CS 726: Samples questions on VAEs

1. Let $\mathbf{x} = [x_1, x_2, x_3]$ and z be real random variables where $p(z) \sim N(0,1)$. Also, we know that $[x_1, x_2, x_3] = [3z+2, 4z+1, z-2] + \alpha[\epsilon_1, \epsilon_2, \epsilon_3]$ where $\epsilon_i \sim N(0,1)$. With this knowledge, we will plug in the optimal parameters of a VAE defined over them where encoder $q_\phi(z|\mathbf{x}) \sim N(\mu_{z|\mathbf{x}}, \sigma_{z|\mathbf{x}})$ and decoder $p_\theta(\mathbf{x}|z) \sim N(\mu_{\mathbf{x}|z}, \Sigma_{\mathbf{x}|z})$. Assume both encoder and decoder are linear layers where $\mu_{\mathbf{x}|z} = [\theta_1 z + \theta_2, \theta_3 z + \theta_4, \theta_5 z + \theta_6]$ and $\mu_{z|\mathbf{x}} = \phi_1 x_1 + \phi_2 x_2 + \phi_3 x_3 + \phi_4$.

   (a) Provide the optimal parameters $\theta_1 \ldots \theta_6$ of the decoder mean. ..1 These will match the corresponding coefficients defined in $[x_1, x_2, x_3] = [3z+2, 4z+1, z-2]$

   (b) Provide the optimal parameters of the encoder means $\phi_1 \ldots, \phi_4$ ..3 Since the components of $\mathbf{x}$ are independent of each other given z, we can estimate $q(z|\mathbf{x})$ as $1/3[(x_1-2)/3 + (x_2-1)/4 + (x_3+2)]$ This when simplified will give us values for $\phi_1, \phi_2, \phi_3, \phi + 4$.

   (c) What is the ideal NN to estimate $\Sigma_{\mathbf{x}|z}$ and $\sigma_{z|\mathbf{x}}$ from their respective inputs? ..3 The variance $\sigma_{z|\mathbf{x}}$ to be reliably estimated requires the sample itself and square of individual samples. Thus the σ logic in the encoder must depend on x_i and square of x_i. The variance of each x_i is a constant function of its input and thus any linear layer with a bias parameter could suffice.

2. Consider a VAE for which prior $p(z)$ is Bernoulli distributed with parameter α. The α now needs to be learned jointly with the encoder parameter ϕ and decoder parameters θ. What will be the function at the last layer of the encoder? Let e_i denote the output from the last encoder layer. Write the formulae for the $D_{KL}(q_\phi(z|x^i) \| p_\alpha(z))$ in terms of α and e_i. ..2 Applying the formula for the KL distance between two Bernoulli distributions, one with parameter e_i and second with parameter α gives us the answer as: $e_i \log \frac{e_i}{\alpha} + (1-e_i) \log \frac{1-e_i}{(1-\alpha)}$

$\boxed{\textbf{Total: 9}}$