LCCDE ← suitable mathematical description for many physical systems.

Ex. Mass-spring

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0 \quad F(t) \leftarrow \delta(t)$$

$$\Rightarrow x(t) = h(t)$$

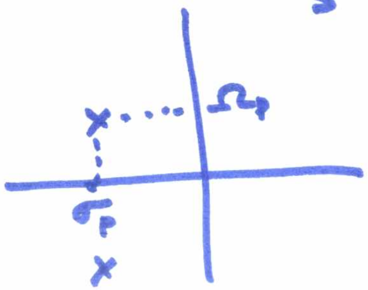
$$\Rightarrow H(s) = \frac{1}{ms^2+cs+k}$$

Under-damped system

Vocal-tract model for a vowel :

Consider a 2nd order under-damped system

Let the 2 poles be $s_p = -\sigma_p \pm j\Omega_p$



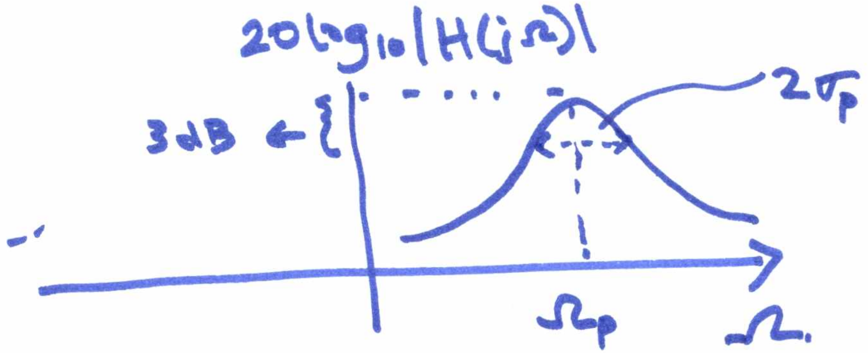
$$H(s) = \frac{G}{(s+\sigma_p+j\Omega_p)(s+\sigma_p-j\Omega_p)}$$

$$= \frac{G}{(s+\sigma_p)^2+\Omega_p^2}$$

How the formant param... relate to pole loc's

Single-formant resonator

$$H(j\Omega) = \frac{G}{(j\Omega+\sigma_p)^2 + \Omega_p^2}$$



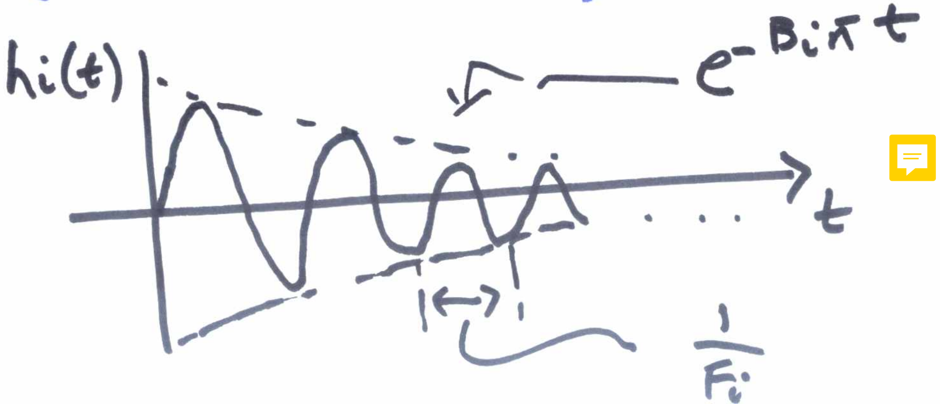
$H(s)$ $H(j\Omega)$ $h(t)$

$$h(t) = c_1 e^{(-\sigma_p + j\Omega_p)t} + c_2 e^{(-\sigma_p - j\Omega_p)t} \cdot u(t)$$

$$= e^{-\sigma_p t} A \sin(\Omega_p t + \theta) \cdot u(t)$$

$\Omega_p = 2\pi F_i$

$2\sigma_p = 2\pi B_i$



Equivalent digital resonator model

$s \to z$ transform^n using impulse-invariance

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$$h[n] = T.\underbrace{h_c(nT)}_{h_c(t)}$$

transforms so that the following is the mapping of
$H(s) \to H(z)$ poles of s plane to poles of z-plane.

$$\boxed{z = e^{sT}}, \quad T = \text{samp. interval}$$

$s_p = -\sigma_p \pm j\Omega_p$

$z_p = re^{j\theta}$

$\neq$ i^{th} resonance $\longrightarrow$ $r_i = e^{-B_i \pi T}$, $\theta_i = 2\pi F_i T$

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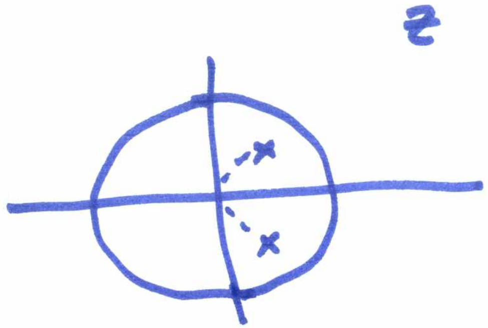

$re^{j\pm\theta}$

$$r_i = e^{-B_i \pi T}$$

$$\theta_i = 2\pi F_i T$$

→ F_i : formant freq (Hz)

→ B_i : formant b.w. (Hz).

$$H(z) = \frac{K}{(1-re^{j\theta}z^{-1})(1-re^{-j\theta}z^{-1})} = \frac{1}{1-2r\cos\theta z^{-1}+r^2z^{-2}}$$

↓

$H(e^{j\omega})$