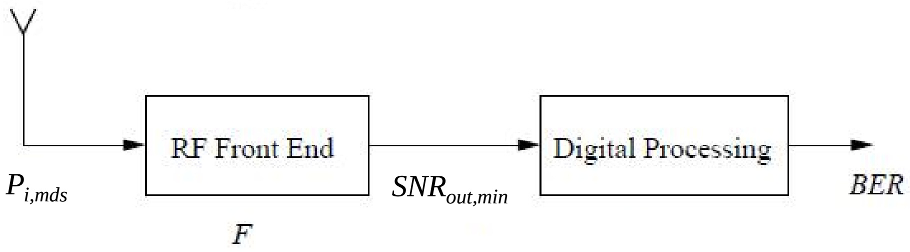
# Low Noise Amplifiers

# Transceiver Performance

**Typical GSM Receiver**

- Minimum detectable signal ($P_{i,mds}$) of -102 dBm
- A min BER of $10^{-3}$
- BER of $10^{-3}$ corresponds to a $SNR_{out}$ of about 9-12 dB (GMSK)

$$P_{i,mds} = -174\ dBm + 10\log[\Delta f\ (Hz)] + F\ (dB) + SNR_{out,min}\ (dB),\ \Delta f = 1\ MHz$$




Required range for the noise figure of the receiver < 3 dB

# Gain

- Gain large enough to minimize noise contribution of subsequent stages.
- This leads to compromise between NF and linearity . Higher gain will degrade linearity but improve NF.
- For heterodyne output of LNA matched to i/p of mixer (50 ohms). Here gain implies power gain.
- However where uniform matching across the chain cannot be done, voltage gain is used.

# Input and Output Matching

- Maximum gain occurs for simultaneous conjugate match at the input and output (if the device is stable).
- Input matching : 50 ohms
  - Reflection coefficient $\Gamma_{\mathrm{in}} = \dfrac{Z_{in} - R_0}{Z_{in} + R_0}$
  - For $\mathrm{Z_{in}} = \mathrm{R_0} + \Delta\mathrm{R}$ we have $\Gamma_{\mathrm{in}} = \dfrac{\Delta R}{2R_0 + \Delta R}$
  - For $\Gamma_{\mathrm{in}}$ of around -17 dB we need $\Delta\mathrm{R} \approx 15$ ohms.
- Output matching : 50 ohms for heterodyne transceiver.

# Stability

Stern Stability factor:

$$K = \frac{1+|\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{21}||S_{12}|} \quad \text{with } \Delta = S_{11}S_{22} - S_{12}S_{21}$$

If $K > 1$ and $\Delta < 1$ for all frequencies the circuit is unconditionally stable for all passive sources and loads.
Unconditional stability is not required if the source and load impedances $Z_S$ and $Z_L$ are known e.g. heterodyne receiver.

*Stability is achieved if*

$$\mathrm{Re}\left[Z_{in} + Z_S\right] > 0 \quad \text{and} \quad \mathrm{Re}\left[Z_{out} + Z_L\right] > 0$$

In practice good grounding and power supply decoupling (using decoupling capacitors) is required to reduce the wire inductance and establish the AC grounding.

# Linearity

Peak power may be 2 dB above average. Hence LNA should have high enough linearity to avoid spreading from Tx to Rx till -18 dBm i/p power. So say $P_{1dB}$ of -15dBm can provide good compromise between spreading and Rx signal detection.

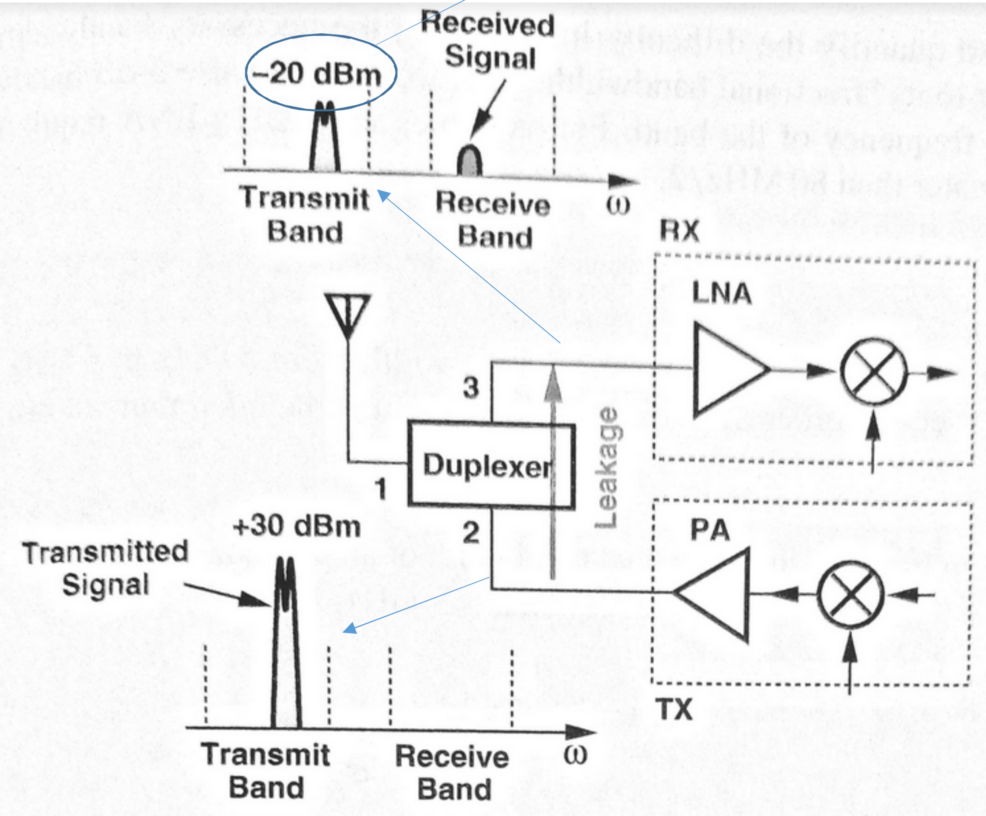



**Figure 5.5** *TX leakage to RX in a full-duplex system.*

- With i/p power levels being lower than $P_{1dB}$, usually linearity is not a problem.
- Wideband receivers like UWB, SDR and cognitive can pose a problem on linearity since a strong interferer in the presence of IM distortion can spread and affect the desired band.

# Bandwidth

- BW should be large enough to accommodate band.
- Less than 1 dB variation over band.
- BW may be switched using various techniques like N path filtering, switching tank.

# BJT equations

**Table 4.2 SUMMARY OF THE BJT CURRENT–VOLTAGE RELATIONSHIPS IN THE ACTIVE MODE**

$$i_C = I_S\, e^{v_{BE}/V_T}$$

$$i_B = \frac{i_C}{\beta} = \left(\frac{I_S}{\beta}\right) e^{v_{BE}/V_T}$$

$$i_E = \frac{i_C}{\alpha} = \left(\frac{I_S}{\alpha}\right) e^{v_{BE}/V_T}$$

Note: For the *pnp* transistor, replace $v_{BE}$ with $v_{EB}$.

$$i_C = \alpha\, i_E \qquad i_B = (1-\alpha) i_E = \frac{i_E}{\beta + 1}$$

$$i_C = \beta\, i_B \qquad i_E = (\beta + 1) i_B$$

$$\beta = \frac{\alpha}{1-\alpha} \qquad \alpha = \frac{\beta}{\beta + 1}$$

$$V_T = \text{thermal voltage} = \frac{kT}{q} \cong 25 \text{ mV at room temperature}$$

$$r_o = V_A / I_C$$

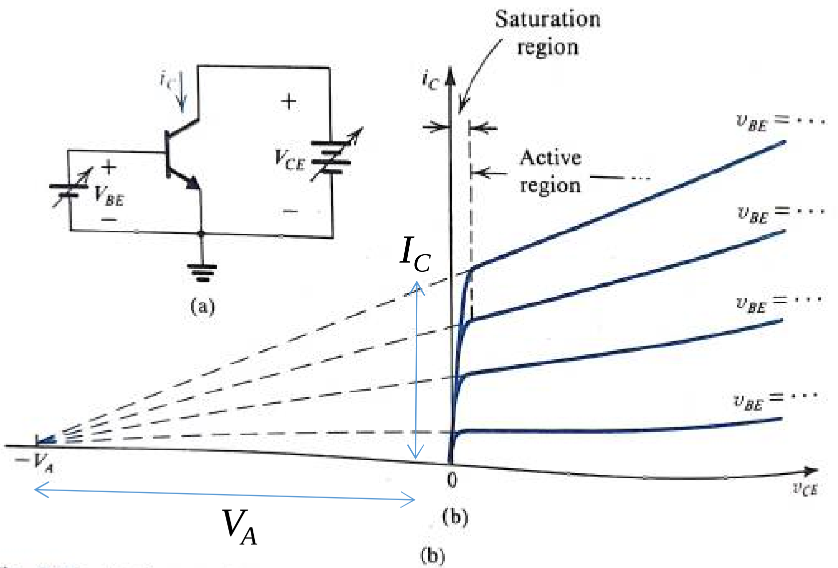



Fig. 4.15 (a) Conceptual circuit for measuring the $i_C$–$v_{CE}$ characteristics of the BJT. (b) The $i_C$–$v_{CE}$ characteristics of a practical BJT.

**Table 4.3** RELATIONSHIPS BETWEEN THE SMALL-SIGNAL MODEL PARAMETERS OF THE BJT

**Model Parameters in Terms of DC Bias Currents:**

$$g_m = \frac{I_C}{V_T} \qquad r_e = \frac{V_T}{I_E} = \alpha\left(\frac{V_T}{I_C}\right)$$

$$r_\pi = \frac{V_T}{I_B} = \beta\left(\frac{V_T}{I_C}\right) \qquad r_o = \frac{V_A}{I_C}$$

**In terms of $g_m$:**

$$r_e = \frac{\alpha}{g_m} \qquad r_\pi = \frac{\beta}{g_m}$$

**In terms of $r_e$:**

$$g_m = \frac{\alpha}{r_e} \qquad r_\pi = (\beta + 1)r_e \qquad g_m + \frac{1}{r_\pi} = \frac{1}{r_e}$$

**Relationships between $\alpha$ and $\beta$:**

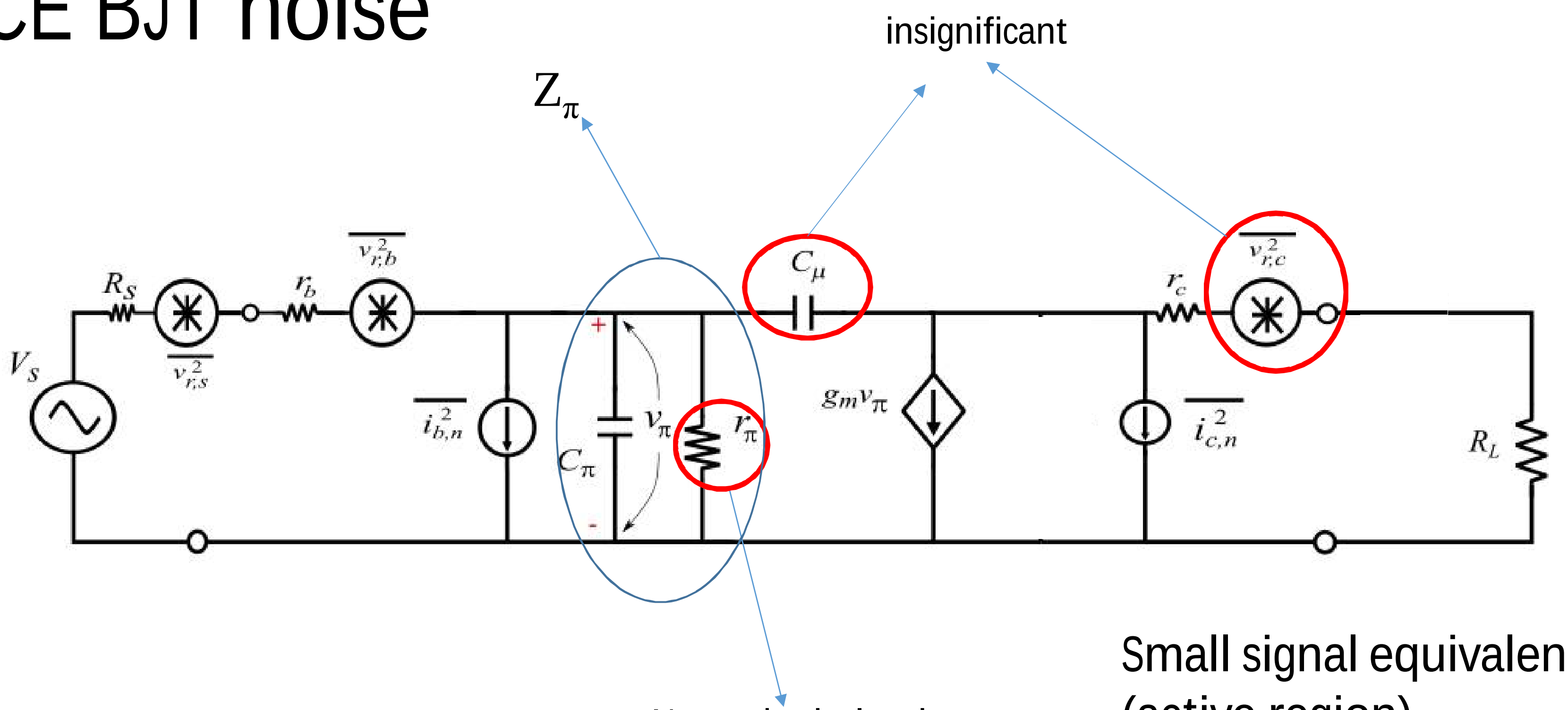
$$\beta = \frac{\alpha}{1-\alpha} \qquad \alpha = \frac{\beta}{\beta+1} \qquad \beta + 1 = \frac{1}{1-\alpha}$$

More generally,

$$|Z_\pi| = \frac{\beta_{complex}}{g_m}$$

| Parameter | Typical values |
|---|---|
| $r_b$ | 100 ohms |
| $g_m$ | 40 mA/V |
| $C_\mu$ | 12 fF |
| $C_\pi$ | 1fF |
| $r_o$ | 100 Kohms |
| $r_c$ | low |
| $r_\pi$ | 1 Kohms |

# CE BJT noise

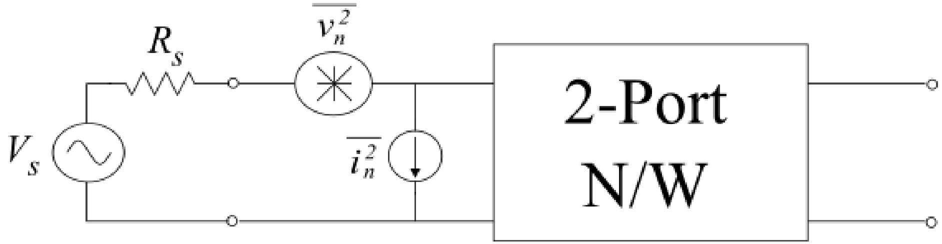



Small signal equivalent circuit (active region)

$$\overline{i_{b,n}^2} = 2qI_B\Delta f \quad , \quad \overline{i_{c,n}^2} = 2qI_C\Delta f = 2kTg_m\Delta f,$$

q $\Rightarrow$ charge of electron (absolute value) $= 1.6 \times 10^{-19}$ Coulomb

# Equivalent noise current




When I/$P$ is open,

$$\overline{i_n^2} = \left(\overline{i_{c,n}^2} + \beta_{complex}^2 \overline{i_{b,n}^2}\right) / \beta_{complex}^2 = \overline{i_{b,n}^2} + \frac{\overline{i_{c,n}^2}}{\beta_{complex}^2} \approx \overline{i_{b,n}^2} = 2qI_b = \frac{2qI_c}{\beta_{complex}} = \frac{2kTg_m}{\beta_{complex}}$$

When I/P is shorted

$$\overline{i_o^2} \approx g_m^2 \overline{v_n^2} \left|\frac{Z_\pi}{Z_\pi + r_b}\right|^2 = \frac{\overline{v_n^2}}{\left|Z_\pi + r_b\right|^2} \left|\frac{g_m Z_\pi}{\beta_{complex}}\right|^2 = \frac{\beta_{complex}{}^2 \overline{v_{r,b}^2}}{\left|Z_\pi + r_b\right|^2} + \overline{i_{c,n}^2}$$

$$\Rightarrow \overline{v_n^2} = \overline{v_{r,b}^2} + \left(\overline{i_{c,n}^2}\right) \frac{\left|Z_\pi + r_b\right|^2}{\beta_{complex}{}^2} \approx \overline{v_{r,b}^2} + \left(\overline{i_{c,n}^2}\right) \frac{\left|Z_\pi\right|^2}{\beta_{complex}{}^2} = 4kTr_b + \frac{2kT}{g_m}$$

*Ignoring* $r_b$

# BJT Noise Figure

$$F=\frac{SNR_{in}}{SNR_{out}}=\frac{S_i/N_i}{S_{out}/N_{out}}=\frac{1}{G_A}\frac{G_AN_i+N_a}{N_i}=1+\frac{N_a/G_A}{N_i}=1+\frac{\overline{v_n^2}+R_s^2\overline{i_n^2}}{\overline{v_s^2}}$$

*Input referred o/p noise*

At lower frequencies, $\beta_{complex}=\beta$

$$F=1+\frac{\overline{v_n^2}+R_s^2\overline{i_n^2}}{\overline{v_s^2}}=1+\frac{4kTr_b+\frac{2kT}{g_m}}{4kTR_s}+\frac{R_s^2 2qI_c}{\beta 4kTR_s}$$

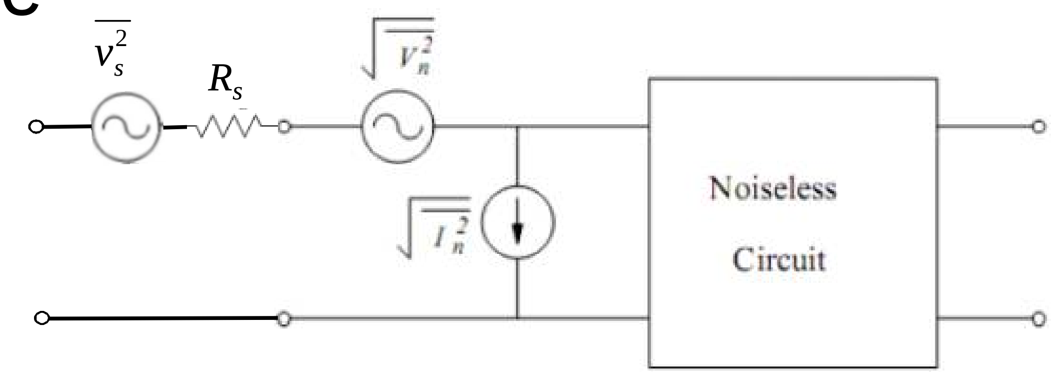
$$F=1+\frac{r_b}{R_s}+\frac{1}{2g_mR_s}+\frac{g_mR_s}{2\beta}, \text{Taking } 2kTg_m=2qI_c, \quad g_m=\frac{I_c}{V_T}=\frac{I_cq}{kT}$$

At higher frequencies,

$$F\approx 1+\frac{r_b}{R_s}+\frac{1}{2g_mR_s}+\frac{g_mR_s}{2\beta_{complex}}\approx 1+\frac{r_b}{R_s}+\frac{1}{2g_mR_s}+\frac{g_mR_s\beta}{2\beta_{complex}^2}$$

$$=1+\frac{r_b}{R_s}+\frac{1}{2g_mR_s}+\frac{g_mR_s}{2\beta}\left[1+\omega^2r_\pi^2C_\pi^2\right]$$

# Optimal Source Impedance




We had earlier seen that, $F(Y_s)$

$$= F_{\min} + \frac{R_n}{G_s}\left|Y_{opt} - Y_s\right|^2 = F_{\min} + \frac{4R_n\left|\Gamma_s\text{-}\Gamma_{opt}\right|^2}{\left(1-\left|\Gamma_s\right|^2\right)\left|1+\Gamma_{opt}\right|^2}$$

*Proof:*

Let, $\overline{v_s^2} = 4\,kTR_s, v_n^2 = 4kTR_n\,,\ i_n^2 = 4kTG_n$

$$F = 1 + \frac{N_a/G_A}{N_i} = 1 + \frac{\overline{v_n^2} + \overline{i_n^2}R_s^2}{\overline{v_s^2}} = 1 + \frac{R_n + G_nR_s^2}{R_s} = 1 + \frac{R_n}{R_s} + G_nR_s$$

For optimum value of $R_s, \frac{dF}{dR_s} = 0$, from which the optimum value of Rs is given by, $\Rightarrow$

$$R_{s,opt} = \sqrt{\frac{R_n}{G_n}} = \sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}}$$

$$\Rightarrow F_{\min} = 1 + G_nR_{s,opt} + \frac{R_n}{R_{s,opt}} = 1 + 2\sqrt{R_nG_n}$$

$$\Rightarrow F - F_{\min} = \frac{R_n}{R_s} + G_nR_s - 2\sqrt{R_nG_n} = \frac{R_n}{R_s}\left[1 + \left(\frac{R_s}{R_{opt}}\right)^2 - 2\frac{R_s}{R_{opt}}\right] = \frac{R_n}{R_s}\left|\frac{R_s}{R_{opt}} - 1\right|^2 = R_nR_s\left|G_{opt} - G_s\right|^2$$

$$\Rightarrow F = F_{\min} + R_nR_s\left|G_{opt} - G_s\right|^2$$

More generally $F\left(Y_s\right) = F_{\min} + \frac{R_n}{G_s}\left|Y_{opt} - Y_s\right|^2$

# Optimal $R_s$ for a BJT

$$\overline{v_n^2} = 4kTR_n = 2kT\left[2r_b + \frac{1}{g_m}\right] \Rightarrow R_n = \frac{1}{2}\left[2r_b + \frac{1}{g_m}\right]$$

$$\overline{i_n^2} = 4kTG_n = \frac{2kTg_m}{\beta_{complex}} \Rightarrow G_n = \frac{g_m}{2\beta_{complex}}$$

$$R_{s,opt} = \sqrt{\frac{R_n}{G_n}} = \frac{\sqrt{\beta_{complex}(1+2g_m r_b)}}{g_m}, NF_{\min} = 1 + 2\sqrt{R_n G_n} = 1 + \sqrt{(1+2g_m r_b)/\beta_{complex}}$$

To decrease $NF_{min}$ we need to:

1. Decrease $r_b$ (increase transistor size)
2. Decrease $g_m$ (decrease $I_C$ )
3. Increase β (very little scope)

# Conjugate Matching and Noise Matching

- Noise matching does not yield the maximum gain (conjugate match)
- Ideal Target:
  - $R_{S,opt} = Z_{in}^{*}$ = 50 ohms for simultaneous conjugate, noise and 50 ohm impedance match.
- Methods:
  - Adjust transistor size and bias to obtain $R_{S,opt}$ = 50 ohms (noise match)as much as possible.

  - If no further improvement can be done, then simply match i/p so that $Z_{in}$=50 ohm

# CE BJT linearity

$$
\begin{aligned}
I_C &\simeq I_S \exp\left(\frac{V_{BE0} + V_{in}}{V_T}\right) \\
&= I_S \exp\frac{V_{BEO}}{V_T}\left[1 + \frac{V_{in}}{V_T} + \frac{1}{2}\left(\frac{V_{in}}{V_T}\right)^2 + \frac{1}{6}\left(\frac{V_{in}}{V_T}\right)^3 + \cdots\right]
\end{aligned}
$$

We identify the non-linear coefficients:

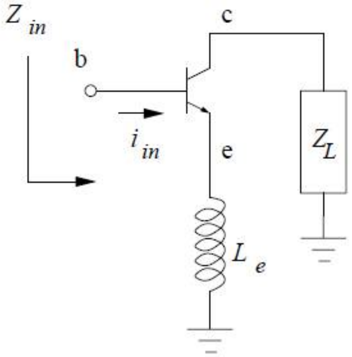
$$
\begin{aligned}
\alpha_1 &= I_S \exp\frac{V_{BEO}}{V_T}\ \frac{1}{V_T} \\
\alpha_3 &= I_S \exp\frac{V_{BEO}}{V_T}\ \frac{1}{6}\left(\frac{1}{V_T}\right)^3
\end{aligned}
$$

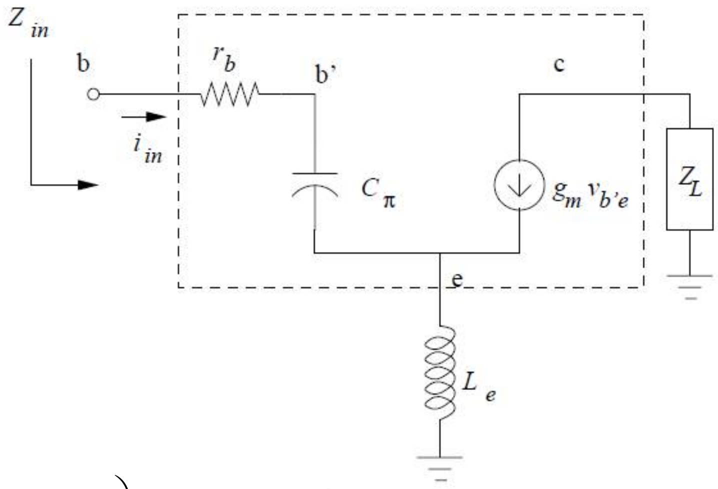
It results that we have:

$$
A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|} = 2\sqrt{2}V_T
$$

This voltage corresponds to -23 dBV or -10 dBm across 50 ohms. Additional linearization is required for larger IIP3.

# Inductive Degeneration




$$\frac{v_e}{L_e s} = \frac{v_b - i_{in}\left(r_b + \frac{1}{sC_\pi}\right)}{L_e s} = i_{in} + g_m v_{b'e} = i_{in}\left(1 + \frac{g_m}{sC_\pi}\right)$$

$$Z_{in} = \frac{v_b}{i_{in}} = r_b + \frac{1}{sC_\pi} + sL_e + g_m \frac{L_e}{C_\pi}$$

with proper choice of $L_e$, $g_m$ and $C_\pi$ we can select :

$$sL_e + \frac{1}{sC_\pi} = 0$$

$$Z_{in} = r_b + g_m \frac{L_e}{C_\pi} = 50\,\Omega$$

$G_m = i_c/v_{in}$ can be made dependent only on $L_e$

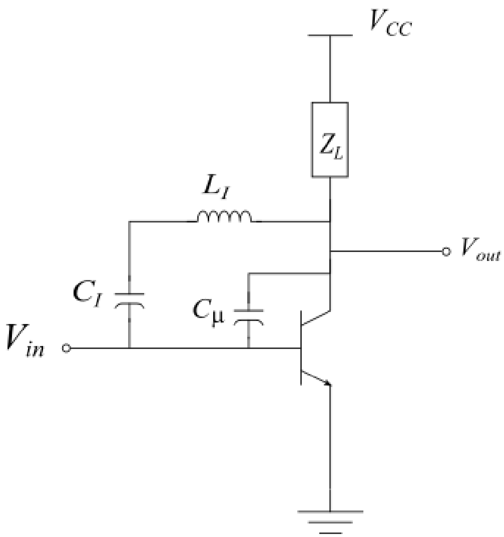
(Prove it)

Hence linearity is improved

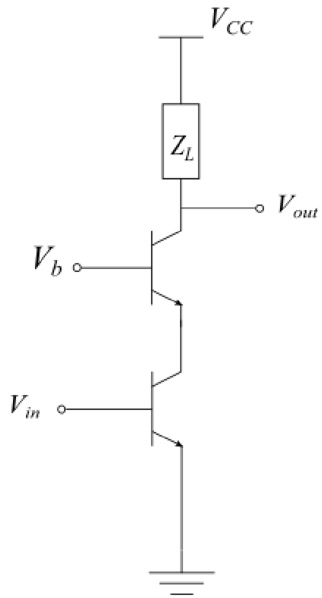
# Neutralization and Cascoding

- K indicates that stability improves as $|S_{12}|$ or ($|z_{12}|$ or $|y_{12}|$) decreases.
- This can be accomplished by neutralizing the input-output capacitance path:

  $L_1$ is selected to resonate with $C_\mu$ at the frequency of interest Problem: In RFIC the floating inductor introduces parasitic capacitances loading the input and output nodes.
- Reduced feedback can be achieved with the cascode configuration.

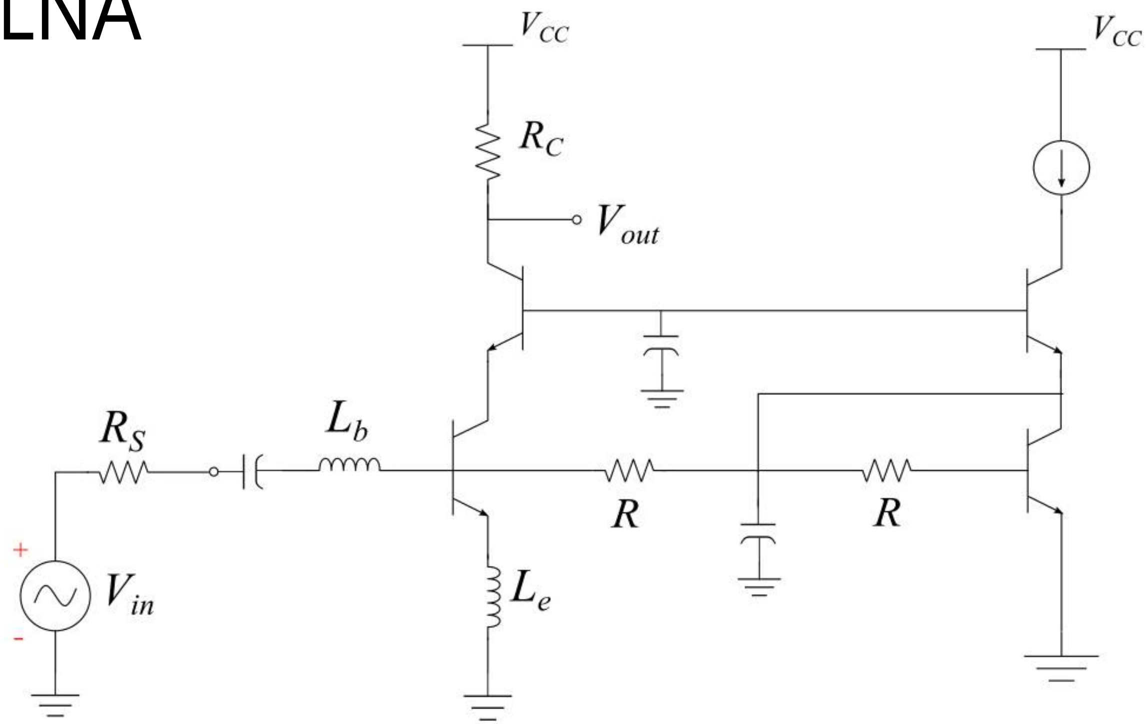



Neutralization



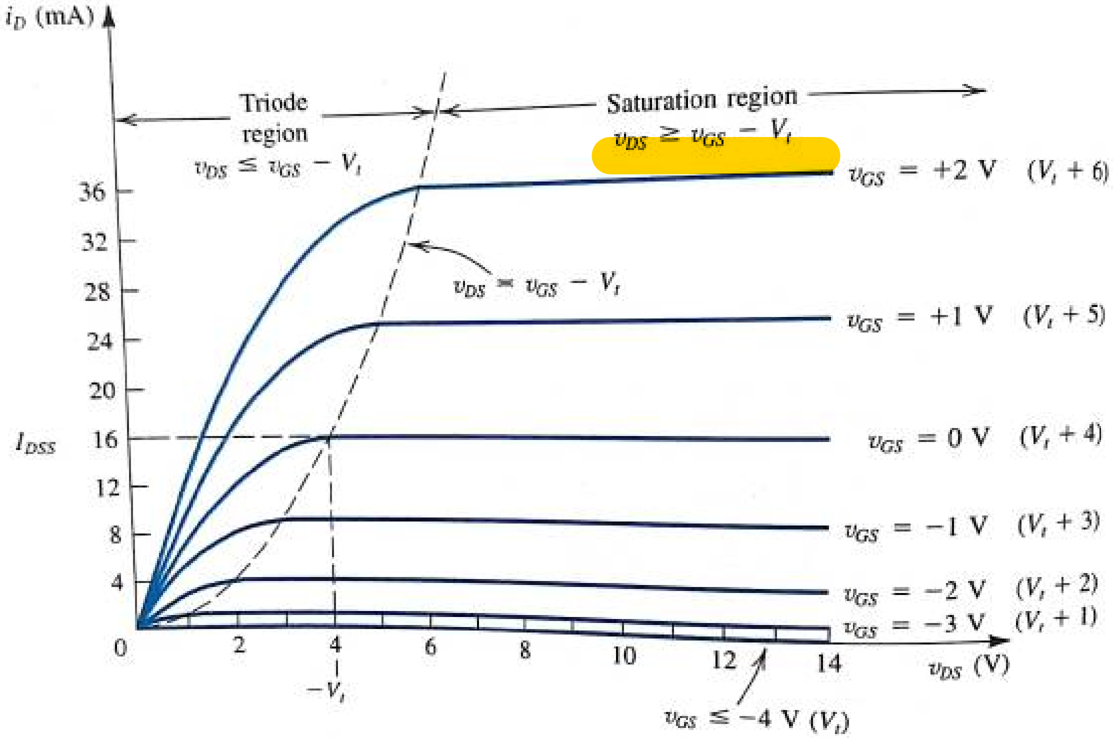

Cascoding

# Cascode BJT LNA




- Current fed to both transistor by the same bias line.
- Low distortion due to the inductance $L_e$.
- Stability improves (since back propagation of of signal is minimized).
- Slight degradation in Noise Figure.

# MOSFET characteristics



(b)

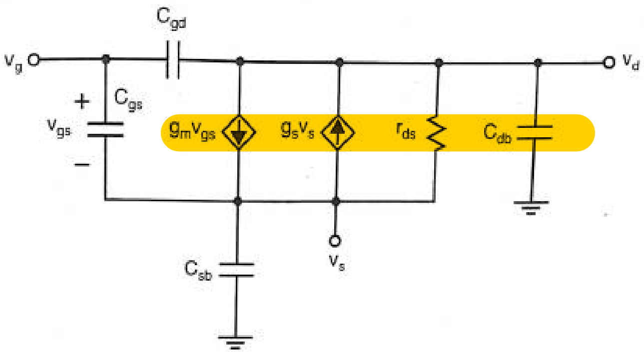
Model as given in the book by Johns and Martin

Saturation region

*Active (or Pinch-Off) Region* $(V_{GS} > V_{tn},\ V_{DS} \ge V_{eff})$

| | |
|---|---|
| $I_D = \frac{\mu_n C_{ox}}{2} \frac{W}{L} (V_{GS} - V_{tn})^2 [1 + \lambda(V_{DS} - V_{eff})]$ | |
| $\lambda \propto \frac{1}{L\sqrt{V_{DS} - V_{eff} + \Phi_0}}$ | $V_{tn} = V_{tn\text{-}0} + \gamma(\sqrt{V_{SB} + 2\phi_F} - \sqrt{2\phi_F})$ |
| $V_{eff} = V_{GS} - V_{tn} = \sqrt{\frac{2I_D}{\mu_n C_{ox} W/L}}$ | |

*Small-Signal Model (Active Region)*




Model as given in the book by Johns and Martin

| | |
|---|---|
| $g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)V_{eff}$ | $g_m = \sqrt{2\mu_n C_{ox}(W/L)I_D}$ |
| $g_m = \frac{2I_D}{V_{eff}}$ | $g_s = \frac{\gamma g_m}{2\sqrt{V_{SB}+\lvert 2\phi_F\rvert}}$ |
| $r_{ds} = \frac{1}{\lambda I_D}$ | $g_s \cong 0.2 g_m$ |
| $\lambda = \frac{k_{rds}}{2L\sqrt{V_{DS}-V_{eff}+\Phi_0}}$ | $k_{rds} = \sqrt{\frac{2K_s\varepsilon_0}{qN_A}}$ |
| $C_{gs} = \frac{2}{3}WLC_{ox} + WL_{ov}C_{ox}$ | $C_{gd} = WL_{ov}C_{ox}$ |
| $C_{sb} = (A_s + WL)C_{js} + P_s C_{j\text{-}sw}$ | $C_{js} = \frac{C_{j0}}{\sqrt{1+V_{SB}/\Phi_0}}$ |
| $C_{db} = A_d C_{jd} + P_d C_{j\text{-}sw}$ | $C_{jd} = \frac{C_{j0}}{\sqrt{1+V_{DB}/\Phi_0}}$ |

*Typical Values for a 0.8-μm Process*

| | |
|---|---|
| $V_{tn} = 0.8\ \text{V}$ | $V_{tp} = -0.9\ \text{V}$ |
| $\mu_n C_{ox} = 90\ \mu\text{A/V}^2$ | $\mu_p C_{ox} = 30\ \mu\text{A/V}^2$ |

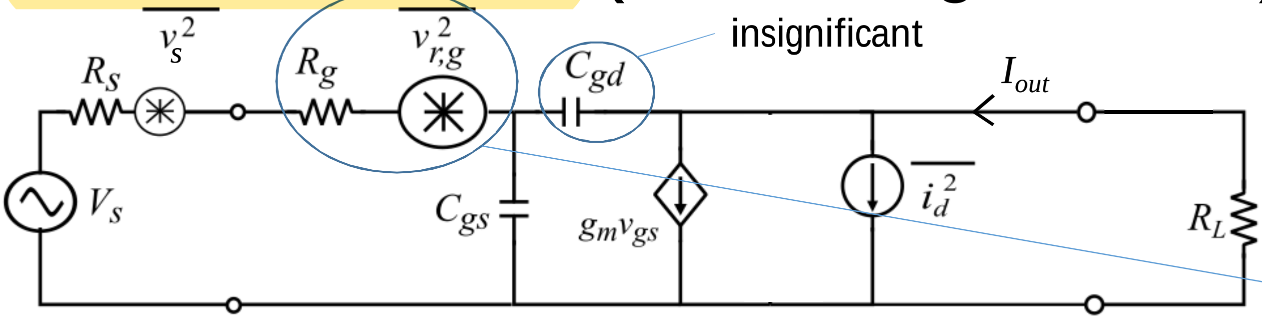
| | |
|---|---|
| $C_{ox} = 1.9\times10^{-3}\ \text{pF}/(\mu\text{m})^2$ | $C_j = 2.4\times10^{-4}\ \text{pF}/(\mu\text{m})^2$ |
| $C_{j\text{-}sw} = 2.0\times10^{-4}\ \text{pF}/\mu\text{m}$ | $C_{gs(overlap)} = 2.0\times10^{-4}\ \text{pF}/\mu\text{m}$ |
| $\phi_F = 0.34\ \text{V}$ | $\Phi_0 = 0.9\ \text{V}$ |
| $\gamma = 0.5\ \text{V}^{1/2}$ | $t_{ox} = 0.02\ \mu\text{m}$ |
| $N_B = 6\times10^{21}\ \text{impurities}/\text{m}^3$ | |

$g_m \cong 0.42\ mS$ for $W/L=10$, $I_D = 100\ \mu A$, $\mu_n C_{ox} = 90\ \mu A/V^2$

$$r_{ds} = r_o = \frac{1}{\lambda I_D} = \frac{1}{0.02\times100\times10^{-6}} = 500\ \ Kohms$$

# MOSFET noise (CS configuration)

Saturation region small signal model




$$\overline{i_d^2} = 4kT\gamma g_m$$

Gate induced thermal noise

$\overline{v_{r,g}^2} = 4kTR_g$, $R_g \approx \frac{1}{5g_{d0}}$ (Typical value 20 ohms), (where $g_{d0}$ is the drain - source conductance at zero $V_{DS}$) is not a real resistor, It only responds to noise currents and voltages but not to deterministic signals.

$$\overline{v_n^2} = \frac{\overline{i_d^2} + \left(\overline{v_{r,g}^2}\right) g_m^2}{g_m^2},\ g_m^2 \overline{i_n^2} |Z_{in}|^2 = \overline{i_d^2} \Rightarrow \overline{i_n^2} = \overline{i_d^2} / \left(g_m^2 |Z_{in}|^2\right)$$

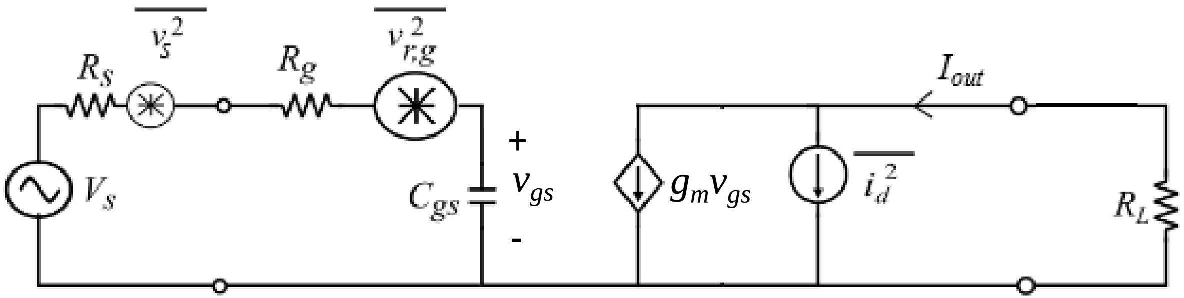
$$\Rightarrow F = 1 + \frac{\overline{v_n^2} + \overline{i_n^2}\left(R_s + R_g\right)^2}{\overline{v_s^2}} = 1 + \frac{\overline{v_{r,g}^2}}{\overline{v_s^2}} + \frac{\overline{i_d^2}}{g_m^2 \overline{v_s^2}} + \frac{\left(R_s + R_g\right)^2 \overline{i_n^2}}{\overline{v_s^2}} = 1 + \frac{R_g}{R_s} + \frac{4kT\gamma g_m}{4kTR_s g_m^2} + \frac{\left(R_s + R_g\right)^2 4kT\gamma g_m}{4kTR_s g_m^2 |Z_{in}|^2}$$

$$\approx 1 + \frac{R_g}{R_s} + \frac{\gamma}{R_s g_m} + \frac{(R_s + R_g)^2 \gamma}{R_s g_m |Z_{in}|^2}$$

At dc $|Z_{in}| = \infty \Rightarrow F = 1 + \frac{R_g}{R_s} + \frac{\gamma}{R_s g_m}$, If gate induced thermal noise is ignored, $F = 1 + \frac{\gamma}{R_s g_m}$

At higher frequencies we consider $|Z_{in}| = \frac{1}{\omega C_{gs}}$, $F = 1 + \frac{R_g}{R_s} + \frac{\gamma}{R_s g_m} + \frac{\gamma \left(R_s + R_g\right)^2 \omega^2 C_{gs}^2}{R_s g_m} = 1 + \frac{R_g}{R_s} + \frac{\gamma}{R_s g_m}\left[1 + \omega^2 \left(R_s + R_g\right)^2 C_{gs}^2\right]$

# Alternate Derivation




$$I_{out} = g_m v_{gs} + i_d \qquad \text{- - - - (1)}$$

$$V_s = (R_s + R_g) \times v_{gs} \times sC_{gs} + v_{gs} = v_{gs}\left[sC_{gs}(R_s + R_g) + 1\right]$$

Now substituting $v_{gs}$ from eqn (1) above we get,

$$V_s = \frac{(I_{out} - i_d)}{g_m}\left[sC_{gs}(R_s + R_g) + 1\right]$$

When $i_d = 0$ (noiseless case)

$$\frac{I_{out}}{V_s} = \frac{g_m}{sC_{gs}(R_s + R_g) + 1} = \frac{g_m}{1 + j\omega C_{gs}(R_s + R_g)} = G_m \text{ (say!)}$$

# Contd ....

$G_m$ represents the overall transcond uctance between i/p voltage and output current.

Hence total o/p referred noise current due to $\overline{v_{r,g}^2}$ and $\overline{v_s^2}$ is given by,

$$4kT(R_s + R_g) \times |G_m|^2$$

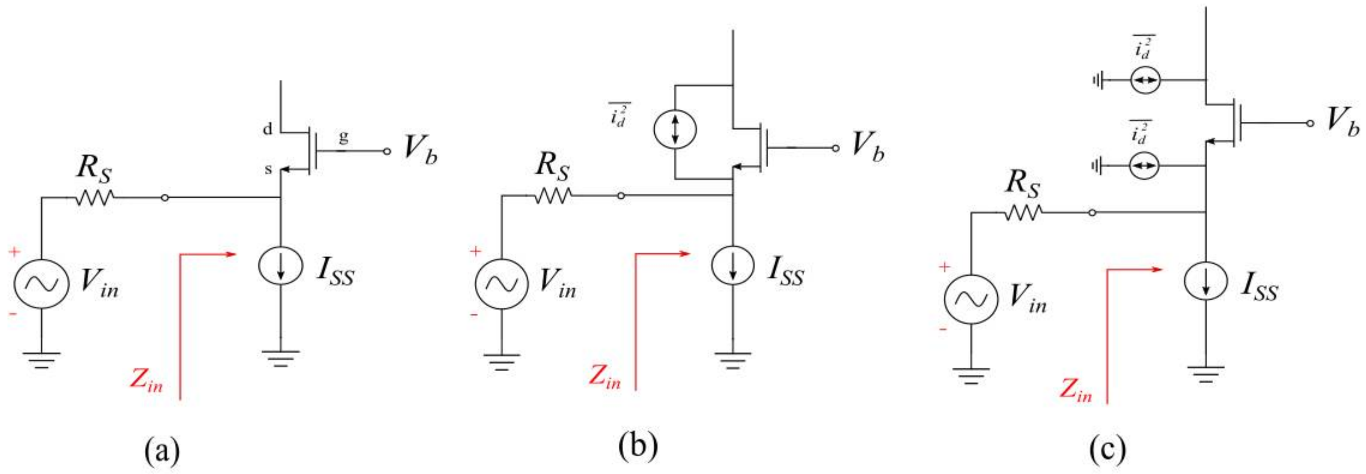
Hence, $NF = \dfrac{4kT(R_s + R_g) \times |G_m|^2 + \overline{i_d^2}}{4kTR_s \times |G_m|^2}$

$$= 1 + \frac{R_g}{R_s} + \frac{\gamma}{R_s g_m}\left[1 + \omega^2 C_{gs}^2 (R_s + R_g)^2\right]$$

# CS MOSFET

- Higher Linearity  using inductive degeneration.
- Wideband matching difficult.

# CG Noise

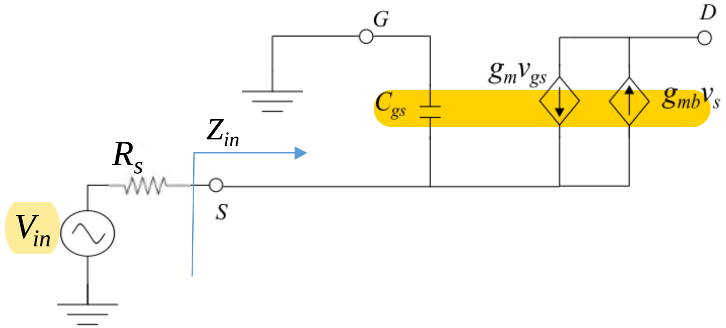



Input impedance is low :

$$Z_{in} = \frac{1}{g_m + g_{mb} + sC_{gs}} \quad \text{(prove it)}$$

$$F = 1 + \frac{\underbrace{\overline{v_n^2}}_{\textit{equal to } 0} + \overline{i_n^2}R_s^2}{\overline{v_s^2}} = 1 + \frac{\overline{i_n^2}R_s^2}{4kTR_s} = 1 + \frac{\overline{i_d^2}R_s^2}{4kTR_s}$$

$$= 1 + \frac{4\gamma kTg_m}{4kTG_s} = 1 + \frac{\gamma g_m}{G_s} > \underbrace{1 + \gamma}_{\textit{assuming that } G_s = g_m \ [\textit{I/P matching}]} = 2.2\,dB\ (\text{for } \gamma = 2/3)$$




- $\gamma$ is (2/3) only for long channel devices.
- Short channel MOSFETs have larger value of $\gamma$.
- Parasitics and other noise sources will add additional noise.
- Easier 50 ohms matching over wideband. However resistive matching will add 3 dB noise.

# Scaling Rules

In classical noise theory the noise figure is a function of the source admittance $Y_s$ and can be written as,

$$F(Y_s) = F_{\min} + \frac{R_n}{G_s}\left|Y_s - Y_{opt}\right|^2 \qquad g_m = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right) I_D} = \frac{2I_D}{V_{GS} - V_{Tn}}$$

$$F_{\min} \approx 1 + \frac{f}{f_T}\sqrt{\gamma\,\delta\left(1-|c|^2\right)} \qquad f_T = \frac{1}{2\pi}\frac{g_m}{C_{gs}}$$

$\gamma = 2/3, \delta = 4/3$, c = j0.395 (Typical values) (see book by Thomas Lee)

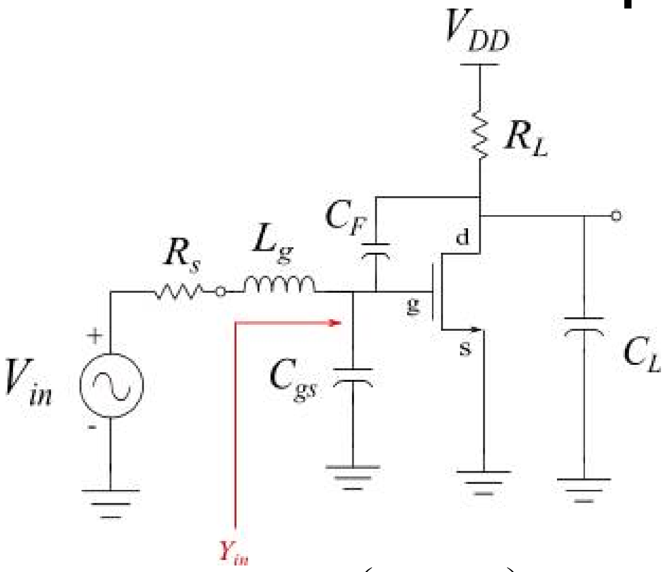
These expressions indicate that $F_{\min}$ is minimized for large $f_T \Rightarrow$ large $g_m$. This indicates that we should:

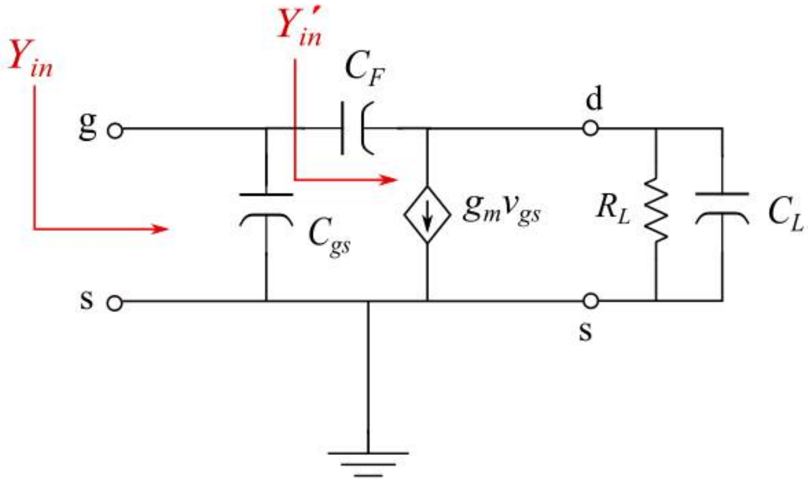
- Select the shortest gate length available.
- Use the largest current allowable by the power budget.
- Use the largest width allowable within power budget.

# Conjugate Matching and Noise Matching

- Noise matching does not yield the maximum gain (conjugate match)
- Ideal Target:
    - $Z_{S,opt} = Z_{in}^{*}$ = 50 ohms for simultaneous conjugate, noise and 50 ohm impedance match.
- Methods:
    - Adjust transistor size and bias to obtain $Z_{S,opt}$ = 50 ohms (noise and conjugate match) as much as possible.
    - If no further improvement can be done, then simply match i/p so that $Z_{in}$=50 ohm (impedance match)

# Common Source Input Matching with feedback capacitor




Starting from: $v_{dg}sC_F + v_{ds}\left(\frac{1}{R_L} + sC_L\right) + g_m v_{gs} = 0, v_{dg} = v_{ds} - v_{gs}$

and rearranging: $v_{gs}(sC_F - g_m) = v_{ds}(sC_F + \frac{1}{R_L} + sC_L)$

$$i_{in} = v_{gs}sC_{gs} + v_{gd}sC_F = v_{gs}\left(sC_{gs} + sC_F - \frac{v_{ds}}{v_{gs}}sC_F\right) = Y_{in}v_{gs}$$

results in the impedance $Y_{in}$-$sC_{gs}$ being:

$$Y'_{in} = Y_{in} - sC_{gs} = sC_F \frac{1 + sC_L R_L + g_m R_L}{1 + s(C_L + C_F)R_L}$$

# ...Contd

$$\mathrm{Re}\left[Y_{in}^{'}\right] = R_L C_F \omega^2 \frac{C_F + g_m R_L (C_L + C_F)}{1 + R_L^2 (C_L + C_F)^2 \omega^2}$$

$$\mathrm{Im}\left[Y_{in}^{'}\right] = C_F \omega \frac{R_L^2 C_L (C_L + C_F)\omega^2 + 1 + g_m R_L}{1 + R_L^2 (C_L + C_F)^2 \omega^2}$$

If $\mathrm{g_m R_L >> 1, C_L >> C}_F$ and $\omega \approx 1/(\mathrm{R_L C_L})$, the expression reduces to :

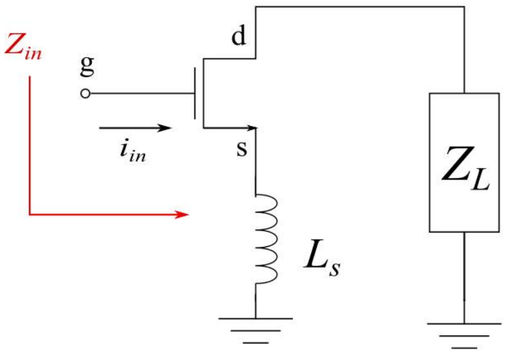
$$\mathrm{Re}\left[Y_{in}^{'}\right] = \frac{g_m}{2}\frac{C_F}{C_L} = \mathrm{Re}\,[Y_{in}] \quad \rightarrow$$ Need to make this equal to $1/50\ \mathrm{ohms}^{-1}$

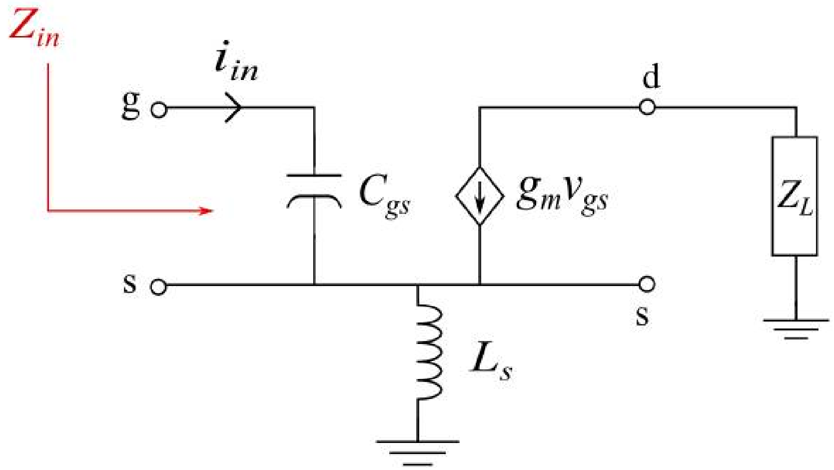
Alternatively a resistance in series/parallel can be added but that will increase NF.

$$\mathrm{Im}\left[Y_{in}^{'}\right] = C_F \omega \left(1 + \frac{g_m R_L}{2}\right) \Rightarrow \quad \mathrm{Im}\left[Y_{in}\right] = \omega \left[C_F \left(1 + \frac{g_m R_L}{2}\right) + C_{GS}\right] \rightarrow$$ Need to design $\mathrm{L_g}$ such that this value is canceled.

- Matching is narrowband
- $C_L$ can change o/p loading.

# Inductive Degeneration




$$\frac{v_s}{L_s s} = \frac{v_g - i_{in}(1/C_{gs}s)}{L_s s} = i_{in} + g_m v_{gs} = i_{in}\left(1 + \frac{g_m}{C_{gs}s}\right)$$

$$Z_{in} = \frac{v_g}{i_{in}} = \frac{1}{C_{gs}s} + L_s s + g_m \frac{L_s}{C_{gs}}$$

With proper choice of $L_s$, $g_m$ and $C_{gs}$ we can select :

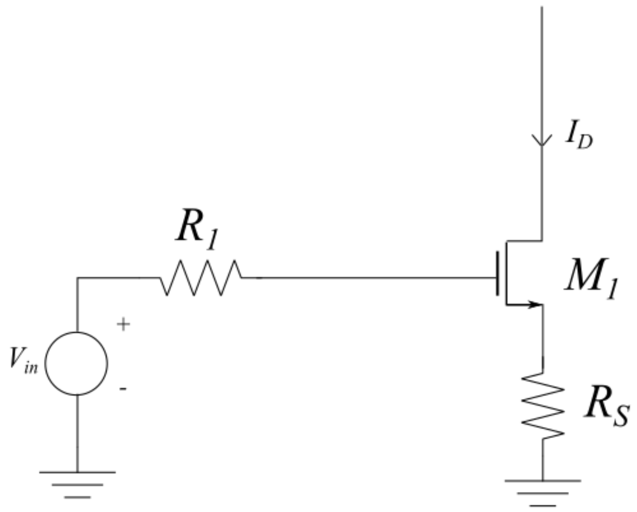
$$L_s s + \frac{1}{C_{gs}s} = 0$$

$$Z_{in} = g_m \frac{L_s}{C_{gs}} = 50\,\Omega \begin{bmatrix} \text{or add a resiatance in series to} \\ \text{match i/p to 50 ohms} \end{bmatrix}$$

$$G_m = \frac{i_D}{v_{in}} = \frac{g_m}{(1-\omega^2 C_{gs}L_s) + j\omega g_m L_s} = \frac{1}{j\omega_r L_s} \text{ for } \omega_r L_s = \frac{1}{\omega_r C_{gs}}$$

- Improves linearity (due to negative feedback).
- Matching does not lead to increase in NF since components are reactive.
- Try to derive the NF of the inductively degenerated CS LNA.

# Linearity of degenerated CS stage




$$I_D = \alpha_1 V_{in} + \alpha_2 V_{in}^2 + \alpha_3 V_{in}^3$$ Here $V_{in}$ refers to the ac component

$$\alpha_1 = \left.\frac{\partial I_D}{\partial V_{in}}\right|_{V_{in0}}, \alpha_2 = \frac{1}{2}\left.\frac{\partial^2 I_D}{\partial V_{in}^2}\right|_{V_{in0}}, \alpha_3 = \frac{1}{6}\left.\frac{\partial^2 I_D}{\partial V_{in}^2}\right|_{V_{in0}}$$

$$I_D = K(V_{GS} - V_{TH})^2, g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{in0}} = 2K(V_{GS0} - V_{TH}) = 2K(V_{in0} - R_S I_{D0} - V_{TH})$$

where $V_{in0}$ and $I_{D0}$ are the dc bias values

$$V_{GS} = V_{in} - R_S I_D \Rightarrow I_D = K(V_{in} - R_S I_D - V_{TH})^2$$

$$\Rightarrow \frac{\partial I_D}{\partial V_{in}} = 2K(V_{in} - R_S I_D - V_{TH})\left(1 - R_S \frac{\partial I_D}{\partial V_{in}}\right) \Rightarrow \left.\frac{\partial I_D}{\partial V_{in}}\right|_{V_{in0}} = 2K(V_{in0} - R_S I_{D0} - V_{TH})\left(1 - R_S \left.\frac{\partial I_D}{\partial V_{in}}\right|_{V_{in0}}\right)$$

$$\Rightarrow \alpha_1 = \left.\frac{\partial I_D}{\partial V_{in}}\right|_{V_{in0}} = \frac{g_m}{1 + g_m R_S},$$

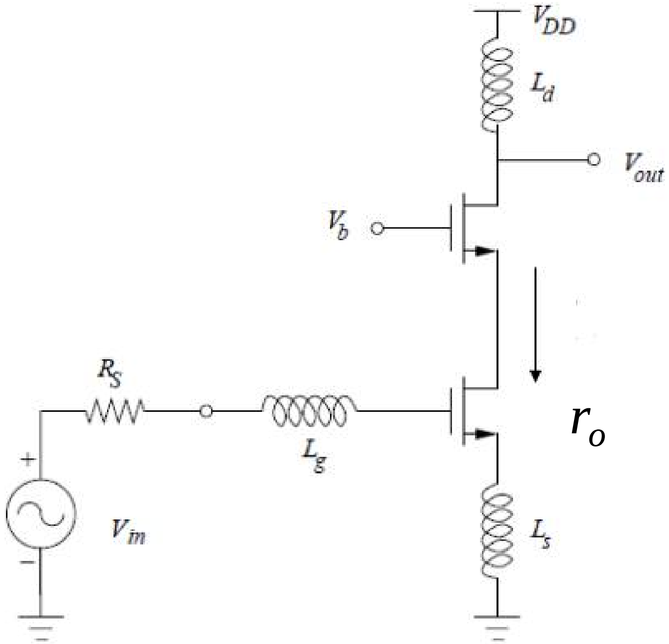
Similarly, we can find,

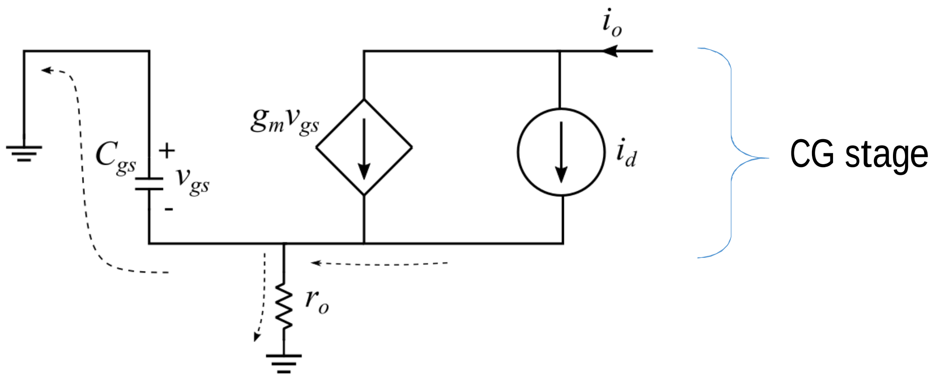
$$\alpha_2 = \frac{1}{2}\left.\frac{\partial^2 I_D}{\partial V_{in}^2}\right|_{V_{in0}} = \frac{K}{(1 + g_m R_S)^3}, \alpha_3 = \frac{1}{6}\left.\frac{\partial^3 I_D}{\partial V_{in}^3}\right|_{V_{in0}} = \frac{-2K^2 R_S}{(1 + g_m R_S)^5}$$

From these equations, IIP3 and IIP2 can be obtained as

$$A_{IIP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|} = \frac{(1 + g_m R_S)^2}{K}\sqrt{\frac{2}{3}\frac{g_m}{R_S}}, \quad A_{IIP2} = \frac{\alpha_1}{\alpha_2} = \frac{g_m}{K}(1 + g_m R_S)$$

# MOSFET Cascode LNA




For $\omega$ high, $C_{gs}$ acts as a short. Hence noise current will be shorted.

For $\omega$ medium, $C_{gs}$ acts as low impedance

$$v_{gs} = -(i_d + g_m v_{gs})\frac{1}{j\omega C_{gs}} \Rightarrow v_{gs} = \frac{-i_d}{g_m + j\omega C_{gs}}$$

$$g_m v_{gs} = \frac{-i_d}{1+\frac{j\omega C_{gs}}{g_m}} = \frac{-i_d}{1+\frac{j\omega}{\omega_T}} \approx -i_d$$

Hence the drain noise current does not pass through $r_o$.

For $\omega$ low, $C_{gs}$ acts as high impedance,

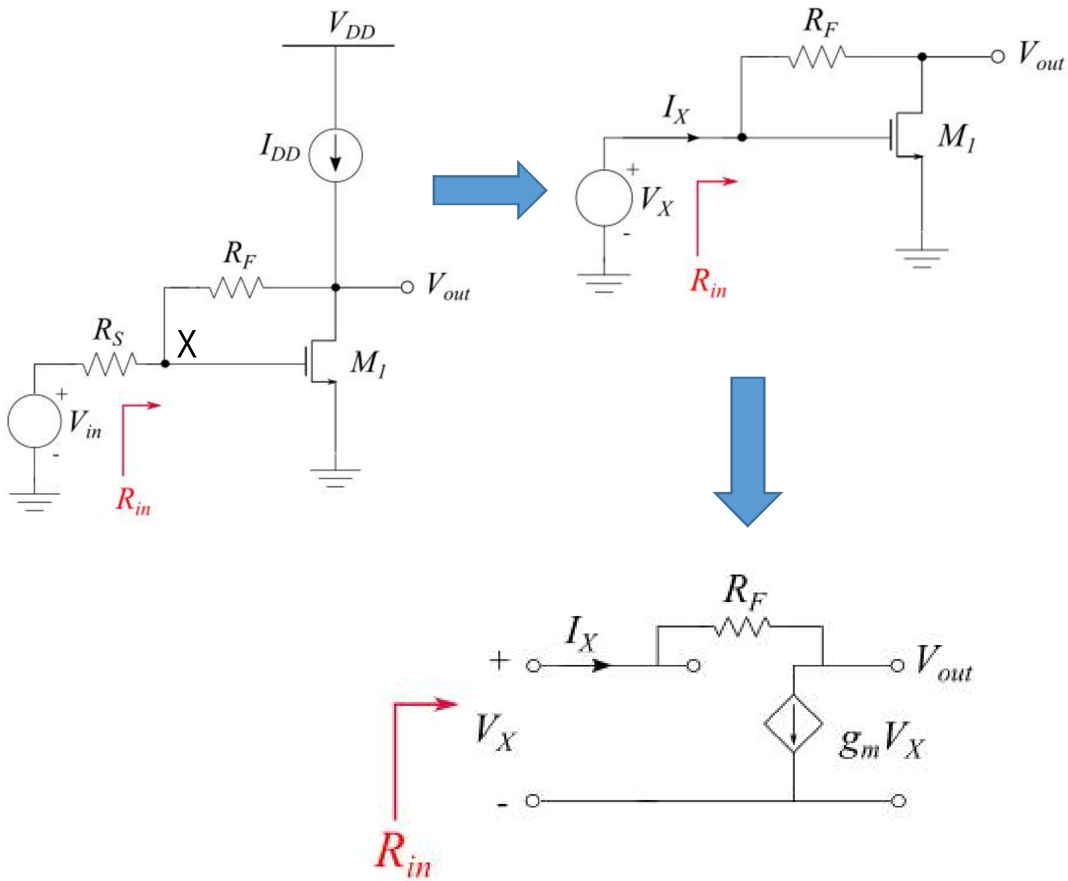
$$v_{gs} = -(i_d + g_m v_{gs})\, r_o \Rightarrow v_{gs} = \frac{-i_d r_o}{1+g_m r_o}$$

$$g_m v_{gs} = \frac{-i_d g_m r_o}{1+g_m r_o} \approx -i_d$$

Hence again the drain noise current does not pass through $r_o$.

- Cascode noise does not appear at o/p so noise analysis same as inductor degenerated CS stage.
- Cascode device is unilateral so stability improves
- Linearity improved by $L_s$.

# Common Source with Resistive Feedback




$$I_X = g_m V_X$$

$$\Rightarrow \frac{V_X}{I_X} = \frac{1}{g_m} = R_{in}, \text{ For input matching, } R_S = R_{in} \Rightarrow R_S = \frac{1}{g_m}$$

$$\text{Also, } V_{out} = V_X - I_X R_F = V_X - g_m V_X R_F$$

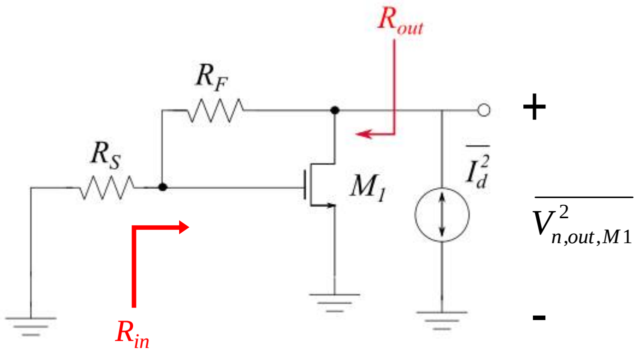
$$\Rightarrow \frac{V_{out}}{V_X} = 1 - \frac{R_F}{R_S} \qquad (1)$$

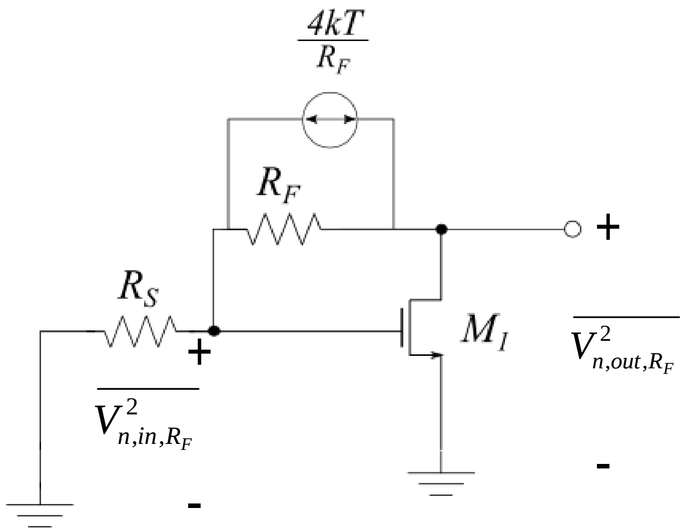
$$\text{Further, } V_X = \frac{V_{in} R_{in}}{R_{in} + R_S} \Rightarrow \frac{V_X}{V_{in}} = \frac{R_{in}}{R_{in} + R_S} \qquad (2)$$

From (1) and (2),

$$A_v = \frac{V_{out}}{V_{in}} = \left(1 - \frac{R_F}{R_S}\right)\left(\frac{R_{in}}{R_{in} + R_S}\right) = \frac{1}{2}\left(1 - \frac{R_F}{R_S}\right)$$

# Common Source with Resistive Feedback (Noise analysis)




$$R_{out} = \frac{1}{2}\left(R_F + R_S\right)(\text{Prove it !})$$

$$\overline{V^2_{n,out,M_1}} = \overline{I_d^2} R_{out}^2 = 4kTg_m \frac{\left(R_F + R_S\right)^2}{4},$$

$$\overline{V^2_{n,in,R_F}} = \frac{4kT}{R_F} R_S^2, \ \overline{V^2_{n,out,R_F}} = \overline{V^2_{n,in,R_F}} A_v^2 = \frac{4kT}{R_F} R_S^2 \frac{1}{4}\left(1 - \frac{R_F}{R_S}\right)^2 = \frac{kTR_S^2}{R_F}\left(1 - \frac{R_F}{R_S}\right)^2$$

Hence,

$$NF = 1 + \frac{\overline{V^2_{n,out,M_1}} + \overline{V^2_{n,out,R_F}}}{A_v^2\left(4kTR_S\right)} = 1 + \frac{4kT\gamma g_m \frac{\left(R_F + R_S\right)^2}{4} + \frac{kTR_S^2}{R_F}\left(1 - \frac{R_F}{R_S}\right)^2}{\left(1 - \frac{R_F}{R_S}\right)^2 \left(kTR_S\right)}$$

$$= 1 + \gamma \ g_m \ R_S \left(\frac{R_S + R_F}{R_S - R_F}\right)^2 + \left(\frac{R_S}{R_F}\right)$$

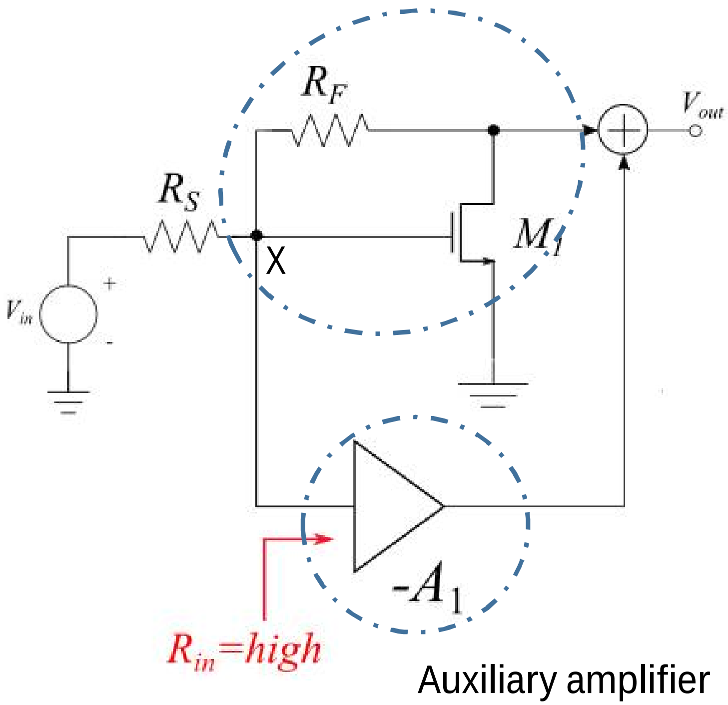
$$= \infty \quad \left[\text{For } R_S = R_F = 50\,\Omega\right]$$

$$= 1 + \gamma \quad \left[\text{For } R_F \text{ high}\right]$$

For, $\gamma = 2/3$, and $R_F$ high, the NF is equal to 2.2 dB

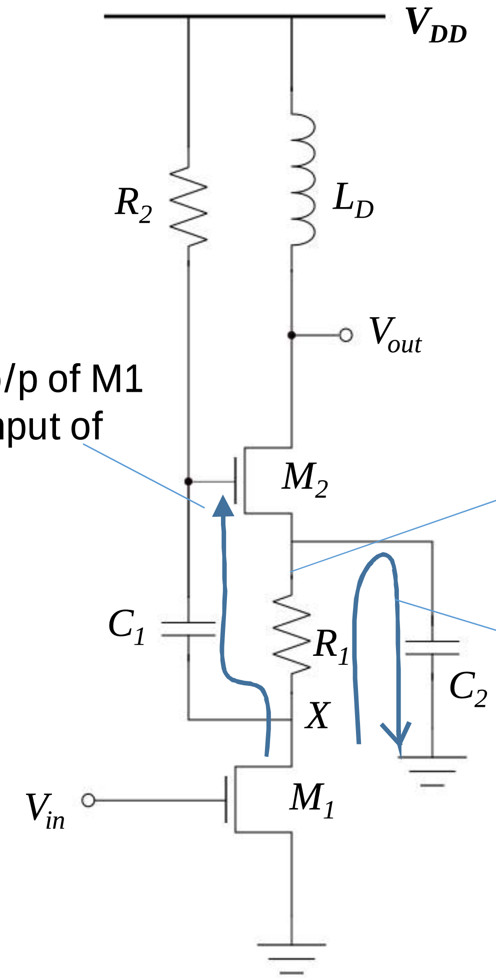
# Some other topologies

# Noise cancelling LNA




$A_1$ is the gain of noise voltage of $M_1$ at the o/p.
It can be shown that the noise voltage of auxillary amplifier can be used to cancel the noise of $M_1$.

# Current reuse LNA




RF o/p of M1 as input of M2

- Not cascode. Both M1 and M2 act as CS stages.
- Additional gain due to 2 stages but no additional bias current needed.
- Voltage head room reduced.
- Less reverse isolation than cascode

Same bias current for M1 and M2 hence the term "current reuse"

No RF coupling between drain of M1 and source of M2