16/10/2023 Write out the inadequacies of classical mechanics that leads to quantum mechanics. Dr Akala

Answer,

Classical mechanics. while incredibly successful in explaining macroscope phenomena, faced inadequacies when dealing with microscope scales and high speeds. These limitations ultimately led to the development of quantum mechanics.

1 Failure at the quantum level.

Classical mechanics fails to explain phenomena at the atomic and sub-atomic levels such as the behavior of electrons with atoms, where discrete energy levels and quantized behaviors are observed.

2 Wave- particle duality

Classical mechainics could not account for the dual nature of particles which exhibits both wave-like and particle like properties. The wave particle duality became evident in experiments like the double-slit experiment, Challenging classical ideas of particle behavior

3. Uncertainly principle :- Classical mechanics assumed precise measurement was possible yet the Heisenberg uncertainty Principle revealed that it's impossible to Simultaneously measure certain pairs of properties like position and momentum with absolute accuracy

4. Quantization of energy

Classical mechanics did not account for the quantization of energy levels observed in phenomena like black body radiation, and the photoelectric effect where energy is discrete units or quanta

5 Need for statistical interpretation

Classical mechanics struggled with explaining Statistical behavior observed in systems of many Particles which become foundational in quantum mechanics with the development of statistical mechanics

These limitations prompted the development of quantum mechanics, a theory that revolutionized our understanding of the miscropic world by introducing probabilistic descriptions wave functions and a fundamentally different set of principles govering the behavior of particles at the quantum Scale

Annihilation of Electrons.

Electrons annihilation refers to a process where an electrons and its antimatter counterpart, a positron, collide and cease to exist, transforming their mass into energy.

When they annihilate, they release photons (gamma rays) with energies equivalent to the mass of the annihilating particles, following Einstein's famous equation, $E = mc^2$. This process conserves both energy and momentum.

The resulting gamma-ray ~~particles~~ photons carry the energy previously held in the mass of the electron and positron. This phenomena is crucial in various fields, including particle physics, astrophysics (such as in the interactions of cosmic rays) and medical imaging techniques like PET scans, where positron emission is used to produce images of internal body structures.

In a Simpler terms

Electrons annihilation means the process when an electron and its opposite, a positron meet, they disappear and what is left is energy in the form of light. It is like when you have an equation where you start with two things on one side and they turn into something different on the other side. In this case, the electrons and positron combine, and the result is energy specifically light called gamma rays. This process is called ANNIHILATION, and it happens according to Eistein's famous equation where mass can change into energy

$$E = mc^2$$

22-11-27

* Inadequacies of Classical mechanics and the birth of quantum ~~of quar~~ mechanics

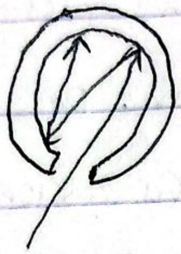
Up to the late 19th Century, the law of Physics was based on Classical (Newtonian) mechanics and maxwell's equat for electricity and magnetism. This Classical laws were early thougth to be Sufficient for describing all physical phenom however, as new physical phenomena at ~~sub~~ atomic and Sub-atomic emerged in the late 19 and early 20th Centurary, These laws became consistently unsuccessful in proving Satisfactory physical explanation to the newly em Phenomena. This Inadequacies Lead to the development of quantum mechanics. Reasons →

1. Black body radiatin
ii. photoelectric emission
" atomic Spectra
iii Compton effect
iv Specific heat Capacity of solid

Max planck (1858 - 1947)

1. Black body radiation

A black body is an ideal system that absorb all radiation that incident on it. The electromagnetic radiation emitted by a black body is called black body radiation.



It depends on the temperature of the cavity.

Characteristics of the system depends on the temperature of the body. At room temperature, the wavelength of the thermal radiation are mainly in the infrared region. However as the surface temperature increases, the wavelength change the object glow red, ~~orange~~ Yellow and blue.

Light is EM wave that is produced when an electric charge vibrates.

In a hot object, electrons vibrates in random motion as a result, light is produce. A hot object means more energetic vibration and so more light ~~by~~ is emitted as it glows ~~brighter~~ larger a it vibrate.

Classical physics predicted that each frequency of vibrat should have the same energy

Experimental result show that the black body spectrum allways become short wavelength (high frequency)

In 1900, Planck propose that each frequency of vibration Should have the same energy instead he Suggest that energy is not shared equally with electrons that Vibrate. Planck explain that energy come in packet called quatum.

Stephan - ~~Boltzmann~~ Boltzman law : the total power of the emitted radiation increases with temperature. Joseph Stefan formulated a law which state that the radiant energy of a black body is proportional to the fourth power of its temperature

$$J = \sigma T^4$$

Stefan's law was one of the ~~the~~ first important steps toward

The Same law was theorically derived by Ludwig boltzman

* Wilhelm kliens displacement law

: he used theory about heat and electromagnetism to detected the Wien's displacement which calculates the emission of a black body at any temperature from the emission at any $\omega$

Wien's displacement law states that black body radiation curve for different temperature peaks at the wavelength inversely proportional to the temperature

$$\lambda = \frac{B}{T}$$

$$\lambda T = B$$

$$\lambda_{max} T = 2.898 \times 10^{-3} \, mK$$

The Intensity of a black body radiation increases with increasing temperature.

Rayleigh – Jean's law :- he derived a ~~dependence~~ dependence $\lambda^{-4}$. It reveal an important error in physics theory at that tim the law predicted an energy output that diverted toward infinity as wavelength approaches zero ($F \to \infty$). However, measurement of energy at short wavelength disagreed with the rayleigh – Jean's law. It is given by,

$$I(\lambda, T) = \frac{2\pi c K_B T}{\lambda^4}$$

Max Planck explanation of black body radiation

Classical physics predicted that all object will instantly radiate all their heat into E.M.W. Experiments showed that the radiation rate went to infinity as E.M.W approaches zero, and it is called the UV ~~catta~~ catastrophe. Max planck solve d problem by ~~saying~~ postulation that E.M energy was emitted in quanta with $hf$

Two major assumption

d energy of d oscillators can hv only discrete value

$E = nhf$ $n = 1, 2, 3, 4$ $n$ = quantum no

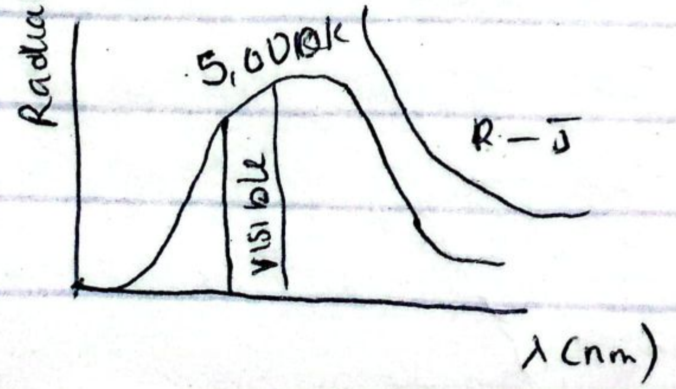
d oscillators emit or absorb energy when making a transition from one quantum state to another.

$$E_2 - E_1 = nhf_2 - nhf_1$$

Plank's distribution function

UV catastrophe solution

$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^2} \cdot \frac{1}{(e^{hc/\lambda k_B T} - 1)}$$



Class-work

Compute the black body radiation Intensity using the Planck's radiation distribution formula at varying wavelength 100:25:1800nm, For three set of temperature (T) 4000K, 6000K, 7000K.

b Plot on a composite ~~of~~ axis for the Planck radiation Intensity against wavelength for temperature 4000K, 6000K and 7000K. Hold the Composite axis, and plot the rayleigh Jean's radiation against wavelength for 7000K

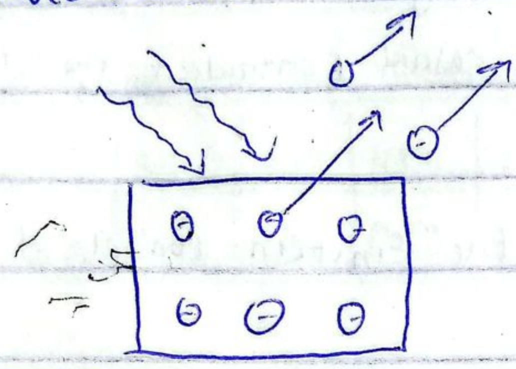
c Comment on the rayleigh Jean's radiation Intensity Curve and planck radiation Intensity Curve!

What is the position of ~~d~~ peaks of the ~~radiatio~~ planck's radiation intensity wrt to the origin of the wavelength axis, 4000K, 6000K and 7000K.

Estimate the order of Increment in the radiation Intensity peaks for 4000K, 6000K, 7000K

PHOTOELECTRIC EFFECT

It occur when light is incident on a metal Surface causing electron to be emitted from such surface. The emitted electrons are called Photo electrons



Frequency
The light Coming must be equal and greater than the Surface Frequency.

\# Classical ~~experimentation~~ experimentation and quantum experimentation.

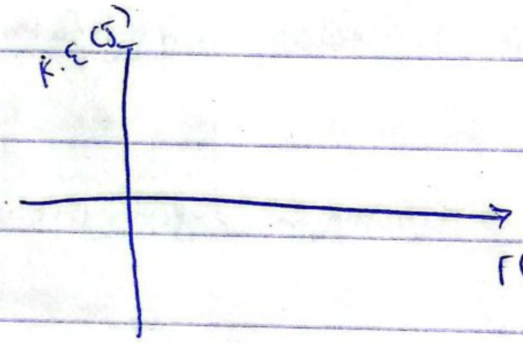
Classical mechanics predicted that the K.E of photons depends on light Intensity. As d light Intensity Incident on a metal is Increased, electrons expected to ejected with more energy. But experimentation Shows that the ~~mass~~ maximum K.E is Independent of light energy but proportional to the Stopping potentials. The Max K.E Increases with Increase in light freq.

further more, electrons are expected to be eject at any frequency as long as light Intensity is high enough. However, experimental result shows that electrons are emitted if d Freq is below the Cut off Freq. The Cut off Frequency is a Characteristics of Photo electric material to electron is emitted below the Cut off Freq.

At low intensities measurable time interval should pass btw the instant, d light is turned on.

The experimental result contradict the classical mechanic prediction.

This nessi. the Einstein concept of quantization to Electromagnet waves




$E = hf$

kinetic

$K.E = hf - \phi$ work function

Workfunction $\phi$ of metals

| Metal | $\phi$ eV |
|---|---|
| Sodium | 2.46 |
| Al | 4.18 |
| Fe | 4.50 |
| Cu | 4.70 |
| Zn | 4.31 |
| Ag | 4.73 |
| Pt | 6.75 |
| Pb | 4.14 |

We know the plot of K.E Vs f is always linear

## Assignment:

A Certain photo electric surface was illuminated with light of different wavelength. And the following stopping potential are as follow

| $\lambda$ nm | 336 | 405 | 436 | 492 | 546 | 579 |
|---|---|---|---|---|---|---|
| stopping potential (eV) | 1.48 | 1.15 | 0.93 | 0.62 | 0.36 | 0.24 |

Plot a graph of f against eV and det

1. threshold frequency and wavelength
2. The workfunction of the materia
3. The value of the Planck Constant
4. Comment on your result when the intensity of the light is assumed to be Increased by a factor of 4.

## Compton Effect

~~The wave length~~

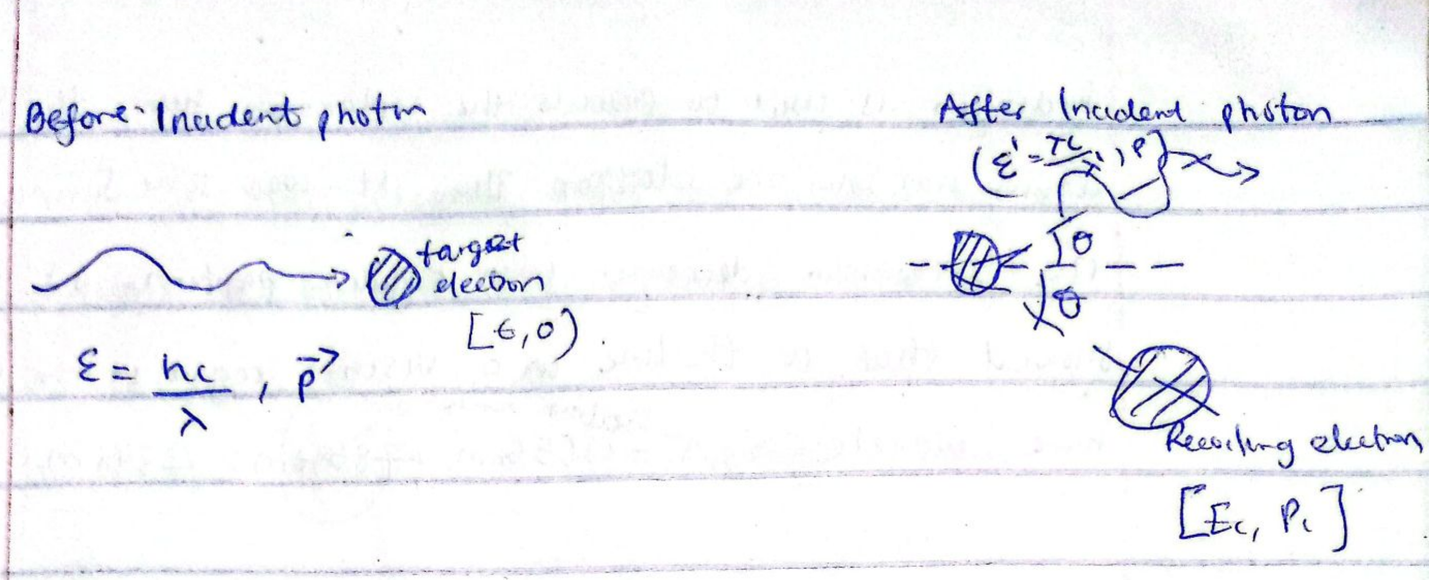
In 1923, Compton reported the result of his experiment on Scattering of monochromatic hard X-ray from graphite target. The wave length of the Scattered radiation ~~experiment~~ was measured using a Bragg's Spectrometer. The Scattered radiation at a given scattered angle was passed through the Collimator

### Failure of Classical mechanics

According to the Classical theory of Scattering of electromagnetic radiation from electrons, the oscillating field of the incident X-ray acts on the electrons that are contain in the atom of the target material. This Interaction forces the electrons to oscillate with d same freq at the Incident radiation. The Oscillating electrons in turn radiate E.M wave of the Same Frequency.

Therefore, the classical Scattering theory ~~electron~~ predict that the wavelength of the Scattering radiation ~~ratio~~ must be exactly be d same as dat of d Incident radiation. Thus, it fails to explain the Change in wavelength observed in Compton Scattering.

The Compton Scattering is elastic or coherence Scattering in Incident photon to electrons




$$\Delta h = h - \lambda' = \frac{\lambda}{m_e c}(1-\cos\theta)$$

## Atomic theory

31/11/23

After Rutherford found that repulsive charge in atoms is in a thin nucleus - classical mechanics predicted that the electrons orbiting a nucleus would radiate their energy away and spiral into the nucleus. Certainly, this did not happen.

Rydberg formula : $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

hydrogen is used to provide the explanation being the Simplest atom has only one electron. Thus it was the Simplest Spectral. The spectrum decreases in a regular pattern. JJ balmer Showed that to the line in a visible region in the spectrum have wavelength $\lambda =$ (balmer series) 656nm, 486nm, 434nm.

for $n=1$, it produce lyman series of line, $\lambda = 91nm - 122nm$ and it is the ultra-violet region

for $n=3$, it produce the paschen series of line of wavelength of region Infra-red

NB:- In all the series, $n=2$ must start at

i.e $n_2 = n_1 + 1$

Bohrs action.

In 1913, electron orbit the nucleus like planent orbiting a sun. He prostulate the hydrogen atom $(Z=1)$.

A typical Bohrs model

$$F_c = \frac{mv^2}{r} \quad , \quad F_c = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2} \rightarrow \text{Coulombs}$$

centripetal

$$F_c = F_c$$

$$\frac{mv^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$$

$$L = mvr = n\hbar$$

$$\therefore \frac{1}{r} = \frac{mZe^2}{4\pi\varepsilon_0\hbar^2} \cdot \frac{1}{n^2}$$

$$\therefore r_n = \frac{n^2h^2\varepsilon_0}{\pi mZe^2} = \frac{n^2}{Z} r_1$$

$$r_1 = \frac{h^2\varepsilon_0}{\pi me^2} = \text{Bohr's radius.}$$

$$r_1 = 0.529 A^\circ$$

The potential energy $U = -eV = -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r}$

The total energy of an electron in n orbit is the sum of PE and KE

$$T.E\,(E_n) = \frac{1}{2}mv^2 - \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r_n}$$

$$E_n = -\left(\frac{Z^2e^2m}{8\varepsilon_0^2h^2}\right)\left(\frac{1}{n^2}\right) \quad n = 1, 2, 3 \ldots$$

$$E_n = (-13.6eV)\left(\frac{Z^2}{n^2}\right)$$

when $n = 1$ and $Z = 1$, $E_n =$ ~~13eV~~ $-13.6eV$ → lowest energy level.

Construct Energy level diagram for Hydrogen $He^{2+}$ and $n = 1$ : 10, comment on the proximaty as n increase.

5 Validate the failure of Bohr's ~~fal~~ theory to Uncertainity principle take $r_a = 5.3 \times 10^{-11}m$, electron velocity $= 2.2 \times 10^6 ms^{-1}$, $m_e =$ ~~7.22 x 10~~ $9.11 \times 10^{-31}kg$

Assuming 10% of uncertainity is associated with ~~d~~ velocity

## Failures of Bohr Model

1- It fails to explain why certain spectral lines are brighter than other - and there is no mechanism for the calculation of transition probability

2. The Bohr model treat the electrons as if it were a minature planet with definite radius and momentum. This violated uncertainty principle which state that "position and momentum can not be simultaneously determine"

## Specific Heat Capacity

With classical mechanics prediction, the molar heat capacity at constant volume = 3R where R = the molar gas constant.

$= 8.314 J/K/mol.$

In thermodynamics, the ~~energy~~ average K.E per atom in each degree of freedom is $\frac{1}{2}KT$

i.e $K.E = \frac{1}{2}KT$

An atom

Wave particle duality

light can behave like a wave and like a particle.

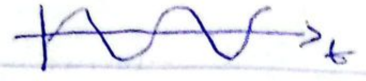
Diffraction experiment → a wave 

photo electric effect → a particle 

louis de broglie (1892-1987) he postulate that light have both wave and particles characteristics. Matter also have wave property.

he assumed $E = hf$ for photons and other particles. He used lorentz inverance to derived the wave length for particle the debrogle wavelength

$$\lambda = \frac{h}{P(mv)} \quad - ① \qquad \frac{h}{mu}$$

λ and F
wave like property

In term of freq, he postulate that $f = \frac{E}{h}$ - ② Pare 2
Particle-like properties

from equ 1 and 2 we can have the dual nature of matter

Condition

1. we can have wave like behavior of light

# Heinsenberg Uncertainity principle (HUP)

The uncertainity pairs of physical properties such as position and speed cannot be simultaneously measure.

$$\hbar = \frac{h}{2\pi}$$

$$\Delta x \, \Delta P_x \geq \frac{\hbar}{2}$$

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

~~Ast~~ A student is examing a batenum under a microscope. The E. coli bacteria cell has a mass of 0.1 Fectogram (0.1Fg) and is swimming at a velocity of $10 \mu ms^{-1}$ with uncertainity ~~and uncertai~~ of ~~10~~ of velocity of 10%.

Hint:- E coli baterra cell are around $1 \mu m$ in length. what is the uncertainity of position of the bacteria?

Fectr $= 10^{-15}$

soln

mass $= 0.1 fg = 0.1 \times 10^{-18} kg$

Velocity $= 10 \mu ms^{-1} = 10 \times 10^{-6} m/s.$

Uncertainity in vel $= 10\% = 0.1 m/s$

$$\Delta x \, \Delta P_x \geq \frac{h}{4\pi}$$

$$\Delta p_x = mv = (0.1 \times 10^{-18})(10 \times 10^{-6}) \times 0.1$$

$$\Delta x \geq \frac{h}{4\pi} \cdot \frac{1}{\Delta p_x} = \frac{6.626 \times 10^{-34}}{4\pi (0.1 \times 10^{-18}) \times (10 \times 10^{-6}) \times 0.1}$$

$$\Delta x \geq 0.53 \times 10^{-9} m$$

## Basic fundamental of Q. M

i. wave functin ( $\Psi$ )

ii Probability

iii operators

iv Expectation value

v Uncertainity value

vi mathematical formalize

### Wave function ( $\Psi$ ) →

In Q. M, the instantaneous state of a system is represented by a complex wave function $\Psi(x,t)$. A wave function contains information about where a particle is located and it must be well behaved. i.e it must ~~sta~~ satisfy the following -

1 A wave function is in general complex eg $\Psi(x,t) = a + ib$

2 ~~A wave funct~~ $|\Psi(x,t)|^2$ is always real and not negative in other word, it is always $\geq 0$

$$|\Psi(x,t)|^2 = |a + ib|^2 \geq 0$$

3. A wave-function must be single valued i.e for each points on a domain, the wave function must be a unique value.

4. A wave function must be continuous i.e the small change in the input will translate to a small change in the output. The function has finite value at any point in a given space.

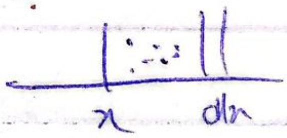
5. A wave-function must be differentiable i.e the derivative exists and it is a continuous function.

6. A wave function must be square integrable - for normalization condition to be satisfied, the given wave function must be square integrable.

i.e $\left[ \int_{-\infty}^{\infty} \psi \psi^* dx = 1 \right]$

If the wavefunction has a value at point $x$, the probability of finding the particle between $x$ and $x + dx$ is proportional to $dx$

$P_r \propto dx$




## Probability and Normalization in Quantum mechanics

The wave function that describes a quantum state provides the most complete description about the state.

Wave function = $\psi$

Wavefunction cannot be measured, and therefore it is seen not to have physical meaning. However, the information that can be derive from the wave function is related to the probabilities of the possible outcome of the measurement that are made on the quantum system.

$$\hat{H} Q = \sum H_i Q_i$$

$$\psi_n = \rightarrow \text{Eign values}$$

$E_i = E_1, E_2, E_3$ are Eigen energy

The eigen functions are said to be orthonormal

* The Eigen function is normalized so that the probability of finding electron ($e^-$) any where in space will be unity.
* Two eigen energy vectors with different eigen values are said to be orthogonal.

$$\int_{-\infty}^{\infty} \psi_i^*(x)\, \psi_j(x)\, dx = \delta_{ij}$$

$i = j$ = orthonormal, $\delta_{ij} = 1$

$$\delta_{ij} = \begin{cases} =1, & i = j \\ =0, & i \neq j \end{cases} \qquad \psi = A \sin\left(\frac{n\pi x}{2}\right)$$

$$P_r = \int_{x_1}^{x_2} \psi_n^*(x)\, \psi_n(x)\, dx = 1 \quad \text{normalization constant}$$

## Quantum mechanic Operator

I Quantum Mechanics, application of operators are so vital because waves are being used to described dynamical, systems.

"A Quantam Mechanics operator is a defined as a mathematical Instruction that can be performed on a wave function in order to extract a useful Information about a wave function" These operators are associated with measurable quantities i.e observables

For Instance, the energy of a particle in a quantum system can be measured. Hence energy is observed.

other common observables in QM are position, momentum and angular momentum. Each quantum mechanics operators is associated with observable

$\psi_n$; eigen value $a_n$, observable $A$, operator $\hat{A}$

$$\hat{A}\psi_n = \underbrace{a_n}_{\text{eigen value}}\psi_n$$

| SN | Operators | Observable | Description or remark |
|---|---|---|---|
| 1. | $\hat{A} = \hat{A}$ | Unknown (generic) Observable A | This Operator is for an unknown Observable A |
| 2 | $\hat{x} = \hat{x}$ | Position | This operator is for position, $x$ in 1-D and $r$ in 3D |
| 3 | $\hat{P}_x = i\hbar\frac{\partial}{\partial x}$ | momentum | The operator is for momentum in $x$ direction, same for y and z direction. |

| S/N | Operator | Observation | Remarks |
|---|---|---|---|
| 4. | $\hat{T} = \frac{-\hbar^2 \partial^2}{2m\partial x^2}$ | Kinetic energy | For K.E of a quantum System |
| 5 | $\hat{V}(x) = \hat{V}(x)$ | Potential energy | For P.E of a quantum system |
| 6 | $\hat{H} = i\hbar \frac{\partial}{\partial t}$ | Time dependent Hamilton | In addition of its ~~state~~ role in determing the quantum System, energies, the hamilton can also operate on a wave-funct to generate the time revolution of wave function. |
| 7. | $H = \frac{-\hbar^2 \partial^2}{2m\partial x^2} + V(x)$ | Time independent Hamilton | The time independent ~~in addition~~ Schrodinger equation can be constructed when time independent hamilton operates on wave function |
| | $\hat{L}_z = \frac{-i\hbar\partial}{\partial \phi}$ | Angular momentum | Anguler momentum in Z direction |

NB Schrodinger wave equation

$$\hat{H}\psi = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + \hat{V}\psi(x)$$

EXPECTATION VALUES AND UNCERTAINTY VALUE

1 Expectation value :- The estimate of the mean.

2 The uncertainity value :- The spread of the value around the mean i.e standard deviation. Suppose that an observable quantity A, has small discrect values; $a_1, a_2, a_3 \ldots a_n$, then we can prepare 'n' identical system i.e ensemble of systems, all in the same state and measure the value of "a" in each system.

In general, we will get a spread of results. In this regard the best estimate of quantity A is to get its mean value and standard deviation can be used as a measure of spread of uncertainty in the measurement.

| S/N | Operator | Observable | Expectation value Integral |
|---|---|---|---|
| 1 | $\hat{A}$ | | $\langle A \rangle = \int_{-\infty}^{\infty} \psi_n^*(x)\, \hat{A}\, \psi_n(x)\, dx$ |
| 2. | $\hat{x}$ | | $\langle x \rangle = \int_{-\infty}^{\infty} \psi_n^*(x)\, \hat{x}\, \psi_n(x)\, dx$ |
| 3. | $\hat{P}$ | momentum | $\langle P \rangle = \int_{-\infty}^{\infty} \psi_n^*(x)\, \hat{P}\, \psi_n(x)\, dx$ |
| 4 | $\hat{T}$ | K.E | $\langle T \rangle = \int_{-\infty}^{\infty} \psi_n^*(x)\, \hat{T}\, \psi_n(x)\, dx$ |

5 $\hat{V}$ $\langle V \rangle = \int_{-\infty}^{\infty} \psi^*_{(n)}(x)\, \hat{V}\, \psi_n(x)\, dx$

6. $\hat{H} = \hat{T} + \hat{V}$ Hamilton $\langle H \rangle = \int_{-\infty}^{\infty} \psi^*_n(x) \left( \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \hat{V}_{(x)} \right) \psi_n$

$\psi^*_n(x) \Rightarrow$ Bra $A\psi_n(x) = K_r\, et$

Uncertainty $\Delta A = \left[ \langle A^2 \rangle - \langle A \rangle^2 \right]^{1/2}$.

The wave function $\psi_n(x)$ cannot describe a particle that has a sharp momentum and is localized in space. The uncertainty in the measurement : $\Delta x \Delta p \geq \frac{\hbar}{2}$

$\Delta x = \left[ \langle x^2 \rangle - \langle x \rangle^2 \right]^{1/2}$

$\Delta p = \left[ \langle p^2 \rangle - \langle p \rangle^2 \right]^{1/2}$

$\therefore \left[ \langle x \rangle^2 - \langle x \rangle^2 \right]^{1/2} \left[ \langle p \rangle^2 - \langle p \rangle^2 \right]^{1/2} \geq \frac{\hbar}{2}$

Assignment

1 A particle of mass $m$ is in the state $\psi_n(x,t) = Ae^{-a}$

$\psi_n(x,t) = Ae^{-a\left[\frac{mx^2}{\hbar} + it\right]}$, where "$A$" and "$a$" are real constants

determine the normalisation constant $A$

Calculate the expression value of momentum and position, hence justify the validation and non-validation of HUP

# Mathematical fundamental of Quantum Mechanics

The basic mathematical background necessary to robust the understanding of Quantum mechanics is ~~lea~~ linear algebra in infinite space ie hillbert Space. With the concept of linear algebra, the state-function which is in complex domain and other mathematical operation upon it are represented in simple and compact mathematical notations. In complex field theory, complex number $z$ is denoted by $a + ib$, where $a$ and $b$ are real numbers and $i = \sqrt{-1}$. The

the complex conjugate $z^*$ of $z$ is $a - ib$. This implies that the product $z z^*$ is a real number.

For a set of vectors $\{x_n\}$ belongs to $V$ $[\in V]$, $V$ can be written as the linear combination of $V$ ie

$V = a_1 x_1 + a_2 x_2 + \dots a_n x_n$ where $a_1, a_2$ are complex no while $X_n$ are vectors and they are linearly independent, if

$a_1 x_1 + a_2 x_2 + \dots a_n x_n = 0$

The vector $x_1 + x_2 \dots x_n$ are called the basis and $n$ is unique and must have the dimension of $V$

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$a_1 x_1 + a_2 x_2 + \cdots a_n x_n = 0$

The vector $x_1 + x_2 \ldots X_n$ are called the basis and n is unique and must have the dimension of V

Dirac Function. (notation)

In Quantum mechanics, Bra and Ket notation also known as dirac notation is a standard notation for describing quantum mechanical state. The notation can also be used to denote abs vectors in Linear algebra. In such ~~t~~ terms, the scaler product or action of a linear function on a vector in a complex vector space ($C^n$) can be represented as $\langle b|a\rangle$

In linear algebra the abstract vector $C|a\rangle \rightarrow$ to be $|a'\rangle$ where $|a'\rangle$ is a new vector. Similarly, $|a\rangle + |b\rangle = |c\rangle$. Complex number are themselve vector space i.e they are column vector

$$|a\rangle = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{pmatrix} \qquad |b\rangle = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{pmatrix} \qquad C|a\rangle = \begin{pmatrix} Ca_1 \\ Ca_2 \\ Ca_3 \\ \vdots \\ Ca_n \end{pmatrix}$$

$$|a\rangle + |b\rangle = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ \vdots \\ a_n + b_n \end{pmatrix}$$

Every complex vector, represented as the ket vector has a complex conjugate vector in a dual vector space to the original vector space (known as bra - a row vector)

$$|a\rangle = \text{column wise} \rightarrow \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{pmatrix}$$

$$\langle a| = \text{row wise} \rightarrow (a_1^*, a_2^* + a_3^* \dots a_n^*)$$

The state of a QM is defined as a vector | which belong to a hillbert space and obey ~~d~~ super position principle. is $|\psi_1\rangle, |\psi_2\rangle \dots |\psi_n\rangle$ are ket belonging to ~~the~~ Hillbert Spaces. The linear combination shown below is also a valid state vector belonging to hillbert space.

$$|\psi\rangle = a_1|\psi_1\rangle + a_2|\psi_2\rangle + \dots a_n|\psi_n\rangle$$

So the state function conventional rep. by a ket is a column vector that must be multiply by a row vector known as bra.

| S/N | Operator | Dirac notation for expectation value | Integral notation for expectation value |
|---|---|---|---|
| 1 | ~~Generic~~ | | |
| 1. | Generic $\hat{Q}$ | $\langle\psi|\hat{Q}|\psi\rangle$ | $\langle Q\rangle = \int_{-\infty}^{\infty} \psi_n^*(x)\,\hat{Q}\,\psi_n(x)\,dx$ |
| 2 | Position $\hat{x}$ | $\langle\psi|\hat{x}|\psi\rangle$ | $\langle x\rangle = \int_{-\infty}^{\infty} \psi_n^*(x)\,\hat{x}\,\psi_n(x)\,dx$ |
| 3. | Momentum $\hat{P}_x$ | | |

INNER PRODUCT, ORTHOGONALITY AND ORTHONORMALITY.

The inner product of the two vector is defined as

$$\langle b|a\rangle = (b_1^* \; b_2^* \; \dots \; b_n^*) \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$

$$= b_1^* a_1 + b_2^* a_2 + \dots b_n^* a_n.$$

If the vectors are perpendicular to each other in orthogonality form. The inner product can be expressed as

$$\langle \alpha|\beta\rangle = \int_{-\infty}^{\infty} \phi^*(x) \; \varphi_\beta(x) \, dx = 0$$

However generally the orthonormality condition is the general condition orthonormality. $\langle \phi_n | \varphi_m \rangle = \delta_{nm}$

$$\langle \phi_n | \varphi_m \rangle = 1 \quad \text{If } n=1, m=1$$

$$\langle \phi_n | \phi_m \rangle = 0 \quad \text{If } n \neq m.$$

Normalization Condition

$$\int_{-\infty}^{\infty} \phi^*(x) \, \phi(x) \, dx = 1$$

For an arbituary Quantum mechanics state $\{|\phi_n\rangle\}$, representing orthonormal basis vector in a hillbert space, we can express a given state vector as a linear combination of this basis.

$$|\phi\rangle = \sum_{-\infty}^{\infty} a_n |\phi_n\rangle$$

Since the basis state are orthonormal, we can defined

$|\phi\rangle$ to be equal to $\sum_n a_n|\phi_n\rangle$

$$= \cancel{\sum_n \langle\phi_n|\phi\rangle} \sum_n \langle\phi_n|\phi\rangle|\phi_n\rangle$$

$$= \sum_n \left(|\phi_n\rangle\langle\phi_n|\right)|\phi\rangle$$

$$= \sum_n |\phi_n\rangle\langle\phi_n| = 1$$

The equation above is the completeness equation

$|\phi_n\rangle\langle\phi_n|$ is the outproduct which defined Projection P.

Classwork.

1. The set $\{|1\rangle, |2\rangle, |3\rangle\}$ is an othonormal basis in a three dimensional vector space. Let $|\psi\rangle$ be a vector in the vector space with: $|\psi\rangle = a(|1\rangle + 2i|2\rangle + (1+i)|3\rangle)$ and operator $\hat{Q}$ act on vectors in a 3-D vector space with the following effect on the orthonormal basis set $\{|1\rangle, |2\rangle, |3\rangle\}$

$\hat{Q}|1\rangle = |1\rangle + |2\rangle$

$\hat{Q}|2\rangle = -|1\rangle + |2\rangle$

$\hat{Q}|3\rangle = 0$

If $|\psi\rangle$ is a normalizable state function, find the value of constant a

$|1\rangle \times |1\rangle = 1$

$|1\rangle \times |2\rangle = 0$

ii. What is the representation of the column vector for $|\psi\rangle$ and $Q|\psi\rangle$ in this basis

iii. Write down the matrix representation of $\hat{Q}$ in this basis?

<u>Soln</u>

$a[|1\rangle + 2i|2\rangle + (1+i)|3\rangle) \times a(|1\rangle - 2i.|2\rangle + (1-i)|3\rangle] = 1$

$a^2 \cdot 7 = 1$

$a = \sqrt{\frac{1}{7}} = \frac{1}{\sqrt{7}}$

~~$a^2[1 - 2i + (1-i) + 2i + 4 + 2 + 2$~~

$|\psi\rangle = \frac{1}{\sqrt{7}}\left(|1\rangle + 2i|2\rangle + (1+i)|3\rangle\right)$

ii. $|\psi\rangle = \frac{1}{\sqrt{7}}\begin{pmatrix} |1\rangle \\ 2i|2\rangle \\ (1+i)|3\rangle \end{pmatrix}$

<u>Correction</u>

(ii) (i) $|\psi\rangle = \begin{pmatrix} \langle 1|\psi\rangle \\ \langle 2|\psi\rangle \\ \langle 3|\psi\rangle \end{pmatrix} = \frac{1}{\sqrt{7}}\begin{pmatrix} i \\ 2i \\ 1+i \end{pmatrix}$

ii. $Q|\psi\rangle = \begin{bmatrix} \langle 1|\hat{Q}|\psi\rangle \\ \langle 2|\hat{Q}|\psi\rangle \\ \langle 3|\hat{Q}|\psi\rangle \end{bmatrix} = \frac{1}{\sqrt{7}}\begin{pmatrix} 1-2i \\ 1+2i \\ 0 \end{pmatrix}$

iii. $\hat{Q} = \begin{bmatrix} \langle 1|\hat{Q}|1\rangle & \langle 1|\hat{Q}|2\rangle & \langle 1|\hat{Q}|3\rangle \\ \langle 2|\hat{Q}|2\rangle & \langle 1|\hat{Q}|2\rangle & \langle 1|\hat{Q}|3\rangle \\ \langle 3|\hat{Q}|3\rangle & \langle 1|\hat{Q}|2\rangle & \langle 1|\hat{Q}|3\rangle \end{bmatrix}$

$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$