Levinson-Durbin recursion method to solve:

$$\bar{R}A = \bar{r} \quad \text{for unknowns } \{a_k\}$$

$$\equiv \quad r[i] = \sum_{k=1}^{p} a_k r[i-k] \quad \text{given}$$

$r[0] \dots r[p]$



We shall find:

$\{a_k\}$ coeffs & error for the $p^{th}$-order predictor

i.e. $a_1^{(p)}, a_2^{(p)}, \dots a_p^{(p)}$ & $E^{(p)}$ $\quad \boxed{\{a_k\}, E}$

The basic idea for the recursion is to find the soln $\{a_k\}^p$ from $\{a_k\}^{p-1}$.

It is less complex than matrix inversion, exploits Toeplitz nature of $\bar{R}$.

## Steps

$E^{(0)} = r[0]$ $\quad E^{(1)} \quad a_1^{(1)}$

$E^{(2)} \quad a_1^{(2)}, a_2^{(2)}$

$E^{(3)} \quad a_1^{(3)}, a_2^{(3)}, a_3^{(3)}$

$\boxed{i=1}$

reflection coeff $\rightarrow$ $k_1 = \dfrac{r[1]}{E^{(0)}}$

$$a_1^{(1)} = k_1, \quad E^{(1)} = (1-k_1^2)E^{(0)};$$

for $\boxed{i = 2, \ldots p}$

$$k_i = \left(r[i] - \sum_{j=1}^{i-1} a_j^{(i-1)} r[i-j]\right) \cdot \frac{1}{E^{(i-1)}};$$

$$a_{ii} = k_i$$

for $j = 1, 2, \ldots i-1$

$$a_j^{(i)} = a_j^{(i-1)} - k_i a_{i-j}^{(i-1)}$$

end

$$E^{(i)} = (1-k_i^2)E^{(i-1)}$$

$|k_i| \leq 1$

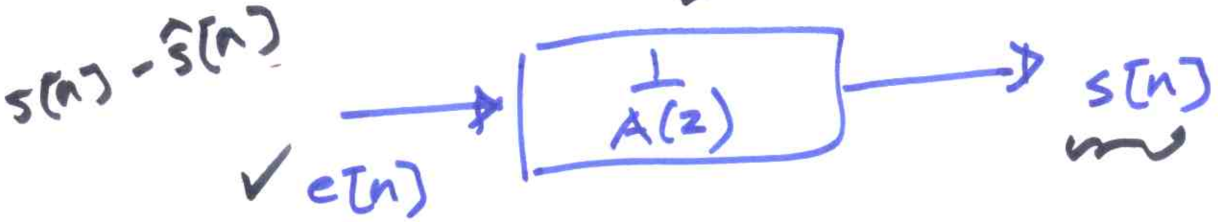
$\Updownarrow$

all roots of $A(z)$ lie within $\bigcirc$

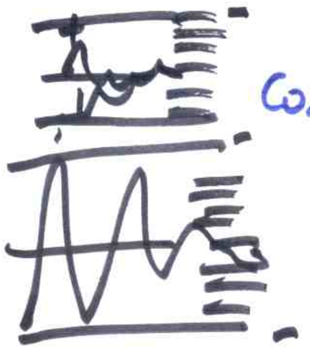
$\boxed{\text{end}}$

We have $e[n] = s[n] - \sum_{k=1}^{p} a_k s[n-k]$ . . . (a)

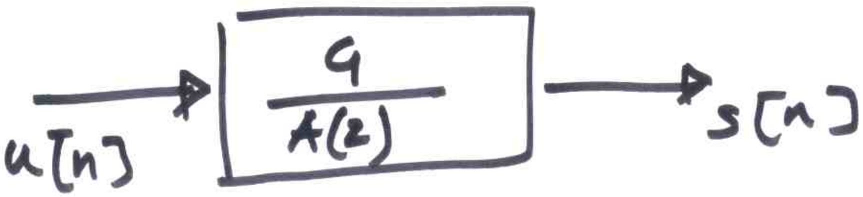
$\Rightarrow \quad E(z) = S(z)\, A(z)$




$A(z) = 1 - \sum_{k=1}^{p} a_k z^{-k}$



Comparing with our speech model:




$Gu[n] = s[n] - \sum_{k=1}^{p} a_k s[n-k]$ . . . (b)

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To compute G :

$$G^2 \sum_{n=0}^{N-1} u^2[n] = \sum_{n=0}^{N-1} e^2[n] = E_{min}$$

Assume $\sum_{n=0}^{N-1} u^2[n] = 1$

$$\Rightarrow \quad G^2 = E_{min} = r[0] - \sum_{k=1}^{p} a_k r[k]$$

Ex. Consider a first-order lin. predictor for $\{x[n]\}$

i.e. $\hat{x}[n] = a\,x[n-1]$

Find $a$ based on least sq error min.



Sol$^n$ :

$$e[n] = x[n] - \hat{x}[n]$$

$$E = \sum_n e^2[n] = \sum_n \left(x[n] - a\,x[n-1]\right)^2$$

$$= \sum x^2[n] - 2a \sum x[n]\,x[n-1] + a^2 \sum x^2[n-1]$$

$$= r_{xx}[0] - 2a\,r_{xx}[1] + a^2\,r_{xx}[0]$$

$$= r_{xx}[0] \left(1 + a^2 - 2a \frac{r_{xx}[1]}{r_{xx}[0]}\right)$$

$$\frac{\partial E}{\partial a} = 0 \quad \Rightarrow \quad r_{xx}[0] \cdot 2a = 2 r_{xx}[1]$$

$$\Rightarrow \quad a = \frac{r_{xx}[1]}{r_{xx}[0]} = \rho_{xx}[1]$$

$$E_{min} = r_{xx}[0] - a \cdot r_{xx}[1]$$

$$= r_{xx}[0] \left( 1 - a \frac{r_{xx}[1]}{r_{xx}[0]} \right)$$

$$= r_{xx}[0] \left( 1 - \rho_{xx}^2[1] \right)$$

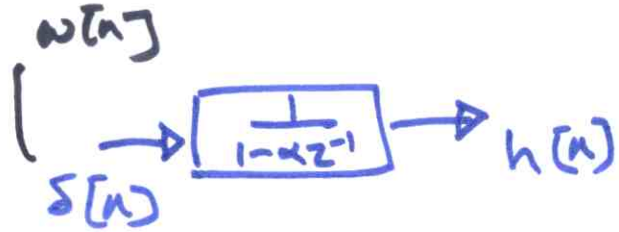
But $\rho_{xx}^2[1] \leq 1$

$\Rightarrow \forall \; \rho_{xx}[1] \neq 0$, $\underset{\uparrow \text{ energy of } \{e[n]\}}{E_{min}} < r_{xx}[0] \leftarrow$ signal energy

Consider a signal $\{h[n]\}$ with acf $r_{hh}[0]$, $r_{hh}[1]$, ....

If we apply LSE minimis$^n$ for the best first-order predictor:

$$a = \frac{r_{hh}[1]}{r_{hh}[0]}$$




Next, let us assume that $h[n]$ is indeed the o/p of a first-order all-pole system with coeff $\alpha$, $|\alpha| < 1$ & impulse input.

$A(z) = 1 - \alpha z^{-1}$

$$\Rightarrow \boxed{h[n] = \alpha h[n-1] + \delta[n]}$$

$$\Rightarrow \sum h[n]h[n-1] = \alpha \sum_n h^2[n-1] + \sum_n \overset{w[n]}{\delta[n]}\, h[n-1]$$

$$\Rightarrow r_{hh}[1] = \alpha r_{hh}[0] + 0 \Rightarrow \alpha = \frac{r_{hh}[1]}{r_{hh}[0]}$$

$$e[n] = h[n] - \hat{h}[n]$$

$$\hat{h}[n] = \alpha h[n-1]$$

$$= h[n] - \alpha h[n-1]$$

$$= \delta[n]$$

$$w[n]$$