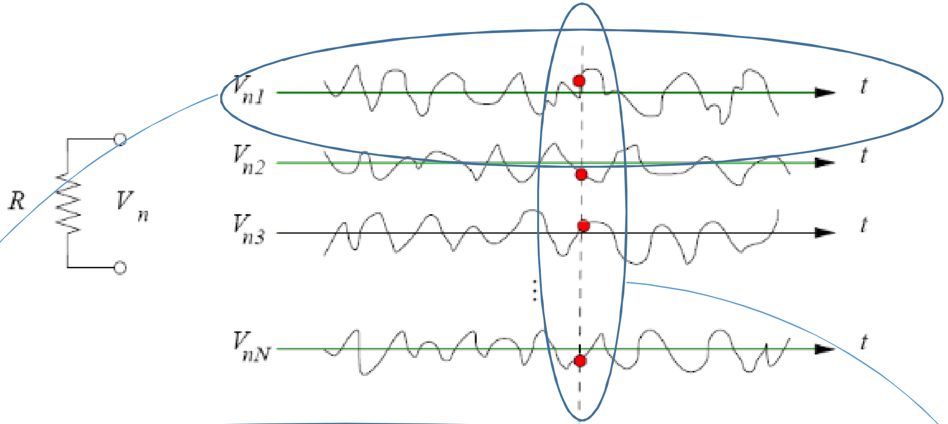
Noise

Noise is represented as a random process

By definition random signals are unknown but they can be characterized by a few parameters or functions:

- Moments: average, variance
- Probability density function (PDF)
- Power Spectral density (PSD)

Statistical Ensemble



Time Average:

$$\langle \mathrm{n(t)} \rangle = \underbrace{\lim}_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} n(t)\,dt$$

Mean square power:

$$\langle \mathrm{n^2(t)} \rangle = \underbrace{\lim}_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} n^2(t)\,dt$$

Ensemble Average:

$$\overline{\mathrm{n(t)}} = \int_{-\infty}^{\infty} n(t) P_n(n)\,dn$$

Mean square power:

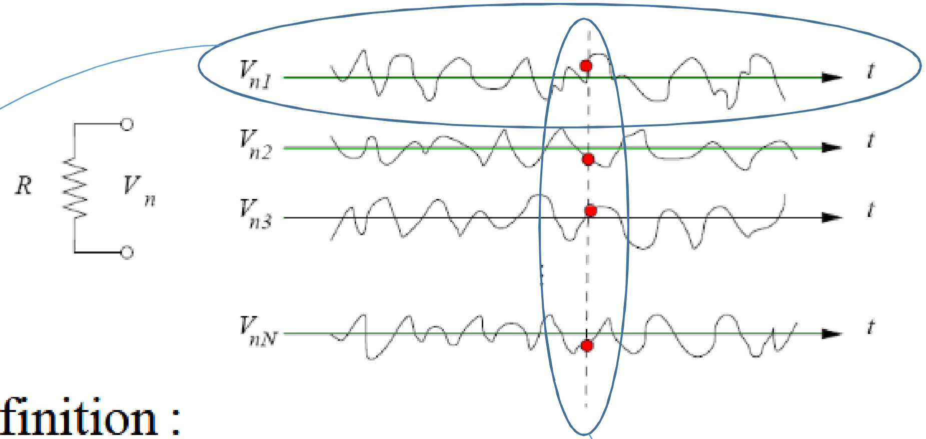
$$\overline{\mathrm{n^2(t)}} = \int_{-\infty}^{\infty} n^2(t) P_n(n)\,dn$$

with $\mathrm{P_n(n)}$ the probability density function (PDF). If $\mathrm{P_n}$ is time-invariant the random process is stationary.

For our purposes:

$$\langle \mathrm{n(t)} \rangle \approx \overline{\mathrm{n(t)}} \approx 0 \qquad\qquad \langle \mathrm{n^2(t)} \rangle \approx \overline{\mathrm{n^2(t)}}$$

Autocorrelation Function



Time average definition :

$$R(\tau) = \int_{-\infty}^{\infty} x^*(t)x(t+\tau)dt$$

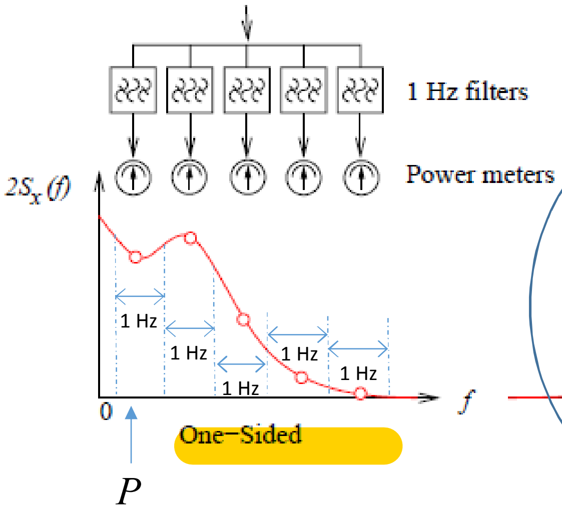
Properties : $R(-\tau) = R(\tau)$ and $R(0) \geq R(\tau)$

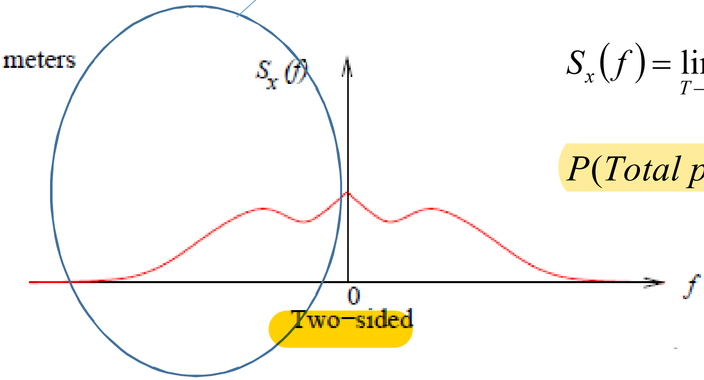
Ensemble average definition for stationary random processes :

$$R(\tau) = \overline{x^*(t)x(t+\tau)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 P_x(x_1, x_2, \tau)dx_1 dx_2$$

Power Spectral density

This part appears only when noise is up converted





$$S_x(f) = \lim_{T\to\infty} \frac{\overline{\left|X_T(f)\right|^2}}{T} \quad \text{where} \quad X_T(f) = \int_0^T x(t)\ \exp(-j2\pi f t)\ dt$$

$$P(\text{Total power in 1 Hz BW at center freq } f\,) \approx S_x(f) \times 1 = S_x(f)$$

Up converted noise power is half of noise power at dc.

noise power at dc

$$A\cos(\omega_1 t)\times A\cos(\omega_0 t) = \frac{A^2}{2}\cos(\omega_1 + \omega_0)t + \frac{A^2}{2}\cos((-\omega_1) + \omega_0)t$$

Positive frequency upconverted

Negative frequency upconverted

Wiener Khintchine Theorem

Relates autocorrelation and power spectral density.

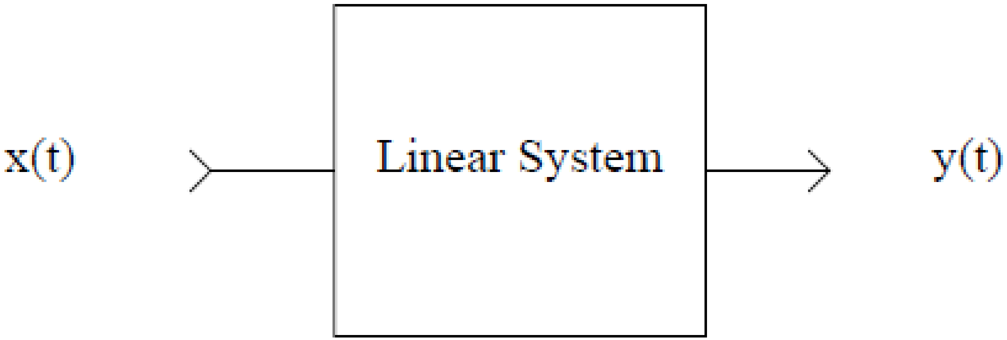
If for a stationary process we have $\int_{-\infty}^{\infty} \left|\tau \, R(\tau)\right| d\tau < \infty$ then,

$$S_x(f) = \int_{-\infty}^{\infty} R(\tau)\exp(-j2\pi f t)d\tau \quad \text{or} \quad R(\tau) = \int_{-\infty}^{\infty} S_x(f)\exp(j2\pi f \tau)\, df$$

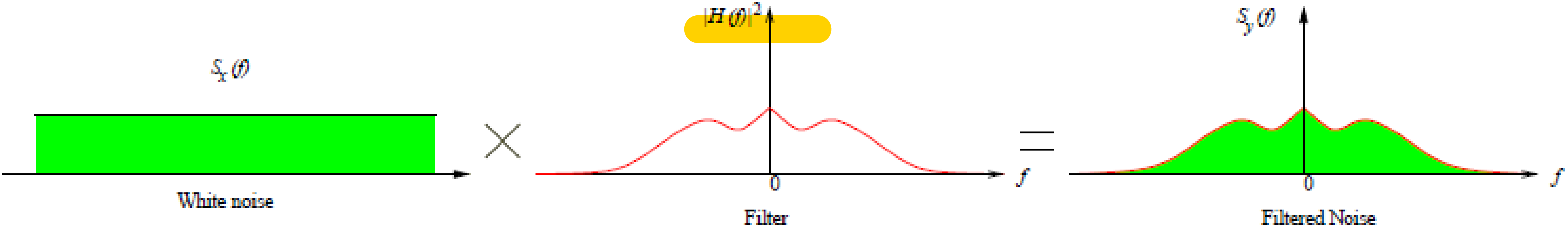
In particular the power in a 1Ω load is :

$$P_L = \overline{x^2(t)} = R(0) = \int_{-\infty}^{\infty} S_x(f)df$$

Noise in Linear Systems



$$S_y(f) = |H(f)|^2 S_x(f)$$



Sources of noise

- External sources of noise (temperature) at the antenna (measured in Kelvin)
  - natural
  - man-made
- Receiver or transmitter noise (temperature or noise figure)

Types of noise:

- Thermal or Johnson or Nyquist noise (PDF: Gaussian, PSD: white noise)
- Shot noise (PDF: Poisson, PSD: white)
- Flicker noise (PDF: Gaussian, PSD: 1/f)
- Quantum noise (PDF: Poisson, PDF: f)

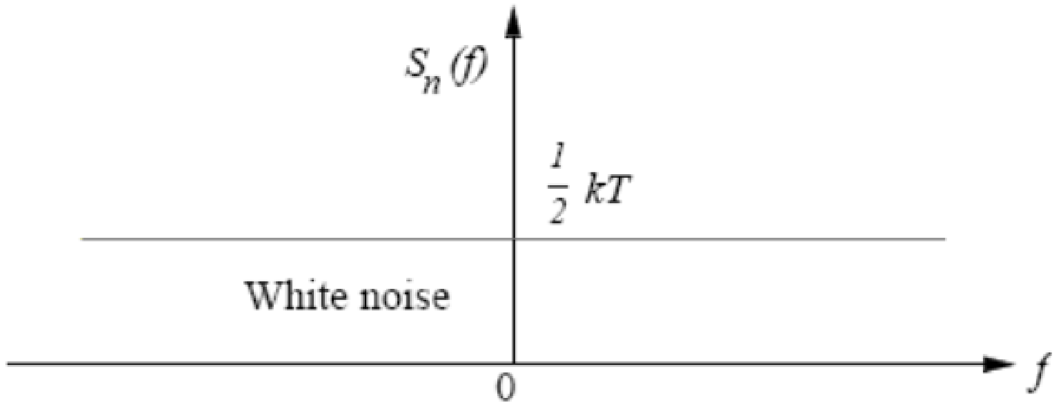
Ideal Thermal Noise

The power spectral density is constant and extends upto infinite frequencies (white noise)

$$S_n(f) = \frac{kT}{2}$$

Autocorrelation :

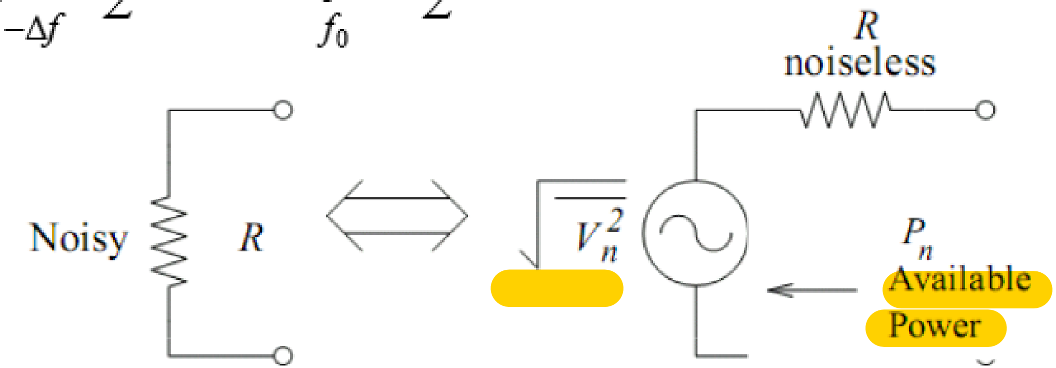
$$R(\tau) = \int_{-\infty}^{\infty} \frac{kT}{2} \exp(j2\pi ft)df = \frac{kT}{2}\delta(\tau)$$



Thermal Noise RMS voltage

Available (maximum) noise power in bandwidth Δf :

$$P_n = \int_{-f_0-\Delta f}^{-f_0} \frac{kT}{2} df + \int_{f_0}^{f_0+\Delta f} \frac{kT}{2} df$$

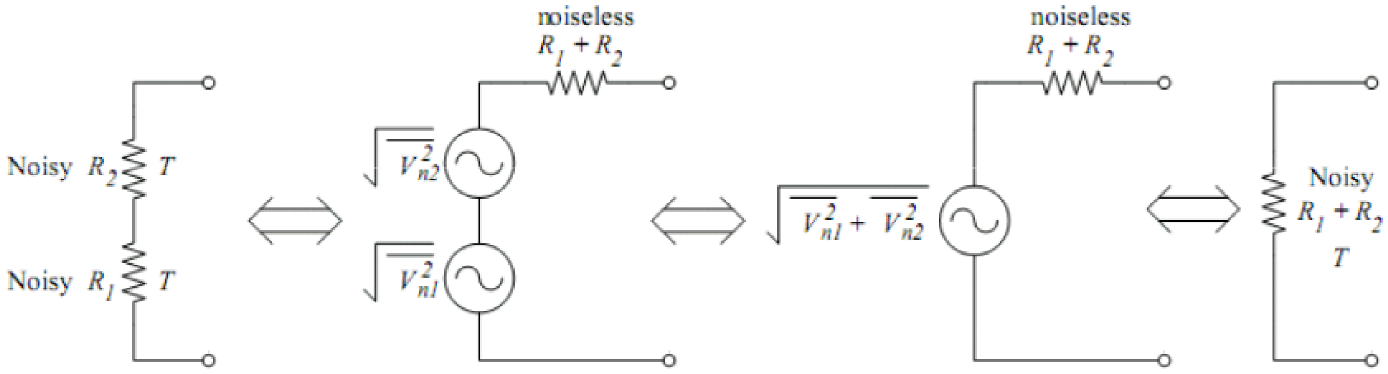


The available (maximum) power is obtained for a congugate matched load R

The maximum power delivered is then given by

$$P_n = kT\Delta f = \left(\frac{\sqrt{\overline{V_n^2}}}{2R} \right)^2 R = \frac{\overline{V_n^2}}{4R} \Rightarrow \sqrt{\overline{V_n^2}} = \sqrt{4kTR\Delta f}$$

Noisy Resistors in series



Total available noise power (need a load $\mathrm{R_1 + R_2}$):

$$\mathrm{P_{n,total}} = (\mathrm{R_1 + R_2})\left(\frac{1}{2}\frac{\sqrt{\overline{V_{n1}^2}}}{(\mathrm{R_1 + R_2})}\right)^2 + (\mathrm{R_1 + R_2})\left(\frac{1}{2}\frac{\sqrt{\overline{V_{n2}^2}}}{(\mathrm{R_1 + R_2})}\right)^2 = kT\Delta f$$

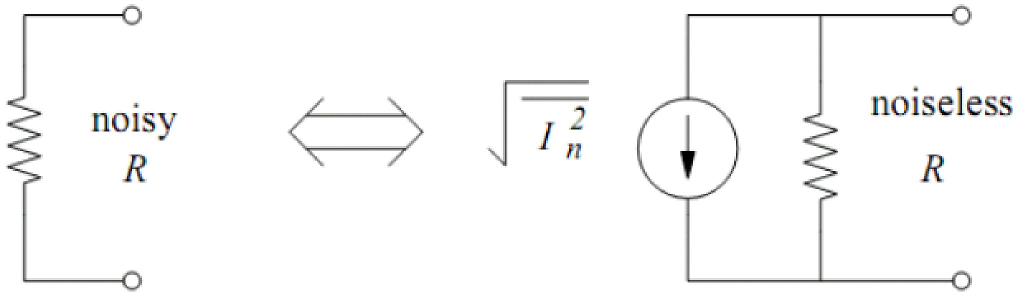
using $\overline{V_{n1}^2} = 4kTR_1\Delta f$ and $\overline{V_{n2}^2} = 4kTR_2\Delta f$

The total rms voltage is obtained by adding the square of the rms voltages:

$$\overline{\mathrm{V_{n,total}^2}} = \overline{|V_{n1} + V_{n2}|^2} = \overline{V_{n1}^2} + \overline{V_{n2}^2} = 4kT(R_1 + R_2)\Delta f$$

This is due to the fact that $\mathrm{V_{n1}}$ and $\mathrm{V_{n2}}$ are uncorrelated

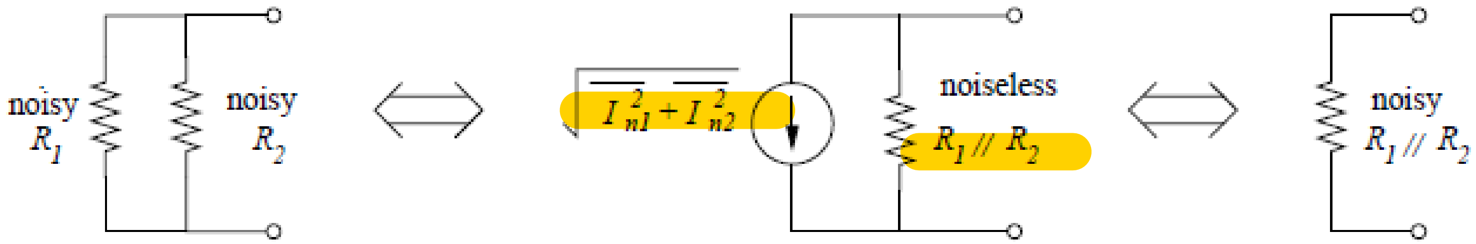
Current Representation



Norton current source representation :

$$\sqrt{\overline{I_n^2}} = \sqrt{4kT\frac{1}{R}\Delta f}$$

Resistors In Shunt



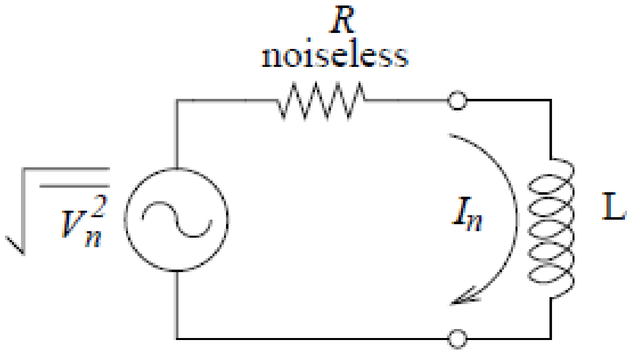
Total available noise power is still $P_n = kT\Delta k$ and is obtained for a load termination $R_1 \| R_2$.

The total rms current is obtained by adding the square of the rms current :

$$\overline{I^2_{n,total}} = \overline{|I_{n1} + I_{n2}|^2} = \overline{I_{n1}^2} + \overline{I_{n2}^2} = 4kT\frac{1}{R_1 \| R_2}\Delta f$$

This is due to the fact that I_{n1} and I_{n2} are uncorrelated

Noise Shaping Example: RL Circuit



The PSD of the voltage V_n generated by the resistor R is : $S_V = 4kTR$

The square of the voltage - current transfer function is : $\left|H(f)\right|^2 = \dfrac{1}{R^2 + \omega^2 L^2}$

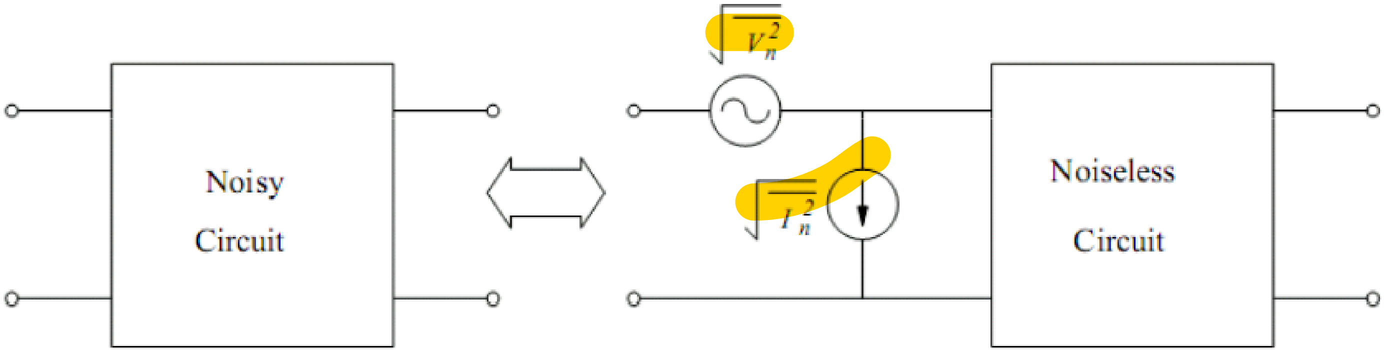
The PSD of the current is then : $S_I = S_V |H(f)|^2 = \dfrac{4kTR}{R^2 + \omega^2 L^2}$ and the rms current $\overline{I_n^2}$ is

(considering + ve frequencies only) :

$$\overline{I_n^2} = \int_0^\infty S_I df = \int_0^\infty S_V |H(f)|^2 df = S_V \int_0^\infty \frac{1}{R^2 + \omega^2 L^2} df = \frac{kT}{L}$$

An inductor is a noise less component but it can store noise energy : $\dfrac{1}{2} L \overline{I_n^2} = \dfrac{1}{2} kT$

Input Referred Noise

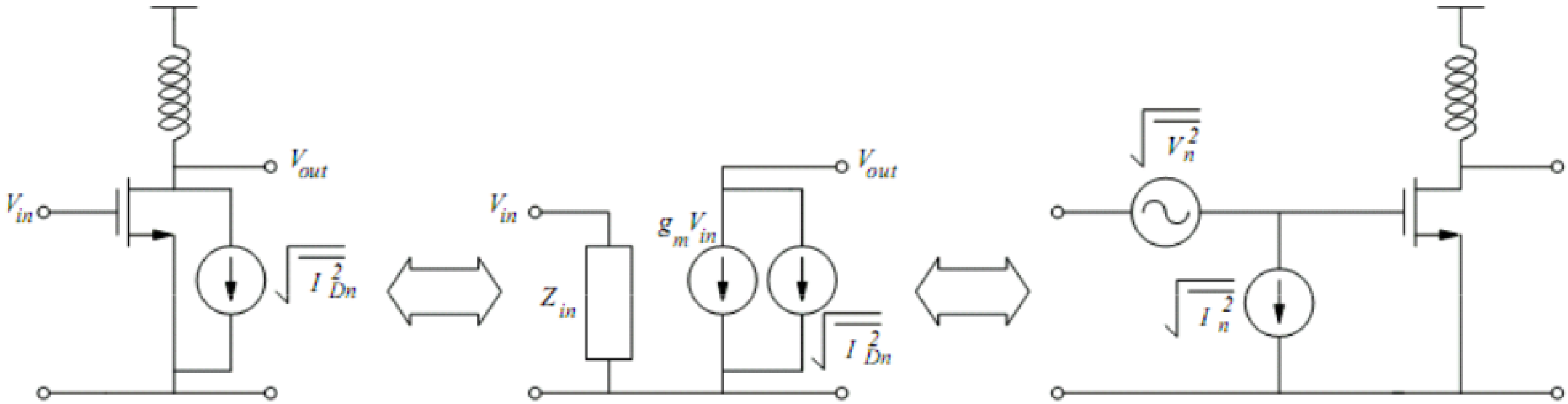


All noise sources in a 2 port network can be moved to the input. See App L of Gonzalez for conversion formula.

Example of Input Referred Noise

For a MOSFET in saturation the dominant noise is thermal noise in the channel :

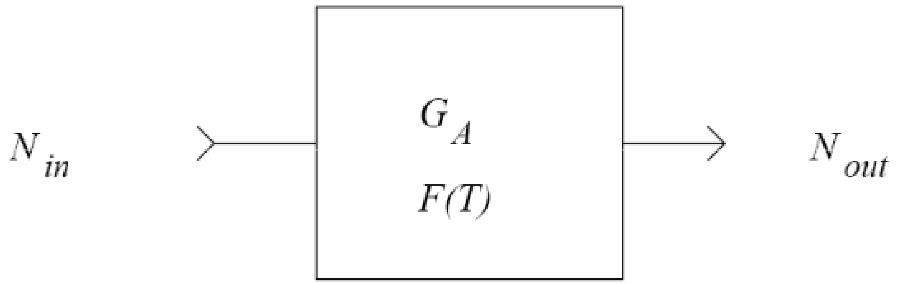
$$\overline{I_{Dn}^2} = 4kT\frac{2}{3}g_m$$



Shorting the input : $g_m^2\overline{V_n^2} = \overline{I_{nD}^2}$ gives $\overline{V_n^2} = \dfrac{8kT}{3g_m}$

Leaving the input open : $g_m^2\overline{I_n^2}\left|Z_{in}\right|^2 = \overline{I_{nD}^2}$ gives $\overline{I_n^2} = \dfrac{8kT}{3g_m\left|Z_{in}\right|^2}$

Noise Figure and SNR ratio



Property : F is equal to the Input to Output Signal to noise ratio :

$$F = \frac{SNR_{in}}{SNR_{out}}$$

Power Available from amplifier at o/p

$$SNR_{out} = \frac{S_{out}}{N_{out}} \quad \text{with } S_{out} \text{ the input signal power}$$

N_{in} and N_{out} are the i/p and o/p noise powers

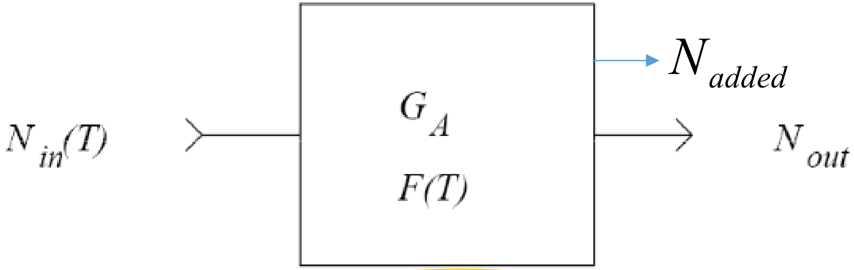
Power Available from Source

$$SNR_{in} = \frac{S_{in}}{N_{in}} \quad \text{with } S_{in} \text{ the output signal power}$$

Available Power gain

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{S_{in}}{N_{in}} \frac{N_{out}}{S_{out}} = \frac{N_{out}}{N_{in} \quad G_A} \quad \text{using : } G_A = \frac{S_{out}}{S_{in}}$$

Noise Figure

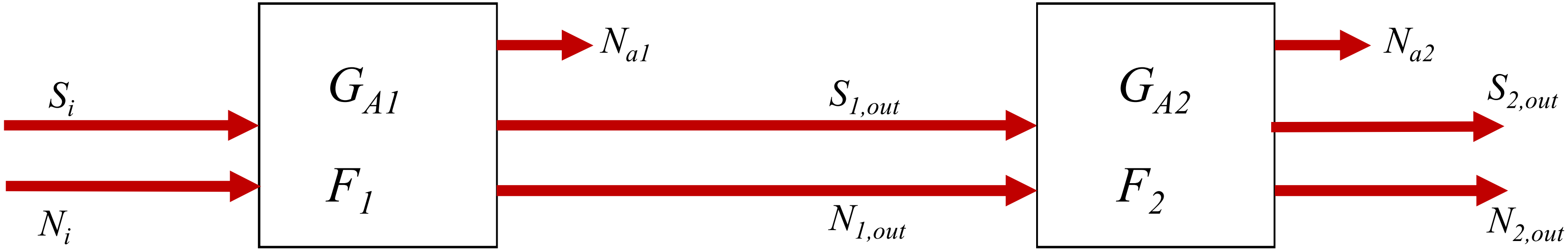


Noise Figure : $F = NF = \dfrac{N_{out}}{N_{in} \quad G_A}$ with :

Properties :

- $F \geq 1$
- $F = 1$ for zero added noise power $N_{added} = 0$ (ideal device)
- $F(T_0)$ is usually given for an input noise source at room temperature
- $N_{out} = G_A N_{in} + N_{added}$

Cascaded Network



$$N_{1,out} = N_{a_1} + G_{A_1} N_i$$

$$N_{2,out} = N_{a_2} + G_{A_2}\left(N_{a_1} + G_{A_1} N_i\right)$$

$$S_{1,out} = G_{A_1} S_i$$

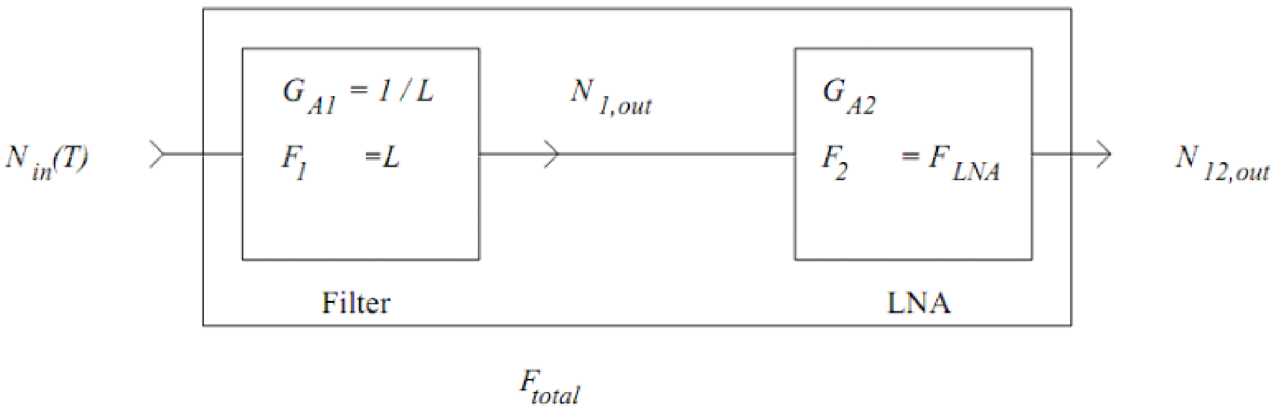
$$S_{2,out} = G_{A_1} G_{A_2} S_i$$

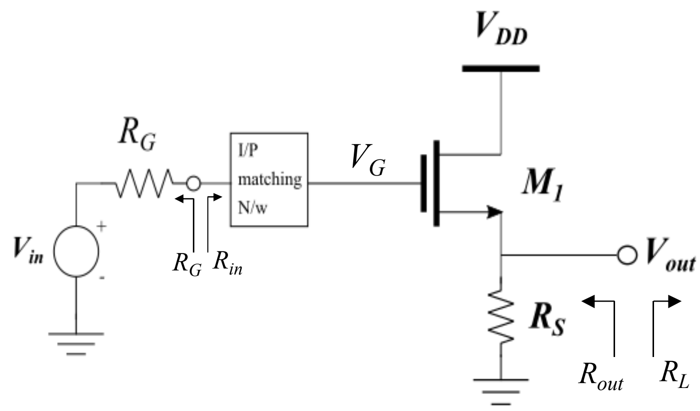
$$F_1 = \frac{SNR_{in}}{SNR_{out,1}} = \frac{S_i / N_i}{S_{1,out} / N_{1,out}} = \frac{N_{a_1} + G_{A_1} N_i}{G_{A_1} N_i} = 1 + \frac{N_{a_1}}{G_{A_1} N_i}$$

Similarly, $F_2 = 1 + \dfrac{N_{a_2}}{G_{A_2} N_i}$

$$F_{total} = \frac{SNR_{in}}{SNR_{out,2}} = \frac{S_i / N_i}{S_{2,out} / N_{2,out}} = \frac{N_{a_2} + N_{a_1} G_{A_2} + G_{A_2} G_{A_1} N_i}{G_{A_2} G_{A_1} N_i} = 1 + \frac{N_{a_1}}{N_i G_{A_1}} + \frac{N_{a_2}}{N_i G_{A_1} G_{A_2}} = F_1 + \frac{F_2 - 1}{G_{A_1}}$$

Filter in cascade with LNA





Source Follower

Total Noise Figure :

$$F_{total} = F_{filter} + \frac{F_{LNA} - 1}{G_{A1}} = L + \frac{F_{LNA} - 1}{1/L} = L \times F_{LNA}$$

using $G_{A1} = G_{filter} = 1/L$

Just need to add the noise figures in dB :

$$F_{total}|_{dB} = L|_{dB} + F_{LNA}|_{dB}$$

Say, $R_G = R_{in} = R_{out} = R_L = 50\Omega$

$V_G = \alpha V_{in} \quad (\alpha < 1)$

$$V_{out} = \frac{g_m(R_S \| R_L)}{1 + g_m(R_S \| R_L)} V_G = \frac{g_m(R_S \| R_L)}{1 + g_m(R_S \| R_L)} \alpha V_{in}, \quad \text{[Neglecting } r_o\text{]}$$

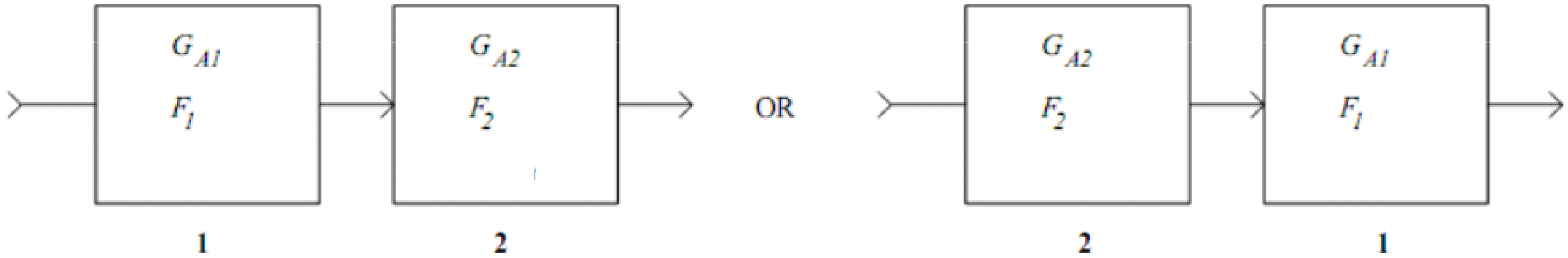
$$\Rightarrow \frac{V_{out}^2}{V_{in}^2} = \frac{g_m^2 (R_S \| R_L)^2}{[1 + g_m (R_S \| R_L)]^2} \alpha^2$$

Attenuation

$$\Rightarrow \frac{P_{AVN}}{P_{AVS}} = G_A = \frac{g_m^2 (R_S \| R_L)^2}{[1 + g_m (R_S \| R_L)]^2} \alpha^2 < 1$$

$$P_{AVN} = \frac{V_{out}^2 R_{out}}{(2R_{out})^2}, \quad P_{AVS} = \frac{V_{in}^2 R_G}{(2R_G)^2}$$

Cascade Ordering and Noise Measure



we need to have :

$$F_{12} < F_{21}$$

$$F_1 + \frac{F_2 - 1}{G_{A1}} < F_2 + \frac{F_1 - 1}{G_{A2}}$$

$$M_1 = \frac{F_1 - 1}{1 - \dfrac{1}{G_{A1}}} < \frac{F_2 - 1}{1 - \dfrac{1}{G_{A2}}} = M_2$$

M_1 and M_2 are called the noise measure.

Optimum source admittance Y_s

The noise figure is a function of the source admittance Y_s and Y_{opt} can be rewriten as :

$$F(Y_s) = F_{min} + \frac{R_n}{G_s}\left|Y_s - Y_{opt}\right|^2$$

where we have :

$$Y_{opt} = G_{opt} + jB_{opt}$$

- Y_{opt} is the optimal source admittance at which the noise figure is minimum :

  $F(Y_{opt}) = F_{min}$
- The locus of constant noise factor in the admittance plane Y_s, are circles centered around Y_{opt}.

Noise Figure in terms of reflection coefficients

The noise figure can be rewritten as a function of normalized admittances y_s and y_{opt} as :

$$F(Y_s) = F_{min} + \frac{r_n}{g_s}\left|y_s - y_{opt}\right|^2$$

where we have :

$$y_{opt} = \frac{Y_{opt}}{Y_0} = g_{opt} + jb_{opt} \qquad , r_n = \frac{R_n}{Z_0}$$

$$y_s = \frac{Y_s}{Y_0} = g_s + jb_s \qquad \frac{Z_{opt} - Z_0}{Z_{opt} + Z_0} = \frac{Y_0 - Y_{opt}}{Y_0 + Y_{opt}} = \frac{y_{opt} - 1}{y_{opt} + 1} \Rightarrow y_{opt} = \frac{1 + \Gamma_{opt}}{1 - \Gamma_{opt}}, y_s = \frac{1 + \Gamma_s}{1 - \Gamma_s}$$

The noise figure can then be rewritten as a function of reflection coefficients Γ_s and Γ_{opt} as :

$$F(Y_s) = F_{min} + \frac{4r_n\left|\Gamma_s - \Gamma_{opt}\right|^2}{(1 - |\Gamma_s|^2)|1 + \Gamma_{opt}|^2}$$

Constant Noise Figure Circles

$$\text{The noise figure } F(\Gamma_s) = F_{min} + \frac{4r_n\left|\Gamma_s - \Gamma_{opt}\right|}{(1-\left|\Gamma_s\right|^2)\left|1+\Gamma_{opt}\right|^2}$$

can be rewritten :

$$N_i = \frac{F_i - F_{min}}{4r_n} \times \left|1+\Gamma_{opt}\right|^2 = \frac{\left|\Gamma_s - \Gamma_{opt}\right|^2}{1-\left|\Gamma_s\right|^2}$$

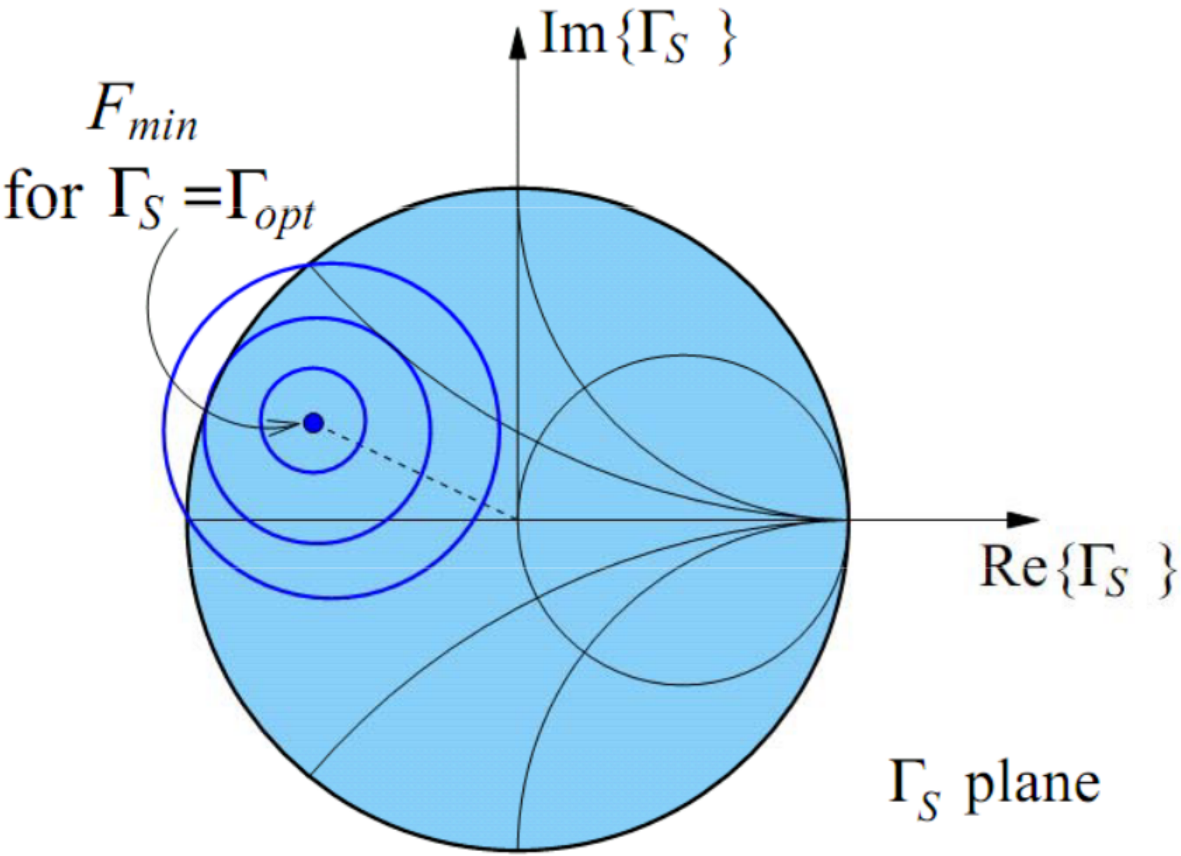
After some mathematical derivations we obtain the equation of a circle :

$$\left|\Gamma_s - C_i\right| = R_i$$

with :

$$C_i = \frac{\Gamma_{opt}}{1+N_i} \quad \text{and} \quad R_i = \frac{\sqrt{N_i^2 + N_i(1-\left|\Gamma_{opt}\right|^2)}}{1+N_i}$$

Constant Noise Figure Circles



Noise, Gain and DC power Trade-Off in RFICs

Need for an input matching trade - off (using for example M) :

· The minimum noise gure occurs for $Y_s = Y_{opt}$

· The maximum available power gain $G_A(Y_s)$ occurs for $Y_s = Y_{s,M}$ (assuming device is stable)

RFIC specic design issues :

· Both the bias point and the device size can be optimized in RFICs to optimize,

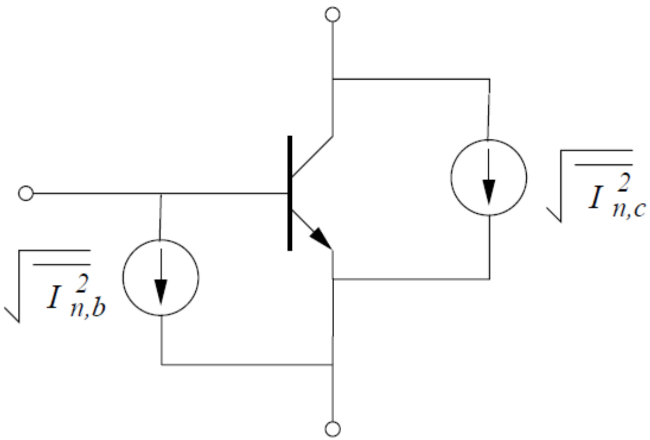
- the maximum power gain
- $F_{\min}$
- IP3 (SFDR)

· In cellular phone, talk time requires that we set a constraint on the power dissipation of the LNA : requiring a power constraint on the optimization.

Other Types of Noise Source: Shot Noise

Shot noise (Poisson process: $\overline{m(t)^2} - \left(\overline{m(t)}\right)^2 = \overline{m(t)}$ noise associated with the corpuscular nature of the electron (charge q) and its emission over a barrier (PN and Schottky diode, BJT). Shot noise is proportional to the DC current I:

$$\overline{I_n^2} = 2qI\Delta f$$



- $\sqrt{\overline{I_n^2}}$ is about 18 pA/$\sqrt{\text{Hz}}$ for 1 mA of I
- Shot noise is to be added to the thermal noise arising from the base r_b and emitter r_e resistance s.

Other Types of Noise Sources: 1/f noise in MOSFETs

Trapping and release of charges in the oxide with different time constants leads to Flicker noise:

$$\overline{I_n^2} = \frac{K}{W_g L_g C_{ox}} \frac{1}{f} \Delta f$$

· The corner frequency is the frequency at which the 1/f noise is equal to the thermal noise.

· A lower corner frequency is desirable.

· MOSFETS have corner frequencies around 10 KHz to 1 MHz (BJT : 10 - 100 Hz).

· 1 / f (pink noise) is to be added to the thermal noise in the FET channel (drain)and gate channel noise.

· See notes from Oslo university about noise calculation in BJT's and MOSFETS's.

Receiver Sensitivity

The sensitivity is the minimum input signal level which can be detected with an acceptable output SNR (SNR_{min}) for an input noise at room temperature T_0.

$$SNR_{out} \geq SNR_{min}$$

using $F = \dfrac{SNR_{in}}{SNR_{out}}$ this gives

Min SNR needed for baseband processing

$$SNR_{out} = \frac{S_{out}}{N_{out}} = \frac{1}{F} SNR_{in} = \frac{1}{F} \frac{S_{in}}{N_{in}} \geq SNR_{min}$$

So the minimum distinguishable signal :

$$P_{i,mds} = S_{in,min} = N_{in} \cdot F \cdot SNR_{min} = kT_0 \Delta f \cdot F \cdot SNR_{min}$$

In dBm this gives a sensitivity of :

$$S_{in,min}\big|_{dB} = 10\log(kT_0) + 10\log(\Delta f\big|_{Hz}) + F\big|dB + SNR_{min}\big|_{dB}$$

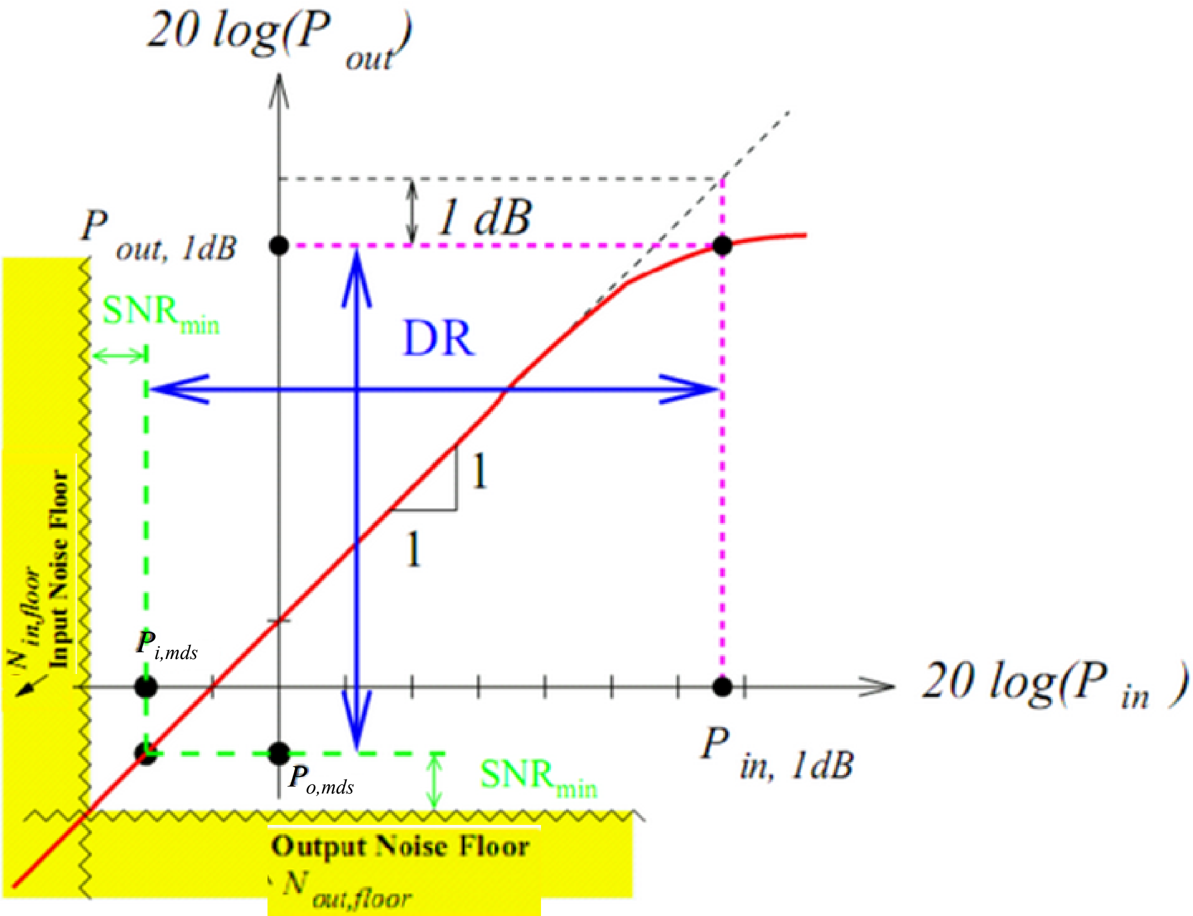
$$= N_{in,floor}\big|_{dB} + SNR_{min}\big|_{dB} = P_{i,mds}\big|_{dB}$$

where Nin, floor is the input referred noise floor ($SNR_{min} = 0$)

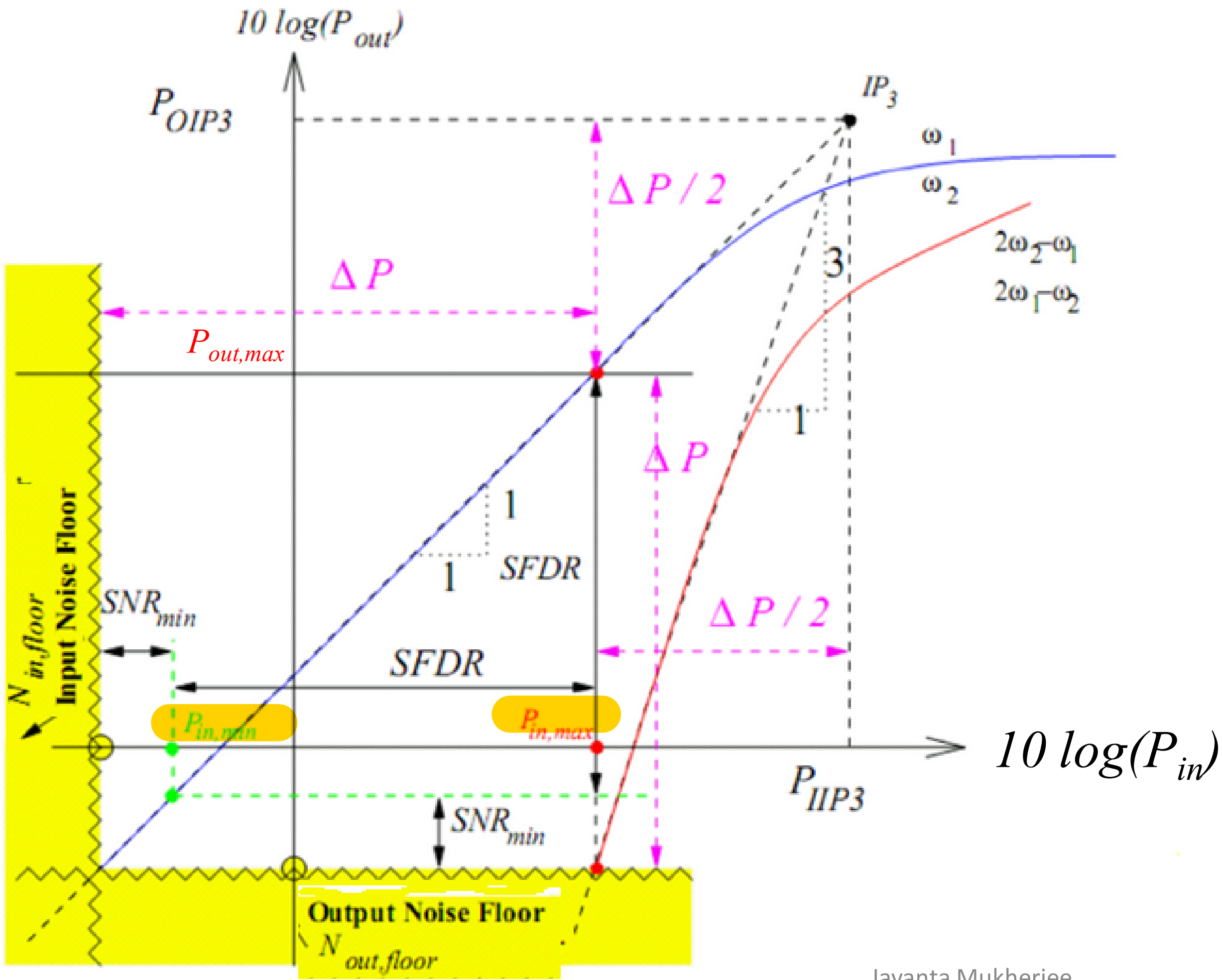
$$N_{in,floor}\big|_{dBm} = -174 dBm/Hz + 10\log(\Delta f\big|_{Hz}) + F\big|_{dB}$$

Dynamic Range

$$DR = P_{out,1dB} - P_{o,mds} = P_{in,1dB} - P_{i,mds}$$



Spurious Free Dynamic Range (SFDR)



Spurious Free Dynamic Range (Contd..)

$$SFDR = P_{in,\max}\Big|_{dB} - P_{in,\min}\Big|_{dB}$$

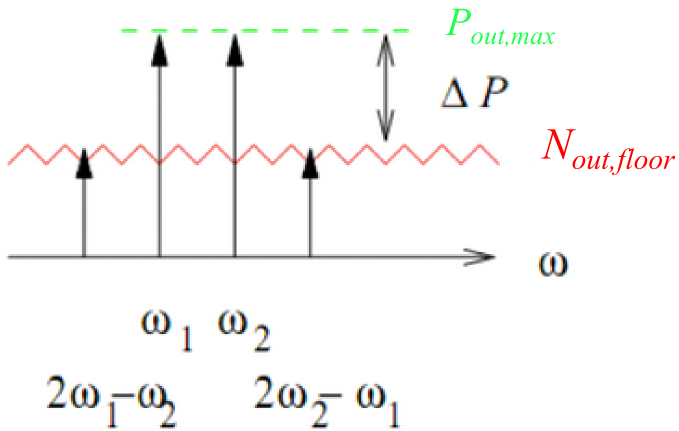
Since we have (all in dB)

$$P_{IIP3} = P_{in,max} + \frac{\Delta P}{2} = P_{in,max} + \frac{P_{in,max} - N_{in,floor}}{2}$$

we can therefore solve for $P_{in,max}$:

$$P_{in,max} = \frac{2P_{IIP3} + N_{in,floor}}{3}$$

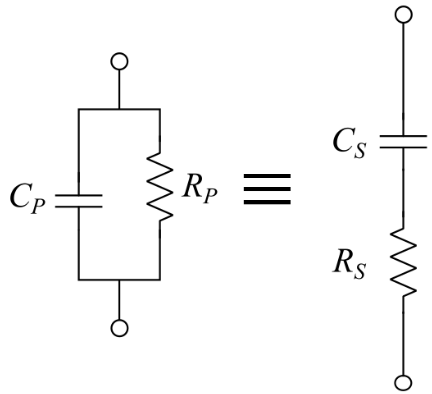
Resulting in the SFDR



$$SFDR = P_{in,max}\Big|_{dB} - P_{in,\min}\Big|_{dB} = \frac{2(P_{IIP3}\big|_{dB} - N_{in,floor}\big|_{dB})}{3} - SNR_{\min}\Big|_{dB}$$

Matching Networks

Consider the following circuit with $R_S << R_P$:

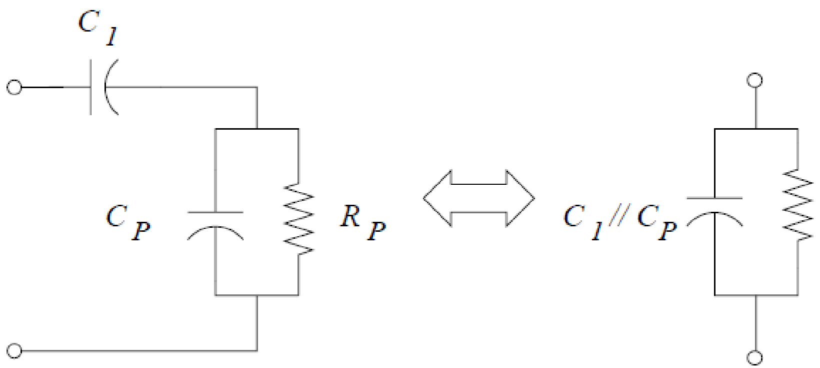


Both circuits have the same impedance at ω when :

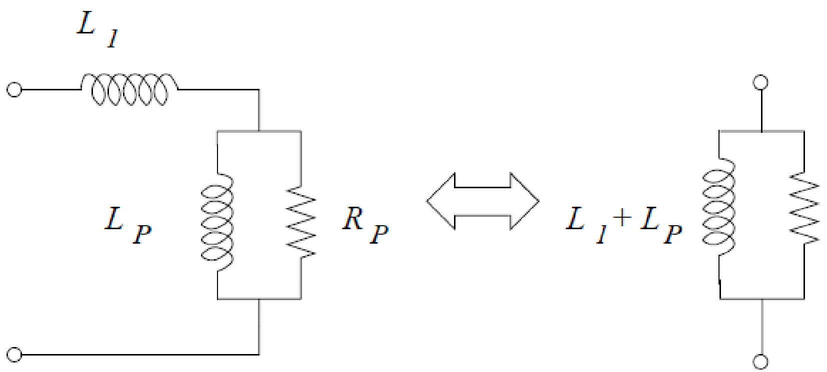
$$C_S \approx C_P \quad \text{and} \quad R_S \approx \frac{1}{R_P (C_P \omega)^2} \Rightarrow R_P \approx \frac{1}{R_S (C_S \omega)^2}$$

· Can be used to decrease R_P to a lower value or increase R_S to a higher value.

High Q Circuits Used for Increasing R_P



$$R_{tot} = (1 + C_P/C_1)^2 R_P$$



$$R_{tot} = (1 + L_1/L_P)^2 R_P$$