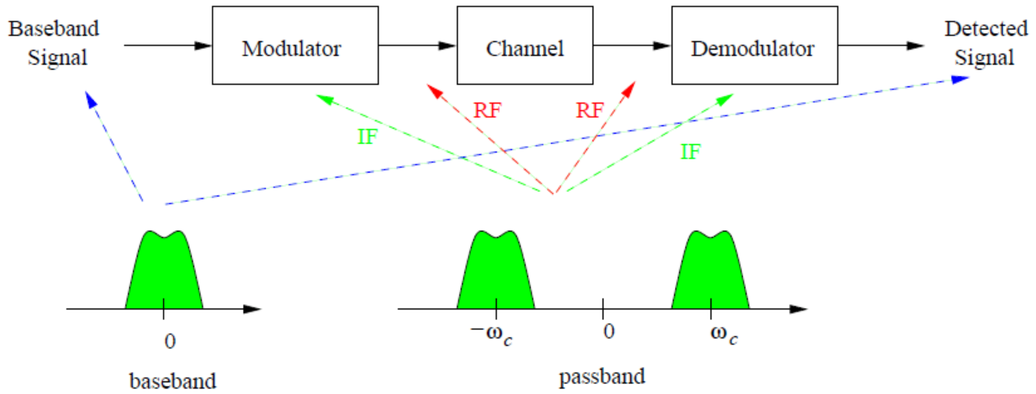
Modulation and Detection

- Analog and digital modulation
- Modulation is the conversion from baseband to passband
- Vary the amplitude or/and phase of the carrier : $x(t) = a(t)\ \cos[\omega_c t + \theta(t)]$
- Demodulation is the reverse process



Note : $\omega_c t + \theta(t)$ is the total phase and θ(t) the excess phase.

$\omega_c + \dfrac{d\theta(t)}{dt}$ is the total frequency and $\dfrac{d\theta(t)}{dt}$ is the excess frequency.

Important Attributes of Modulation

Modem (Modulation + Demodulation) are compared on the basis of :

Signal Quality (SNR)

· Channel loss

· Tolerance to noise

· Tolerance to interferers

Metrics for assessing a modulation scheme

· Spectral efficiency : bandwith of modulated signal compared to baseband

· Power efficiency : linear power amplifier are less efficient

Channel will be assumed to be corrupted by additive white Gaussian noise (AWGN).

Amplitude Modulation

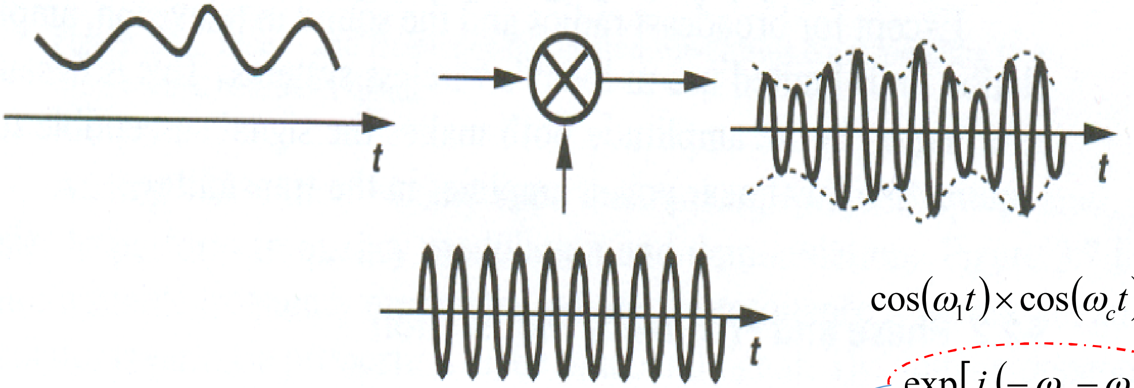
Types of AM modulation :

- Single sideband suppressed carrier (SSB)
- Double sideband suppressed carrier (DSB - SC)
- Double sideband large carrier (DSB - LC)

DSB - LC AM modulation :

$x_{AM} = A_c\,[1 + m\,x_{BB}(t)]\cos\omega_c t$ with m the amplitude modulation index

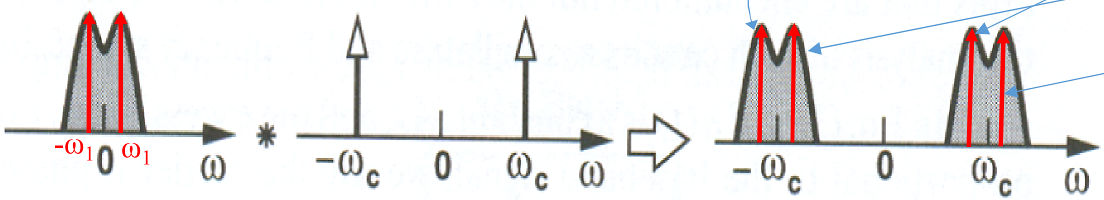
Carrier not suppressed hence large carrier AM



(a)

$$\cos(\omega_1 t)\times\cos(\omega_c t)=\left[\frac{\exp(j\omega_1 t)+\exp(-j\omega_1 t)}{2}\right]\times\left[\frac{\exp(j\omega_c t)+\exp(-j\omega_c t)}{2}\right]$$

$$=\frac{\exp[j(-\omega_1-\omega_c)t]+\exp[j(-\omega_1+\omega_c)t]+\exp[j(\omega_1-\omega_c)t]+\exp[j(\omega_1+\omega_c)t]}{4}$$



(b)

Figure 3.4 Amplitude modulation in (a) time domain, (b) frequency domain.

AM Detectors

Type of AM Dectectors:

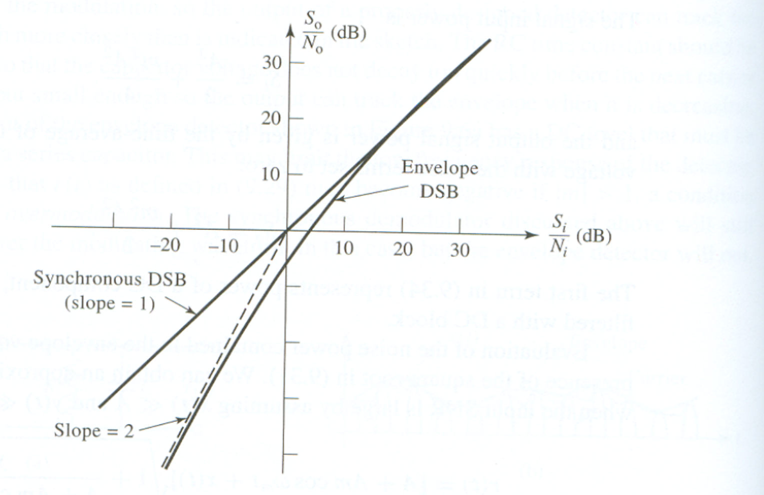
- *Coherent detection:* (requires a phase lock of the LO on the carrier)

$$\frac{SNR_{out}}{SNR_{in}} = \frac{2m^2\overline{x_{BB}^2}}{1+2m^2\overline{x_{BB}^2}} < \frac{2}{3}$$

$$SNR_{out} \approx SNR_{in}\, 2m^2\overline{x_{BB}^2} = \frac{A_c^2\sqrt{2}}{2N_0B} 2m^2\overline{x_{BB}^2} \quad \text{for} \quad mx_{BB} << 1$$

with $B = 2f_{BB,\,\max}$, $f_{BB,\,\max}$ the bandwidth of $x_{BB}(t)$.

- *Non-coherent detection: peak detector*
  - Peak detector does not work for $1 + mx_{BB} < 0$
  - Coherent detection and non-coherent detection have the same SNR_{out} for $\text{SNR}_{\text{in}} > 15$ dB. Typically 25 dB is targeted for quality AM signals.

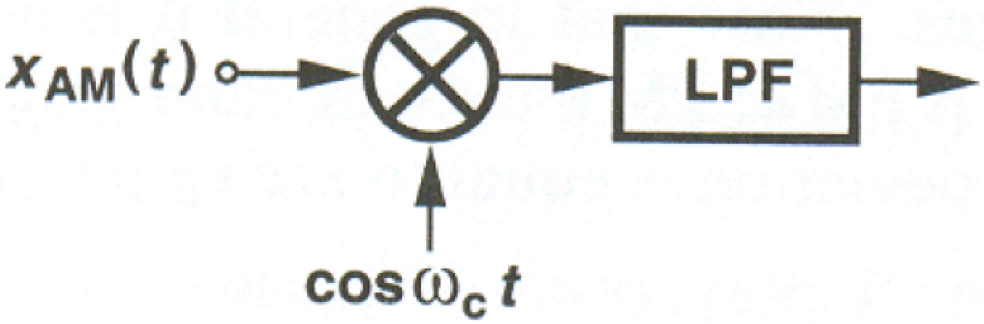


Output versus input SNR for synchronous and envelope detection of a DSB-LC AM signal.

Attributes of AM modulation:

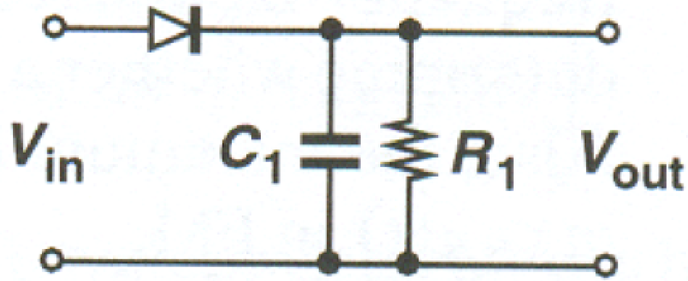
- Succeptible to noise.
- Requires a linear amp in the transmitter.

Coherent



(a)

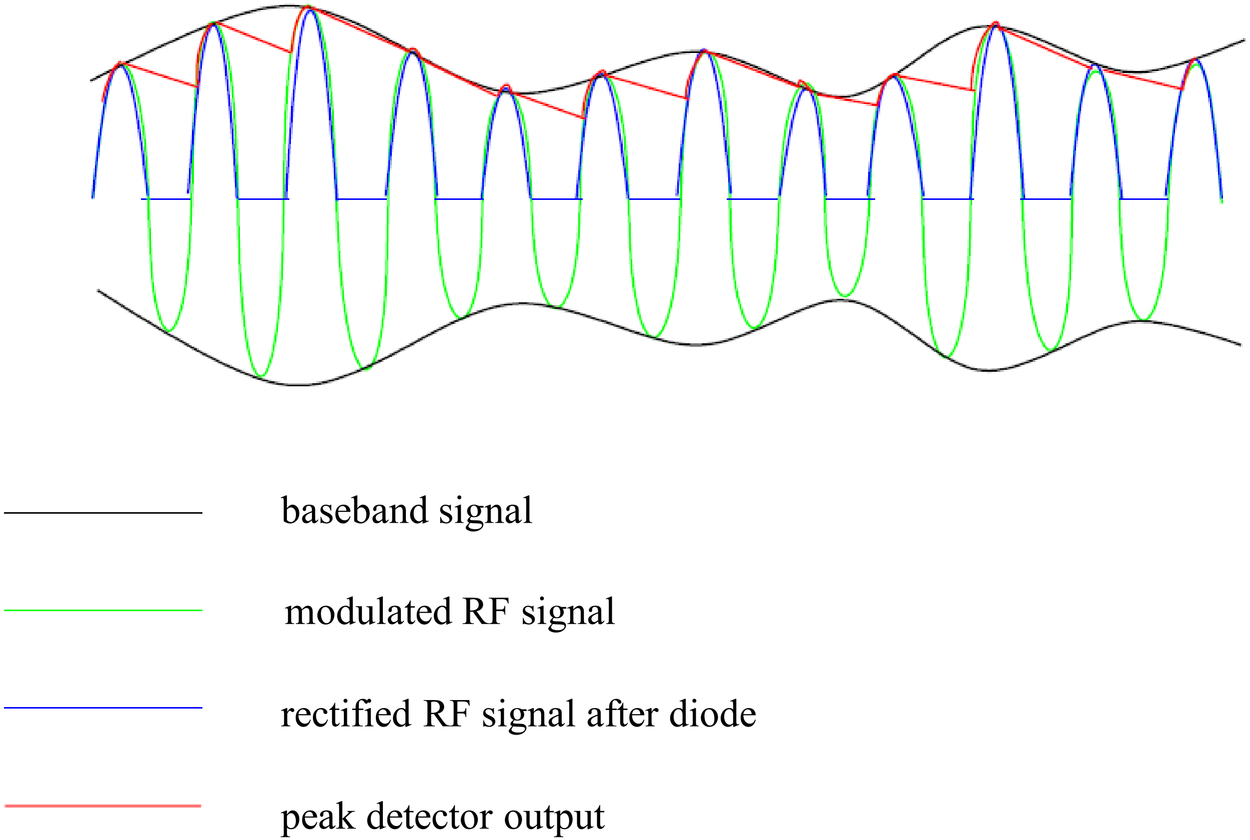
Peak detector



(b)

Figure 3.5 AM detectors.

Peak Detector Waveforms



Phase and Frequency Modulation

Phase modulation :

$x_{PM} = A_c \cos[\omega_c t + m x_{BB}(t)]$ with m the phase modulation coefficient

Frequency modulation :

$x_{FM} = A_c \cos\left[\omega_c t + m\int_{-\infty}^{t} x_{BB}(\tau)d\tau\right]$ with m the frequency modulation coefficient

Note : $\dfrac{d\theta(t)}{dt}$ is proportional to x_{BB}.

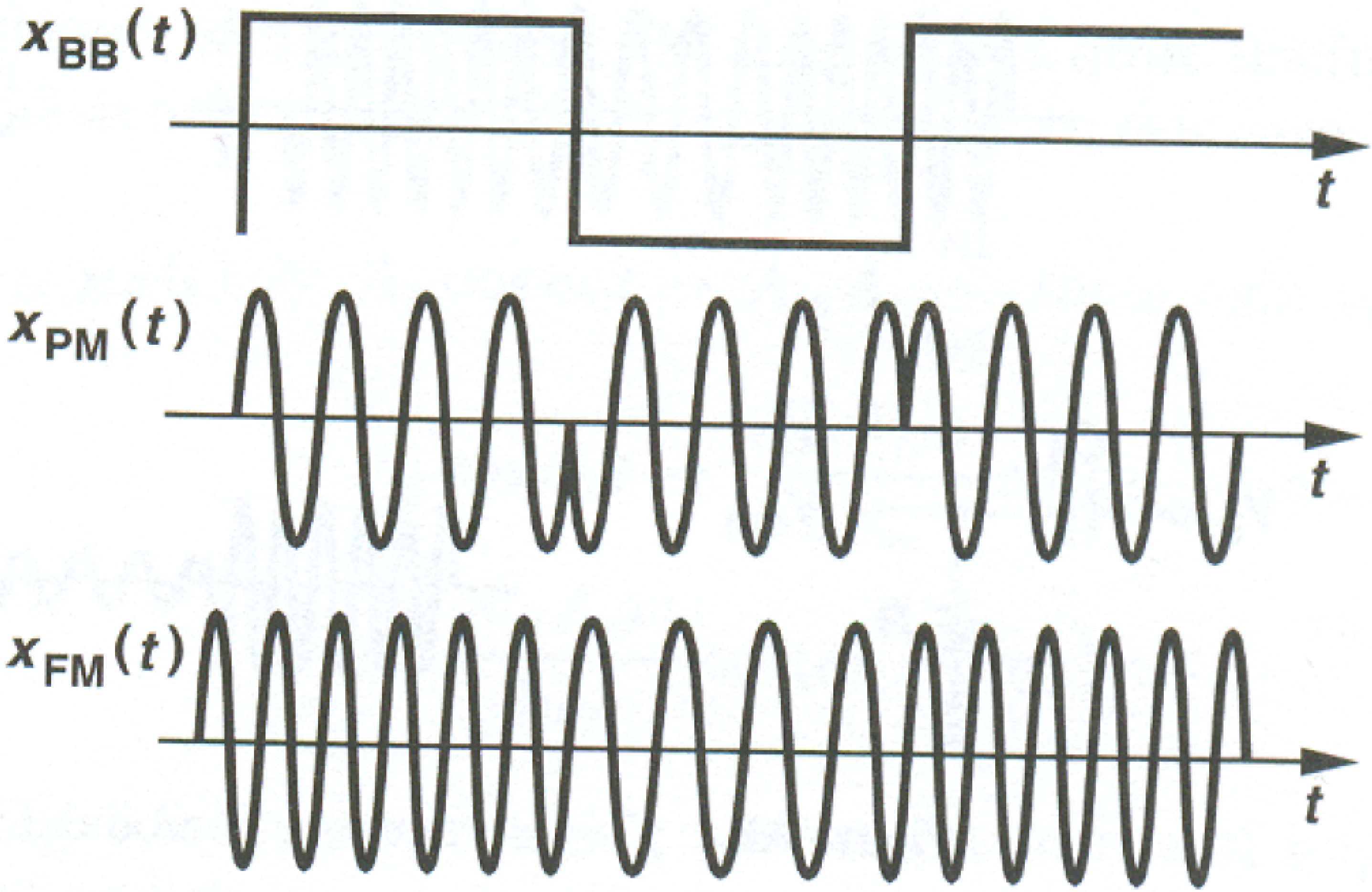


Figure 3.6 Phase- and frequency-modulated waveforms corresponding to a square wave baseband signal.

FM Modulation and Demodulation

Modulation:

A varactor (voltage controlled capacitor) can be used in a LC resonator to tune the frequency realizing a VCO.

Demodulation:

A high - pass RC filter can be used as differentiator to convert FM to AM

$$\mathrm{v}_{\mathrm{out}} \approx A_c R_1 C_1 \left[\omega_c + m x_{BB}(t)\right] \times \sin\left[\omega_c t + \int_{-\infty}^{t} x_{BB}(\tau) d\tau\right]$$

If $\left[\omega_c + m x_{BB}(t)\right]$ always remains positive an envelope detector can then be used to recover the amplitude modulation.

A limiter is required before the differentiator to prevent variation in A_c to corrupt the demodulation.

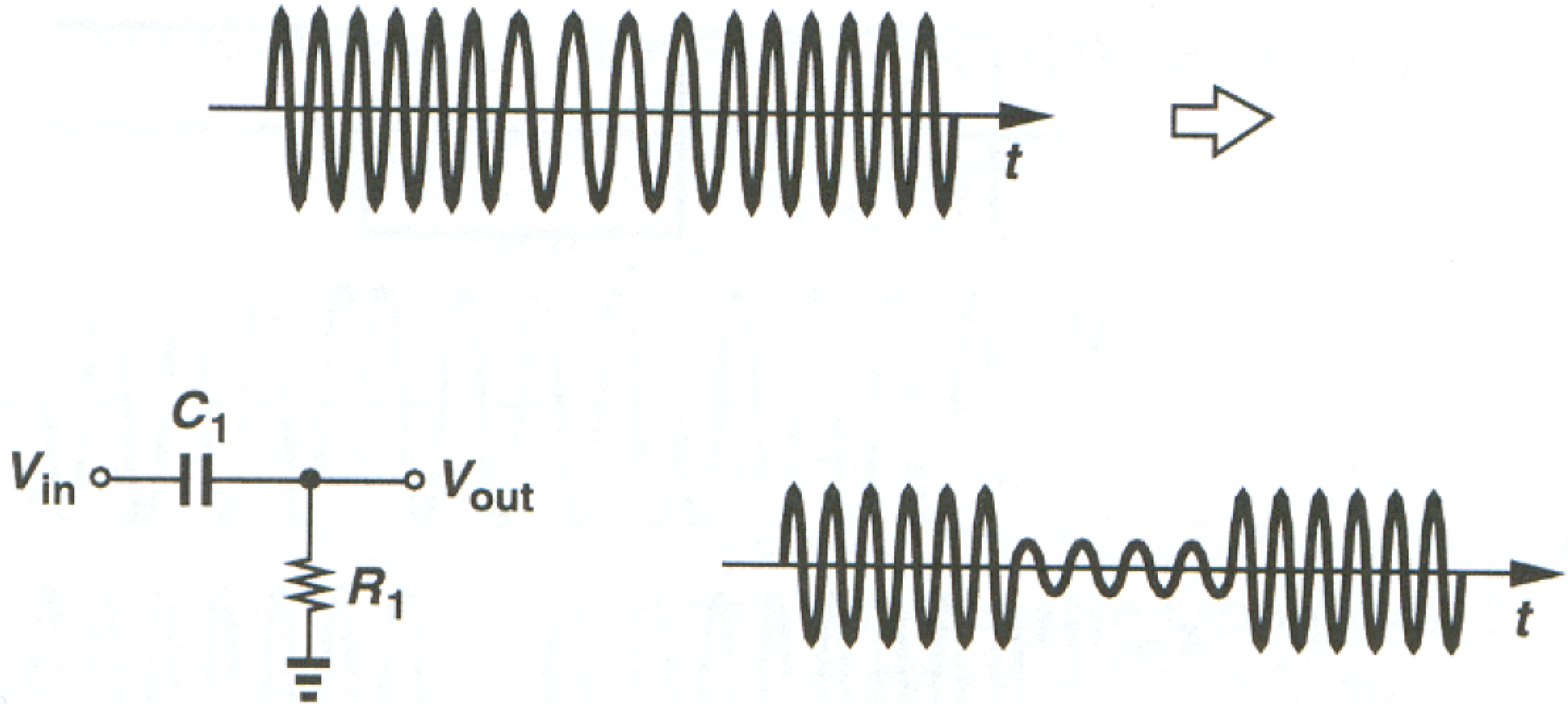


Figure 3.8 Simple frequency demodulator.

Bandwidth and Frequency Dependence

Let the baseband be : $x_{BB}(t) = A_m \cos \omega_m t$

For $mA_m \ll \omega_m$ (Narrow band FM) we have

$$x_{FM}(t) \approx A_c \cos \omega_c t - \frac{mA_m A_c}{2\omega_m} \cos(\omega_c - \omega_m)t + \frac{mA_m A_c}{2\omega_m} \cos(\omega_c + \omega_m)t$$

Bandwidth : $[\omega_c - \omega_m, \omega_c + \omega_m]$ (if ω_m is the maximum frequency of the baseband x_{BB}).

Note the frequency dependence (in ω_m) of the sidebands' amplitude.

For $mA_m \geq \omega_m$ (Broad band FM) we have

$$x_{FM}(t) = A_c \sum_{n=-\infty}^{n=\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t \quad \text{with } \beta = \frac{mA_m}{\omega_m} \text{ (modulation index)}$$

98% Power Bandwidth : $(2\beta + 1)B_{BB}$ with $B_{BB} = \omega_m / 2\pi$ the baseband bandwidth.

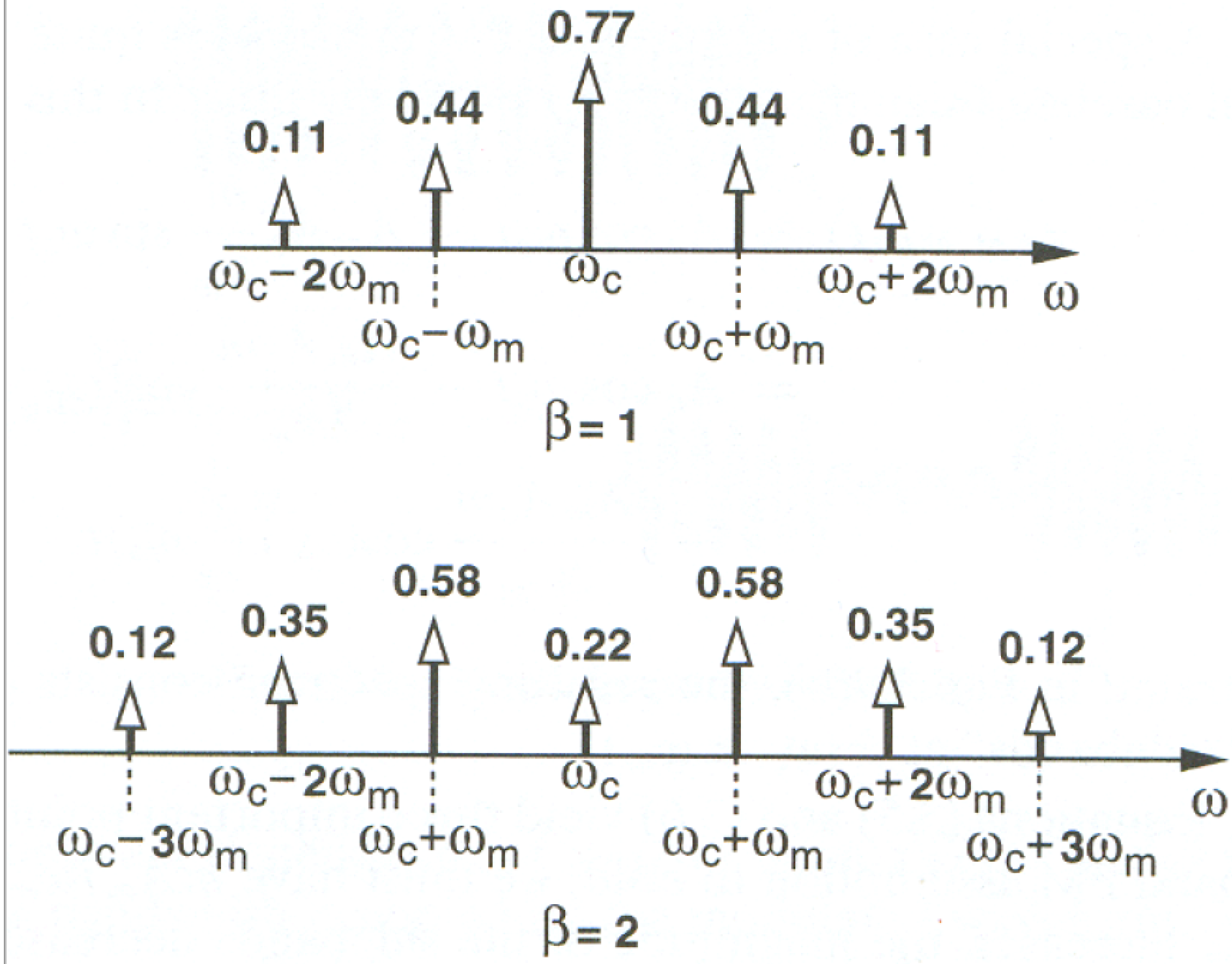


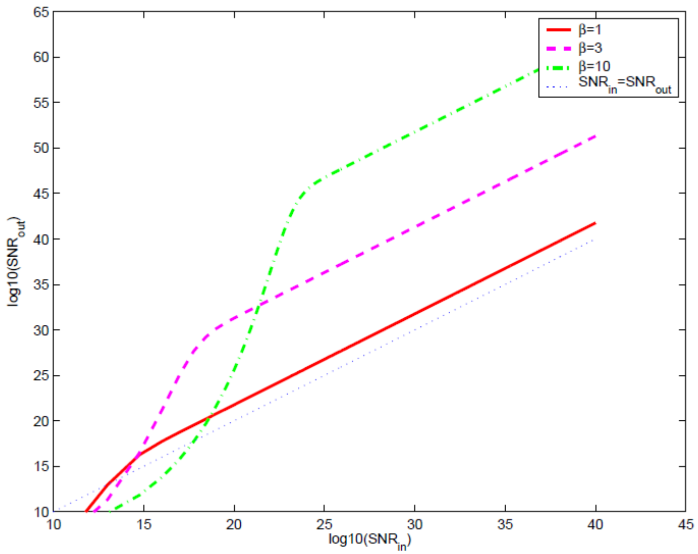
Figure 3.10 FM spectra for two different values of β.

SNR for FM Modulation

$$\frac{SNR_{out}}{SNR_{in}} = \frac{3}{2}\beta^2$$

In book $\frac{SNR_{out}}{SNR_{in}}$ given as

$6\beta^2(\beta+1)\frac{\overline{x_{BB}^2(t)}}{V_P^2}$ depending on the detector



Threshold Effect

with V_P the peak value of $x_{BB}(t)$.

See reference in textbook for further (10 - 15 dB) improvemen t using pre - emphasis and de - emphasis filter to compensate for the decrease of the high - frequency base - band components .

FM Modulat ion Attrib utes:

- The performanc e strongly depends on β (trade - off between signal quality and bandwidth efficiency)
- FM signal can be processed by a non - linear amplier (power efficient)

Digital Modulation

The digital counter parts of AM, PM and FM are

- Amplitude shift keying (ASK)
- Phase shift keying (PSK)
- Frequency shift keying (FSK)

Initial observations :

- PSK and FSK nd wider usage than ASK due to higher noise immunity.
- Comparison of digital modems will again be based upon :
  - signal quality
  - spectral efficiency
  - power efficiency
- Signal quality is measured by the bit error rate (BER) dened as the average erroneous bits received at the output divided by the total number of bit received in a unit time.

Performance Advantages of Digital Modulation

1. Digital transmission produces fewer data errors than analog transmission:

   A. Data integrity & noise immunity: Easier to detect and correct information-bearing data errors, since transmitted data is binary (1's & 0's : only two distinct values) .

   B. Error coding is used to detect and correct digital transmission errors.

   C. Regenerative capability: Regenerative digital repeaters placed along the transmission channel can detect a distorted digital signal and retransmit a new, clean digital data signal. These repeaters minimize the accumulation of noise and signal distortion along the transmission channel.

2. Permits higher transmission data rates: Economical to build transmission links of very high bandwidth. Optical fiber designed for digital transmission.

3. Better spectral efficiency: Effective use of limited frequency resources (narrow bandwidth) to send a large amount of data.

4. Security & privacy: Enables encryption algorithms in information-bearing digital bit stream signals. Deters phone cloning and eavesdropping.

5. Easy to multiplex multiple sources of information: Voice, video and data in a single transmission channel, since all signals are made up of 1's and 0's.

6. Easy to integrate computer/communication systems.

7. Digital equipment consumes less DC power in a smaller physical size.

Disadvantages

1. More Bandwidth Needed:

   A. Transmission of digitally encoded analog signals requires significantly more bandwidth than simply transmitting the original analog signal.

2. Circuit complexity:

   A. Analog signals must be converted to digital pulses prior to transmission and converted back to their original analog form at the receiver: Additional encoding/decoding circuitry needed.

3. Synchronization:

   A. Requires precise time synchronization between the clocks in the transmitter and in the receiver.

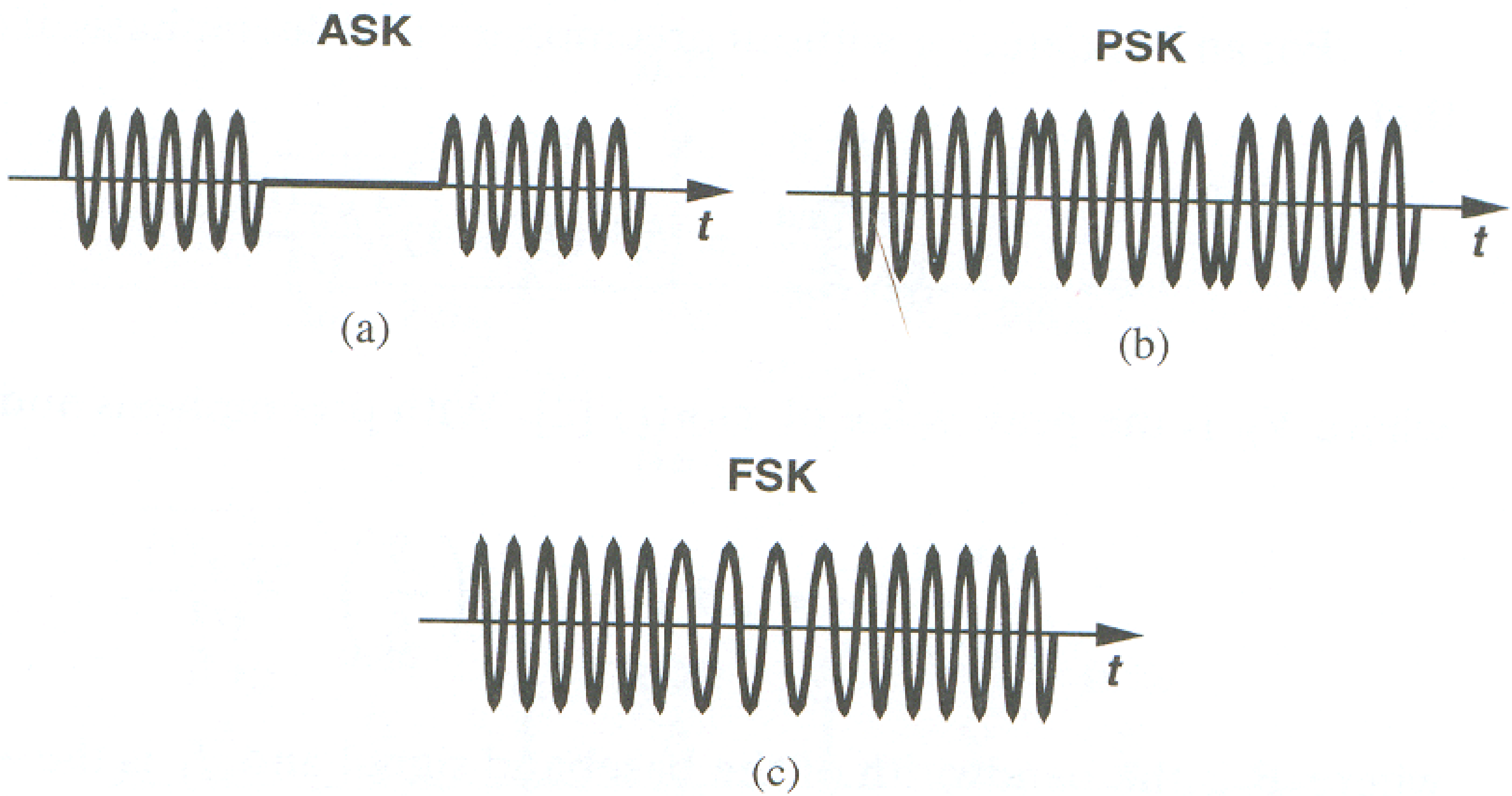


Figure 3.12 (a) Amplitude, (b) phase, and (c) frequency shift keying.

Binary and M-ary Signaling

A binary digital baseband signal x_{BB} is synthesized from the binary data bn using :

$$x_{BB} = \sum_{n} b_n p(t\text{-}nT_b)$$

where $\mathrm{b_n}$ can take either the value 0 and 1 or 1 and - 1 in the interval $[nT_b,\ (n+1)T_b]$

p(t) is the pulse shape (e.g, rectangular pulse).

A M - ary digital baseband signal x_{BB} is synthesized by first converting the binary sequence in M different levels using a D/A converter.

· require higher amplitude resolution in detector

· symbol rate is decreased slightly due to D/A conversion.

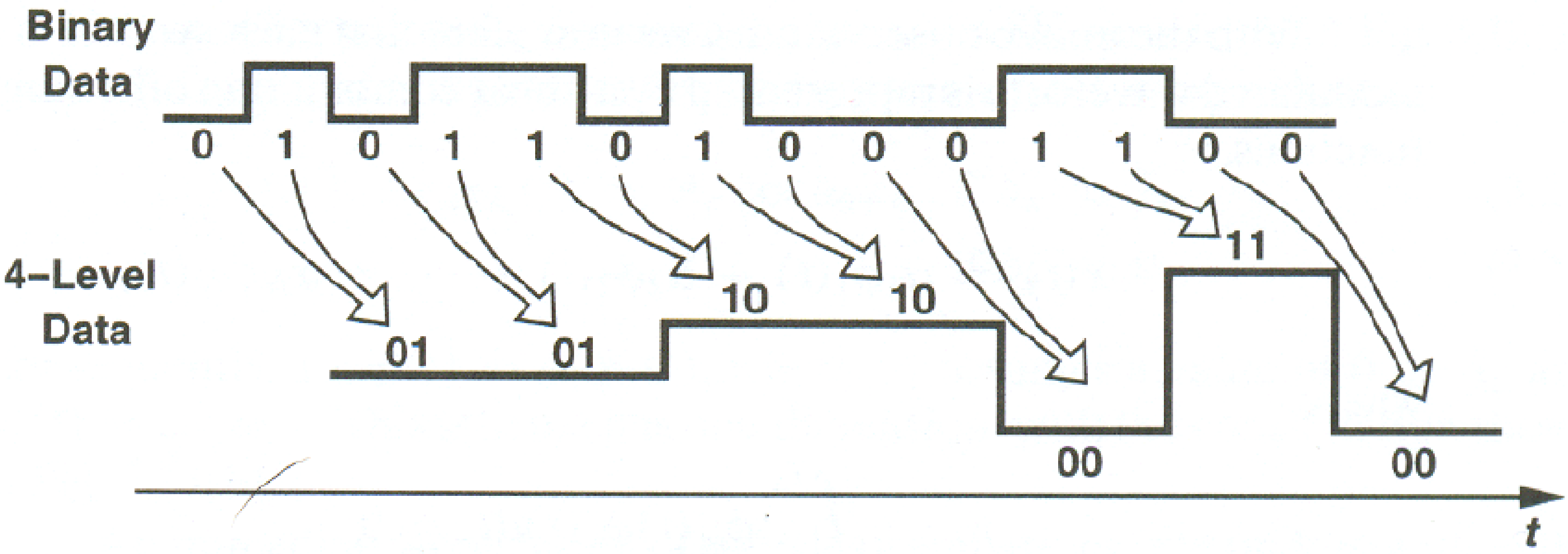


Figure 3.13 Four-level digital representation of a binary data stream.

Basis Function for Modulation

Each symbol in a digitally modulated waveform is presented by a linear superposition of orthogonal basis functions:

$$x_{RF,\mathrm{mod}} = \sum_{k=1}^{N} \alpha_k \varphi_k(t) \quad \text{with} \quad \int_0^{T_S} \varphi_m(t)\,\varphi_n(t)\,dt = \delta_{nm} \begin{cases} 0 & if\ m \neq n \\ 1 & if\ m = n \end{cases}$$

where T_S is the symbol period and N the basis dimension.

Example # 1 : binary FSK signal

$$x_{FSK}(t) = \begin{cases} A_c \cos \omega_1 t & if\ b_n = 0 \\ A_c \cos \omega_2 t & if\ b_n = 1 \end{cases}$$

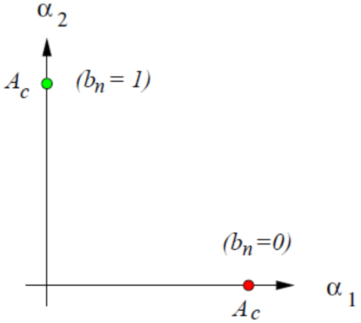
can be represented by :

$$\mathrm{x}_{\mathrm{FSK}}(\mathrm{t}) = [\alpha_1 \quad \alpha_2] \cdot \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = [\alpha_1 \quad \alpha_2] \cdot \begin{bmatrix} \cos(\omega_1 t) \\ \cos(\omega_2 t) \end{bmatrix}$$

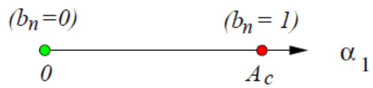
$$\text{with } [\alpha_1 \quad \alpha_2] \begin{cases} (A_c \quad 0) \ \text{if } \mathrm{b_n} = 0 \\ (0 \quad A_c) \ \text{if } \mathrm{b_n} = 1 \end{cases}$$

Signal Constellation

The signal constellation is a plot of all possible values of $\begin{bmatrix} \alpha_1 & \dots & \alpha_2 \end{bmatrix}$



FSK



ASK

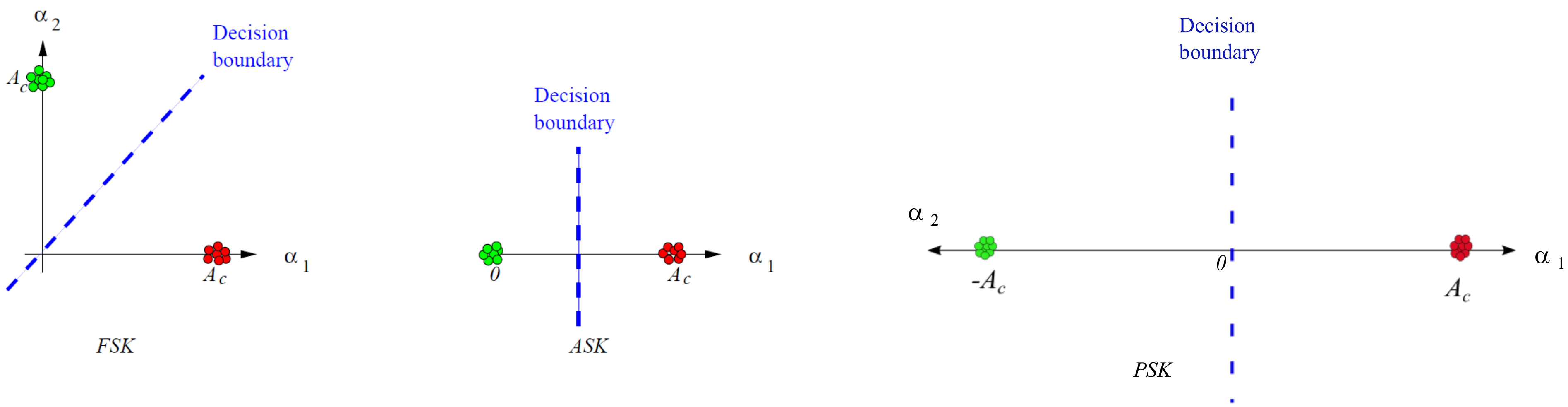
Example # 2: Binary ASK signal

$$x_{ASK}(t) = \begin{cases} A_c \cos \omega_1 t & \text{if } b_n = 1 \\ 0 & \text{if } b_n = 0 \end{cases}$$

Example # 3: Binary PSK signal

$$x_{PSK}(t) = \begin{cases} A_c \cos \omega_1 t & \text{if } b_n = 1 \\ -A_c \cos \omega_1 t & \text{if } b_n = 0 \end{cases}$$

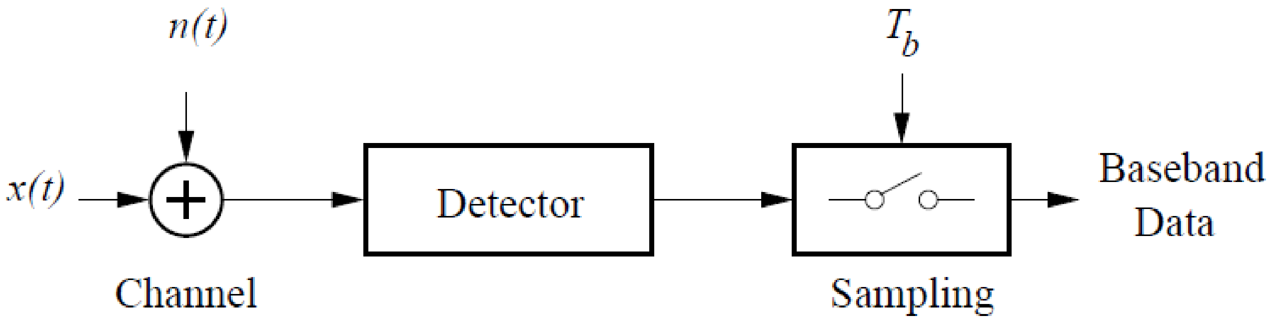
Impact of Noise and Decision Boundary



The distance between the points on the constellation is an indication of the robustness of the modulated signal to noise.

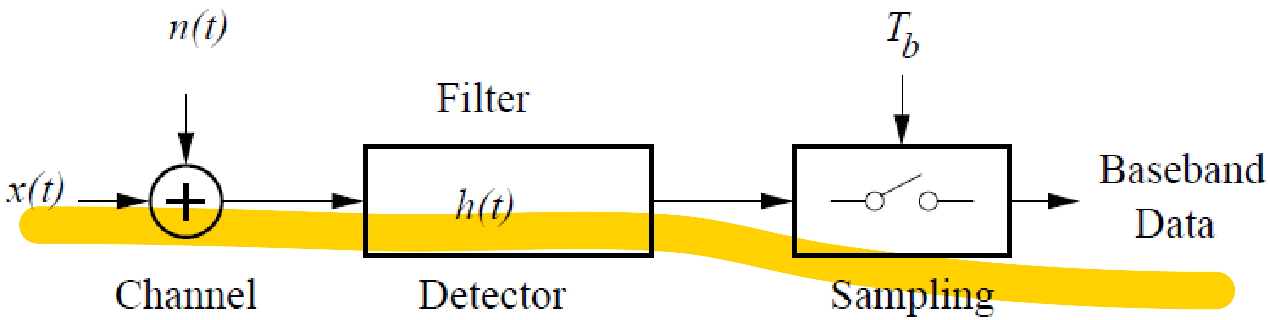
Pulse Detection

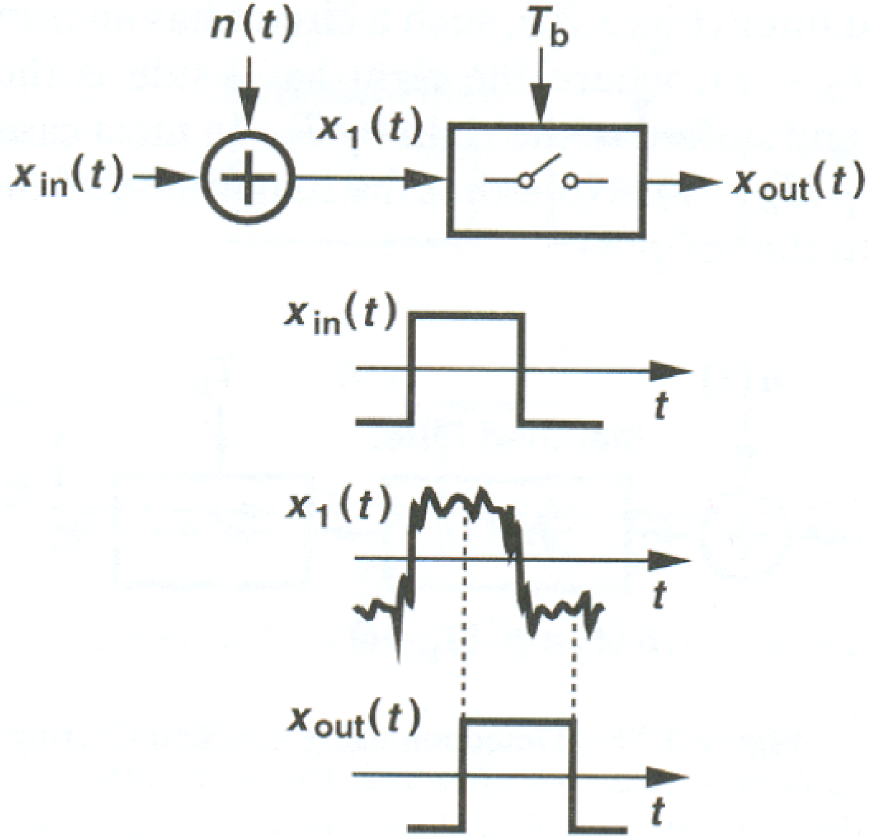
General scheme for detecting a pulse:



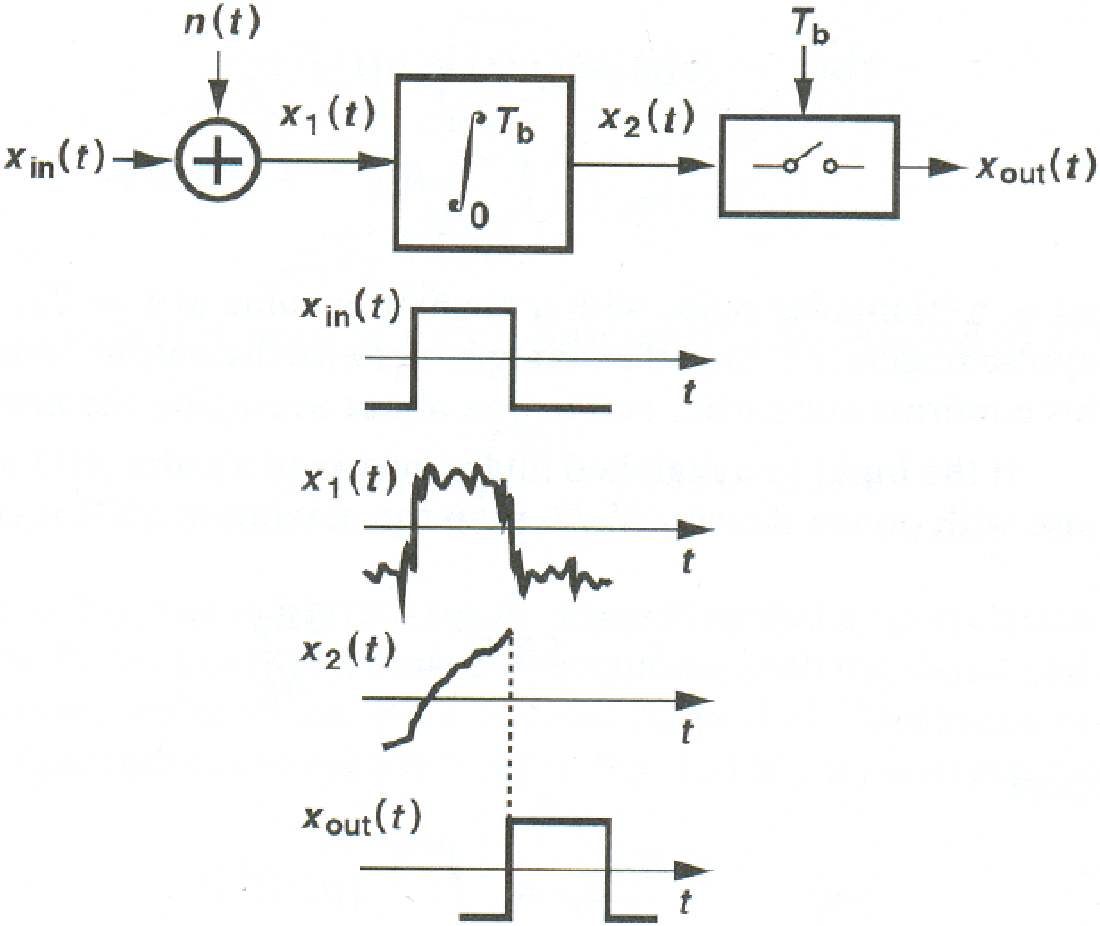
We assume the sampling is synchronized to the input data.

What is the optimal filter for detection?





(a)

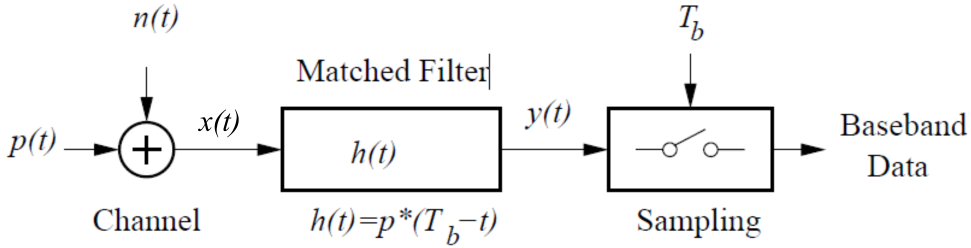


(b)

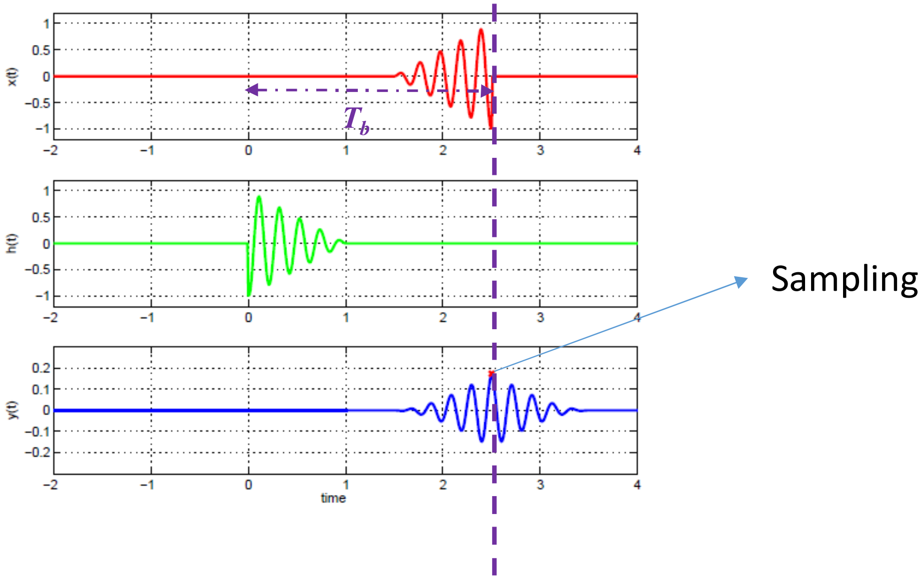
Figure 3.19 Detector with (a) sampling the peak value, (b) integration over one bit prior to sampling.

Optimal Detection with a Matched Filter

The optimal detector for AGWN ($x(t) = p(t) + n(t)$)is a matched filter:

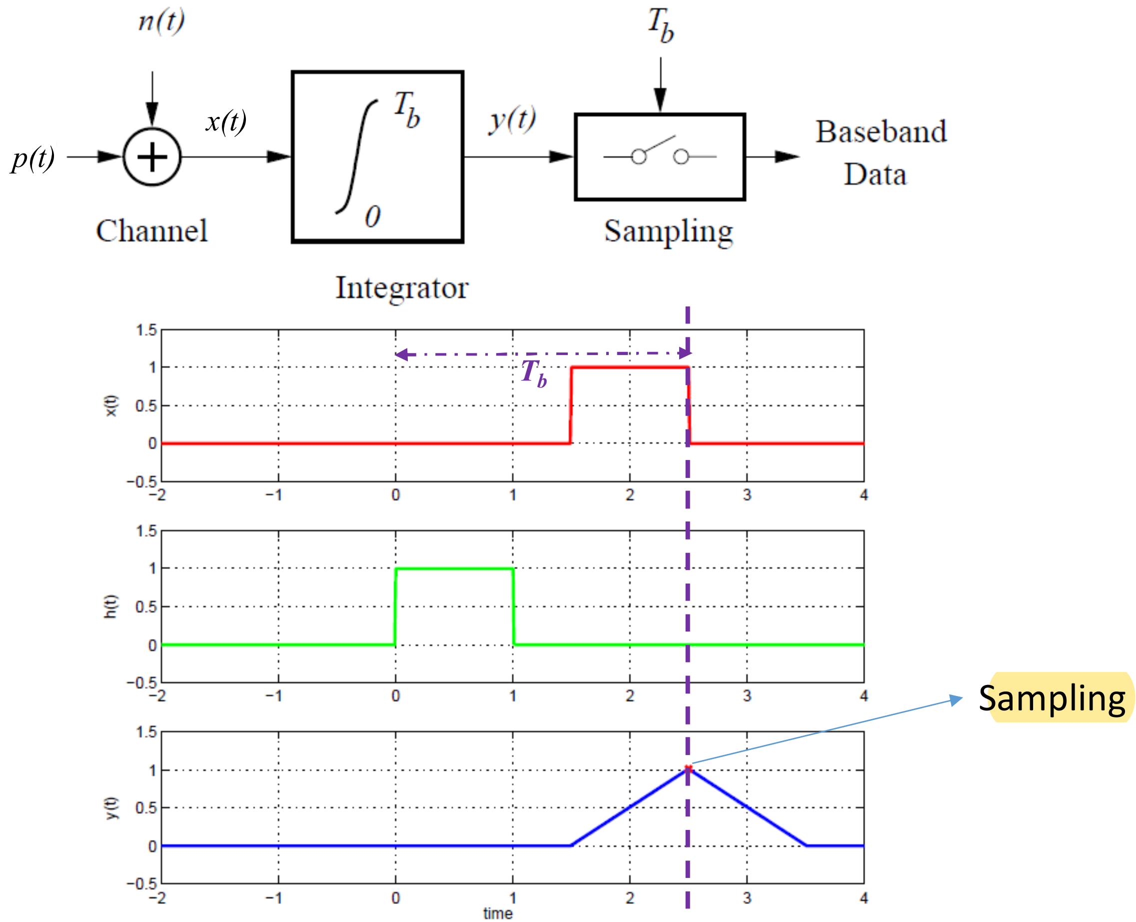


$$y(t) = p(t) * h(t) = \int_{-\infty}^{\infty} p(t-\tau)h(\tau)\, d\tau \qquad \text{with} \quad h(t') = p^*(T_b - t')$$

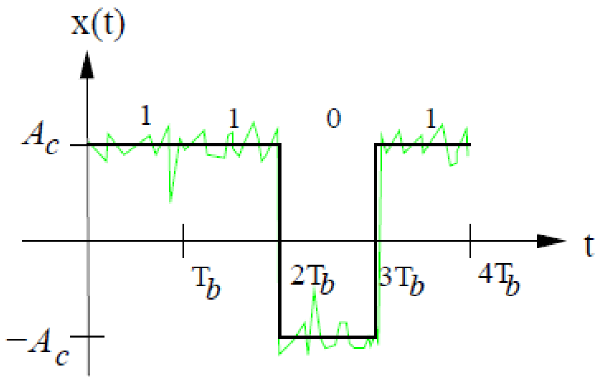


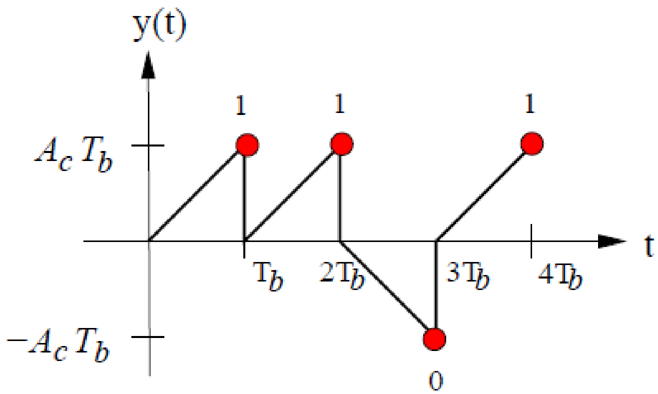
Optimal Detection for Square Pulses

For square pulses a matched filter is realized by integrating the signal from $[0 \quad T_b]$:



Example with a Train of Square Pulses





The correlator (integration) is reseted after each sampling.

Implementation of Matched Filter with a Correlator

The output of the matched filter is:

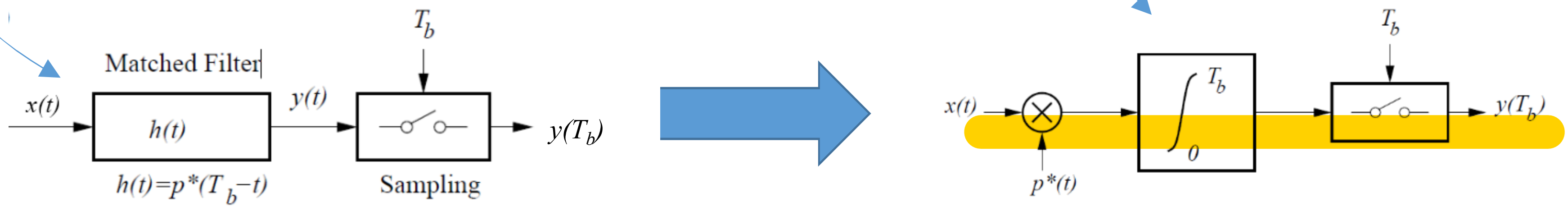
$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(t-\tau)\; x(\tau)\; d\tau \quad \text{with} \quad h(t') = p^*(T_b - t')$$

Since the pulse $p(t)$ only takes value in the interval $[0,T_b]$ we can rewrite the output as:

$$y(t) = \int_0^{T_b} h(t-\tau)\; x(\tau)\; d\tau = \int_0^{T_b} p^*(T_b - t + \tau)\; x(\tau)\; d\tau$$

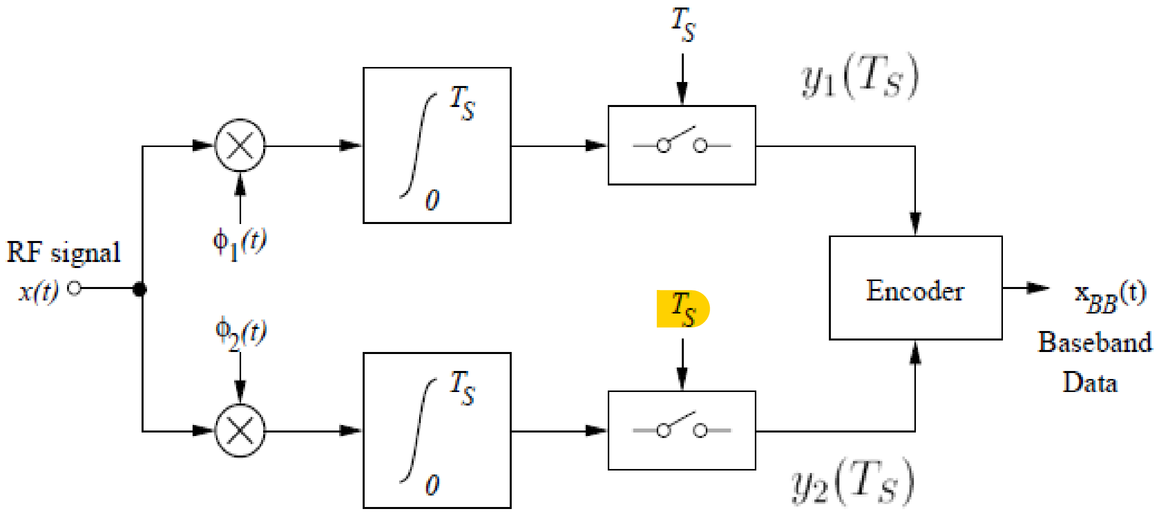
The maximum output is obtained at $t = T_b$ leading to the correlator implementation:

$$y(T_b) = \int_0^{T_b} p^*(\tau)\; x(\tau)\; d\tau$$



Correlation Receiver for 2D Signal Space

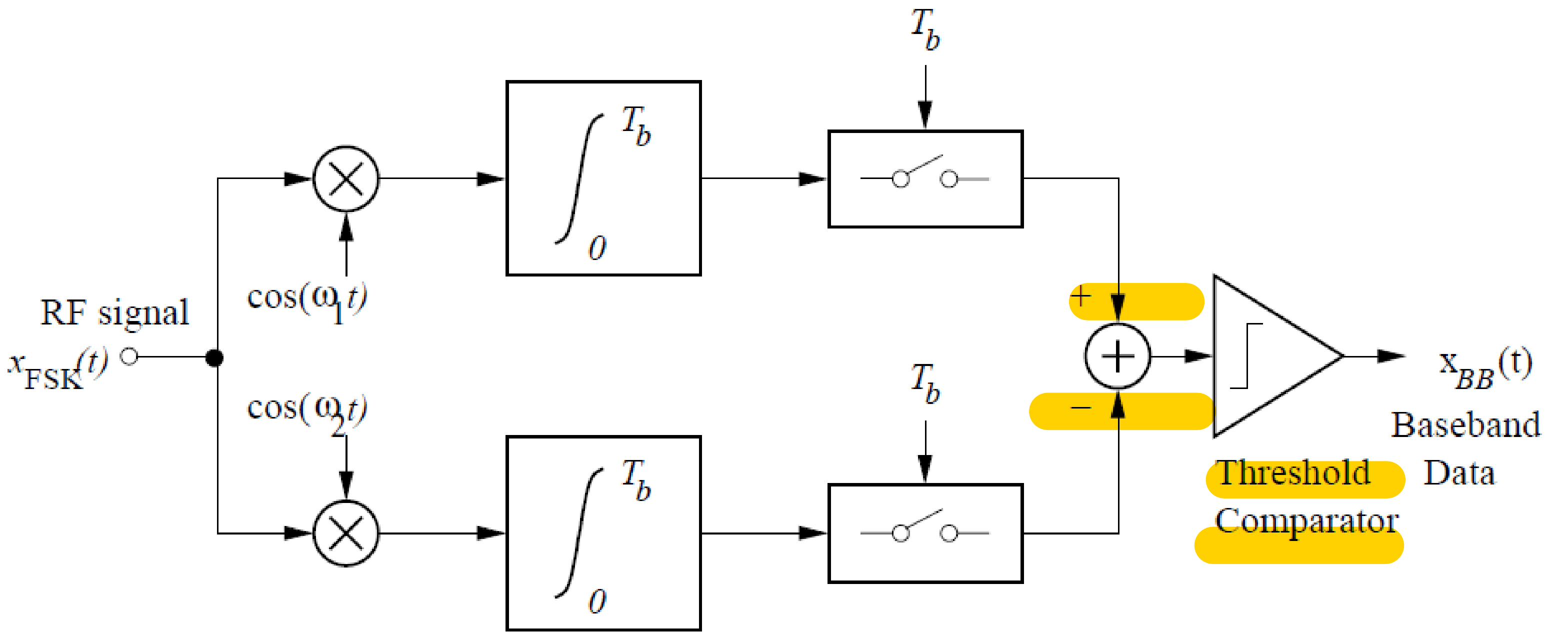
Application to digitally modulated waveforms $x(t)$:



The encoder compares the orthogonal projection of $x(t)$:

$$y_1(T_S) = \int_0^{T_S} x(t)\,\varphi_1(t)\,dt \quad \text{and} \quad y_2(T_S) = \int_0^{T_S} x(t)\,\varphi_2(t)\,dt$$

Example: Coherent FSK Detector



The threshold comparator compares:

$$y_1(T_b) = \int_0^{T_b} x_{FSK}(t)\cos(\omega_1 t)dt \qquad \text{and} \qquad y_2(T_b) = \int_0^{T_b} x_{FSK}(t)\cos(\omega_2 t)dt$$

Carrier Synchronization for Coherent Receiver

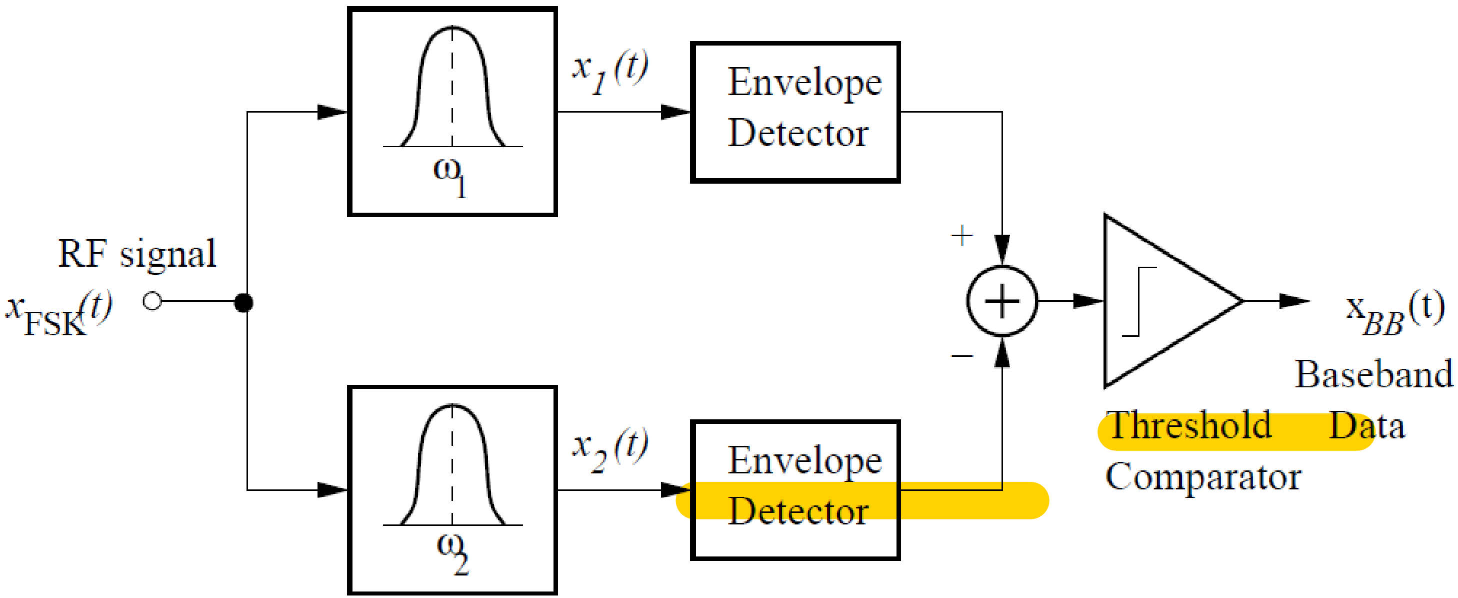
Typically phase error of less than 45 degrees required since 90 degrees gives incorrect reading.

An envelop detector can be used for ASK and FSK with a penalty in BER.

Method used for synchronization:

- Transmit a low level carrier for phase locking LO.
- Carrier recovery from received signal using a phase locked loop circuit.

Non-Coherent FSK Detector

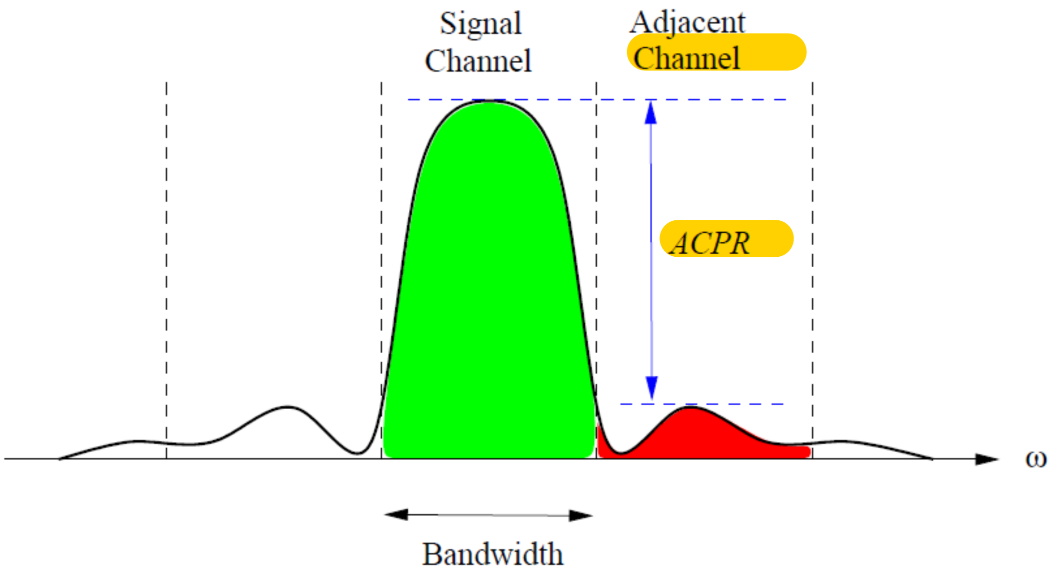


Attributes:

- Simpler implementation (no synchronization required) than coherent FSK detector
- Bit error rate is higher

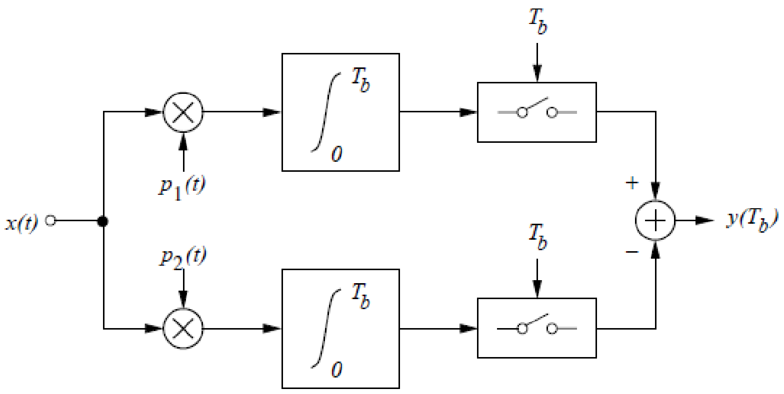
99% Bandwidth and ACPR Definition

The 99% bandwidth is the bandwidth containing 99% of the signal power.

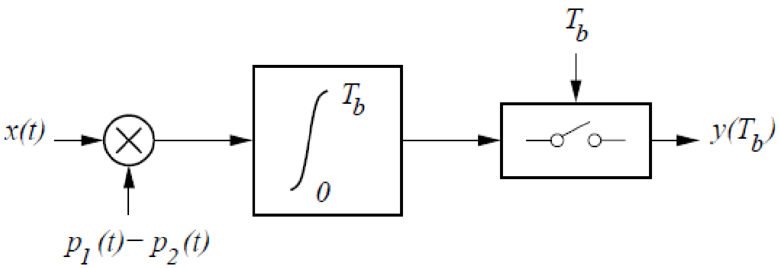


- The 99% bandwidth is used to determine the channel spacing
- ACPR: Ratio in dB of the power in the adjacent channel divided by the power in the channel (e.g. -60 dBc).
- The power used in ACPR are the entire channel bandwidth but typically the peak power can be used.

Coherent Binary Receiver for Modulated Signals



Coherent binary receiver



Equivalent receiver

In the absence of noise $x(t)$ is either $p_1(t)$ or $p_2(t)$ and the integrator output is either A_1 or A_2:

$$y(T_b) = \begin{cases} A_1 & \text{for } x(t) = p_1(t) \\ A_2 & \text{for } x(t) = p_2(t) \end{cases}$$

with
$$A_1 = \int_0^{T_b} p_1 \times p_1 \; dt - \int_0^{T_b} p_1 \times p_2 \; dt = \int_0^{T_b} p_1 \times (p_1 - p_2) \; dt$$

$$A_2 = \int_0^{T_b} p_2 \times p_1 \; dt - \int_0^{T_b} p_2 \times p_2 \; dt = \int_0^{T_b} p_2 \times (p_1 - p_2) \; dt$$

so that we have $A_1 - A_2 = \int_0^{T_b} [p_1(t) - p_2(t)]^2 \; dt = E_d$

Integrator Output

For example:

$$\left.\begin{array}{l} p_1(t) \\ p_2(t) \end{array}\right\} \underset{\text{BASK}}{=} \left\{\begin{array}{l} A_c\cos\omega_c t \\ 0 \end{array}\right. \underset{\text{BPSK}}{=} \left\{\begin{array}{l} A_c\cos\omega_c t \\ -A_c\cos\omega_c t \end{array}\right. \underset{\text{BFSK}^*}{=} \left\{\begin{array}{l} A_c\cos\omega_1 t \\ A_c\cos\omega_2 t \end{array}\right.$$

In the absence of noise $x(t)$ is either $p_1(t)$ or $p_2(t)$ and the integrator output is either A_1 or A_2:

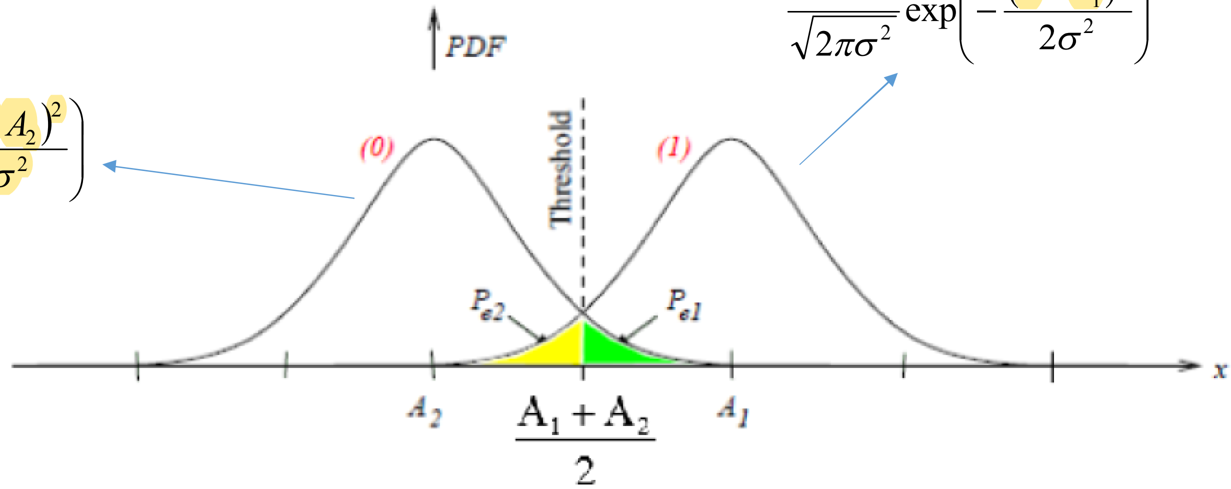
$$\left.\begin{array}{l} A_1 \ \text{for bit 1} \\ A_2 \ \text{for bit 0} \end{array}\right\} \underset{\text{BASK}}{\simeq} \left\{\begin{array}{l} \frac{1}{2}A_c^2T_b \\ 0 \end{array}\right. \underset{\text{BPSK}}{\simeq} \left\{\begin{array}{l} A_c^2T_b \\ -A_c^2T_b \end{array}\right. \underset{\text{BFSK}}{\simeq} \left\{\begin{array}{l} \frac{1}{2}A_c^2T_b \\ -\frac{1}{2}A_c^2T_b \end{array}\right.$$

* we assume that p_1 and p_2 are orthogonal for BFSK.

Probability of Error

$$\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(x-A_1)^2}{2\sigma^2}\right)$$

$$\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(x-A_2)^2}{2\sigma^2}\right)$$



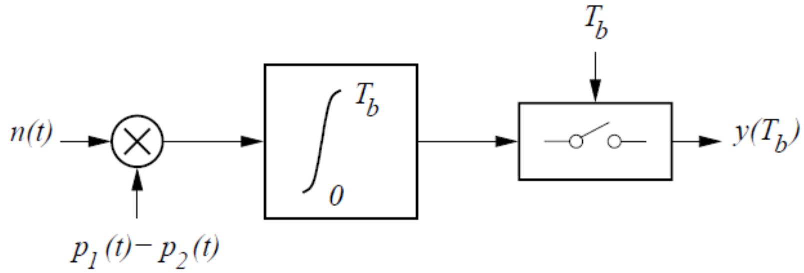
The threshold is : $\frac{A_1+A_2}{2}$

The probability of error (BER) $P_e = \frac{1}{2}P_{e1}^{'} + \frac{1}{2}P_{e2}^{'} = P_{e1}^{'} = P_{e2}^{'}$ is given by,

$$P_e = \int_{\frac{A_1+A_2}{2}}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{(x-A_2)^2}{2\sigma^2}\right]dx = \frac{1}{\sqrt{2\pi}}\int_{\frac{A_1-A_2}{2\sigma}}^{\infty}\exp\left(-\frac{u^2}{2}\right)du = Q\left(\frac{A_1-A_2}{2\sigma}\right) = Q\left(\frac{E_d}{2\sigma}\right)$$

with E_d given by, $E_d = A_1 - A_2 = \int_0^{T_b}\left[p_1(t) - p_2(t)\right]^2 dt$

Variance of Output Noise



Average value of noise at integrator o/p is always zero (in time domain)

$$\Rightarrow \overline{y(T_b)} = \overline{\int_0^{T_b} n(t)dt} = \int_0^{T_b} \overline{n(t)}\, dt = 0$$

Variance σ^2 of the noise at the integrator o/p :

$$\sigma^2 = \overline{\mathrm{y}^2(\mathrm{T_b})} - \overline{y(T_b)}^2 = \overline{\mathrm{y}^2(\mathrm{T_b})} = \overline{\int_0^{T_b} n(t)[p_1(t) - p_2(t)]\,dt \int_0^{T_b} n(t')[p_1(t') - p_2(t')]\,dt'}$$

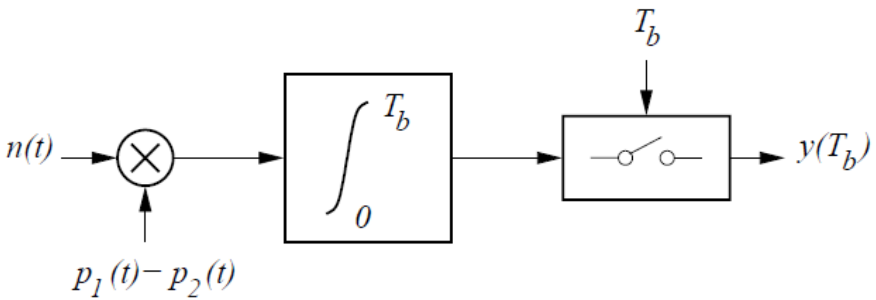
$$= \int_0^{T_b}\int_0^{T_b} \overline{n(t)\, n(t')}\, [p_1(t) - p_2(t)][p_1(t') - p_2(t')]dt\, dt' = \int_0^{T_b}\int_0^{T_b} \frac{1}{2}N_0\, \delta(t-t')\,[p_1(t) - p_2(t)][p_1(t') - p_2(t')]\,dt\, dt'$$

$$= \frac{1}{2}N_0 \int_0^{\mathrm{T_b}} [p_1(t) - p_2(t)]^2 dt = \frac{1}{2}N_0 E_d$$

where we used the autocorrelation property of white noise at temperature $\mathrm{T_0}$:

$$R(t\text{-}t') = \overline{n(t)\, n(t')} = \frac{1}{2}k\, T_0\, \delta(t-t') = \frac{1}{2}N_0\, \delta(t-t')$$

Variance of Output Noise (...Contd)



The variance of the output noise is given by,

$$\sigma^2 = \frac{1}{2} N_0 E_d \quad (\mathrm{N}_0 = \mathrm{kT} \quad \text{for white noise})$$

BER is calculated as follows,

$$\left(\frac{E_d}{2\sigma}\right)^2 = \frac{E_d^2}{4\sigma^2} = \frac{E_d^2}{4 \cdot \frac{1}{2} N_0 E_d} = \frac{E_d}{2N_0}$$

$$\Rightarrow Q\left(\frac{E_d}{2\sigma}\right) = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) = BER$$

$$E_d = A_1 - A_2 = \int_0^{T_b} [p_1(t) - p_2(t)]^2 \, dt$$

Peak and Average Bit Energy

The bit energy is the energy of signal $p_1(t)$ or $p_2(t)$

For bit 1 $\quad E_b(\text{bit } 1) = \int_0^{T_b} [p_1(t)]^2\,dt = \begin{cases} \frac{1}{2}A_c^2T_b & \text{for ASK} \\ \frac{1}{2}A_c^2T_b & \text{for PSK} \\ \frac{1}{2}A_c^2T_b & \text{for FSK} \end{cases}$

For bit 0 $\quad E_b(\text{bit } 0) = \int_0^{T_b} [p_2(t)]^2\,dt = \begin{cases} 0 & \text{for ASK} \\ \frac{1}{2}A_c^2T_b & \text{for PSK} \\ \frac{1}{2}A_c^2T_b & \text{for FSK} \end{cases}$

The peak bit energy is the maximum value of E_b for all bits:

$$E_b(\text{peak}) = \max[E_b(\text{bit } 0), E_b(\text{bit } 1)]$$

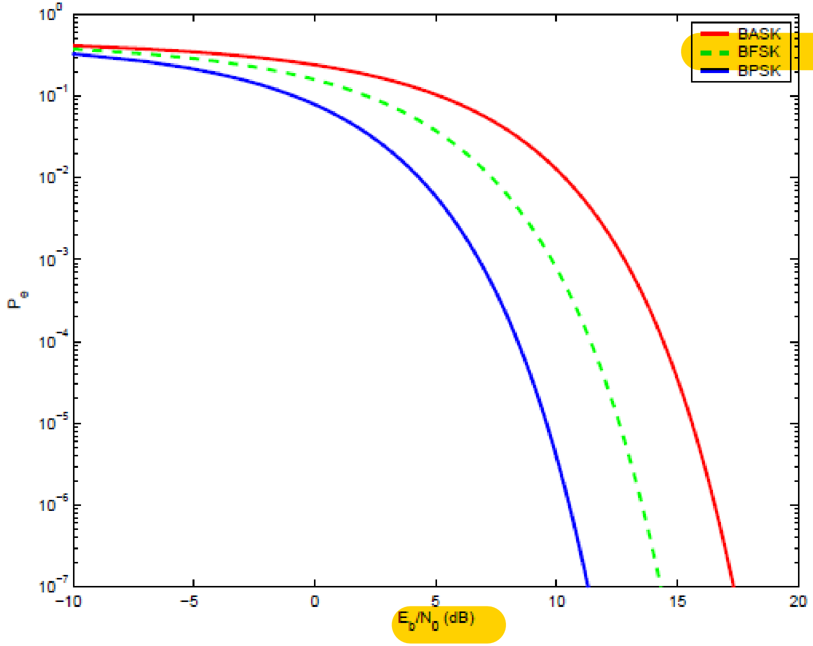
The average bit energy is the average value of E_b of all bits:

$$\begin{aligned} E_b(\text{average}) &= \frac{1}{2}E_b(\text{bit } 0) + \frac{1}{2}E_b(\text{bit } 1) \\ &= \begin{cases} \frac{1}{2}E_b(\text{peak}) & \text{for ASK} \\ E_b(\text{peak}) & \text{for PSK} \\ E_b(\text{peak}) & \text{for FSK} \end{cases} \end{aligned}$$

$$E_d = A_1 - A_2 = \int_0^{T_b} [p_1(t) - p_2(t)]^2\,dt$$

$$E_d = \begin{cases} \frac{1}{2}A_c^2T_b & \text{BASK} \\ 2A_c^2T_b & \text{BPSK} \\ A_c^2T_b & \text{BFSK} \end{cases}$$

BER for Coherent Binary Digital Modulation



Binary Modulation	$E_b(\text{peak})/N_0$ for $P_e = 10^{-5}$
coherent PSK	9.6 dB
coherent FSK	12.6 dB
coherent ASK	15.6 dB

	BASK	BPSK	BFSK
$E_b(\text{peak})$	$\frac{1}{2}A_c^2T_b$	$\frac{1}{2}A_c^2T_b$	$\frac{1}{2}A_c^2T_b$
$P_e = Q\left(\sqrt{\frac{E_d}{2N_0}}\right)$	$Q\left(\sqrt{\frac{E_b(\text{peak})}{2N_0}}\right)$ $Q\left(\sqrt{\frac{E_b(\text{average})}{N_0}}\right)$	$Q\left(\sqrt{\frac{2E_b(\text{peak})}{N_0}}\right)$ $Q\left(\sqrt{\frac{2E_b(\text{average})}{N_0}}\right)$	$Q\left(\sqrt{\frac{E_b(\text{peak})}{N_0}}\right)$ $Q\left(\sqrt{\frac{E_b(\text{average})}{N_0}}\right)$

BER

Note: E_b is the bit energy.

The bit error rate (BER) can be decreased by

- increasing the signal power (larger A_c)
- decreasing the data rate (larger T_b)

since we have: $\frac{E_b}{N_0} = \frac{S}{R_b N_0} = \frac{S}{N}\frac{\Delta f}{R_b} = \frac{SNR}{\eta_B}$

Bandwidth efficiency

with the noise power $N = 2\Delta f \frac{N_0}{2}$ and Δf the channel bandwidth.

Additional Attributes of BPSK

Spectrum:

The BPSK waveform is given by: $x_{BPSK} = x_{BB}(t) \cdot A_c \cos \omega_c t$

If $S_x(\omega)$ is the spectrum of x_{BB} then the spectrum S_{BPSK} of x_{BPSK} is

$$S_{BPSK}(\omega) = \frac{1}{2} S_x(\omega + \omega_c) + \frac{1}{2} S_x(\omega - \omega_c)$$

The bandwidth B_T is on the order of: $B_T \simeq \dfrac{2}{T_b}$

Bandwidth efficiency:

$$\eta = \frac{\text{Bit Rate}}{\text{Bandwidth}} = \frac{R_b}{B_T} \simeq \frac{1}{2}$$ with the bit rate given by $R_b = \dfrac{1}{T_b}$

Additional Attributes of BFSK

Note: $p_1(t)$ are $p_2(t)$ must be orthogonal for optimal demodulation.

$$\int_0^{T_b} \cos\omega_1 t \cos\omega_2 t \; dt = 0$$

For $\omega_1 + \omega_2 >> \omega_1 - \omega_2$ this occurs for $\quad f_1 - f_2 = \frac{n}{2T_b}$

98% Bandwidth for FSK:

$$B_T = 2(\Delta f + \frac{1}{T_b}) \quad \text{with} \quad \Delta f = f_1 - f_2$$

$f_1 = 2f_2$

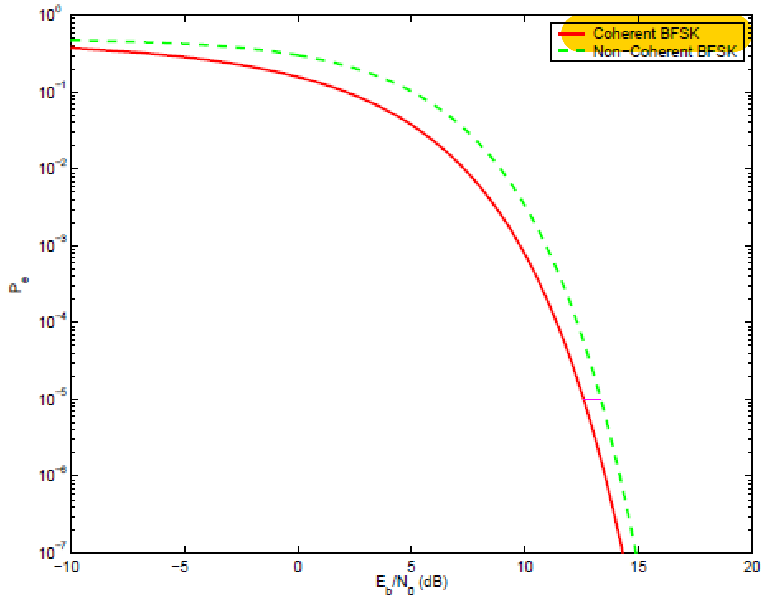
For orthogonal FSK, $\Delta f = \frac{1}{2T_b}$ and $B_T = \frac{3}{T_b}$

Bandwidth efficiency: $\quad \eta = \frac{R_b}{B_T} \simeq \frac{1}{3}$

Note: BFSK can be used with a non-coherent receiver.

Comparison of Coherent and Non-Coherent BFSK Demodulation

- $P_e(\text{Non-coherent BFSK}) \simeq P_e(\text{Coherent BFSK})$
- At $P_e = 10^{-5}$ the degradation in E_b/N_0 is of 0.75 dB for Non-Coherent.
- Non-Coherent BFSK is basically always used
- Application: data modem, teletype and fax



M-ary Digital Modulation

Principle:

Transmit $M = 2^n$ symbols over each symbol interval T_S.

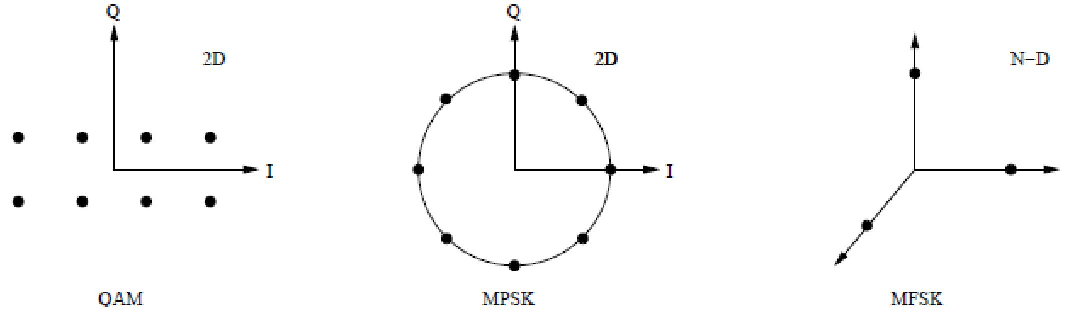
- Each symbol is realized with n bits.
- The symbol interval is $T_S = nT_b$ with T_b the bit interval.
- The symbol rate is $R_S = 1/T_S = R_b/n$ with R_b the bit rate

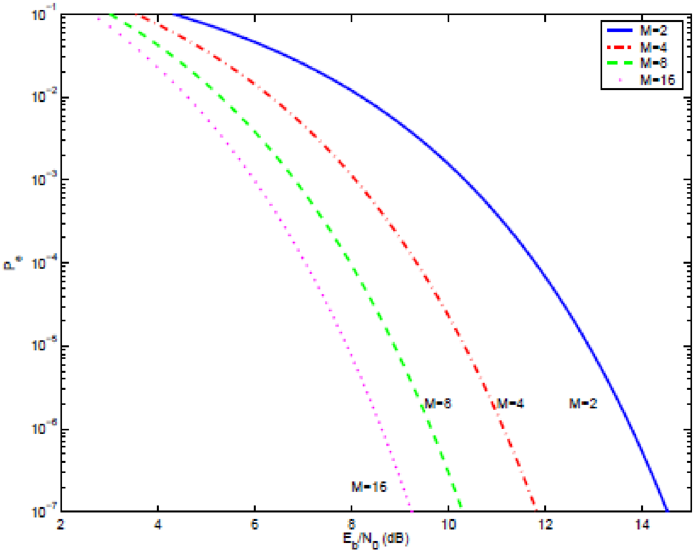
Examples:

- M-ary PSK: The bandwidth efficiency R_b/B_T (Bit/sec/Hz) increases as $\log_2(M)$ with increasing M but at the cost of a degradation in BER.

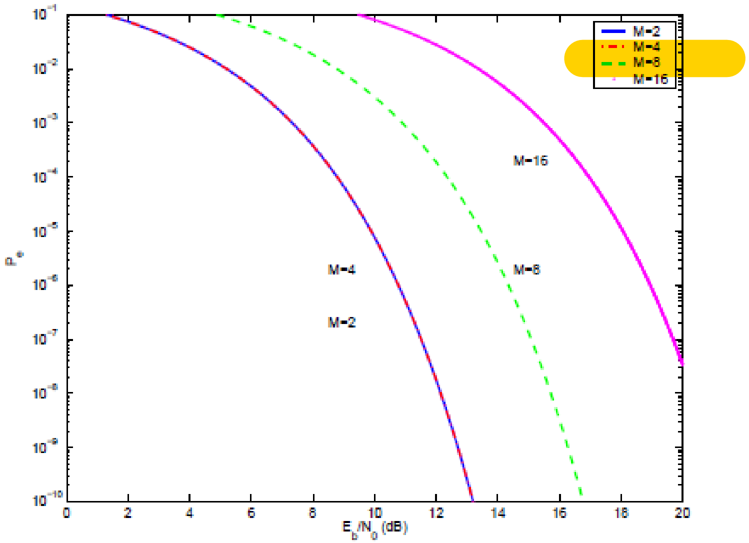
- M-ary FSK: The BER decreases with increasing M but at the cost of increase in bandwidth.

BER for MFSK and MPSK



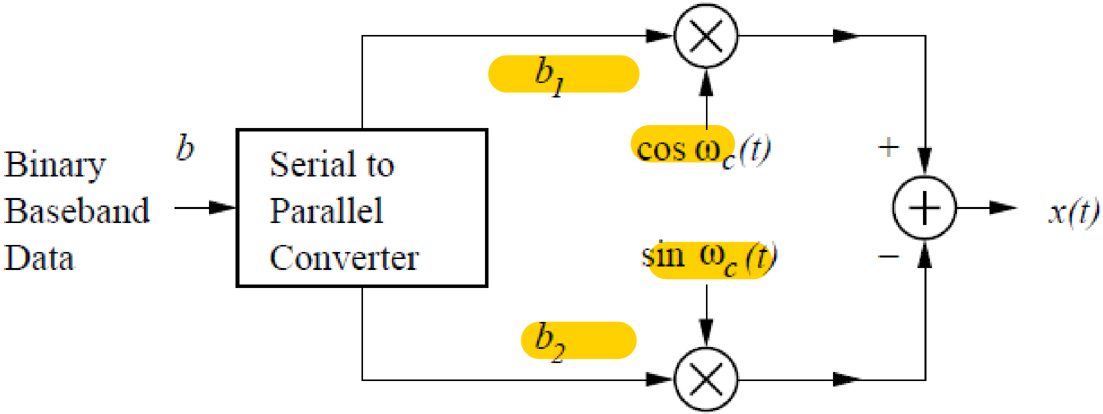


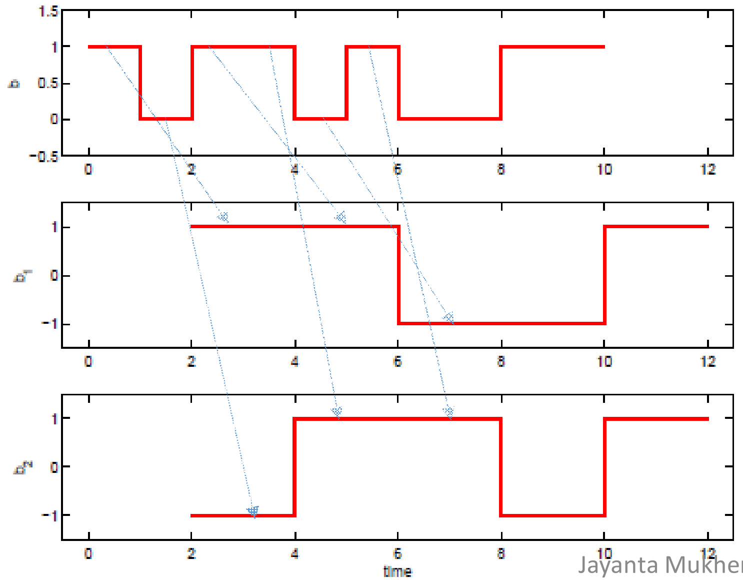
MFSK



MPSK

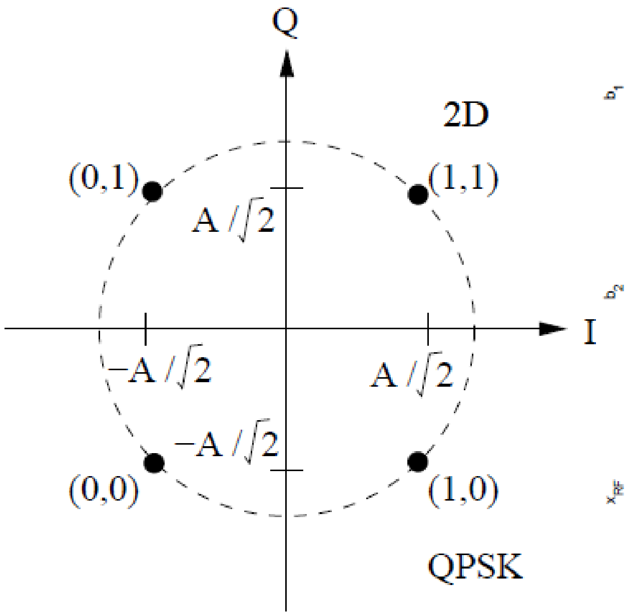
Quadrature Modulation



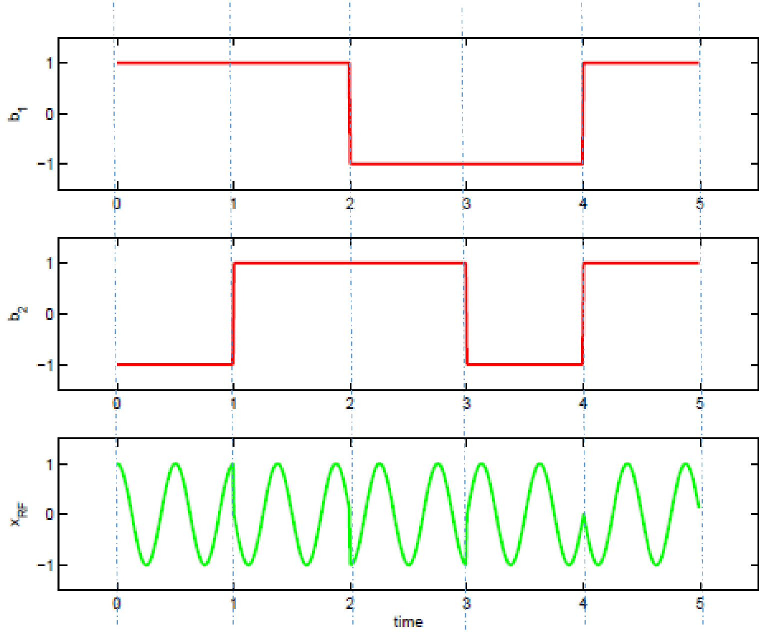


b_1	b_2	$x(t)$
-1	-1	$\sqrt{2}\cos\left(\omega_c t - \frac{3\pi}{4}\right)$
-1	1	$\sqrt{2}\cos\left(\omega_c t + \frac{3\pi}{4}\right)$
1	-1	$\sqrt{2}\cos\left(\omega_c t - \frac{\pi}{4}\right)$
1	1	$\sqrt{2}\cos\left(\omega_c t + \frac{\pi}{4}\right)$

QPSK Constellation and BER



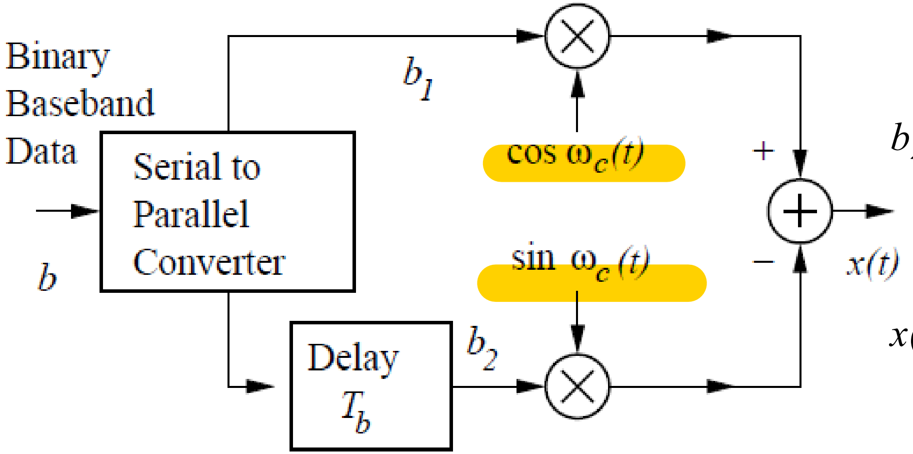
Constellation

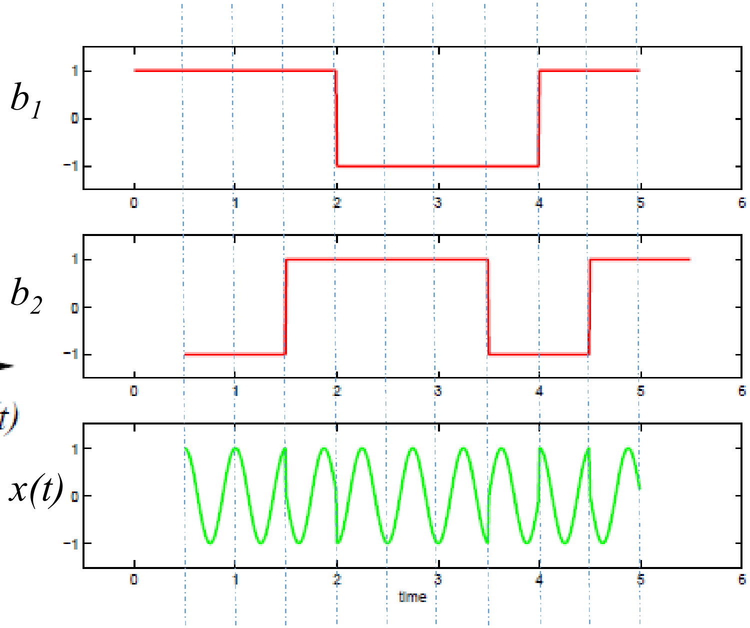


- Constellation is tighter but symbol time is doubled
- The BER of QPSK is the same as that of BPSK: $P_e = Q\left(\frac{2E_b}{N_0}\right)$
- QPSK reduces the bit rate R_b by 2 compared to BPSK and is therefore preferred.

Offset QPSK Modulation

- A delay of T_b reduces the phase shifts to 90^o at the most.
- Cannot be used with differential encoding and therefore requires a coherent demodulator.





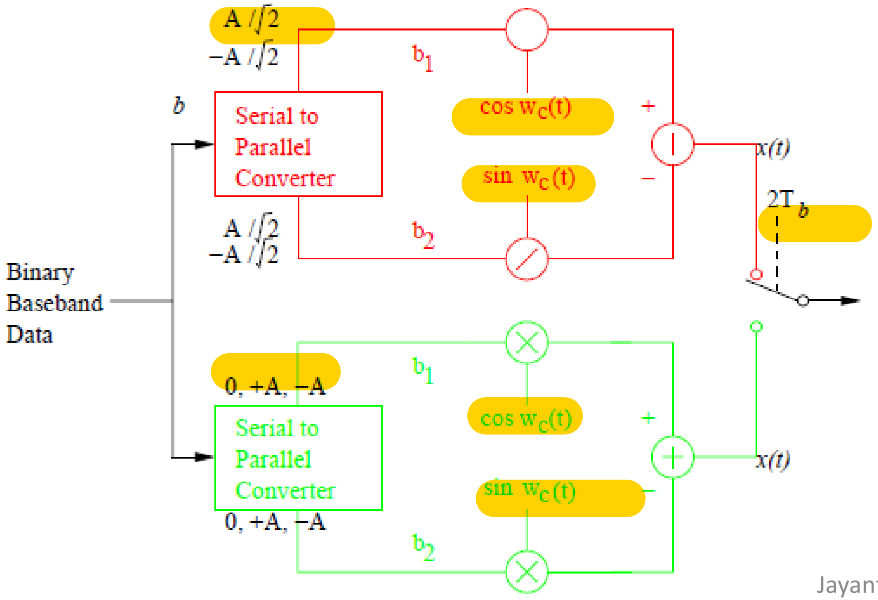
The BER of OQPSK is the same as that of QPSK: $P_e = Q\left(\frac{2E_b}{N_0}\right)$

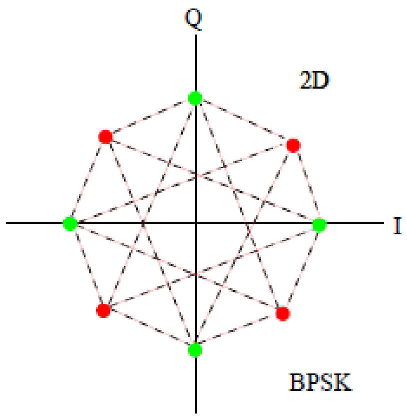
π/4-QPSK Modulation

$\pi/4$ is a superposition of 2 QPSK signals offset by 45^o. This is a compromise between QPSK and OQPSK:

- QPSK transitions are up to 180^o
- OQPSK transitions are up to 90^o
- $\pi/4$-QPSK transitions are up to 135^o

Advantage: it can be detected differentially with a non-coherent receiver.





Differential Phase Shift Keying

In PSK the phase of the RF signal relates to the time origin and has no absolute meaning

An exclusive-NOR (XNOR) is used

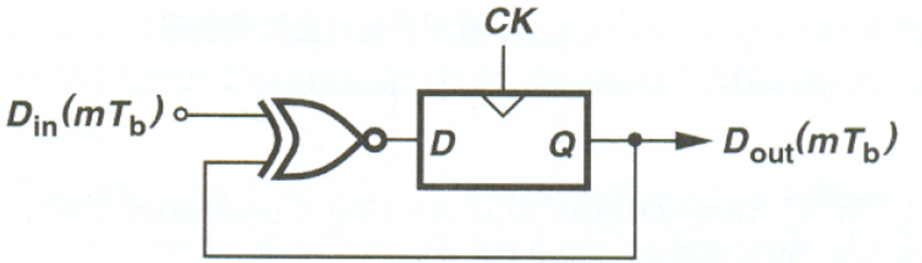
$$D_{out}[(m+1)T_b] = \overline{D_{in}(mT_b) \bigoplus D_{out}(mT_b)}$$

$$= \begin{cases} D_{out}[(m+1)T_b] = D_{out}(mT_b) & \text{if } D_{in}(mT_b) = 1 \\ D_{out}[(m+1)T_b] = \overline{D_{out}(mT_b)} & \text{if } D_{in}(mT_b) = 0 \end{cases}$$

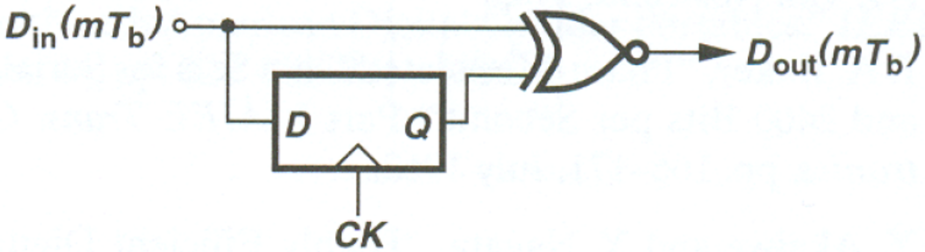
An extra arbitrary bit is used to initiate the coding and decoding.

$$\text{BER}: \quad P_e = \frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$$

This is because *two* noisy signals are compared instead of a *single* noisy signal compared to the low-noise local oscillator reference.



(a)



(b)

Input Data		0	1	1	1	0	0	1	1	0	1
Encoded Data	1	0	0	0	0	1	0	0	0	1	1
Decoded Data		0	1	1	1	0	0	1	1	0	1

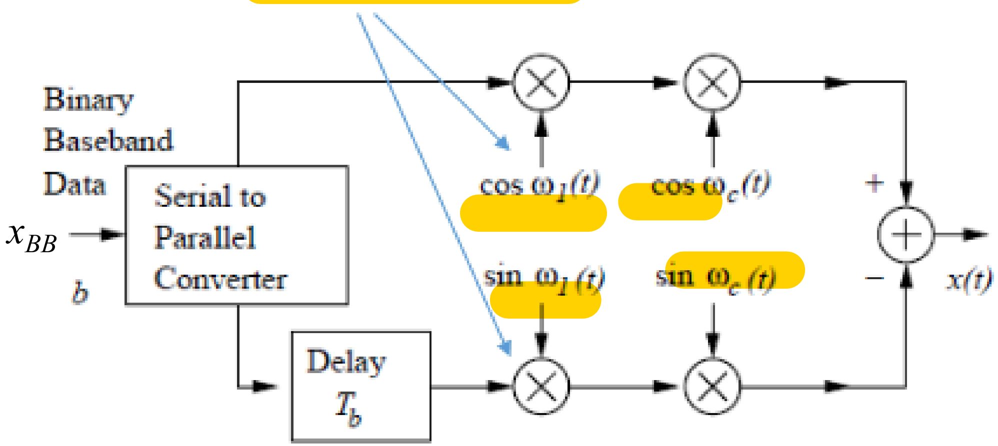
(c)

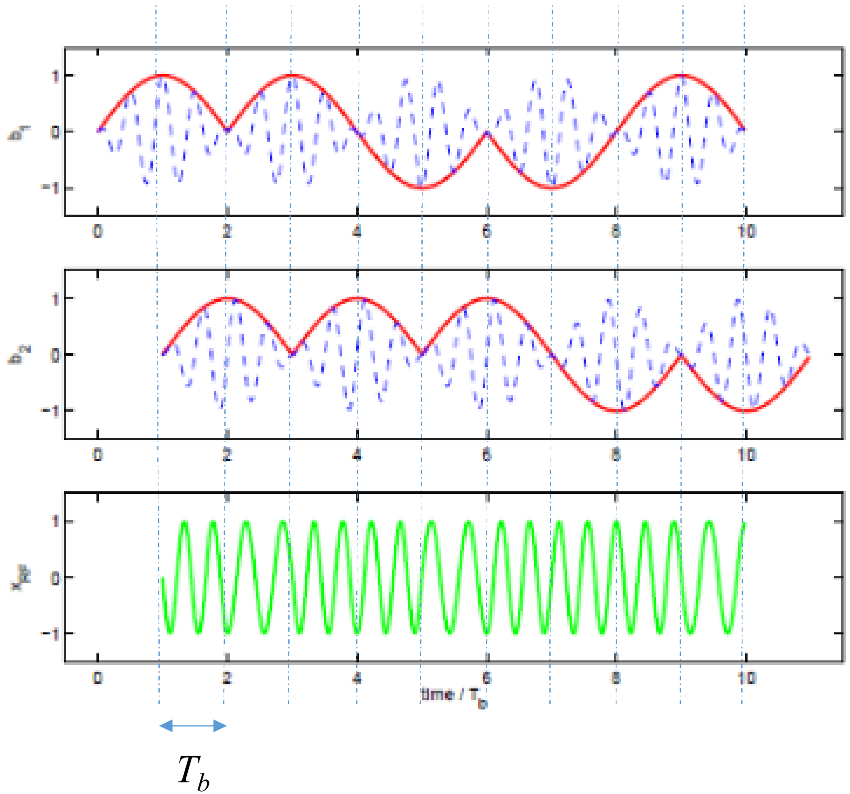
Figure 3.47 (a) Differential encoding, (b) differential decoding, (c) example of encoded and decoded sequence.

Minimum Shift Keying

MSK is similar to Offset PSK but with square pulses replaced by sine function

Variable phase shift also added





The sine function has frequency $\omega_1 = (2\pi/4T_b)$

The smooth phase transition lowers the signal power in the side lobes of the spectrum

MSK Representation in terms of FSK

$$S_{MSK}(t) = \sum_{n=0}^{N\text{-}1} m_I(t)\, p(t\text{-}2nT_b) \cos 2\pi f_c t + \sum_{n=0}^{N\text{-}1} m_Q(t)\, p(t\text{-}2nT_b\text{-}T_b) \sin 2\pi f_c t \;,$$

where,

$$p(t) = \begin{cases} \sin\left(\dfrac{\pi t}{2T_b}\right) & 0 \le t \le 2T_b \\ 0 & elsewhere \end{cases}$$

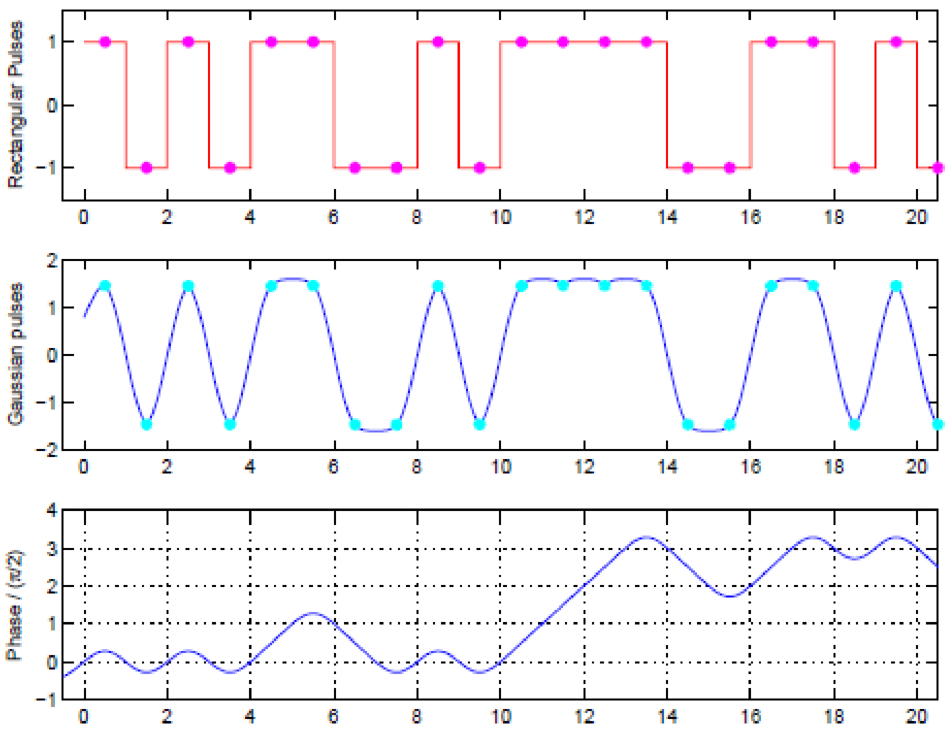
- The phase is continuous.
- With rectangular pulses p(t) we obtain MSK
- With Gaussian pulses p(t) we obtain GMSK.
- MSK and GMSK can be viewed as frequency modulation, hence a constant envelop.
- Detection is either coherent or non coherent

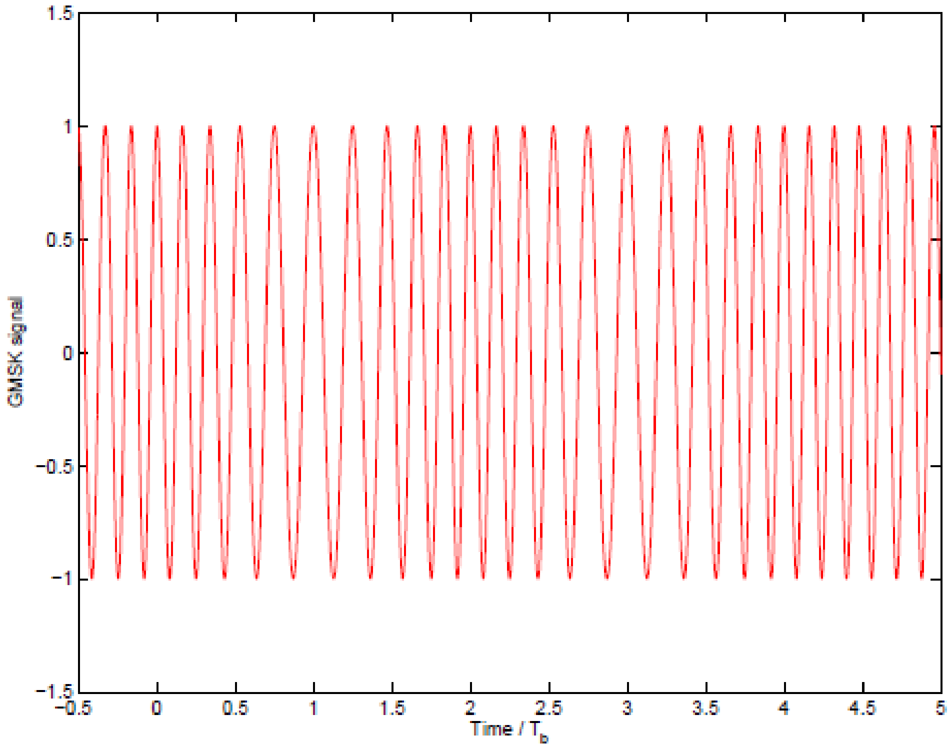
Generation of GMSK

A Gaussian filter is used

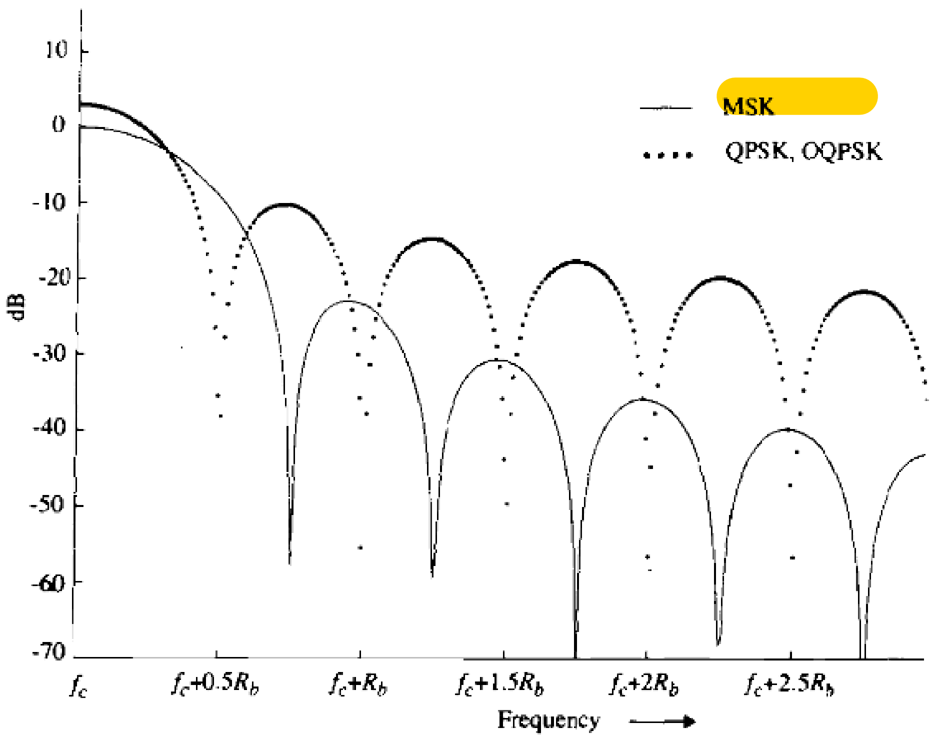
- Impulse response $h(t) = K \exp(-\alpha t^2)$. A type of Low Pass filter.
- Impulse response is usually truncated to $[-T_b \ \ T_b]$
- 99% bandwidth is less than $(1.2/T_b)$

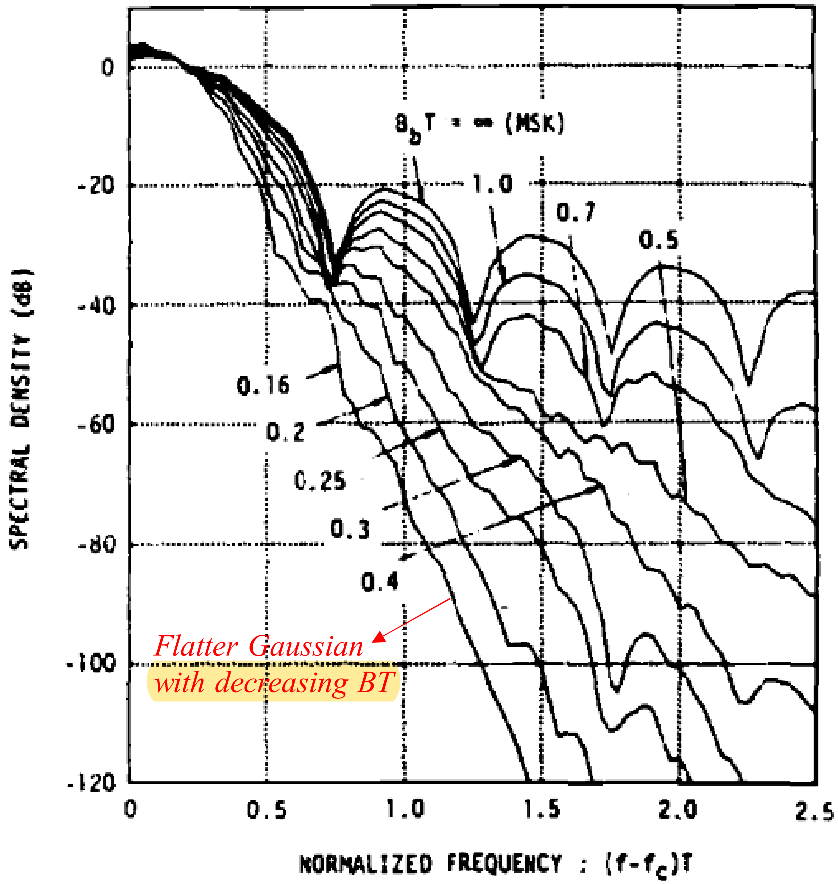
Intermediate Signals





Note the constant envelope and smooth phase variation (superior to MSK)





Rappaport - Wireless Communications, Principles and Practice, 2002 edition

Power Efficiency

Constant Envelope Signals $x(t) = A_c \cos(\omega_c t + \phi(t))$

Consider a system with memory-less 3rd order non-linearity

$$\begin{aligned} y(t) &= \alpha_3 x^3(t) + \cdots = \alpha_3 A_c^3 \cos^3[\omega_c t + \phi(t)] + \cdots \\ &= \frac{3}{4}\alpha_3 A_c^3 \cos[\omega_c t + \phi(t)] + \frac{1}{4}\alpha_3 A_c^3 \cos[3\omega_c t + 3\phi(t)] + \cdots \end{aligned}$$

No additional mixing and hence no spreading

There will be still some spreading due to IM distortion

No inband distortion is observed. But filtering can generate PM to AM modulation.

Variable Envelope Signals $x(t) = x_I(t)\cos(\omega_c t) - x_Q(t)\sin(\omega_c t)$

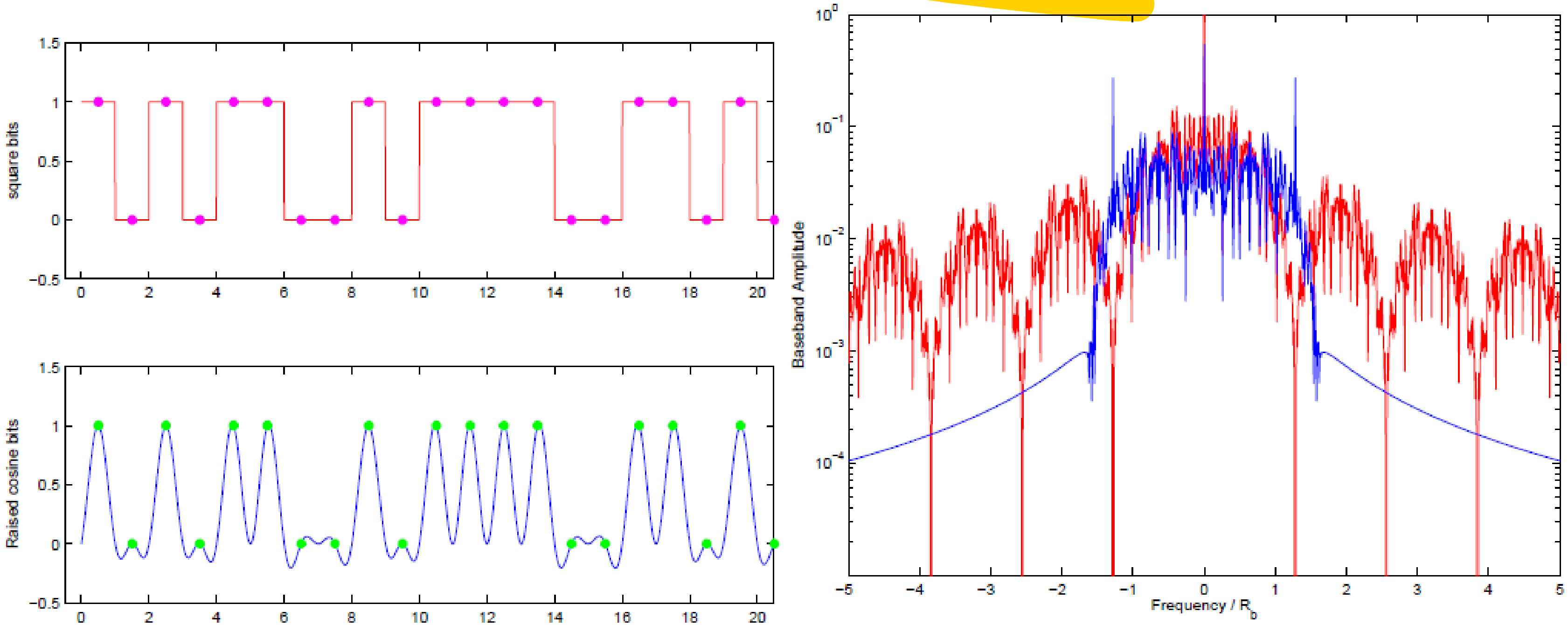
$$\begin{aligned} y(t) &= \alpha_3 x^3(t) + \cdots = \alpha_3 [x_I(t) \cos\omega_c t - x_Q(t) \sin\omega_c t]^3 + \cdots \\ &= \alpha_3 x_I^3(t) \frac{3}{4} \cos\omega_c t - \alpha_3 x_Q^3(t) \frac{3}{4} \sin\omega_c t + \cdots \end{aligned}$$

The terms $x_I^3(t)$ and $x_Q^3(t)$ usually lead to spectral regrowth.

Mixing of BW of x_I and x_Q with $\omega_c \Rightarrow$ additional BW or spreading

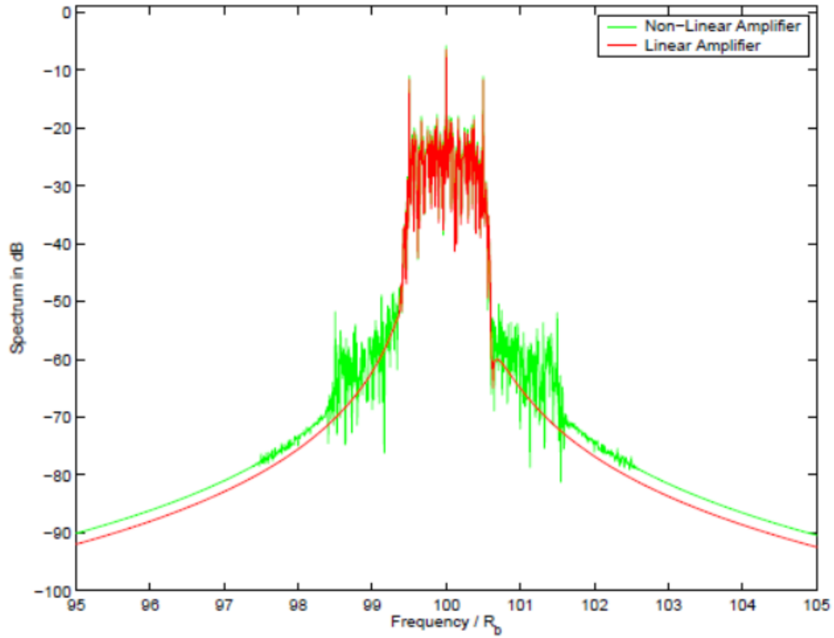
Raised Cosine Filtering of Baseband

Raised Cosine Filtering permits to reduce the bandwidth



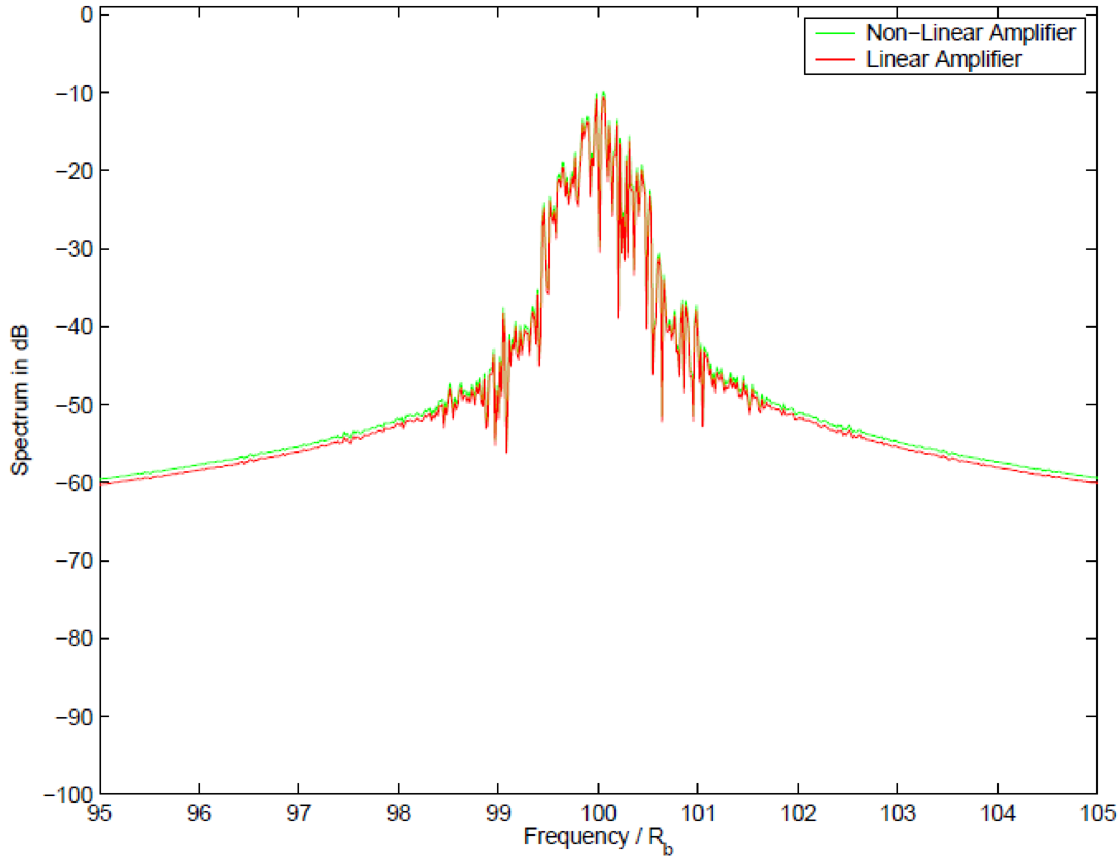
The cost is a non constant envelope...

Spectrum Regrowth in QPSK when using Raised Cosine Filtering



The non constant envelope leads to spectral regrowth once the signal is amplified.
A linear amplifier is required!

Spectral Regrowth Advantages of GMSK



No spectral regrowth is observed but a wider bandwidth is required.