CS663 Assignment 4 Question 1 Report

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In this question, we have implemented a MATLAB schedule named MySVD.m to find out the singular value decomposition of any given $m \times n$ matrix **A**. We have only used the the inbuilt eig() function in MATLAB. Below the algorithm for the code is discussed.

Given any A, we can compute the square matrices AA^T and A^TA. From the properties of singular value decomposition, we know that for $A = USV^T$, the U an V are orthonormal matrices. Thus we will have:

$$AA^T = U(SS^T)U^T$$
$$A^TA = V(S^TS)V^T$$

Hence, the $m \times m$ matrix U and $n \times n$ matrix V contains the eigen vectors corresponding to the matrices AA^T and A^TA respectively.

Then we use the in-built feature eig() in MATLAB to get U and V along with the diagonal matrix of eigen values D_U and D_V respectively. Note that all the eigen values will be positive as they will be square of the singular values:

$$D\_U = SS^T$$
$$D\_V = S^TS$$

Next we rearrange the columns of both U and V such that the eigen vectors corresponding to the highest eigen value comes in the beginning and so on for other columns too in a decreasing order of eigen values.

Now, we have obtained our U and V whose columns are rearranged, and we know S is such that:

$$A = USV^T$$

Thus, to get the value of S, we use the above equation:

$$S = U^TAV$$

As we have calculated the U and V as independent eigen value problems, there might be chances that the values of diagonals in S might be positive or negative. In order to fix this, we invert the signs of negative entries of S and invert the sign of the corresponding columns of V. Then we again take $S = U^TAV$, which gives us the correct results.

In order to check our implementation, we have taken some example matrices A, calculated its corresponding SVD and checked if: $A = USV^T$ and both U and V are orthonormal or not.

NOTE: This implementation will not give the exact same U,V as of when done with the inbuilt svd() in MATLAB, because the matrices U and V can vary up to a sign of the respective columns. Anyhow, the columns for which the singular value is zero will be redundant and hence it can be arbitrary.