REFRACTIVE INDEX OF A TRANSPARENT LIQUID

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Abstract. This paper describes an alternative method to determine the refractive index of transparent liquids. The method only requires a laser pointer, a ruler and modified fish tank.

1. Introduction

Measuring the refractive index n of a substance is part of every introductory physics and organic chemistry lab course, but the usage of this measurement extends beyond the classrooms. Industrially, for example, it is most often used to determine the concentration of a dissolved solute in liquid samples. The most common application is measuring the concentration of sugar dissolved in water, such as in fruit juices and juice concentrates. For the case of transparent liquids, the refractive index can be determined using different methods. However, most of these methods make use of lenses [1] and diffraction gratings [2], requiring plenty of mathematical approximations and conceptual knowledge of optical physics prior to doing the experiment. These facts might distract the students from the main objective of the experiment.

In this paper we propose an alternative and simple method for measuring the refractive index of a transparent liquid (for a specific wavelength) by measuring the lateral displacements that a laser beam suffers due to refraction while passing through a liquid of different thicknesses. To test the method validity, we assessed the refractive index of a water-based sugar solution as a function of its concentration (measured in percentage by mass). Since the entire experiment execution time is considerably long (approx. 3 hours), it can be split into several parts, so that some of them are suitable for either a large lecture or a small class. Alternatively, the entire experiment could be assigned to students as a project. The only materials required are a laser pointer, a ruler, graph paper, and a modified fish tank. We shall commence by describing the experimental setup, followed by the results of the measurements, and the conclusions.

2. Description of the Method

A schematic diagram of the experimental setup used to determine the refractive index of a transparent liquid is depicted in figure 1. It consists of a fish tank in the form of a rectangular parallelepiped and a laser pointer. The tank with dimensions $a \times h \times c$ is made out of transparent glass and stands on a sheet of graph paper that serves to locate the position of the laser beam spot. Figure 2 shows a front view of the tank and the path followed by the laser beam.

When the tank is empty, the laser beam follows the path $\overline{qop}$. As the height x of the liquid in the tank changes, the distance y of the refracted beam spot measured from the right wall also changes. Applying Snells law at the interface air-liquid we have

$$n_1 \sin\theta = n_a \sin\beta \tag{1}$$

where n_1, n_a are the refractive indices of air and the liquid respectively; θ y β are the angles of incidence and refraction measured with respect to the normal line to the liquid surface.

From the geometry of the system

$$\tan\theta = \frac{a}{h} = \frac{m+y}{x} \tag{2}$$

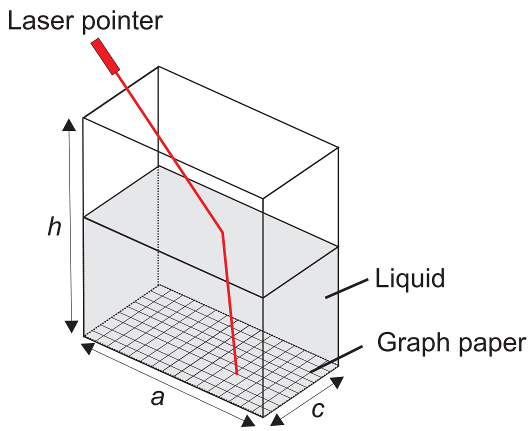

Figure 1. Schematic diagram of the experimental setup used to determine the refractive index of a liquid

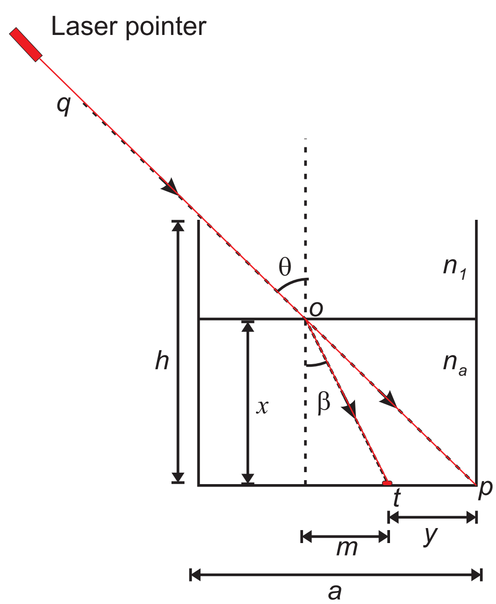

Figure 2. Front view of the tank

$$\tan\beta = \frac{m}{x} \tag{3}$$

Now let us manipulate these three equations. From equations (2) and (3) we get

$$ax = h[y + x\tan\beta] \tag{4}$$

From equation (1) we can build the triangle showed in figure 3 and from this triangle

$$\tan\beta = \frac{\sin\theta}{\sqrt{n_a^2 - \sin^2\theta}} \tag{5}$$

From equation (2) we can build the triangle showed in figure 4 and from this triangle

$$\sin\theta = \frac{a}{\sqrt{a^2 + h^2}} \tag{6}$$

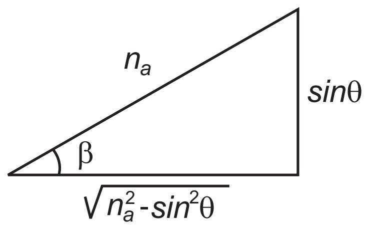

Figure 3.

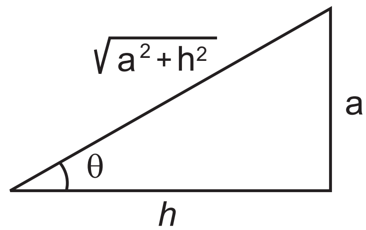

Figure 4.

Substituting the value of $\sin\theta$ from equation (6) into equation (5) we get

$$\tan\beta = \frac{a}{\sqrt{n_a^2(a^2+h^2)-a^2}} \tag{7}$$

Substituting the value of $\tan\beta$ from equation (7) into equation (4) we get

$$ax = h[y + \frac{xa}{\sqrt{n_a^2(a^2+h^2)-a^2}}] \tag{8}$$

From equation (8) we finally obtain

$$a^2[h^2x^2 + (ax-hy)^2] = n_a^2(a^2+h^2)(ax-hy)^2 \tag{9}$$

Equation (9) clearly shows that the relationship between the x and y variables is not so simple. This equation simply tells us that the value of n_a can be determined if the quantities a, h, x and y are measured. However, our intention is to be able to determine n_a in a systematic fashion. This can be done by making the following changes of variable: $Y = a^2[h^2x^2 + (ax-hy)^2]$ and $X = (a^2+h^2)(ax-hy)^2$. As a result, we have a linear relationship, $Y = mX + b$, where the $Y-$intercept is $b = 0$ and the slope of the line is related to the refractive index of the liquid as, $n_a = \sqrt{m}$.

3. Results

Figure 5 shows the actual equipment used to determine the refractive index of a liquid. The dimensions of the tank used were a = 196.5 mm, c = 50 mm and h = 200 mm. It should be noted that the fish tank width given by c is irrelevant to carry out the experiment. In a PYREX glass beaker-1000mL containing 500 g of tap water, regular table sugar was added until 16.7%, 28.6%, 37.5%, 50.0% and 55.0% levels of concentration were obtained. For each of these water-based solutions, the y lateral displacements of the laser beam spot (1-mW He-Ne laser, wavelength 0.6328 μm) were measured as a function of the x height in the tank. It should be noted that the relationship between these variables is not linear as shown in figure 6 for the 37.5% concentration. Figure 8 summarize the results for all concentrations mentioned above.

From these data, the quantities Y and X were input in an Excel spreadsheet. Figure 7 shows the plot of Y against X for the 37.5% concentration; and figure 9 shows the same dependencies for all concentrations mentioned above and their corresponding best-fit line. The refractive index of each solution was found as the square root of

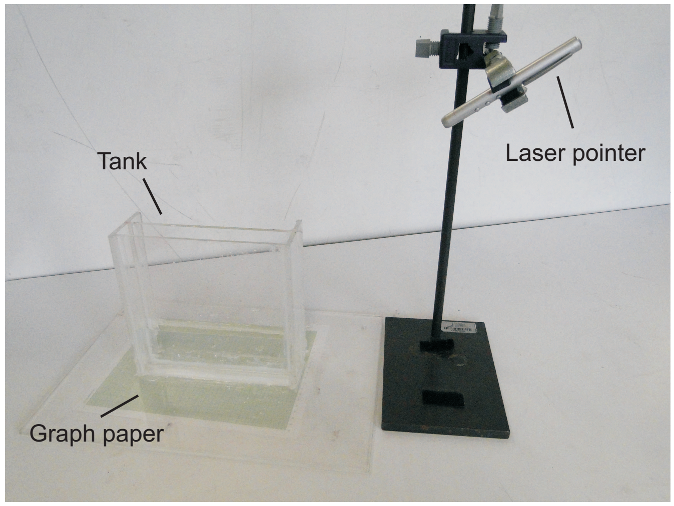

Figure 5. Equipment used to determine the refractive index of a transparent liquid. It consists of a modified fish tank and a laser pointer.

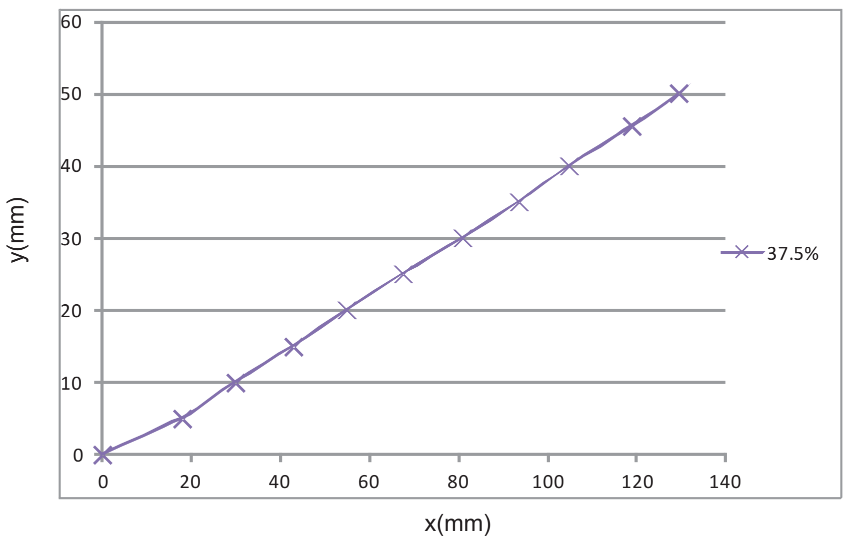

Figure 6. Lateral displacements of the laser beam spot as a function of the solution height in the fish tank for a 37.5% concentration. Crosses: represent the experimental data. Solid line is the line that joins the crosses.

the slope of the tangent lines. Figure 10 shows the refractive index variations with concentration for the water-based solution. Thus, for regular table sugar solutions containing less than 60% sugar there exists a linear relationship between the refractive index and sugar concentration, $n_a = 1.60 \times 10^{-3}C + 1.30$. The same linear tendency is exhibited with different kinds of sugars using a different technique [3]. Parenthetically, we could not increase the sugar concentration to values higher than 60% because the laser beam spot at the bottom of the fish tank started taking a blurred shape. Moreover, there was no difference in the results when a standard laser pointer (red light) was used, which is convenient in case the laboratory does not have enough lasers available.

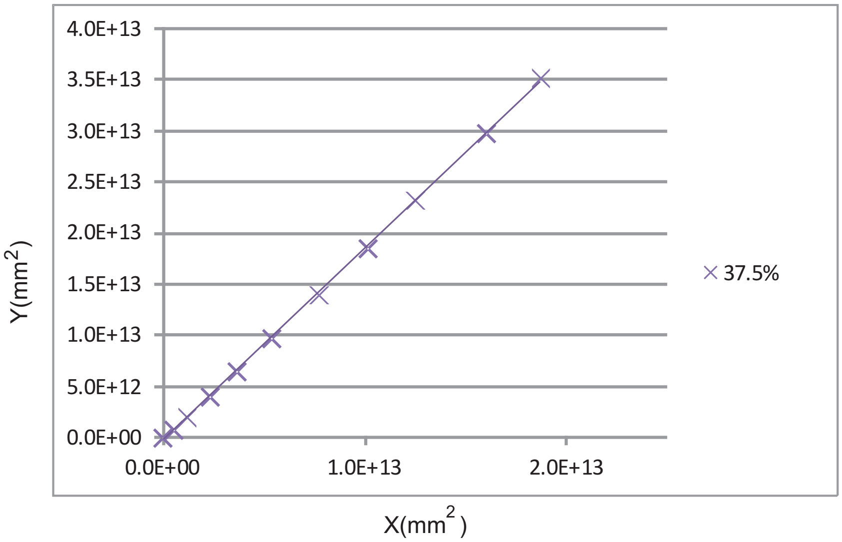

Figure 7. Crosses: experimental data. Solid line is the best-fit line for a 37.5% concentration: $Y = 1.85X$, where $Y = a^2[h^2x^2 + (ax - hy)^2]$ and $X = (a^2 + h^2)(ax - hy)^2$.

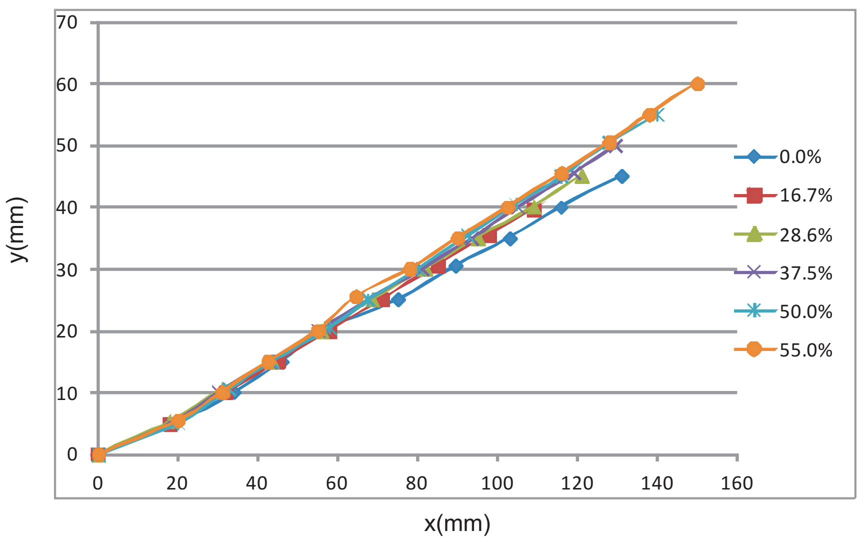

Figure 8. Lateral displacements of the laser beam spot as a function of the solution height in the fish tank for different sugar concentrations. The diamond, square, triangle, cross, star and circle symbols are the experimental data for the concentrations indicated and the solid lines are the lines that join the experimental points.

4. Conclusions

The experiment described in this paper gives a simple and effective method for measuring the refractive index of a transparent liquid using a modified fish tank and a laser. This experiment is suitable for undergraduate physics laboratories. It has been carried out successfully in the lab providing reasonable results. Finally, we should mention that our students felt a sense of accomplishment, having discovered the relationship between

Figure 9. Solid lines are the best-fit lines for different sugar concentrations: diamond: $Y = 1.70X$, square: $Y = 1.76X$, triangle: $Y = 1.80X$, cross: $Y = 1.85X$, star: $Y = 1.89X$, circle: $Y = 1.92X$, where $Y = a^2[h^2x^2 + (ax - hy)^2]$ and $X = (a^2 + h^2)(ax - hy)^2$.

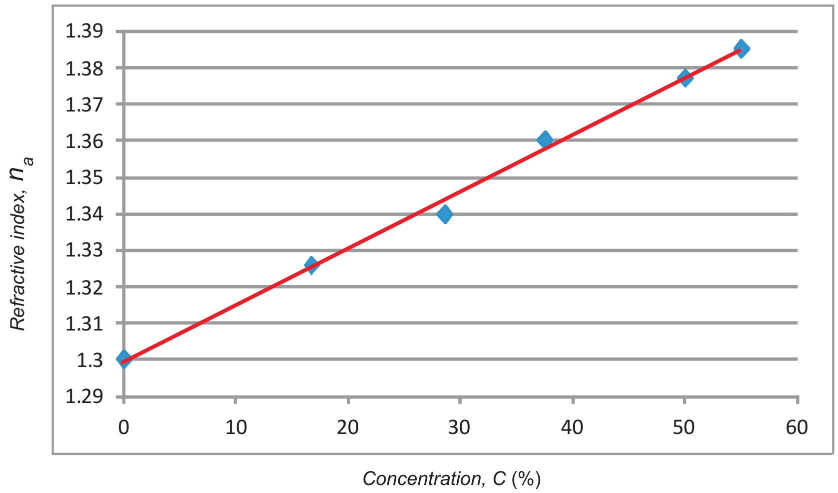

Figure 10. Refractive index of a water-based solution as a function of the sugar concentration (solid line is the best-fit line: $n_a = 1.60 \times 10^{-3}C + 1.30$)

refractive index and concentration for a water-based solution on their own.

5. Acknowledgments

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[2] Wojewoda F web site: http://www.euhou.net/index.php/exercises-mainmenu-13/classroom-experiments-and-activities-mainmenu-186/203-determination-of-the-index-of-refraction-using-a-laser-pointer

[3] Mahmood W and Rahman A 1988 *Applied Optics* **27** 3341