PHS 313:

MAGNETOSTATICS:

-force due to Magnetic field:

There are at least three ways in which Force due to Magnetic fields can be experienced.

1. Due to a moving charged particle in a B field
2. On a current element in an external B field or
3. Between two current elements.

1. Force on a charged particle:

Recall from Coulomb's experiment law that

$$F_e = QE \qquad (1)$$

F_e, the electric force on a moving or stationary electric charge Q in an electric field is related to the electric field intensity E as shown in equation (1)

A magnetic field can exert force ONLY on a moving charge. From experiment it is found that the magnetic force F_m experienced by a charge Q moving with a velocity U in a magnetic field B is

$$F_m = Q U \times B \qquad (2)$$

This clearly show that F_m is perpendicular to both U and B.

From equations (1) and (2), a comparison between the electric force F_e and the magnetic form F_m can be made.

F_e is independent of the velocity of the charge and can perform work on the charge and change its kinetic energy unlike F_e, F_m depends on the charge velocity and is normal to it.

F_m cannot perform work because it is at right angle to the direction of motion of the charge ($F_m \cdot dl = 0$); it does not cause an increase in kinetic energy of the charge. The magnitude of F_m is generally small compared to F_e except at high velocities.

For a moving charge Q in the presence of both electric and magnetic fields, the ~~total~~ force on the charge is given by

$$F = F_e + F_m$$

or

$$F = Q(E + U \times B) \quad \text{——— ③}$$

This is known as Lorentz force equation.

If the mass of charged particle is m, by Newton's second law of motion

$$F = m\frac{du}{dt} = Q(E + U \times B) \quad \text{——— ④}$$

The solution to this equation is important in determining the motion of charged particles in E and B fields.

② FORCE ON A CURRENT ELEMENT.

To determine the force on a current element Idl of current-carrying conductor due to the magnetic field B, we modify equation ② using the fact that for convection current

$$J = \rho_v U \quad \text{——— ⑤}$$

Also recall that the relationship between current elements

$$Idl = Kds = Jdv \quad \text{——— ⑥}$$

Combining equations ⑤ and ⑥ yields

$$Idl = \rho_v U dv = dQ\,U$$

Alternatively
$$Idl = \frac{dQ}{dt} dl = dQ\frac{dl}{dt} = dQ\,U$$

Hence

$$\boxed{I\,dI = dQ\,u} \quad \text{---} (7)$$

The force on a current element $I\,dI$ in a magnetic field B is found from eqn. (5) by merely replacing Qu by $I\,dI$; that is

$$dF = I\,dI \times B \quad \text{---} (8)$$

If through a closed path L or circuit, the force on the circuit is given by

$$F = \oint_L I\,dI \times B \quad \text{---} (9)$$

The B-field in equation (8) and (9) is external to the current element $I\,dI$.

If instead of of line current element $I\,dI$, we have surface current element $K\,ds$ or a volume current element $J\,dv$, we simply make use of (6) so that ~~eqn~~ equation (8) becomes.

$$dF = K\,ds \times B \quad \text{or} \quad dF = J\,dv \times B \quad \text{---} (8a)$$

While eqn. (9) becomes

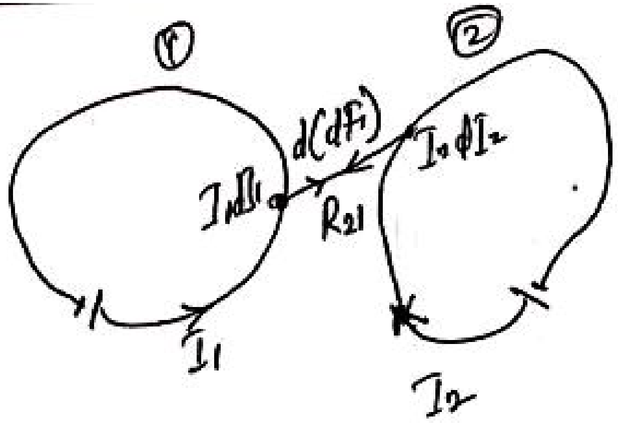
$$F = \int_s K\,ds \times B \quad \text{or} \quad F = \int_v J\,dv \times B \quad \text{---} (9a)$$

$\therefore$ The Magnetic field is defined as the force per unit current element.

(3) Force between Two Current Elements:

Consider the force between two element $I_1 dI_1$ and $I_2 dI_2$. According to Biot-Savart Law, both current elements produce magnetic fields.

We may find the force $d(dF_1)$ on element $I_1 dI_1$ due to field dB_2 produced by element $I_2 dI_2$ as shown below from eqn (8)



: Force between two current loops

$$d(dF_1) = I_1 dl_1 \times dB_2 \quad \text{—— (10)}$$

But from Biot – Savart law

$$dB_2 = \frac{\mu_0 I_2 dl_2 \times a_{R_{21}}}{4\pi R_{21}^2} \quad \text{—— (11)}$$

Hence

$$d(dF_1) = \frac{\mu_0 I_1 dl_1 \times (I_2 dl_2 \times a_{R_{21}})}{4\pi R_{21}^2} \quad \text{—— (12)}$$

This equation is essentially the law of force between two current elements and is analogous to Colomb's law" from equation (12), we obtain the total force F_1 on current loop 1 due to current loop 2 shown in the figure above as

$$F_1 = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{L_1} \oint_{L_2} \frac{dl_1 \times (dl_2 \times a_{R_{21}})}{R_{21}^2} \quad \text{—— (13)}$$

Example: 1

A charged particle of mass 2kg and charge 3C starts at point $(1, -2, 0)$ with velocity $4a_x + 3a_z$ m/s in an electric field $12a_x + 10a_y$ V/m. At time $t = 1s$, determine

(a) The acceleration

(b) Velocity

(c) Kinetic energy

(d) Its position

Solution

According to Newton's second law,

$$F = ma = QE$$

Where a is the acceleration of the particle, Hence,

$$a = \frac{QE}{m} = \frac{3}{2}(12a_x + 10a_y) = 18a_x + 15a_y \ m/s^2$$

$$a = \frac{du}{dt} = \frac{d}{dt}(u_x, u_y, u_z) = 18a_x + 15a_y$$

(b) Equating Components gives

$$\frac{du_x}{dt} = 18 \rightarrow u_x = 18t + A \quad \text{—— (1)}$$

$$\frac{du_y}{dt} = 15 \rightarrow u_y = 15t + B \quad \text{—— (2)}$$

$$\frac{du_z}{dt} = 0 \rightarrow u_z = C \quad \text{—— (3)}$$

Where $A, B, \& C$ are integration Constants. But at $t=0$, $u = 4a_x + 3a_z$, Hence

$$u_x(t=0) = 4 \rightarrow 4 = 0 + A \text{ or } A = 4$$

$$u_y(t=0) = 0 \rightarrow 0 = 0 + B \text{ or } B = 0$$

$$u_z(t=0) = 3 \rightarrow 3 = C$$

Substituting the values of A, B, and C into (1) ~~to (3)~~ (3) gives

$$u(t) = (u_x, u_y, u_z) = (18t + 4, 15t, 3)$$

Hence $u(t = 1s) = \underline{22a_x + 15a_y + 3a_z} \ m/s$

(c) Kinetic energy $(K.E) = \frac{1}{2}m|u|^2 = \frac{1}{2}(2)(22^2 + 15^2 + 3^2)$

$$= 718 \text{ J}.$$

$$U = \frac{dr}{dt} = \frac{d}{dt}(x, y, z) = (18t + 4, 15t, 3)$$

$$\frac{dx}{dt} = U_x = 18t + 4 \rightarrow x = 9t^2 + 4t + A_1 \quad (4)$$

$$\frac{dy}{dt} = U_y = 15t \longrightarrow y = 7.5t^2 + B_1 \quad (5)$$

$$\frac{dz}{dy} = U_z = 3 \longrightarrow z = 3t + C_1 \quad (6)$$

At $t = 0$, $(x, y, z) = (1, -2, 0)$; hence

$x(t=0) = 1 = 0 + A_1$ or $A_1 = 1$

$y(t=0) = -2 \rightarrow -2 = 0 + B_1$ or $B_1 = -2$

$z(t=0) = 0 \longrightarrow 0 + C_1$ or $C_1 = 0$

Substituting the values of A_1, B_1 and C_1 into eqns (4) to (6)

$$(x, y, z) = (9t^2 + 4t + 1, \; 7.5t^2 - 2, \; 3t)$$

Hence at $t = 1$, $(x, y, z) = (14, 5.5, 3)$

Example: 2

A charge particle moves with a Uniform Velocity $4a_x$ m/s in a region where $E = 20a_y$ V/m and $B = B_0 a_z$ Wb/m². Determine B_0 such that the velocity of the particle remains constant.

Solution;

If the particle moves with a constant Velocity, it implies that its acceleration is zero. The particle experiences no net force. Hence

$$0 = F = ma = Q(E + U \times B)$$

$$0 = Q(20a_y + 4a_x \times B_0 a_z)$$

$$-20a_y = -4B_0 a_y \Rightarrow B_0 = 5$$

Example 3 (9)

A charge particle of mass 2kg and 1 C starts at the origin with velocity $3a_y$ m/s and travels in a region of uniform magnetic field $B = 10a_z$ Wb/m^2, calculate.

(a) The velocity and acceleration of the particle
(b) The magnetic force on it
(c) Its kinetic and location
(d) Find the particle's trajectory by eliminating t.
(e) Show that its K.E. remains constant.

Solution:

(a) $F = m\frac{du}{dt} = Qu \times B$

$$a = \frac{du}{dt} = \frac{Q}{m} u \times B$$

$$\frac{d}{dt}(u_x a_x + u_y a_y + u_z a_z) = \frac{1}{2}\begin{vmatrix} a_x & a_y & a_z \\ u_x & u_y & u_z \\ 0 & 0 & 10 \end{vmatrix} = 5(u_y a_x - u_x a_y)$$

By equating components, we get

$$\frac{du_x}{dt} = 5u_y \quad \text{———} (1)$$

$$\frac{du_y}{dt} = -5u_x \quad \text{———} (2)$$

$$\frac{du_z}{dt} = 0 \rightarrow u_z = C_0 \quad \text{———} (3)$$

We can eliminate u_x or u_y in eqns (1) and (2) by taking 2nd derivatives of one equation and making use of the others. Thus

$$\frac{d^2u_x}{dt^2} = 5\frac{du_y}{dt} = -25u_x$$

or

$$\frac{d^2u_x}{dt^2} + 25u_x = 0$$

Which is a linear differential equation with solution

$$u_x = C_1 \cos 5t + C_2 \sin 5t \quad \text{—— (4)}$$

from eqns (1) and (4)

$$5u_y = \frac{du_x}{dt} = -5C_1 \sin 5t + 5C_2 \cos 5t \quad \text{—— (5)}$$

or $$u_y = -C_1 \sin 5t + C_2 \cos 5t$$

We now determine constant C_0, C_1 and C_2 using the initial conditions.

At $t = 0$, $u = 3a_y$.

Hence

$$u_x = 0 \rightarrow 0 = C_1 \cdot 1 + C_2 \cdot 0 \rightarrow C_1 = 0$$

$$u_y = 3 \rightarrow 3 = -C_1 \cdot 0 + C_2 \cdot 1 \rightarrow C_2 = 3$$

$$u_z = 0 \rightarrow 0 = C_0$$

Substituting the values of C_0, C_1 and C_2 into (3) to (5) gives

$$u = (u_x, u_y, u_z) = (3 \sin 5t, 3\cos 5t, 0) \quad \text{—— (6)}$$

Hence $$u(t=4) = (3\sin 20, 3\cos 20, 0)$$

$$= (2.739a_x + 1.224a_y) \text{ m/s}$$

$$a = \frac{du}{dt} = 15\cos 5t, -15\sin 5t, 0)$$

at $(t=4) = 6.101a_x, -13.703a_y$ m/s^2

$$F = ma = 12.2a_x - 27.4a_y \text{ N}$$

$$F = Qu \times B = (1)(2.739a_x + 1.224a_y) \times 10a_z = 12.2a_x - 27.4a_y \text{ N}$$

c) $K.E = \frac{1}{2} m |u|^2 = \frac{1}{2}(2)\left(2.739^2 + 1.224^2\right) = 9J.$

$u_x = \frac{dx}{dt} = 3\sin 5t \rightarrow x = \frac{-3}{5}\cos 5t + b_1$ ——— (7)

$u_y = \frac{dy}{dt} = 3\cos 5t \longrightarrow y = \frac{3}{5}\sin 5t + b_2$ ——— (8)

$u_z = \frac{dz}{dt} = 0 \rightarrow z = b_3$ ——— (9)

b_1, b_2 and b_3 are integration constants. ~~At t=0~~ At $t=0$, $(x, y, z) = (0, 0, 0)$ and hence:

$x(t=0) = 0 \longrightarrow 0 = \frac{-3}{5} \cdot 1 + b_1 \rightarrow b_1 = 0.6$

$y(t=0) = 0 \rightarrow 0 = \frac{3}{5} \cdot 0 + b_2 \Rightarrow b_2 = 0$

$z(t=0) = 0 \rightarrow 0 = b_3$

Substituting the values of b_1, b_2, and b_3 into eqns (7) to (9), we obtain:

$(x, y, z) = (0.6 - 0.6\cos 5t,\ 0.6\sin 5t,\ 0)$ ——— (10)

At $t = 4s$

$(x, y, z) = (0.3552,\ 0.5478,\ 0)$

d) from eqn (10), we eliminate t by noting that

$(x-0.6)^2 + y^2 = (0.6)^2\left(\cos^2 5t + \sin^2 5t\right), \quad z=0$

or $(x-0.6)^2 + y^2 = (0.6)^2, \quad z=0$

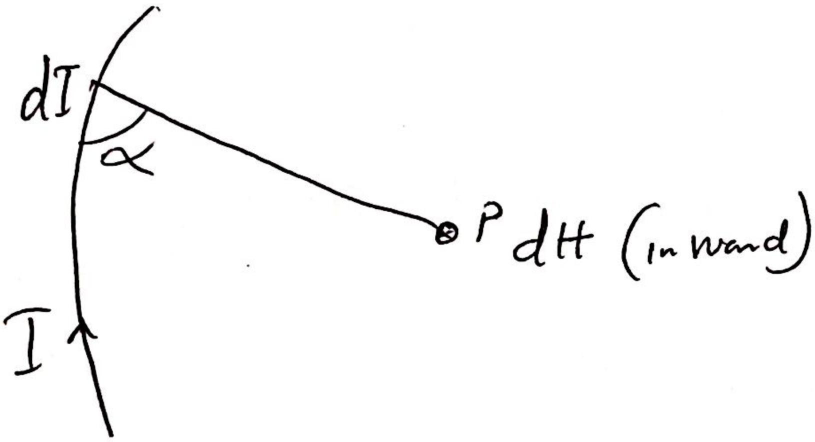
which is a circle on plane $z=0$, centered at $(0.6, 0, 0)$ and of radius 0.6m. Thus the particle gyrates in an orbit about a magnetic field.

e) $K.E = \frac{1}{2} m |u|^2 = \frac{1}{2}(2)\left(9\cos^2 5t + 9\sin^2 5t\right) = 9J$

N.B. Same K.E at $t=0$ and $t=4$, thus uniform magnetic field has no effect on the K.E of the particle.

BIOT - SAVART'S LAW:

Biot Savart's law states that the differential Magnetic field Intensity dH produced at a point P as shown in the figure below, by the differential current element Idl is proportional to the product Idl and the sine of the angle α between the element and the line joining P to the element and is inversely proportional to the square of the distance R between P and the element.



That is,

$$dH \propto \frac{I\,dl \sin\alpha}{R^2} \qquad (14)$$

or

$$dH = \frac{K I\,dl \sin\alpha}{R^2} \qquad (15)$$

Where K is the constant of proportionality. In SI units $K = 1/4\pi$, so equation (15) becomes

$$dH = \frac{I\,dl \sin\alpha}{4\pi R^2} \qquad (16)$$

Equation (16) in vector form is therefore

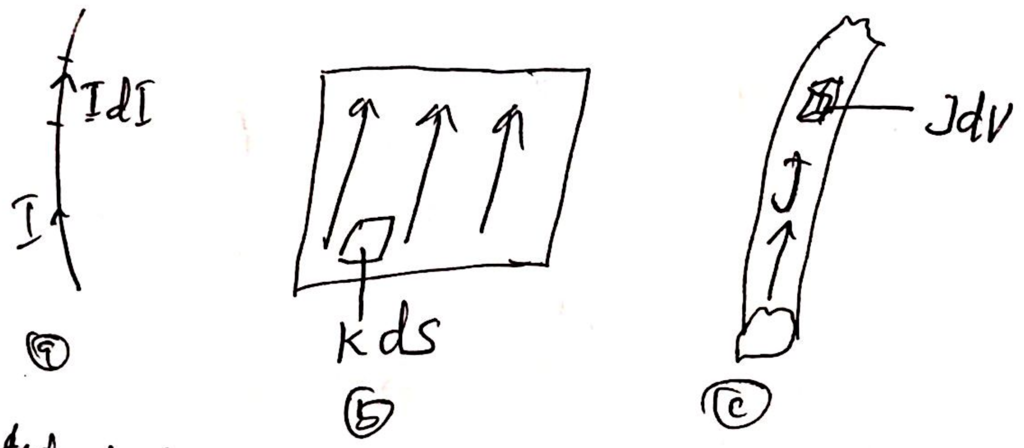
(11)

$$dH = \frac{I\,dl \times a_R}{4\pi R^2} = \frac{I\,dl \times R}{4\pi R^3}$$

Where $R = |R|$ and $a_R = \frac{R}{R}$

Just like we have different charge configuration, we can also have different current distributions: Line current, Surface current, and Volume current as shown below. If we define K as the surface current density in amperes per meter and J as the Volume current density in amperes per meter squared, the source elements are related as

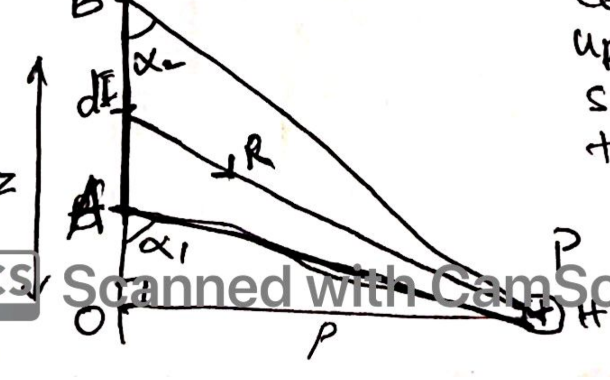
$$I\,dl \equiv K\,ds \equiv J\,dV \quad \text{—— (17)}$$



Current distribution (a) Line distribution (b) Surface current (c) Volume current.

Field at point P due to a straight filamentary Conductor:

Fig Z



Conductor is along the z-axis with its upper and lower ends, respectively, substending angles α_2 and α_1 at P, the point at which H is to be determined. The current flow from A, where $\alpha = \alpha_1$, to point B, where $\alpha = \alpha_2$. We Consider the Contribution dH at

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P due to an element dI at $(0,0,z)$

$$dH = \frac{I\,dI \times R}{4\pi R^3} \quad \text{——— (18)}$$

But $dI = dz\,a_z$ and $R = \rho a_\rho - z a_z$, so

Hence

$$dl \times R = \rho\, dz\, a_\phi \quad \text{——— (19)}$$

$$H = \int \frac{I\rho\, dz}{4\pi[\rho^2+z^2]^{3/2}} a_\phi \quad \text{——— (20)}$$

Letting $z = \rho\cot\alpha$, $dz = -\rho\csc^2\alpha\, d\alpha$, $(\rho^2+z^2)^{3/2} = \rho^3\csc\alpha^3$ and equation (20) becomes

$$H = -\frac{1}{4\pi}\int_{\alpha_1}^{\alpha_2} \frac{\rho^2\csc^2\alpha\, d\alpha}{\rho^3\csc^3\alpha} a_\phi$$

$$= \frac{-I\, a_\phi}{4\pi\rho}\int_{\alpha_1}^{\alpha_2} \sin\alpha\, d\alpha$$

Or

$$\boxed{H = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\, a_\phi} \quad \text{——— (21.)}$$

This expression is generally applicable for any straight filamentary conductor.

If the conductor is semi-infinite (with respect to P) so that point A is now at O (0,0,0) while B is at (0,0,∞), $\alpha_1 = 90°$, $\alpha_2 = 0°$ and equation (21) becomes

$$H = \frac{1}{4\pi\rho} a_\phi \quad \text{——— (22)}$$

If the conductor is infinite in length. For this case, point A is at $(0, 0, -\infty)$ while B is at $(0, 0, \infty)$, $\alpha_1 = 180^\circ$, $\alpha_2 = 0^\circ$. The equation (21) reduces to

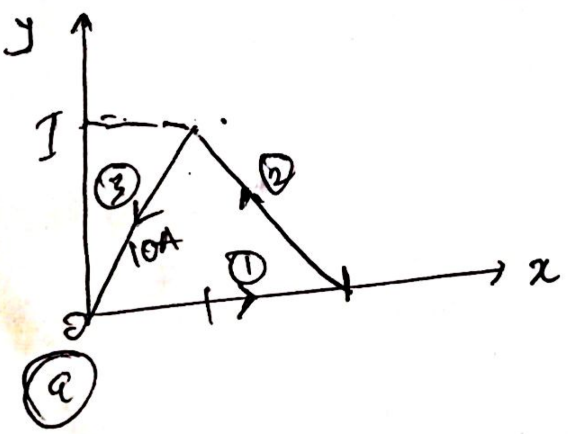
$$H = \frac{1}{2\pi\rho} a_\phi \quad \text{——— (23)}$$

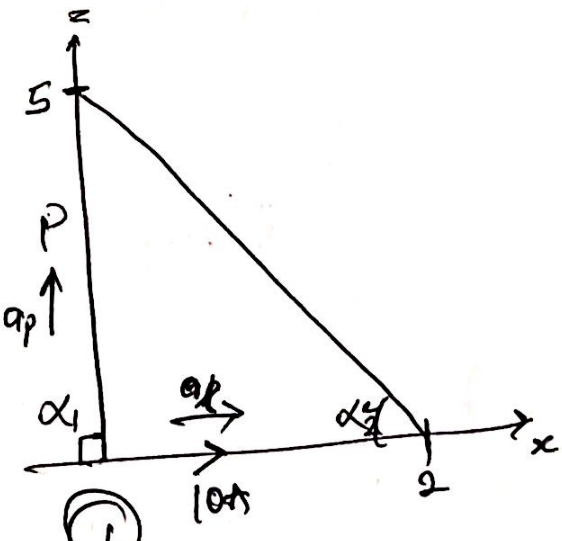
$$a_\phi = a_\ell \times a_\rho \quad \text{——— (24)}$$

Where a_ℓ is a unit vector along the ~~line~~ current and a_ρ is a unit vector along the perpendicular line from the line current to the field point.

Example:

The conducting triangular loop shown below carries a current of 10A. Find H at (0,0,5) due to side I of the loop.





Note:

In figure (b) Side I is treated as a straight conductor. Notice that we join the point of ~~interest~~ interest (0,0,5) to the begining and the end of the line current. Observe that α_1, α_2 and ρ are assigned in the same manner as in fig. Z on which

Question (21) is based.

$$\cos\alpha_1 = \cos 90^\circ = 0, \quad \cos\alpha_2 = \frac{2}{\sqrt{29}}, \quad \rho = 5$$

To determine a_ϕ.

$$a_l = a_x \text{ and } a_\rho = a_z, \text{ so}$$

$$a_\phi = a_x \times a_z = -a_y$$

Hence

$$H_1 = \frac{I}{4\pi\rho}\left(\cos\alpha_2 - \cos\alpha_1\right)a_\phi = \frac{10}{4\pi(5)}\left(\frac{2}{\sqrt{29}} - 0\right) - a_y$$

$$= -59.1 a_y \text{ mA/m}$$

AMPERE'S CIRCUIT LAW — MAXWELL'S EQUATION:

Ampere's circuit law states that the line integral of H around a closed path is the same as the net current I_{enc} enclosed by the path.
That is

$$\boxed{\oint_L H \cdot dl = I_{enc}} \quad \text{——— (25)}$$

Ampere's law is similar to gauss's law.
Equation (25) can be used to determine only when a symmetrical distribution exists.
By applying stoke's theorem to the left hand side of (25) we obtain

$$I_{enc} = \oint_L H \cdot dl = \int_S (\nabla \times H) \cdot ds \quad \text{——— (26)}$$

But

$$I_{enc} = \int_s J \cdot ds \quad \text{———} \quad (27)$$

Comparing the surface integrals in equations (26) and (27) clearly reveals that

$$\boxed{\nabla \times H = J} \quad \text{———} \quad (28)$$

This is the third Maxwell's equation. It is Ampere's law in differential form or point form. Whereas equation (25) is the integral form.

From equation (28), we should observe that $\nabla \times H \neq 0$, that is, a magnetostatic field is not conservative.

For an infinite sheet of current density K A/m

$$\boxed{H = \frac{1}{2} K \times a_n} \quad \text{———} \quad (29)$$

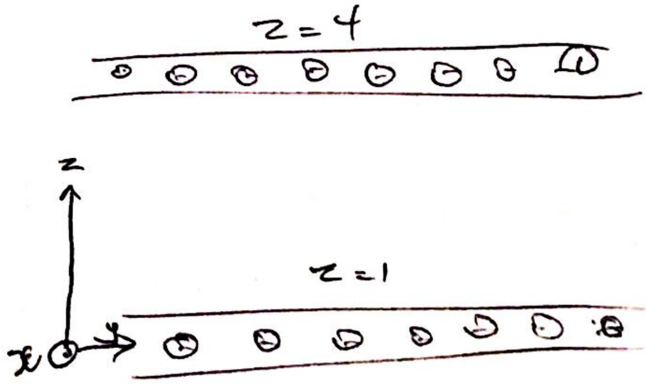
Where a_n is a unit normal vector directed from the current sheet to the point of interest.

Example:

Plane $z=0$ and $z=4$ carry current $K = -10a_x$ A/m and $K = 10a_x$ A/m respectively. Determine H at

(a) $(1,1,1)$

(b) $(0,-3,10)$



Solution: Let $H = H_0 + H_4$

Where H_0 and H_4 are the contribution due to the current sheets $z=0$ and $z=4$ respectively.

using equation 28

$$H_0 = \frac{1}{2} K \times a_n = \frac{1}{2}(-10a_x) \times a_z = 5a_y \ A/m$$

$$H_4 = \frac{1}{2} K \times a_n = \frac{1}{2}(10a_x) \times (-a_z) = 5a_y \ A/m$$

Hence

$$H = 10a_y \ A/m$$

(b) At $(0, -3, 10)$, which is above the two sheets $(z = 10 > 4 > 0)$

$$H_0 = \frac{1}{2}(-10a_x) \times a_z = 5a_y \ A/m$$

$$H_0 = \frac{1}{2}(10a_x) \times a_z = -5a_y \ A/m$$

Hence

$$H = 0 \ A/m$$

Assignment

Plane $y = 1$ carries current $K = 50a_z$ mA/m. Find H at

(a) $(0, 0, 0)$

(b) $(1, 5, -3)$

MAGNETIC FLUX DENSITY – MAXWELL'S EQUATION

The magnetic flux density B is similar to the electric flux density D. As $D = \varepsilon_0 E$ in free space, the magnetic flux density B is related to the magnetic field ~~[illegible]~~ intensity H according to

$$\boxed{B = \mu_0 H} \qquad (30)$$

Where μ_0 is a Constant known as the permeability of free Space. In henrys per meter (H/m) and has the value of

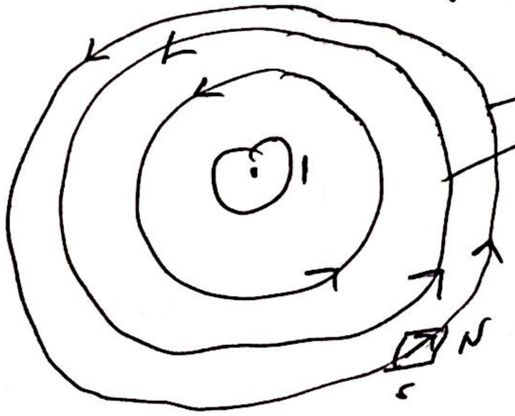
$$\mu_0 = 4\pi \times 10^{-7} \, H/m \qquad (31)$$

The magnetic flux through a Surface S is given by

$$\psi = \int_S B \cdot ds \qquad (32)$$

Where the magnetic flux ψ is in Webers (Wb) and the magnetic flux density is in Webers per Square meter (Wb/m^2) or Tesla (T)

Magnetic flux line is a path to which B is tangential at every point on the line. Magnetic flux line are closed and do not cross each other regardless of the current distribution



Magnetic flux lines

It is possible to have isolated electric charge, however It is not possible to have isolated magnetic poles or magnetic charges.

An isolated magnetic charge does not exist.

Thus the total flux through a closed path in a magnetic field must be zero, that is

$$\oint_S B \cdot ds = 0 \qquad (33)$$

This equation is referred to as the law of conservation of

of Magnetic flux or Gauss's law for magnetostatic fields
Although the magnetostatic is not conservative, Magnetic flux is conserved.

By applying the divergence theorem to (33), we obtain

$$\oint_S B \cdot ds = \int_v \nabla \cdot B \, dv = 0$$

$$\boxed{\nabla \cdot B = 0} \quad (34)$$

This is the Maxwell's fourth equation. Equation (33) and (34) shows that magnetostatic fields have no sources or sinks. Equation (34) suggests that magnetic field are always continuous.

Maxwell's Equations For Static Electric & Magnetic Field

Differential Form	Integral form	Remarks
$\nabla \cdot D = \rho_v$	$\oint_S D \cdot ds = \int_v \rho_v \, dV$	Gauss law
$\nabla \cdot B = 0$	$\oint_S B \cdot ds = 0$	Non existence of magnetic monopoles
$\nabla \times E = 0$	$\oint_L E \cdot dl = 0$	Conservative nature of electrostatic field
$\nabla \times H = J$	$\oint_L H \cdot dL = \int_S J \cdot ds$	Ampere law

MAGNETIC SCALAR AND VECTOR POTENTIALS.

Magnetic potential Could be scalar V_m or Vector A. To define V_m and A involves recalling two important identities

$$\nabla \times (\nabla V) = 0 \quad \text{—— (35a)}$$

$$\nabla \cdot (\nabla \times A) = 0 \quad \text{—— (35b)}$$

Which must always hold For any Scalar field V and vector field A.

Just as $E = -\nabla V$, we define the magnetic Scalar potential V_m in (Amperes) as related to H according to

$$\boxed{H = -\nabla V_m} \quad \text{—— (36)}$$

IF $J = 0$

The Condition $J=0$ is important.

Combining equation (36) and equation (28) gives

$$J = \nabla \times H = \nabla \times (-\nabla V_m) = 0 \quad \text{—— (37)}$$

Since V_m must satisfy the Condition in equation (35a). Thus the magnetic Scalar potential V_m is only defined in a region Where $J=0$ as in equation (36).

V_m also Satisfies laplace's equation,

$$\nabla^2 V_m = 0, \quad (J=0) \quad \text{—— (38)}$$

We know that For a magnetostatic field, $\nabla \cdot B = 0$ as stated in equation (34). To satisfy equation (34) and (35b) Simultaneously, we can define the vector magnetic potential A (Wb/m) Such that

$$\boxed{B = \nabla \times A} \quad \text{—— (39)}$$

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Just as we defined

$$V = \int \frac{dq}{4\pi\varepsilon_0 R} \quad \text{———} \quad (40)$$

we define

$$A = \int_L \frac{\mu_0 I \, dl}{4\pi R} \quad \text{———} \quad (41)$$

For line Current

$$A = \int_S \frac{\mu_0 K \, dS}{4\pi R} \quad \text{———} \quad (42)$$

For Surface Current

$$A = \int_V \frac{\mu_0 J \, dV}{4\pi R} \quad \text{———} \quad (43)$$

for Volume Current

Magnetic Flux Can be found using $\boxed{\Psi = \oint_L A \cdot dl}$ ——— (44)

Example: Given the magnetic vector potential $A = -\frac{\rho^2}{4} a_z$ Wb/m. Calculate the total magnetic flux crossing the Surface $\phi = \pi/2$, $1 \leq \rho \leq 2\,\text{m}$, $0 \leq z \leq 5\,\text{m}$.

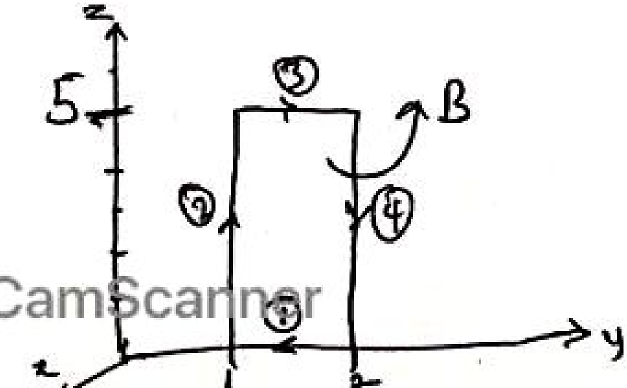
Solution

Using

$$\cancel{B = \nabla \times A = -\frac{\partial A_z}{\partial \rho} a_\phi}$$

$$\Psi = \oint_L A \cdot dl = \Psi_1 + \Psi_2 + \Psi_3 + \Psi_4$$

Where L is the path bounding Surface S; Ψ_1, Ψ_2, Ψ_3 and Ψ_4 are respectively, the evaluation of $\int A \cdot dl$ along Segments of L.



Since A has only a Z-Component.

$$\Psi_1 = 0 = \Psi_3$$

$$\Psi = \Psi_2 + \Psi_4 = -\frac{1}{4}\left[(1)^2\int_0^5 dz + (2)^2\int_5^0 dz\right]$$

$$= -\frac{1}{4}\left[5 + 4\times(0-5)\right]$$

$$= -\frac{1}{4}\left[-15\right] = \frac{15}{4}$$

$$= \underline{3.75\,\text{Wb}}$$

Note that the direction OR the path must agree with that of ds'

MAGNETIC SUSCEPTIBILITY AND PERMEABILITY.

In free space, $M=0$, (M = magnetic polarization density) and we have

$$\nabla\times H = J_f \quad \text{or} \quad \nabla\times\left(\frac{B}{\mu_0}\right) = J_f \qquad (45)$$

Where J_f is the free current volume density. In a material medium $M \neq 0$, and as a result, B changes so that

$$\nabla\times\left(\frac{B}{\mu_0}\right) = J_f + J_b = J$$

$$= \nabla\times H + \nabla\times M$$

or

$$B = \mu_0(H+M) \qquad (46)$$

Equation (46) holds for all materials whether they are linear or not.

for linear Material, M [in A/m] depends linearly on H such that

$$\boxed{M = \chi_m H} \qquad (47)$$

Where χ_m is a dimensionless quantity (ratio of M to H) called magnetic susceptibility of the medium. It is a measure of how sensitive the material is to a magnetic field.

Substituting (47) into equation (46), yields

$$B = \mu_0 (1 + \chi_m) H = \mu H \qquad (48)$$

or

$$\boxed{B = \mu_0 \mu_r H} \qquad (49)$$

Where

$$\mu_r = 1 + \chi_m = \frac{\mu}{\mu_0} \qquad (50)$$

The quantity $\mu = \mu_0 \mu_r$ is called the permeability of the material and is measured in henrys/meter. The dimensionless quantity μ_r is the ratio of the permeability of a given material to that of free space and is known as the relative permeability of the material.