CS663 Assignment 4 Question 5 Report

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NOTE: The codes that are implemented for question 5 are independent from the ones used in question 4.

1 Overview

In this question, we calculate the necessary matrices after applying SVD and PCA methods in the images present in the ORL data-set using the function SVD_PCA_ORL.m. Then we use the values obtained to show the reconstruction of some particular images and the first 25 eigen faces for both SVD and PCA method using the function eig_face_and_recon.m.

2 Code Implementation

2.1 SVD_PCA_ORL.m

In this code, we need to provide the number of basis/eigen faces we want(num_basis), the number of persons we need to consider and the number of images per person.

Given the set of images, we first vectorize each of the N(= persons*images) images, and call the matrix storing these training image vectors in its columns as X. Then we subtract the mean of the training images(X_avg) from all the images and the matrix thus obtained is X_bar.

2.1.1 SVD Implementation

We simply calculate the SVD of X_bar and the obtained matrix of left singular vectors(denoted as U) will the the matrix of the eigen faces. Out of the d columns of U, we take the first **num_basis** vectors(assuming them to be corresponding to largest eigen values, otherwise rearrange appropriately). Thus the final matrix obtained is U_cap(size = $d \times k$). From this we calculate the co-ordinates for each of the training images as $alpha\_svd = U\_cap' * X\_bar$.

2.1.2 PCA Implementation

In the PCA implementation, we first calculate a matrix $L = X\_bar' * X\_bar$. Then we calculate a matrix W which consists of the the eigen vectors of L, this W is then arranged as decreasing order of eigen values(note: all eigen values are non-negative).

Then we define a new vector $V\_pca = X\_bar * W$ which is the matrix of eigen vectors of $X\_bar * X\_bar'$. Normalizing the columns of the matrix V_pca and truncating it to he given number of basis elements/eigen vectors, we get V_pca_cap. then we calculate the co-ordinates for each of the training images as $alpha\_pca = V\_pca\_cap' * X\_bar$.

2.2 eig_face_and_recon.m

In this function, we simply approximate one of the images in the training data set as:

$$x_{aprrox.} = alpha\_svd(1,n) \times U\_cap(:,1) + alpha\_svd(2,n) \times U\_cap(:,2) + ... + alpha\_svd(k,n) \times U\_cap(:,k).$$

here, n is the image number we want to approximate and k is the number of basis vectors available with us. The similar procedure is carried for PCA based algorithm. Also the 25 eigen faces corresponding to the largest eigen values/ singular values are plotted. These are obtained by simply turning the first 25 columns of U_cap and V_pca_cap into images of appropriate size.

3 Results

3.1 Eigen Faces

In the below figures 1 and 2, we have shown the eigen faces obtained for SVD and PCA implementations respectively.

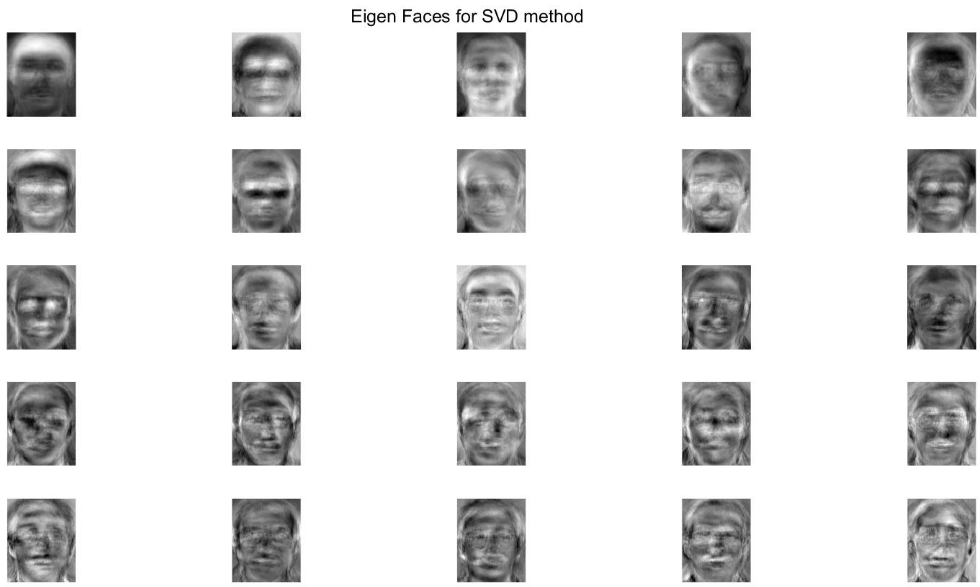

Figure 1: Eigen faces for SVD implementation

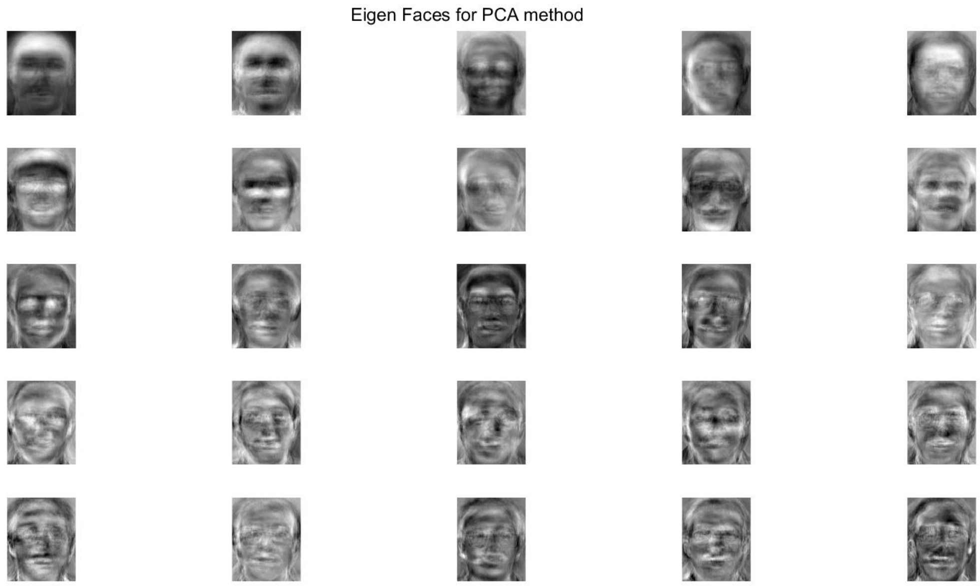

Figure 2: Eigen faces for PCA implementation

From the above images, we can observe that as the eigen value or singular values increases, the images start to show more sharp and high frequency components. Where as the initial images(eg. first five) have mostly blurry and low frequency components. The images obtained are similar for both the SVD and PCA cases.

3.2 Reconstruction of Training Images

In the below images of figure 3 and 4, we have reconstructed two images which are part of the training data set. The number of basis used are $\{2; 10; 20; 50; 75; 100; 125; 150; 175\}$.

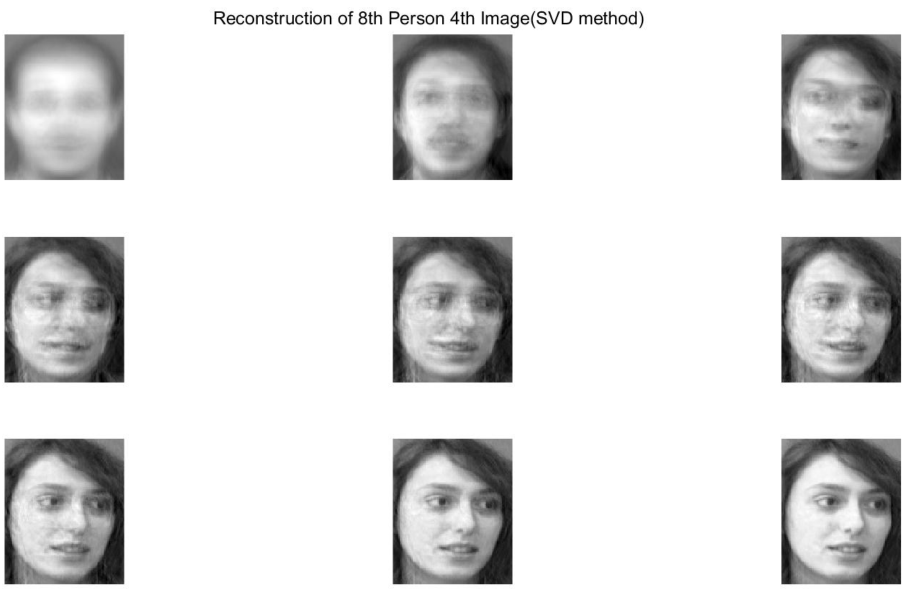

Figure 3: Reconstruction of 8th person 4th image

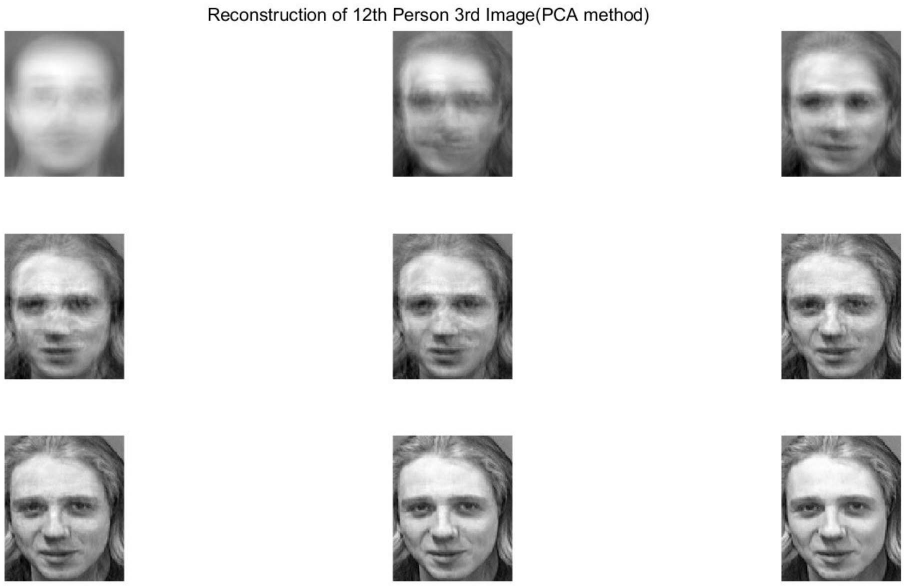

Figure 4: Reconstruction of 12th person 3rd image

From the above two set of images we can observe that by the 4th image, i.e. first 50 basis vectors used, we have a somewhat distinguishable reconstruction of the images. By the point, we start using 125 basis vectors, the reconstructed images are of good enough quality, though there are some visual difference which are eliminated when we take 175 basis vectors. The final images are as good as the original images.