$$E = \sum_{n=-\infty}^{\infty} \left(x[n] - \hat{x}[n] \right)^2$$

$$= \sum_{n=-\infty}^{\infty} \left\{ x[n] - \sum_{k=1}^{p} a_k \, x[n-k] \right\}^2$$

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To solve for the unknown model paramts $\{a_k\}$ (for chosen p):

we set

$$\frac{\partial E}{\partial a_i} = 0 \quad \text{for } i = 1, \ldots p.$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} \left(x[n] - \sum_{k=1}^{p} a_k \, x[n-k] \right) \times - x[n-i] = 0$$

5 $\Rightarrow \sum_{n} x[n] \cdot x[n-i] = \sum_{k=1}^{p} a_k \sum_{n} x[n-i] \, x[n-k] \ldots$ 5

$$\Rightarrow \quad r[i] = \sum_{k=1}^{p} a_k \, r[i-k], \quad i = 1 \ldots p$$

(p eq^{n}s in p unknowns)

Using $r[i] = r[-i]$

$$\begin{bmatrix} r[0] & r[1] & \cdots & r[p-1] \\ r[1] & r[0] & & r[p-2] \\ & & & \\ r[p-1] & \cdots & & r[0] \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} = \begin{bmatrix} r[1] \\ r[2] \\ \vdots \\ r[p] \end{bmatrix}$$

(6) ... $\boxed{\bar{R} A = \bar{r}}$ ← Solve for A

$p \times p$ matrix of elements $r[|i-k|]$, $1 \le i, k \le p$

$(r[1], r[2], \ldots r[p])^T$

$A = (a_1, a_2 \ldots\ldots a_p)^T$

"Toeplitz" matrix ← R

Now, let us consider E at optimal $\{a_k\}$

$$E = \sum_n \left(x[n] - \sum_{k=1}^{p} a_k x[n-k] \right)^2$$

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Substitute (5) in above : we can show

$$E_{min} = \sum x^2[n] - \sum_{k=1}^{p} a_k \sum_n x[n-k]\, x[n]$$

$$= r[0] - \sum_{k=1}^{p} a_k r[k]$$