#1

$$x - \frac{\alpha}{2} x^2 =$$

$$U(x) = x - \frac{\alpha x^2}{2}$$

$$x \sim N(\mu, \sigma^2)$$

Calculate:

- $E[U(x)] = \mu - \frac{\alpha}{2}(\mu^2 + \sigma^2)$
- x_{CE} : solution to $-\frac{\alpha}{2} x_{CE}^2 + x - \mu + \frac{\alpha}{2}(\mu^2 + \sigma^2) = 0$
- π_A :

$t = 10^6$

$z \in [0, 10^6]$ in risky

$10^6 - z$ in safe asset

$R \sim N(\mu, \sigma^2)$: risky return

r : safe return.

We want to maximize.

$$\mathbb{E}\left[U\left(R\cdot z + r(10^6 - z)\right)\right].$$

$$= \mathbb{E}\left[Rz + r(10^6 - z) - \frac{\alpha}{2}\left(Rz + r(10^6 - z)\right)^2 \right]$$

$$= \mathbb{E}[R]z + \frac{\alpha}{2}\left(\mathbb{E}\left[R^2 z^2 + 2Rzr(10^6 - z)\right] + r^2(10^6 - z)^2\right)$$

$$= \mu z + \frac{\alpha}{2} z^2(\sigma^2 + \mu^2) + 2\mu z r(10^6 - z) + r^2(10^6 - z)^2$$

$$= \left(\frac{\alpha}{2}(\sigma^2 + \mu^2) - 2\mu r + r^2\right) z^2$$

$$+ \left(\mu + 2\times 10^6 \mu r - 2\times 10^6 r^2\right) z,$$

$$+ 10^{12} r^2$$

This is a quadratic fct. of z.
Maximizing it is easy.