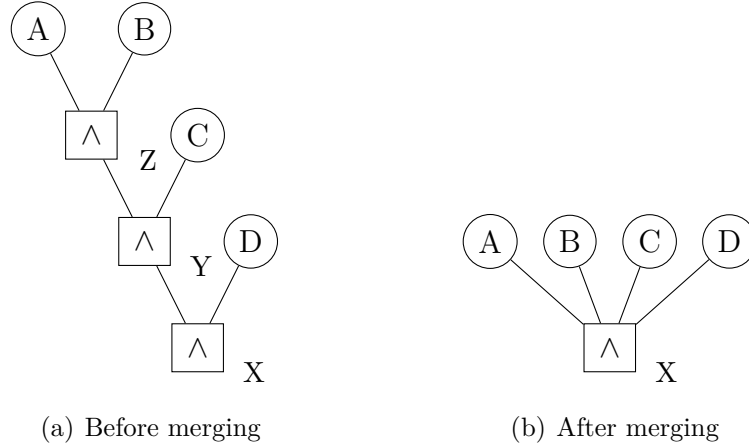


Figure 3.1: Comparing the effect of operator merging on the boolean circuit resulting from the expression $X = A \& B \& C \& D$.

3.2 Expression encoding

The automatic and transparent conversion from unmodified source code to SAT instance via operator overloading and boolean circuits may lead to suboptimal encoding of various expressions. For example, the *choice* round function in *SHA-1*

$$Ch(x, y, z) = (x \wedge y) \oplus (\bar{x} \wedge z)$$

leads to an encoding with three variables and ten clauses (three clauses for each *and* gate and four for the *xor* gate) if we apply the Tseitin transformation directly:

$$\begin{array}{ll}
 (\bar{a} \vee x) \wedge (\bar{a} \vee y) \wedge (a \vee \bar{x} \vee \bar{y}) & \text{Left AND gate} \\
 (\bar{b} \vee \bar{x}) \wedge (\bar{b} \vee z) \wedge (b \vee x \vee \bar{z}) & \text{Right AND gate} \\
 (c \vee a \vee \bar{b}) \wedge (c \vee \bar{a} \vee b) \wedge (\bar{c} \vee a \vee b) \wedge (\bar{c} \vee \bar{a} \vee \bar{b}) & \text{XOR gate}
 \end{array}$$

A better encoding using just six clauses and one variable is shown in [32]:

$$\begin{aligned}
 &(\bar{a} \vee \bar{x} \vee y) \wedge (\bar{a} \vee x \vee z) \wedge (\bar{a} \vee y \vee z) \\
 &\wedge (a \vee \bar{x} \vee \bar{y}) \wedge (a \vee x \vee \bar{z}) \wedge (a \vee \bar{y} \vee \bar{z})
 \end{aligned}$$

In fact, it is possible to encode this expression with just four clauses and one variable:

$$(a \vee \bar{x} \vee \bar{y}) \wedge (a \vee x \vee \bar{z}) \wedge (\bar{a} \vee x \vee z) \wedge (\bar{a} \vee \bar{x} \vee y)$$

To solve this issue we provide an expression optimization function in our library. Any n -ary expression can be wrapped using this function to replace the standard Tseitin encoding with a (potentially) smaller one, in terms of number of extra variables and clauses required.

CHAPTER 4

Experiments

In this chapter we describe various experiments that were performed to evaluate the effectiveness and necessity of optimizations described in the previous chapter. These experiments compare the time required to solve particular instances across two or more variants. Each variant can be either a modification to the instance itself (by changing the way it is generated), or the instance can be unchanged but rather the behavior of the SAT solver itself is modified.

4.1 Methodology

Since the running time of a SAT solver is not deterministic it is not enough to simply perform a single run for every instance. In fact, given the same instance and the same SAT solver the running time can differ significantly, even by several orders of magnitude. This is because the random heuristic decisions can sometimes find the satisfying assignment by "luck" and other times keep exploring unsatisfiable parts of the search space for a long time. Multiple samples therefore have to be gathered and sound statistical methods used to discover patterns in this noisy data.

Sampling procedure

To counter the high variability in time even for one instance we will solve each instance several times.

Additionally, not all instances for particular parameters (such as number of rounds) are equally hard. For example, reduced hash functions with significantly lower number of rounds than the full hash function do not behave sufficiently as random functions. Some output digests might be significantly less likely to be produced than others, or might not be even be possible. When we are performing a partial preimage attack by fixing several digest bits to be equal to a reference digest (obtained by first hashing a random message) some instances might have fewer solutions than others.

For this reason we also perform all such experiments on multiple instances that have identical parameters but the random reference messages are independently generated.

Censoring

Sometimes we might obtain an instance that takes very long time to solve. To ensure that experiments finish in a reasonable time it is common to enforce a time limit and abort all computations that exceed it. Since in such cases we can not know the true running time we must discard these aborted measurements.

Data sets with such discarded measurements are called *censored*. They make statistical analysis complicated if not impossible, since even trivial statistics such as mean can not be computed without knowing the true censored values.

For this reason we avoided censoring in our experiments. While we did have a time limit in place, all experiments that contained censored runs were discarded and not considered for analysis. To obtain valid experiments we either increased the time limit or reduced some of the parameters (such as number of rounds or length of the preimage) until we got a configuration that finished without any censoring.

Statistical analysis

For plots of solving time versus some parameter of the instance the most useful statistic is the mean. It shows the general trend in behavior and the rate with which the time increases. However it is by itself unreliable since the random variability can affect it significantly.

For this reason we also include 95% confidence intervals in our plots. By performing more and more repetitions and obtaining more samples we can reduce the size of those intervals and obtain reliable results.

It is important to note that standard techniques for computing means and confidence intervals require either normal distribution of the data, or at least some knowledge about the distribution. With SAT solving times we have neither – although the solving times for some cryptographic instances have been shown [2] to have a log-normal distribution we would still have to verify this assumption for our data sets.

Instead we use the BC_a bootstrap procedure [10] for calculating means and confidence intervals that does not require any knowledge or assumptions about the distribution.

To evaluate the effect of some optimization on the running time we need a statistical procedure to decide if there is a significant difference in the running time distribution or if all differences can be attributed to chance. Since experiments can have varying parameters (such as number of rounds) and for each parameter combination we run multiple independent samples a pairwise sample comparison is required. There are several such procedures, such as the Student's t-test or the Tukey's test.

However, both of those make assumptions about the underlying distributions which are not met in our case. For that reason we use the rather less known *Games-Howell* procedure [16] which does not require these assumptions.

Both used procedures are implemented by the *R* statistical computing language [35]. Availability of implementations was also a factor when deciding on methodology – using a tested implementation greatly reduces the chance of errors.

4.2 Merging operators

First we will describe the effects of the operator merging optimization described in section 3.1. Since it is less versatile and powerful than the *Espresso* expression optimization described in the next section we will only evaluate it on one hash function, the *SHA-1*.

While there is a place in the *SHA-3* hash function that could benefit from operator merging (the θ step, where exclusive or of five values is computed), the method used in section 4.3.2 is also faster. This is because the whole boolean circuit does not have to be traversed and modified.

As for *SHA-1* there are two places that can benefit from this optimization. First one is the computation of the extended chunk (1.1 in section 1.2.1). Here four values are combined using exclusive or. Second is the *majority* round function where three values are again combined using exclusive or.

On a full 80 round *SHA-1* instance this reduces the number of variables from 43 232 to 37 216 and increases the number of clauses from 212 463 to 220 655. The reduction in number of variables is quite significant and could be thought to make solving faster.

However, after measuring the running time to find an 8-bit preimage with both with and without this optimization and performing the Games-Howell procedure gives us a very high p-value of 0.43. Therefore we can't reject the null hypothesis that any difference in running times between these variants is not due to chance.

From this we conclude that this operator merging optimization is neither helpful nor required, at least in cases such as cryptographic hash functions where repeated use of the same operator is rare.

4.3 Expression encoding evaluation

In this section we discuss the effect of expression encoding optimizations described in section 3.2. We performed this experiment on two hash functions, the *SHA-1* and *SHA-3*.

Optimizations		Time	Number of		Optimizations		Time	
χ step	θ step	Mean	Clauses	Variables	χ step	θ step	t	p
Off	Off	7.5	717 865	243 904	Off	On	3.21	< 0.01
					On	Off	0.52	0.95
					On	On	2.11	0.15
Off	On	8.5	828 927	220 864	On	Off	3.63	< 0.01
					On	On	0.98	0.75
On	Off	7.4	605 805	167 104	On	On	2.56	0.05
On	On	8.2	716 867	144 064				

Table 4.1: Pairwise comparison of four *SHA-3-512* optimization combinations using the Games-Howell procedure. Only measurements for more than 12 rounds were used to avoid randomness in timing, for a total of $n = 949$ samples per strategy. Numbers of clauses and variables are given for the full 24 round instances. Times are given in seconds.

messages. We considered two possible optimizations in the round function and tested four variants – all combinations of turning on or off these two optimizations. A total of 2000 samples was collected for each variant.

The first optimization was minimizing the expression $x \oplus (\bar{y} \wedge z)$ in the χ step, which is used to fill the 5×5 state matrix S in each round.

The second optimization was in the θ step, where the C vector is filled using an exclusive or of five different values. Without optimization this leads to four extra variables and 16 extra clauses. Using the Espresso minimization will lead to 32 clauses but only one extra variable.

Using the Games-Howell procedure again we obtain the pairwise comparison shown in table 4.1.

We can see that the *xor* optimization – which reduces the number of variables but requires twice as many clauses – in fact leads to higher solving time (mean difference $t = 3.21$) at significance level of $p < 0.01$. Thus the default behavior is more efficient. On the other hand, from the high p-value of 0.75 we can’t reject the hypothesis that optimizing the χ step makes no difference at all.

4.3.3 Discussion

Even with large number of samples to eliminate the intrinsic randomness in SAT solving times we were unable to reject hypotheses that the tested optimizations do not lead to an improvement. From this we conclude that they do not provide significant benefits.

While our library provides this optimization feature to users as we saw in our measurements it is not necessary to use it. Therefore minimal changes to existing, off-the-shelf implementations of hash functions (without the need to identify expressions with non-optimal Tseitin representation) are sufficient to match the hand-optimized instances such as in [32].

(with a mean time of 2.7s) all strategies with $i \geq 5$ had instances which took more than two minutes to solve, after which we aborted the computation. As described in the methodology section of this chapter, we wish to avoid censoring for various reasons. Therefore only the default strategy and strategies with $0 \leq i < 5$ were further evaluated.

For these strategies we collected $n = 256$ samples each. We observed the biggest difference in mean solving time between the strategy with $i = 3$ and $i = 4$ which took 2.6s and 3.1s to solve on average respectively. However the variance in running time is too high and performing the Games-Howell pairwise test on this data gives no statistically significant results. In fact even for the two strategies mentioned here the significance level is $p = 0.22$.

Therefore we can't reject the hypothesis that this kind of optimization does not lead to faster solving times.

4.4.2 SHA-3 analysis

For *SHA-3-512* we tested various branching orders on 8-bit reference preimage attack with number of rounds ranging from 1 to 24 (the maximum for this hash function). For every number of rounds 10 different instances were generated and each was solved 10 times, for a total of 100 samples per round per strategy. We compared the following branching order strategies:

none: No branching order was specified. This is the default unmodified MiniSat behavior.

r0-S-x-y: The S matrix from the first round is branched on first, in column-major order.

r0-S-y-x: Same as previous one, but in row-major order.

rlast-S-x-y and ***rlast-S-y-x***: Same as previous two, but the S matrix from the last round is used instead.

The figures 4.2 and 4.3 show *violin plots* of the distribution of ratios of the solving time and the number of conflicts. Each *violin* also show the mean, median and extreme values.

From these plots we see that while the ratios for the number of conflict are mostly below 1 (meaning that the *r0-S-x-y* strategy leads to fewer conflicts), the time ratios are often higher than 1.

The Games-Howell procedure (figure 4.2) confirms these findings, with the mean difference for number of conflicts between the *none* and *r0-S-x-y* strategies of $t = 17$ at significance level $p < 0.01$. However, for the solving time the high p-value does not let us reject the hypothesis that any difference is due to chance. Note that once again only samples for more than 12 rounds (out of maximum of 24) were included to avoid the strong effect of randomness for instances that solve in very short time.

Strategy	Time	Conflicts	Strategy	Time		Conflicts	
	Mean			t	p	t	p
<i>none</i>	7.8	731	<i>r0-S-x-y</i>	0.65	0.97	17	< 0.01
			<i>r0-S-y-x</i>	0.58	0.98	17	< 0.01
			<i>rlast-S-x-y</i>	0.18	> 0.99	958	> 0.99
			<i>rlast-S-y-x</i>	0.03	> 0.99	958	> 0.99
<i>r0-S-x-y</i>	7.5	250	<i>r0-S-y-x</i>	0.07	> 0.99	0	< 0.01
			<i>rlast-S-x-y</i>	0.83	0.92	17	< 0.01
			<i>rlast-S-y-x</i>	0.68	0.96	17	< 0.01
<i>r0-S-y-x</i>	7.5	250	<i>rlast-S-x-y</i>	0.76	0.94	17	< 0.01
			<i>rlast-y-x</i>	0.60	0.97	17	< 0.01
<i>r0-last-x-y</i>	7.8	731	<i>rlast-S-y-x</i>	0.15	> 0.99	0	> 0.99
<i>r0-last-y-x</i>	7.8	731					

Table 4.2: Pairwise comparison of various branching order strategies for *SHA-3-512* using the Games-Howell procedure. Only measurements for more than 12 rounds were used to avoid randomness in timing for a total of $n = 480$ samples per strategy. Times are in seconds.

The *r0-S-y-x* strategy behaves the same as *r0-S-x-y* – the number of conflicts for every instance is identical and the differences in time are negligible. On the other hand the strategies starting with the last round’s *S* matrix lead to the same behavior as not providing any branch ordering at all (the *none* strategy).

Similar experiments were performed with the auxiliary vectors and matrices *B*, *C* and *D* with the same results – enforcing branching order did not lead to better solving times.

4.4.3 Discussion

The fact that these branching orders do not change the solving time significantly must mean that either their choice does not lead to any forced assignments and conflicts (which is highly unlikely) or that the default MiniSat heuristic is also picking them for branching first. The second case means that this optimization is unnecessary and that the default SAT solver heuristic is sufficient in this case.

Conclusion

We have created a library for modeling various problems as SAT instances that can then be solved with SAT solvers. While the library is fairly universal, we have specifically focused on cryptographic problems such as modeling preimage attacks on hash functions. To simplify this use case we make use of operator overloading and dynamic typing that allows the use of our library with existing implementations with minimal changes.

Previous works in this area used handwritten code to generate instances of a specific hash function that were not easily modifiable. On the other hand, this allowed them to optimize the resulting instances by reducing the number of variables and clauses required compared to a naïve Tseitin transformation. To also benefit from such optimizations in our library, we added an option to optimize specific expression and evaluated the effects of such optimizations.

While we found that the optimized instances can be smaller (in terms of number of variables and clauses) and lead to fewer conflicts, the solving time was not improved in any statistically significant way. This leads us to conclude that these optimizations are not required and therefore the minimal changes to existing implementations mentioned above are sufficient to create a reasonable instance.

We then modified the MiniSat solver to see if overriding the default branching order heuristic with the help of additional information about the problem structure will lead to speed improvements. However, same as with previous optimizations, we found that only the number of conflicts was reduced in this way. From this we conclude that the existing heuristics employed by modern solvers behave reasonably on these instances and therefore using any supported off-the-shelf solver is sufficient.

Our work can provide basis for future research in several ways. Firstly, while we have focused on preimage attacks on hash functions, different problems could be easily modeled using our library, which we release with full source code and several usage samples. We believe our approach would prove successful in all areas where SAT solvers have been used previously and even problems not related to cryptography could benefit from the ease of use of our library.

Furthermore, more optimizations of the resulting instance could be implemented and evaluated. And lastly, the *HashToolkit* command line tool could be extended to support more hash function, attack modes and so on.

APPENDIX A

Library

The library we developed is available at

<https://github.com/lacop/master-thesis-code>

or on the CD attachment [33].

The attachment includes:

Modeling library *library/instance.py*

The most important part of the library, used to model boolean circuits, solve them using SAT solver and read the solutions.

Optimizations *library/optimizations.py*

Implementation of the operator merging (section 3.1) and the *Espresso* expression minimization (section 3.2) optimizations.

HashToolkit *hashtoolkit.py, hashes.py*

The *HashToolkit* command line tool (appendix B).

Samples *samples/*

Sample implementations of several hash functions.

Statistics and plots *statistics/*

Code used for statistical analysis and to generate plots in this text.

APPENDIX B

HashToolkit

The *HashToolkit* command line tool can be used to generate, solve and verify (partial) preimage or collision attacks on hash functions. Supported hash functions are *MD5*, *SHA-1* and *SHA-3-512*.

Input and output bits can be individually set to arbitrary values. Output bits can additionally be set to be the same as a randomly generated reference digest.

Moreover, the length of the input message and number of rounds can also be customized. Any SAT solver that supports the *DIMACS* format (section 1.1.2) can be used for solving.

For more details about all the commands line options run the tool with the `--help` flag.

Bibliography

- [1] AJ Alt: *An Implementation of the SHA-1 Hashing Algorithm in Pure Python*. GitHub repository, <https://github.com/ajalt/python-sha1>. 2015.
- [2] Gregory V. Bard, Nicolas Courtois, Chris Jefferson.: *Efficient Methods for Conversion and Solution of Sparse Systems of Low-Degree Multivariate Polynomials over $GF(2)$ via SAT-Solvers*. In: *IACR Cryptology ePrint Archive 2007* (2007), pp. 24–37. eprint: <http://eprint.iacr.org/2007/024>.
- [3] Guido Bertoni, Joan Daemen, Michaël Peeters, GV Assche: *The Keccak reference*. In: *Submission to NIST (Round 3)* (2011).
- [4] Armin Biere, Alessandro Cimatti, Edmund M. Clarke, Yunshan Zhu: *Symbolic Model Checking without BDDs*. In: *Tools and Algorithms for Construction and Analysis of Systems, 5th International Conference, TACAS '99, Held as Part of the European Joint Conferences on the Theory and Practice of Software, ETAPS'99, Amsterdam, The Netherlands, March 22-28, 1999, Proceedings*. 1999, pp. 193–207. DOI: 10.1007/3-540-49059-0_14.
- [5] Sagar Chaki, Edmund M. Clarke, Alex Groce, Somesh Jha, Helmut Veith: *Modular Verification of Software Components in C*. In: *IEEE Trans. Software Eng.* 30.6 (2004), pp. 388–402. DOI: 10.1109/TSE.2004.22.
- [6] Byron Cook, Daniel Kroening, Natasha Sharygina: *Cogent: Accurate Theorem Proving for Program Verification*. In: *Computer Aided Verification, 17th International Conference, CAV 2005, Edinburgh, Scotland, UK, July 6-10, 2005, Proceedings*. 2005, pp. 296–300. DOI: 10.1007/11513988_30.
- [7] Stephen A. Cook: *The Complexity of Theorem-Proving Procedures*. In: *Proceedings of the 3rd Annual ACM Symposium on Theory of Computing, May 3-5, 1971, Shaker Heights, Ohio, USA*. 1971, pp. 151–158. DOI: 10.1145/800157.805047.
- [8] Martin Davis, George Logemann, Donald W. Loveland: *A Machine Program for Theorem-Proving*. In: *Commun. ACM* 5.7 (1962), pp. 394–397. DOI: 10.1145/368273.368557.
- [9] Martin Davis, Hilary Putnam: *A Computing Procedure for Quantification Theory*. In: *J. ACM* 7.3 (1960), pp. 201–215. DOI: 10.1145/321033.321034.

- [10] Thomas J DiCiccio, Bradley Efron: *Bootstrap Confidence Intervals*. In: *Statistical science* (1996), pp. 189–212.
- [11] Morris J Dworkin: *SHA-3 Standard: Permutation-Based Hash and Extendable-Output Functions*. In: *Federal Inf. Process. Stds. NIST FIPS-202* (2015).
- [12] Niklas Een, Niklas Sörensson: *MiniSat: A SAT Solver with Conflict-Clause Minimization*. In: *Sat 5* (2005).
- [13] Tobias Eibach, Enrico Pilz, Gunnar Völkel: *Attacking Bivium Using SAT Solvers*. In: *Theory and Applications of Satisfiability Testing - SAT 2008, 11th International Conference, SAT 2008, Guangzhou, China, May 12-15, 2008. Proceedings*. 2008, pp. 63–76. DOI: 10.1007/978-3-540-79719-7_7.
- [14] R. T. Faizullin, I. G. Khnykin, V. I. Dylkey: *The SAT Solving Method as Applied to Cryptographic Analysis of Asymmetric Ciphers*. In: *CoRR abs/0907.1755* (2009).
- [15] Niels Ferguson, Bruce Schneier, Tadayoshi Kohno: *Cryptography Engineering - Design Principles and Practical Applications*. Wiley, 2010.
- [16] Paul A Games, John F Howell: *Pairwise Multiple Comparison Procedures with Unequal n 's and/or Variances: a Monte Carlo Study*. In: *Journal of Educational and Behavioral Statistics* 1.2 (1976), pp. 113–125.
- [17] Ekawat Homsirikamol, Pawel Morawiecki, Marcin Rogawski, Marian Srebrny: *Security Margin Evaluation of SHA-3 Contest Finalists through SAT-Based Attacks*. In: *Computer Information Systems and Industrial Management - 11th IFIP TC 8 International Conference, CISIM 2012, Venice, Italy, September 26-28, 2012. Proceedings*. 2012, pp. 56–67. DOI: 10.1007/978-3-642-33260-9_4.
- [18] Dejan Jovanovic, Predrag Janicic: *Logical Analysis of Hash Functions*. In: *Frontiers of Combining Systems, 5th International Workshop, FroCoS 2005, Vienna, Austria, September 19-21, 2005, Proceedings*. 2005, pp. 200–215. DOI: 10.1007/11559306_11.
- [19] Philipp Jovanovic, Martin Kreuzer: *Algebraic Attacks Using SAT-Solvers*. In: *Groups Complexity Cryptology* 2.2 (2010), pp. 247–259. DOI: 10.1515/gcc.2010.016.
- [20] Roberto J. Bayardo Jr., Robert Schrag: *Using CSP Look-Back Techniques to Solve Real-World SAT Instances*. In: *Proceedings of the Fourteenth National Conference on Artificial Intelligence and Ninth Innovative Applications of Artificial Intelligence Conference, AAAI 97, IAAI 97, July 27-31, 1997, Providence, Rhode Island*. 1997, pp. 203–208.
- [21] Jonathan Katz, Yehuda Lindell: *Introduction to Modern Cryptography*. Chapman and Hall/CRC Press, 2007.

- [22] Bin Li, Michael S. Hsiao, Shuo Sheng: *A Novel SAT All-Solutions Solver for Efficient Preimage Computation*. In: *2004 Design, Automation and Test in Europe Conference and Exposition (DATE 2004), 16-20 February 2004, Paris, France*. 2004, pp. 272–279. DOI: 10.1109/DATE.2004.1268860.
- [23] Fabio Massacci, Laura Marraro: *Logical Cryptanalysis as a SAT Problem*. In: *J. Autom. Reasoning* 24.1/2 (2000), pp. 165–203. DOI: 10.1023/A:1006326723002.
- [24] Kenneth L. McMillan: *Applying SAT Methods in Unbounded Symbolic Model Checking*. In: *Computer Aided Verification, 14th International Conference, CAV 2002, Copenhagen, Denmark, July 27-31, 2002, Proceedings*. 2002, pp. 250–264. DOI: 10.1007/3-540-45657-0_19.
- [25] Alfred Menezes, Paul C. van Oorschot, Scott A. Vanstone: *Handbook of Applied Cryptography*. CRC Press, 1996.
- [26] Ralph Charles Merkle, Ralph Charles, et al.: *Secrecy, Authentication, and Public Key Systems*. PhD thesis. 1979.
- [27] Ilya Mironov, Lintao Zhang: *Applications of SAT Solvers to Cryptanalysis of Hash Functions*. In: *Theory and Applications of Satisfiability Testing - SAT 2006, 9th International Conference, Seattle, WA, USA, August 12-15, 2006, Proceedings*. 2006, pp. 102–115. DOI: 10.1007/11814948_13.
- [28] Pawel Morawiecki, Marian Srebrny: *A SAT-based Preimage Analysis of Reduced Keccak Hash Functions*. In: *Inf. Process. Lett.* 113.10-11 (2013), pp. 392–397. DOI: 10.1016/j.ip1.2013.03.004.
- [29] Gi-Joon Nam, Fadi A. Aloul, Kareem A. Sakallah, Rob A. Rutenbar: *A Comparative Study of Two Boolean Formulations of FPGA Detailed Routing Constraints*. In: *IEEE Trans. Computers* 53.6 (2004), pp. 688–696. DOI: 10.1109/TC.2004.1.
- [30] Gi-Joon Nam, Kareem A. Sakallah, Rob A. Rutenbar: *Satisfiability-Based Layout Revisited: Detailed Routing of Complex FPGAs via Search-Based Boolean SAT*. In: *FPGA*. 1999, pp. 167–175. DOI: 10.1145/296399.296450.
- [31] National Institute of Standards and Technology: *FIPS PUB 180-1: Secure Hash Standard*. 1995.
- [32] Vegard Nossrum: *SAT-based Preimage Attacks on SHA-1*. MA thesis. 2012.
- [33] Ladislav Pápay: *Modeling Library, Usage Samples and Analysis Code*. <http://dx.doi.org/10.5281/zenodo.50622>. Apr. 2016. DOI: 10.5281/zenodo.50622.
- [34] Constantinos Patsakis: *RSA Private Key Reconstruction from Random Bits Using SAT Solvers*. In: *IACR Cryptology ePrint Archive 2013 (2013)*, p. 26.
- [35] R Core Team: *R: A Language and Environment for Statistical Computing*. ISBN 3-900051-07-0. R Foundation for Statistical Computing. Vienna, Austria, 2013.

- [36] Phillip Rogaway, Thomas Shrimpton: *Cryptographic Hash-Function Basics: Definitions, Implications, and Separations for Preimage Resistance, Second-Preimage Resistance, and Collision Resistance*. In: *Fast Software Encryption, 11th International Workshop, FSE 2004, Delhi, India, February 5-7, 2004, Revised Papers*. 2004, pp. 371–388. DOI: 10.1007/978-3-540-25937-4_24.
- [37] Richard L. Rudell, Alberto L. Sangiovanni-Vincentelli: *Multiple-Valued Minimization for PLA Optimization*. Tech. rep. 5. 1987, pp. 727–750. DOI: 10.1109/TCAD.1987.1270318.
- [38] Bruce Schneier: *Applied Cryptography - Protocols, Algorithms, and Source Code in C (2. ed.)* Wiley, 1996.
- [39] Gustav Sennton: *On Conflict Driven Clause Learning – a Comparison between Theory and Practice*. MA thesis. 2014.
- [40] João P. Marques Silva, Karem A. Sakallah: *GRASP - a New Search Algorithm for Satisfiability*. In: *ICCAD*. 1996, pp. 220–227. DOI: 10.1109/ICCAD.1996.569607.
- [41] Mate Soos, Karsten Nohl, Claude Castelluccia: *Extending SAT Solvers to Cryptographic Problems*. In: *Theory and Applications of Satisfiability Testing - SAT 2009, 12th International Conference, SAT 2009, Swansea, UK, June 30 - July 3, 2009. Proceedings*. 2009, pp. 244–257. DOI: 10.1007/978-3-642-02777-2_24.
- [42] G. S. Tseitin: *Automation of Reasoning: 2: Classical Papers on Computational Logic 1967–1970*. In: Springer Berlin Heidelberg, 1983. Chap. On the Complexity of Derivation in Propositional Calculus, pp. 466–483. DOI: 10.1007/978-3-642-81955-1_28.

Colophon

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