#1

$W \in \mathbb{R}$, at $t=0$

$I \in \mathbb{Z}$

$0 \leq t \leq T$

$$V(t, S_t, W, I) = \mathbb{E}\left[-e^{-r(W+I S_T)} \middle| (t, S_t)\right]$$

$dS_t = \sigma \, dz_t$. for some fixed $\sigma \in \mathbb{R}_+$.

$$S_{t_2} \sim N(S_{t_1}, \sigma^2 \cdot (t_2 - t_1)) \quad \forall \; 0 \leq t_1 \leq t_2.$$

$$\mathbb{E}\left[-e^{-r(W+I S_T)} \middle| (t, S_t)\right]$$

$$= -\int_{-\infty}^{\infty} e^{-r(W+I S_T)} \cdot \underset{S_T \sim N(S_t, \sigma^2(T-t))}{f(S_T)} \, dS_T$$

$$= -\frac{1}{\sqrt{2\pi \sigma (T-t)}} \int_{-\infty}^{\infty} e^{-r(W+I S_T) - \frac{1}{2} \frac{(S_T - S_t)^2}{\sigma^2 (T-t)}} \, dS_T.$$

$$= -\frac{1}{\sqrt{2\pi\sigma(T-t)}} e^{-rW + \frac{S_t^2}{\sigma^2(T-t)}} \int_{-\infty}^{\infty} e^{\left(-rI + \frac{S_t}{\sigma^2(T-t)}\right) S_T - \frac{S_T^2}{\sigma^2(T-t)}} \, dS_T$$

(stuck)