Power Amplifiers

Typical PA Performance

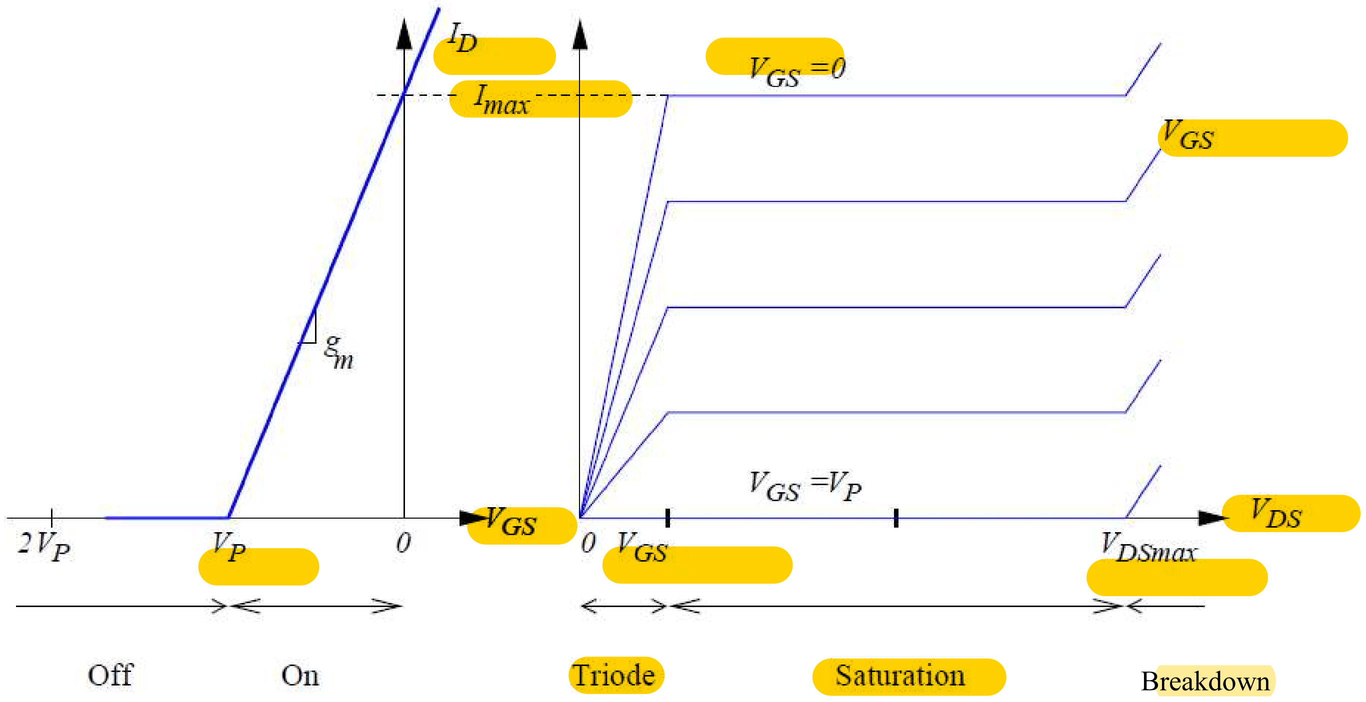
Output Power	20 to 30 dBm (.1 to 1 W)
Efficiency	30 % to 60 %
IMD	- 30 dBc
Supply Voltage	3.8 to 5.8 V
Gain	20 to 30 dB
Output Spurs & Harmonics	- 50 to - 70 dBc
Power Control	On-Off (TDMA) or 1 dB-steps (CDMA)
Output Thermal Noise	< -130 dBm/Hz
Stability Factor	>1

PA efficiency

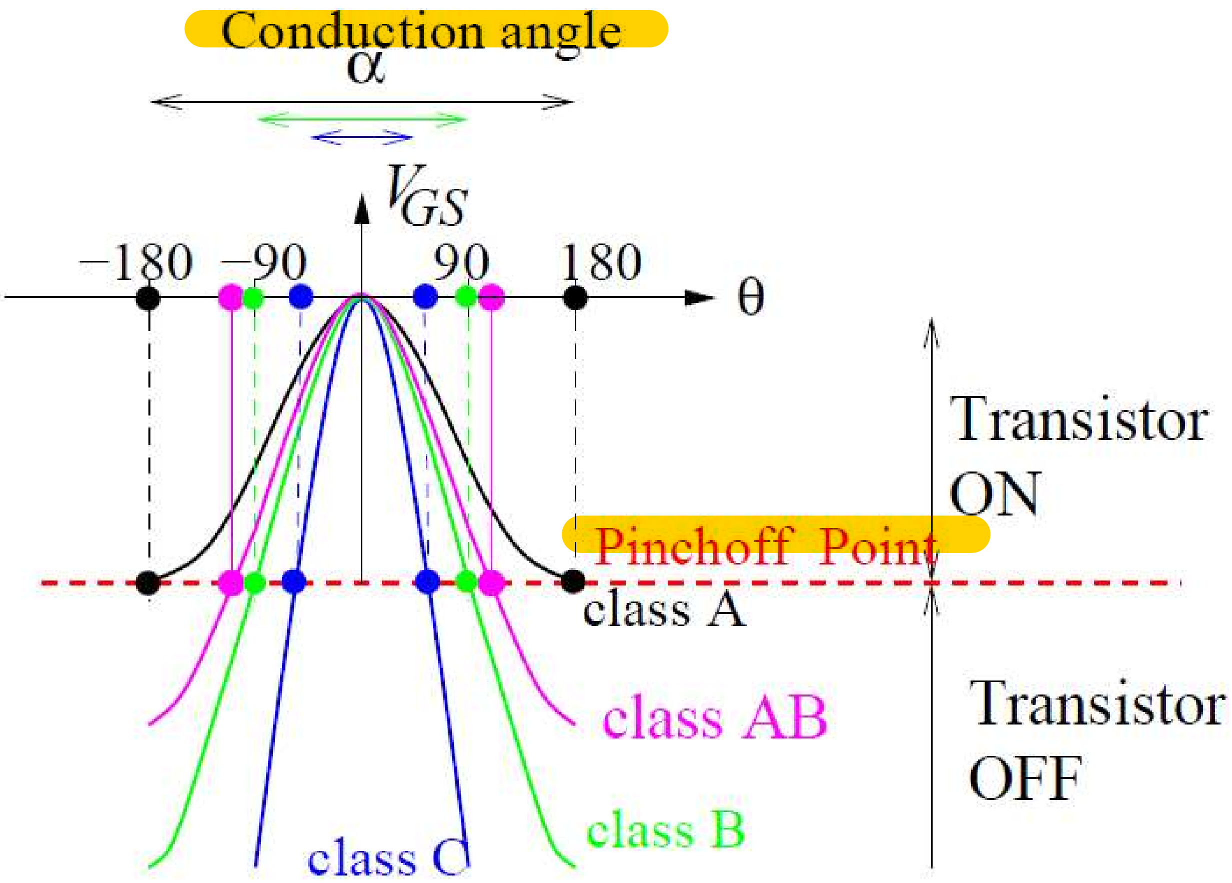
Drain Efficiency, $\eta = \dfrac{P_{RF}}{P_{DC}}$

Power Added Efficiency, $\mathrm{PAE} = \dfrac{P_{RF} - P_{in}}{P_{DC}}$

Ideal FET Characteristics



Conduction Angle Definition (α)



Class Definition

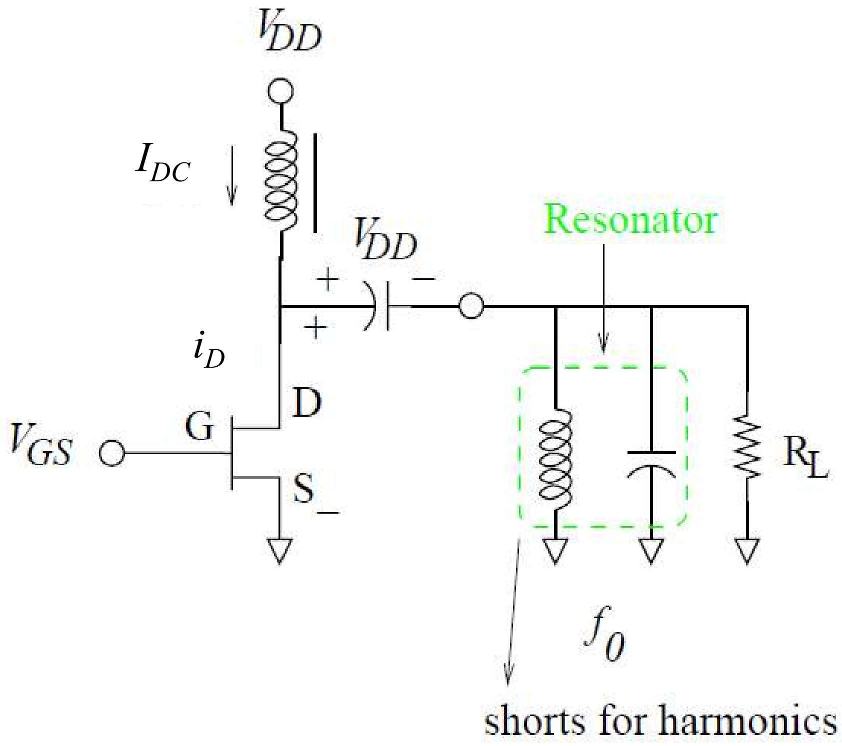
Amplifier classes are defined using the conduction angle:

- Class A: $\alpha = 360^o$ (On 100% of the cycle)
- Class AB: $180 < \alpha < 360$ (On 50 - 100% of the cycle)
- Class B: $\alpha = 180^o$ (On 50 % of the cycle)
- Class C: $\alpha < 180^o$ (On less than 50% of the cycle)

Note:

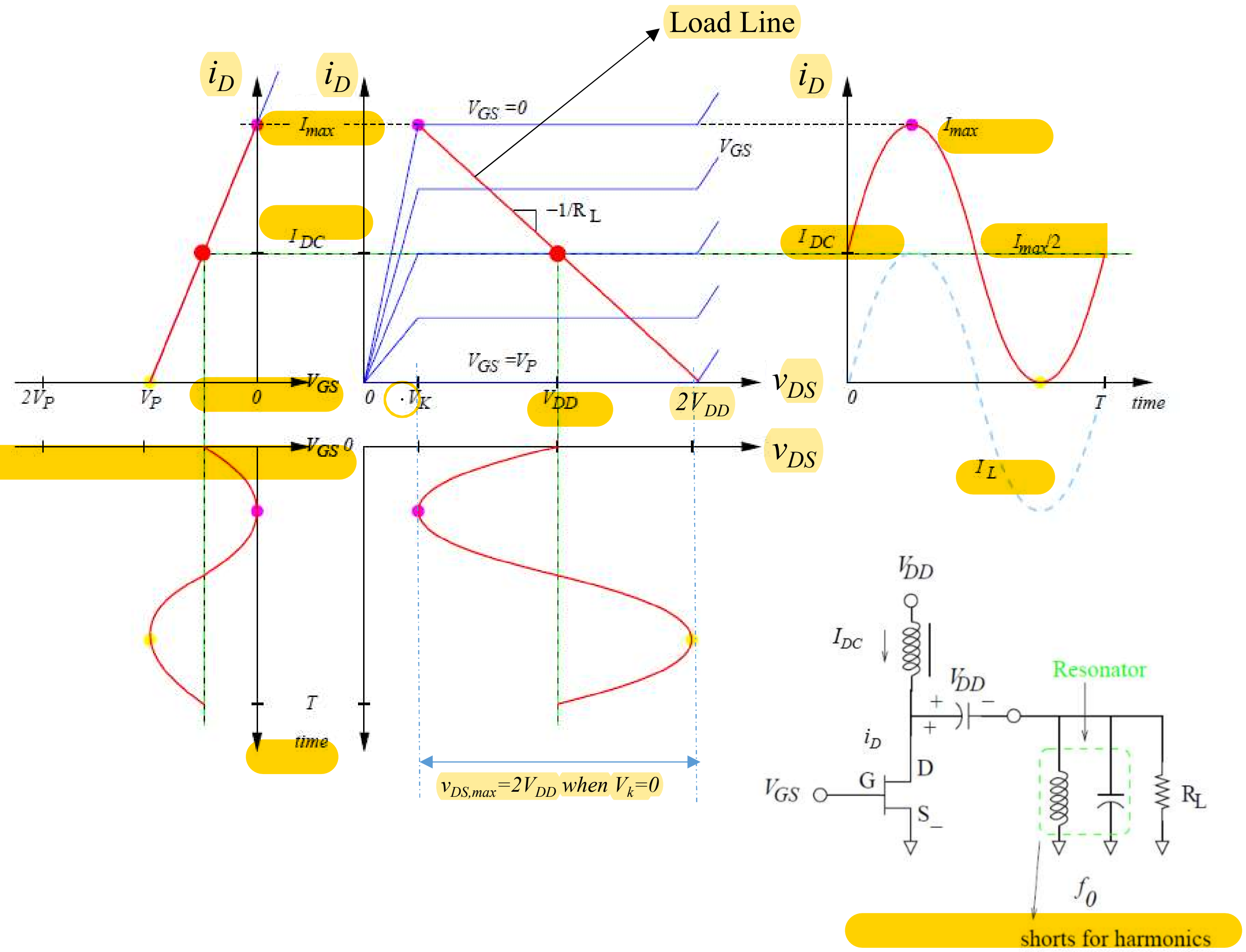
- Low noise amplifiers operate in class A.
- Power amplifiers operate in class A to F

Class A Amplifier



- The RF choke maintains a constant DC current and provides an RF open.
- The power supply voltage V_{DD} appears across the DC block capacitor.
- V_{DS} can swing from 0 to $2V_{DD}$.

Class A Amplifier Operation (V_K=0)



$$i_D = I_{DD} + \underbrace{\left(-i_{pk}\cos(\omega_0 t)\right)}_{I_L} = I_{DD} + \underbrace{\left(i_{pk}\sin(\omega_0 t)\right)}_{I_L}$$

$\Rightarrow\ I_{DC}$ (dc value of i_D) $= I_{DD}$

$I_{DC} = V_{DD}/R_L \Rightarrow I_{DD} = V_{DD}/R_L$

because V_{DD} appears directly across R_L when $v_{DS}=0$

when $i_D = 0 \Rightarrow |i_{pk}| = I_{DD} = V_{DD}/R_L$

max value of $i_D = I_{\max} = 2|i_{pk}| = 2I_{DC}$

Taking $V_k = 0$,

$v_{DS} = V_{DD} + i_{pk}R_L\cos(\omega_0 t)$

$\Rightarrow v_{DS,\max} = 2V_{DD}$

Further,

$v_{DS} = 2V_{DD} - R_L i_D = v_{DS,\max} - R_L i_D \rightarrow$ Load line eqn

when, $i_D = 0,\ v_{DS} = v_{DS,\max}$

when $v_{DS} = 0,\ i_D = I_{\max} \Rightarrow I_{\max} = \dfrac{2V_{DD}}{R_L} = 2I_{DD} = 2I_{DC}$

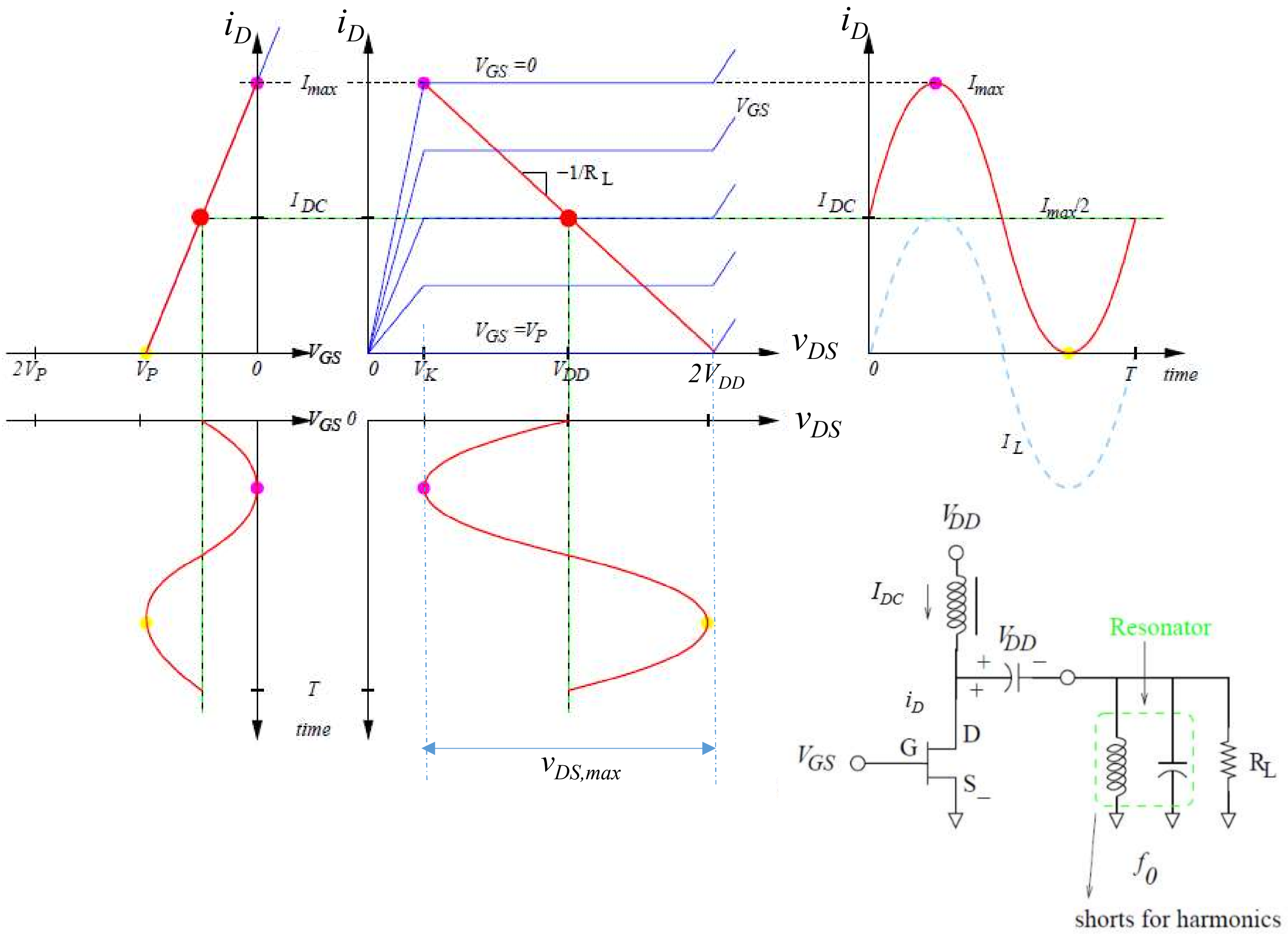
$$P_{rf,\max} = \frac{i_{pk}^2 R_L}{2} = \frac{I_{DD}^2 R_L}{2} = \frac{V_{DD}^2}{2R_L},$$

$$P_{DC} = I_{DD}\ V_{DD} = I_{DD}^2 R_L = \frac{V_{DD}^2}{R_L}$$

Drain Efficiency, $\eta = \dfrac{P_{RF,MAX}}{P_{DC}} = \dfrac{1}{2} = 50\%$

assuming V_K is very small

If V_K (Knee Voltage) ≠0 (Class A)



$$V_{RF} = \frac{1}{2}(v_{DS\max} - V_K)$$

The rf voltage swing across D-S is centered at V_{DD}

Hence, $V_{DD} = \frac{1}{2}(v_{DS\max} + V_K)$

$$\Rightarrow v_{DS\max} = 2V_{DD} - V_K$$

amplitude of rf output current, $i_{pk} = I_{DC} = I_{DD}$

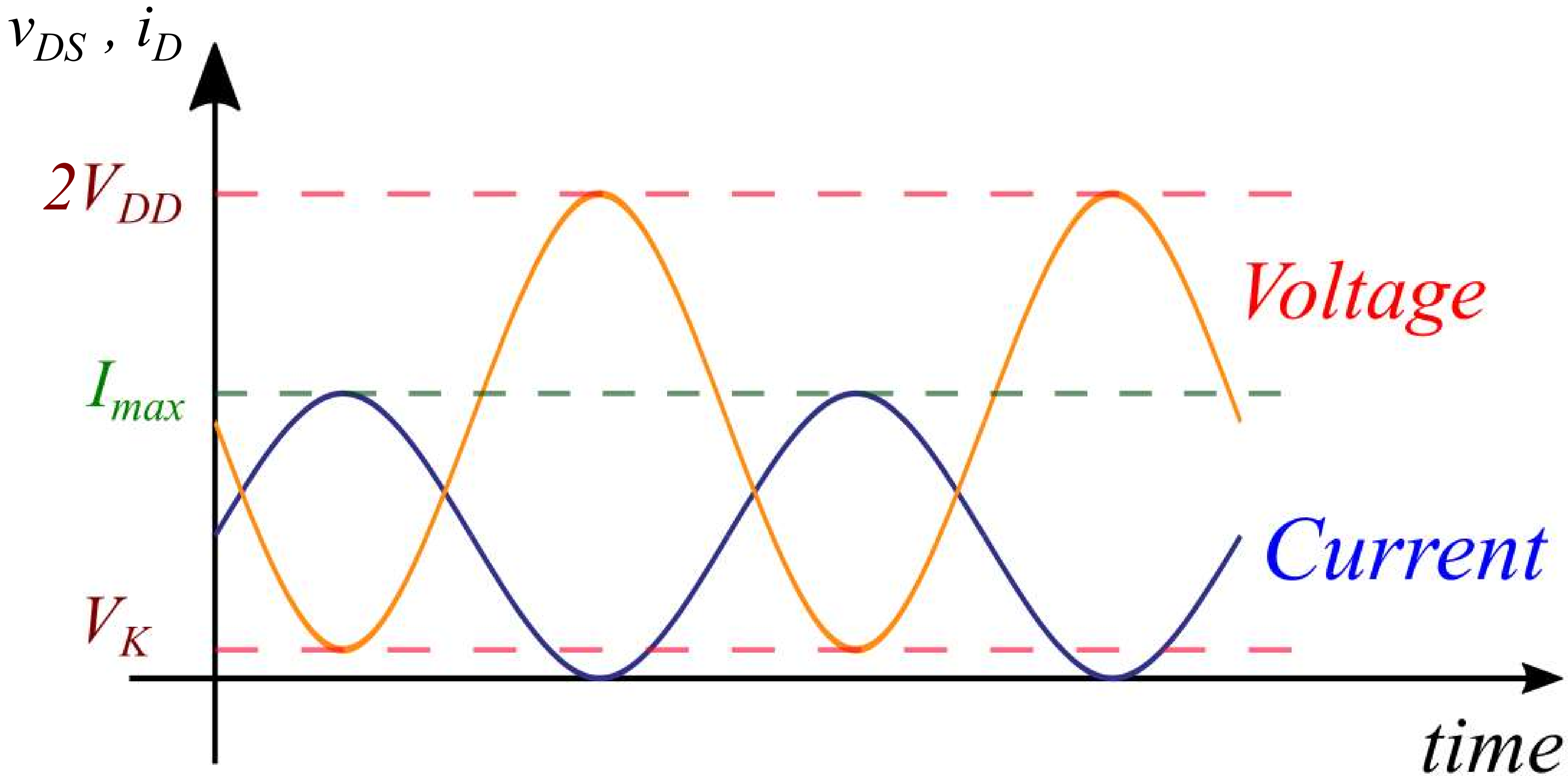
$P_{RF} = \frac{1}{2} V_{RF} i_{pk}$ (Assuming swing is between $v_{DS\max}$ and V_K)

$$= \frac{1}{4}(v_{DS\max} - V_K) I_{DC}$$

$$P_{DC} = V_{DD} I_{DC}$$

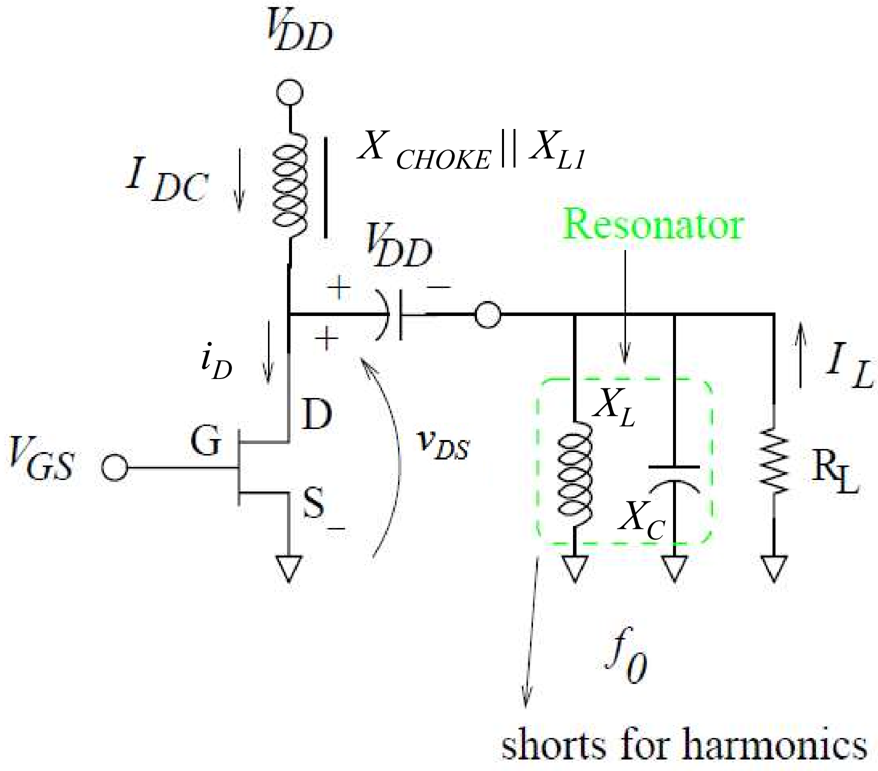
$$\eta = \frac{P_{RF}}{P_{DC}} = \frac{\frac{1}{4}(v_{DS\max} - V_K) I_{DC}}{\frac{1}{2}(v_{DS\max} + V_K) I_{DC}}$$

$$= \frac{1}{2} \frac{v_{DS\max} - V_K}{v_{DS\max} + V_K} = 50\% \text{ when } V_K = 0$$



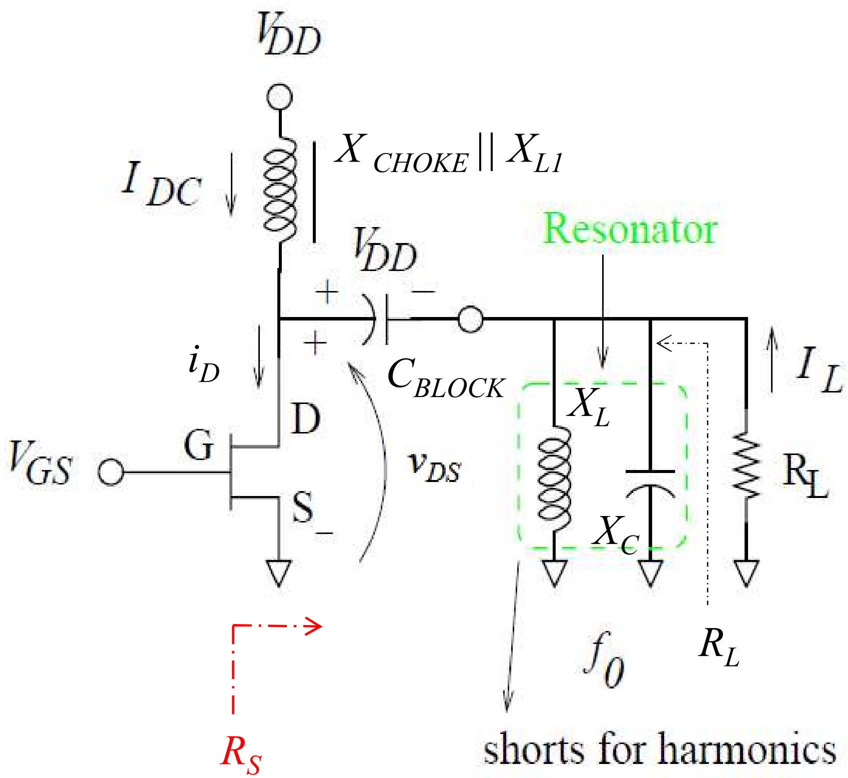
Drain voltage and current for ideal Class A amplifier

Class A amplifier design example



- Required max o/p power, P_{max} = 1 watt, V_{DD} = 3.3 V.
- $R_L=V_{DD}^2/2P_{max} = (3.3)^2/(2 \times 1)=5.4$ ohms.
- Peak RF current $i_{pk} = V_{DD}/R_L = 3.3/5.4 = 0.611$ mA.
- $I_{DC} = i_{pk} = 0.611$ mA.
- Drain efficiency, $\eta = P_{max}/P_{DC} = 1/(0.611 \times 3.3) = 49.6\ \% < 50\%$.
- Say f = 1 GHz. Required Q = 10. Hence $2\Delta f = 100$ MHz.
- $Q = \omega C/G \Rightarrow \omega C=(1/50) \times 10$
  $\Rightarrow C = (1/5) \times (1/(2 \times \pi \times 10^9) = 31.8$ pF
- $X_L= X_C =1/\omega C=1/(2 \times \pi \times 10^9 \times 31.8 \times 10^{-12})$
  $\Rightarrow L=X_L/(2 \times \pi \times 10^9) = 0.79$ nH

Class A amplifier design example (..Contd)

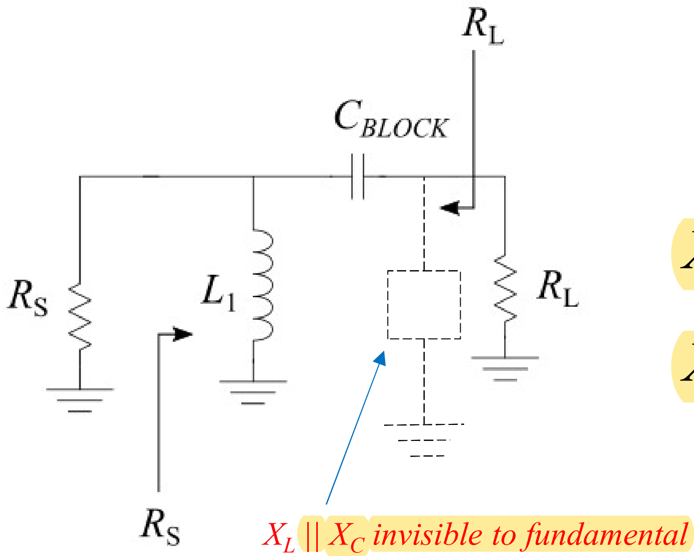


$X_{CHOKE} = 10R_L = 10 \text{ x } 5.4 \text{ ohms} = 54 \text{ ohms} \Rightarrow L_{CHOKE} = 8.6 \text{ nH}$

$M \cong sqrt(R_S/R_L) = sqrt(50/5.4) = 3.04$

$L_1 = R_S / (2 \text{ x } \pi \text{ x } 10^9 \text{ x } M) = 2.6 \text{ nH} \longrightarrow \text{combined with } L_{CHOKE}$

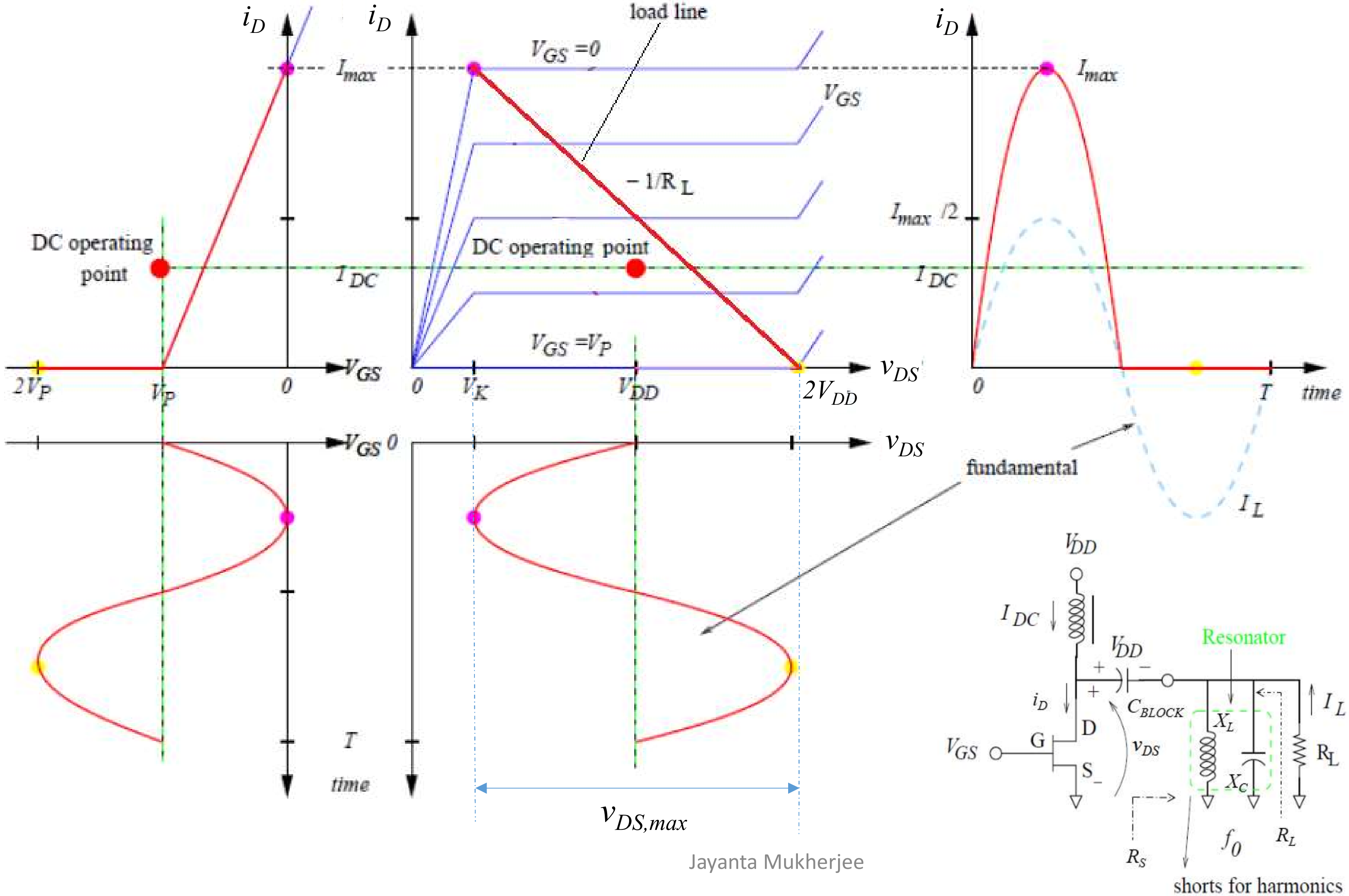
$C_{BLOCK} = 1/(2 \text{ x } \pi \text{ x } 10^9 \text{ x } M \text{ x } R_L) = 9.7 \text{ pF}$



$X_{L1} \cong R_S/M$

$X_{C,BLOCK} \cong M\, R_L$

Class B Amplifiers



Class-B Operation

·Drain current on for half the cycle

$$i_D = \begin{cases} i_{rf}\sin(\omega_0 t) & i_d > 0 \\ 0 & i_d \le 0 \end{cases}$$

$$i_D(t) = \underbrace{\frac{i_{rf}}{\pi}}_{dc} + \underbrace{\frac{i_{rf}}{2}\sin\left(2\pi f_0 t\right)}_{-I_L\,(fundamental)} \underbrace{-\frac{2i_{rf}}{\pi}\sum_{k\ge 1}\frac{\cos\left(4\pi k f t\right)}{4k^2-1}}_{higher\ harmonics\left(\text{sinked by BP filter}\right)}$$

$$I_{DC} = \frac{i_{rf}}{\pi}$$

$$v_{RF} = -\frac{i_{rf}}{2}R_L\sin\left(\omega_0 t\right)\ ,$$

Max value of $v_{RF,\max} = \mathrm{V_{DD}} = v_{DS\max} - \left(v_{DS\max} + V_K\right)/2 = \left(v_{DS\max} - V_K\right)/2$

$\Rightarrow v_{DS\max} \approx 2\mathrm{V_{DD}}$ (Taking $\mathrm{V_K} = 0$)

$$\left|i_{rf,\max}\right| = \frac{2v_{RF,\max}}{R_L} = \frac{2V_{DD}}{R_L} = I_{\max}$$

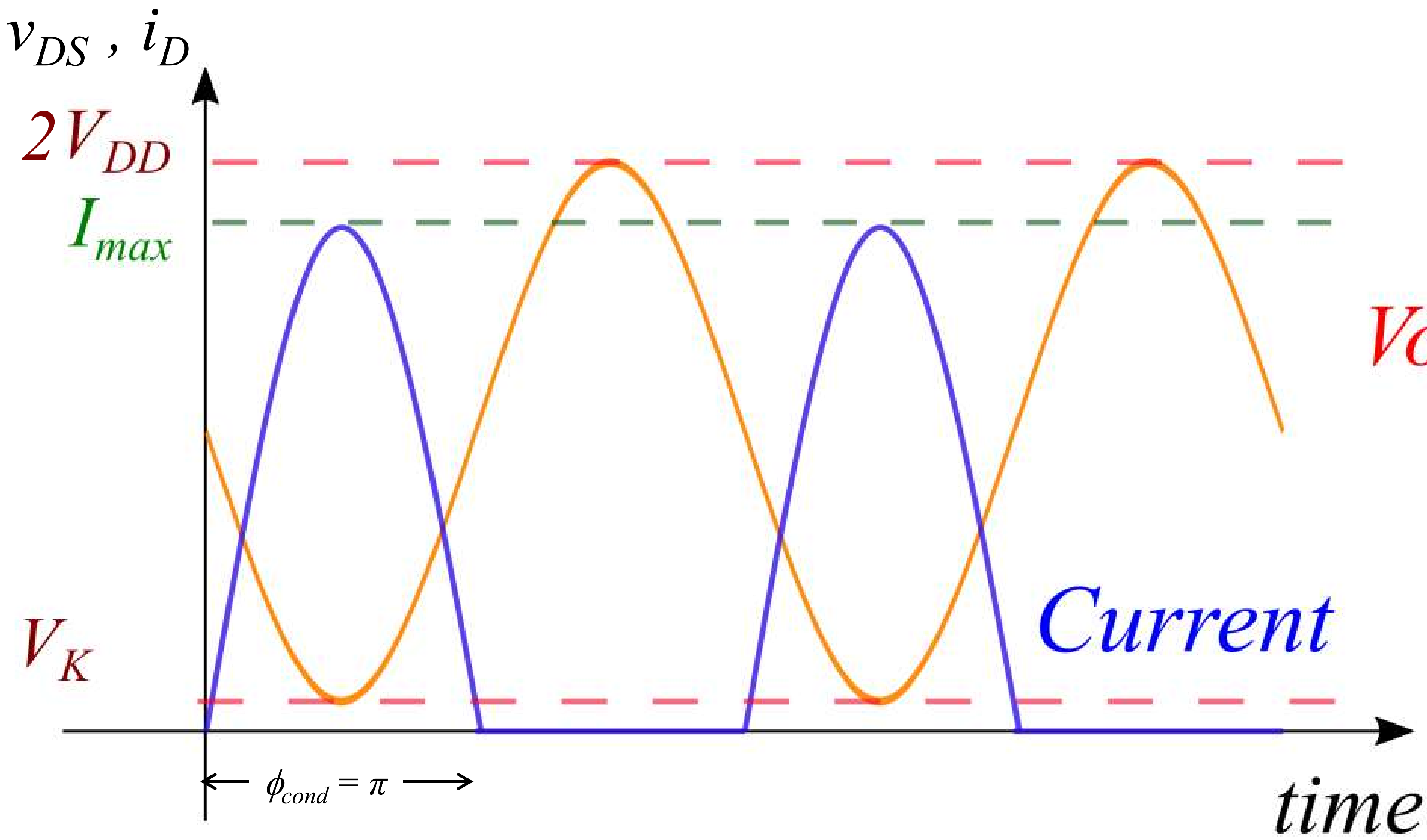
$$P_{RF,\max} = \frac{\left(\dfrac{i_{rf,\max}}{2}R_L\right)^2}{2R_L} = \frac{V_{DD}^2}{2R_L}$$

· Efficiency

$$P_{RF,\max} = \frac{\left(\dfrac{i_{rf,\max}}{2}R_L\right)^2}{2R_L} = \frac{V_{DD}^2}{2R_L}$$

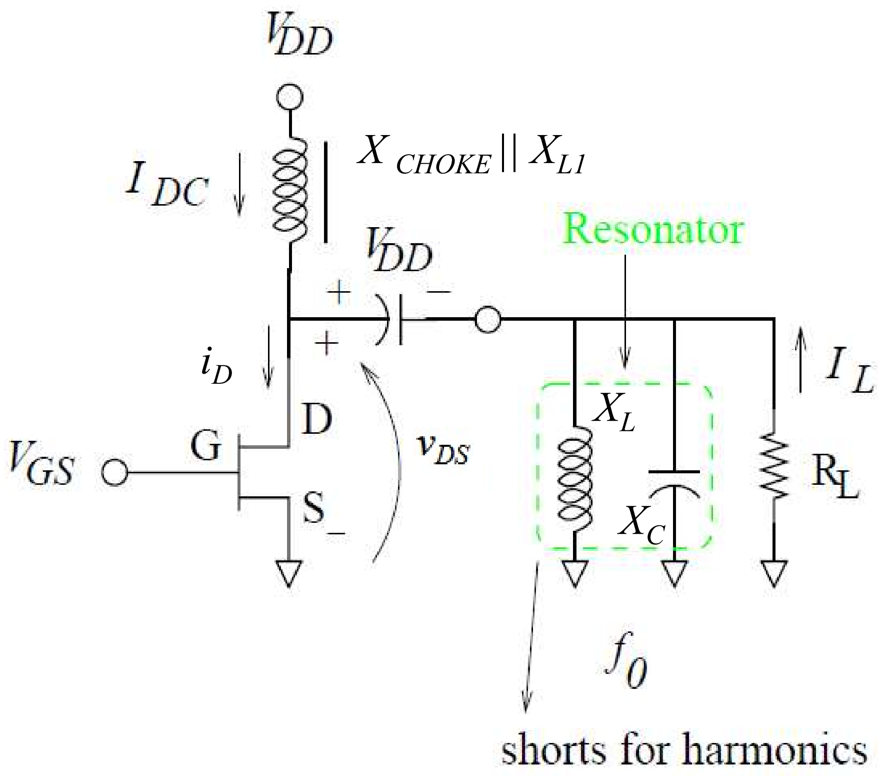
$$P_{DC} = V_{DD}\,I_{DC} = V_{DD}\cdot\frac{i_{rf}}{\pi} = \frac{2V_{DD}^2}{\pi\,R_L}$$

Drain efficiency, $\eta = \dfrac{P_{RF,\max}}{P_{DC}} = \dfrac{\pi}{4} = 78.5\%$



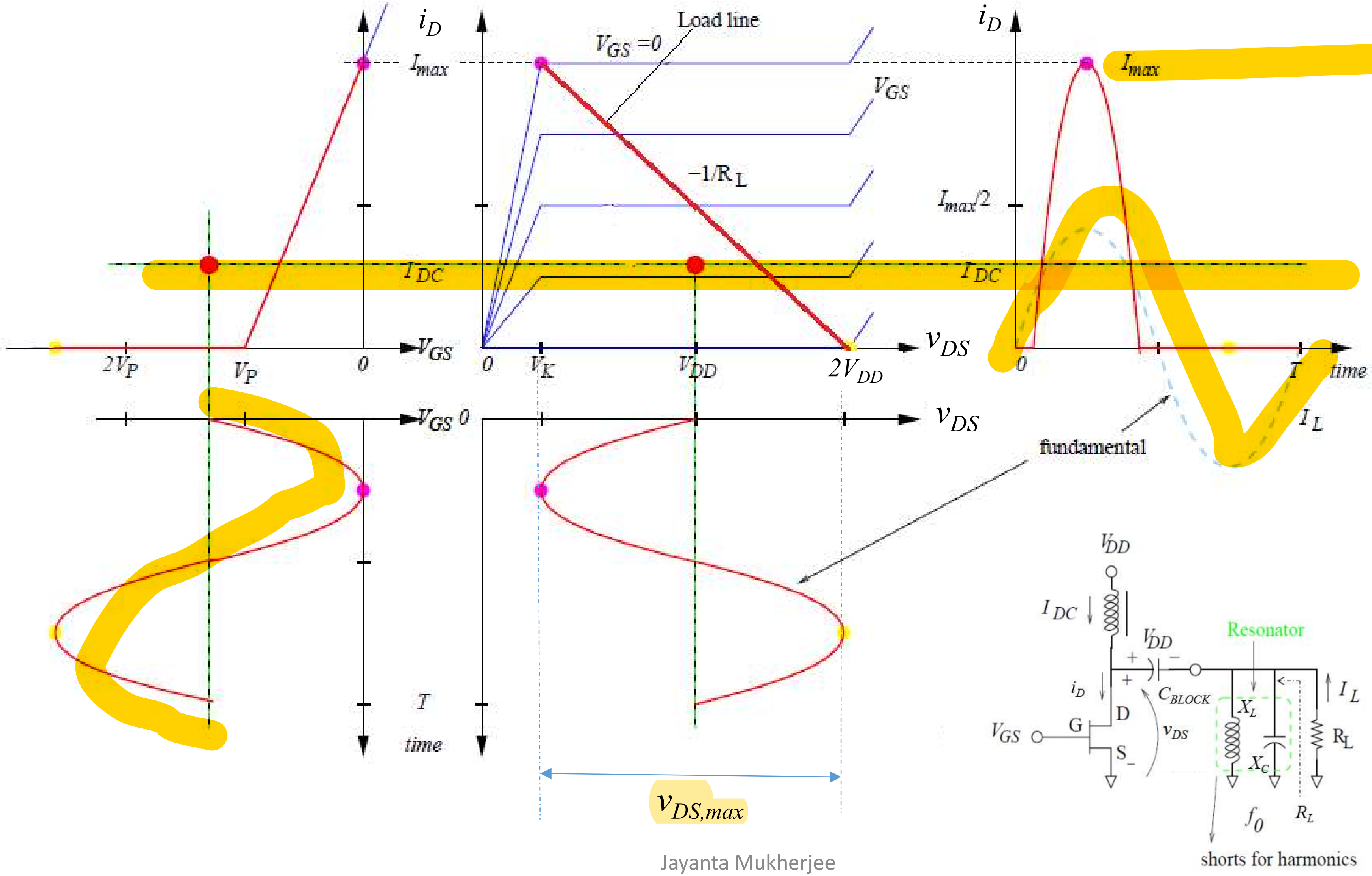
Drain voltage and current for ideal Class B amplifier

Class B amplifier design example

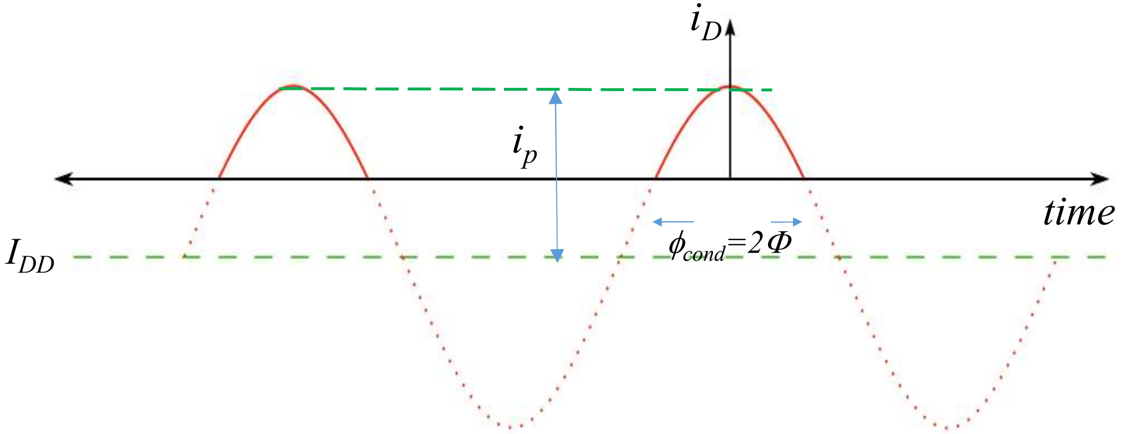


- Required max o/p power, P_{max} = 1 watt=$(1/2)(i_{rf,max}/2)^2R_L$,
- V_{DD} = 3.3 V, $i_{rf,max}=(2V_{DD}/R_L)$
- $R_L=V_{DD}^2/2P_{max}$ = $(3.3)^2/(2 \times 1)$=5.4 ohms, $i_{rf,max}$=1.22 A
- $I_{DC} = i_{rf,max}/\pi$ = 0.388 mA.
- Drain efficiency, $\eta = P_{max}/P_{DC}$ = 1/(0.388 x 3.3) = 78.1 % > 50%.

Class C Amplifier Operation



Class-C Operation



$$i_D = I_{DD} + i_p \cos\ (\omega_0 t)\ , i_D > 0$$
$$= \ 0 \ \text{ otherwise}$$

$$\cos\Phi = -\frac{I_{DD}}{i_p} \Rightarrow I_{DD} = -i_p \cos\Phi,\ \text{- - -}\ (1)$$

Φ is the value of $\omega_0\ t$

when i_D goes to zero.

$$I_{DC} = \frac{1}{2\pi}\int_{-\Phi}^{\Phi}\left(I_{DD} + i_p \cos\varphi\right) d\varphi = \frac{1}{2\pi} 2\varphi\ I_{DD} + \frac{1}{2\pi}\left[i_p \sin\varphi\right]\Big|_{-\Phi}^{\Phi}$$

$$\Rightarrow I_{DC} = \frac{i_p}{\pi}\left[\sin\Phi - \Phi\cos\Phi\right] = \frac{i_p}{\pi}\left[\sin\frac{\varphi_{cond}}{2} - \frac{\varphi_{cond}}{2}\cos\frac{\varphi_{cond}}{2}\right]$$

Fundamenta l component

$$i_{fund} = \frac{2}{T}\int_{-T/2}^{T/2} i_D \cos\left(\omega_0 t\right) = \frac{1}{\pi}\int_{-\Phi}^{\Phi}\left(I_{DD} + i_p \cos\left(\varphi\right)\right)\ \cos\left(\varphi\right) d\varphi = \frac{2}{\pi}\int_{0}^{\Phi}\left(I_{DD} + i_p \cos\left(\varphi\right)\right)\cos\left(\varphi\right) d\varphi$$

$$= \frac{2}{\pi}\left[I_{DD}\sin\varphi + \frac{i_p}{2}\left(\varphi\right) + \frac{i_p}{4}\left(\sin 2\varphi\right)\right]_0^{\Phi} = \frac{2}{\pi}\left[-i_p \cos\Phi \sin\Phi + \frac{i_p}{2}\Phi + \frac{i_p}{4}\left(\sin 2\Phi\right)\right]$$

$$= \frac{i_p}{\pi}\left[\Phi - \frac{1}{2}\left(\sin 2\Phi\right)\right] = \frac{i_p}{\pi}\left[\frac{\varphi_{cond}}{2} - \frac{1}{2}\left(\sin\varphi_{cond}\right)\right]$$

Class-C Operation

$$v_{rf} = i_{fund} R_L = \frac{R_L\, i_p}{\pi}\left[\frac{\varphi_{cond}}{2} - \frac{1}{2}\left(\sin\varphi_{cond}\right)\right]$$

$$P_{rf} = v_{rf}^2 / (2R_L) = \frac{R_L i_p^2}{2\pi^2}\left[\frac{\varphi_{cond}}{2} - \frac{1}{2}\sin\left(\varphi_{cond}\right)\right]^2 = \frac{i_p^2 R_L}{8\pi^2}\left(\varphi_{cond} - \sin\left(\varphi_{cond}\right)\right)^2$$

Maximum value of $v_{rf} = v_{rf,\max} = V_{DD} \Rightarrow i_{p,\max} = \dfrac{2\pi V_{DD}}{R_L\left[\varphi_{cond} - \sin\varphi_{cond}\right]} = -\dfrac{I_{DD}}{\cos\left(\varphi_{cond}/2\right)}$ (From Eqn (1))

$$\Rightarrow I_{DD} = \frac{2\pi V_{DD}\cos\left(\varphi_{cond}/2\right)}{R_L\left[\sin\varphi_{cond} - \varphi_{cond}\right]}$$

$$\mathrm{I}_{\max} = I_{DD} + i_{p,\max} = \frac{2\pi V_{DD}\cos\left(\varphi_{cond}/2\right)}{R_L\left[\sin\varphi_{cond} - \varphi_{cond}\right]} + \frac{2\pi V_{DD}}{R_L\left[\varphi_{cond} - \sin\varphi_{cond}\right]} = \frac{2\pi\, V_{DD}\left[\cos\left(\varphi_{cond}/2\right) - 1\right]}{R_L\left[\sin\varphi_{cond} - \varphi_{cond}\right]} = i_{p,\max}\left[1 - \cos\left(\varphi_{cond}/2\right)\right]$$

Verification:

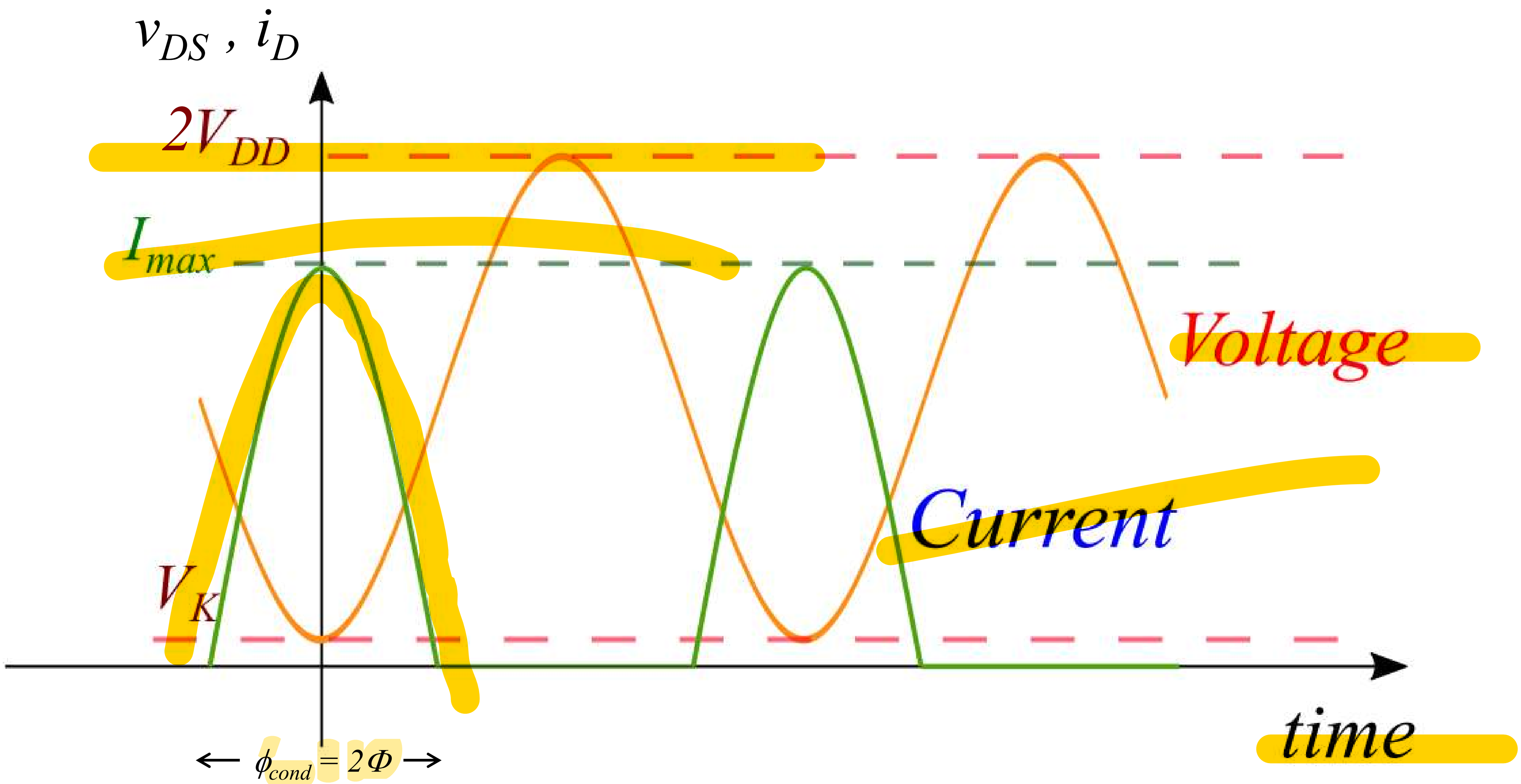
For class A, $\varphi_{cond} = 2\pi$, hence $I_{DD} = \mathrm{V_{DD}/R_L}$, For class B, $\varphi_{cond} = \pi$, hence $I_{DD} = 0$

$I_{\max} = I_{DD} + i_{p,\max}$,

For class A, $\varphi_{cond} = 2\pi$, hence $I_{\max} = I_{DD} + i_{p,\max} = 2\mathrm{V_{DD}/R_L}$,

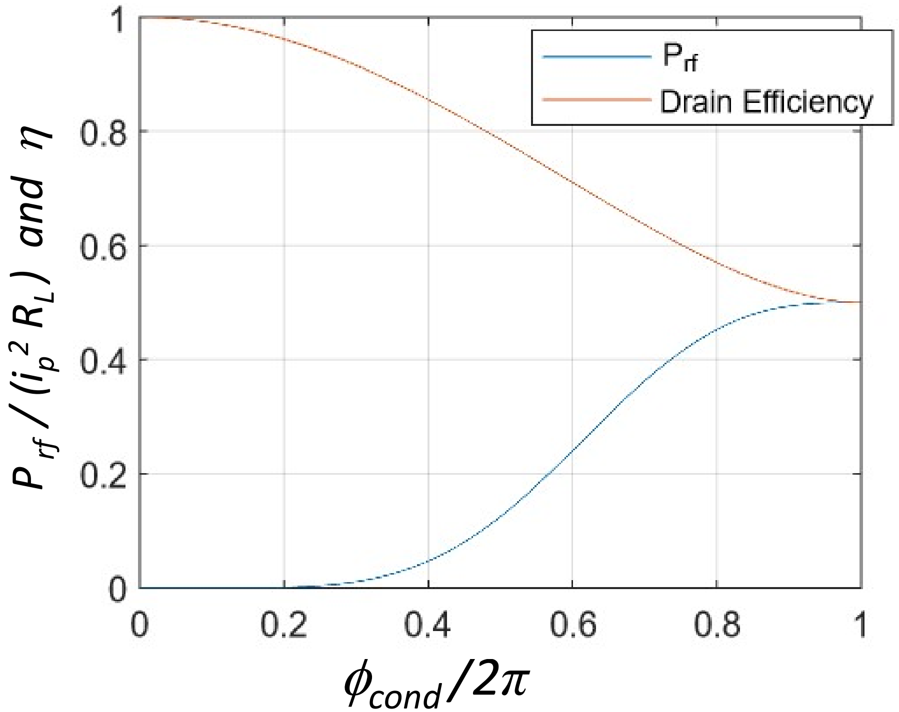
For class B, $\varphi_{cond} = \pi$, hence $I_{\max} = I_{DD} + i_{p,\max} = 0 + 2\mathrm{V_{DD}/R_L} = 2\mathrm{V_{DD}/R_L}$

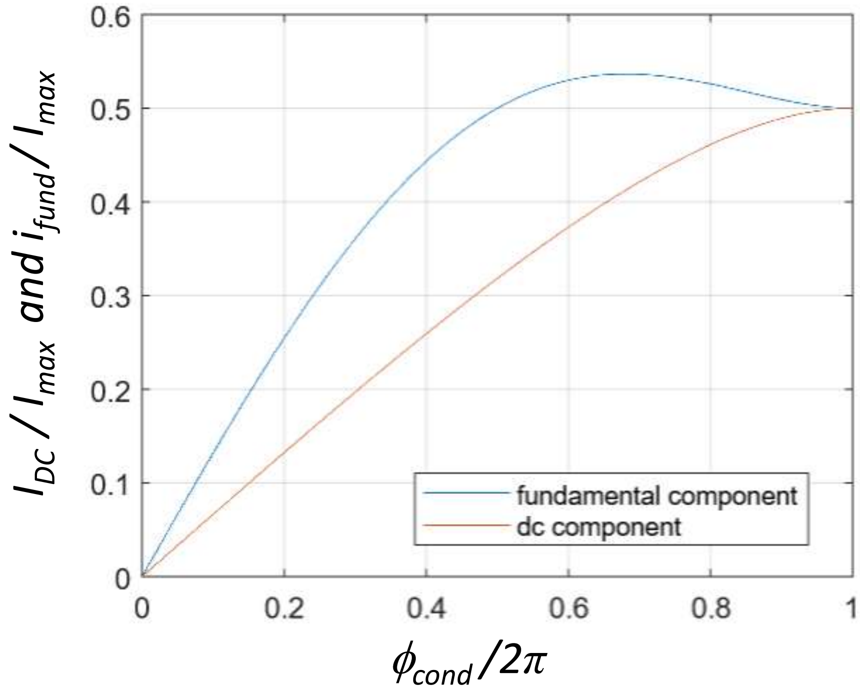
Drain Efficiency, $\eta = \dfrac{P_{rf,\max}}{P_{DC}} = \dfrac{\left(v_{rf,\max}\right)^2/2R_L}{V_{DD}\; I_{DC}} = \dfrac{\left(V_{DD}^2\right)/2R_L}{V_{DD}.\dfrac{i_{p,\max}}{\pi}\left[\sin\dfrac{\varphi_{cond}}{2} - \dfrac{\varphi_{cond}}{2}\cos\dfrac{\varphi_{cond}}{2}\right]} = \dfrac{\left[\varphi_{cond} - \sin\varphi_{cond}\right]}{2\left[2\sin\dfrac{\varphi_{cond}}{2} - \varphi_{cond}\cos\dfrac{\varphi_{cond}}{2}\right]}$



Drain voltage and current for ideal Class C amplifier

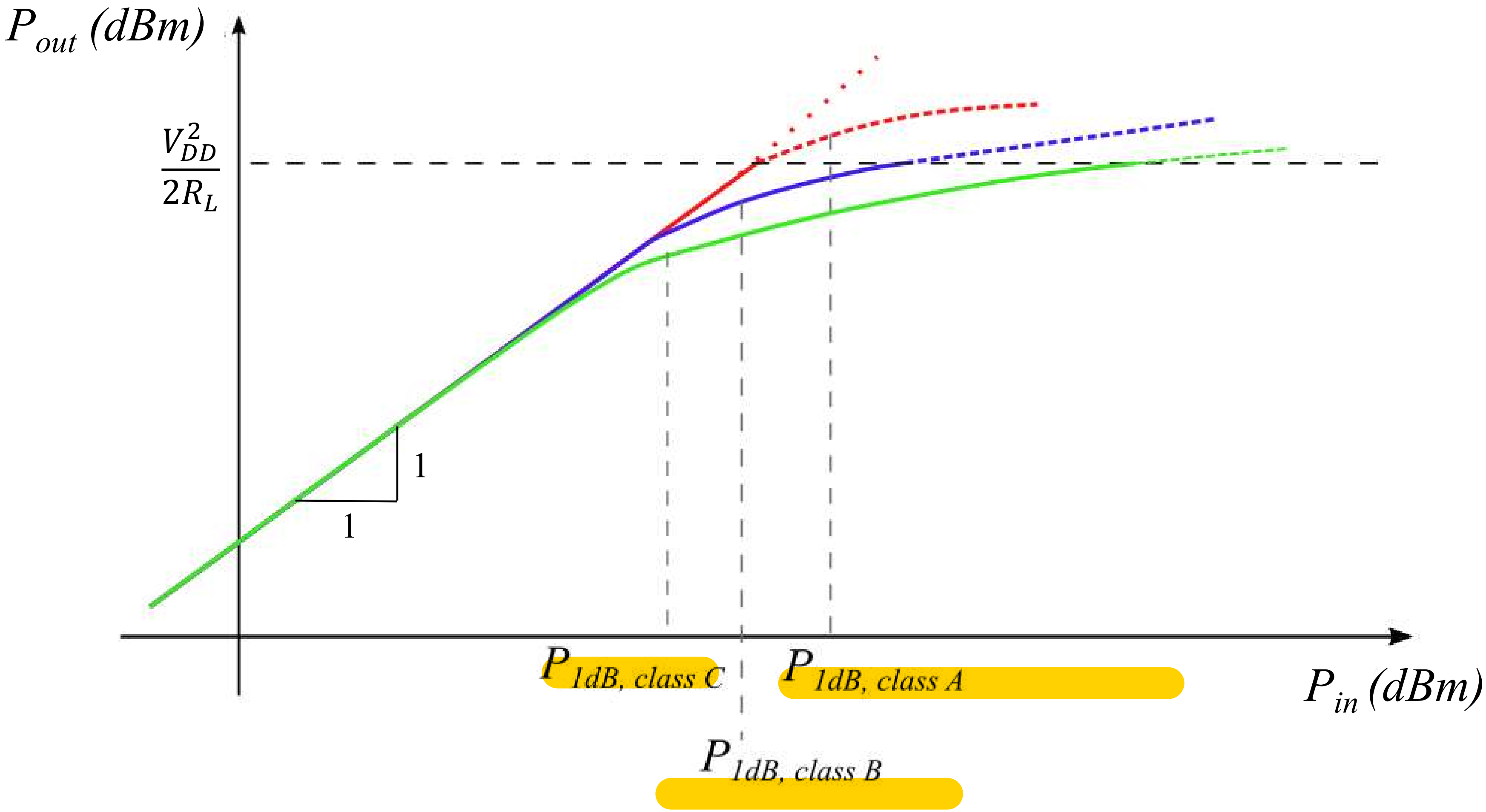
Amplifier Efficiency and Harmonics



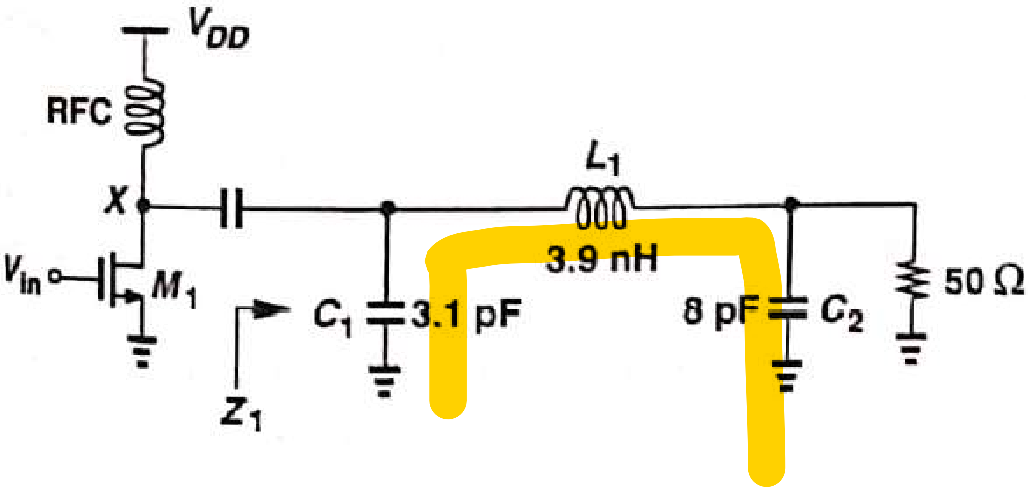


The output power decreases as the power efficiency increases

Linearity

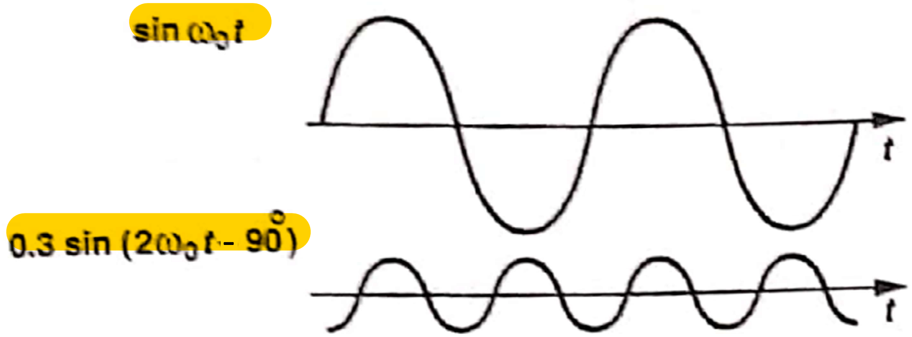

P_{out} (dBm)
$\frac{V_{DD}^2}{2R_L}$
1
1
$P_{1dB,\ class\ C}$
$P_{1dB,\ class\ A}$
$P_{1dB,\ class\ B}$
P_{in} (dBm)

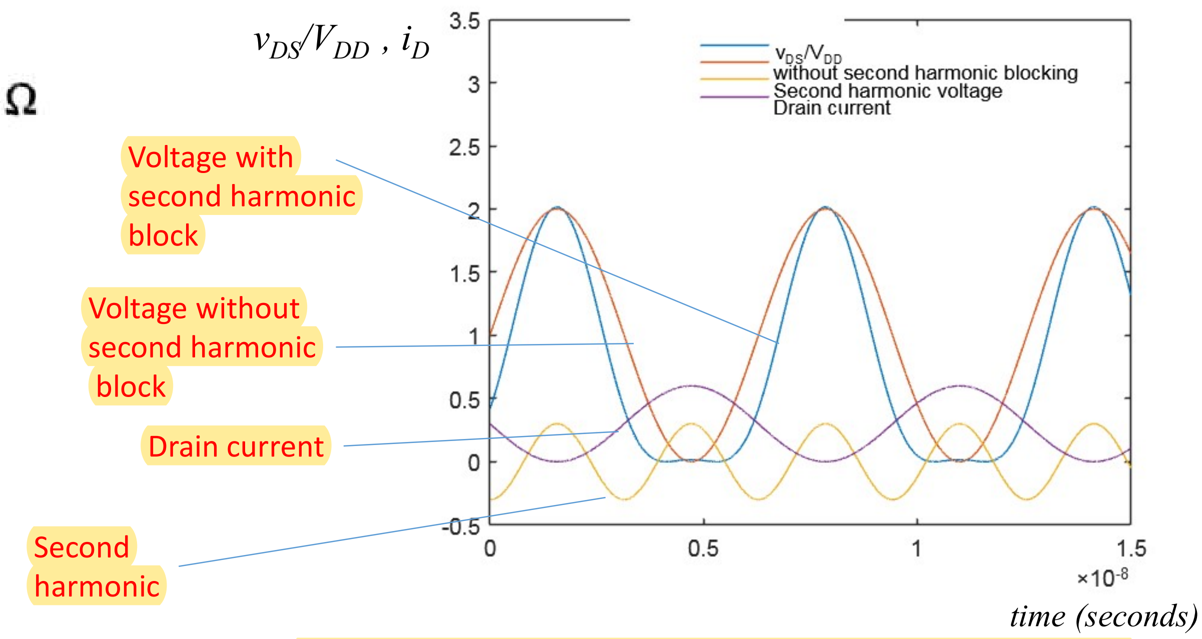
High Efficiency Class A PA



Z_1(850 MHz) = 9 ohms

Z_1(2 x 850 MHz) = 330 ohms





Drain voltage and current for high efficiency Class A amplifier

Class E PA – a switching amplifier

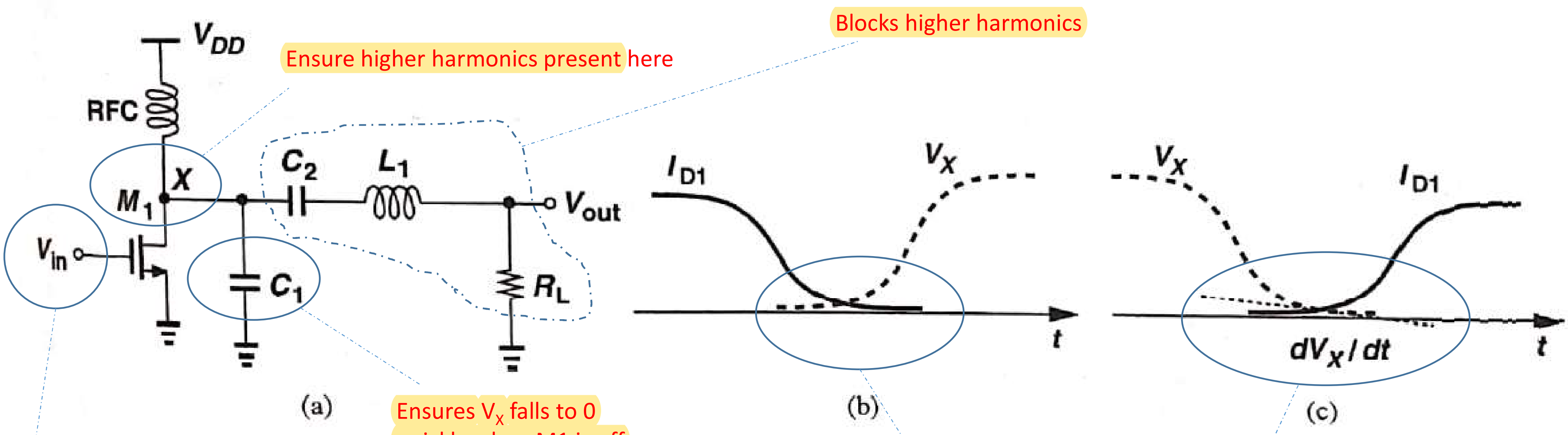


Figure 12.27 *(a) Class E stage, (b) condition to ensure minimal overlap between drain current and voltage, (c) condition to ensure low sensitivity to timing errors.*

Sharp switching, ideally turned on only when $V_X = 0$ to make ideal efficiency 100 %

I_D should be ideally 0 when $V_X \neq 0$
And slope of voltage vs time curve should be 0 ideally to minimize efficiency variation

Class E Design Equations

$$L_1 = \frac{QR_L}{\omega},\ \mathrm{C}_1 = \frac{1}{\omega R_L\left(\frac{\pi^2}{4}+1\right)\left(\frac{\pi}{2}\right)} \approx \frac{1}{\omega\left(R_L.447\right)}$$

$$C_2 \approx C_1\left(\frac{5.447}{Q}\right)\left(1+\frac{1.42}{Q-2.08}\right),\ \text{Q is obtained from the desired BW of the LC n/w}$$

$$\mathrm{P}_{\mathrm{rf,max}} = \frac{2}{1+\pi^2/4}\cdot\frac{V_{DD}^2}{R_L} \approx 0.577\cdot\frac{V_{DD}^2}{R_L}$$

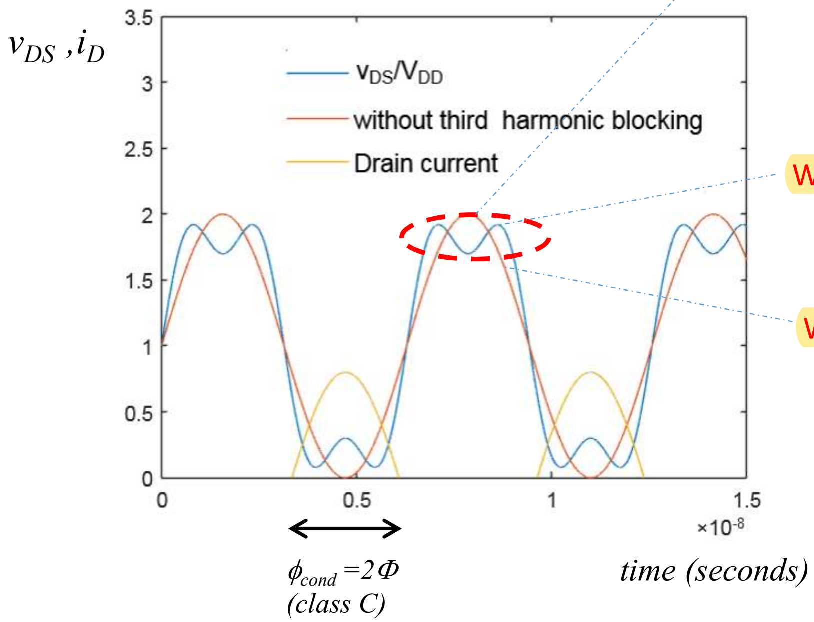
Not easy to find expression for drain current or drain voltage.

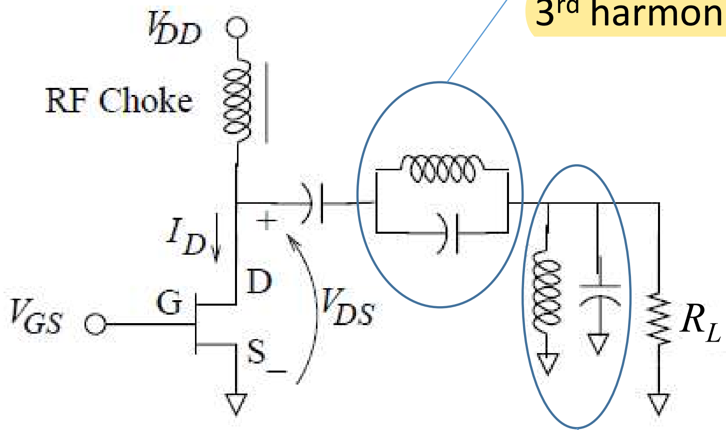
Class E shows poor power o/p capability and reduced efficiency due to switch turn off losses.

Class E-A new class of high-efficiency tuned single-ended switching power amplifiers, N.O. Sokal; A.D. Sokal, IEEE Journal of Solid-State Circuits Year: 1975 | Volume: 10, Issue: 3

Class F PA

3rd harmonic superimposed on 1st harmonic creates this dip and also makes the edges more vertical



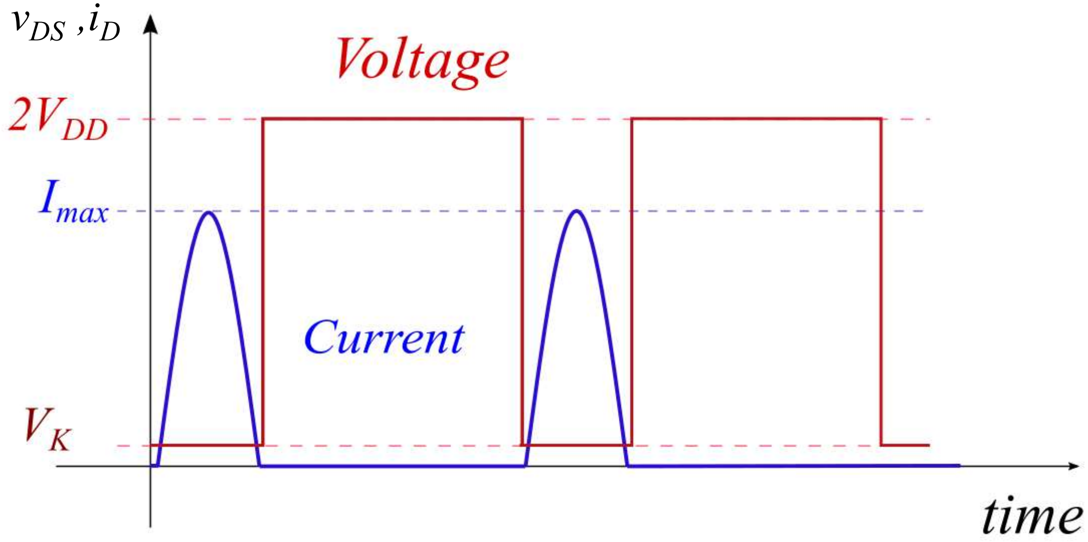


- Drain voltage and current for ideal Class F amplifier.

$$\text{peak to peak voltage of fundamental component of } v_{DS} = \frac{4}{\pi} \cdot 2V_{DD}$$

$$P_{rf,\max} = \left(\frac{4}{\pi} \cdot V_{DD}\right)^2 / 2R_L = 0.81\frac{V_{DD}^2}{R_L},\ i_{fund} = \left(\frac{8}{\pi} \cdot V_{DD}\right) / R_L = 2.54\frac{V_{DD}}{R_L}$$

Ideal Class F voltage and current



- Ideally the waveform approaches a rectangular wave when all odd harmonics are superimposed.
- Theoretical drain efficiency is 100%.

peak to peak voltage of fundamental component of $v_{DS} = \dfrac{4}{\pi} \cdot 2V_{DD}$

$$P_{rf,\max} = \left(\frac{4}{\pi} \cdot V_{DD}\right)^2 / 2R_L = 0.81\frac{V_{DD}^2}{R_L},\ i_{fund} = \left(\frac{8}{\pi} \cdot V_{DD}\right) / R_L = 2.54\frac{V_{DD}}{R_L}$$

Inverse class F or F^{-1}

- Dual of class F, i.e. Voltage and current waveforms are interchanged

O/P power comparison between Class A and B PAs

Max o/p power for both class A and B is comparable

$$P_{RF,A} = \frac{V_{DD}^2}{2R} \quad P_{RF,B} = \frac{V_{DD}^2}{2R}$$

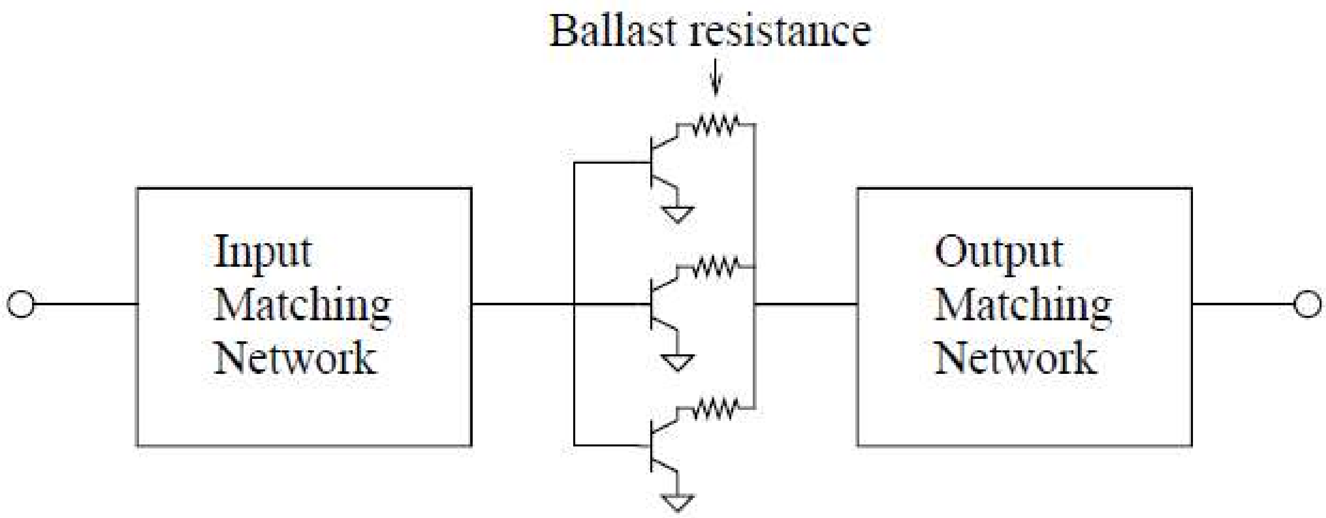
However for same v_{in} the power o/p of class B is lesser

$$P_{RF,class\ B} = \frac{1}{2} \times \left(\underbrace{i_{rf}/2}_{\substack{\text{amplitude of} \\ \text{first harmonic}}} \right)^2 \times R_L = \frac{i_{rf}^2 R_L}{8}$$

$$P_{RF,class\ A} = \frac{1}{2} \times \left(i_{pk}\right)^2 \times R_L = \frac{i_{pk}^2 R_L}{2}$$

Assuming $i_{rf} = i_{pk} = g_m v_{in}$, for the same v_{in}, power o/p of a class B amplifier is 6 dB lesser than a class A amplifier.

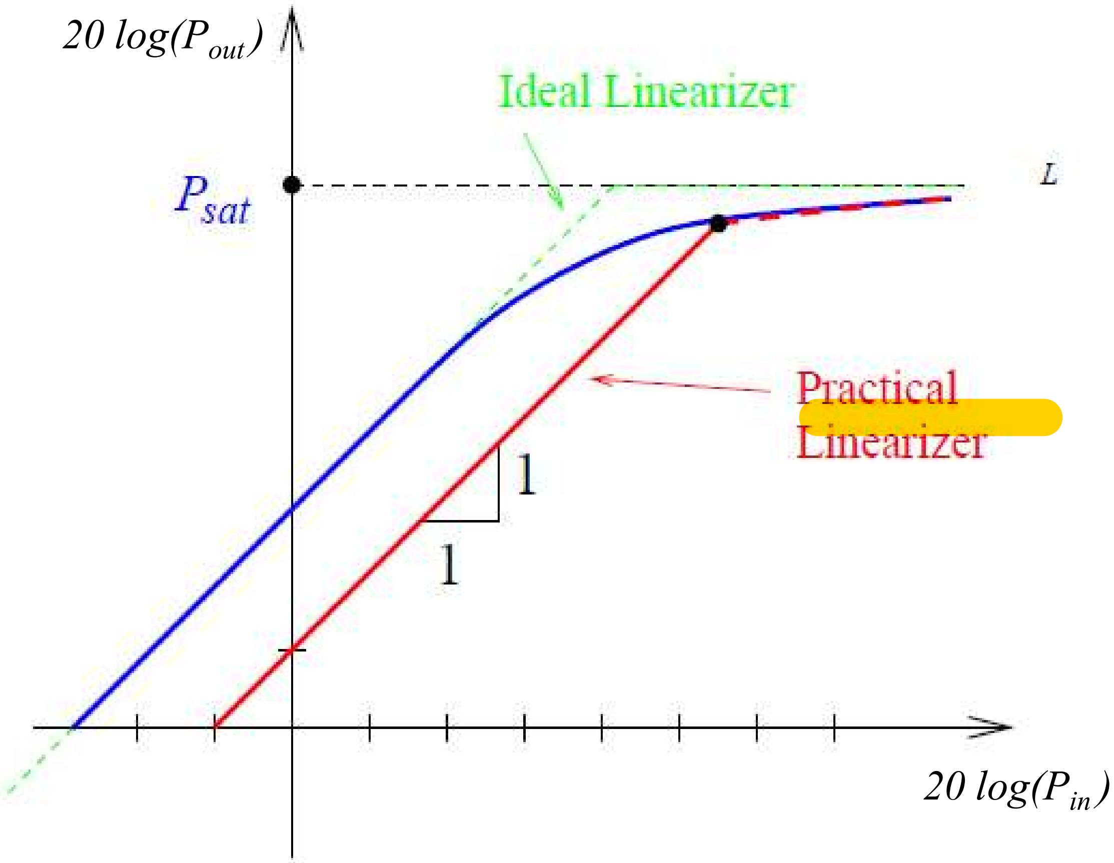
Power Combining



Issues

- Difficult to design: input impedance is very low or very high.
- Load sharing: ballast resistances are needed but reduce the output power.
- Distributed effects: constant delay & corporate tree impedance

Linearization

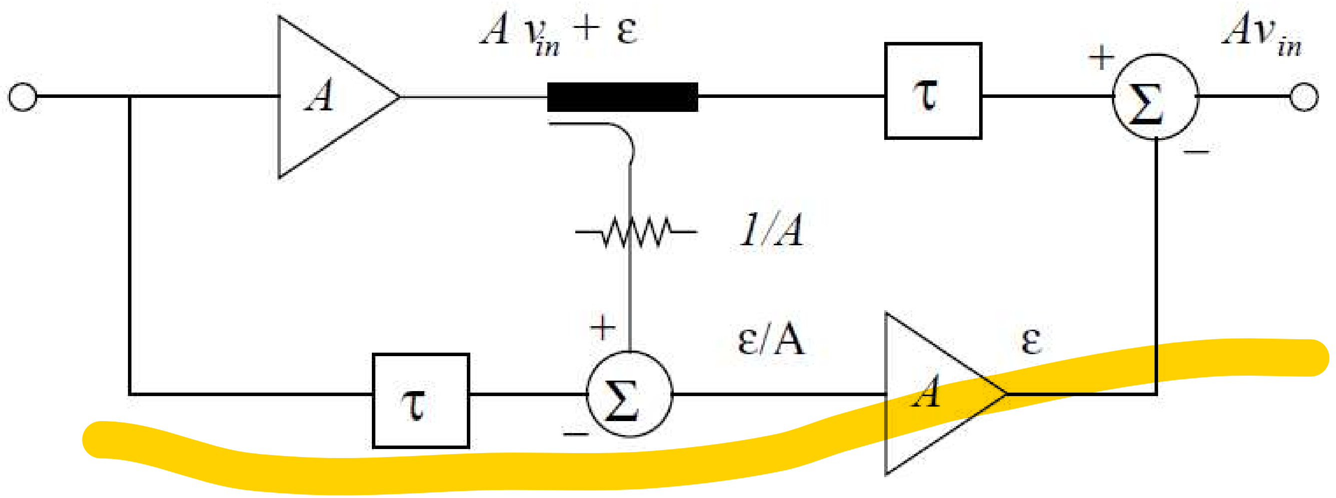


- Power saturation is the same
- Some degradation in gain typical
- Improved IMD3 and ACPR

Various Methods Potentially Applicable

- Feedforward
- Predistortion
- Envelope Elimination and Restoration or Polar Modulation
- LINC (linear amplification with non-linear components): a non-constant envelop v_{in} is expressed in terms of constant envelope signals v_1 and v_2 such that: $v_{in} = v_1(t) + v_2(t)$
- Doherty

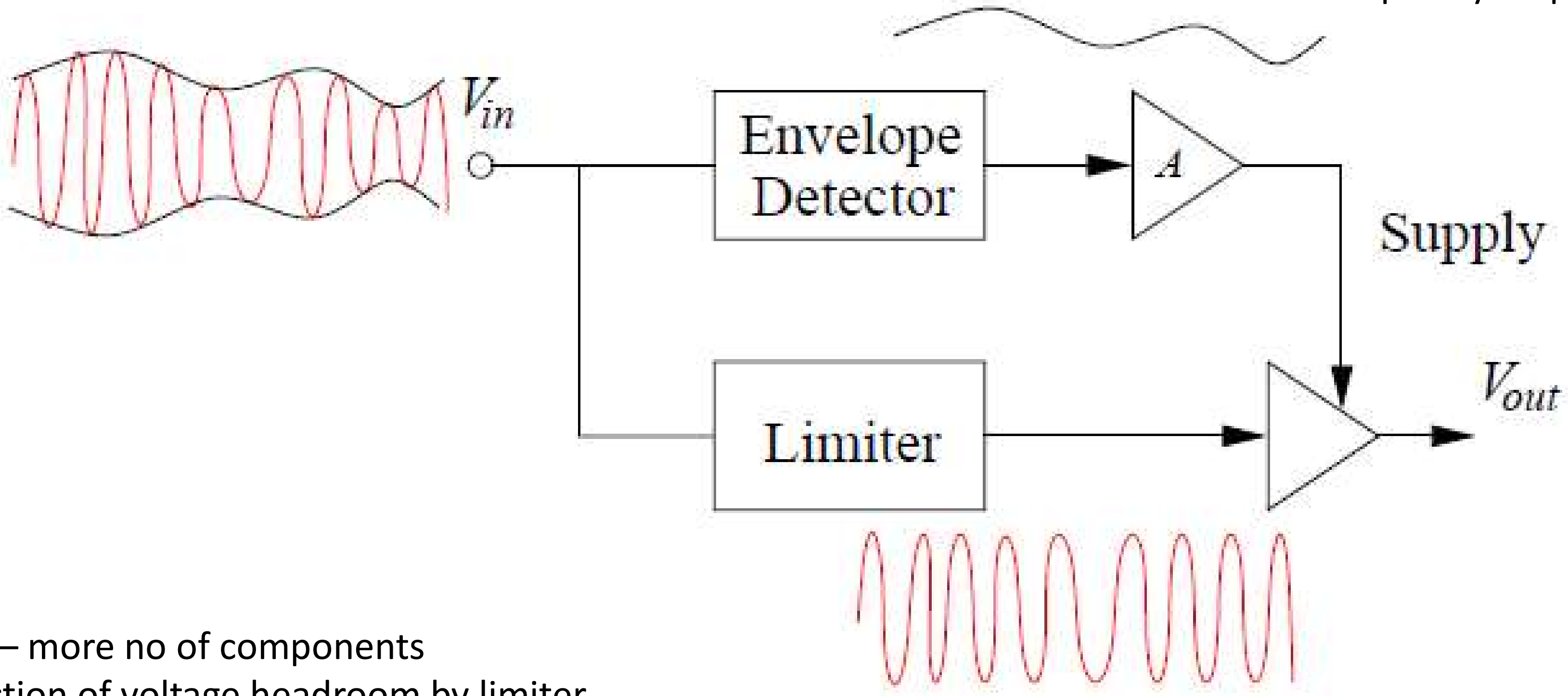
Feed Forward Linearization



Broadband but complex (2 loops) , expensive and inefficient (since 2 PAs used)

Envelope Elimination and Restoration (or Polar Modulation)

Low frequency amplification



High frequency amplification with constant envelope

Issues

1. Efficiency – more no of components
   and reduction of voltage headroom by limiter
2. Mismatch.
3. Linearity of envelope detector.
4. Bandwidth of limiter

Predistortion Linearization

Consider an incoming signal

$x = A\cos(\omega t + \varphi) = I\cos(\omega t) - Q\sin(\omega t)$

$with,\ I = A\cos\varphi \ \ and\ Q = A\sin\varphi$

The action of the predistorter is to modify the phase and amplitude of the input signal to compensate for the in-band AM-AM and AM-PM distortion introduced by the PA.

The output of the predistorter is then,

$$y = A\,B\cos(\omega t + \varphi + \psi) = \left[\underbrace{I \times B\cos\psi - Q \times B\sin\psi}_{I'}\right]\cos(\omega t) + \left[\underbrace{-B \times I\sin\psi - B \times Q\cos\psi}_{Q'}\right]\sin(\omega t)$$

where one can easily verify that I' and Q' are given by:

$$\begin{bmatrix} I' \\ Q' \end{bmatrix} = \begin{bmatrix} B\cos\psi & -B\sin\psi \\ -B\sin\psi & -B\cos\psi \end{bmatrix} \begin{bmatrix} I \\ Q \end{bmatrix} = \begin{bmatrix} \alpha & -\beta \\ -\beta & -\alpha \end{bmatrix} \begin{bmatrix} I \\ Q \end{bmatrix}$$

with $\alpha = B\cos\psi$ and $\beta = B\sin\psi$.

So, say we pre-distort I & Q as follows,

$$\begin{bmatrix} I'' \\ Q'' \end{bmatrix} = \begin{bmatrix} \alpha & -\beta \\ -\beta & -\alpha \end{bmatrix}^{-1} \begin{bmatrix} I \\ Q \end{bmatrix},$$ then I' and Q' become,

$$\begin{bmatrix} I' \\ Q' \end{bmatrix} = \begin{bmatrix} \alpha & -\beta \\ -\beta & -\alpha \end{bmatrix} \begin{bmatrix} I'' \\ Q'' \end{bmatrix} = \begin{bmatrix} \alpha & -\beta \\ -\beta & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & -\beta \\ -\beta & -\alpha \end{bmatrix}^{-1} \begin{bmatrix} I \\ Q \end{bmatrix} = \begin{bmatrix} I \\ Q \end{bmatrix}$$

α and β need to be estimated continuously

Digital Implementation of Predistortion