ISLP — Chapter 4: Conceptual Exercises (Classification)

Conceptual 1 (Exercise 4.8 #1)

Show equivalence between logistic and logit forms.

Solution. Logistic form:

$$p(x) = \frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}.$$

Then

$$\frac{p(x)}{1-p(x)} = e^{\beta_0+\beta_1 x}, \quad \log\frac{p(x)}{1-p(x)} = \beta_0 + \beta_1 x.$$

Interpretation. The logistic and logit forms are two ways of writing the same model: one in probability space, one in log-odds.

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Conceptual 2 (Exercise 4.8 #2)

Show that with equal variances, Bayes classification reduces to LDA.

Solution. If $X|Y = k \sim \mathcal{N}(\mu_k, \sigma^2)$, then

$$\Pr(Y = k|x) \propto \pi_k \exp\left(-\frac{(x-\mu_k)^2}{2\sigma^2}\right).$$

Taking logs and dropping constants,

$$\log \Pr(Y = k|x) = \alpha_k + \beta_k x,$$

a linear function of x. This is exactly the discriminant function of LDA.

Interpretation. When class variances are equal, the Bayes optimal classifier is linear — the same as LDA.

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Conceptual 3 (Exercise 4.8 #3)

Why use logistic regression instead of linear regression for classification?

Solution. Linear regression applied to a binary outcome can predict values below 0 or above 1, which are not valid probabilities. Logistic regression maps any input to $(0, 1)$ via the logistic function, and has a natural interpretation in terms of odds.

Interpretation. Logistic regression ensures coherent probability estimates and allows odds-ratio interpretations.

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Conceptual 4 (Exercise 4.8 #4)

Interpret the coefficient β_j in logistic regression.

Solution. The model is $\log \frac{p}{1-p} = \beta_0 + \beta_1 x_1 + \cdots + \beta_j x_j + \cdots$. A one-unit increase in x_j, holding other variables fixed, changes the log-odds by β_j. In odds scale, the odds are multiplied by e^{β_j}.

Interpretation. If $\beta_j > 0$, higher x_j increases the odds of $Y = 1$; if $\beta_j < 0$, it decreases the odds.