INTENTIONALLY SCUFFED NOTES

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REMEMBER THAT AT ANY POINT, THESE EQUATIONS, FORMULAE AND NOTES IN GENERAL MAY CONTAIN INTENTIONAL ERRORS. WE ARE NOT LIABLE FOR ANYTHING

1 Introduction

Edit later.

2 Proof by Induction

2.1 The Principle of Mathematical Induction

When inducting, we must follow strict protocols. We first have the initial proposition (usually $P(1)$) where we must prove that LHS = RHS. Then we prove the kth case, then the k+1 case.

We then conclude that by principle of mathematical induction, what we said is true because we proved it to be. Simple as that.

2.2 Sequences and Series

3 Functions

Let us focus on functions.

3.1 Linear equation

A linear equation is a polynomial degree 1. It can be shown in several ways.

- General form: $ax + by = c$
- Slope-intercept form: $y = mx + c$

3.2 Dealing With Infinity

For functions such as the inverse function $(1/x)$, then x cannot be zero otherwise we will have $1/0 = \infty$.

Question 3.1. Why is $1/0 = \infty$?

4 Trigonometry

4.1 Pythagorean Theorem

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$\cot^2\theta + 1 = \csc^2\theta$$

4.2 Angle Sum and Difference

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \pm \sin A \sin B$$

4.3 Double Angle Identities

Double angle identities are derived from the angle and sum difference equations.

We know that

$$\sin(2x) = 2\sin(x)\cos(x)$$

therefore, we can say that

$$\sin(4x) = 4\sin^2(x)\cos^2(x)$$

4.4 n Angle Identities

Using De Moivre's theorem it is known that

$$(|z|\cos\theta)^n = |z|^n \cos n\theta$$

The identity $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$ can be applied to deduce identites for sin.

$$\left(|z|\sin\left(\frac{\pi}{2} - \theta\right)\right)^n = |z|^n \sin\left(\frac{\pi}{2} - \theta\right)$$

Since the unit circle has radius 1, $z = 1$. Hence, the following identities are derived.

$$\cos n\theta = (|n|\cos\theta)^n \qquad \sin n\theta = \left(|n|\sin\left(\frac{\pi}{2} - \theta\right)\right)^n$$

How useful!

5 Limits

5.1 Trigonometric Limits

$$\lim_{\theta\to 0} \frac{\sin(\theta)}{\theta} = 1$$

$$\lim_{\theta\to 0} \sin^\theta(\theta) = 1$$

$$\lim_{\theta\to \infty} \sin^\theta(\theta) = 0$$

Question 5.1. What would $\lim_{\theta\to 0}\cos^\theta(\theta)$ be equal to?

6 Calculus of the Differential Kind

6.1 Deriving from First Principles

Deriving from first principles is the fundamentals of calculus. In fact, we refer to this as the fundamental theorem of calculus.

6.1.1 First Principles with $\sin(x)$

Recall the formula for first principles:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\begin{aligned}
\frac{\mathrm{d}\sin(x)}{\mathrm{d}x} &= \lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h} \\
&= \lim_{h\to 0}\frac{\sin x\cosh + \cos x\sinh - \sin x}{h} \\
&= \lim_{h\to 0}\frac{\sin x(\cosh - 1) + \cos x\sinh}{h} \\
&= \sin x\lim_{h\to 0}\frac{\cosh - 1}{h} + \cos x\lim_{h\to 0}\frac{\sinh}{h} \\
&= \cos x
\end{aligned}$$

7 Integration

7.1 Integration Rules

$$\int k \ \mathrm{d}x = kx + C$$

7.1.1 The Sum Rule

$$\int i + j + k \ \mathrm{d}x = \int i \ \mathrm{d}x + \int j \ \mathrm{d}x + \int k \ \mathrm{d}x = ix + jx + kx + 3C$$

7.1.2 The Difference Rule

Similar to the sum rule, the difference rule is essentially the sum of additive inverses:

$$\int i - j - k \ \mathrm{d}x = \int i \ \mathrm{d}x - \int j \ \mathrm{d}x - \int k \ \mathrm{d}x = ix - jx - kx - 3C$$

7.2 Substitution

7.3 By Parts (Product Rule)

Integration by parts is when you split it to solve by parts. Suppose we have a function $f(x) = u(x)v(x)$ which we want to find the integral of:

$$\int f(x) \ \mathrm{d}x = \int u(x)v(x) \ \mathrm{d}x$$

We can use integration by parts, also known as the product rule of integrals:

$$\int u(x)v(x) \ \mathrm{d}x = \int u(x) \ \mathrm{d}x \times \int v(x) \ \mathrm{d}x$$

Exercise 7.1. Solve $\int x^2 \ \mathrm{d}x$ using integration by parts.

8 Complex Numbers

8.1 Euler's Form

We know that

$$\sin^2\theta + \cos^2\theta = 1$$

We also know the famous equation known as Euler's formula. The imaginary number one. With the pi. No, not the one with polyhedrons. To be more precise, we mean Euler's identity, the "beautiful" equation.

$$e^{i\pi} + 1 = 0$$

Therefore, we can then conclude that:

$$e^{i\theta} + \sin^2\theta + \cos^2\theta = 0$$

$$\frac{e^{i\theta} + \sin^2\theta}{-\cos^2\theta} = 1$$

$$-\frac{e^{i\theta}}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} = 1$$

$$-e^{i\theta}\sec^2\theta - \tan^2\theta = 1$$

$$-e^{i\theta}\sec^2\theta = 1 + \tan^2\theta$$

$$-e^{i\theta}\sec^2\theta = \sec^2\theta$$

$$-e^{i\theta} = 1 \Rightarrow e^{i\theta} = -1$$

What a beautiful conclusion!

9 Linear Algebruh

9.1 Vectors