CS 726: Problems on Bayesian Networks

Write your answers in the space provided. You are expected to solve each question on your own. Do not try to search the answers from any external sources, like the web. You are allowed to discuss a few questions with your classmates provided you mention their names.

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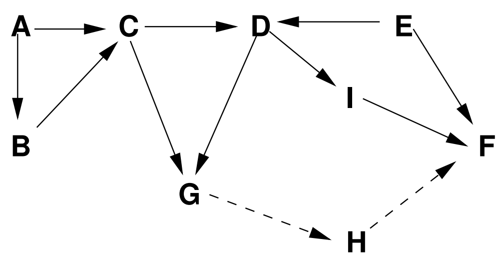
1. Draw a Bayesian network over five variables $x_1, \ldots, x_5$ assuming the variable order x_1, x_2, x_3, x_4, x_5. For this ordering, assume that the following set of local CIs hold in the distribution: $x_1 \perp\!\!\!\perp x_2$, $x_3 \perp\!\!\!\perp x_2 | x_1$, $x_4 \perp\!\!\!\perp x_1, x_3 | x_2$, $x_5 \perp\!\!\!\perp x_1, x_2 | x_3, x_4$

2. Consider the Bayesian Network represented by the following graph:

   (a) Assume that the vertex H and the dotted edges $G \rightarrow H$ and $H \rightarrow F$ are absent. State with reasons whether the following conditional independence statements hold:

      i. $A \perp\!\!\!\perp G \,|\, F$



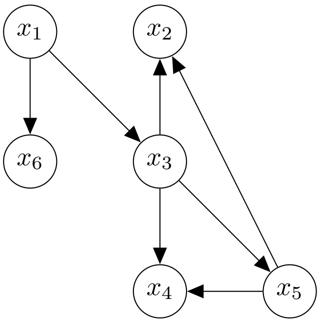
ii. $A \perp\!\!\!\perp D$

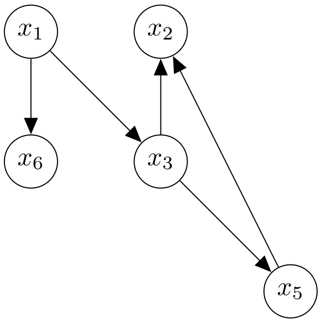
iii. $B \perp\!\!\!\perp G \,|\, C, F$

iv. $F \perp\!\!\!\perp D \,|\, C$

(b) Now add the vertex H along with the two dotted edges to the graph. Find the smallest set S ($A, E \notin S$) such that $A \perp\!\!\!\perp E \,|\, S$. Provide a proof if no such S exists.

3. Let L be a BN with n variables. Let R be obtained from L by removing a node w that has no children. For example, if we call the graph on the left side below as L, and node w as x_4 (Note x_4 has no children), we will get the rightside graph R where node x_4 and the edges incident on it are dropped.





Let X, Y, Z denote disjoint subsets of variables in R. We use $\text{dsep}(X, Y, Z, R)$ to denote that Z d-separates X and Y in R. For example, $\text{dsep}(\{x_1, x_6\}, \{x_2, x_5\}, \{x_3\}, R)$ is true.

(a) Justify briefly that $\text{dsep}(X, Y, Z, R)$ implies $\text{dsep}(X, Y, Z, L)$.

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w can only appear as a V-node in any path between X and Y and it is not a part of Z.

(b) Let P denote $\text{pa}(w) - Z$ where $\text{pa}(w)$ denotes parents of w. Justify briefly that $\text{dsep}(w, Y, Z, L)$ implies $\text{dsep}(P, Y, Z, R)$.

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Assume by contradiction that there is path p between P and Y that is not blocked by Z in G. p extended with w will also be unblocked in L since no node in P can be a v-node on the w to Y path. Thus, $\text{dsep}(w, Y, Z, L)$ will be violated.

4. Consider a distribution $P(x_1, x_2, x_3)$ as follows:

$$\begin{aligned} \Pr(x_1, x_2, x_3) &= 0.5 \;\text{ if }\; x_1 = x_2 = x_3 \\ &= 0 \;\text{ otherwise.} \end{aligned} \tag{1}$$

For this distribution $x_1 \perp\!\!\!\perp x_2 | x_3$ but say we change the distribution slightly to

$$\begin{aligned} \Pr(x_1, x_2, x_3) &= \frac{1-\epsilon}{2} \;\text{ if }\; x_1 = x_2 = x_3 \\ &= \frac{\epsilon}{6} \;\text{ otherwise.} \end{aligned} \tag{2}$$

For what values of ϵ is $x_1 \not\perp\!\!\!\perp x_2 | x_3$?

..3 We look for conditions under which $P(x_1 = 1|x_2 = 1, x_3 = 1) = P(x_1 = 1|x_2 = 0, x_3 = 1)$. That is, $\frac{\frac{1-\epsilon}{2}}{\frac{1-\epsilon}{2}+\frac{\epsilon}{6}} = \frac{\frac{\epsilon}{6}}{\frac{\epsilon}{6}+\frac{\epsilon}{6}}$ This holds only for $\epsilon = 0$ and for $\epsilon = \frac{3}{4}$. The latter corresponds to the case where the distribution is uniform over all 8 possible combinations. Students who point this point should get +2 extra credit.