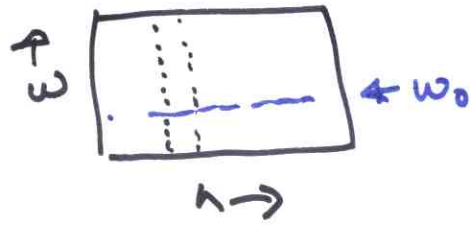
STFT: Filtering viewpoint

$$X(n, \omega_0) = \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, e^{-j\omega_0 m}$$

w: fin. dur window of dur = L samples

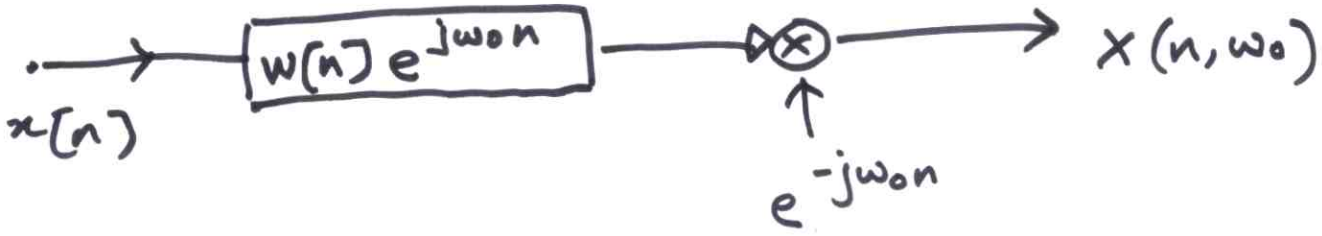
$$= \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, e^{-j\omega_0 m}\, e^{j\omega_0 n}\, e^{-j\omega_0 n}$$

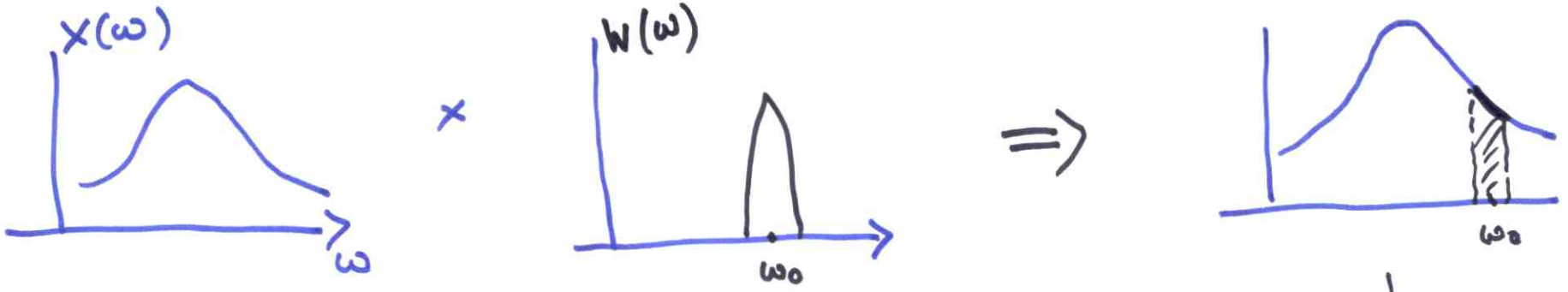


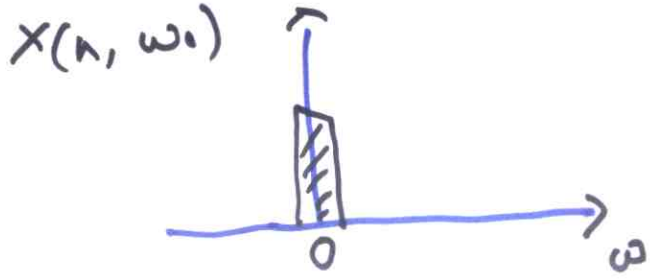
$$= e^{-j\omega_0 n} \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, e^{j\omega_0 (n-m)}$$

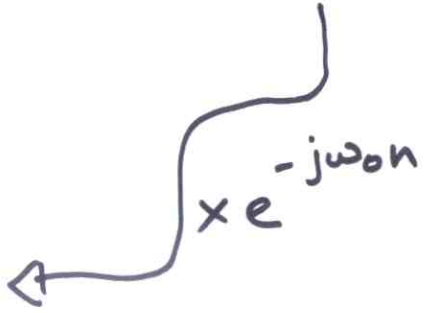
$$= e^{-j\omega_0 n} \left(x[n] * w[n] e^{j\omega_0 n} \right)$$

$$X(n,\omega_0) = e^{-j\omega_0 n}\left(x[n] * w[n]e^{j\omega_0 n}\right)$$



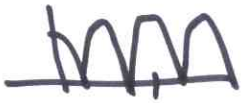


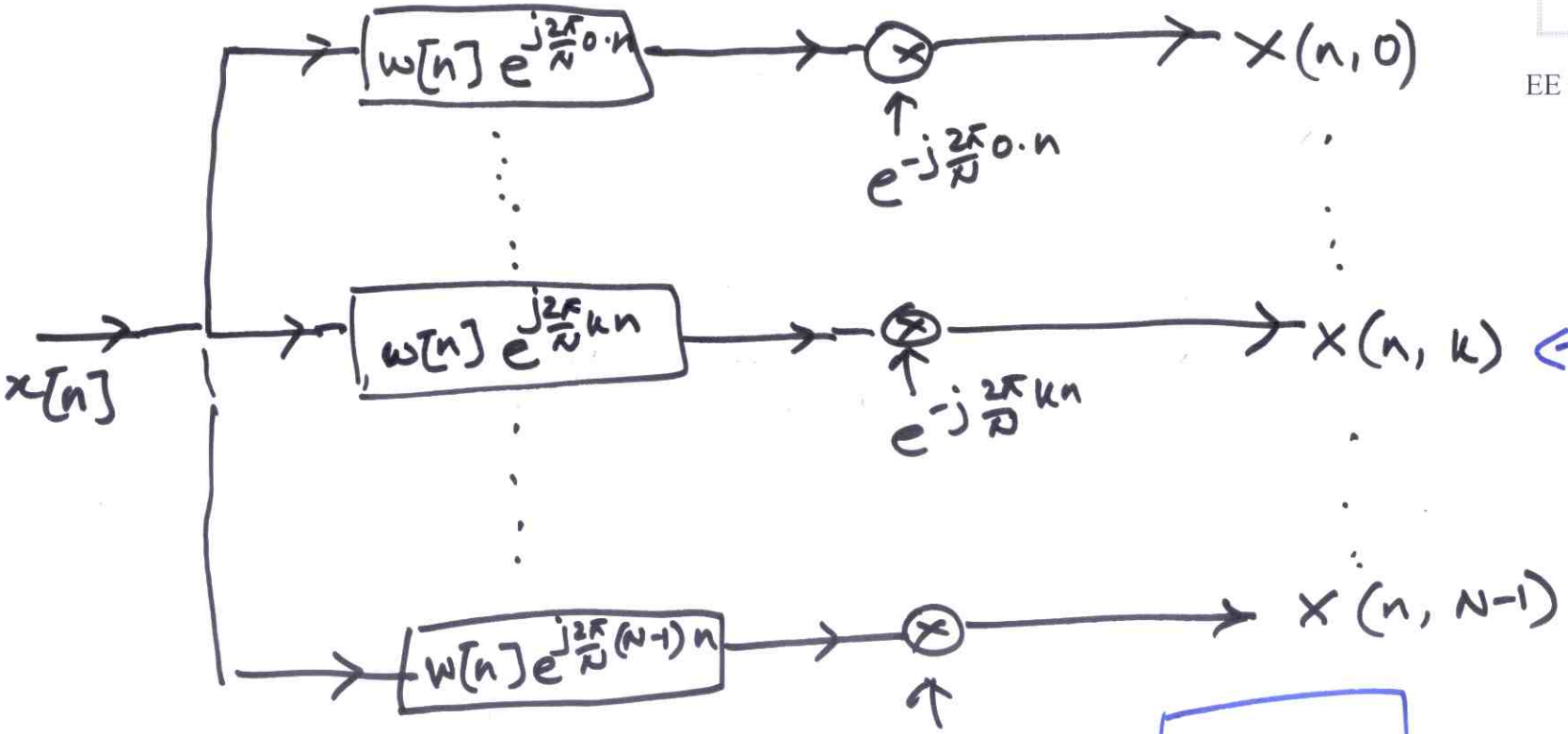




For the DFT component $e^{j\frac{2\pi}{N}k}$, $k: 0 \ldots N-1$

$X(n,\omega)$

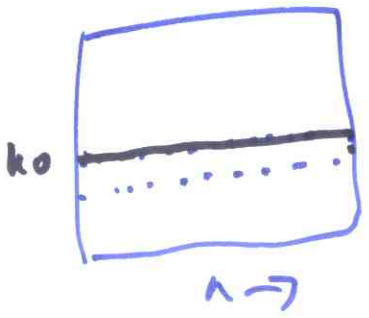




low-pass time-signal

"Bank of b.p.f.s"

$X(n,k) = \underset{\sim}{A_0} e^{j(\omega_0-\omega_k)n}$



$$X(n,k) = \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, e^{-j\frac{2\pi}{N}km}$$

$$\Rightarrow \frac{1}{N}\sum_{k=0}^{N-1} X(n,k)\, e^{j\frac{2\pi}{N}kn} = \sum_{m=-\infty}^{\infty} x[m]\, w[n-m]\, \frac{1}{N}\sum_{k=0}^{N-1} e^{-j\frac{2\pi}{N}km}\cdot e^{j\frac{2\pi}{N}kn}$$

Sum of the BPF outputs at each "n"

$= 1$ for $(n-m) = rN$

$= 0$, else

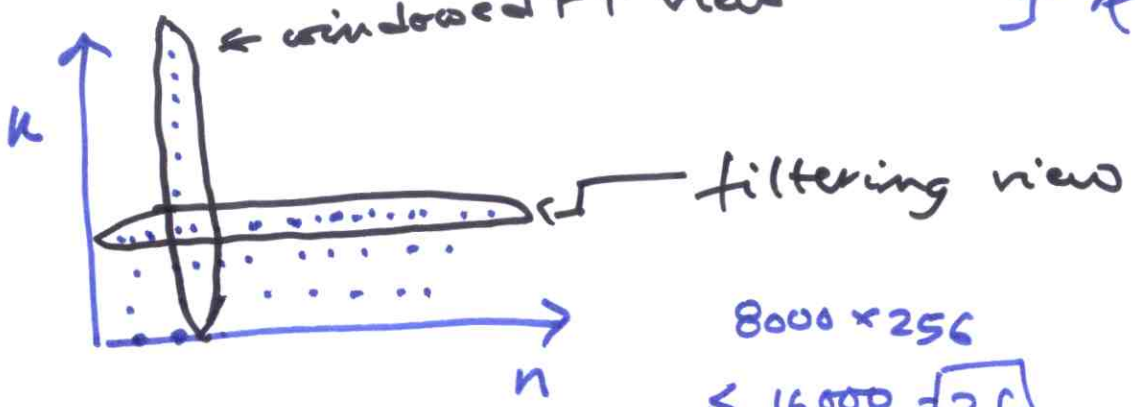
$= x[n]w[0] + x[n-N]w[N] + \ldots$ (the $w[N]$ term $= 0$)

for $N \geq L$ (window len)

The 2 interpretations

$X(n, k)$ "ω"

$\uparrow$ $\uparrow$ $N \geq L$



8000×256

$\leq 16000 = 2f_s$

$N \geq L$

Consider increasing the window hop upward from $n = 1$ sample

Based on the 2 properties

- a time signal bandlimited to B Hz must be sampled at 2B samples/sec
- a finite dur of L samples requires the DFT to be computed at L unit spaced freqs in $(-\pi, \pi]$

window len = L & b.w. = B Hz

Samp. rate of STFT = $2B \cdot L$

$\frac{2\pi}{L} = 2\pi \Delta f$

$\Delta f_a = \Delta f \cdot f_s$

(filter bw in Hz) $= \frac{f_s}{L}$ — f_s : samp freq (Hz)

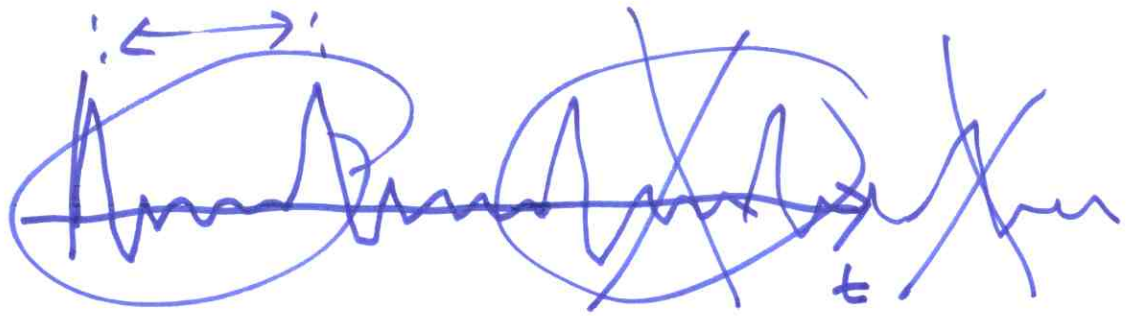
$B = \frac{2\pi}{L}$ ← d-t freq.

Hamming → $4\pi/L$

∴ Sampling rate of STFT $= 2 \cdot \frac{f_s}{L} \cdot L = 2 f_s$

$L = N$

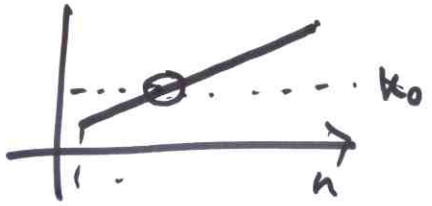
$4 f_s$

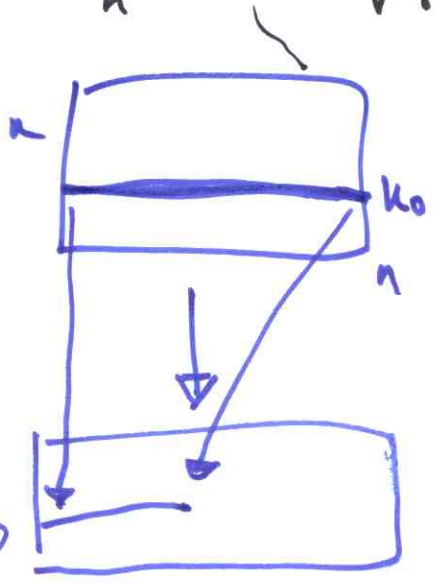


Control signals

$A_k(n), \theta_k(n)$

$A_k \cdot e^{j\theta_{kn}} \cdot e^{j\omega_0 n}$





$A_k e^{j\theta_k} = e^{j\frac{2\pi}{N}k_0}$

$Y(n,k) = H(k)\, X(n,k)$

$Y(n,k) = H(n,k)\, X(n,k)$

time-varying filter