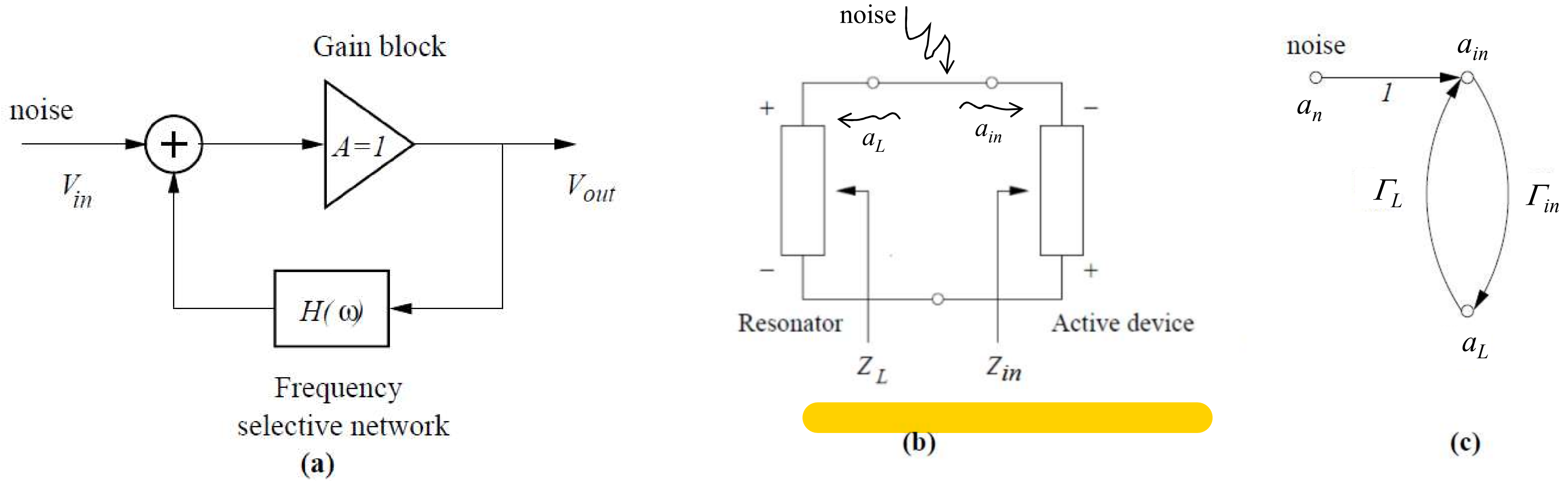
Oscillators

Oscillator Models

- Feedback oscillator system (a)
- Negative resistance oscillator (b)
- Equivalence of representation using S-parameters (c)



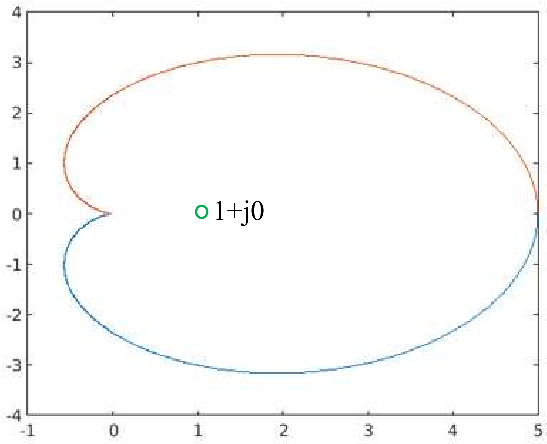
Note: the impedances shown are for the first harmonic (fundamental).

Condition for Starting the Oscillations: Nyquist Test

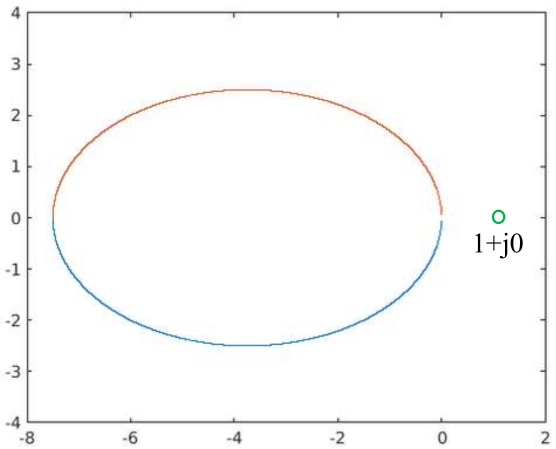
Consider the transfer functions:

$$\frac{V_{out}}{V_{in}} = \frac{A}{1-H(s)} \quad or \quad \frac{a_L}{a_n} = \frac{\Gamma_{in}}{1-\Gamma_{in}\Gamma_L}$$

The number of encirclements of the point $(1+j0)$ should be non-zero. For example:



$$\Gamma_{in}\Gamma_L = \frac{90}{(s+3)\,(s+6)}$$



$$\Gamma_{in}\Gamma_L = \frac{10\,(s+3)}{(s+2)\,(s-2)}$$

Operation Point: Barkhausen's Criteria

- The oscillation needs to sustain themselves in the absence of noise. This occurs for:

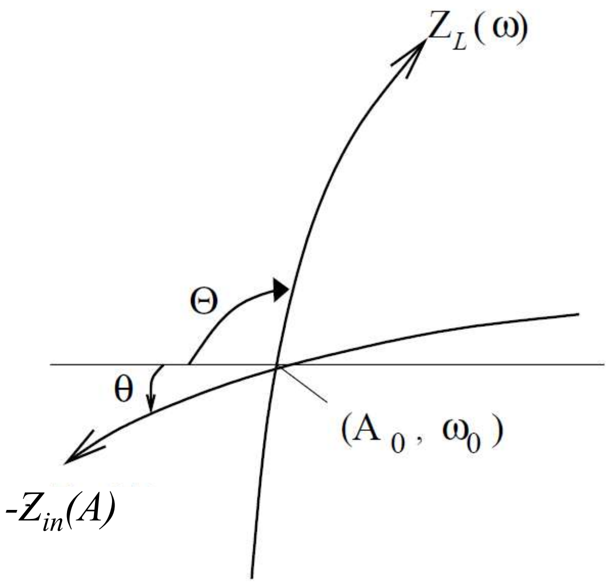
$$H(s = j\omega_0, A_0) = \Gamma_{in}(\omega_0, A)\ \Gamma_L(\omega_0) = 1$$

where A_0 is the amplitude of oscillation and ω_0 the oscillation frequency in steady state.

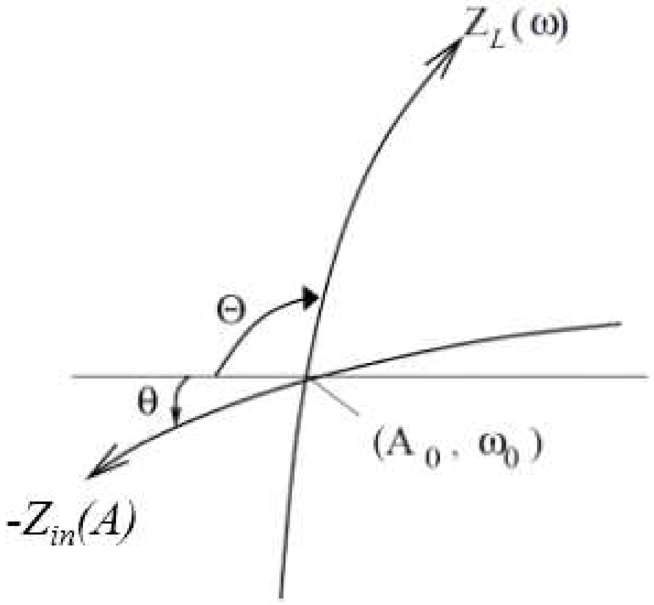
$$\Gamma_{in}(A_0, \omega_0) = \frac{1}{\Gamma_L(\omega_0)}$$

$$\Rightarrow Z_{in}(A_0, \omega_0) = -Z_L(\omega_0)$$

$$\Rightarrow Z_{in}(A_0, \omega_0) + Z_L(\omega_0) = 0$$



Stability



An oscillator is stable if at the operating pt,

$$\frac{\partial R_{in}(A_0,\omega_0)}{\partial A}\frac{\partial X_L(\omega_0)}{\partial \omega}-\frac{\partial X_{in}(A_0,\omega_0)}{\partial A}\frac{\partial R_L(\omega_0)}{\partial \omega}>0$$

Kurokawa condition of stability

- It can be shown that this translates to,

$$\pi > (\theta + \Theta) > 0 \quad \text{(prove it)}$$

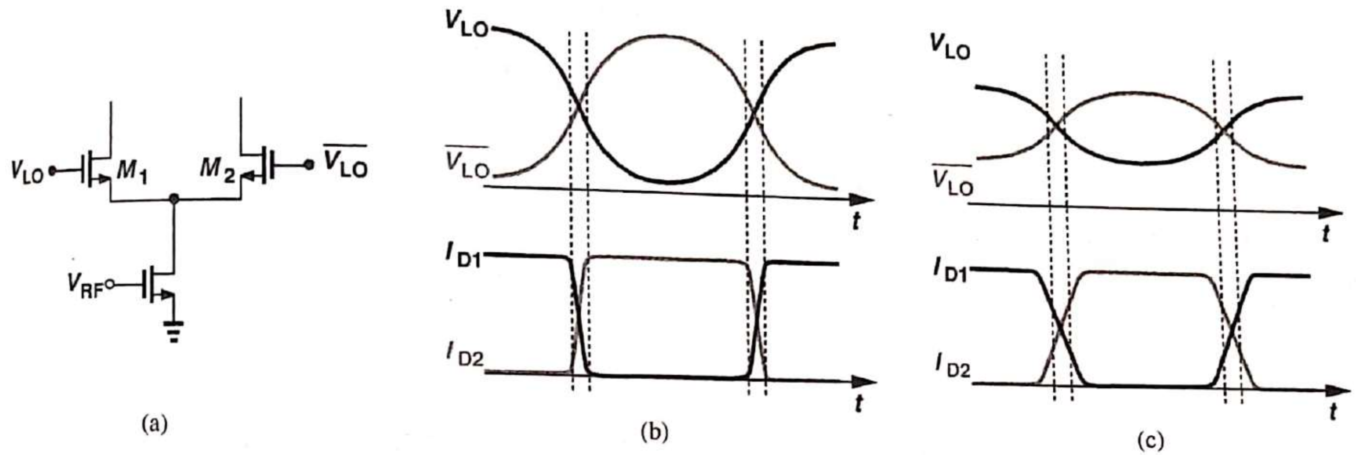
Performance Parameters

1. **Frequency Range**

- e.g. GSM 935 – 960 MHz tuning range.
- Allowance for temperature variation and modeling inaccuracies.
- Quadrature topology or Injection pulling can necessitate higher frequency generation.

2. **O/P voltage swing**

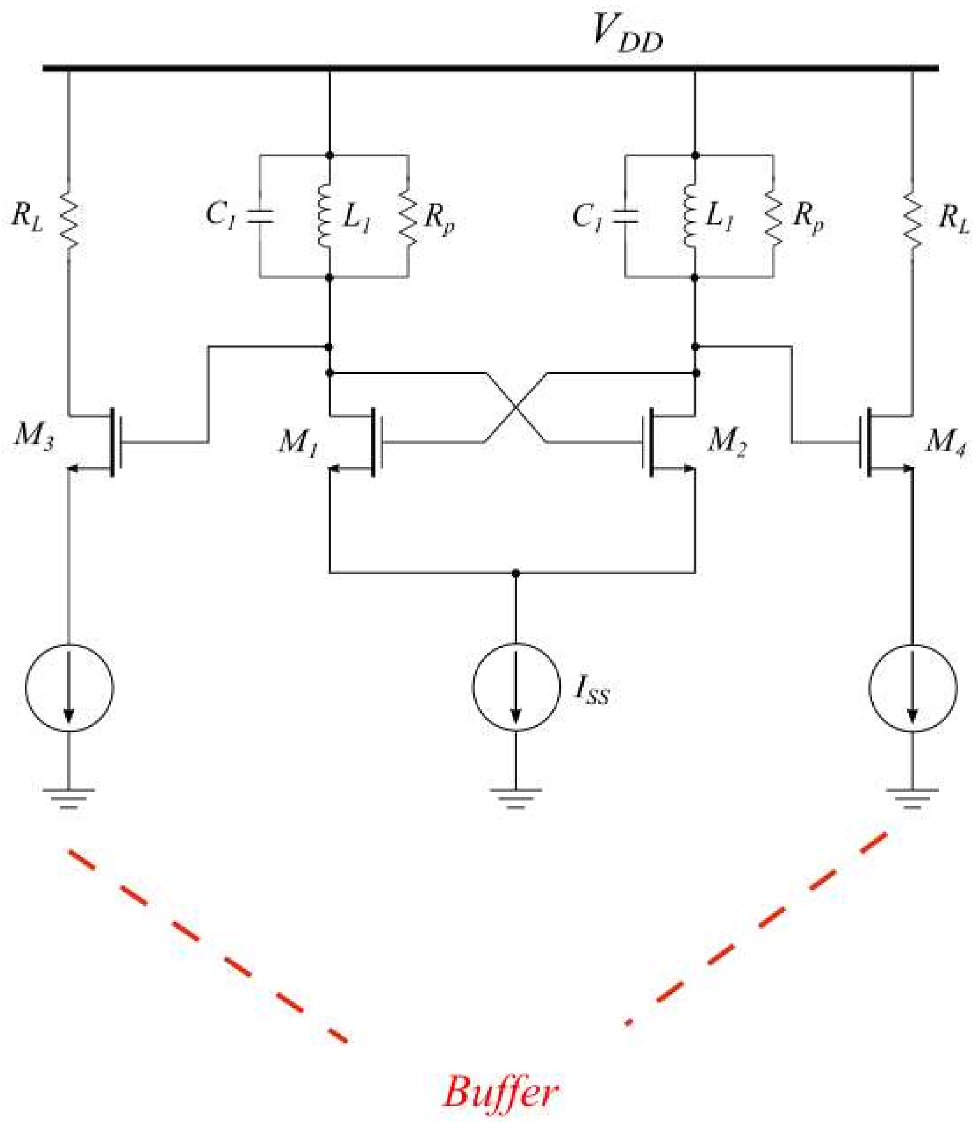
- Higher swing ensures sharp switching.
- Lower swing can amplify internal oscillator noise.
- For 1 V supply swing should be between 0.6 to 0.8 Vpp.



Performance Parameters

3. **Load Pulling**

- May need to supply to more than one load like frequency divider and mixer.
- Problem amplified in Tx where PA i/p capacitances can cause PA pulling.
- Usually a buffer stage is used to decouple oscillator from o/p loading.
- Buffers can also amplify o/p of oscillator thereby increasing voltage swing.



Performance Parameters

4. **Power dissipation** – Often in conflict with phase noise and tuning range. Since tuning range depends on range of control voltage. For increasing control voltage supply voltage and hence power consumption needs to be increased.
   Phase Noise is related to voltage swing $\rightarrow$ lower noise requires higher voltage swing and hence higher power consumption.

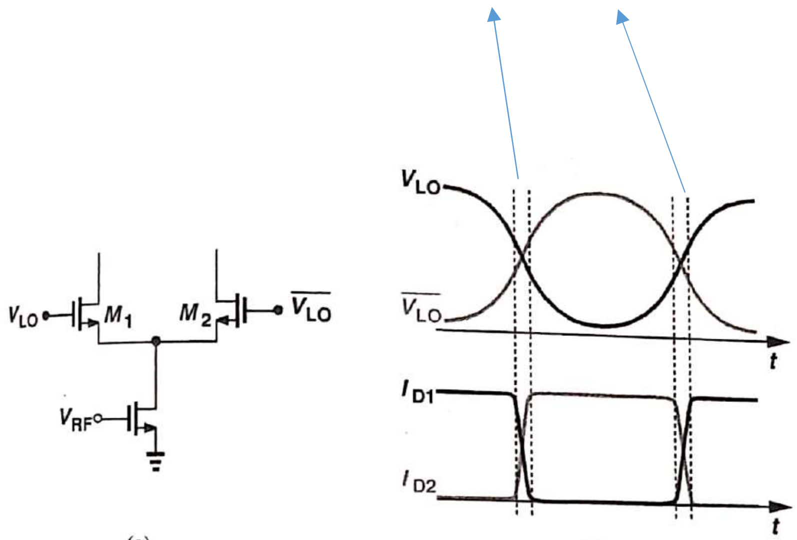
5. **Supply Sensitivity** – Supply variation is equivalent to flicker noise which affects oscillator frequency. Flicker noise cannot be easily removed by bypass capacitors.

Performance Parameters

6. **Output Waveform**

- Abrupt LO transitions reduce noise and increase conversion gain.
- Improve frequency divider performance.
- Differential signals with 50% duty cycle reduce second order non linear effects and also dc feedthrough.
- Pure square waves difficult to achieve due to harmonic suppression due to narrowband nature of oscillator tank and the o/p parasitics of buffer.
- Hence the next best approach is to increase O/P voltage swing (create large I_D changes in M1 and M2) by increasing V_{LO} swing or increasing size of M1 and M2.

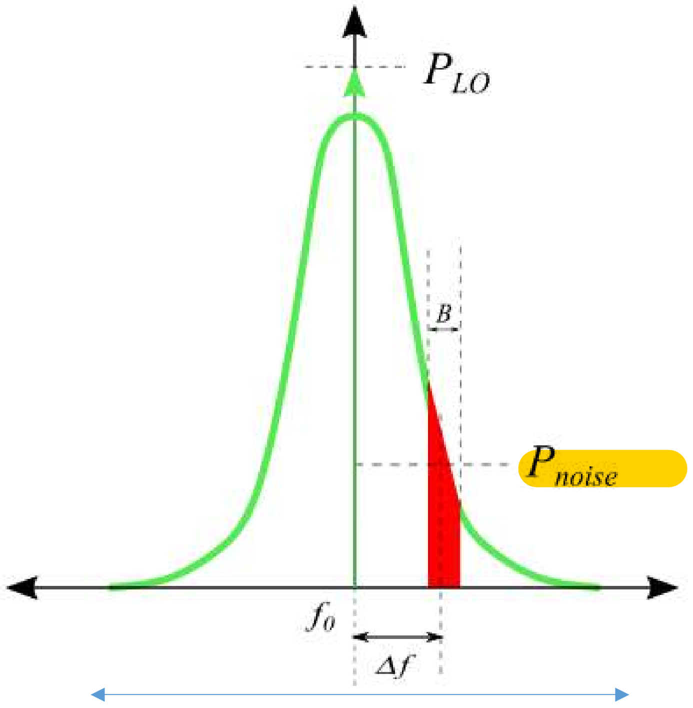
Both M1 and M2 on. Only common mode gain provided.
Also both M1 and M2 contribute to o/p noise.



(a) (b)

Performance Parameters

7. Phase Noise



Random change in frequency

Oscillator Output spectrum

$$Say,\ x_{LO}(t) = A_{LO} \cos\left[\omega_c t + \underbrace{\varphi_n(t)}_{\substack{\text{change is random,} \\ \text{causes random} \\ \text{change in frequency}}} \right]$$

$$L_n(f_0 + \Delta f)\Big|_{dBc/Hz} = P_{noise}\Big|_{dBm} - B\Big|_{dBHz} - P_{LO}\Big|_{dB}$$

Effect of LO Phase Noise On Receiver

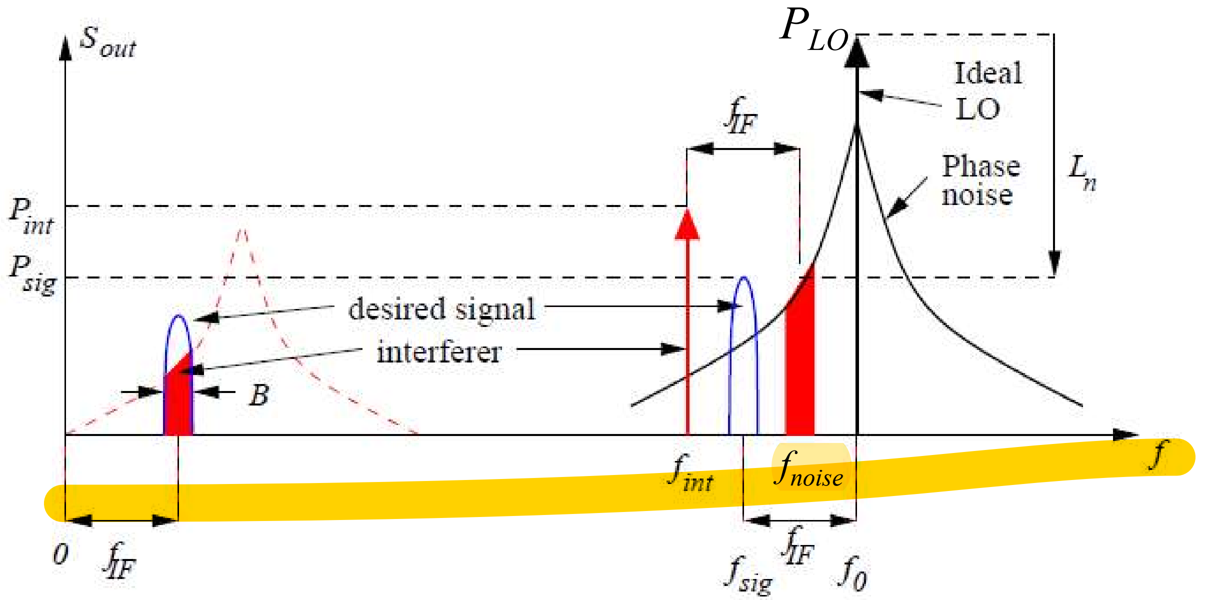
- Degradation (decrease) of SNR → increase of BER
- Degradation of selectivity (in addition to IMD)
- For small bandwidth B the noise power is:

$$P_{noise} \approx P_{\text{int}} B P_{LO} L_n (f_{noise} = f_{\text{int}} + f_0 - f_{sig})$$

We must have at worst: $SNR_{\min} = \dfrac{P_{sig} \times P_{LO}}{P_{noise}} = \dfrac{P_{sig}}{B P_{int} L_n(\Delta f)}$

Giving the maximum allowable phase noise at f $= \Delta f$ as,

$$L_n(\Delta f)\big|_{dBc} = P_{sig}\big|_{dBm} - P_{\text{int}}\big|_{dBm} - B\big|_{dBHz} - SNR_{\min}\big|_{dB}$$



$B|_{dBHz}$=10*log10(B)

GSM Example

Frequency Offset Δf *(MHz)*	*Interfering Signal Level (dBm)*	$L_n(\Delta f)$ *dBc/Hz*
3.0	-23	-138
1.6	-33	-128
0.6	-43	-118

The channel Bandwidth is 200 kHz. The carrier signal is -99 dBm. The required SNR_{min} is 9 dB.

Effect of LO Phase Noise On Transmitter

SNR at f_2 reduced due to phase noise of user 1

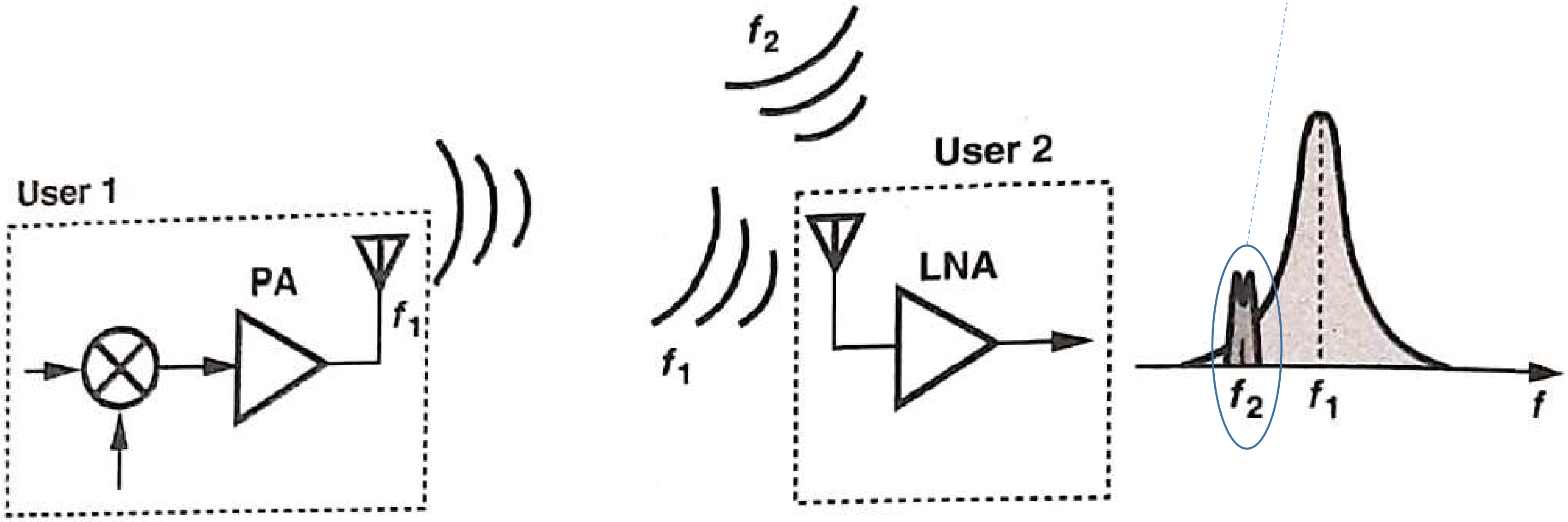


Figure 8.52 *Received noise due to phase noise of an unwanted signal.*

Effect of LO Phase Noise On Modulation and Demodulation

Overall phase is random

$$x_{QPSK}(t) = A\cos\left[\overbrace{\omega_c t + (2k+1)\frac{\pi}{4} + \underbrace{\varphi_n(t)}_{\text{From Phase Noise}}}^{\text{Overall phase is random}}\right] \quad k = 0, \dots, 3$$

From Phase Noise

Amplitude remains same but phase is random.

If noise is very high then symbol will be detected falsely.

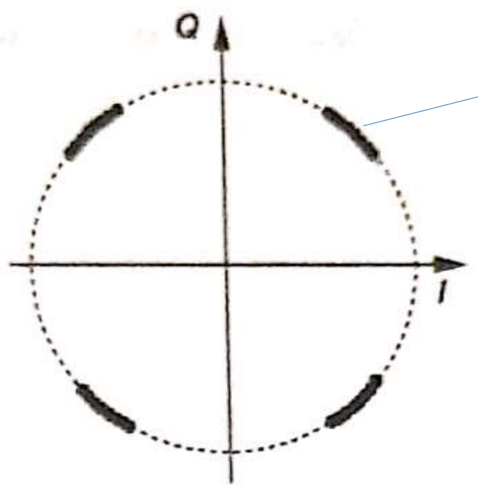
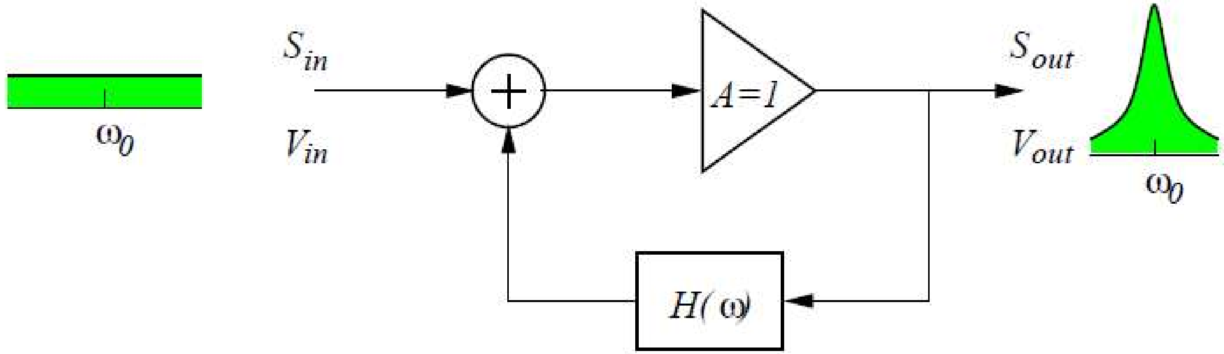


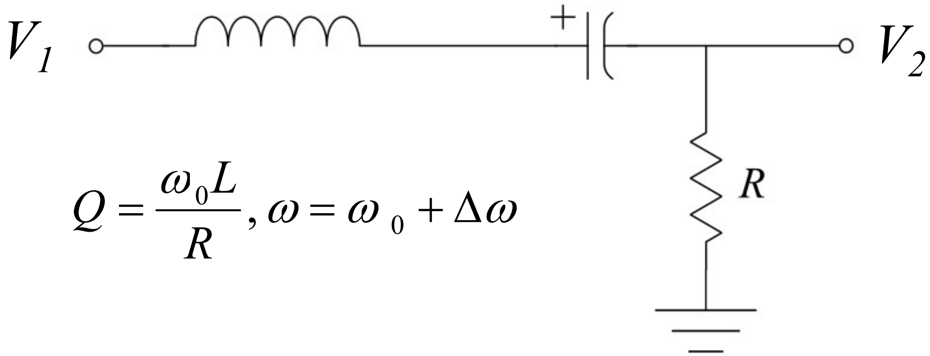
Figure 8.53 *Corruption of a QPSK signal due to phase noise.*

Phase Noise Model



Transfer function of a parallel RLC resonator

$$H(\omega) = \frac{V_2}{V_1} = \frac{1}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)} \approx \frac{1}{1 + jQ\frac{2\Delta\omega}{\omega_0}}$$



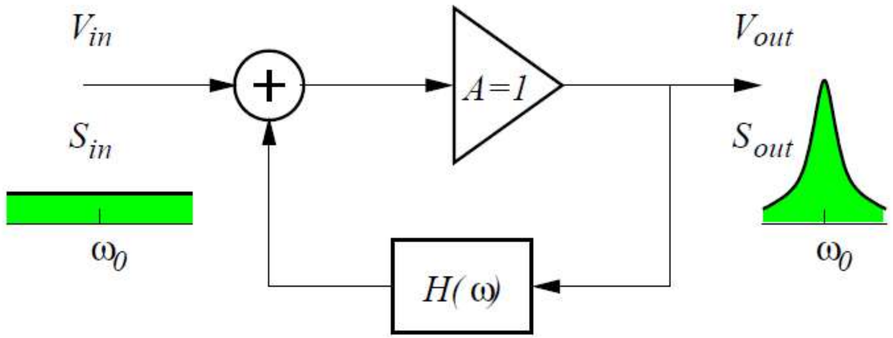
$$Q = \frac{\omega_0 L}{R}, \omega = \omega_0 + \Delta\omega$$

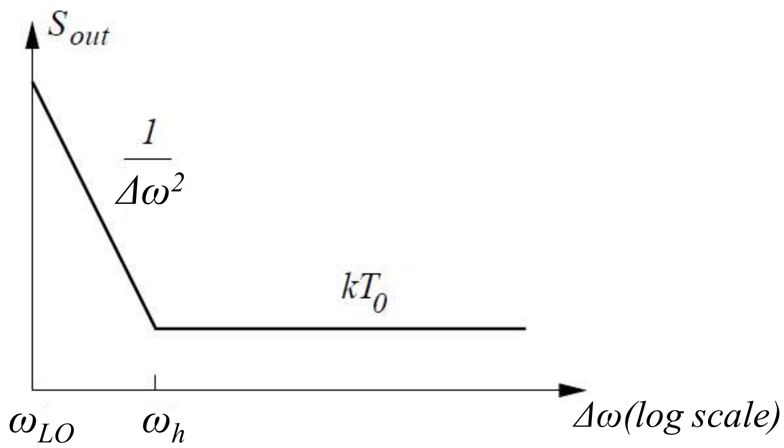
For calculating the output oscillator PSD we need,

$$\frac{S_{out}(\omega)}{S_{in}(\omega)} = \left|\frac{1}{1 - H(\omega)}\right|^2 = 1 + \left(\frac{\omega_0}{2Q\Delta\omega}\right)^2 = 1 + \left(\frac{\omega_h}{\Delta\omega}\right)^2$$

where, $\omega_h = \frac{\omega_0}{2Q}$

Leeson's Model





$$S_{out} = S_{in}\left|1/(1-H(\omega))\right|^2 = S_{in}\left[1+\left(\frac{\omega_0}{2Q\Delta\omega}\right)^2\right] \rightarrow \text{o/p PSD}$$

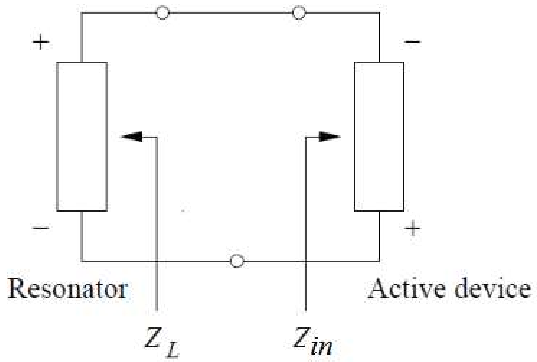
$S_{in} = kT_0F \rightarrow$ i/p PSD, where F is an empirical noise factor

$$L(\Delta\omega) = 10\log\left[\frac{S_{out}}{P_{LO}}\right] = 10\log\left\{\frac{FkT_0}{P_{LO}}\left[1+\left(\frac{\omega_0}{2Q\Delta\omega}\right)^2\right]\right\}$$

where P_{LO} is the oscillator output power.

Better phase noise characteristics are achieved with a high Q resonator.

Basic LC Oscillator Topologies (Colpitt's and Clapp's)



At operating point,

$$R_S = -R_{IN} = \frac{g_m}{C_1C_2\omega^2}$$

$$X_L = j\omega_0 L = -X_{IN} = \frac{j}{\omega_0}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{L_1\dfrac{C_1C_2}{C_1+C_2}}}$$

Condition of startup,

$$|R_{IN}| > R_S$$

$$\Rightarrow \frac{g_m}{C_1C_2\omega^2} > R_S \text{ (which is usually satisfied at dc)}$$

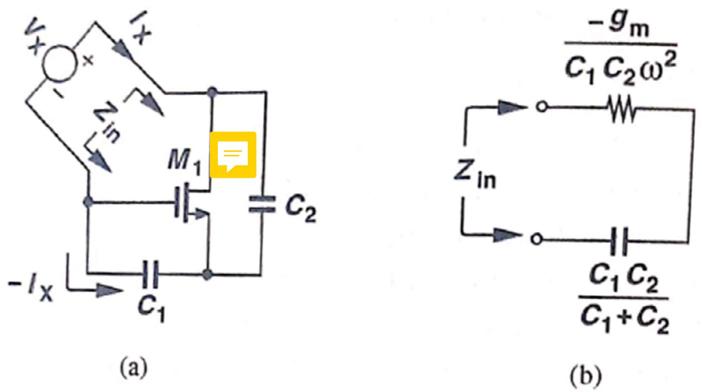


Figure 8.14 *(a) Circuit providing negative resistance, (b) equivalent circuit.*

$$i_d = g_m v_{gs}, v_{gs} = -I_X \times \frac{1}{sC_1}$$

$$I_{C1} = I_X, I_{C2} = I_X - i_d = I_X + \frac{g_m I_X}{sC_1}$$

$$V_X = I_X\left(1+\frac{g_m}{sC_1}\right)\times\frac{1}{sC_2} + I_X \times \frac{1}{sC_1}$$

$$\frac{V_X(j\omega)}{I_X(j\omega)} = \underbrace{\frac{1}{jC_1\omega}+\frac{1}{jC_2\omega}}_{X_{IN}} \underbrace{-\frac{g_m}{C_1C_2\omega^2}}_{R_{IN}}$$

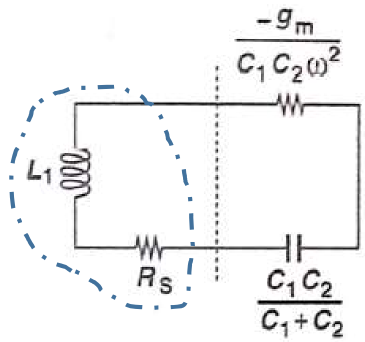
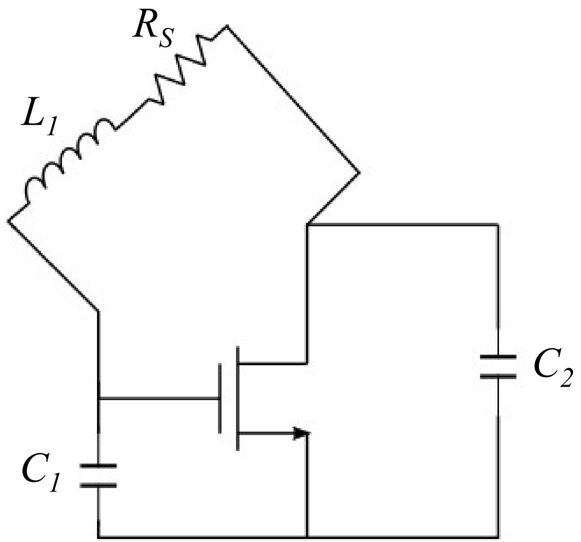
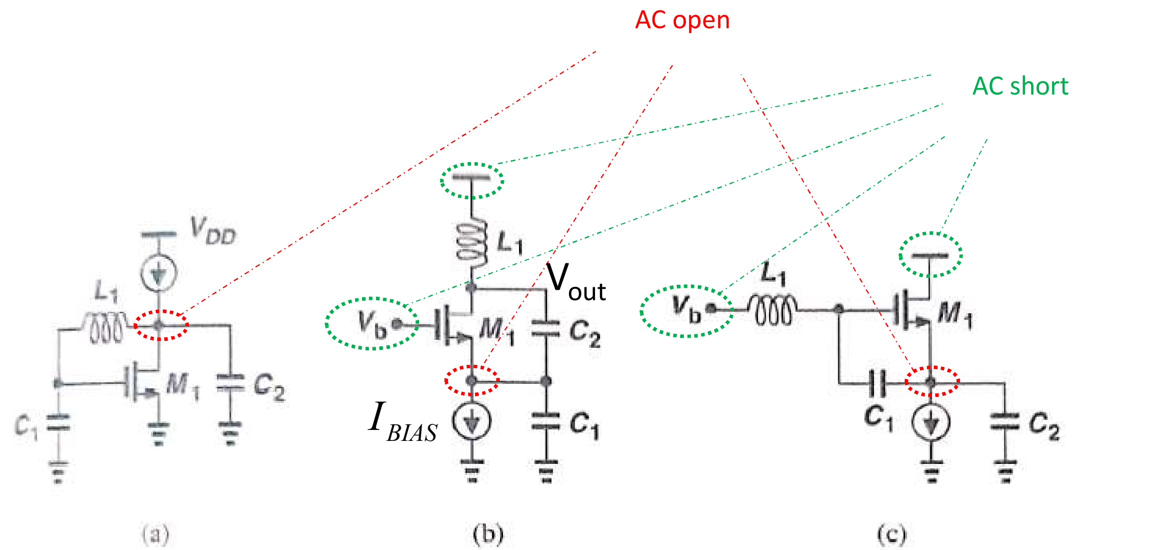


Figure 8.15 *Connection of lossy inductor to negative-resistance circuit.*

Variants of 3 point oscillator





igure 8.22 *Variants of three-point oscillator, (a) with source grounded, (b) with gate grounded (Colpitts oscillator), (c) with drain grounded (Clapp oscillator).*

For Colpitt's oscillator,

$$V_{tank} \propto I_{BIAS}$$
$$\propto R_P$$

$$V_{out} = \frac{2}{\pi} I_{BIAS} R_p$$

where, $R_P = (L_s \omega_0)^2 / R_S$, is the equivalent shunt resistance of the inductor at ω_0.

Differential Negative Gm Oscillators

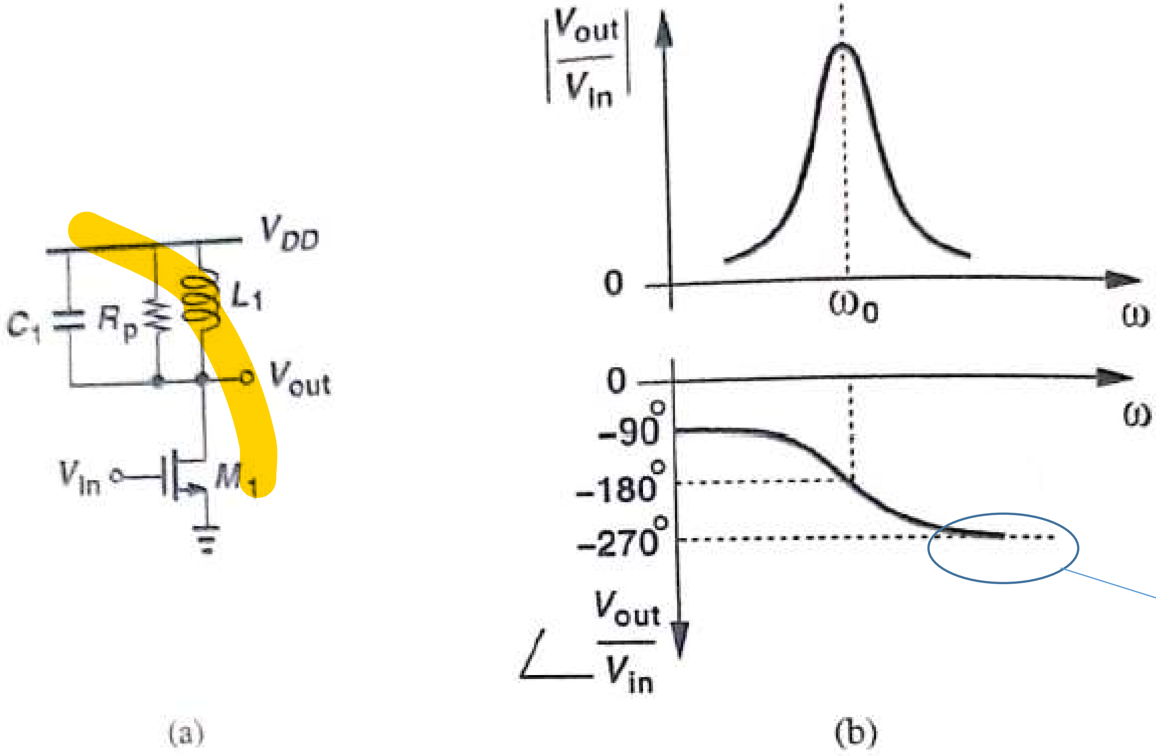
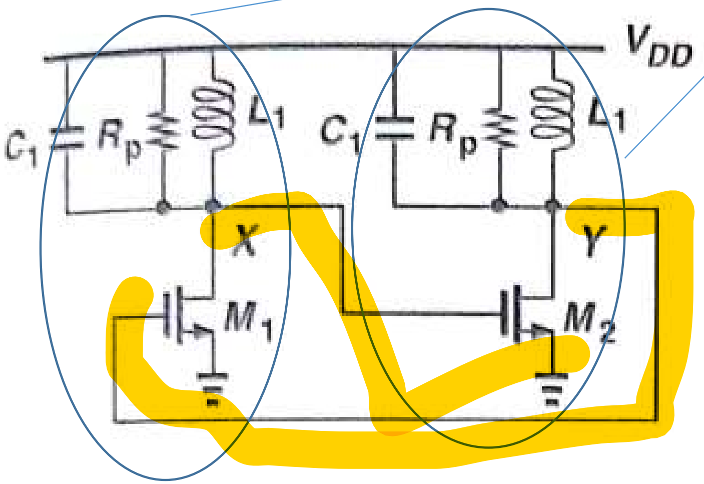


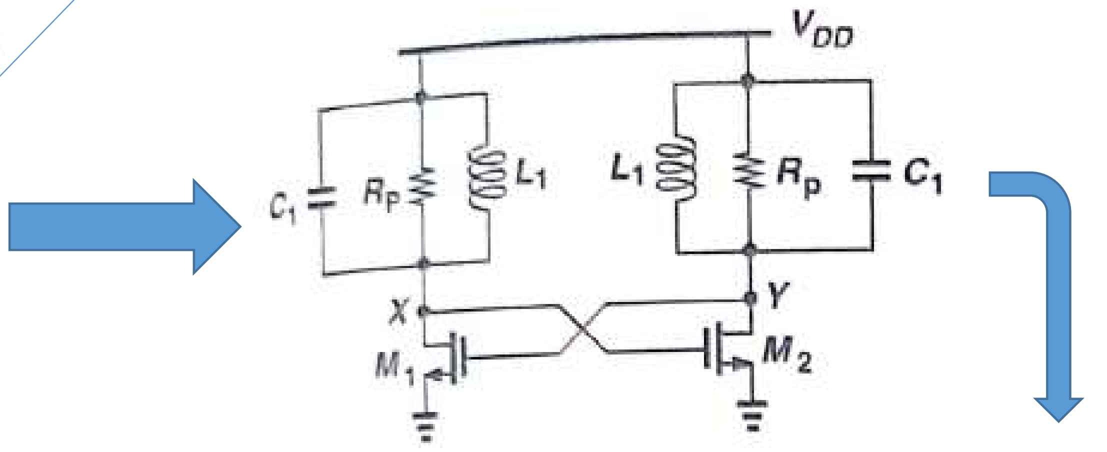
Figure 8.16 *(a) Tuned amplifier, (b) frequency response.*

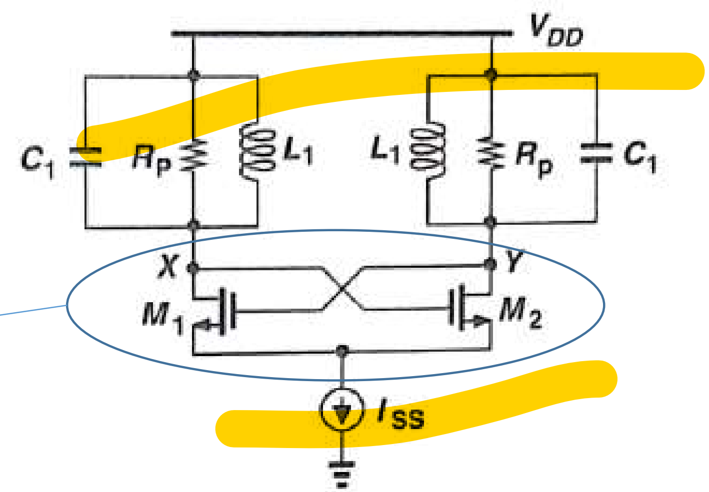
Max phase shift of 270 degree, Not sufficient to meet Bark hausen criteria

Differential Negative G_m Oscillators

Each stage provides 180 degree phase shift



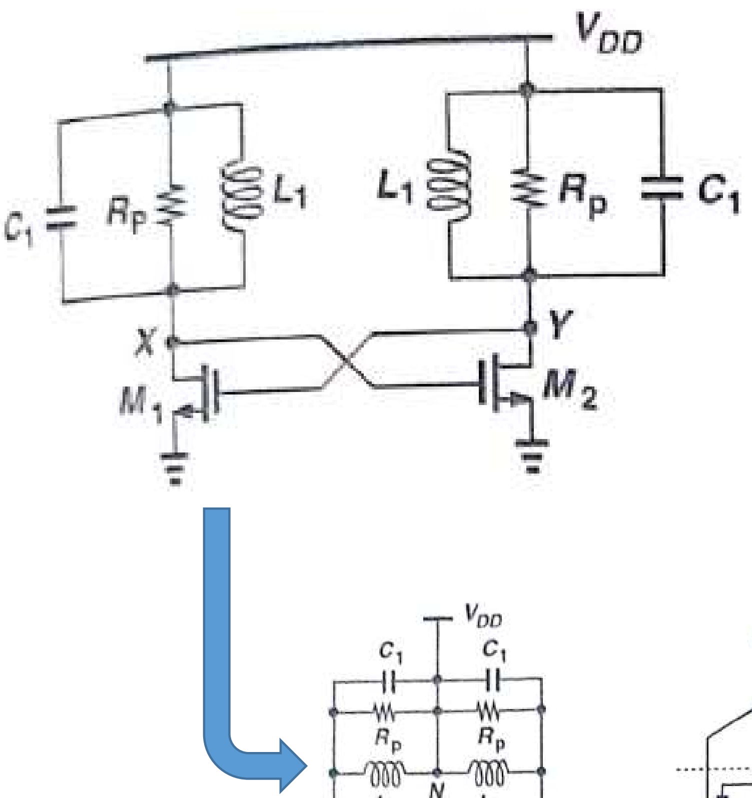




Less affected by V_T variation when I_{SS} is present

Voltage swing $V_{XY} \cong (4/\pi)\, I_{SS} R_p$

(See e.g. 8.11 in razavi new edition)



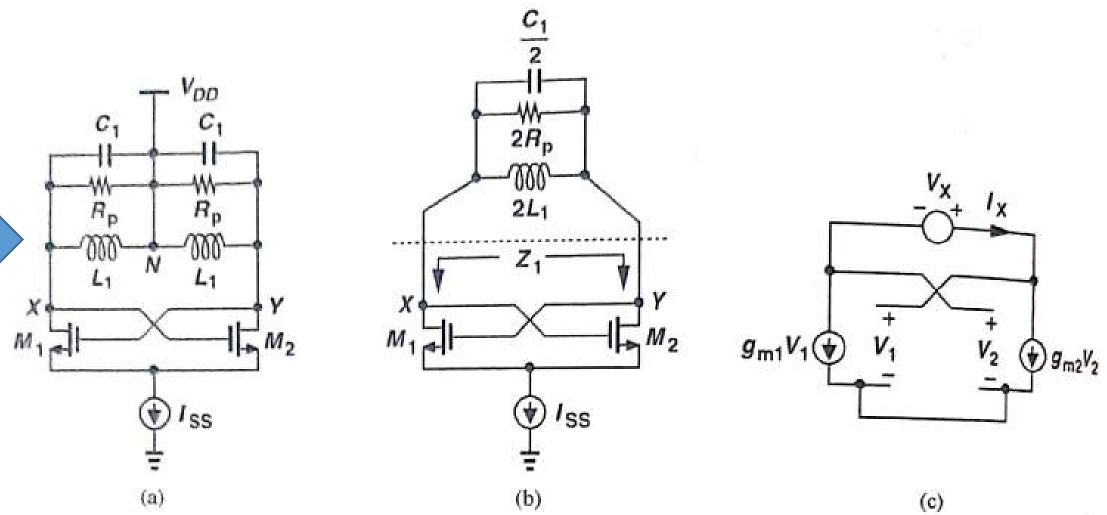


Figure 8.21 *(a) Redrawing of cross-coupled oscillator, (b) load tanks merged, (c) equivalent circuit of cross-coupled pair.*

$$I_X = -g_{m1}V_1 = g_{m2}V_2$$

$$\frac{V_X}{I_X} = \frac{V_1 - V_2}{I_X} = \frac{-I_X / g_{m1} - I_X / g_{m2}}{I_X}$$

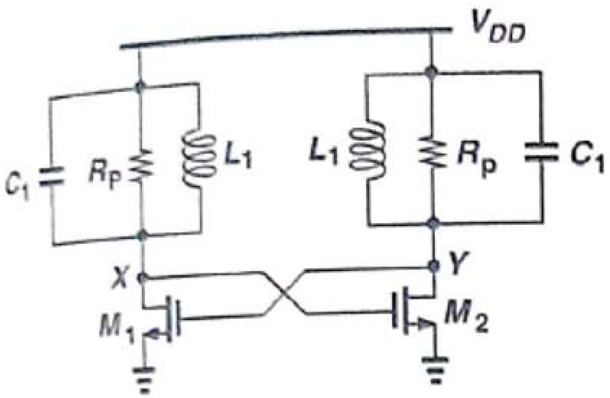
$$= -\left(\frac{1}{g_{m1}} + \frac{1}{g_{m2}}\right) = -\frac{2}{g_m}$$

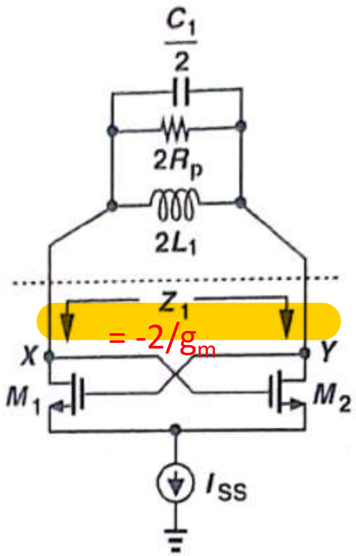
At steady state,

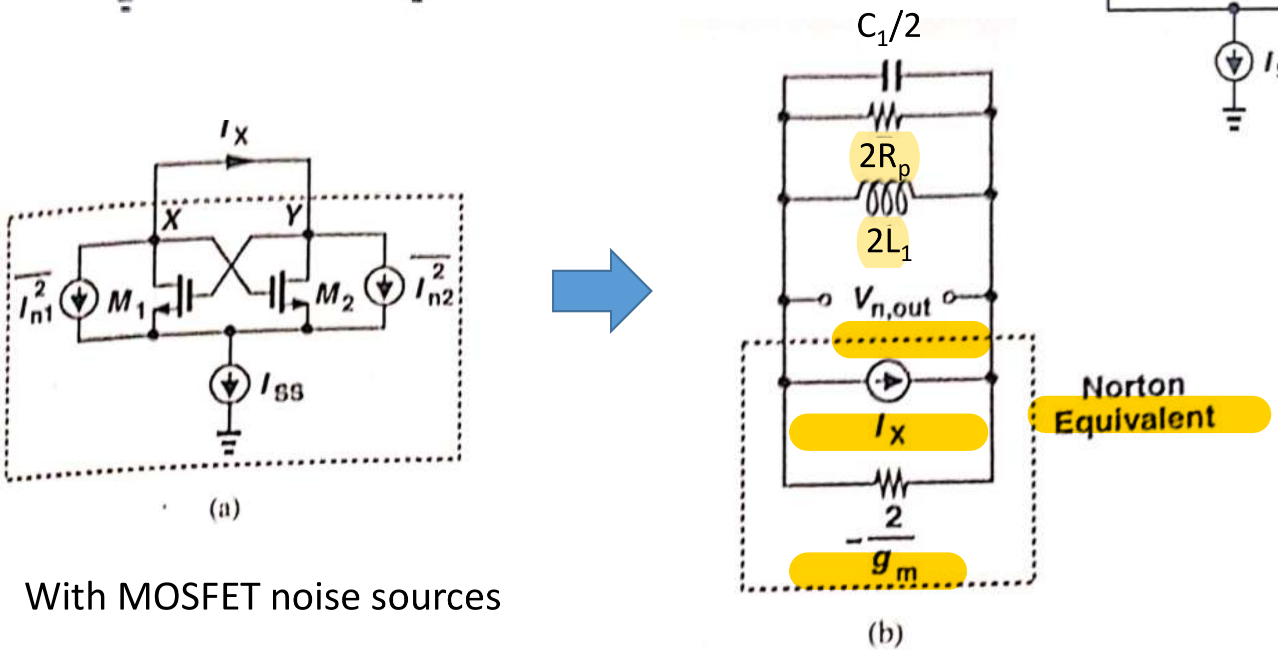
$$2R_p = \frac{2}{g_m} \Rightarrow R_p = \frac{1}{g_m}$$

$$\Rightarrow R_p g_m = 1$$

Phase Noise of differential oscillator







With MOSFET noise sources

Say net conductance $G=1/2R_p - g_m/2 = 0$

Hence net resistance $R = 1/G = \infty$

$$I_X = \frac{I_{n1} - I_{n2}}{2}, \overline{I_{n1}^2} = \begin{cases} 2kT\gamma g_m & \text{(for homodyne)} \\ kT\gamma g_m & \text{(for heterodyne)} \end{cases}$$

Hence total noise current psd in one branch,

$$= \overline{\mathrm{I}_X^2} + \frac{4kT}{2R_p}$$

Total noise voltage psd, $\overline{V_{n,out}^2} = \left(\frac{\overline{\mathrm{I}_{n1}^2}}{2} + \frac{2kT}{R_p} \right) |Z_{\text{tank}}|^2$

$$= \left(\frac{\overline{\mathrm{I}_{n1}^2}}{2} + \frac{2kT}{R_p} \right) \frac{L_1^2\omega^2}{\left(1 - L_1C_1\omega^2\right)^2} \quad (\text{Taking R} = \infty)$$

$$= \left(\frac{\overline{\mathrm{I}_{n1}^2}}{2} + \frac{2kT}{R_p} \right) \frac{L_1^2\omega^2}{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2} \approx \left(\frac{\overline{\mathrm{I}_{n1}^2}}{2} + \frac{2kT}{R_p} \right) \frac{1}{4C_1^2\Delta\omega^2}$$

$where, \omega = \omega_0 + \Delta\omega$ and $\omega_0 >> \Delta\omega$

LO power

$$= \left[\frac{4}{\pi} \left(I_{SS} R_p \right) \right]^2 / 2$$

Hence, Phase noise

$$L(\Delta\omega) = \frac{\overline{V_{n,out}^2}}{\text{LO power}} = \frac{\left(\frac{\overline{\mathrm{I}_{n1}^2}}{2} + \frac{2kT}{R_p} \right) \frac{1}{4C_1^2\Delta\omega^2}}{\left[\frac{4}{\pi} \left(I_{SS} R_p \right) \right]^2 / 2}$$

$$= \frac{\pi^2}{32} \left(\frac{\overline{\mathrm{I}_{n1}^2}}{2} + \frac{2kT}{R_p} \right) \frac{1}{I_{SS}^2 R_p^2 C_1^2 \Delta\omega^2},$$

$\mathrm{R_p} = \mathrm{QL_1}\omega_0$ (parallel tank)

Hence, $L(\Delta\omega) = \frac{\pi^2}{32} \left(\frac{\overline{\mathrm{I}_{n1}^2}}{2} + \frac{2kT}{R_p} \right) \frac{1}{I_{SS}^2 Q^2 \Delta\omega^2}$

- $L(\Delta\omega)$ is inversely proportional to I_{SS}^2 (which is proportional to voltage swing).
- $L(\Delta\omega)$ is inversely proportional to $\Delta\omega^2$ (the offset frequency).
- $L(\Delta\omega)$ is inversely proportional to Q^2.

Voltage controlled oscillator

- Range of C_{var} depends on channel length.
- Lesser the value of L higher should be the range.
- However overlap capacitance(Cox) limits the range.
- Need to increase L for optimum C_{var} variation.
- Increasing L decreases the Q of the varactor.

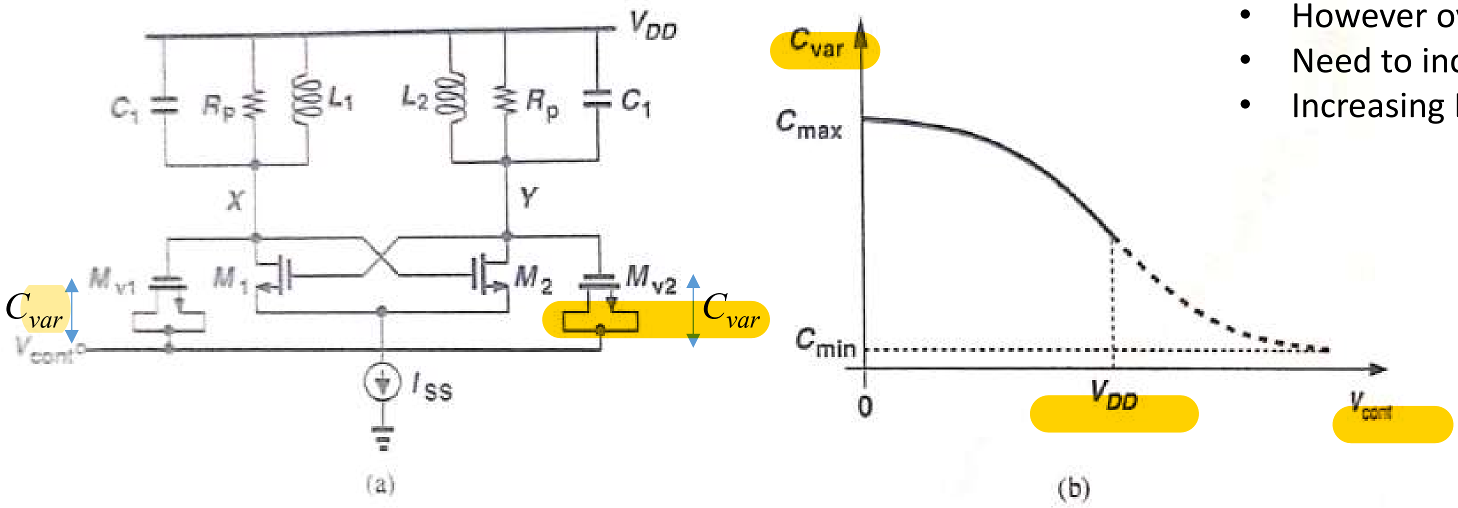


Figure 8.25 (a) VCO using MOS varactors, (b) range of varactor capacitance used in (a).

$$\omega_0 = \frac{1}{\sqrt{L_1(C_1 + C_{var})}}$$

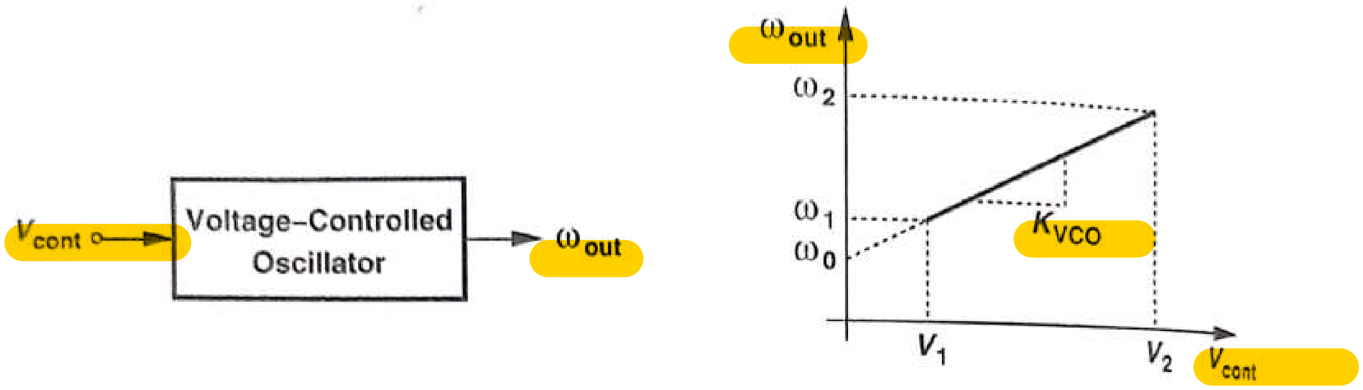


Figure 8.24 VCO characteristic.

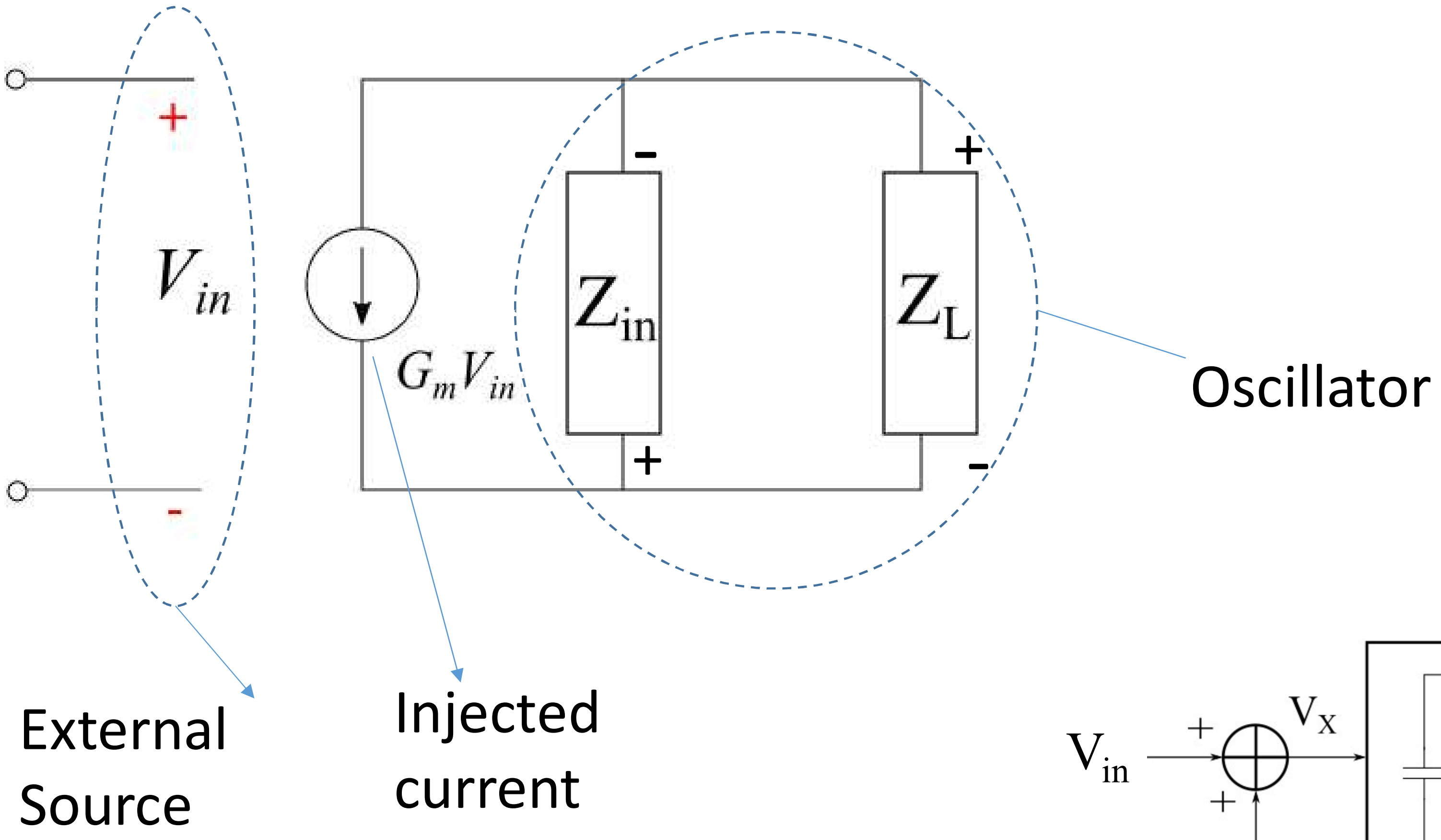
constant

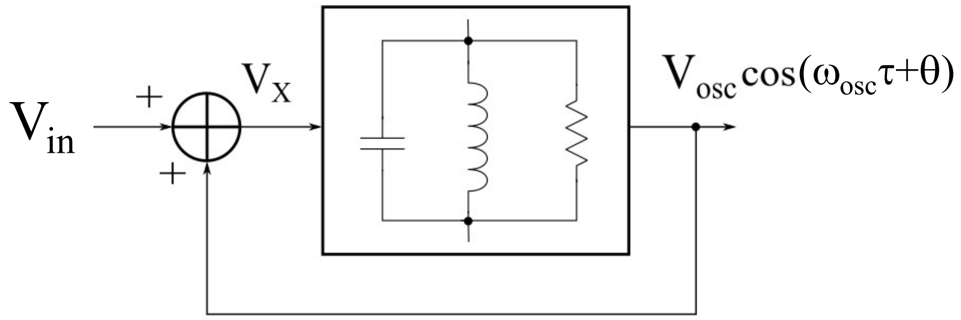
$$\underbrace{\omega_{out} = \omega_0 + K_{VCO} V_{cont}}_{\textit{Ideal characteristics}}$$

Advantages of Differential topology

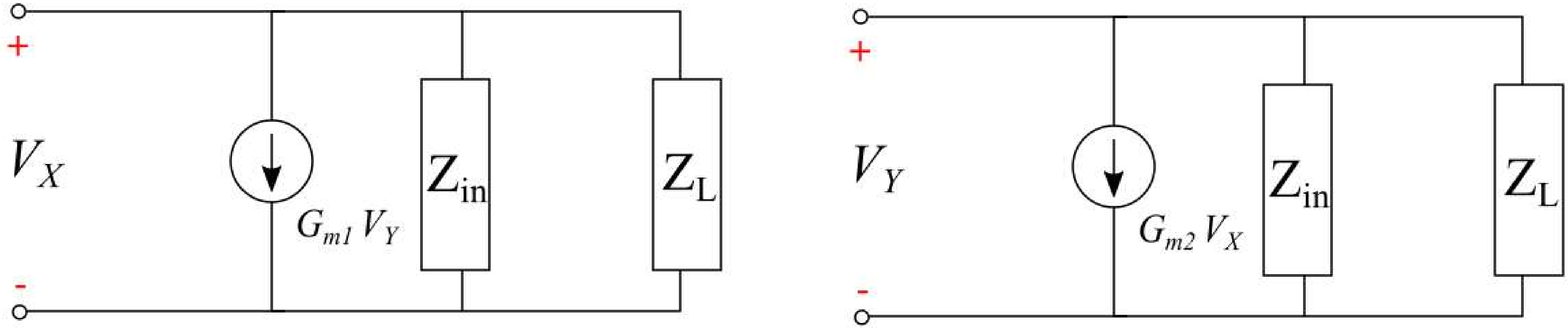
- Less affected by power supply variations due to symmetry.
- Higher voltage swing
- Less affected by V_T and temperature variations

Injection Locking in oscillators





Quadrature LC oscillator



$$G_{m2} V_X \frac{Z_L Z_{in}}{Z_L + Z_{in}} = V_Y \qquad (1)$$

$$G_{m1} V_Y \frac{Z_L Z_{in}}{Z_L + Z_{in}} = V_X \qquad (2)$$

Equating (1) and (2) we get,

$$\left(G_{m2}V_X^2 - G_{m1}V_Y^2\right)\frac{Z_L Z_{in}}{Z_L + Z_{in}} = 0$$

Since $Z_L \| Z_{in}$ cannot be zero,

$$G_{m2}V_X^2 = G_{m1}V_Y^2$$

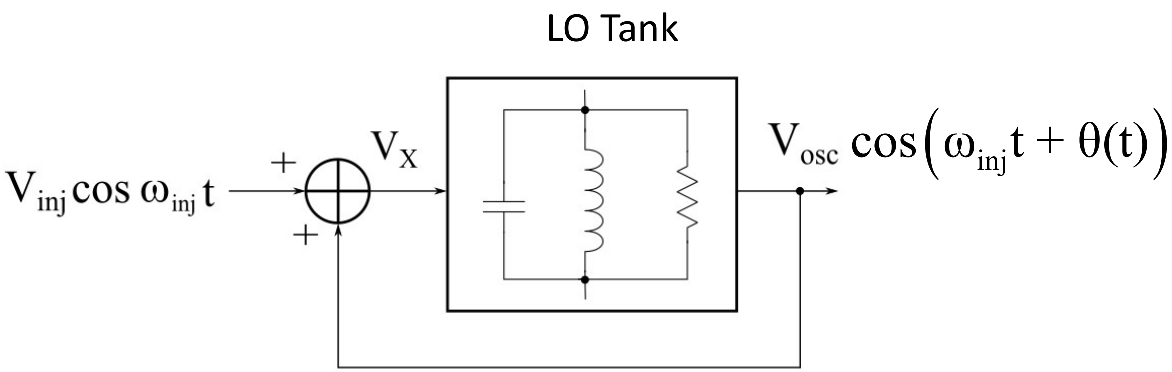
If $G_{m1} = G_{m2}$

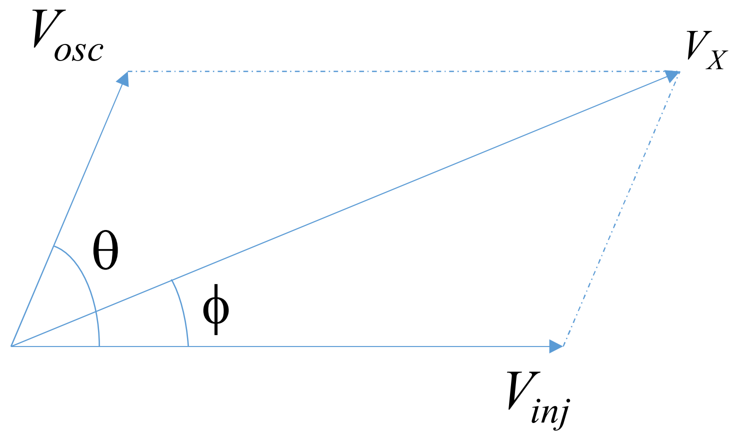
$\Rightarrow \quad V_X = \pm V_Y$ i.e. a oscillator with 180° phase shift .

If $G_{m1} = -\,G_{m2}$

$\Rightarrow \quad V_X = \pm\, j\, V_Y$ i.e. a quadrature oscillator

Injection Pulling (Discussed earlier in Transmitter Architectures)





$\dfrac{d\theta(t)}{dt}$ is a measure of the injection pulling.

If ϕ is small, $(|V_{inj}| >> |V_{osc}|)$, V_X will follow V_{inj} even if $\dfrac{d\theta}{dt}$ is small.

Conversely if ϕ is near $\dfrac{\pi}{2}$, $(|V_{inj}| << |V_{osc}|)$, V_X will not follow V_{inj} even if $\dfrac{d\theta}{dt}$ is large.

In Phase coupling $(V_A = -V_C)$

Z_A and Z_B are resonators

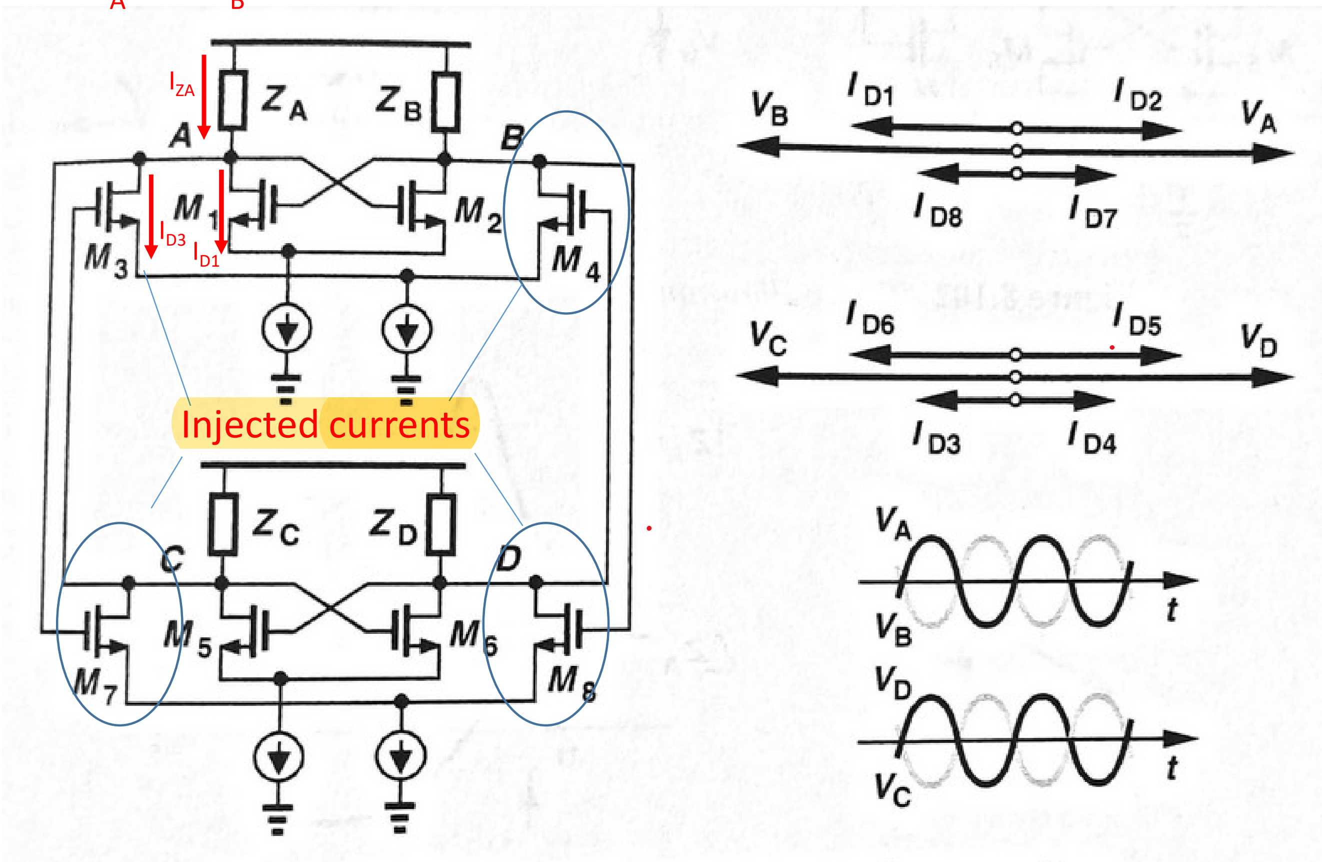


Figure 8.101 *Phasor diagrams for in-phase coupling.*

$$V_A = -I_{ZA} Z_A$$

$$I_{ZA} = |I_{ZA}| \exp(j\pi), Z_A = |Z_A| \exp(0)$$

Hence, $V_A = -I_{ZA} \; Z_A = |I_{ZA}||Z_A|$,

Thus V_A can have phase = 0,

Cross Coupling ($V_C = j\,V_A$)

Z_A and Z_B are resonators

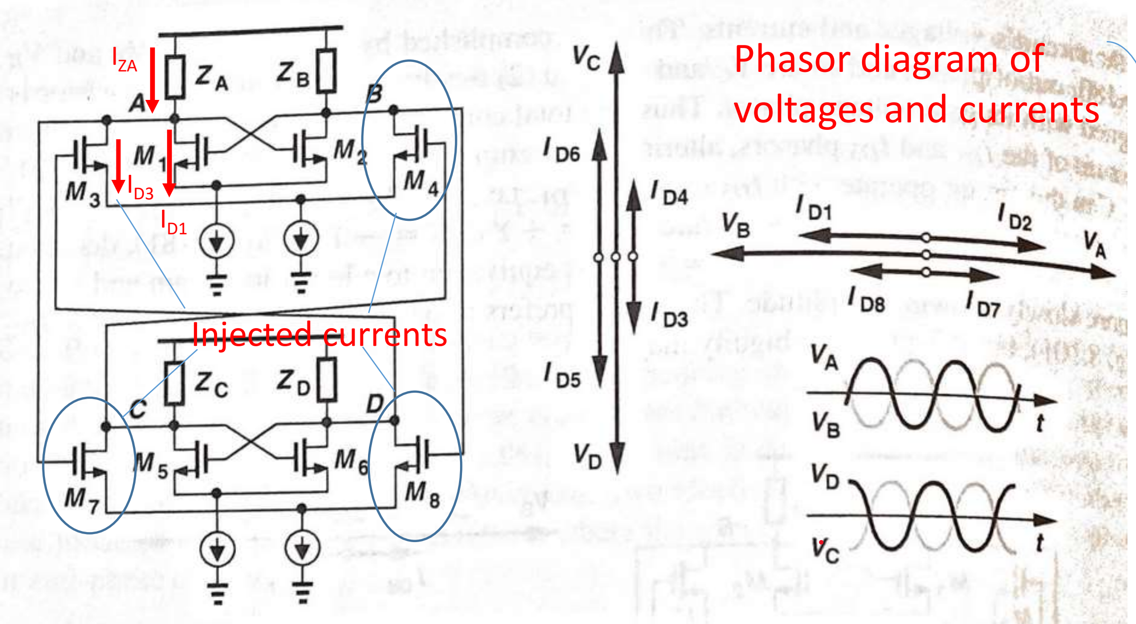


Figure 8.102 *Phasor diagrams for anti-phase coupling.*

Currents in transistors will be along the direction of voltages

$$\Delta\omega = \omega_{osc1} - \omega_0 = \frac{\omega_0}{2Q_{\text{tank}}} \tan^{-1}\frac{I_{D3}}{I_{D1}}$$

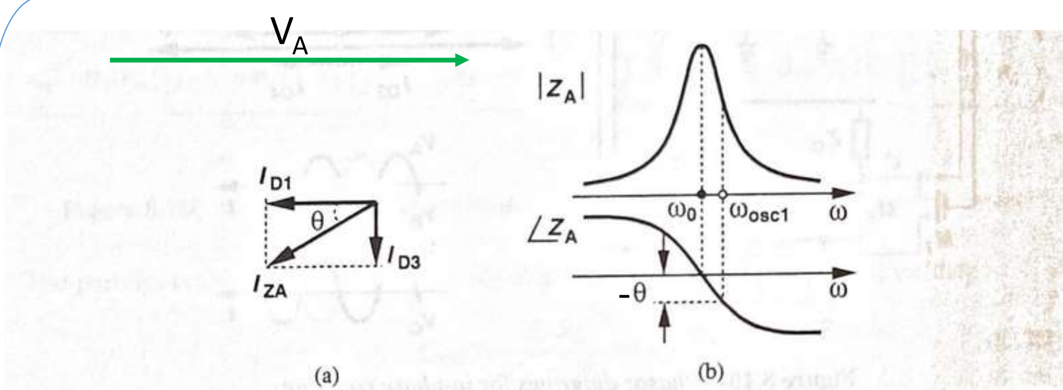


Figure 8.103 *(a) Vector summation of core and coupling currents, (b) resulting departure from resonance.*

$V_A = -I_{ZA} Z_A \quad I_{ZA} = |I_{ZA}|\exp(j(\pi+\theta)), Z_A = |Z_A|\exp(-j\theta)$

Hence, $V_A = -I_{ZA}\ Z_A = |I_{ZA}||Z_A|$,

Thus V_A can have phase = 0, even though I_{D1} and I_{D3} are normal to each other.

Injection based frequency division

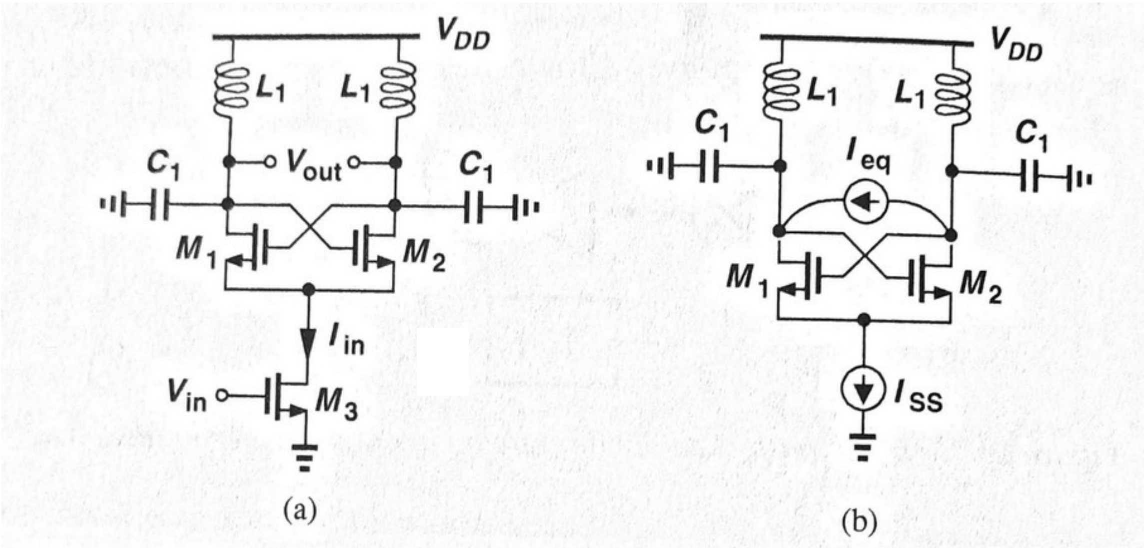


Figure 10.74 *(a) Example of injection-locked divider, (b) equivalent view.*

O/P frequencies over which lock can be achieved,

$$\Delta\omega_{out} = \frac{\omega_0}{2Q_{\text{tank}}}\tan^{-1}\frac{2\Delta I_{in}}{\pi I_{osc}}$$

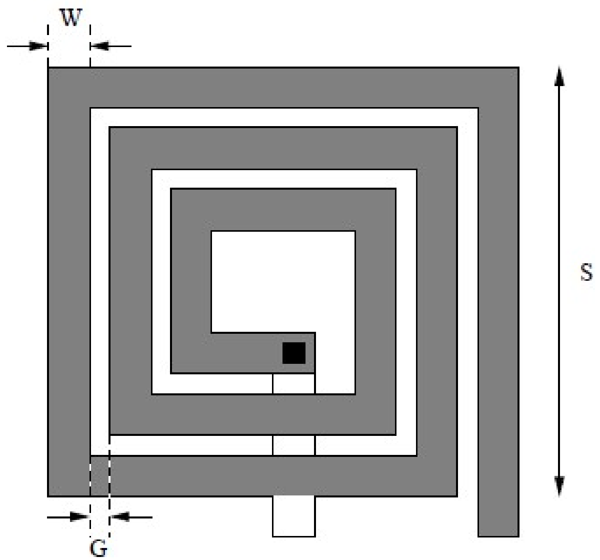
$$\omega_{osc} = \omega_0 \pm \Delta\omega_{out}$$

By adjusting $\Delta \mathrm{I}_{\mathrm{in}}$ the required ω_{osc} can be achieved.

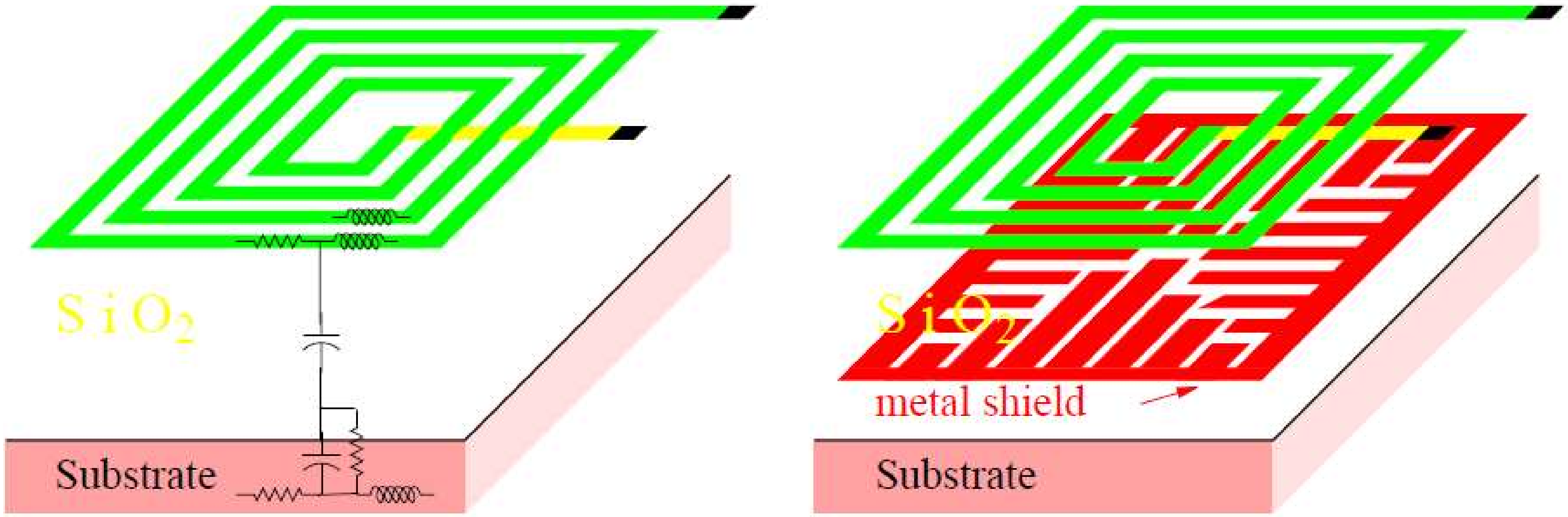
Monolithic Inductors

- Empirical formula for spiral inductor $L \approx 1.3 \times 10^{-7} \dfrac{A_m^{5/3}}{A_{tot}^{1/6}\; W^{1.75} (W+G)^{1/4}}$

  where A_m is the metal area, $A_{tot} \approx S^2$ is the inductor area, W the line width, G the line spacing. Formula is accurate from 5 to 50 nH.
- Image currents in a lossy substrate decrease the inductor Q (eddy currents).



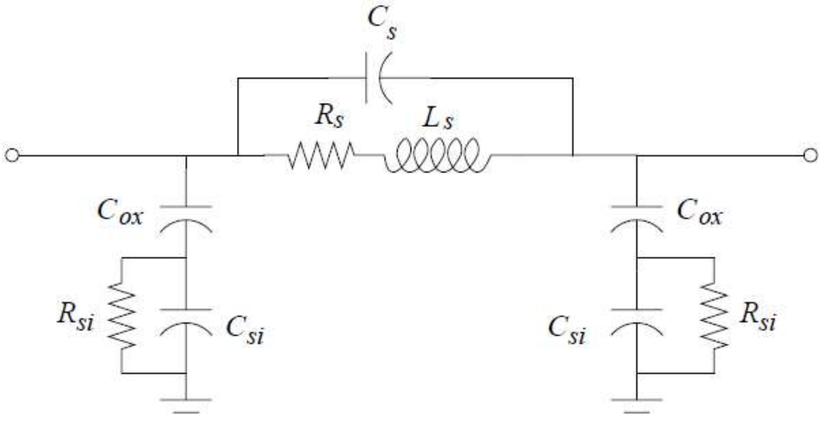
Shielded and Not Shielded Inductors



(a) Not shielded (b) Shielded

- The shield suppresses the substrate loss and noise coupling. A patterned shield reduces the image current.

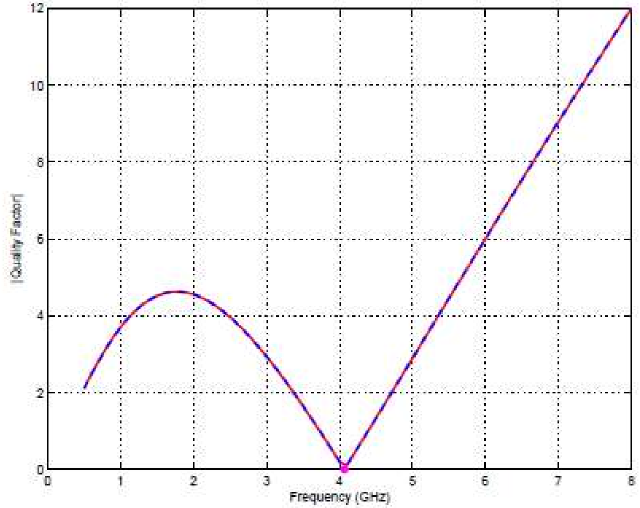
Electrical Model for Non-Shielded Inductors



$$Q = -\frac{\mathrm{Im}[y_{11}]}{\mathrm{Re}[y_{11}]} = \frac{\omega L_s}{R_s} \times \text{Substrate Loss Factor} \times \text{Self-resonance Factor}$$

$$= \frac{\omega L_s}{R_s} \times \frac{R_p}{R_p + R_s\left[\left(\frac{\omega L_s}{R_s}\right)^2 + 1\right]} \times \left[1 - \frac{R_s^2(C_s + C_p)}{L_s} - \omega^2 L_s(C_s + C_p)\right]$$

using $R_p = \frac{1}{\omega^2 C^2 R_{si}} + \frac{R_{si}(C_{ox} + C_{si})^2}{C_{ox}^2}$ $\quad C_p = C_{ox}\frac{1 + \omega^2(C_{ox} + C_{si})C_{si}R_{si}^2}{1 + \omega^2(C_{ox} + C_{si})^2 R_{si}^2}$



IEEE JOURNAL OF SOLID-STATE CIRCUITS, VOL. 33, NO. 5, MAY 1998, "On-Chip Spiral Inductors with Patterned Ground Shields for Si-Based RF IC's" Patrick Yue, and S. Simon Wong

Effective Inductance

$$L_{eff} = \frac{\mathrm{Im}\left[\frac{1}{y_{11}}\right]}{2\pi f}$$