CS 726: Problems on Markov Random Field (Graded)

Write your answers in the space provided. You are expected to solve each question on your own. Do not try to search the answers from any external sources, like the web. You are allowed to discuss a few questions with your classmates provided you mention their names.

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1. Let $P(x_1, \ldots, x_4)$ be a distribution defined over binary variables as follows

$$P(x_1, x_2, x_3, x_4) = \frac{1}{Z} e^{x_1 \oplus x_2 \oplus x_3} e^{x_3 \oplus x_4} \tag{1}$$

$\psi_{123}(x_1, x_2, x_3)$ $\psi_{34}(x_3, x_4)$ $\psi_C(x_C)$

where $\oplus$ denotes the XOR operation. XOR of two binary variables is 0 when both its arguments are the same and 1 otherwise. The value of the numerator for some of the entries have been filled in. You need to fill in the five missing entries.

x_1	x_2	x_3	x_4	$ZP(\mathbf{x})$
0	0	0	0	1
0	0	0	1	e
0	0	1	0	e^2
0	0	1	1	e
0	1	0	0	e
0	1	0	1	e^2
0	1	1	0	e
0	1	1	1	1
1	0	0	0	e
1	0	0	1	e^2
1	0	1	0	e
1	0	1	1	1
1	1	0	0	1
1	1	0	1	e
1	1	1	0	e^2
1	1	1	1	e

$\psi_{123}(0,0,0)\psi_{34}(0,0)$

$Z = \sum_{x} \prod_C \psi_C(x_C)$

eg: $X \perp\!\!\!\perp Y \mid W$ & $Z \perp\!\!\!\perp Y \mid W$ $\Rightarrow (X, Z \perp\!\!\!\perp Y \mid W)$? for MRFs.

$\psi_{123}(1,0,1)\psi_{34}(1,1)$

$x_1 \perp\!\!\!\perp x_4 \mid x_2, x_3$

$x_2 \perp\!\!\!\perp x_4 \mid x_1, x_3$

$x_4 \perp\!\!\!\perp \{x_1, x_2\} \mid x_3$

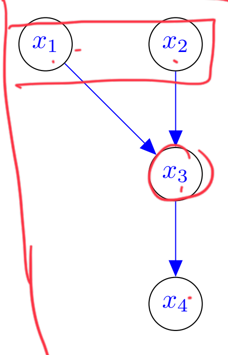
..1

(a) Calculate the value of Z ..1 Missing entries are e, e^2 and e. The value of Z is the sum of all table entries and is $Z = 4e^2 + 4 + 8e$

(b) Draw a minimal Bayesian network representing the above distribution using the variable order x_1, x_2, x_3, x_4 to the right of the above table. ..2

$x_1 \perp\!\!\!\perp x_2$? Yes

$x_3 \perp\!\!\!\perp \{x_1\} \mid x_2$



$x_4 \perp\!\!\!\perp \{x_1, x_2\} \mid x_3$

BN

It is easy to see that $x_1 \perp\!\!\!\perp x_2$ but x_3 depends on both of them. Also, the form of the factorization implies that $x_4 \perp\!\!\!\perp x_1, x_2 | x_3$.

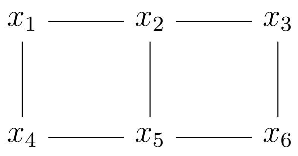
(c) Write the CPD for $\Pr(x_1|\text{Pa}(x_1)), \Pr(x_2|\text{Pa}(x_2)), \Pr(x_3|\text{Pa}(x_3))$ in your Bayesian network above. ..3

$\Pr(x_1|\text{Pa}(x_1)) = \Pr(x_1)[0.5, 0.5]$

$\Pr(x_2|\text{Pa}(x_2)) = \Pr(x_2) = [0.5, 0.5]$

$$\Pr(x_3|x_1, x_2) = \begin{array}{c c} & \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\ \begin{array}{c} 0 \\ 0 \end{array} & \begin{bmatrix} \frac{1}{1+e} & \frac{e}{1+e} & \frac{e}{1+e} & \frac{1}{1+e} \\ \frac{e}{1+e} & \frac{1}{1+e} & \frac{1}{1+e} & \frac{e}{1+e} \end{bmatrix} \end{array}$$

2. For the undirected graphical model H below, perform the following operations

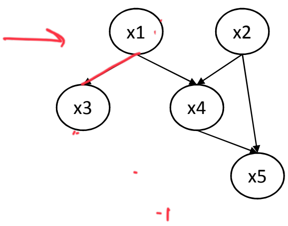


(a) Convert it into a BN G using variable order $x_1, x_4, x_5, x_2, x_3, x_6$. ..2

(b) Choose a different variable order that leads to adding more edges in G than in the above ordering. ..2

(c) List two CIs that holds in H but do not hold in G. ..2

3. For the Bayesian network G below, perform the following operations



$x_1 \perp\!\!\!\perp x_2$

$x_2 \perp\!\!\!\perp x_3$

(a) Convert it into a undirected graphical model H ..2

(b) List two CIs that holds in G but do not hold in H. ..2