CS663 Assignment 5 Question 5 Report

Shreya Laddha, Archishman Biswas, Shreyan Jabade, Rishabh Arya

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Question 5
Case I - Images not noisy

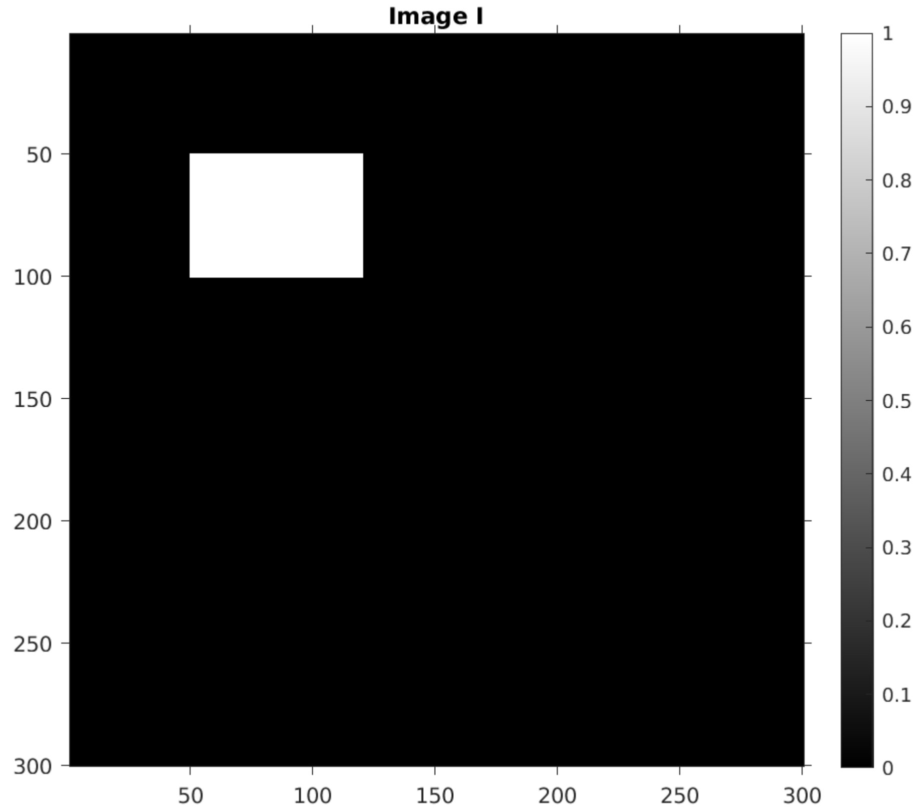

Figure 1: Image 1 - size 300x300 with 50×70 white rectangle starting at (50, 50)

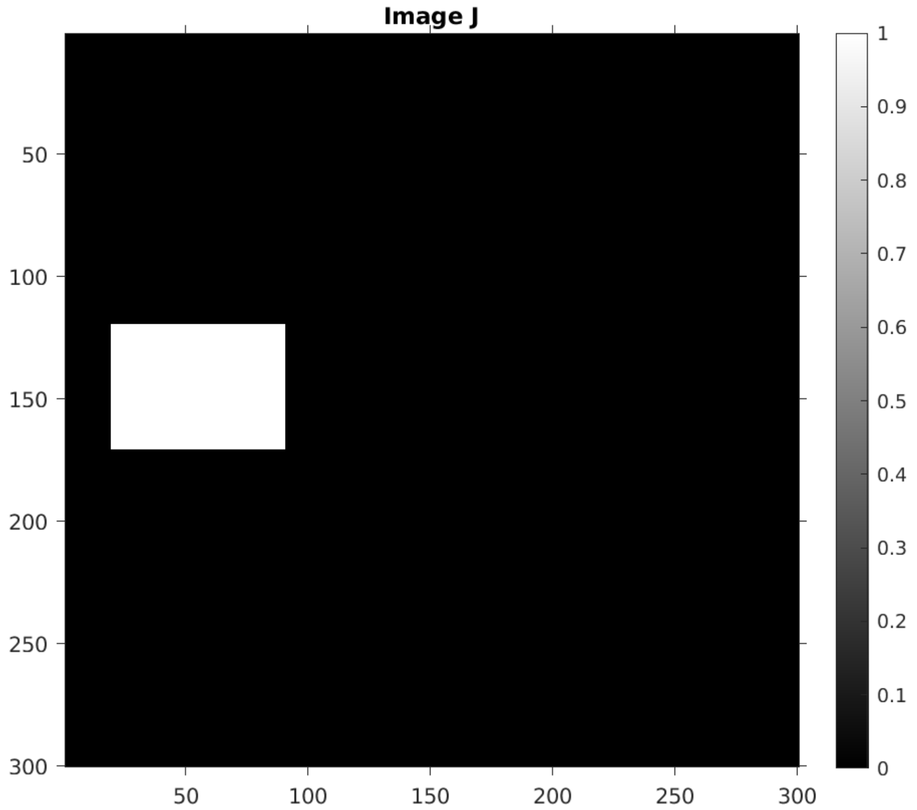

Figure 2: Image J - size 300x300 with 50x70 white rectangle starting at (20, 120)

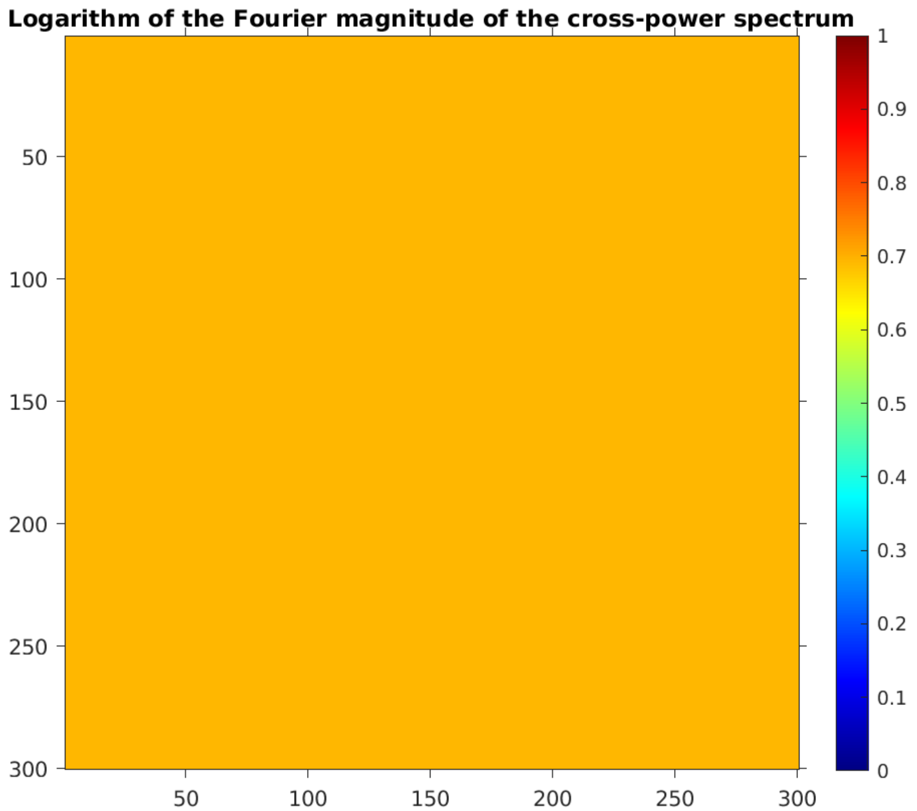

Figure 3: log(abs(F(u,v)) + 1) plot

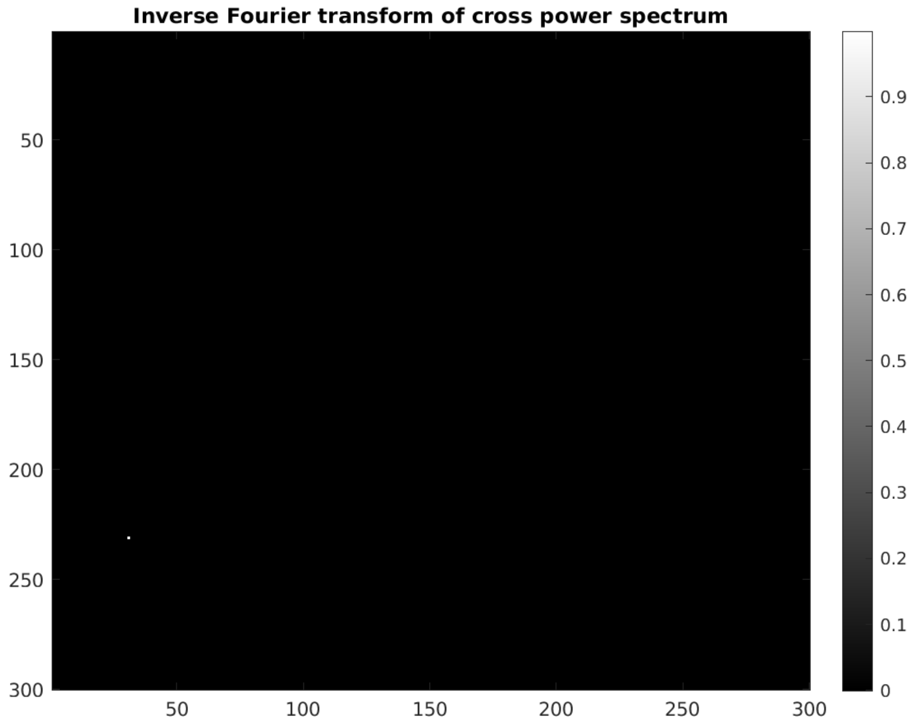

Figure 4: Displacement needed on J image to optimally register the two images

Case II - Noisy images

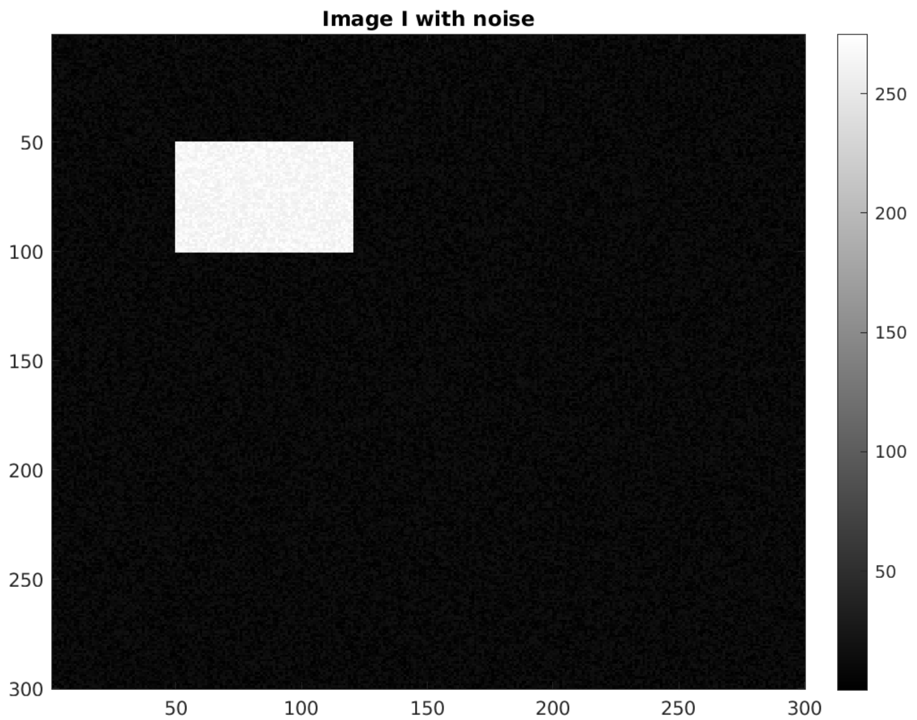

Figure 5: Image I with noise added (gaussian with mean 0 and var=20)

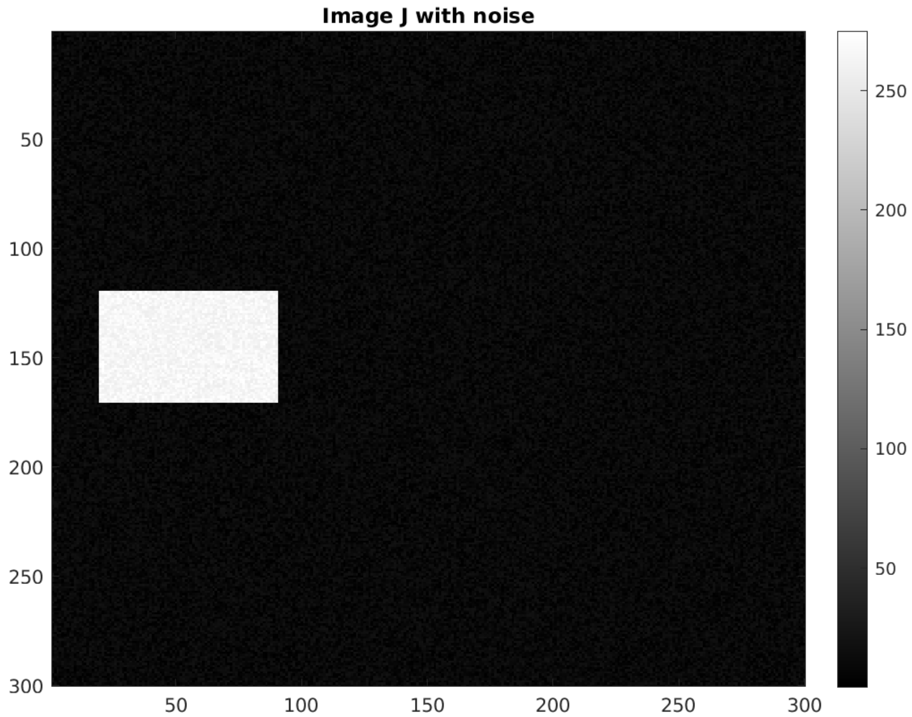

Figure 6: Image J with noise added (gaussian with mean 0 and var=20)

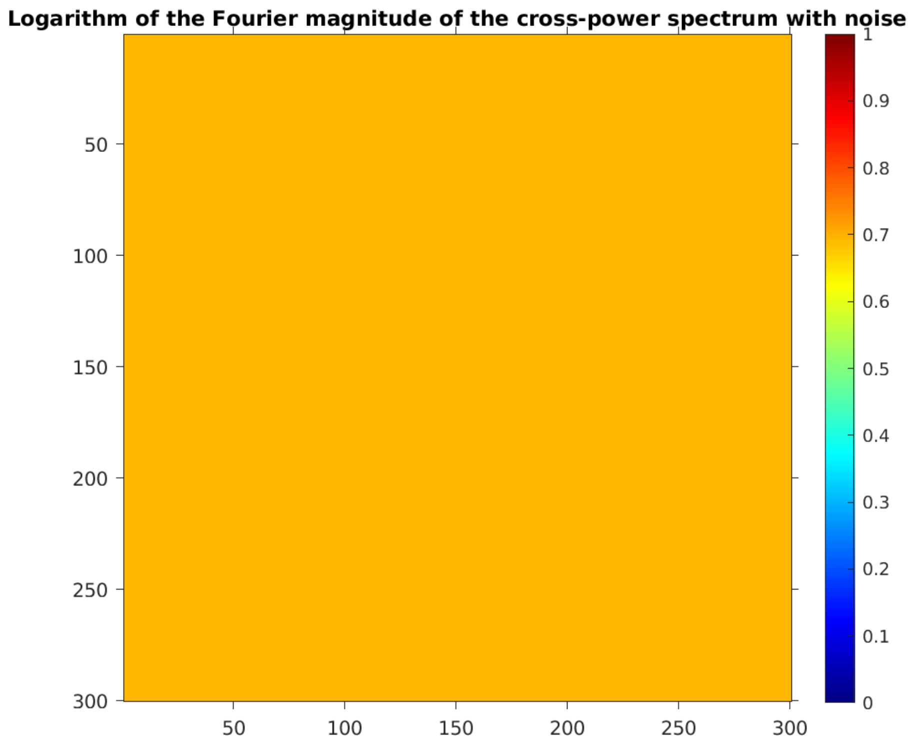

Figure 7: log(abs(F(u,v)) + 1) plot in noisy case

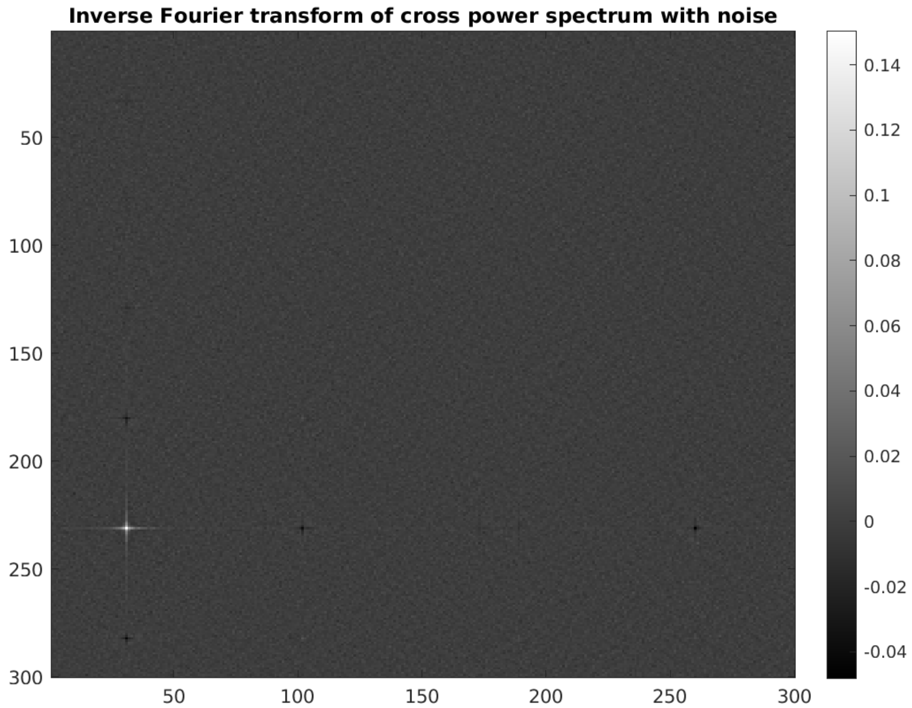

Figure 8: Displacement needed on noisy image of J to optimally register the two images

- The logarithm of the Fourier magnitude of the cross-power spectrum in both the cases is a constant of value log(2) = 0.6931 since the result of cross-power spectrum is a complex number of unit magnitude. In both the cases, by taking inverse Fourier transform of the representation in the frequency domain of cross power spectrum, we get an impulse; that is approximately zero everywhere except at the displacement that is needed to optimally register the two images.

- For non noisy case (Case I), there is a clean spike at (31, 231) which can be thought of as (31, -71) when applying to a 300x300 image J while translation. Image J is obtained by applying (-30, 70) translation to image I. Due to MATLAB indexes starting from 1, we get a displacement (31, -71) to be applied to image J as expected.

- For noisy case (Case II), there is a spike at (31, 231) which means (31, -71) displacement needed on the image noisy J to get noisy image I, similar to that in first case. But here, the spike intensity is less as compared to Case I and it is also surrounded by other frequencies of non-zero magnitude.

- This algorithm involves - calculation of Fourier transform [O(N log(N)] of each image, conjugation of the FFT [O(N)], pointwise multiplication and division [O(1)]. So total complexity is O(N log(N)).

- The time complexity of pixel-wise image comparison procedure for predicting the translation is O(N2).

Approach for correcting for rotation between two images (Section II in paper)

If f2(x,y) is a rotated replica of f1(x, y) with rotation θo (considering pure rotation, and no translation and scaling), we can write in fourier domain, F2(x, y) = F1(xcos(θo) + ysin(θo), -xsin(θo) + ycos(θo)). Thus, the magnitudes of both the spectra are the same, but one is a rotated replica of the other. Using polar coordinates, we can convert the rotational movement into translation, and then use the concept of cross power spectrum to obtain the rotation θo

$$f2(\rho, \theta) = f1(\rho, \theta - \theta o)$$
$$F2(u, v) = exp(-2\pi jv\theta o) * F1(u, v)$$
$$M1(\rho, \theta) = M2(\rho, \theta - \theta o)$$

where M is magnitude of Fourier transform.
Now, we can get the exp($2\pi jv\theta o$) term from the cross power spectrum of the two fourier transform, which gives us the value of rotation.

In case of translation + rotation, f2(r, θ) = $f1(\rho - \rho o, \theta - \theta o)$, i.e the translation will lead to change in ρ by ρo. We will get exp($2\pi j(u\rho o + v\theta o)$ after applying the cross power spectrum. Thus we can get both translation and rotation values. The exact (x,y) coordinates can be deduced from the polar coordinates using the corresponding relations.

In case of scale change (without rotation and translation), if f1 is scaled replica of f2 with scale factors (a,b) for the horizontal and vertical directions then, F2(x,y) = F1(x/a,y/b)/$\mid ab \mid$ or F2 (logx, logy) = F2 (logx - loga, logy - logb), where we can find loga and logb using the phase correlation technique mentioned above.

If f2 is translated, rotated, and scaled replica of f1, then we have in polar coordinates M2 (logρ, θ) = M1(logρ - loga, $\theta - \theta o$), where θo is the rotation and a is the scale magnitude. Thus we can find the rotation and scale using phase correlation technique. The translation is inside the ρ term, which can be found out by applying phase correlation technique on the image which we have scaled by a and rotated by θo, i.e removed the effects of scaling and rotating.