# CS663 Assignment 5 Question 2 Report

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November 15, 2020

## 1 Solution:

At first we assume that gradients we have are not noisy.
For 1D image of size N it is given that g= h*f , where g is gradient image , h is convolution kernel and f is our original image.

Since image gradients are computed discretely using

g(x) = f(x+1) - f(x) for x∈ (1,2,3,4.........,N), the convolution kernel h for computing the gradient image can be given by [-1,1]

Our motive is to find f from given g & h. We can proceed using the principle of applying Discrete Fourier Transform on equation and using fourier shift theorem

$$G(u) = F(u)(e^{\frac{2\pi u j}{N}} - 1)$$

$$F(u) = \frac{G(u)}{(e^{\frac{2\pi u j}{N}} - 1)}$$

so when $e^{\frac{2\pi u j}{N}}$ -1 = 0 denominator becomes 0 and F(u) becomes undefined. It will happen when u=0. Thus we cannot recover the DC component of the original 1D image using this method and must use another methods of determining the DC component.

In the case of a 2D Image of size N× N, same problem as above is observed while trying to use DFT to obtain f(x,y) from g(x,y) & h(x,y)

Let $f_x$(x,y) and $f_y$(x,y) be the gradients of f(x,y) in the spatial domain with $F_x$(u,v) & $F_y$(u,v) as the corresponding Fourier domain represntations.

In the fourier domain

$$F_x(u,v) = F(u,v)(e^{\frac{2\pi u j}{N}} - 1)$$

$$F_y(u,v) = F(u,v)(e^{\frac{2\pi v j}{N}} - 1)$$

Problems similar to the 1D case arise at u = 0 for $\boldsymbol{F_x}$ and at v = 0 for $\boldsymbol{F_y}$. Even of we know both, the intersection (u = 0, v = 0) will still remain unknown. Thus, we will again have to estimate the DC component using some other technique. This is a big trouble with this approach. ANd till now,in our analysis we have assume gradients to be noise-free. But in reality, they will be not. The noisy gradients will pose greater problems in the reconstruction of the original image (both 1D  2D).