Consider a continuous-time signal $x_c(t)$

We have its Fourier spectrum:

$$X_c(j\Omega) = \int_{-\infty}^{\infty} x_c(t) e^{-j\Omega t} dt$$

Discrete-time signal obtained by sampling

$$x[n] = x_c(nT), \quad -\infty < n < \infty$$

Now the F.T. of the d-t signal is the DTFT

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

A continuous function of 'ω' in $[-\pi, \pi]$

Q. How does the DTFT represent the c-t signal spectrum?

A. If the sampling rate is adequate, we have:

$$X(e^{j\omega}) = \frac{1}{T} X_c\left(j\frac{\omega}{T}\right)$$ ← linear mapping of freq axes

$$\boxed{\Omega = \frac{\omega}{T}}$$

$X_c(j\Omega)$

$\left(-\frac{\Omega_s}{2}, \frac{\Omega_s}{2}\right]$ or $\left(-\frac{f_s}{2}, \frac{f_s}{2}\right]$ $\quad \Omega = 2\pi f_a$ ← Hz

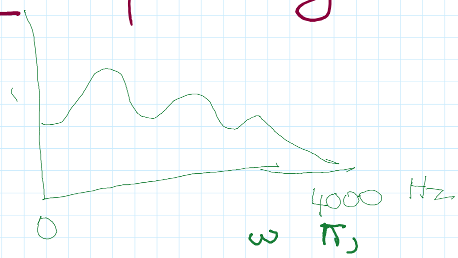
$\Downarrow$

$X(e^{j\omega})$

$X(e^{j\omega})$

$(-\pi, \pi]$ or $(-0.5, 0.5]$ $\omega = 2\pi f$

Example: speech signal sampled at 8000 samp/sec.



$f_s/2$

Source filter modeling

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Excitation Source $\longrightarrow$ Vocal tract Filter $\longrightarrow$ Speech signal

$x[n]$ $\qquad$ $h[n]$ $\qquad$ $s[n]$

$$s[n] = x[n] * h[n]$$ $\leftarrow$ convolution

$$S(e^{jw}) = X(e^{jw}) \times \underbrace{H(e^{jw})}_{\text{freq. resp.}}$$ $\leftarrow$ product

A discrete-time periodic signal:

$$x[n] = \sum_{k=0}^{N-1} c_k e^{j\frac{2\pi}{N}kn}$$

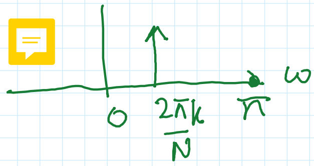
Period = N samples

$$\omega_0 = \frac{2\pi}{N}$$

Given signal - DTFT pair:

$e^{j\omega_0 n}$ $\quad e^{j\omega n}$

$$e^{j\frac{2\pi}{N}kn} \longleftrightarrow 2\pi\,\delta\left(\omega - \frac{2\pi}{N}k\right)$$

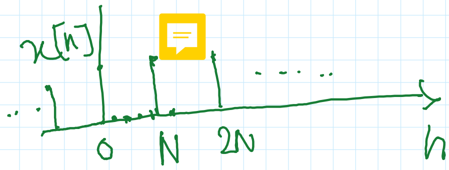


We have:

$$X(e^{j\omega}) = \sum_{k=0}^{N-1} 2\pi\, c_k\, \delta\left(\omega - \frac{2\pi}{N}k\right)$$

A special d-t periodic signal: "impulse train"

$$x[n] = \sum_{m=-\infty}^{\infty} \delta[n - mN]$$



$$\Rightarrow c_k = \frac{1}{N}\sum_{k=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}nk} = \frac{1}{N}$$

$$\therefore X(e^{j\omega}) = \frac{2\pi}{N}\sum_{k=0}^{N-1} \delta\left(\omega - \frac{2\pi}{N}k\right)$$