Some basic definitions

1. $dBW = 10\log_{10}(Power$ (power expressed in watts))

So 1 watt of power $\equiv$ 0 dBW

10 watt of power $\equiv$ 10 dBW

2. $\text{dBm} = 10\log_{10}(Power$ (power expressed in milliwatts))

So 1 mW of power $\equiv$ 0 dBm

10 mW of power $\equiv$ 10 dBm

1000 mW (1 Watt) $\equiv$ 30 dBm

3. $\text{dBV} = 20\log_{10}(voltage$ expressed in volts)

So 1 V $\equiv$ 0 dBV

10 V $\equiv$ 20 dBV

1000V $\equiv$ 60 dBV

4. $\text{dB} = 20\log_{10}$(ratio of voltage or currents)

or $= 10\log_{10}$ (ratio of power quantities)

5. Neper(Np) = ln (ratio of voltage or currents)

$$\text{Np} = \ln\left(\frac{x_1}{x_2}\right)$$

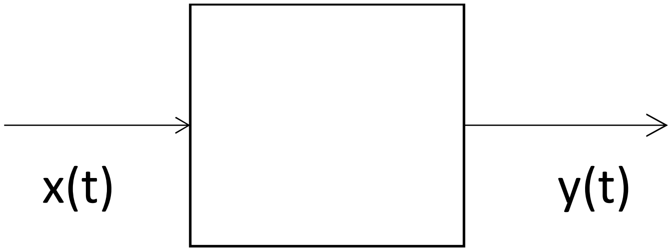
$$1dB = 10\log_{10}\left(\frac{x_1^2}{x_2^2}\right)$$

$$= 20\log_{10}\left(\frac{x_1}{x_2}\right) \Rightarrow \frac{x_1}{x_2} = 10^{(1/20)}$$

Again, $\frac{x_1}{x_2} = e^L$ where L is the Np value of $\frac{x_1}{x_2}$

Hence, $10^{(1/20)} = e^L \Rightarrow L = \ln(10^{(1/20)}) = 0.115$ Np

Non Linearity and Time Variance



This system is linear if for :

$$x_1(t) \to y_1(t)$$
$$x_2(t) \to y_2(t)$$

then we have :

$$a_1 x_1(t) + a_2 x_2(t) \to a_1 y_1(t) + a_2 y_2(t)$$

This system is time invariant if for

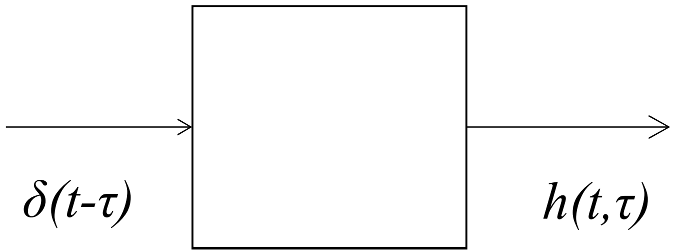
$$x(t) \to y(t)$$

we have

$$x(t-\tau) \to y(t-\tau)$$

Impulse Response of a Linear System

The impulse response for a linear system is obtained from:



The output $y(t)$ for a general input $x(t)$ can be expressed in terms of $h(t,\tau)$:

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t,\tau)\, d\tau$$

For a time - invarying and causal system :

$$y(t) = h(t) * x(t) = \int_{-\infty}^{t} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{t} x(t-\tau)\, h(t)\, d\tau$$

In the frequency domain this gives :

$$Y(j\omega) = H(j\omega)\, X(j\omega)$$

with $H(j\omega) = \Im[h(t)],\ X(j\omega) = \Im[x(t)],\ Y(j\omega) = \Im[y(t)]$,

where $\Im[x]$ is the Fourier transform :

$$\Im[x(t)] = \int_{-\infty}^{\infty} x(t)\ \exp(-j\omega t)\, dt$$

Memoryless Non-Linear System

A system is called memoryless if its output does not depend on the past values of the input :

$$y(t) = f\left[x(t)\right]$$

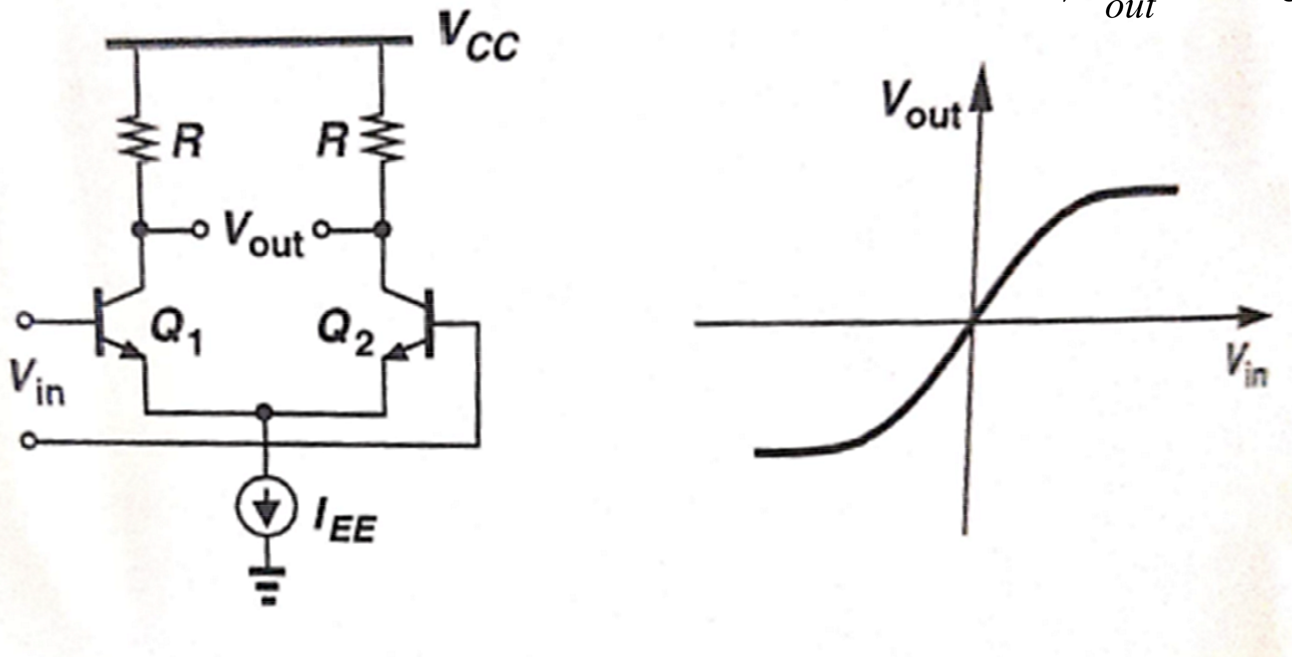
Example :

$$y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x(t)^2 + \alpha_3 x(t)^3$$

Example: A Differential Pair at low Frequency

This is a memoryless circuit if the transistor capacitances are neglected.

$$V_{out} = \alpha R \, I_{EE} \tanh\left(\frac{V_{in}}{2V_T}\right)$$



(a) (b)

- Odd symmetry
- Differential or "balanced"

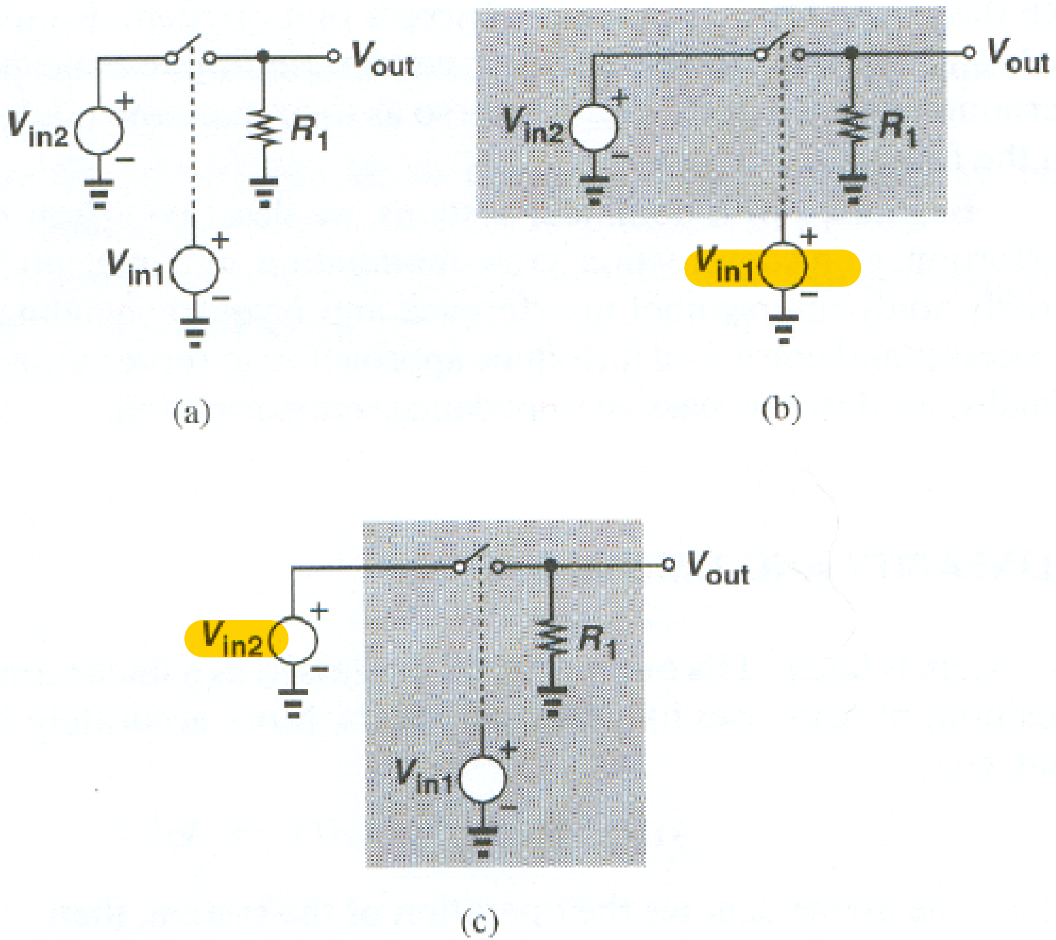
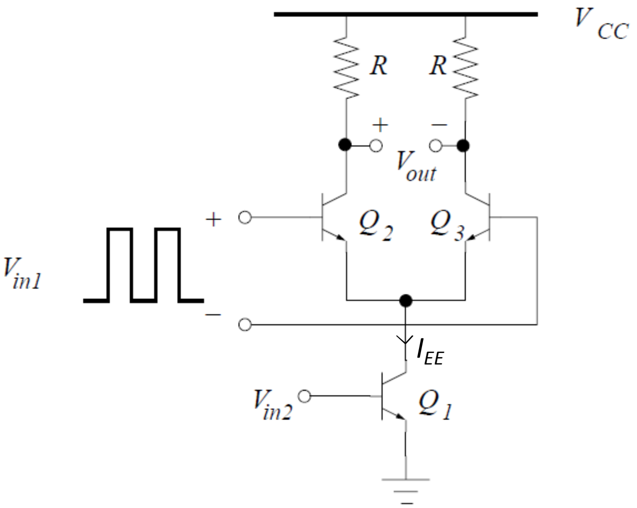
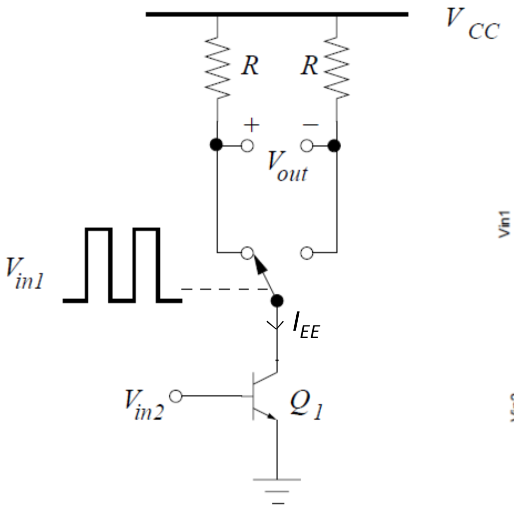
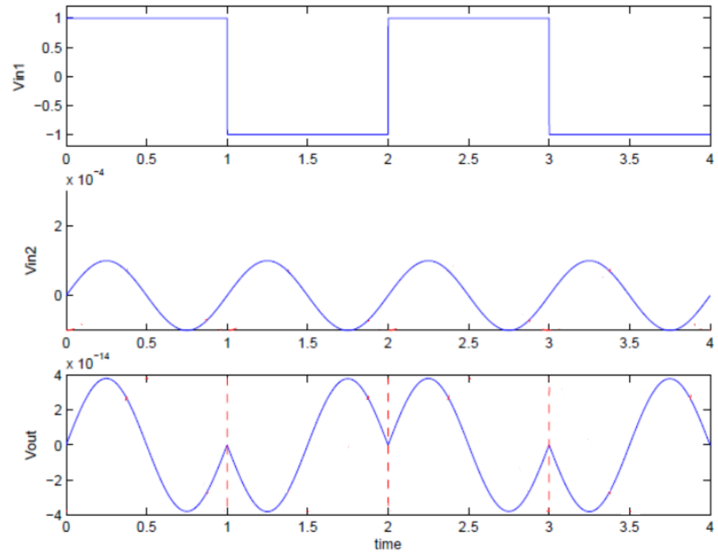


Figure 2.1 (a) Simple switching circuit, (b) nonlinear time-variant system, (c) linear time-variant system.

Implementation of a Mixer with a Differential Pair







$$V_{out} \approx \alpha R I_{EE} \left[\exp\left(\frac{V_{in2}(t)}{2V_T} \right) \right] sign[V_{in1}(t)]$$

Effect of Non-Linearity: Harmonics

Consider a memoryless system given by :

$$y(t) = \alpha_0 + \alpha_1 x(t) + \alpha_2 x(t)^2 + \alpha_3 x(t)^3$$

For $x(t) = A_{in} \cos \omega t$ the output $y(t)$ is :

$$y(t) = \alpha_0 + \frac{\alpha_2 A_{in}^2}{2} + \left(\alpha_1 A_{in} + \frac{3\alpha_3 A_{in}^3}{4} \right) \cos \omega t + \frac{\alpha_2 A_{in}^2}{2} \cos 2\omega t + \frac{\alpha_3 A_{in}^3}{4} \cos 3\omega t$$

Harmonics are generated.

At the fundamental frequency the output amplitude is :

$$A_{out} = \alpha_1 A_{in} + \frac{3\alpha_3 A_{in}^3}{4}$$

and the gain at the fundamental frequency varies (decreases since α_3 is usually negative) with the input signal amplitude :

$$G(A_{in}) = \left| \alpha_1 + \frac{3\alpha_3 A_{in}^2}{4} \right|$$

Gain Compression

The 1 dB compression point is defined as the point $(A_{in,1dB}, A_{out,1dB})$ where the gain $G(A_{in})$ drops by 1 dB :

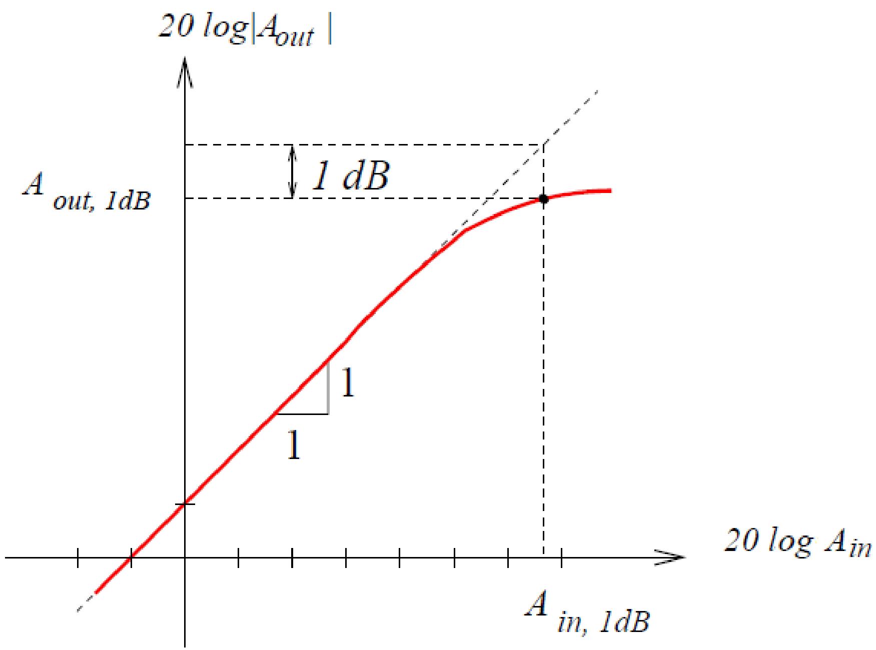
$$20\ \log\left[G(A_{in,1dB})\right] = 20\log[\alpha_1] - 1dB$$

$$\Rightarrow 20\log\left(\alpha_1 + \frac{3}{4}\alpha_3 A_{in,1dB}^2\right) = 20\log[\alpha_1] - 20\log 10^{(1/20)} = 20\log\left(\frac{\alpha_1}{10^{(1/20)}}\right)$$

$$\Rightarrow \alpha_1 + \frac{3}{4}\alpha_3 A_{in,1dB}^2 = \frac{\alpha_1}{10^{(1/20)}} \Rightarrow \frac{3}{4}\alpha_3 A_{in,1dB}^2 = -0.1087\alpha_1$$

$$\Rightarrow A_{in,1dB}^2 = -0.145\frac{\alpha_1}{\alpha_3}\left[\because \alpha_1 \text{ and } \alpha_3 \text{ are of opposite signs}\right]$$

$$\Rightarrow A_{in,1dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}$$

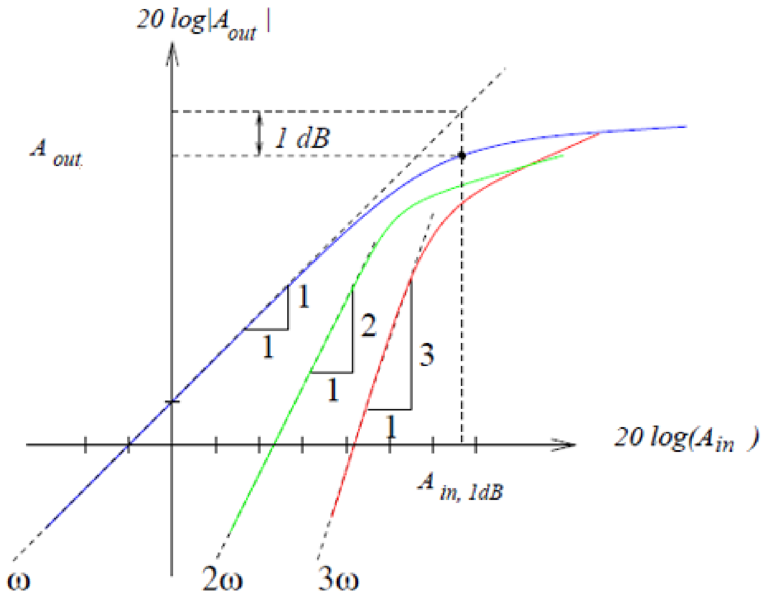


The input amplitude at the 1dB compression point is then: $A_{in,1dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}$

Harmonics

$$A_{out}(2\omega)\big|_{dB} = 20\log\left[A_{out}(2\omega)\right] = 20\log\left[\frac{\alpha_2 A_{in}^2}{2}\right] = \frac{\alpha_2}{2}\bigg|_{dB} + 2 * A_{in}\big|_{dB}$$

$$A_{out}(3\omega)\big|_{dB} = 20\log\left[A_{out}(3\omega)\right] = 20\log\left[\frac{\alpha_3 A_{in}^3}{4}\right] = \frac{\alpha_3}{4}\bigg|_{dB} + 3 * A_{in}\big|_{dB}$$



Desensitization and Blocking

If at the input we have a weak desired signal (A_1, ω_1) in the presence of a strong interferer (A_2, ω_2) with $\omega_1 \neq \omega_2$:

$$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$$

high

the output y(t) is then :

$$y(t) = \left(\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2 \right) \cos \omega_1 t + ...$$

$$\approx \left(\alpha_1 + \frac{3}{2}\alpha_3 A_2^2 \right) A_1 \cos \omega_1 t + ... \text{ for } A_1 << A_2.$$

For α_3 negative, the strong interferer signal (A_2, ω_2) can completely cancel or block the desired weak signal (A_1, ω_1) :

$$\alpha_1 + \frac{3}{2}\alpha_3 A_2^2 = 0$$

A receiver may need to withstand blocking signal 60 to 70 dB greater.

Cross Modulation

If together with the weak desired signal (A_1, ω_1) we have a strong interferer which is modulated at the input (A_2, ω_2)

$$x(t) = A_1 \cos \omega_1 t + A_2 (1 + m \cos \omega_m t) \cos \omega_2 t$$

high

the output y(t) is then :

$$y(t) = \left[\alpha_1 + \frac{3}{2} \alpha_3 A_2^2 (1 + m \cos \omega_m t)^2 \right] A_1 \cos \omega_1 t + \ldots.$$

The desired signal (A_1, ω_1) contains at the output the modulation of the interferer signal (A_2, ω_2).

Intermodulation

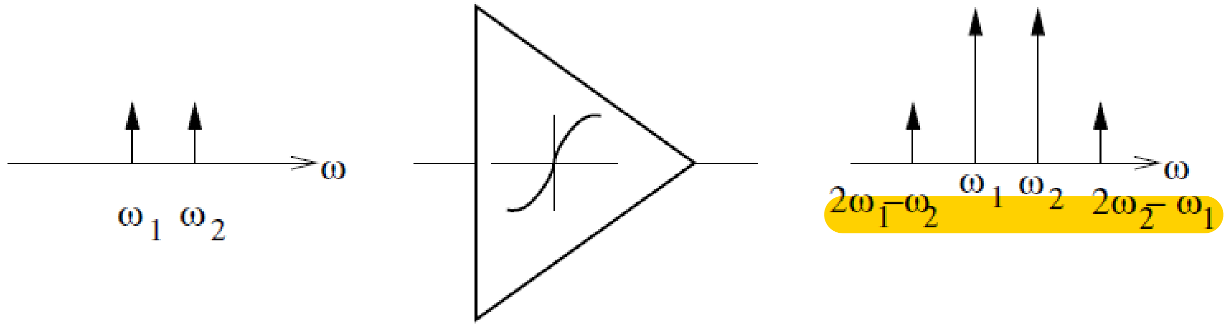
If at the input we have two signals (A_1, ω_1) and (A_2, ω_2)

$$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$$

the output contains terms of frequencies $n\omega_1 + m\omega_2$. Undesired terms appear at $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$ frequencies:

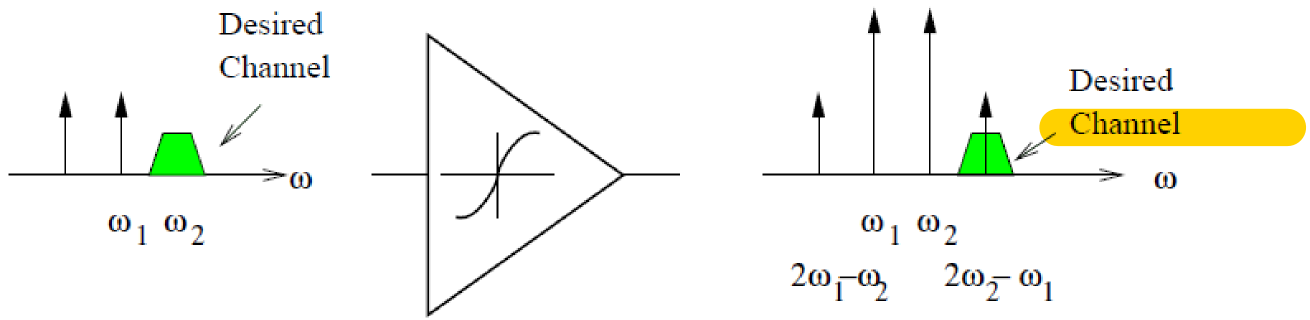
$$y = (\alpha_1 A_1 + \ldots) \cos \omega_1 t + (\alpha_1 A_2 + \ldots) \cos \omega_2 t$$

$$+ \frac{3}{4} \alpha_3 A_1^2 A_2 \cos(2\omega_1 - \omega_2) + \frac{3}{4} \alpha_3 A_1 A_2^2 \cos(2\omega_2 - \omega_1)$$

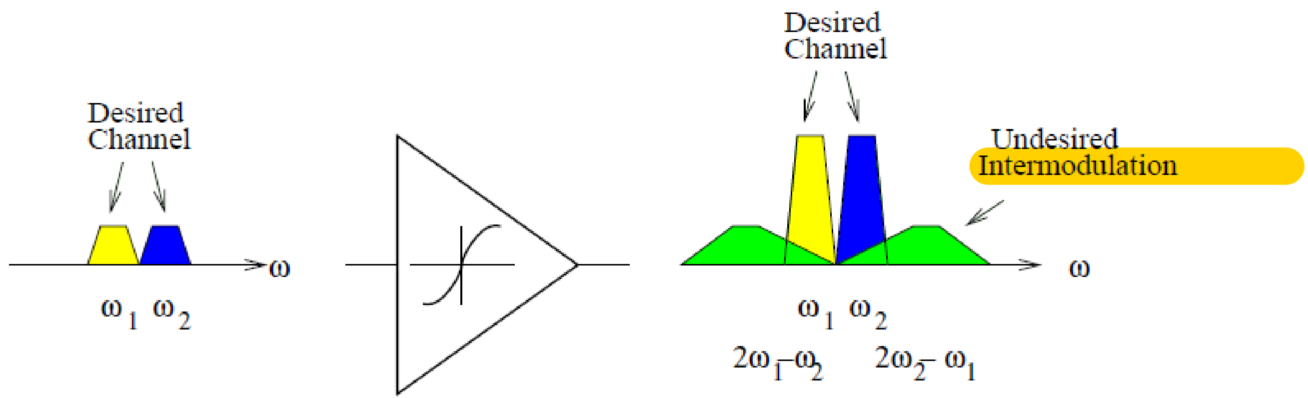


Undesirable Impacts of Intermodulation

Scenario 1: Two strong interferers perturbs a weak desired channel (LNA)



Scenario 2: Two channels blending (PA)



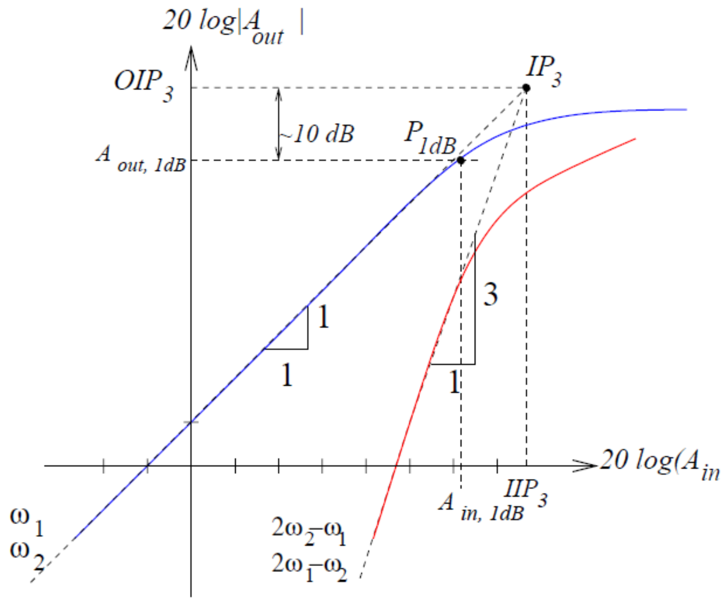
IP3: 3rd Order Intercept Point

That i/p power where the o/p power of fundamental = o/p power of 3rd order IM products

$$\alpha_1 A_{in,IP3} + \frac{3}{4}\alpha_3 A_{in,IP3}^3 = 0 \Rightarrow A_{in,IP3}^2 = -\frac{4}{3}\frac{\alpha_1}{\alpha_3} (\because \alpha_1 \text{ and } \alpha_3 \text{ are of opposite signs})$$

$$\Rightarrow A_{in,IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$$

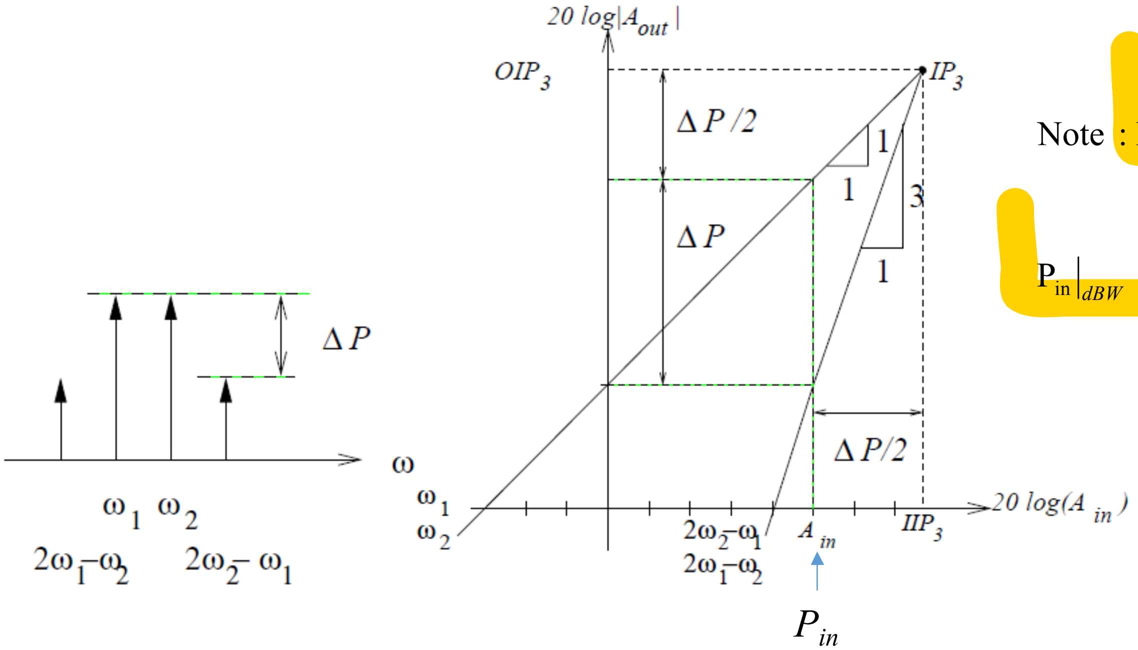
$$\frac{A_{out,IP3}}{A_{out,1dB}} \approx \frac{A_{in,IP3}}{A_{in,1dB}} = \sqrt{\frac{4/3}{0.145}} \approx 10\ dB$$



Quick IP3 Calculation

Let P be the IMD (intermodu lation distortion measured in dB or - dBc). From the graph below :

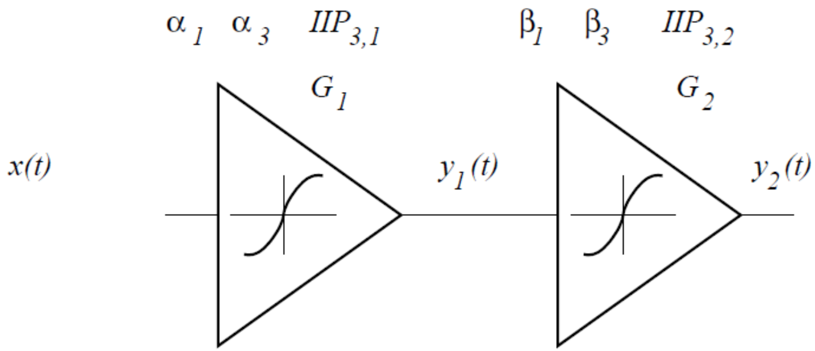
$$IIP3\big|_{dBV} = \Delta P\big|_{dB}/2 + A_{in}\big|_{dBV} \qquad IIP3\big|_{dBm} = \Delta P\big|_{dB}/2 + P_{in}\big|_{dBm}$$



Note : $P_{in}\big|_{dBm} = 10\log P_{in}\big|_{mW} = 10\log\left[\dfrac{\left(A_{in,rms}\big|_V\right)^2}{50\ \Omega\times 10^{-3}}\right] = A_{in,rms}\big|_{dBV} + 13\,dB$

$P_{in}\big|_{dBW} = 10\log P_{in}\big|_{W} = 10\log\left[\dfrac{\left(A_{in,rms}\big|_V\right)^2}{50\ \Omega}\right] = A_{in,rms}\big|_{dBV} - 17\,dB$

IP3 of Cascaded Non-Linear Stages



$$y_1 = \alpha_1 x + \alpha_3 x^3 \qquad y_2 = \beta_1 y_1 + \beta_2 y_1^3 = \alpha_1 \beta_1 x + \left(\alpha_3 \beta_1 + \alpha_1^3 \beta_3\right) x^3 + \ldots.$$

The total IP3 is then :

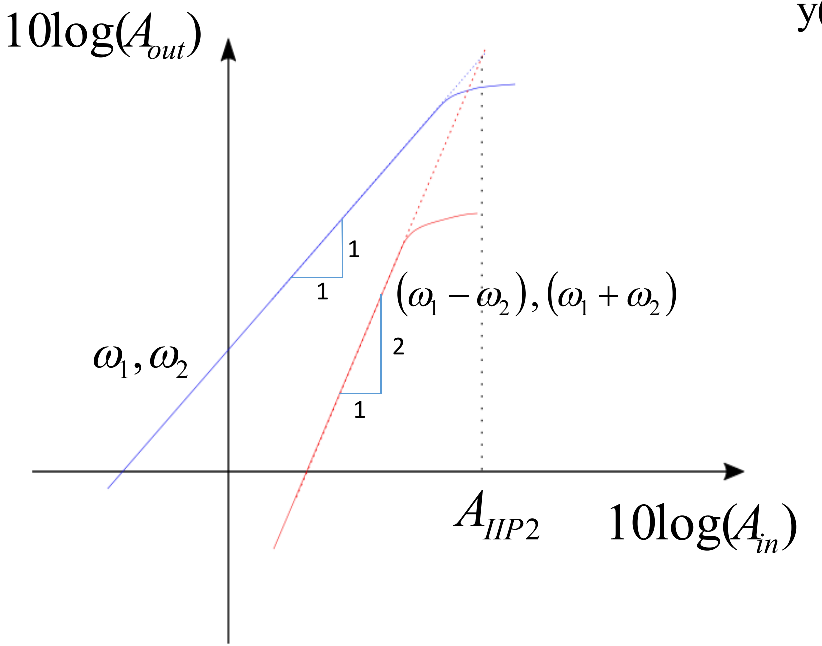
$$\frac{1}{A^2_{\text{in, IP3, total}}} = \frac{3}{4}\left|\frac{\left(\alpha_3\beta_1 + \alpha_1^3\beta_3\right)}{\alpha_1\beta_1}\right| < \frac{3}{4}\frac{|\alpha_3\beta_1| + |\alpha_1^3\beta_3|}{|\alpha_1\beta_1|} = \frac{1}{A^2_{in,IP3,1}} + \frac{\alpha_1^2}{A^2_{in,IP3,2}}$$

In terms of power P $= A^2_{\text{in}}/(2R)$ and power gain G $= \alpha^2$ this is written (using also $P_{\text{OIP3}} = P_{\text{IIP3}} * G$ and $G_{\text{total}} = G_1 G_2$)

$$\frac{1}{P_{IIP3,total}} = \frac{1}{P_{IIP3,1}} + \frac{G_1}{P_{IIP3,2}} \quad \text{or} \quad \frac{1}{P_{OIP3,total}} = \frac{1}{G_2 P_{OIP3,1}} + \frac{1}{P_{OIP3,2}}$$

α_2 and β_2 are neglected because they contribute via higher harmonics which are filtered out.

IP2: 2nd Order Intercept Point



Let $x(t) = A\cos\omega_1 t + A\cos\omega_2 t$

$$y(t) = \alpha_1 x(t) + \alpha_2 x^2(t)$$

$$= \alpha_1 A(\cos\omega_1 t + \cos\omega_2 t) + \alpha_2 A^2 \cos(\omega_1+\omega_2)t$$

$$+ \alpha_2 A^2 \cos(\omega_1-\omega_2)t + \frac{\alpha_2 A^2}{2}\cos(2\omega_1)t + \frac{\alpha_2 A^2}{2}\cos(2\omega_2)t$$

Power at these harmonics is half that at $(\omega_1\text{-}\omega_2)$ or $(\omega_1\text{-}\omega_2)$

$$\alpha_1 A_{IIP2} = \alpha_2 A_{IIP2}^2$$

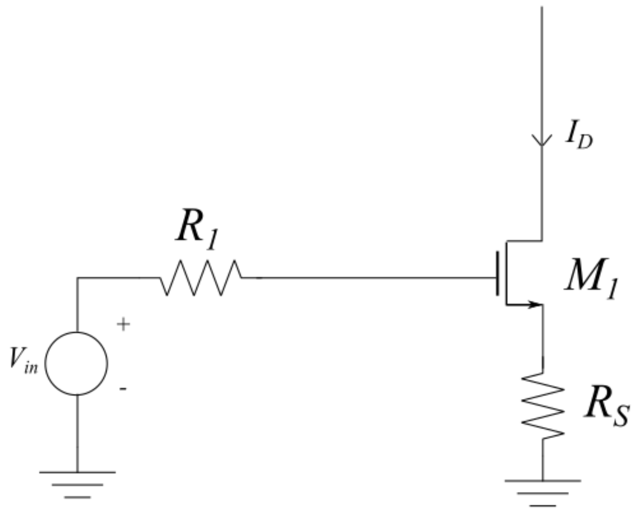
$$\Rightarrow A_{IIP2} = \frac{\alpha_1}{\alpha_2}$$

In practice, the $(\omega_1 - \omega_2)$ terms may originate from some other part of the circuit (feed through, leakage). In that case the eqn above is modified as follows,

$$\alpha_1 A_{IIP2} = \underbrace{k}_{\substack{coupling\\factor}} \alpha_2 A_{IIP2}^2$$

$$\Rightarrow A_{IIP2} = \frac{\alpha_1}{k\alpha_2}$$

Linearity of degenerated CS stage



$$y = \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3$$

$$\alpha_1 = \left.\frac{\partial y}{\partial x}\right|_{x=0}, \alpha_2 = \left.\frac{1}{2}\frac{\partial^2 y}{\partial x^2}\right|_{x=0}, \alpha_3 = \left.\frac{1}{6}\frac{\partial^3 y}{\partial x^3}\right|_{x=0}$$

$$I_D = K(V_{GS} - V_{TH})^2,$$

$$V_{GS} = V_{in} - R_S I_D \Rightarrow I_D = K(V_{in} - R_S I_D - V_{TH})^2$$

$$\Rightarrow \frac{\partial I_D}{\partial V_{in}} = \underbrace{2K(V_{in} - R_S I_D - V_{TH})}_{g_m}\left(1 - R_S \frac{\partial I_D}{\partial V_{in}}\right)$$

$$\Rightarrow \alpha_1 = \left.\frac{\partial I_D}{\partial V_{in}}\right|_{V_{in}=0} = \frac{g_m}{1 + g_m R_S}, \text{ where } g_m = \frac{\partial I_D}{\partial V_{GS}} = 2K(V_{GS} - V_{TH})$$

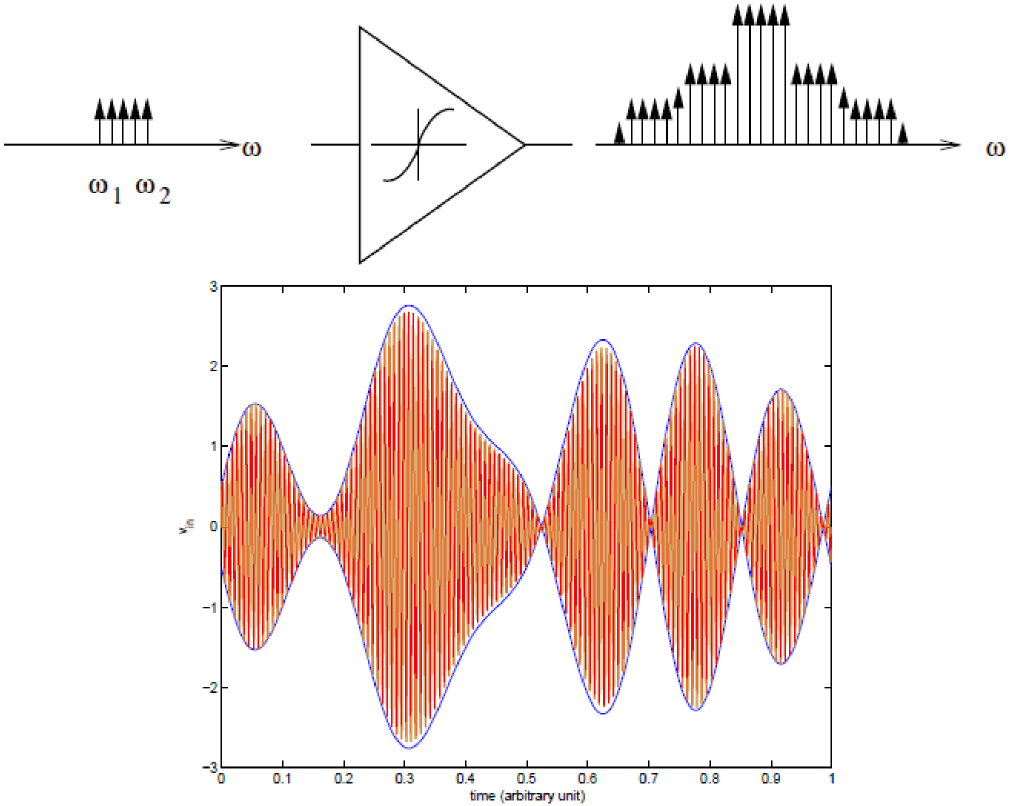
Similarly, we can find,

$$\alpha_2 = \left.\frac{1}{2}\frac{\partial^2 I_D}{\partial V_{in}^2}\right|_{V_{in}=0} = \frac{K}{(1 + g_m R_S)^2}, \alpha_3 = \left.\frac{1}{6}\frac{\partial^2 I_D}{\partial V_{in}^2}\right|_{V_{in}=0} = \frac{-2K^2 R_S}{(1 + g_m R_S)^5}$$

From these equations, IIP3 and IIP2 can be obtained as

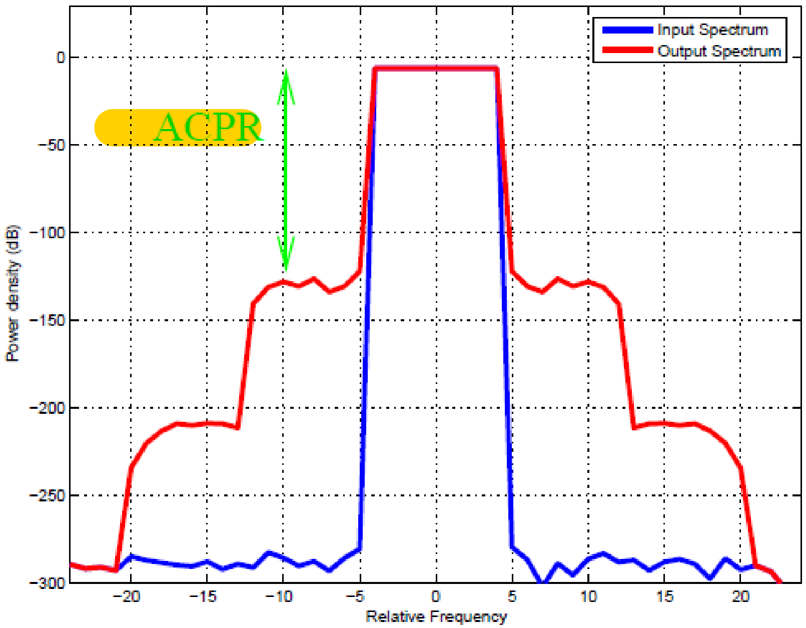
$$A_{IIP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|} = \frac{(1 + g_m R_S)^2}{K}\sqrt{\frac{2}{3}\frac{g_m}{R_S}}, \quad A_{IIP2} = \frac{\alpha_1}{\alpha_2} = \frac{g_m}{K}(1 + g_m R_S)$$

Multitone excitation: Multisine

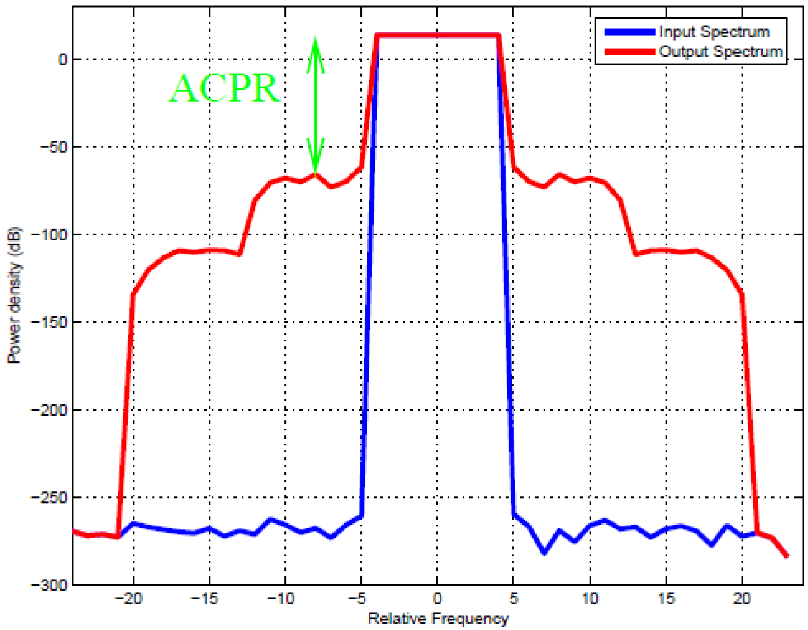


Input and Output Multisine

Spectral Regrowth



Low input power



High input power

Intersymbol Interference

The problem:
Distortion of a signal can also arise from insufficient bandwidth. For digital signals this leads to intersymbol interference" (ISI) (see next slide).

The solutions:
- Equalization in the receiver
- Pulse shaping (Nyquist signaling)

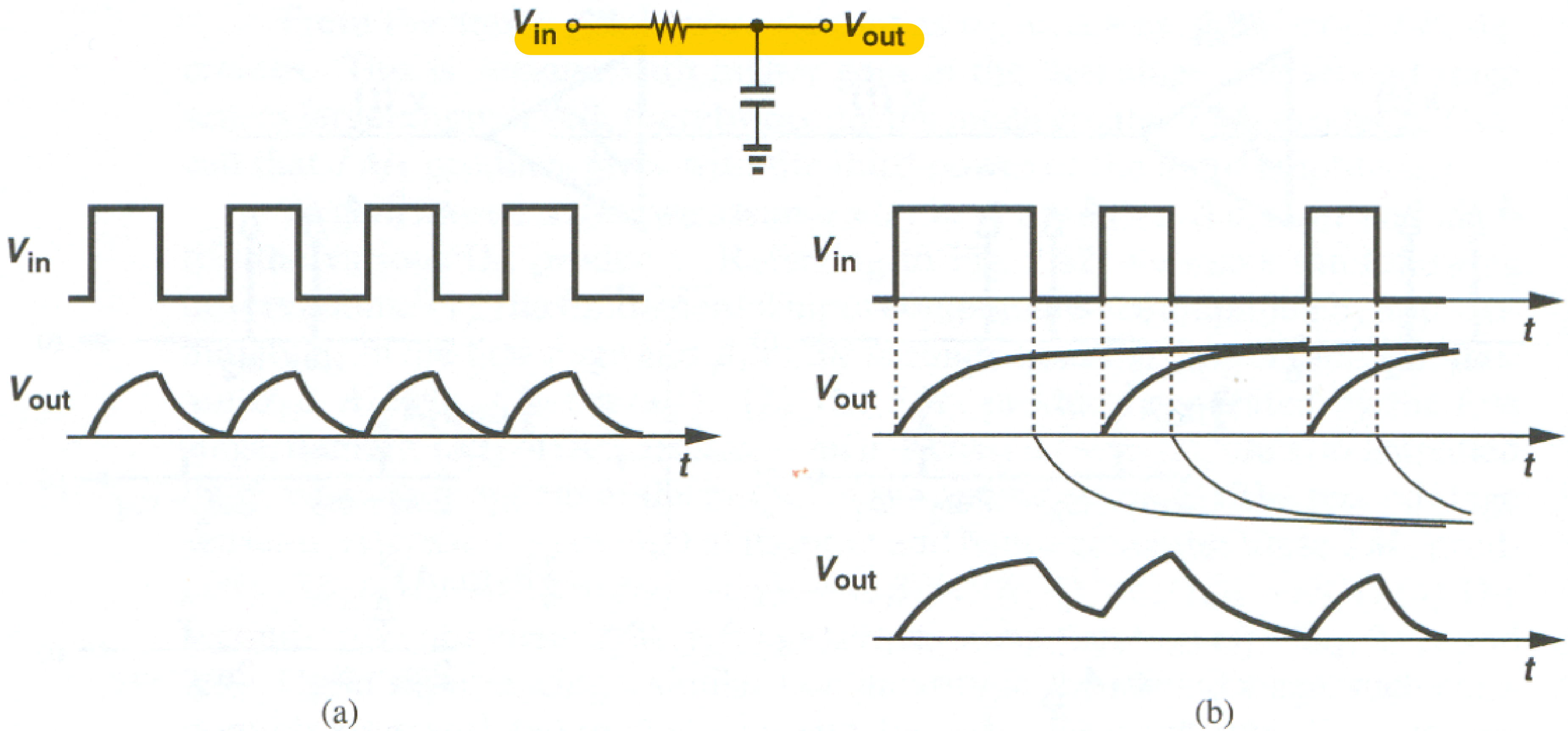


Figure 2.13 Response of a low-pass filter to (a) a periodic square wave, (b) a random sequence of ONEs and ZEROs.

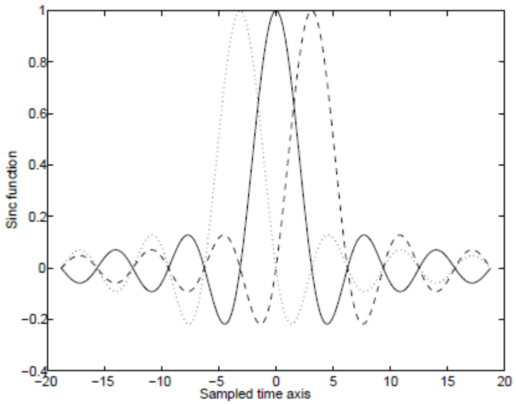
Nyquist Signaling

In Nyquist signaling, each pulse p(t) is allowed to overlap with past and future pulses but the shape is selected such that the pulse is zero at the sampled time t = kT_S so that no ISI exists:

$$p(kT_S) = \begin{cases} 1 & \text{if } k = 0 \\ 0 & \text{if } k \neq 0 \end{cases}$$

Example #1: sinc function (rectangular spectrum calls for complex filter):

$$p(t) = \mathrm{sinc}(t - kT_S)$$



Example #2: raised-cosine pulse (smoother but truncated spectrum):

$$p(t) = \frac{\sin(\pi t/T_S)}{\pi t/T_S} \frac{\cos(\pi \alpha t / T_S)}{1 - 4\alpha^2 t^2 / T_S^2}$$

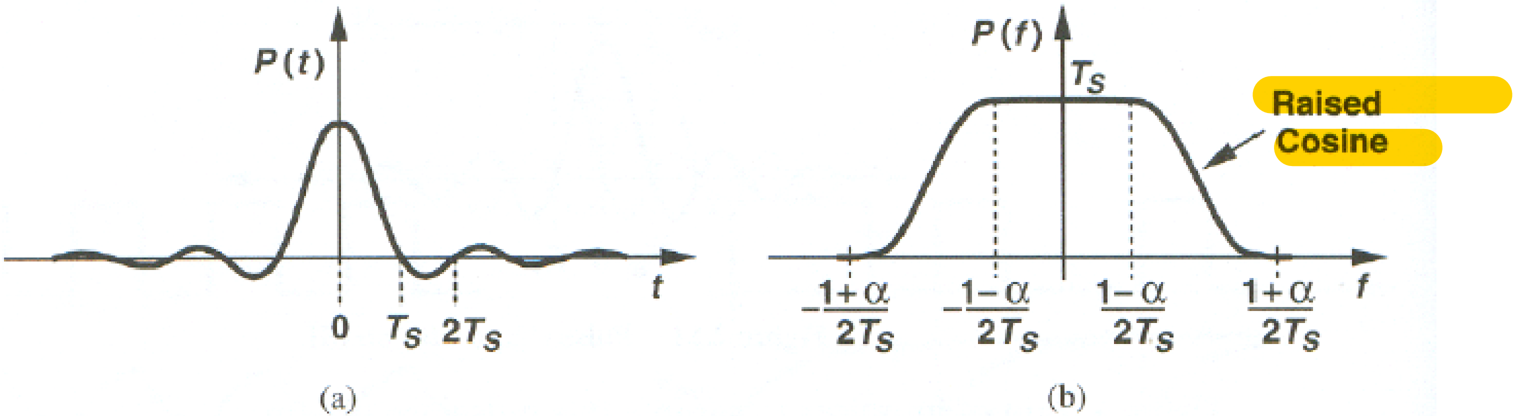
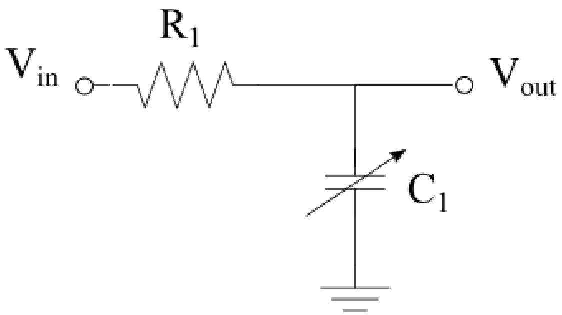


Figure 2.16 Raised-cosine pulse: (a) in time domain, (b) in frequency domain.

and

$$
\begin{aligned}
P(f) &= T_S \quad 0 < |f| < \frac{1-\alpha}{2T_S} \\
&= \frac{T_S}{2}\left[1 + \cos\frac{\pi T_S}{\alpha}\left(|f| - \frac{1-\alpha}{2T_S}\right)\right] \quad \frac{1-\alpha}{2T_S} < |f| < \frac{1+\alpha}{2T_S} \\
&= 0 \quad |f| > \frac{1+\alpha}{2T_S},
\end{aligned}
\tag{2.54}
$$

Dynamic Nonlinearity (α's change dynamically in the circuit)



$$V_{out}(t) = \sum_{n=1}^{\infty} a_n \cos(n\omega_1 t + \theta_n) + \sum_{n=1}^{\infty} b_n \cos(n\omega_2 t + \phi_n)$$

$$+ \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} c_{m,n} \cos(n\omega_1 t + m\omega_2 t + \phi_{n,m})$$

Assuming weak non linearity,

$$V_{out}(t) = a_1 \cos(\omega_1 t + \phi_1) + b_1 \cos(\omega_2 t + \phi_2) + c_1 \cos[(\omega_1 + \omega_2)t + \phi_3]$$

$$+ c_2 \cos[(\omega_1 - \omega_2)t + \phi_4]$$

$$C_1 = C_0 (1 + \alpha V_{out})$$

Only these non linear effects have been considered

$$R_1 C_0 (1 + \alpha V_{out}) \frac{dV_{out}}{dt} + V_{out} = V_{in}$$

Substitute here and equate LHS and RHS frequency wise to find a's, b's and c's.

This method is known as harmonic balance

Dynamic Nonlinearity

- The number of equations will equal the number of unknowns.
- Problem is that there are too many equations to solve.
- A better method is using Volterra series.