CS663 Assignment 4 Question 2 Report

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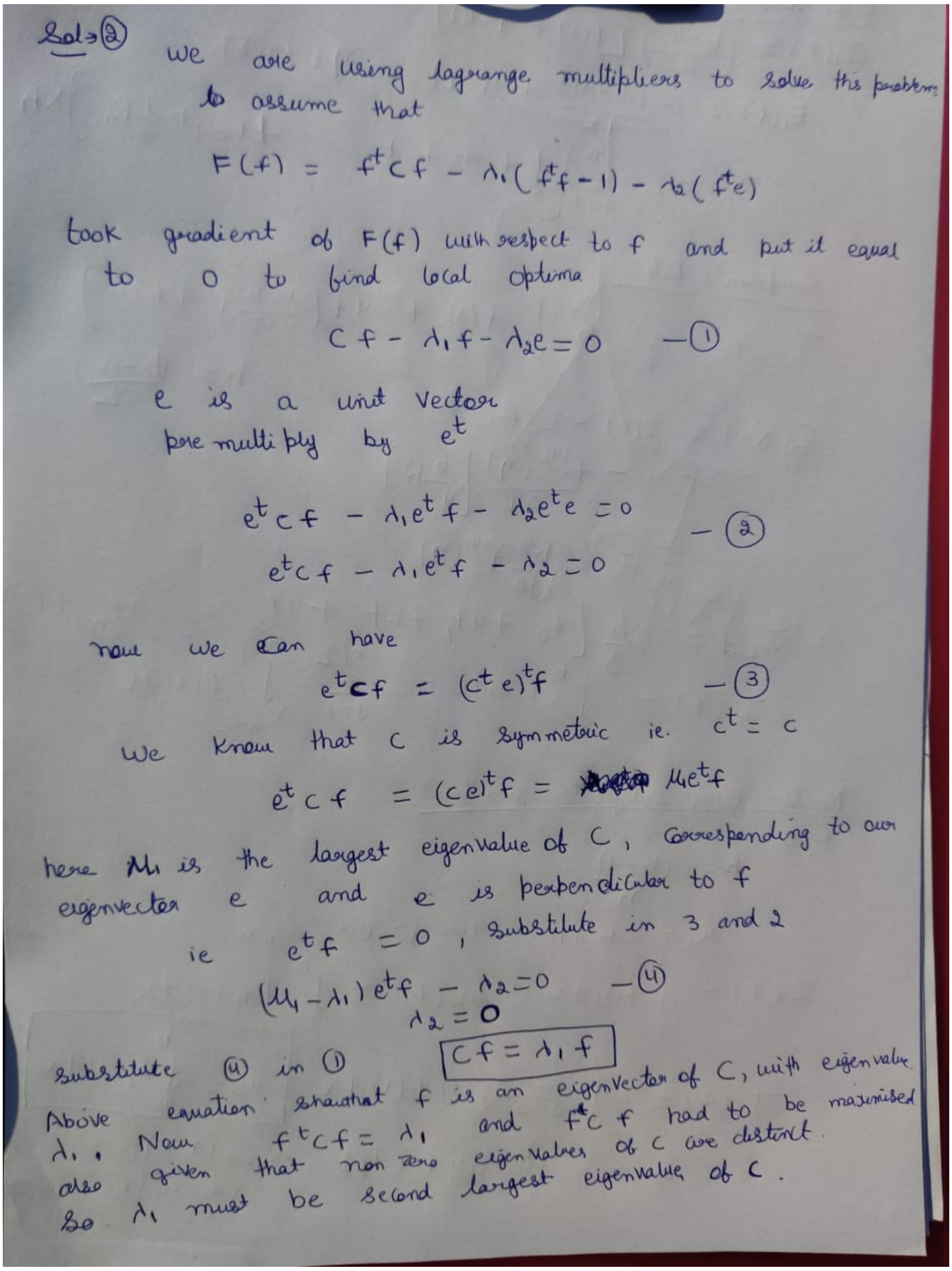
November 7, 2020



Sol→ 2 we are using lagrange multipliers to solve this problem to assume that

$$F(f) = f^t C f - \lambda_1 (f^t f - 1) - \lambda_2 (f^t e)$$

took gradient of $F(f)$ with respect to f and put it equal to 0 to find local optima

$$Cf - \lambda_1 f - \lambda_2 e = 0 \quad - (1)$$

e is a unit vector

pre multiply by e^t

$$e^t C f - \lambda_1 e^t f - \lambda_2 e^t e = 0 \quad - (2)$$

$$e^t C f - \lambda_1 e^t f - \lambda_2 = 0$$

now we can have

$$e^t C f = (C^t e)^t f \quad - (3)$$

We know that C is symmetric ie. $C^t = C$

$$e^t C f = (Ce)^t f = \mu_1 e^t f$$

here μ_1 is the largest eigenvalue of C, corresponding to our eigenvector e and e is perpendicular to f

ie $e^t f = 0$, substitute in 3 and 2

$$(\mu_1 - \lambda_1) e^t f - \lambda_2 = 0 \quad - (4)$$

$$\lambda_2 = 0$$

substitute (4) in (1) $\boxed{Cf = \lambda_1 f}$

Above equation shows that f is an eigenvector of C, with eigenvalue λ_1. Now $f^t C f = \lambda_1$ and $f^t C f$ had to be maximised also given that non zero eigenvalues of C are distinct. So λ_1 must be second largest eigenvalue of C.

Figure 1: Question2