Assignment 5 Question 1

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Given : $g_1 = f_1 + h_2 * f_2$

$g_2 = h_1 * f_1 + f_2$.

$h_1 h_2 \rightarrow$ blur.

Taking D. Fourier Transform of above and using the fact that $\mathcal{F}(h_2 * f_2) = \mathcal{F}(h_2) \cdot \mathcal{F}(f_2)$
Here onwards, $\mathcal{F}(h_2) = H_2(u,v)$

$$\therefore \quad G_1 = F_1 + H_2 F_2 \quad \ldots (1)$$

$$G_2 = H_1 F_1 + F_2 \quad \ldots (2)$$

$$\therefore \quad F_2 = G_2 - H_1 F_1 \quad \text{Put in } (1) \Rightarrow$$

$$G_1 = F_1 + H_2 G_2 - H_2 F_1 F_2.$$

$$\therefore \quad F_1 = \frac{G_1 - H_2 G_2}{1 - H_1 H_2}.$$

$$f_1(x,y) \xleftrightarrow{DTFT} F_1(u,v) = \frac{G_1(u,v) - H_2(u,v) G_2(u,v)}{1 - H_1(u,v) H_2(u,v)}$$

$\therefore$ Taking IDFT we get

$$\boxed{f_1(x,y)} = \mathcal{F}^{-1}(F_1(u,v)) = \boxed{\mathcal{F}^{-1}\left\{\frac{G_1(u,v) - H_2(u,v) G_2(u,v)}{1 - H_1(u,v) H_2(u,v)}\right\}}$$

$F_1 = G_1 - H_2 F_2$ Put in ② ⇒

We have $H_1 G_1 - G_2 = H_1 H_2 F_2 - F_2$

$$\therefore F_2 = \frac{G_2 - H_1 G_1}{1 - H_1 H_2}.$$

$$\therefore f_2(x,y) = F_2(u,v) = \frac{G_2(u,v) - H_1(u,v)\,G_1(u,v)}{1 - H_1(u,v)\,H_2(u,v)} \rightarrow \text{Taking IDFT}$$

$$\Rightarrow \boxed{f_2(x,y) = F^{-1}\left(\frac{G_2(u,v) - H_1(u,v)\,G_1(u,v)}{1 - H_1(u,v)\,H_2(u,v)}\right)}$$

We note the following problems with the formula:

It is given that h_1 & h_2 are blur kernels. Thus $H_1(u,v)$ and $H_2(u,v)$ are low-pass in nature.
Thus, for lower frequencies, $H_1(u,v)$ & $H_2(u,v) \rightarrow 1$.
Thus $H_1 \cdot H_2 \rightarrow 1$. Thus $1 - H_1 H_2 \rightarrow 0$ & $\frac{1}{1 - H_1 H_2} \rightarrow \infty$

1) This implies if at ~~some~~ place, if $H_1 = H_2 = 1 \Rightarrow$ i.e. $1 - H_1 H_2 = 0$, the denominator becomes zero and we can't use this approach to ~~find~~ reconstruct f_1, f_2.
Thus, this method can't extract the $\boxed{\text{low frequency}}$ component perfectly.

2) Also at low freq (where $\frac{1}{1-H_1H_2} \to \infty$), errors in measurement (of $g_1 g_2$) will get magnified due to division by small quantity.