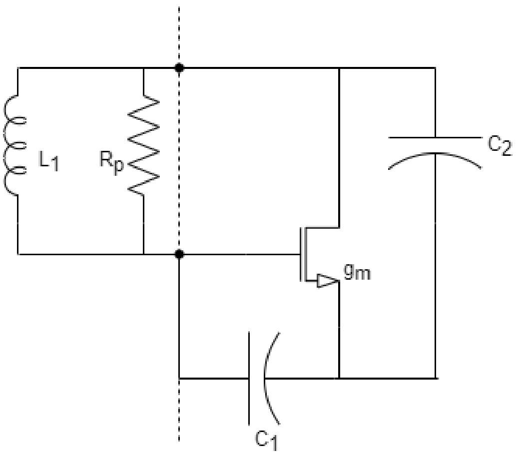
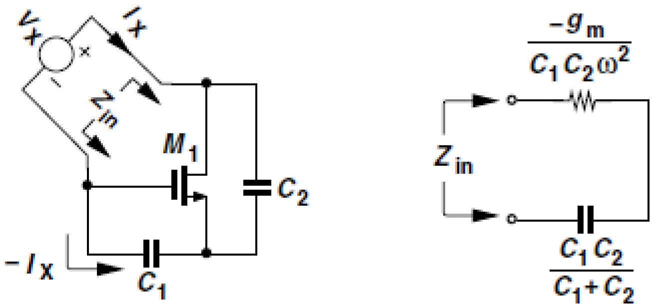
Q1. For the given collpits oscillator, prove that the condition of oscillation is $g_m.R_P=4$ and $C_1=C_2$ **(3M)**



Solution:

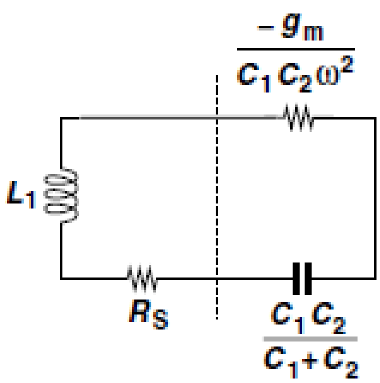


$$-\frac{I_X}{C_1 s} + V_X = \left(I_X + I_X \frac{g_m}{C_1 s}\right) \frac{1}{C_2 s}.$$

$$\frac{V_X}{I_X}(s) = \frac{1}{C_1 s} + \frac{1}{C_2 s} + \frac{g_m}{C_1 C_2 s^2}.$$

For a sinusoidal input, $s = j\omega$,

$$\frac{V_X}{I_X}(j\omega) = \frac{1}{jC_1\omega} + \frac{1}{jC_2\omega} - \frac{g_m}{C_1 C_2 \omega^2}.$$



It is simpler to model the loss of the inductor by a series resistance, RS. The circuit oscillates if

$$R_S = \frac{g_m}{C_1 C_2 \omega^2}.$$

Under this condition, the circuit reduces to L_1 and the series combination of C_1 and C_2, exhibiting an oscillation frequency of

$$\omega_{osc} = \frac{1}{\sqrt{L_1 \dfrac{C_1 C_2}{C_1 + C_2}}}. \qquad \text{.....................(1)}$$

$$\frac{L_1\, \omega}{Rs} \approx \frac{R_P}{L_1\, \omega}$$

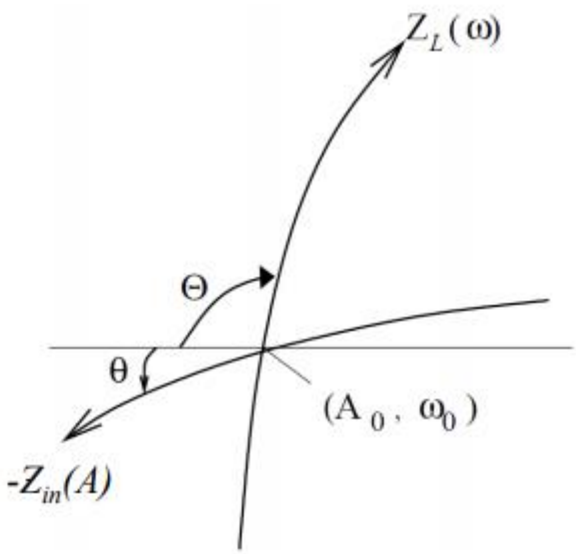
Thus,

$$\frac{L_1{}^2 \omega^2}{R_P} = \frac{g_m}{c_1\, c_2\, \omega^2}$$

Moreover, we can replace ω^2 with the value given in Eq.1, arriving at the startup condition:

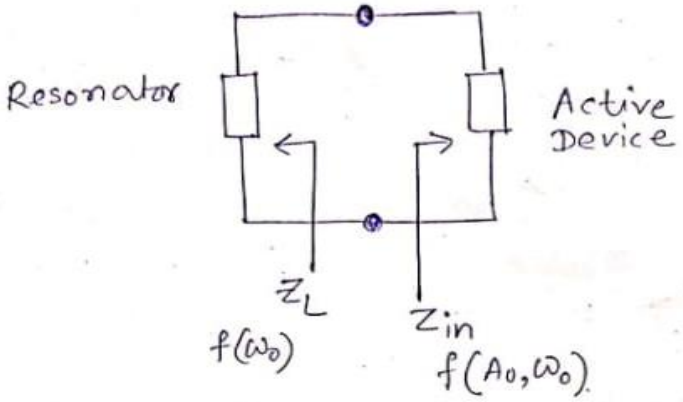
$$g_m.R_P = \frac{(c_1 + c_2)^2}{c_1\, c_2} = \frac{c_1}{c_2} + \frac{c_2}{c_1} + 2 = 4(\text{Proved})$$

Q2. For oscillation stability prove that Θ+Θ is between 0 and 180 degree. (give the fig of ZL and Zin and define what is Θ and θ) **(3 M)**

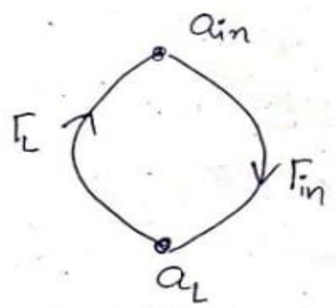


Solution:

Total oscillator circuit can be modelled into



This can be modelled as SFG:



a_{in}, a_L → Normalised voltages & currents.

$$\frac{a_L}{a_{in}} = \frac{\Gamma_{in}}{1 - \Gamma_{in}\Gamma_L}$$

For oscillation, the TF must satisfy Barkhausen criteria

$$\Gamma_{in}\Gamma_L = 1$$

$$\Rightarrow \Gamma_{in}(A_0, \omega_0) = \frac{1}{\Gamma_L(\omega_0)}$$

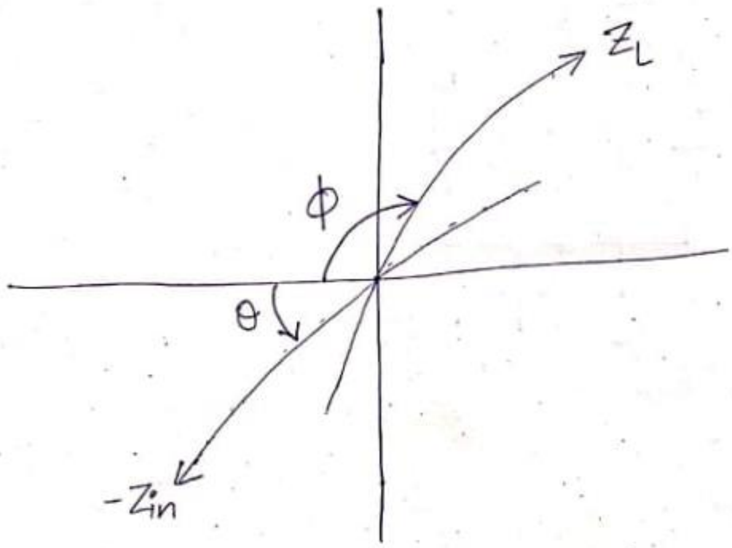
$$\Rightarrow \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{Z_L + Z_0}{Z_L - Z_0}$$

$$\Rightarrow Z_{in}(A_0, \omega_0) + Z_L(\omega_0) = 0$$

Now, Kurokawa has derived that for a stable oscillator,

$$\left[\frac{\partial R_{in}(A_0, \omega_0)}{\partial A} \cdot \frac{\partial X_L(\omega_0)}{\partial \omega}\right] - \left[\frac{\partial X_{in}(A_0, \omega_0)}{\partial A} \cdot \frac{\partial R_L(\omega_0)}{\partial \omega}\right] > 0 \quad (1)$$

where $R_{in} = Re\{Z_{in}\}$, $X_{in} = Img\{Z_{in}\}$
$R_L = Re\{Z_L\}$, $X_L = Img\{Z_L\}$.



$$R_L = Z_L \cos(180-\phi) = -Z_L \cos\phi$$
$$X_L = Z_L \sin(180-\phi) = Z_L \sin\phi$$
$$-R_{in} = -Z_{in}\cos\theta \Rightarrow R_{in} = Z_{in}\cos\theta$$
$$-X_{in} = -Z_{in}\sin\theta \Rightarrow X_{in} = Z_{in}\sin\theta$$

Substituting into ①,

$$\frac{\partial}{\partial A}(Z_{in}\cos\theta)\cdot\frac{\partial}{\partial \omega}(Z_L\sin\phi) - \frac{\partial}{\partial A}(Z_{in}\sin\theta)\frac{\partial}{\partial \omega}(-Z_L\cos\phi) > 0$$

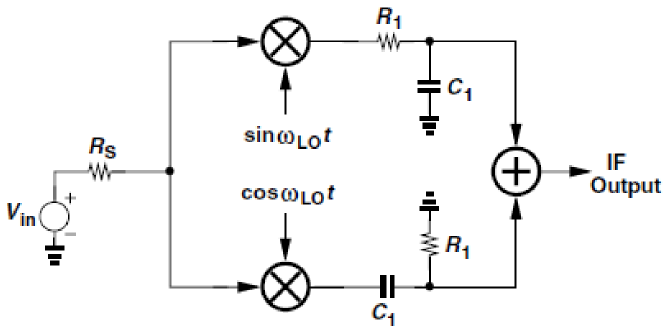
$$\Rightarrow \left[\cos\theta.\sin\phi + \sin\theta\cos\phi\right]\left[2\frac{\partial Z_{in}}{\partial A}\frac{\partial Z_L}{\partial \omega}\right] > 0$$

$$\Rightarrow \underbrace{2\frac{\partial Z_{in}}{\partial A}\cdot\frac{\partial Z_L}{\partial \omega}}_{>0 \text{ always}}\sin(\theta+\phi) > 0$$

$$\therefore \sin(\theta+\phi) > 0 \Rightarrow \boxed{0 < (\theta+\phi) < \pi}$$

(Proved)

Q3. The simplified Hartely architecture shown incorporates mixers having a voltage conversion gain of A_{mix} and infinite input impedance. Taking into account only the noise of the two resistors, compute the noise figure of the receiver with respect to a source resistance of R_S at an IF of $1/R_1C_1$. **(3M)**

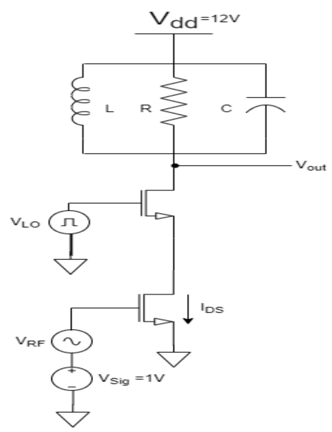


Solution:

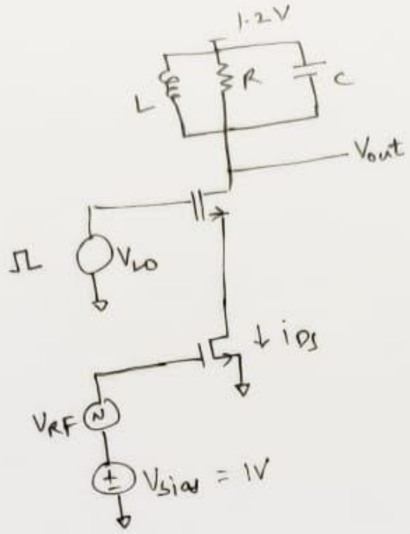
- Noise,$_{Out}$ =$4KTR_1$*2
- Finding the gain:
  - For upper-branch: A_{mix} *0.5*0.5
  - For bottom-branch: A_{mix} *0.5*0.5
  - Total gain=0.5*A_{mix}
- NF=$1+\frac{4KTR_1*2}{(0.5*A_{mix})^2}*\frac{1}{4KTR_S}=1+\frac{8R_1}{R_S}\frac{1}{A_{mix}{}^2}$

Q4. In the given figure, find the voltage conversion gain (A_{VC}) of the mixer in dB. Assume that the resonant tank circuit (LRC) will sufficiently allow the (ω_{RF} - ω_{LO}) frequency component and reject others. Given, g_m of the MOSFETS is 2mS, R=3KΩ, V_{bias}=1V, V_{thn}=0.5V, Amplitude of the square wave of LO=1.2V,(LO oscillated between 0 and 1.2 with frequency ω_{LO})

(3 M)



Solution:



$R = 3\,K\Omega$

$V_{th} = 0.5V$

V_{LO} = square wave toggling between 0 & 1 with frequency f_{LO}, $A_{LO} = 1.2V$

$V_{RF} = V_{RF}\, \cancel{\sin} \cos(\omega_{RF} t)$

$$i_{DS}(t) = \beta \left[V_{RF} \cos(\omega_{RF} t) + (V_B - V_{TH}) \right]^2$$

$$= \beta \left[V_{RF}^2 \cos^2(\omega_{RF} t) + (V_B - V_{TH})^2 + 2V_{RF}(V_B - V_{TH}) \cos(\omega_{RF} t) \right]$$

F-series of LO – $\left[\frac{1}{2} + \frac{2}{\pi} \cos(\omega_{LO} t) + \frac{2}{3\pi} \cos(3\omega_{LO} t) + \dots \right] A_{LO}$

i_{DS} flowing through Load –

$$A_{LO}\, \beta \left[V_{RF}^2 \cos^2(\omega_{RF} t) + (V_B - V_{TH})^2 + 2V_{RF}(V_B - V_{TH}) \cos(\omega_{RF} t) \right] \cdot \left[\frac{1}{2} + \frac{2}{\pi} \cos(\omega_{LO} t) \dots \right]$$

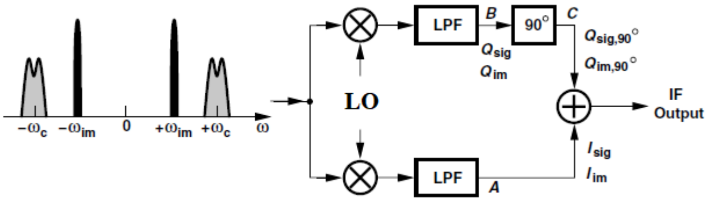
Since, the Tank is designed to pass only $\omega_{LO} - \omega_{RF}$

$$\therefore\ 2V_{RF}(V_B - V_{TH}) \cos((\omega_{RF} - \omega_{LO}) t) \cdot \frac{A_{LO} \beta R}{\pi} = V_{out}$$

$$\therefore\ \text{Conversion Gain} = \frac{A_{LO} \beta}{\pi} \cdot \frac{2V_{RF}(V_{gs} - V_{TH}) R}{V_{RF}} = \frac{A_{LO}}{\pi} \times g_m R$$

Q5. For the given architecture, derive the expression of **I**mage **R**ejection **R**atio (IRR) as a function of ε and ΔΘ. Hence prove that IRR≈$\frac{4}{\varepsilon^2+\Delta\theta^2}$ if ΔΘ<<1rad and ε<<1rad.

(Assume one LO waveform is expressed as $\sin\omega_{LO}t$ and the other as $(1+\varepsilon)\cos(\omega_{LO}t+\Delta\Theta)$ due to mismatches) **(4M)**



Solution:

$$x_A(t) = \frac{A_{sig}}{2}(1+\epsilon)\cos\left[(\omega_c-\omega_{LO})t+\phi_{sig}+\Delta\theta\right] + \frac{A_{im}}{2}(1+\epsilon)\cos\left[(\omega_{im}-\omega_{LO})t+\phi_{im}+\Delta\theta\right]$$

$$x_{sig}(t) = \frac{A_{sig}}{2}(1+\epsilon)\cos\left[(\omega_c-\omega_{LO})t+\phi_{sig}+\Delta\theta\right] + \frac{A_{sig}}{2}\cos\left[(\omega_c-\omega_{LO})t+\phi_{sig}\right]$$

$$x_{im}(t) = \frac{A_{im}}{2}(1+\epsilon)\cos\left[(\omega_{im}-\omega_{LO})t+\phi_{im}+\Delta\theta\right] - \frac{A_{im}}{2}\cos\left[(\omega_{im}-\omega_{LO})t+\phi_{im}\right]$$

We know, avg. power of the vector sum $a\cos(\omega t+\alpha)+b\cos\omega t$ is $\frac{a^2+2ab\cos\alpha+b^2}{2}$.

$$\left.\frac{P_{im}}{P_{sig}}\right|_{out} = \frac{A_{im}^2}{A_{sig}^2}\,\frac{(1+\epsilon)^2-2(1+\epsilon)\cos\Delta\theta+1}{(1+\epsilon)^2+2(1+\epsilon)\cos\Delta\theta+1}$$

$$\therefore \boxed{IRR = \frac{(1+\epsilon)^2+2(1+\epsilon)\cos\Delta\theta+1}{(1+\epsilon)^2-2(1+\epsilon)\cos\Delta\theta+1}} \qquad (1)$$

Since,

$$\cos \Delta\theta \approx 1 - \frac{\Delta\theta^2}{2} \text{ for } \Delta\theta \ll 1 \text{ rad},$$

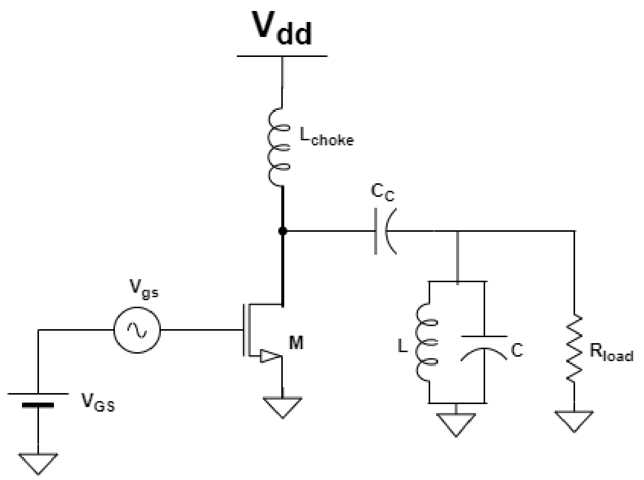
1 is reduced to,

$$IRR \approx \frac{4 + 4\epsilon + \epsilon^2 - (1+\epsilon)\Delta\theta^2}{\epsilon^2 + (1+\epsilon)\Delta\theta^2}$$

In the numerator, the first term dominates and in the denominator $\epsilon \ll 1$ giving,

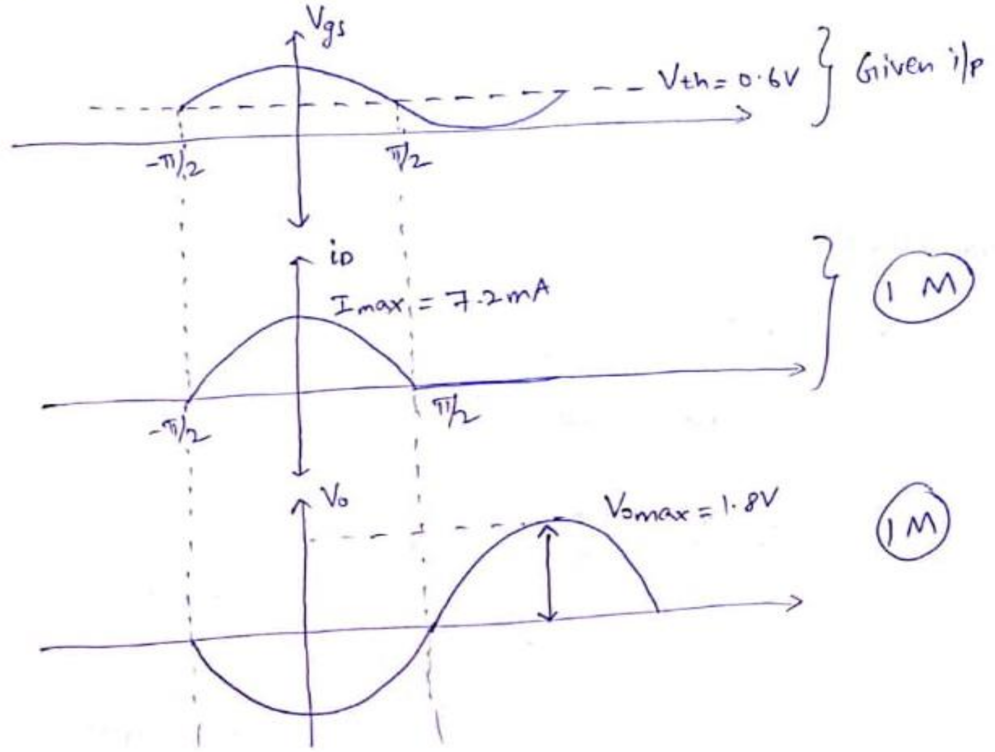
$$\boxed{IRR \approx \frac{4}{\epsilon^2 + \Delta\theta^2}}$$

Q6. Design a class-B RF power amplifier to deliver an output power of 4dBW at f_0=2.45GHz. Assume V_{DD}=1.8V, V_{th}=0.6V (Threshold voltage of NMOS) $\mu_n C_{ox}$=120µA/V^2. Tabulate the values of R_{load}, L, C, W/L of transistor M. **(6M)**



Solution:

Waveforms at V_o and I_D -



Class B P.A -

Assume: $V_{GS} = V_{th} = 0.6V$

$V_{dsat} = V_{GS} + v_{gsmax} - V_t = 0.2V$

The maximum output voltage: $V_{o_{max}} \cong V_{DD} = 1.8V$

Load Resistance = $R_{Load} = \dfrac{V_{omax}^2}{2 \cdot P_o}$ [Given $P_o = 5.1 dBm$]

$\therefore \boxed{R_{Load} = 500\Omega}$ (1 M)

Amplitude of fundamental component of Drain current -

$$I_L = \frac{V_{omax}}{R_{Load}} = \frac{1.8}{500} = 3.6 \text{ mA}$$

$$I_{DC} = \frac{2}{\pi} \cdot I_L = \frac{2}{\pi} \times 3.6 \text{ mA} = 2.3 \text{ mA}$$

Max. Drain current: $I_{max} = \pi \cdot I_{DC} = 7.2 \text{ mA}$

To calculate (W/L)

$$\frac{W}{L} = \frac{2 \times I_{max}}{\mu_n C_{ox} V_{Dsat}^2} = 1800$$ (1 M)

To calculate L, C

Given: f, BW

$$\therefore Q = \frac{f}{BW} = \frac{2.45 \times 10^9}{500 \times 10^6} = 4.9$$

$$\therefore X_L = X_C = \frac{R}{Q} = \frac{500}{4.9} \cong 102$$

$$\therefore L = \frac{102}{\omega_c} = \frac{102}{2\pi \times 2.45 \times 10^9} = 6.62\, nH$$ (1 M)

$$C = \frac{1}{102 \times 2\pi \times 2.45 \times 10^9} = 0.636\, pF$$ (1 M)

Q7. A hilbert transform of a signal m(t) <-> m_H(t) with the relation : $m_H(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(t)}{t-\tau} d\tau$
and the Fourier Transform relation: $M_H(f) = -j\, sgn(f)\, M(f)$
Show that for a upper sideband modulated signal resulting from m(t) and $\cos(2\pi f_c t)$ is given by : $s(t) = \frac{1}{2}[m(t)\cos(2\pi f_c t) - m_H(t)\cos(2\pi f_c t)]$ **(2M)**

Solution:

$$s(t) = \frac{1}{2}[m(t)\cos(2\pi f_c t) - m_H(t)\sin(2\pi f_c t)]$$

Applying Fourier transform to the above equation, we get

$$S(f) = \frac{1}{4}[M(f - f_c) + M(f + f_c)] - \frac{1}{4j}[M_H(f - f_c) - M_H(f + f_c)]$$

From the defenition of Hilbert Transform, we have

$$M_H(f) = -j\ sgn(f)M(f)$$

where $sgn(f)$ is the signum function. Equivalently, we may write

$$\frac{-1}{j}M_H(f - f_c) = sgn(f - f_c)M(f - f_c)\frac{-1}{j}M_H(f + f_c) = sgn(f + f_c)M(f + f_c)$$

From the definition of signum function, we note the following for $f > 0$ and $f > f_c$

$$sgn(f - f_c) = sgn(f + f_c) = 1$$

Correspondingly, the equation for $S(f)$ reduces to,

$$S(f) = \frac{1}{4}[M(f - f_c) + M(f + f_c)] + \frac{1}{4}[M(f - f_c) - M(f + f_c)] = \frac{1}{2}M(f - f_c)$$

In words, the above result means that, except for a scaling factor, the spectrum of the modulated signal $s(t)$ is the same as that of a DSB-SC modulated signal for $f > f_c$.
For $f > 0$ and $f < f_c$, we have

$$sgn(f - f_c) = -1 sgn(f + f_c) = 1$$

Correspondingly, $S(f)$ reduces to,

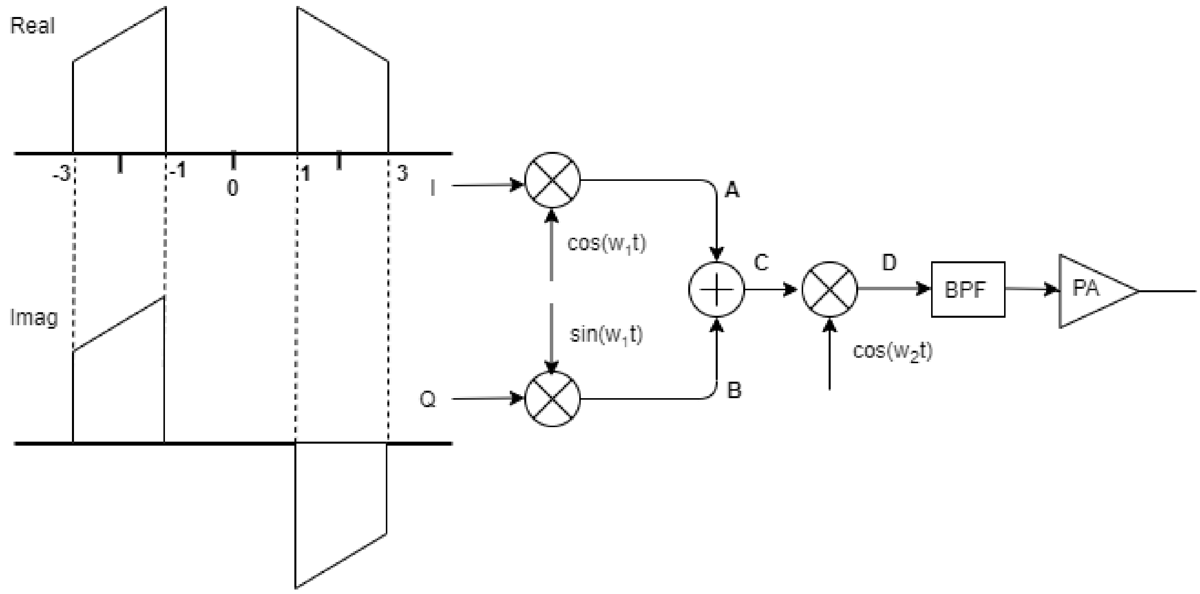
$$S(f) = \frac{1}{4}[M(f - f_c) + M(f + f_c)] + \frac{1}{4}[-M(f - f_c) - M(f - f + c)] = 0$$

In words, for $f < f_c$, the modulated signal $s(t)$ is zero.

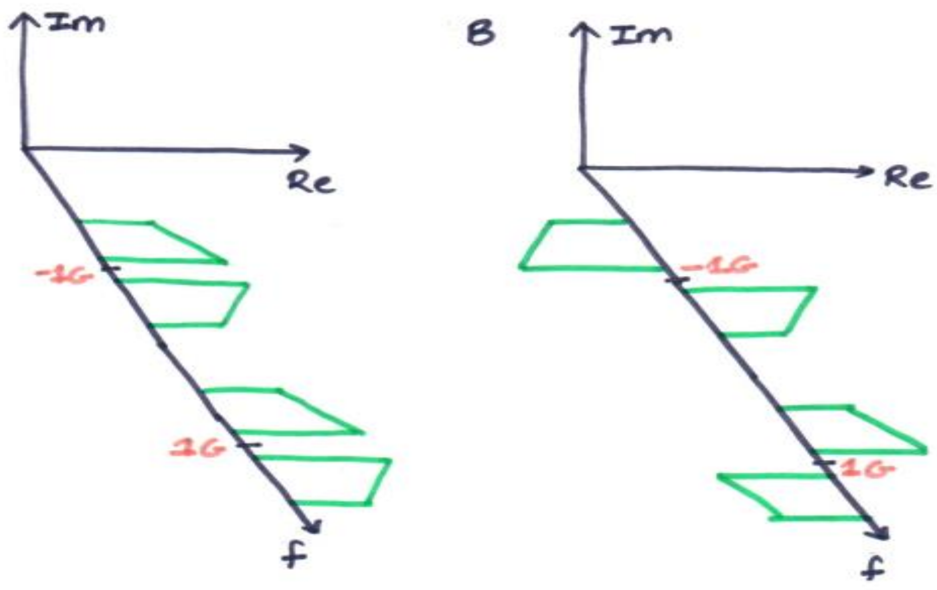
Q8. The below figure shows the concept of Heterodyne Transmitter. Signal conversion happens in two steps as shown. The frequency of up-conversion is f_0=5GHz. Units in spectrum are in MHz. Assume that f_1=1GHz and f_2=4GHz . The spectrum of I signal is real. The Q spectrum is 90 degrees phase shifted version of I spectrum (Imaginary). The I and Q signals are at base band(Close to DC). Our goal is to have final PA transmitter spectrum at 5GHz.

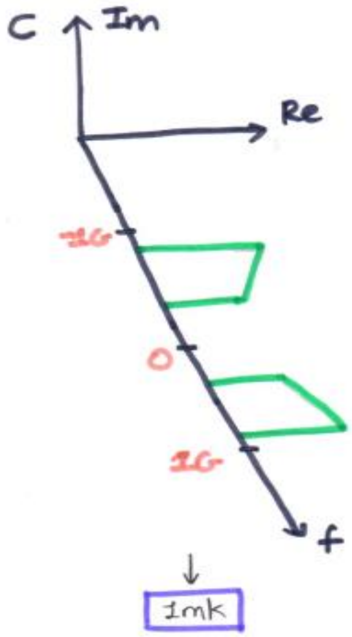
a. Draw the 3D spectrum at A,B,C, and D **(4M)**

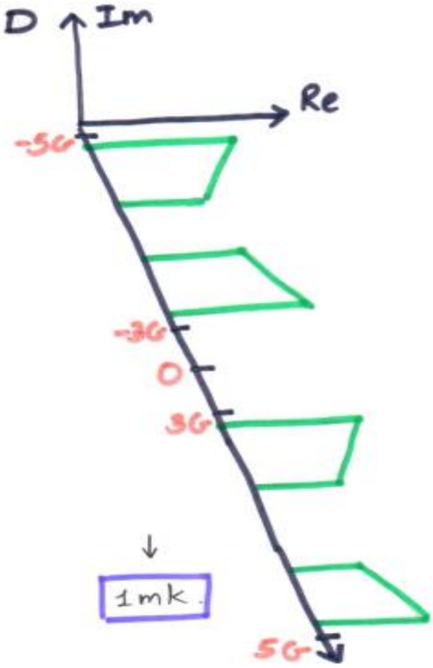
b. Sketch the characteristics of the desired BPF?(Center frequency & Stopband) **(2M)**



Solution:







Bandpass characteristics.

As seen in spectrum of D, in addition to the required band at 5 GHz, we also have sidebands at 3 GHz.

centre frequency = 5 GHz − 2 MHz

stopband region =

should reject the sideband at 3 GHz.

1 mk.

specifically make sure that the 3 GHz sideband is cancelled

1mk