DSP: A Statistically-Principled Structural Polarization Measure: Supplementary Material

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Abstract

Social and information networks may become polarized, leading to echo chambers and political gridlock. Accurately measuring this phenomenon is a critical challenge. Existing measures often conflate genuine structural division with random topological features, yielding misleadingly high polarization scores on random networks, and failing to distinguish real-world networks from randomized null models. We introduce DSP, a Diffusion-based Structural Polarization measure designed from first principles to correct for such biases. DSP removes the arbitrary concept of "influencers" used by the popular Random Walk Controversy (RWC) score, instead treating every node as a potential origin for a random walk. To validate our approach, we introduce a set of desirable properties for polarization measures, expressed through reference topologies with known structural properties. We show that DSP satisfies these desiderata, being near-zero for non-polarized structures such as cliques and random networks, while correctly capturing the expected polarization of benchmark topologies such as split-color symmetrical networks. Finally, on real-world datasets of legislative behavior in the U.S. Congress, DSP uncovers trends of increasing polarization in recent years. By integrating a null model into its core definition, DSP provides a reliable and interpretable diagnostic tool, highlighting the necessity of statistically-grounded metrics to analyze societal fragmentation.

CCS Concepts

• **Human-centered computing → Social network analysis**; • **Theory of computation** → *Graph algorithms analysis*; • **Applied computing** → *Sociology*;

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A Structural Polarization Measures

The Betweenness Centrality Controversy (BCC) [4] measure compares the edge betweenness centrality of boundary and non-boundary links, by computing the KL-divergence d_{KL} between the two corresponding distributions:

$$\mathrm{BCC} \doteq 1 - \exp^{-d_{\mathrm{KL}}} .$$

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The intuition is that if the two communities are strongly separated, then the links on the boundary are expected to have high edge betweenness centralities.

The Boundary Polarization (BP) [5] compares the concentration of high-degree nodes within the communities (I) and their concentration in the boundary (C):

$$\mathrm{BP} \doteq \frac{1}{|C|} \sum_{s \in C} \frac{d_I(s)}{d_C(s) + d_I(s)} - 0.5 ,$$

where $d_C(s)$ is the degree of s restricted to neighbors in C. The intuition is that the further away authoritative users are from the boundary, the larger the amount of polarization present in the network.

The Dipole Moment (DP) [9] measure applies label propagation for quantifying the distance between the top-k influencers of each community, which are assigned the extreme opinion scores of -1 or 1. Let gc^+ and gc^- denote the average of positive and negative opinion scores. The DP measure is a function of the distance between the means of the opposite opinion score distributions, rescaled to penalize differences in the community sizes:

$$\mathrm{DM} \doteq \left(1 - \frac{n^+ - n^-}{n^+ + n^-}\right) \frac{|gc^+ - gc^-|}{2} .$$

The Krackhardt E/I Ratio (EI) [8] is defined as the relative density of intra-edges compared to the number of inter-edges:

$$\mathrm{EI} \doteq \frac{\mathrm{EL} - \mathrm{IL}}{\mathrm{EL} + \mathrm{IL}} ,$$

where EL is the set of edges with endpoints belonging to different communities, and IL is the set of edges with endpoints belonging to the same community.

The Adaptive E/I Index (AEI) [1] extends the EI measure to account for communities with different sizes:

$$\mathrm{AEI} \doteq \frac{\sigma_{aa} + \sigma_{bb} - 2 * \sigma_{ab}}{\sigma_{aa} + \sigma_{bb} + 2 * \sigma_{ab}} ,$$

where σ_{aa} is the ratio between the number of intra-edges in community a and the number of potential intra-edges, and σ_{ab} is the ratio of the number of inter-edges between community a and b and the number of potential inter-edges.

Modularity (Q) [10] compares the connectivity of the communities to that observable in random graphs extracted from the configuration model:

$$\mathrm{Q} \doteq \frac{1}{2|E|} \sum_{i,j \in V} \left(A_{i,j} - \frac{d(i)d(j)}{2|E|}\right) \delta(i, j) ,$$

where A is the adjacency matrix of the graph and $\delta(i, j)$ equals one only when i and j belong to the same community.

B Additional Datasets

GARIMELLA [3] contains 10 controversial and 10 non-controversial Twitter retweet networks, constructed from Tweets collected from Feb 27 to Jun 15, 2015, using sets of related hashtags, each anchored by one manually selected hashtag. CONOVER [2] consists of two networks constructed from tweets collected between Sep 14 and Nov 1, 2010, ahead of the U.S. congressional midterm elections. Starting from the two popular political hashtags #p2 ("Progressives 2.0") and #tcot ("Top Conservatives on Twitter"), a set of 66 related hashtags was identified and used to create a retweet and a mention network. For node labels, in CONOVER we use the node labels provided by the authors, and for GARIMELLA, we generate node labels using the Kernighan–Lin (KLIN) [7] and METIS [6] partitioning algorithms.

C Additional Experiments

More on Classification Performance. Figure 1 shows that the performance of DSP is comparable to that of RWC.

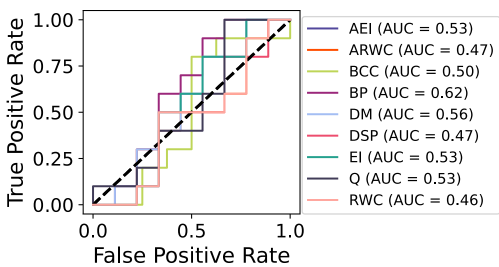

Figure 1: ROC curves and AUC values for GARIMELLA.

Similar results can be observed for the CONOVER networks (Table 1): higher values are recorded for the polarized RETWEET network, while values close to the minimum for the non-polarized MENTION network (see Figure 2).

Table 1: Polarization scores for the CONOVER networks.

	Metric								
Dataset	AEI	ARWC	BCC	BP	DM	EI	Q	RWC	DSP
MENTION	0.306	0.150	0.609	-0.030	0.643	0.315	0.101	0.101	0.057
RETWEET	0.959	0.837	0.832	0.257	0.768	0.954	0.475	0.869	0.410

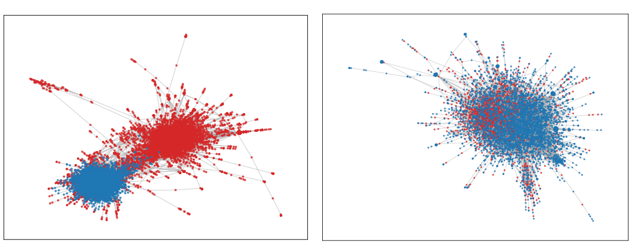

Figure 2: RETWEET (left) and MENTION (right) networks.

Impact of α on DSP. Figure 3 displays the DSP scores computed on bi-colored alternating cycles with various numbers of vertices, evenly split between red and blue, using different values of the parameter α that controls the restart probability in the PPR computation. Lower values of α correspond to more frequent restarts, making the PPR vector more influenced by the immediate neighbors of the starting node. Higher values of α emphasize more central nodes. We observe that DSP is sensitive to the choice of α: as α decreases, the DSP score also decreases. This behavior is expected, as each node's immediate neighbors belong to the opposite community. When the diffusion process is more strongly influenced by neighbors, more probability mass flows across community boundaries, leading to a lower polarization score.

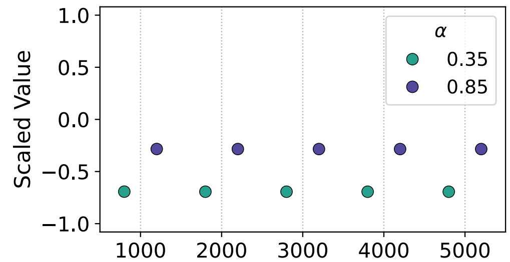

Figure 3: DSP in bi-colored alternating cycles with various numbers of vertices, 50% red and 50% blue. We show the rescaled values for different values of α.

In addition, Figure 4 shows the value of DSP for the SALLOUM networks, for two values of α. Although the polarization scores are slightly lower for larger values of α, the classification performance of DSP remains stable (0.68 using $\alpha = 0.85$ and 0.64 using $\alpha = 0.35$).

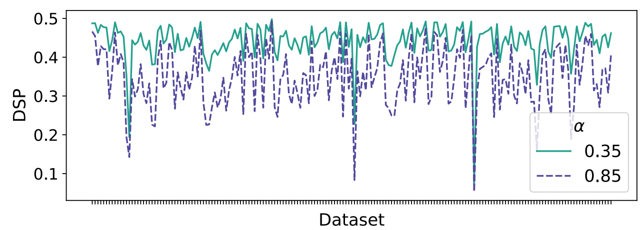

Figure 4: DSP for SALLOUM, varying the value of α.

More on Polarization and Assortativity. Figure 5 shows the distribution of the DSP values in 100 graphs sampled using Polaris, together with the color assortativity of the observed datasets. We report results for a subset of datasets for visualization purposes. This chart helps understand whether higher assortativity tends to correspond to higher polarization scores. We find that this relationship generally holds across datasets.

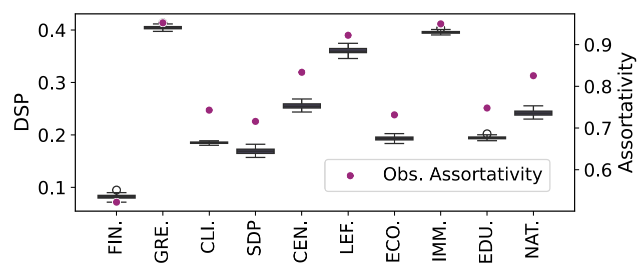

Figure 5: SALLOUM: DSP scores in the samples generated using Polaris and assortativity of the observed datasets.

References

[1] Ted Hsuan Yun Chen, Ali Salloum, Antti Gronow, Tuomas Ylä-Anttila, and Mikko Kivelä. 2021. Polarization of climate politics results from partisan sorting: Evidence from Finnish Twittersphere. *Global Environmental Change* 71 (2021), 102348. 1

[2] Michael Conover, Jacob Ratkiewicz, Matthew Francisco, Bruno Gonçalves, Filippo Menczer, and Alessandro Flammini. 2011. Political polarization on twitter. In *Proceedings of the international aaai conference on web and social media*, Vol. 5. 89–96. 2

[3] Kiran Garimella, Gianmarco De Francisci Morales, Aristides Gionis, and Michael Mathioudakis. 2016. Quantifying Controversy in Social Media. In *International Conference on Web Search and Data Mining (WSDM)*. ACM, 33–42. 2

[4] Kiran Garimella, Gianmarco De Francisci Morales, Aristides Gionis, and Michael Mathioudakis. 2018. Quantifying Controversy on Social Media. *ACM Transactions on Social Computing* 1, 1 (2018), 3. 1

[5] Pedro Guerra, Wagner Meira Jr, Claire Cardie, and Robert Kleinberg. 2013. A measure of polarization on social media networks based on community boundaries. In *Proceedings of the international AAAI conference on web and social media*, Vol. 7. 215–224. 1

[6] George Karypis and Vipin Kumar. 1997. METIS: A software package for partitioning unstructured graphs, partitioning meshes, and computing fill-reducing orderings of sparse matrices. (1997). 2

[7] Brian W Kernighan and Shen Lin. 1970. An efficient heuristic procedure for partitioning graphs. *The Bell system technical journal* 49, 2 (1970), 291–307. 2

[8] David Krackhardt and Robert N Stern. 1988. Informal networks and organizational crises: An experimental simulation. *Social psychology quarterly* (1988), 123–140. 1

[9] Alfredo Jose Morales, Javier Borondo, Juan Carlos Losada, and Rosa M Benito. 2015. Measuring political polarization: Twitter shows the two sides of Venezuela. *Chaos: An Interdisciplinary Journal of Nonlinear Science* 25, 3 (2015). 1

[10] Andrew Scott Waugh, Liuyi Pei, James H Fowler, Peter J Mucha, and Mason A Porter. 2009. Party polarization in congress: A network science approach. *arXiv preprint arXiv:0907.3509* (2009). 1