Project Description

1 Introduction

Spectroscopic galaxy surveys play an indispensable role in precision cosmology. The measurements of the Baryon Acoustic Oscillation (BAO) peak position at various redshifts put tight constraints on the expansion rate and geometry of the Universe. The Redshift-Space Distortion (RSD) signal, which has its origins in the growth of structure over cosmic time, can constraints the growth rate as well as a function of redshift. These measurements have been pushed to a very high precision. BAO are now measured at a one per cent precision, while the best RSD measurements stand at about a five per cent precision. The next generation of galaxy surveys, such as Dark Energy Spectroscopic Instrument (DESI), Euclid satellite mission, and the Wide Field Infrared Survey Telescope (WFIRST), will extend these measurements to higher redshifts and even better precision.

When these measurements are combined with a Cosmic Microwave Background (CMB) data the resulting constraints on Dark Energy (DE) and theories of gravity (TG) are overall consistent with the standard cosmological ΛCDM model. The constraints however at this point are not tight enough to conclusively rule out alternative models of DE and TG that are arguably better motivated from a fundamental physics point of view than the standard cosmological model. In addition, there are a number of 2 to 3 σ discrepancies (such as e.g. the discrepancy with the local Hubble constant measurements, or the tension with the clustering amplitude measured by weak lensing surveys), but they are not statistically significant enough to make a conclusive case in favour of non-standard models.

Since we have only one vintage point for observing the Universe the amount of information that is available to us is fundamentally limited. Both BAO and RSD tests, as currently implemented, rely on two-point statistics measurements of the galaxy field the precision of which is limited by the total observable volume. Expected DESI data will almost saturate this "cosmic variance" limit at lower redshifts where DE dominates the evolution. Hopefully, we will be able to either strongly rule out alternative scenarios or find a convincing non-ΛCDM signal. If we, however, end up with a number of nagging discrepancies that are suggestive but not statistically significant, the only way to resolve the issue will be to find an alternative source of robust and precise cosmological constraints in the same data.

There are a number of ways to extract more information from spectroscopic galaxy data. We can try to enhance the standard BAO analysis by (by now standard) method of reconstructing the linear field or trying to come up with more optimal weighting schemes. Other approaches include looking at stacked voids, joint multi-tracer analysis, and going to smaller scales by means of simulation assisted modeling. All these approaches have their advantages and disadvantages and all of them must be tried out on real data. In this proposal we would like to pursue the same goal by going to the third-order statistics of galaxy fields.

The main objective of this proposal is to develop methods for **extracting robust BAO and RSD measurements from the bispectrum of galaxies on linear and semi-linear scales**. We argue that, even though this is a difficult endeavour that has a number of inherent risks, it also has a number of potential advantages over alternatives (e.g. using a two-point statistics of reconstructed fields, or extracting RSD signal from stacked objects such as stacked voids), and is worth pursuing. The main appeal of bispectrum is that, at least for some future surveys, it has a potential to **significantly enhance** the standard BAO/RSD analysis. We hope to convince our colleagues that the benefits outweighs the risks and this line of research merits a modest investment of resources and that our preliminary work this direction places us in a good position to perform this research.

2 Bispectrum overview

Spectroscopic galaxy surveys provide us with highly accurate measurements of 3D positions of millions of galaxies in large cosmic volumes. The specific distribution of galaxies that we observe is just one out of a large number of possible distribution that could have been observed in the same cosmological model if the stochastic initial conditions were slightly different. The most popular way of describing this stochasticity in the large-scale structure community has been to use the n-point correlation functions. The two-point correlation function $\xi(\vec{r})$ is a function of a separation vector $\vec{r}$ and describes a "clustering strength" at a certain scale (average number of neighbours at distance $\vec{r}$ from a randomly selected galaxy in access of a purely random distribution). Three-point function $\eta(\vec{r}_1, \vec{r}_2)$, similarly, describes a probability of finding three galaxies in a specific triangular configuration. This information is sometimes expressed in terms of Fourier transforms of these two functions (power-spectrum $P(\vec{k})$ and bispectrum $B(\vec{k}_1, \vec{k}_2)$ respectively). From now on we will use the term "bispectrum" to generically refer to the three-point function both in Fourier and configuration spaces.

For purely Gaussian fields the two-point statistics contains all of the information and the expectation value of bispectrum is zero. Even though the seed cosmological fluctuations are believed to be very close to Gaussian gravitational instability and halo/galaxy bias generate non-zero bispectrum at late times. Non-linear terms in the Euler equation will inevitably couple initially independent Fourier modes and will therefore generate bispectrum. The relationship between DM and galaxy overdensities is linear on very large scales but non-linear and non-local effects become more important on smaller scales. Because of this non-linear mapping the galaxy distribution would have a non-zero bispectrum even if it was generated form a purely Gaussian DM field. This immediately suggests that bispectrum should be sensitive to the exact nature of TG and higher order bias parameters. **We will later argue that the sensitivity to DE through projection effects is even stronger.** Planck data put very tight upper limit on local primordial non-Gaussianity parameter – $f_{\rm nl}$ – at least for some triangular configurations. This is not a problem for our research program since we do not intend to use bispectrum as means to measure $f_{\rm nl}$, in fact the small value of $f_{\rm nl}$ makes the modeling part significantly easier. Our goal is to measure BAO/RSD from the distortions of the bispectrum shape and as we will later show for this goal the bispectrum generated by gravitational instability and higher order bias is sufficient.

The measured bispectrum is a function of of five variables: three variables describing the size of the triangle (e.g. wavelengths k_1, k_2, k_3 corresponding to three sides) and two variables describing its orientation with respect to the line of sight (e.g. angle that one of the sides makes with respect to the line of sight, and the internal rotational angle of the triangle). The potential to constrain cosmological parameters comes from the fact that the exact shape of this five dimensional function depends on initial conditions, the nature of gravity that generated it, and the higher order bias. The accuracy of bispectrum measurements is related to the number of independent galaxy triangles in the survey. The dominant contributions to the uncertainty in measured bispectrum come from "cosmic variance" and "shot-noise". Shot-noise stems from the fact that sparsely sampled galaxy populations have smaller number of triangles and for the bispectrum (assuming Poisson distribution) scales as $1/n^2$, where n is the number density of galaxies. The cosmic variance scales as $1/V$ with the survey volume and reflects the fact that larger volumes have more independent triangular configurations. Large scale measurements are limited by the cosmic variance and small scale measurements by the shot-noise. The overall variance of the bispectrum measured in wavevector bins scales as (up to a normalization factor, and under a number of simplifying assumptions)

$$\mathrm{Var}\left[B(\vec{k}_1,\vec{k}_2,\vec{k}_3)\right] \propto \frac{k_1k_2k_3\Delta k_1\Delta k_2\Delta k_3}{V}\left[\frac{P(\vec{k}_1)P(\vec{k}_2)+P(\vec{k}_2)P(\vec{k}_3)+P(\vec{k}_3)P(\vec{k}_1)}{n}+\frac{1}{n^2}\right], \quad (1)$$

where Δk-s are the widths of wavevector bins. For comparison the power spectrum variance scales as

$$\mathrm{Var}\left[P(\vec{k})\right] \propto \frac{k^2\Delta k}{V}\left[P(\vec{k})+\frac{1}{n}\right]. \quad (2)$$

This rough comparison demonstrates that the signal to noise of measured bispectrum benefits much more from increasing sampling density compared to the power spectrum (n^{-2} vs n^{-1}). It also benefits much more by extending the analysis to smaller scales (higher wavenumbers, k^6 vs k^3). So, the higher the number density is and simpler the non-linear modeling is more competitive the bispectrum constraints become. Our forecasts suggest that, if we manage to control systematic effects, bispectrum is capable of delivering impressively competitive constraints for DESI Bright Galaxy Survey (BGS) and the lower redshift range of WFIRST. Both of these samples have a high number density and WFIRST is additionally at a high redshift (hopefully allowing to push the analysis to higher values of k_{max}). The projected bispectrum constraints from DESI main samples and Euclid are also very interesting.

3 BAO/RSD analysis

The primordial shape is imprinted very cleanly in the initial distribution. Later gravitational evolution mixes the modes and moves some of the shape information from power spectrum to bispectrum. If cosmological constraints were coming **only from the initial shape** contribution of bispectrum to the power spectrum would be modest. Reconstruction of the initial linear field would then move all of this higher order information back to the power spectrum. Fortunately, we have other effects in the observed clustering that are **far more sensitive to DE and TG then the initial shape**. Most of the BAO constraints on DE actually come from so called Alcock-Paczynski (AP) distortions of the feature rather than actual determination of its physical scale. Similarly, most of TG constraints come from radial RSD signal.

BAO is a preferred scale set by processes in early Universe. It manifests itself as a peak in the correlation function and an oscillatory pattern in the power spectrum. The BAO scale has been measured with an exquisite precision by CMB experiments and convincingly detected in low redshift galaxy samples. Since baryonic matter only comprises a small fraction of non-relativistic matter, the BAO feature is not as strong in galaxy distribution as it is in CMB. **The constraining power of the BAO in galaxy distribution is strongly enhanced by the AP effect**. The AP effect results from the fact that we observe galaxies in the space of angular coordinates and redshifts. To link these to theoretical predictions we need to translate them into physical distances. To do this we need to know the angular diameter distance D_A (for angular distances) and the Hubble parameter H (for radial distances) at the redshift of our galaxy sample. If we pick wrong D_A and H our scaling will be off. Since we know the position of the BAO peak at a very high precision from CMB data, we can use AP distortions to find a correct value of D_A that aligns BAO in galaxy distribution with the correct physical scale. Performing this alignment in the radial direction similarly constraints H. These D_A and H measurements can then be translated into strong and robust DE constraints. In principle, AP effect would work even without the BAO feature, but the presence of a well calibrated peak (or for the power spectrum oscillatory pattern with known frequency) makes it easier to detect small shifts.

RSD is an anisotropic signal in the galaxy distribution that results from the fact that galaxies on average tend to fall into each other and because of this coherent extra Doppler shift their radial distances seem on average to be smaller than they really are. Since we belie that there is no fundamental reason for galaxy distributions to have a different statistical pattern in radial and angular directions, we can use measured RSD to constrain infall velocities of galaxies. And since gravitational interactions source the infall velocities this information can then be translated into strong constraints on TG.

BAO are considered to be very robust and virtually systematics free measurements. RSD modeling is more involved and significant systematic effects can be present already on semi-linear scales. There is no straightforward way of isolating the RSD feature in the analysis, so the RSD measurements must always incorporate AP measurements on the full power spectrum shape. However, if the systematics can be brought under control RSD enhance the constraining power of galaxy surveys by an order of magnitude, and make it possible to constrain TG in addition to DE.

Our flagship BAO/RSD measurements have so far come from two point statistics. The BOSS measurement is so far the most accurate (1% at $z \sim 0.57$). Large-scale BOSS measurements stand at 5%. DESI, Euclid, and WFIRST are projected to make multiple similar sub-percent level measurements up to a redshift of $z \sim 2$.

The full fledged equivalent BAO/RSD bispectrum measurements on linear and semi-linear scales have not so far. Early works have been mostly concerned with the small scale bispectrum with the purposes of estimating bias parameters.

4 Bispectrum projections

At this point, it would be interesting to check how much cosmologically relevant information is in the potential bispectrum measurements from future surveys. We expect bispectrum to perform well on densely sampled surveys (e.g. DESI BGS or low-redshift WFIRST). We use conventional Fisher information matrix techniques to perform these forecasts. We assume that bispectrum is analysed in exactly the same way as the full BAO/RSD power spectrum analysis, i.e. it is measured in bins that are narrow enough not too loose a lot of information to in-bin averaging, the nonlinear effects are separated by choosing a value of $k_{\max}$ appropriate for each redshift (more scales are included for higher redshifts), uncertainties in the DM to galaxy biasing and intra-halo motions are parametrized by bias parameters and a fingers of god dispersion term. The parameters of interest are the two AP distortion parameters that correspond to D_A and H and an RSD parameter ($f\sigma_8$) that can be related to the growth rate.

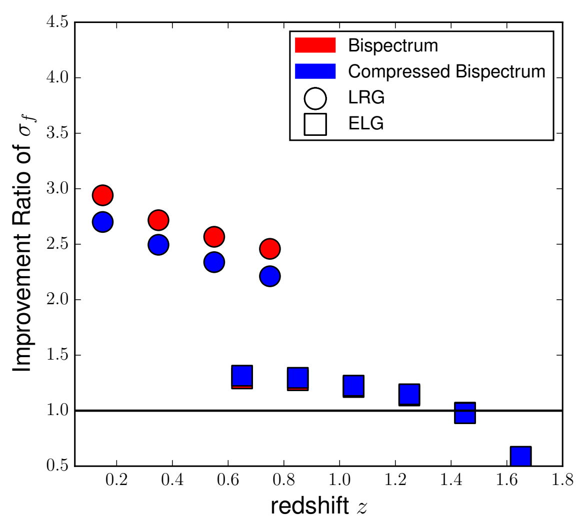


Figure above shows such prediction for $f\sigma_8$ parameter for DESI survey divided in redshift bins of $\Delta z \sim 0.1$. Plotted is the ratio of variance from power spectrum to that of the bispectrum. It is interesting and encouraging to see that the bispectrum analysis outperforms the standard power spectrum in this regime by almost a factor of 3! At higher redshifts the improvement is more modest but even there the bispectrum RSD constraints are comparable to the ones derived from power spectrum. It is worth highlighting that these predictions are for the bispectrum only. Combining $P(\vec{k})$ and $B(\vec{k}_1, \vec{k}_2)$ Fisher matrices is complicated because of the presence of cross-correlation terms and we currently do not have a reliable code to perform these computations. Nevertheless, it is clear that the addition of the bispectrum would strongly enhance overall constraints even after-accounting for the cross-correlations (Our mock tests suggest a cross-correlation of about 30 per-cent, consistent with findings of Ref. []). We see similar improvements for BAO measurements and for Euclid and WFIRST samples.

5 Bispectrum advantages

In this section we will compare bispectrum analysis to alternative ways of extracting extra information from galaxy surveys on top of standard two-point BAO/RSD. The intent here is not to argue that the bispectrum approach is superior but rather to show that despite its intrinsic difficulties it has advantages that merit further development and research.

A safe option (one that is not likely to introduce additional systematics) for improving standard BAO/RSD measurements is to implement weighting schemes that are in some ways more optimal than the standard Feldman-Kaiser-Peacock prescription. Weighting schemes based on relative bias of galaxy sub-samples and redshift evolution have recently been studied in literature. Even though they have a potential to somewhat improve the measurements they could never achieve a factor of three improvement that is potentially present in the bispectrum.

A very popular methods, that has by now become standard, is to undo some of the effects of non-linear evolution on large scales by reconstructing the initial field. This sharpens the power spectrum shape around BAO and makes it more sensitive to the AP test (sensitivity to small distortions scales as $\mathrm{d}P/\mathrm{d}k$ to leading order). The effectiveness of reconstruction is limited by two factors. At high redshifts the BAO shape is closer to linear and there is not much to gain by making it sharper. Also, it is not completely clear how the reconstruction interplays with RSD. Even though it is possible to run a reconstruction algorithm without removing RSD signal, some kind of RSD modeling will have to be adopted, and it is not entirely clear that this procedure will not bias the extracted RSD constraints. Reconstruction is therefore suitable for BAO only constraints but does not help with the RSD analysis.

Bispectrum is an independent (although somewhat correlated) measurement that can be used for the BAO/RSD measurements. Small distortions in the bispectrum shape can be used to constraint D_A and H in a way that is identical to the power spectrum (even if the modeling of former can be more complicated). While reconstruction linearises BAO peak in power spectrum it also reduces the bispectrum amplitude. For the number densities that are typical for current surveys (order of 10^{-4} Mpc/h) the gain in the power spectrum sensitivity to AP is approximately equal to the loss in the bispectrum sensitivity. For current surveys this implies that the BAO constraints from reconstructed power spectrum are roughly equal to the kind of constraints that are obtainable from a joint power spectrum and bispectrum analysis, and since the former is simper there is no convincing case for the later. This is not however true at higher densities and higher redshifts where the sharpening of power spectrum BAO feature does not gain as much information as adding a bispectrum function to the analysis.

In summary, **the information content of reconstructed power spectrum is not always**

equal to the information content of joint power spectrum and bispectrum (plus higher order correlators) analysis. They would be equal for a field in a box, where the shape of the power spectrum of initial nearly Gaussian field contains all the information and gravitational evolution only serves to couple the phases and dilute this information into higher order terms. Our constraints however are coming from AP distortions of observed quantities and having an extra distorted function (bispectrum) is in some cases more profitable than having a slightly "sharper" power spectrum. *This is easy to see for a hypothetical case of a universe without a BAO feature (perhaps a Universe with trace amounts of baryonic matter). In that universe reconstruction would not really add anything to the power spectrum, it would simply slightly tilt the power law. But having an extra function (bispectrum) for the AP analysis would obviously increase the constraining power.*

Another popular technique for going beyond standard BAO/RSD analysis is to use stacked voids (or clusters) that are assumed to be isotropic in physical space because of the statistical homogeneity. The observed anisotropies than are generated by AP and RSD and can be used to obtain DE and TG constraints. The method has a huge statistical promise, the number of objects after all scales linearly with volume. The main challenge is the modeling. Voids (and clusters) are small scale objects that are strongly affected by non-linear evolution and do not lend themselves to perturbative treatments like large scale n-point statistics do.

Yet another extremely promising option is to use statistical measures on small extremely non-linear scales. These scales are too non-linear to be modeled from first principles and therefore theoretical modeling will have to be aided by high quality and resolution cosmological simulations. The main question is whether we will in fact have suitable (in terms of quality and numbers) simulations that are accurate enough for this purpose, large enough to have appropriately small errorbars, and diverse enough to meaningfully cover large parameter space which, in addition to cosmological parameters, should now include extra parameters describing small scale physics (Halo-galaxy connection parameters, baryonic effects, environmental effects, etc.), **in time for DESI/Euclid/WFIRST analysis.**

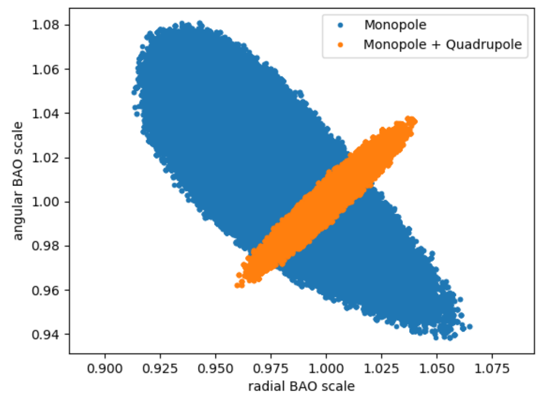
All these are excellent ways of significantly enhance cosmological information coming from spectroscopic galaxy surveys, and the entire community hopes that they will all be mature enough in time to be applicable to DESI/Euclid/WFIRST data. They should all be pursued to make sure that we have multiple complementary ways of looking at the data and spot possible systematics. It is clear that BAO/RSD from bispectrum on linear and semi-linear scales has it's role in this joint effort. Some potential advantage are that, unlike voids, the modeling is bound to work at least on extremely large scales where the perturbative approach will eventually work (the real question is how far we can slides this boundary down the scale ladder); Unlike small scale clustering the modeling will not rely as much on simulations (although the validation most definitely will, and some calibration on simulations may be necessary); And compared to reconstruction it has a theoretical potential to deliver significantly stronger enhancements.

6 Bispectrum – Status Quo

Early bispectrum measurements in galaxy distribution have been made in Refs. [1–12]. Most of these works were focused on determining bias parameters and RSD on smaller scales. The galaxy samples used in these studies were relatively small and the modeling was not tested on the precision demanded by DESI/Euclid/WFIRST data.

More recent bispectrum measurements have been performed in WiggleZ, Vipers, and BOSS galaxy samples. BOSS three-point function was used to detect a BAO peak [13] and to constrain baryon to DM velocity [14]. BOSS bispectrum monopole was used to supplement power spectrum analysis of RSD [15–17]. Small scale three-point function was used to constrain growth and bias in WiggleZ [18], and to study dependence on galaxy type in Vipers [19].

Our group has published a number of recent papers related to galaxy bispectrum analysis. In Ref. [20] we studied the ways of reducing bispectrum information without sacrificing information content and identified three harmonic angular modes that contain most relevant information. In Ref. [21] we detected a BAO signature in BOSS CMASS bispectrum monopole at a 4.1σ confidence level. This allowed us to get a 1.6 per cent distance measure. We are currently working on extending our work to anisotropic bispectrum analysis. This will allow us to constrain both angular and radial BAO parameters. Preliminary MCMC chains from BOSS bispectrum monopole and quadrupole measurement are displayed on the figure to the right, and clearly show the improvement from the quadrupole.



To summarize, existing high precision real-data based measurements are currently coming only from bispectrum monopole BAO. While earlier works have looked at RSD measurements they are based on a significantly smaller data sets and the methods are not extendable to DESI/Euclid/WFIRST. Our proposed work would take over from here and would extend the existing work in two complementary directions. We will continue working on the anisotropic BAO measurements and validate them for the precision required by future surveys, and we will add large-scale bispectrum RSD to the mix.

7 Research Plan

Our overall goal is to develop methods of measuring BAO and RSD in large-scale bispectrum of galaxy surveys that would be accurate enough to be applicable to DESI/Euclid/WFIRST data. Achieving this goal schematically involves five steps: measuring bispectrum, estimating covariance matrices, modeling or removing observational systematics, theoretical modeling, and likelihood analysis. Our tentative plans for each of these directions are briefly described below.

measurements

Recent works have demonstrated that even wide-angle bispectrum measurements can be reduced to a series of Fast Fourier Transforms (FFT) [22, 23]. Our group has developed a fast GPU implementation of bispectrum multipole algorithm that has been validated on controlled mocks and can process order of few thousand mocks (of the size of BOSS CMASS sample) in order of few hours. This part of the work will be concerned with further high precision testing of our pipeline. We will produce particle distributions with known input power spectrum and bispectrum following e.g. Refs [24–26]. We will then check that our pipeline accurately reproduces the input. To accomplish this task we will not need to evolve the initial conditions under gravity, which will make the task computationally inexpensive.

In addition, we will test methods of modeling the shot-noise and window effects on the measurements. We expect the simple Poisson shot-noise model to work sufficiently well on large scales but this expectation will be tested on controlled simulations. Exact window convolution for bispectrum requires computing a six-dimensional double convolution integrals which is computationally challenging, especially considering the fact that the convolution will have to be performed for every model in Monte Carlo Markov Chains. We will extend approximate methods that have been tried on power spectrum (e.g. [27]) and check whether they are accurate enough for our purposes and fast enough to be implement in the likelihood pipeline.

covariances

The covariance matrices of measured bispectra provide another challenge. The standard method of estimating covariance matrices from a sample variance of a large number of mocks is complicated by the fact that there are a very large number of bins and an accurate estimation of covariance and precision matrices will require a large number of mocks [28**?** –33]. This problem becomes even more acute for a joint power spectrum and bispectrum analysis. The covariance matrices in this case are large and clearly non-diagonal. We plan to tackle this problem from two complementary directions. One is to somehow reduce the size of data vector so that computing covariances from the mocks becomes a possibility. Another route is to estimate covariances at least partially theoretically and calibrate theoretical estimates with mocks.

There are tested ways of compressing measured bispectrum to a significantly smaller vector without loosing too much information [34–38]. This approach is very effective but requires the knowledge of covariance matrix and is model dependent. Other approaches include further reduction of bispectrum multipoles into e.g. first few multipoles in triangular configuration angle, or identifying few principal components and only using them in the analysis. All these approaches can be tested in a straightforward manner on mocks.

The exact theoretical estimation of bispectrum covariance would require computing a six-point function and is probably an unachievable goal. It is however likely that, especially at high redshifts, the covariance is dominated by the leading order terms which are reasonably easy to compute. The biggest effect would than be the effect of the mask on covariance and the coupling between shot-noise and mask effects. We will borrow some recent ideas that were studied in the context of power spectrum (e.g. Refs. [39–43]) and will generalize them for the bispectrum. Taking into account the fact that fractional error in bispectrum in individual bins is lower than for the power spectrum and that our requirement on the accuracy of covariance matrix is not as stringent as a similar requirement on the model, we will identify approximations that are good enough for our purposes and do not bias likelihood fitting strongly.

observational systematics

All observational systematics affecting the standard two-point BAO/RSD analysis are also likely to affect the bispectrum. Most observational systematics affect one point distribution (e.g. certain areas of sky have a lower efficiency of having galaxies detected or the efficiency is a function of emission line flux). These systematic inefficiencies are usually expressed either in terms of visibility cubes (implemented in practice as random catalogs) or as sets of per object weights. The mitigation of observational systematics is a very high priority task for DESI/Euclid/WFIRST collaborations. We will closely follow this work (PI Samushia is actively involved with most of this work) and will implement the known mitigation techniques in our bispectrum analysis. It is difficult to imagine observational systematics that would be either exclusive to the bispectrum or significantly more severe.

Some systematics may not directly translate from the power spectrum analysis and may require a separate investigation. These are the systematics that affect the configuration space and can not be split into a product of one-point weights. A good example of this type of systematics is e.g. fiber assignment effects in DESI and slitless-spectroscopy effects in Euclid []. Two proposed methods of dealing with fiber assignment effects are to remove the most affected modes or to use a pair-wise weighting. Even though the power spectrum specific versions of these methods can not be directly applied to bispectrum they can be easily generalised. It should be reasonably straightforward to identify the most affected bispectrum configurations and to exclude them from the analysis. It should also be reasonably straightforward to generalise pair-wise weighting schemes to triplet-wise weighting. In fact, the bit-wise weights as implemented in Ref. [] can be very easily translated into triplet-wise weights.

In summary, our main concern here is to make sure that we are on top of ongoing systematics work and to generalise and validate developed methods for higher order statistics.

modeling

Accurate modeling of various nonlinear effects that contribute to higher order clustering is arguably the most important step towards deriving competitive BAO/RSD constraints form real data. The modeling can be schematically divided into three semi-independent steps: modeling non-linear gravitational evolution of DM, nonlinearities in halo-to-DM and galaxy-to-halo relationship, and nonlinearities in redshift-to-real space mapping.

Nonlinear evolution of DM field is an extensively studied subject. Pioneering works have shown that perturbative approaches work on large scales. The rate of convergence of standard perturbation theory (STP) is slow, which prompted a development of alternative approaches including Lagrangian perturbation theory, closure theory, and various resummation and renormalization schemes. Perturbation theory for bispectrum is generally more challenging because of numerical issues associated with higher dimensional integrals and even slower convergence than for a second order statistics. The nonlinear effects in DM growth however are expected to become milder with redshift. For high redshift samples form DESI/Euclid/WFIRST the modeling of pure DM nonlinearities will be a significantly easier task. We plan to work on a project similar to Ref. [] where the outcomes of various perturbation theory approaches will be tested on simulations on the same footing and a careful test of validity will be performed and a safe minimal scale will be determined for all models.

The modeling of halo and galaxy bias is complicated and does not necessarily become easier at higher redshifts. The hope is a few parameter models (e.g. linear and second order bias to describe halo biasing supplemented with Fingers-of-God dispersion term to describe the interhalo motions of galaxies) will be found sufficiently accurate on large scales. We plan to work on a project similar to Ref. [] and test various biasing schemes on simulations. Following Ref. [] we will measure the DM and halo clustering in real space and will use them to check how well various biasing schemes work and to what extent is inclusion of higher order bias and non-local bias necessary. The main goal of our project is to derive unbiased BAO/RSD constraints from linear scales which makes our job slightly easier in a sense that bias models that do not necessarily accurately describe the physics of halo-DM and galaxy-halo interaction may be good enough for us as long as they do not systematically bias our BAO and RSD results.

RSD effects do not respond well to perturbative approaches. The Most popular methods for modeling them belong to a family of “streaming” models that describe redshift-to-real space mapping geometrically without relying on linearisation. Unfortunately, one has to refer to approximate perturbative schemes at some point since the streaming models require knowledge of velocity field statistics (e.g. probability distribution of infall velocities). Even so, the streaming models have been demonstrated to work much better than plain perturbation theory and most recently have been used to model BOSS data. One popular model along these lines is the TNS model that has been generalised to bispectrum. Ref. [] showed that the model works well up to scales of $k_{\rm max} \sim 0.2$ on low resolution simulations. We envision two possible projects here. One is to test this Fourier space bispectrum model at higher precision level demanded by future surveys and to check whether it works for lighter halos that will host most of DESI/Euclid/WFIRST galaxies. The second direction is to generalize a real-space work of Ref. [] to higher orders. We will start with the simplest approximation in which the real space and redshift space three-point functions are related by a triple infall velocity distribution convolution which is multivariate Gaussian. We will be able to test how well this approximation works (and on what scales) by directly measuring three-point infall velocity distribution in simulations and convolving it with a measured three-point function. We will then try probability distribution functions of increasing complexity and check if any of them work better and at the same time have a conveniently parametrizable shape.

likelihood

Fitting model to data will result in a likelihood surface in D_A, H, $f\sigma_8$, and nuisance parameters. These measurements in turn can be used to derive DE and TG constraints in exactly the same way as for the power spectrum. The treatment of likelihood will be standard and we do not expect it to be significantly different from the standard BAO/RSD analysis.

The only potential open question is combining these measurements with the power spectrum BAO/RSD. In the ideal case one would like to perform a joint fit to power spectrum and bispectrum. In this case the joint BAO/RSD constraints would internally account for all the cross-correlations. This may turn out to be complicated because of the need for larger covariance matrices. The covariance of bispectra and power spectra individually is significantly easier to model. An alternative approach is to get the BAO/RSD correlations between power spectrum and bispectrum by running fits to a suit of mocks and then combining measurements based on empirical correlations. This approach will work as long as the two measurements are consistent. The amount of cross-correlation will be driven by the shot-noise and volume and is not likely to be very sensitive to cosmological parameters.

Simulations

Many aspects of the research plan will require two types of high quality simulations. The first type is where the input bispectrum and power spectrum of the mocks are not necessarily very accurate but are known to high accuracy. We already have a validated pipeline for creating distributions with known anisotropic power spectrum and bispectrum based on the ideas put forward in Refs. []. These types of simulations are non-expensive to produce. We already have order of 10,000 of them for our ongoing work and could easily increase the number if required. We will use these simulations to test our measuring algorithms, and effects of the mask on measurements and covariances.

Testing theoretical models will require high quality N-body simulations that can be used to link underlying cosmological parameters to measured bispectrum. There are number of publicly available N-body simulations that are suitable for this task []. They span a wide range of cosmological parameters, mass resolutions, and total volume and are ideal for testing the robustness of theoretical models.

Application to real data

We will be applying our methods to existing BOSS data as they develop. Even though because of lower number density we do not expect the results to be competitive, BOSS is a very well studied highly complete survey and the application of methods and testing outcomes will have a great value.

DESI is on schedule to start collecting data in late 2019. PI Samushia is a collaboration member and can get the PhD student and the Postdoc access to collaboration tools and data. By the time our methods are well tested on simulations we will be able to apply them to DESI early data. Main DESI samples are expected to give better results than BOSS. They will also be easier to model because they are at a higher redshift and more linear (at least the DM fields). We expect very strong constraints from DESI BGS because of its extremely high sampling density. The challenge there is that the low redshift sample will be more susceptible to nonlinear effects. If we manage to calibrate our models to required accuracy there is a potential to more than double standard BAO/RSD results form DESI BGS.

Eventually, after the project is completed, the methodology will be applied to Euclid/WFIRST samples. Lower redshift range of WFIRST, which also has a very high sampling density, can potentially be very strongly enhanced by the bispectrum analysis.

8 Research Team

Our core team will consist of two senior (PI and Co-PI) and two junior (PhD student and postdoc) members. PhD student (supervised by PI) will be working mostly on the topics related to bispectrum estimators, and observational systematics. The Postdoc (jointly supervised by PI and Co-PI) will lead the efforts on theoretical modeling of bispectrum measurements and covariances. These are of course very tentative plans. All team members will be working collaboratively on all aspects of the project and the exact distribution of work will organically emerge from this collaborative process.

In addition, we will involve two undergraduate students in the research. Undergraduate students will be employed on part time basis over school period and will help in the software pipeline development. PI already has experience of involving undergraduates with appropriate programming skills in this type of research. E.g. physics majors James Minton and David Coria have worked with PI to develop a set of tools for analysing N-body simulation data (power-spectrum, mass functions, etc.) and REU student Peter Klinge has similarly worked with the PI on developing codes for fast likelihood sampling. The students will be selected from Computational Physics class that PI is teaching every year. This will at the same time benefit the core team and help the students get a hands-on physics research experience.

9 Relevant Prior Research

PI Samushia has an extensive experience of working with spectroscopic galaxy survey data. He was involved with the BOSS survey, led a number of key projects related to the BAO and RSD analysis of two-point statistics [44, 45], and significantly contributed to many others [46–49]. He has also worked on systematic mitigation techniques including wide-angle effects [50–52], mode-nulling techniques [53, 54], and selection function related systematics [55].

PI is actively involved in three major near future surveys: DESI, Euclid, and WFIRST. He is leading observational systematics work-package for Euclid, the BAO proto-pipeline development efforts in WFIRST, and large-scale structure catalogue pipeline in DESI. He will transfer his knowledge of intricacies involved with working on real data from surveys to junior team members, which will tremendously benefit our project.

We briefly described our bispectrum related recent work in section 6. We are currently working on making the bispectrum BAO measurements more accurate and extending them to bispectrum quadrupole. Tests on mock catalogues suggested that our bispectrum monopole BAO measurement was not strongly affected by theoretical systematics at the level of BOSS errorbars. A more careful investigation is merited, however, if the same analysis were to be applied to DESI data, which has significantly stronger statistical power. Another direction of our recent research is related to finding better ways of reducing bispectrum data. By looking at most sensitive combinations we were able to cut our data vector size by a factor of more than five without sacrificing constraining power. This is relevant to the accurate estimation of covariance matrices. Ongoing efforts to extend BAO analysis to bispectrum quadrupole are also promising. Extending analysis to quadrupole allows us to significantly shrink the size of errorbars and constrain D_A and H separately (see figure in section 6). We are currently working on making sure that the measurements are not affected by systematics at least at the level relevant to BOSS data. The project that we propose naturally follows from our published and ongoing work. We will have to refine analysis methods being applied to BOSS data to make sure that they will be suitable for DESI/Euclid/WFIRST data; And we would like to extend analysis to include RSD measurements.

10 Known Risk Factors

All nontrivial research endeavours are inherently risky. In this section we identify the main risk factors for our project and ways of mitigating them. We will argue that while some risk factors can effectively lower the constraining power of our measurements, it is very unlikely that they will prevent us from getting results.

Measuring higher order statistics involves computationally difficult algorithms and requires significantly higher CPU budget than standard two-point measurements. In fact, if this proposal were written ten years ago, risk factors associated with measuring bispectrum from a large number of mocks would have been one of the major risk factors. Fortunately, algorithmic developments in last few years combined with advances in computing hardware make this risk almost non-existent. Our KSU group has a number of alternative pipelines for efficiently measuring bispectrum from a combination of FFTs. Our GPU and CPU codes have been thoroughly tested on controlled simulations and cross-compared with previously published results. The pipeline has been optimized for available hardware and we can compute bispectrum of all linear ($k_{\rm max} = 0.2$ Mpc) triangle monopoles and quadrupoles from order of few thousand mocks in order of tens of hours on a single GPU equipped workstation. Computing theoretical bispectrum values will be computationally expensive but is unlikely to be prohibitive. Likelihood analysis of bispectrum is not going to be more computationally expensive than the power spectrum.

Accurate estimation of covariance matrices is another potential difficulty that could limit the accuracy of derived constraints. In fact, this was the main limiting factor in our BOSS BAO analysis. We plan to devote significant fraction of our time to this issue. Our preliminary work related to further compression of bispectrum measurements (by selecting most sensitive triangles) makes us optimistic that we can successfully address this issue.

Arguably, the biggest challenge for our program is the ability to theoretically model bispectrum at high accuracy. Our plan is to thoroughly investigate well studied perturbative schemes that have been already been implemented for the power spectrum analysis and partially also for the bispectrum analysis. The possibility that we will not be able to come up with a method that works well on semi-linear scales can not be totally excluded. We would like to note however that the theory is eventually bound to work on extremely large scales where all processes become increasingly linear. The worst case scenario then is that we will not be able to push our $k_{\rm max}$ to very high values and will have to stop at modestly large scales. Future surveys offer so much signal even on largest scales that even for this worst case scenario the resulting constraints will be useful (albeit perhaps not as dramatic as the previously advertised factor of 3 improvement in the growth rate measurements).

Another possible risk factor is that observational systematics associated with specific surveys (DESI, Euclid, WFIRST) may make the application of theory to data difficult. The mitigation of observational effects on standard BAO/RSD analysis is an active area of research in all these surveys (and PI is at the forefront of this research). Our plan is to stay updated with these developments and reimplement them for higher order statistics. It is unlikely that there will be observational systematics that can be mitigated for the power spectrum but can not be dealt with for the bispectrum. It is also not very likely that there will be observational effects that are specific to bispectrum measurements but do not affect the standard power spectrum analysis.

11 Other research activities

PI and Co-PI's other research activities are strongly synergistic to the proposed program.

PI Samushia is partially funded by DOE to work on large-scale structure catalogue creation and observational systematics mitigation for DESI (one month of summer support and half-time PhD student). PI Samushia is also funded by NASA for his involvement in Euclid and WFIRST experiments (cumulative two weeks of summer support). Within Euclid he is working on sample selection algorithms and observational systematics. He is also part of one of the WFIRST Science Investigation Groups. Involvement in these activities makes PI Samushia familiar with the internals of the DESI/Euclid data and up to date with employed systematics mitigation techniques. A know-how that will aid our research group in later parts of the project related to applying developed methods to real data. This project would fund two weeks of PI Samushia's summer research and this project would leverage from PI's participation in DESI/Euclid/WFIRST experiments.

PI Ratra is a world renown expert on DE modeling. He authored seminal early papers on DE modeling [56, 57] and a recent, highly cited review paper on cosmology [58]. His most recent works include analysis of CMB and BAO datasets in light of non-standard models [59–63], DM-baryon interaction in galaxies [64] and inflation models [65]. He is currently partially funded (one month of summer support) by DOE to work on observational implications of non-flat models.

Previous NSF Support

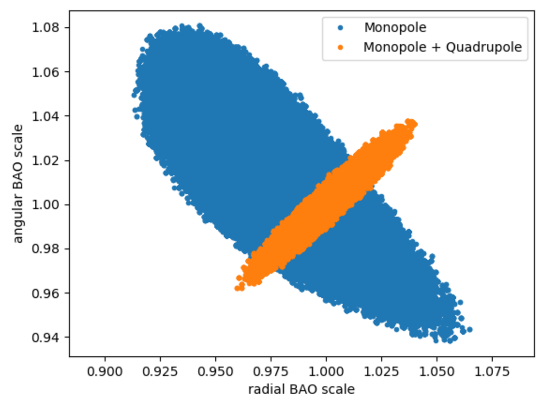
PI Samushia has not previously been funded by the NSF.

Co-PI Ratra has been involved in KSU QuarkNet activities funded by a subaward from the University of Notre Dame via their NSF award "QuarkNet", NSF grant number PHY-1219444 (09/01/2012 – 08/31/2018). Cumulative KSU QuarkNet funding over 05/01/2003—12/31/2018 is $281,910. KSU QuarkNet activities have not resulted in publications.

KSU QuarkNet students and teachers have used Fermilab-loaned cosmic-ray muon detectors to: record the cosmic-ray muon flux over a large part of Kansas and in Kansas City, MO, for many years in some cases; study the efficacy of different shielding material for (and become finalists in) an international NASA high-school competition to research shielding material for a manned mission to Mars; and study the effect of the Moon blockage on muon emission rates from the direction of the Sun by making measurements in the direction of the Sun, before, during, and after the 2017 August 21 Solar-eclipse totality.

The Co-PI is responsible for seven full days of teacher-centered QuarkNet activities each year. Typically more than 15 teachers visit at a time and typically over 100 high school students take part. Teachers are exposed to many different physics activities, recent ones have included classes or workshops on statistics and error analysis; electromagnetic waves, light, lasers, coherence, and polarization; dark matter; gravitational waves. In alternate years teachers visit national laboratories for four full days of activities. In 2017 they went to LIGO in LA and in 2015 it was SURF in SD. Feedback from teachers has been very positive. Every year high-school students spend a day at the KSU campus involved in hands-on physics activities and at this QuarkNet Masterclass they analyze real CERN LHC CMS data, after which they interact via videoconference with other groups of high-school students, typically in Central or South America (because of time zone alignment) and experimental particle physicists at Fermilab. KSU QuarkNet high-school students have become KSU Physics majors.

Intellectual Merit



The main outcome of this project will be the methods of extracting BAO/RSD constraints from measured galaxy bispectrum. The main merit of this enterprise is that these constraints will help to significantly tighten constraints on DE and TG. DE is one of the biggest masteries in modern physics and for now our only experimental handle on it is cosmological data. We have argued that the even in the worst case scenario, in which systematics can only be controlled on large scale, the results would still be helpful in tightening constraints on DE and TG. In addition to this practical gain, the proper modeling of higher order statistics of galaxy field would be interesting in itself. Nonlinearities in gravitational evolution, bias, and RSD are difficult to model mathematically and solving this problem would be a formidable intellectual achievement. The results of the project will have practical application in other fields of observational cosmology, including analysis of higher order clustering of weak lensing, CMB and various cross-correlations.

Broader Impacts

While working on this project we will make a broader impact on society in multiple complementary ways.

We will train a graduate student and a postdoc. The PhD student will be involved in most aspects of this work and will gain invaluable experience in data analysis, theoretical modeling of large-scale structure, and scientific software development. The student will have an opportunity to get involved with biggest cosmology surveys of next decade. By the end of the project the PhD student will be in a very good position to pursue the research career in this very promising field further and to contribute to the next generation of dark energy experiments. The postdoc will also have an opportunity to work with DESI and to produce high impact work that will strongly enhance their career. The PI and Co-PI will devote significant time to mentoring the PhD student and the postdoc.

As part of our project we plan to involve two undergraduate students in our research activities. The students will help core team members in developing software pipeline. This will be a wonderful opportunity for them to get hands-on experience with scientific research and hone their programming, physics, and applied mathematics skills.

KSU cosmology group has a very good track record of involving undergraduate students in research. PI has worked with four undergraduate students in last three years. The Co-PI guides four to five undergraduates in cosmology independent study each semester. In the last four years Co-PI submitted eight papers with undergraduate students [62, 66–73]. A number of these students have been accepted in good PhD programs. For instance, Sara Crandall is now at UC Santa Cruz, Stephen Houston is at the University of Kansas, Drew Johnson joined the PhD program at University of Minnesota, and Sanket Doshi and Varun Mathur just joined the PhD programs at Maryland and Virginia. Both PI and Co-PI have been active in the KSU REU program.

In addition we plan to run an extensive outreach program consisting of middle and elementary school visits and public talks. The Co-PI and colleagues do about an hour of physics demonstrations each year for roughly 20 physics classes in two Manhattan Middle Schools, typically reaching over 450 eighth graders each year. They have done this for the last eight years. They have also put on a school-wide set of physics demonstrations each year at Theodore Roosevelt Elementary, for more than 15 years. Typically, close to 300 students and teachers see these demonstrations.

The Co-PI has organized many public lectures on science at KSU. These attract large, diverse audiences that can include a number of upper elementary school children and townies. On a number of occasions over 400 people have attended. Last year John Preskill spoke about Quantum Computing and Fred Espenak (Mr. Eclipse") talked about the Great American Total Solar Eclipse. The path of totality was less than 100 miles from KSU and the PI, with other colleagues, actively promoted this solar eclipse and KSU responded in a very positive manner, allowing students to take ofrom classes on August 21 and arranging for bus rides to two sites, one in NE Kansas (for 550 people!) and the other in NW Missouri, both close to the path of totality.

Co-PI is a well-known public speaker on cosmology. He has spoken at many Science Cafes, local observatories, planetaria, science museums, and amateur astronomy clubs (including two of the largest, the St. Louis Astronomical Society and the Astronomical Society of Kansas City), in the mid-west, across the US, and internationally. A number of his talks have had audiences exceeding 150, one was attended by over 600 people. The PI has extensive outreach experience. He has contributed to many "stargazing" events at University of Portsmouth, and has presented popular science talks at St. Jude's elementary school. He has supervised work placement of three high school students at University of Portsmouth. After moving to KSU PI gave a number of public talks geared at general undergraduate student audience at Benedictine College and Ilia State University.

PI and Co-PI plan to continue being involved in these rewarding outreach activities.

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