#3

Let's Prove that

$$G_t - V(S_t) = \sum_{i=t}^{T-1} \gamma^{i-t}\left(R_{i+1} + \gamma V(S_{i+1}) - V(S_i)\right)$$

Let's start with the RHS.

$$\sum_{i=t}^{T-1} \gamma^{i-t}\left(R_{i+1} + \gamma V(S_{i+1}) - V(S_i)\right)$$

$$= \sum_{i=t}^{T-1} \gamma^{i-t} R_{i+1} + \sum_{i=t}^{T-1} \gamma^{i-t}\left(V(S_{i+1}) - V(S_i)\right)$$

$$= G_t + \left(\gamma V(S_{t+1}) - V(S_t)\right)$$

$$+ \gamma\left(\gamma V(S_{t+2}) - V(S_{t+1})\right)$$

$$\vdots$$

$$+ \gamma^{T-1-t}\left(\gamma V(S_T) - V(S_{T-1})\right)$$

$$= G_t + \gamma^{T-t} V(S_T) - V(S_t)$$

$$\lim_{T\to\infty}\left(G_t + \gamma^{T-t} V(S_T) - V(S_t)\right)$$

$$= G_t - V(S_t), \text{ if } \gamma \in [0,1)$$

□