# CS350 — Winter 2014
# Homework 2

Due Tuesday 28$^{\text{th}}$ January, on paper (or in d2l's dropbox) by the start of class. This assignment will be graded.

1. **Compute the following sums.**

   (a)
   $$\sum_{i=0}^{2n+3} i(i+1)$$

   (b)
   $$\sum_{j=1}^{n} 2^{j-6}$$

   (c)
   $$\sum_{i=1}^{n}\sum_{j=i}^{n} ij$$

   (d)
   $$\sum_{i=1}^{n} \frac{2}{i(i+2)}$$

2. **Consider the following algorithm.**

   **ALGORITHM**
   **function** ENIGMA($A[0..n-1, 0..n-1]$)
   ▷ Input: A matrix $A[0..n-1, 0..n-1]$ of real numbers
   **for** $i = 0$ to $n-2$ **do**
   **for** $j = i+1$ to $n-1$ **do**
   **if** $A[i,j] \neq A[j,i]$ **then return** false
   **return** true

   (a) What does this algorithm do?
   (b) What is its basic operation?
   (c) How many times is the basic operation executed in the best and worst cases?
   (d) What is the efficiency class of this algorithm?
   (e) Suggest an improvement, or a better algorithm altogether, and indicate it's efficiency class. If you cannot find an improvement, try to prove that the problem can't be solved by an algorithm in a better efficiency class.

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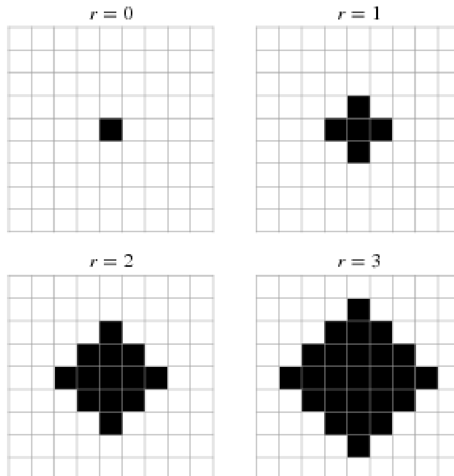
3. **von Neumann neighborhood.** Recall the definition of von Neumann neighborhood from Levitin Ex 2.3, problem 11.




On the $n^{\text{th}}$ iteration, $n$ squares are *added* to each of the four symmetric sides of the von Neumann neighborhood. Hence we obtain the following recurrence for $S(n)$, the total number of squares in the neighborhood after the $n^{\text{th}}$ iteration:

$$S(n) = S(n-1) + 4n \quad \text{for } n > 0 \text{ and } S(0) = 1.$$

Solve this recurrence relation using backwards substitution to find a closed-form formula for the number of cells in the von Neumann neighborhood of range $n$.

**Hint:** Make sure that the working for your solution contains at least one instance of the back-substitution. If it's at all unclear where a line of your derivation comes from, add a comment, such as "replace $i$ by $n-1$", or "substitute for $S(n-1)$".

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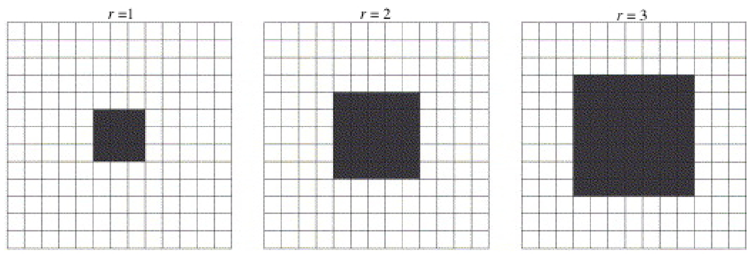
4. **Moore neighborhood**. The Moore neighborhood is similar to the von Neumann neighborhood, except that on each iteration, squares are added on the diagonals as well. The starting point ($0^{\text{th}}$ iteration) is the same; the first three iterations in the development of the Moore neighborhood are shown here:




Write down a recurrence relation for the total number of squares in the Moore neighborhood after the $n^{\text{th}}$ iteration. Also write down the initial condition.

**Hint:** Be sure to describe your reasoning.

5. **Distances.** There are several ways to define a distance between two points $p$ and $q$. In particular, the *Manhattan distance* is defined as

$$d_M(p, q) = |p.x - q.x| + |p.y - q.y|$$

(a) Prove that $d_M$ satisfies the following axioms that every distance function must satisfy:

i. $d_M(p, q) \geq 0$ for any two points $p$ and $q$, and $d_M(p, q) = 0$ if and only if $p = q$;

ii. $d_M(p, q) = d_M(q, p)$; and

iii. $d_M(p, r) \leq d_M(p, q) + d_M(q, r)$ for any $p$, $q$ and $r$

**Hint:** The questions says “Prove”, so be sure to write a formal proof, using the definition of $d_M$.

(b) Sketch all the points in the $x, y$ coordinate plane whose Manhattan distance to the origin $(0, 0)$ is equal to 1. Do the same for the Euclidean distance.

(c) True or false: the answer to an instance of the closest-pair problem does not depend on which of the two metrics — $d_E$ (Euclidean) or $d_M$(Manhattan) — is used.

**Hint:** Justify your answer, for example, by providing a counter-example if you said “false”.