Quiz 6 solutions CS726 2022

1. As θ=0 we have uniform over all possibilities of $\mathbf{y}$ so Z=8.

   (a)
$$dPr(y|x,\theta)/d\theta_1 = Pr(y|x,\theta).dlogPr(y|x,\theta)/d\theta_1$$
$$dPr(y|x,\theta)/d\theta_1 = Pr(y|x,\theta).[f_1(x,y) - E_{Pr(y^1|\theta,x)}f_1(x,y^1)]$$
$$Pr(y|x,\theta) = 1/8$$
$$f_1(x,y) = 1*2+0*0+1*2 = 4$$
$$E_{Pr(y^1|\theta,x)}f_1(x,y^1) = \sum_c \sum_{y_c^1} f_1(x,y_c^1,c)Pr(y_c^1|\theta,x)$$
$$E_{Pr(y^1|\theta,x)}f_1(x,y^1) = \sum_{i=1to3} 1*x_i*1/2+0*x_i*1/2 = 2.5$$

   so answer is 1.5/8.

   (b)
$$dPr(y|x,\theta)/d\theta_2 = Pr(y|x,\theta).dlogPr(y|x,\theta)/d\theta_2$$
$$dPr(y|x,\theta)/d\theta_2 = Pr(y|x,\theta).[f_2(x,y) - E_{Pr(y^1|\theta,x)}f_2(x,y^1)]$$
$$Pr(y|x,\theta) = 1/8$$
$$f_2(x,y) = 1*0+0*1 = 0$$
$$E_{Pr(y^1|\theta,x)}f_2(x,y^1) = \sum_c \sum_{y_c^1} f_2(x,y_c^1,c)Pr(y_c^1|\theta,x)$$
$$E_{Pr(y^1|\theta,x)}f_2(x,y^1) = \sum_{i=1to2} 0*0*1/4+0*1*1/4+1*0*1/4+1*1*1/4 = 0.5$$

   so answer is -0.5/8.

   (c) using bayes rule
$$Pr(y_2|y_1,y_3,\mathbf{x},\theta) = Pr(\mathbf{y}|\mathbf{x},\theta)/\sum_{y_2} Pr(\mathbf{y}|\mathbf{x},\theta)$$

   Denominator doesnot depend on y2
   numerator is proportional to $e^{\theta_1*y_2+\theta_2*y_2*2+4*\theta_1}$
   hence answer is y2=1 at $\theta = [3,-1,-1]$

2.
$$\sum_{y1,y2,y3} \psi_{123}(y_1,y_2,y_3) = \sum_{y_1y_2y_3} e^{\theta(f(y_1,y_2)+f(y_2,y_3)+f(y_3,y_1)})$$
$$\sum_{y1,y2,y3} \psi_{123}(y_1,y_2,y_3) = 4.e^0 for(1,0,0);(0,1,0);(0,0,1);(0,0,0)$$
$$+3.e^{2*1} for(1,1,0);(1,0,1);(0,1,1) + e^{2*3} for(1,1,1)$$

3. let n(x) be no.of adjacent vertices in x that have the same label now we get

$$argmax_{\sigma} logPr(x|\sigma) = argmax_{\sigma} n(x) log\sigma - logZ(\sigma)$$

$$dlogPr(x|\sigma)/d\sigma = n(x)/\sigma - \sum_{(i,j)\in Graph} (\sum_{l} Pr(x_i = l, x_j = l))$$

Take any tree of 4 nodes and draw junction tree, potential for every node in junction tree consists of 2 nodes is xnor table and sigma over any variable for finding message distribution will be uniform, so

$$Pr(x_i = l, x_j = l) = 2\sigma/(2\sigma + 2)$$

and as x=[0,0,0,0] we have n(x) as 3 and 3 edges irrespective of tree structure and making above derivative equals 0 gives $\sigma^2 = \sigma + 1$

4. (a) Consider θ=0 and chain graph of three variables we get Z=8 and Z_A=16
   (b) we have one clique y_1, y_2, y_3

$$f(x, y_c, c) = f(y_1, y_2) + f(y_2, y_3) + f(y_3, y_1)$$

   Now we have to go through all possible values of y and find sum of $e^{\theta * f(x, y_c, c)}$ which is $e^{3\theta} + 3e^{\theta} + 4$