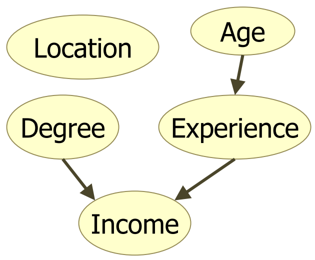
### CS 726: Samples questions on Forward and MCMC Sampling

1. For the Bayesian Network $H$ and corresponding potentials below answer the following questions. [Assume the first letter of each node's name as a shortcut for the variable. For example, A is for Age, D for Degree, and so on.]




$\psi_1(L) = \Pr(L)=$

| NY | CA | Other |
|---|---|---|
| 0.2 | 0.3 | 0.5 |

$\psi_2(A) = \Pr(A)=$

| 0 | 1 |
|---|---|
| 0.6 | 0.4 |

$\psi_3(D) = \Pr(D)=$

| **0** | **1** |
|---|---|
| 0.3 | 0.7 |

$\psi_4(E, A) = \Pr(E|A) =$

| $A \rightarrow$ | **0** | **1** |
|---|---|---|
| **0** | 0.9 | 0.3 |
| **1** | 0.1 | 0.7 |

$\Pr(I|D,E) = \Pr(I|D+E)$

| $D+E \rightarrow$ | **0** | **1** | **2** |
|---|---|---|---|
| **0** | 0.9 | 0.6 | 0.2 |
| **1** | 0.1 | 0.4 | 0.8 |

(a) If we perform MCMC sampling on the full network, what is the value of the following transition probabilities

i. $\Pr(L, A, D, E, I = NY, 0, 1, 1, 0|L, A, D, E, I = NY, 0, 0, 1, 0)$ ..1

$1/5 * \frac{\Pr(D=1)*\Pr(I=0|D=1,E=1)}{\Pr(D=1)*\Pr(I=0|D=1,E=1)+\Pr(D=0)*\Pr(I=0|D=0,E=1)}$

ii. $\Pr(L, A, D, E, I = NY, 0, 0, 1, 0|L, A, D, E, I = NY, 0, 0, 1, 0)$ ..2

$1/5 \Pr(A = NY)$

$+1/5 \frac{\Pr(A=0)*\Pr(E=1|A=0)}{\Pr(A=0)*\Pr(E=1|A=0)+\Pr(A=1)*\Pr(E=1|A=1)}$

$+1/5 \frac{\Pr(D=0)*\Pr(I=0|D=0,E=1)}{\Pr(D=1)*\Pr(I=0|D=1,E=1)+\Pr(D=0)*\Pr(I=0|D=0,E=1)}$

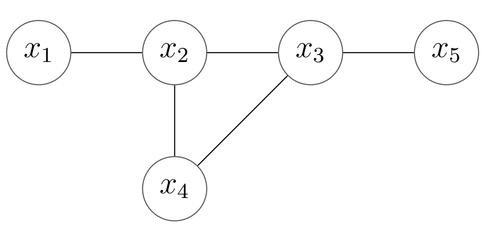
$+1/5 \frac{\Pr(E=1|A=0)*\Pr(I=0|D=0,E=1)}{\Pr(E=0|A=0)*\Pr(I=0|D=0,E=0)+\Pr(E=1|A=0)*\Pr(I=0|D=0,E=1)}$

$+1/5 \frac{\Pr(I=0|D=0,E=1)}{\Pr(I=0|D=0,E=1)+\Pr(I=1|D=0,E=1)}$

(b) Now suppose we wish to try sampling for answering the query: $\Pr(I|E = 1)$,

i. For MCMC, what is the transition probability $\Pr(L, A, D, E, I = NY, 0, 1, 1, 0|L, A, D, E, I = NY, 0, 0, 1, 0)$ ..1 $1/5 * \frac{\Pr(D=1)*\Pr(I=0|D=1,E=1)}{\Pr(D=1)*\Pr(I=0|D=1,E=1)+\Pr(D=0)*\Pr(I=0|D=0,E=1)}$

2. In the graphical model below, assume binary variables and potentials as $\psi_{1,2}(x_1, x_2)$, $\psi_{234}(x_2, x_3, x_4)$, $\psi_{35}(x_3, x_5)$. If we are doing MCMC sampling and the current sample is [0,0,0,0,0], what is the probability that the next sample will be [0,0,0,0,1]. Your expression should involve the minimal number of terms from the potentials and should use exact value (0 or 1) in place of $x_i$.



$1/5 \frac{\psi_{35}(0,1)}{\psi_{35}(0,1)+\psi_{35}(0,0)}$

3. Consider doing Gibbs sampling on a $3 \times 3$ pairwise grid network with binary labels $0, 1$ and all edges having the same potentials of the form $\psi(0,0) = 0.1$, $\psi(1,1) = 0.6$, $\psi(0,1) = 0.2$, $\psi(1,0) = 0.2$. A state $X$ is the assignment of 0/1 labels to the nine variables of the network and we write them in row major order. For example, $X = 111\ 000\ 000$ denotes that the first row has all 1s and the last two rows has all 0s. Work out the following transition probabilities $T(X \to X')$ for this network.

   (a) $T(000\ 000\ 000 \to 000\ 010\ 000)$

$$\begin{aligned} T(000\ 000\ 000 \to 000\ 010\ 000) &= \frac{1}{9} \frac{\psi(0,1)\psi(0,1)\psi(1,0)\psi(1,0)}{\sum_{v=1,0} \psi(0,v)\psi(0,v)\psi(v,0)\psi(v,0)} \\ &= (\frac{1}{9}) \frac{.2^4}{.1^4 + .2^4} \\ &= 0.10457 \end{aligned}$$

..1

   (b) $T(000\ 000\ 000 \to 000\ 000\ 000)$

$$\begin{aligned} T(000\ 000\ 000 \to 000\ 000\ 000) &= \frac{1}{9} \sum_i P(x_i | x_{-i}) \\ &= (\frac{1}{9})(4 \frac{.1^2}{.1^2 + .2^2} + 4 \frac{.1^3}{.1^3 + .2^3} + \frac{.1^4}{.1^4 + .2^4}) \\ &= 0.14480 \end{aligned}$$

..2

4. Suppose we are computing $T()$ via Gibbs sampling on a Bayesian network with $r$ labels, a maximum of $p$ parents per node and $c$ children per node.

   (a) Write the simplied expression for the conditional probability $\Pr(x_i | x_{-i})$ in a Bayesian network. $Pr(x_i | x_{-i}) = \frac{Pr(x_i | Pa(x_i)) \prod_{j \in child(x_i)} Pr(x_j | Pa(x_j))}{\sum_{x_i} Pr(x_i | Pa(x_i)) \prod_{j \in child(x_i)} Pr(x_j | Pa(x_j))}$ ..2

   (b) What in the maximum number of multiplications and additions you need to perform in the above computation? Total number of multiplications $= c(r+1)$
   Total number of additions $= r - 1$ ..1