

Statistical Thinking 2023_v0

Luis Correia

Lecture #1

<To be reviewed with lecture contents>

Lecture #2

<To be reviewed with lecture contents>

A new model is born when we write:

$$W \sim \text{Binomial}(N, p)$$

with

$$p \sim \text{Unif}(0, 1)$$

Simulation of Bayesian Experiment

```
# definegrid
p_grid <-seq(from=0,to=1,length.out=20)

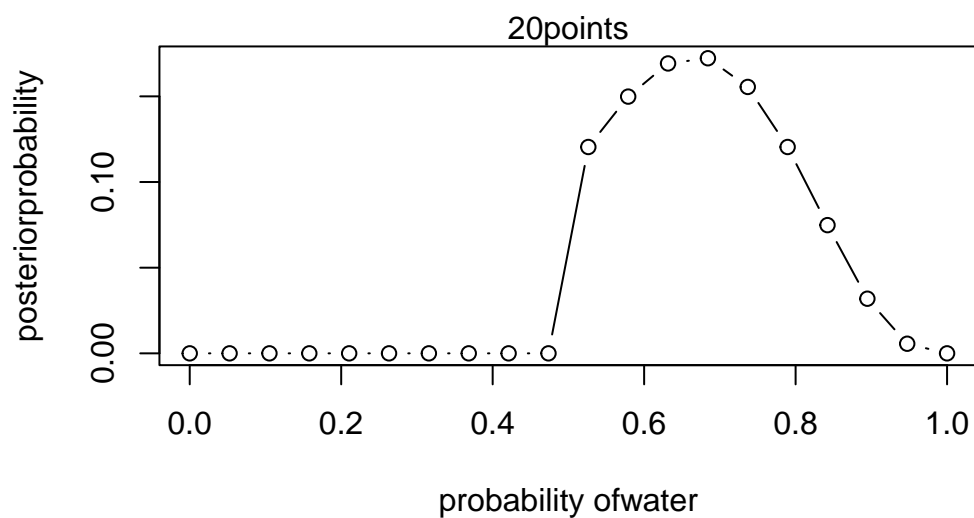
# defineprior
#prior <-rep(1,20)
prior <-ifelse(p_grid<0.5,0,1)

# computelikelihoodateachvalueingrid
likelihood <-dbinom(6,size=9,prob=p_grid)

# computeproductoflikelihoodandprior
unstd.posterior <-likelihood*prior

# standardizetheposterior,soitsumsto1
posterior <-unstd.posterior/sum(unstd.posterior)

plot( p_grid,posterior,type="b",
xlab="probability ofwater",ylab="posteriorprobability")
mtext( "20points")
```

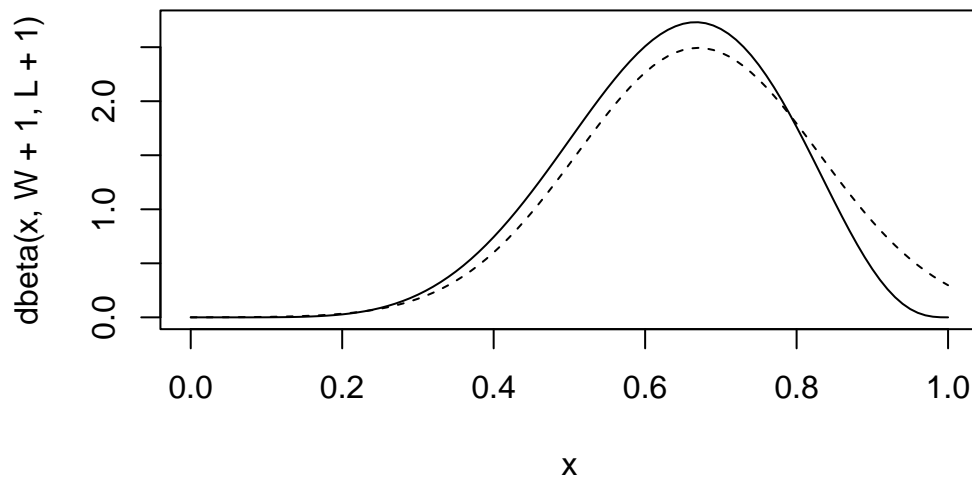


Homework 1

Suppose the globe tossing data had turned into a 4-water and 11-land. Construct the posterior distribution.

```
# 2.7 analytical calculation
W <- 4
L <- 11
curve( dbeta(x,W+1,L+1),from=0,to=1)

# quadratic approximation
curve( dnorm(x,0.67,0.16),lty=2,add=TRUE)
```



```
p <- c(0, .25, .5, .75, 1)
model <- sapply(p, function (p, W, L) return (4*p)^W*(4-4*p)^L)
print(model)
```

```
[1] 0 1 2 3 4
```

```
n_samples <- 1000
p <- rep(NA, n_samples)
p[1] <- 0.5
W <- 6
L <- 3
for (i in 2:n_samples){
  p_new <- rnorm(1, p[i-1], 0.1)
  if (p_new < 0) p_new <- abs(p_new)
  if (p_new > 1) p_new <- 2-p_new
  q0 <- dbinom(W, W+L, p[i-1])
  q1 <- dbinom(W, W+L, p_new)
  p[i] <- ifelse(runif(1) < q1/q0, p_new, p[i-1])
}
```

```
plot(density(p),xlim=c(0,1))  
curve( dbeta(x, W+1, L+1 ),lty=2,add=TRUE)
```

