Matemáticas II
2º Bachillerato
Capítulo 10: Integrales

Respuestas a los ejercicios y problemas propuestos

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Actividades propuestas

1. Calcula las siguientes primitivas

a) $\int 4x^3 dx$ b) $\int 3x^2 dx$ c) $\int 5x^4 dx$ d) $\int (5x^4 - 4x^3 + 3x^2) dx$

a) $\int \boldsymbol{4x^3}\, \boldsymbol{dx} = x^4 + C$

b) $\int \boldsymbol{3x^2}\, \boldsymbol{dx} = \frac{3x^3}{3} + C = x^3 + C$

c) $\int \boldsymbol{5x^4 dx} = \frac{5x^5}{5} + C = x^5 + C$

d) $\int (\boldsymbol{5x^4 - 4x^3 + 3x^2})\boldsymbol{dx} = x^5 - x^4 + x^3 + C$

2. Dada $f(x) = (x^3 - 3x^2 + 2x + 1)$, calcula la primitiva F(x) de f(x) que verifica F(0) = 4

$F(x) = \int (\boldsymbol{x^3 - 3x^2 + 2x + 1})\boldsymbol{dx} = \frac{x^4}{4} - 3\frac{x^3}{3} + 2\frac{x^2}{2} + x + C = \frac{x^4}{4} - x^3 + x^2 + x + C$

Como F(0) = 4 , F(0) = $\frac{0^4}{4} - 0^3 + 0^2 + 0 + C = 4$, luego C = 4 , de donde,

$F(x) = \frac{x^4}{4} - 3\frac{x^3}{3} + 2\frac{x^2}{2} + x + 4$

3. Comprueba si $F(x) = (4x^3 + 2x^2 - x + 5)$ es una primitiva de $f(x) = (12x^2 + 4x + 3)$. En caso negativo, explica por qué.

La derivada de F(x) ha de ser igual a f(x)

Si derivamos F(x) obtenemos $F'(x) = 12x^2 + 4x - 1 \neq f(x)$
Por tanto, F(x) no es una primitiva de f(x)

4. Determina los valores de a, b, c y d para los que $(4a^3 + bx^2 + cx + d)$ es una primitiva de la función $f(x) = (4x^2 - 5x + 3)$

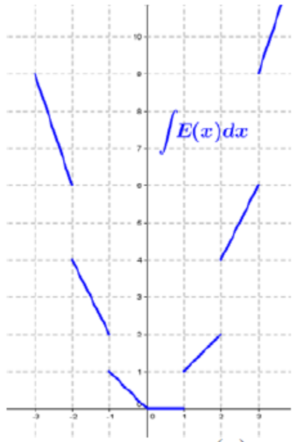
$F(x) = \int (4x^2 - 5x + 3)dx = 4\frac{x^3}{3} - 5\frac{x^2}{2} + 3x + C$ *, por tanto,*

$a = \frac{4}{3}$, $b = \frac{-5}{2}$, $c = 3$, $d = C$

5. Al resolver una primitiva, Javier y Ricardo han utilizado métodos diferentes y, como era de esperar, han obtenido expresiones distintas. Después de revisarlo muchas veces y no encontrar ningún error en los cálculos, le llevan el problema a la profesora para ver quien tiene bien el ejercicio. Para su sorpresa, la profesora les dice que ambos tienen bien el problema. ¿Cómo es posible?

Pueden diferir en constantes y estar los dos bien, además de las posibles simplificaciones.

6. Razona por qué la gráfica siguiente:



es una primitiva de la función "parte entera de x", $E(x)$, (salvo en los puntos de discontinuidad donde no es derivable):

$\int 1dx = x \qquad \int 2dx = 2x \qquad \int 3dx = 3x \qquad \int(-1)dx = -x \qquad \int(-2)dx = -2x$

7. Calcula las siguientes primitivas utilizando el cambio indicado:

a) $\int \frac{\sqrt{x}-\sqrt[3]{x}}{\sqrt[4]{x}}dx$ haciendo $x = t^{12}$.

$x = t^{12}$, $dx = 12t^{11}dt$, sustituyendo, obtenemos $\int \frac{\sqrt{t^{12}}-\sqrt[3]{t^{12}}}{\sqrt[4]{t^{12}}} \cdot 12t^{11}dt$, simplificando,

$$\int \frac{t^6-t^4}{t^3} 12t^{11}dt = \int (t^6 - t^4)12t^8dt = 12\int(t^{14} - t^{12})dt = 12\left(\frac{t^{15}}{15} - \frac{t^{13}}{13}\right) + C$$

$x = t^{12}$, $t = \sqrt[12]{x}$, deshaciendo el cambio, $\int \frac{\sqrt{x}-\sqrt[3]{x}}{\sqrt[4]{x}} \cdot dx = 12\left(\frac{\left(\sqrt[12]{x}\right)^{15}}{15} - \frac{\left(\sqrt[12]{x}\right)^{13}}{13}\right) + C$

b) $\int \frac{dx}{e^x + e^{-x}}$ haciendo $e^x = t$.

$e^x = t$, $e^x dx = dt$, $dx = \frac{dt}{e^x} = \frac{dt}{t}$, $e^{-x} = \frac{1}{e^x} = \frac{1}{t}$ sustituyendo,

$$\int \frac{dx}{e^x+e^{-x}} = \int \frac{\frac{dt}{t}}{t+\frac{1}{t}} = \int \frac{\frac{dt}{t}}{\frac{t^2+1}{t}} = \int \frac{dt}{t^2+1} = arctg(t) + C = arctg(e^x) + C$$

c) $\int \frac{5x^4}{\sqrt{1+2x}}dx$ haciendo $1 + 2x = t^2$

$1 + 2x = t^2$, $2dx = 2tdt$, $dx = tdt$, $x = \frac{t^2-1}{2}$, $t = \sqrt{1+2x}$

$$\int \frac{5x^4}{\sqrt{1+2x}}dx = \int \frac{5\left(\frac{t^2-1}{2}\right)^4}{\sqrt{t^2}}tdt = 5\int \frac{(t^2-1)^4}{2^4}dt = \frac{5}{16}\int(t^8 - 4t^6 + 6t^4 - 4t^2 + 1)dt =$$

$$= \frac{5}{16}\left(\frac{t^9}{9} - 4\frac{t^7}{7} + 6\frac{t^5}{5} - 4\frac{t^3}{3} + t\right) + C =$$

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$$= \frac{5}{16}\left(\frac{(\sqrt{1+2x}\,)^9}{9} - 4\frac{(\sqrt{1+2x}\,)^7}{7} + 6\frac{(\sqrt{1+2x}\,)^5}{5} - 4\frac{(\sqrt{1+2x}\,)^3}{3} + \sqrt{1+2x}\right) + C$$

d) $\int \frac{dx}{x+\sqrt{x^2-1}}$ haciendo $x+\sqrt{x^2-1}=t$

$x+\sqrt{x^2-1}=t$, $\sqrt{x^2-1}=t-x$, $\left(\sqrt{x^2-1}\right)^2=(t-x)^2$,

$x^2-1=t^2+x^2-2xt$, $2xt=t^2+1$, $x=\frac{t^2+1}{2t}$,

$dx=\frac{2t\cdot 2t-(t^2+1)\cdot 2}{4t^2}dt=\frac{(2t^2-2)}{4t^2}dt=\frac{t^2-1}{2t^2}dt$, de donde,

$$\int \frac{dx}{x+\sqrt{x^2-1}} = \int \frac{1}{t}\cdot\frac{t^2-1}{2t^2}dt = \frac{1}{2}\int\left(\frac{t^2}{t^3}-\frac{1}{t^3}\right)dt = \frac{1}{2}\int\left(\frac{1}{t}-t^{-3}\right)dt = \frac{1}{2}\left(ln|t| + \frac{t^{-2}}{2}\right)+C$$

Deshaciendo el cambio, $\int \frac{dx}{x+\sqrt{x^2-1}} = \frac{1}{2}\left(\ln\left|x+\sqrt{x^2-1}\right| + \frac{1}{2\left(x+\sqrt{x^2-1}\right)^2}\right)+C$

e) $\int\left(2\,\text{sen}^3 x+3\,\text{sen}^2 x-\text{sen}\,x+3\right)\cos x\,dx$ haciendo $\text{sen}\,x=t$

$\text{senx}=t$, $\text{cosxdx}=dt$, sustituyendo, nos queda,

$$\int(2t^3+3t^2-t+3)dt = 2\frac{t^4}{4}+3\frac{t^3}{3}-\frac{t^2}{2}+3t+C = \frac{\text{sen}^4x}{2}+\text{sen}^3x-\frac{\text{sen}^2x}{2}+3\text{senx}+C$$

f) $\int\sqrt{1-x^2}dx =$ Haciendo x = sent ; t = arcsenx ; dx = costdt

$$\int\sqrt{1-sen^2t}\;cost\,dt = \int\sqrt{cos^2t}\cdot cost\,dt = \int cost\cdot cost\,dt = \int cos^2t\,dt = \int\frac{1+cos2t}{2}dt =$$

$$= \int\frac{1}{2}dt+\frac{1}{2}\int cos2t\,dt = \int\frac{1}{2}dt+\frac{1}{2}\cdot\frac{1}{2}\int 2cos2t\;dt = \frac{1}{2}t+\frac{1}{4}\cdot sent = \frac{1}{2}arcsenx+\frac{1}{4}x+C$$

8. Elige el cambio que simplifica las siguientes integrales:

a) $\int\frac{2x^3+1}{\left(x^4+2x\right)^3}dx$ $\quad t=x^4+2x$, $dt=(4x^3+2)dx=2(2x^3+1)dx$

b) $\int\frac{e^{\text{tg}\,x}}{\cos^2 x}dx$ $\quad t=tgx$, $dt=\frac{1}{cos^2x}dx$

c) $\int\frac{\ln(\ln x)}{x\cdot\ln x}dx$ $\quad t=ln(lnx)$, $dt=\frac{\frac{1}{x}}{lnx}dx=\frac{1}{xlnx}dx$

d) $\int 2x^3\sqrt{x^4-49}\cdot dx$ $\quad t=x^4-49$, $dt=4x^3dx$

e) $\int \frac{x+1}{\sqrt[3]{x+1}+2}dx$ $\quad t^3 = x+1 \quad , \quad 3t^2dt = dx$

f) $\int \frac{x}{\sqrt{1-4x^2}}dx$ $\quad t = 1-4x^2 \quad , \quad dt = -8xdx$

9. Determina si las siguientes integrales son inmediatas o no:

a) $\int\left(4x^3+3x^3-\frac{1}{x^2}+\sqrt{x}\right)dx$ SÍ.

b) $\int \frac{\ln x}{x}dx$ SÍ $\left(\int \frac{lnx}{x}dx = \int lnx \cdot \frac{1}{x}dx\right)$

c) $\int \operatorname{sen} x \cos x\, dx$ SÍ

d) $\int \frac{e^{arcsenx}dx}{\sqrt{1-x^2}} = e^{arcsenx}+C$ SÍ

e) $\int \frac{\text{arctgx}}{1+\text{x}^2}\text{dx} = \frac{(\text{arctgx})^2}{2}+\text{C}$ SÍ

f) $\int \frac{\ln(x+1)}{x}dx$ NO

g) $\int tgx \cdot cosxdx = \int \frac{senx}{cosx} \cdot cosxdx = \int senxdx = -cosx$+C SÍ

h) $\int \frac{x^2-1}{\sqrt{1-x^2}}dx$ SÍ

i) $\int e^{x^2}dx$ NO

k) $\int \frac{x^4-2x^2+1}{x^2-1}dx$ SÍ $[x^4-2x^2+1 = (x^2-1)^2]$

j) $\int x^2 \cdot e^{x^2}dx$ NO

10. Resuelve las siguientes integrales:

a) $\int\left(e^{3x}+e^{2x}+e^x\right)e^x dx$ $, t = e^x, dt = e^xdx, \int(t^3+t^2+t)dt = \frac{t^4}{4}+\frac{t^3}{3}+\frac{t^2}{2}+C = \frac{e^{4x}}{4}+\frac{e^{3x}}{3}+\frac{e^{2x}}{2}+C$

b) $\int(\ln x+2)\frac{dx}{x}$ $=\frac{(lnx+2)^2}{2}+C$

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c) $\int \ln(\cos x)\operatorname{tg} x\, dx$, $t = \ln(cosx)$, $dt = -\frac{senx}{cosx}dx = -tgxdx =$

$= -\int tdt = -\frac{t^2}{2} + C = -\frac{(\ln(cosx))^2}{2} + C$

d) $\int \frac{x\,dx}{1+x^4}$ $= \frac{1}{2}\int \frac{2x}{1+(x^2)^2}dx = \frac{1}{2}arctg(x^2) + C$

i) $\int \frac{e^x dx}{1+e^{2x}}$ $= \int \frac{e^x}{1+(e^x)^2}dx = arctg(e^x) + C$

j) $\int x\cdot\cos e^{x^2}\cdot e^{x^2}dx$, $t = e^{x^2}$, $dt = 2xe^{x^2}dx$, $\frac{1}{2}\int costdt = \frac{1}{2}sent + C = \frac{1}{2}sen(e^{x^2}) + C$

11. Resuelve las siguientes integrales:

a) $\int (x^2+x+1)e^x dx$ $\begin{Bmatrix} u = x^2+x+1 & \to & du = (2x+1)dx \\ dv = e^x dx & \to & v = \int e^x dx = e^x \end{Bmatrix}$

$\int (x^2+x+1)e^x dx = (x^2+x+1)e^x - \int (2x+1)e^x dx =$

$\begin{Bmatrix} u = 2x+1 & \to & du = 2dx \\ dv = e^x dx & \to & v = \int e^x dx = e^x \end{Bmatrix}$ $= (x^2+x+1)e^x - [(2x+1)e^x - \int 2e^x dx] =$

$= (x^2+x+1)e^x - (2x+1)e^x + 2e^x + C = (x^2-x+2)e^x + C$

b) $\int \ln x\, dx$ $\begin{Bmatrix} u = lnx & \to & du = \frac{1}{x}dx \\ dv = dx & \to & v = \int dx = x \end{Bmatrix}$ $\int lnxdx = ln|x|x - \int x\frac{1}{x}dx = xln|x| - x + C$

c) $\int x\cos x\, dx$ $\begin{Bmatrix} u = x & \to & du = dx \\ dv = cosxdx & \to & v = \int cosxdx = senx \end{Bmatrix}$

$\int xcosxdx = xsenx - \int senxdx = xsenx + cosx + C$

Curiosidad – idea feliz: Resuelve la primitiva $\int \cos(\ln x)dx$.

Para ello, multiplica y divide el integrando por x: $\int \frac{\cos(\ln x)}{x}\cdot x\,dx = \left| \begin{array}{l} u = x \to du = \ldots \\ dv = \frac{\cos(\ln x)}{x}dx \to v = \ldots \end{array} \right\rangle$

$I = \int \cos(lnx)\,dx = \int \frac{\cos(lnx)}{x}\cdot xdx =$ $\begin{Bmatrix} u = x & \to & du = dx \\ dv = \frac{\cos(lnx)}{x}dx & \to & v = sen(lnx) \end{Bmatrix}$

$= xsen(lnx) - \int sen(lnx)dx.$

$\int sen(lnx)dx = \int \frac{sen(lnx)}{x}\cdot xdx =$ $\begin{Bmatrix} u = x & \to & du = dx \\ dv = \frac{sen(lnx)}{x}dx & \to & v = -cos(lnx) \end{Bmatrix}$

$= -xcos(lnx) + \int \cos(lnx)\,dx.$

$I = xsen(lnx) + xcos(lnx) - I\,,\;\; 2I = xsen(lnx) + xcos(lnx)$

$\boldsymbol{I = \frac{1}{2}x(sen(lnx) + cos(lnx)\,) + C}$

d) $\int \boldsymbol{arcsenxdx} =$ $\begin{bmatrix} u = arcsenx \;\rightarrow du = \frac{1}{\sqrt{1-x^2}}dx \\ dv = 1dx \;\rightarrow v = x \end{bmatrix} =$

$arcsenx \cdot x - \int \frac{x}{\sqrt{1-x^2}}dx =$ $\left[t = 1 - x^2 \;\rightarrow dt = -2xdx \rightarrow \frac{dt}{-2} = xdx\right]$

$arcsenx \cdot x - \int \frac{\frac{dt}{-2}}{\sqrt{t}} = arcsenx \cdot x + \frac{1}{2}\int t^{\frac{-1}{2}}dt = arcsenx \cdot x + \frac{1}{2}\frac{t^{\frac{1}{2}}}{\frac{1}{2}} =$

$= arcsenx \cdot x + t^{\frac{1}{2}} = arcsenx \cdot x + \sqrt{1-x^2} + C$

e) $\int \boldsymbol{senax \cdot e^{bx}dx} =$ $\begin{bmatrix} u_1 = sen(ax) \rightarrow du_1 = \mathrm{a} \cdot \cos(ax)dx \\ dv_1 = e^{bx}dx \rightarrow v_1 = \frac{e^{bx}}{b} \end{bmatrix}$

$= \frac{e^{bx}sen(ax)}{b} - \int \frac{ae^{bx}\cos(ax)}{b}dx =$ $\begin{bmatrix} u_2 = \mathrm{a} \cdot \cos(ax) \rightarrow du_2 = -a^2 sen(ax)dx \\ dv_1 = \frac{e^{bx}}{b} \rightarrow v_1 = \frac{e^{bx}}{b^2} \end{bmatrix}$

$= \frac{e^{bx}sen(ax)}{b} - \left(\frac{ae^{bx}\cos(ax)}{b^2} + \int \frac{a^2e^{bx}\,\mathrm{sen}(ax)}{b^2}\right) =$

$= \frac{e^{bx}sen(ax)}{b} - \left(\frac{ae^{bx}\cos(ax)}{b^2} + \frac{a^2}{b^2}\int senax \cdot e^{bx}dx\right)$

$\frac{be^{bx}sen(ax) - ae^{bx}\cos(ax)}{b^2} - \frac{a^2}{b^2}\int senax \cdot e^{bx}dx$

Haciendo $I = \int senax \cdot e^{bx}dx$

$I = \frac{be^{bx}sen(ax) - ae^{bx}\cos(ax)}{b^2} - \frac{a^2}{b^2}I$

$I + \frac{a^2}{b^2}I = \frac{be^{bx}sen(ax) - ae^{bx}\cos(ax)}{b^2}\;;\;\frac{a^2+b^2}{b^2}I = \frac{be^{bx}sen(ax) - ae^{bx}\cos(ax)}{b^2}$

De donde, $I = \frac{be^{bx}sen(ax) - ae^{bx}\cos(ax)}{a^2+b^2} + C$

12. Resuelve las siguientes primitivas.

a) $\int \frac{\mathbf{dx}}{\mathbf{x^2-4}}$ $\qquad x^2 - 4 = 0;\;\; x_1 = 2, x_2 = -2$

$\frac{1}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2} = \frac{A(x+2)+B(x-2)}{(x^2-4)} \;\rightarrow\; 1 = A(x+2) + B(x-2)$

$\mathrm{x} = 2;\;\; 1 = 4\mathrm{A} \rightarrow \mathrm{A} = {}^{1}/_{4}$

$\mathrm{x} = -2;\;\; 1 = -4\mathrm{B} \rightarrow \mathrm{B} = -{}^{1}/_{4}$

$$\int \frac{dx}{x^2-4} = \int \frac{A}{(x-a)}dx + \int \frac{B}{(x-b)}dx = \int \frac{1/4}{x-2}dx + \int \frac{-1/4}{x+2}dx =$$

$$= \frac{1}{4}\int \frac{1}{x-2}dx - \frac{1}{4}\int \frac{1}{x+2}dx = \frac{1}{4}\ln|x-2| - \frac{1}{4}\ln|x+2| = \frac{1}{4}(\ln|x-2| - \ln|x+2|) = \frac{1}{4}\ln\left|\frac{x-2}{x+2}\right| + C$$

b) $\int \frac{dx}{(x+1)^2}$

$$\int \frac{dx}{(x+1)^2} = \int \frac{1}{(x+1)^2}dx = \int (x+1)^{-2}\,dx = \frac{(x+1)^{-1}}{-1} = \frac{-1}{x+1} + C$$

c) $\int \frac{x\,dx}{(x+1)^2}$ $(x+1)^2 = 0;\ \ x_1 = -1, x_2 = -1$

$$\frac{x}{(x+1)^2} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} = \frac{A(x+1)+B}{(x+1)^2} \to x = A(x+1) + B$$

$$x = -1;\ \ B = -1$$

$$x = 0;\ \ A = 1$$

$$\int \frac{x\,dx}{(x+1)^2} = \int \frac{1}{x+1}dx + \int \frac{-1}{(x+1)^2}dx = \int \frac{1}{x+1}dx - \int (x+1)^{-2}\,dx =$$

$$= \ln|x+1| - \frac{(x+1)^{-1}}{-1} = \ln|x+1| + \frac{1}{x+1} + C$$

d) $\int \frac{x^3}{(x+1)^2}\mathbf{dx}$, efectuamos la división, obtenemos C(x) = x – 2 ; R(x) = 3x + 2, luego

$\int \frac{x^3}{(x+1)^2}dx = \int (x-2)dx + \int \frac{3x+2}{x^2+2x+1}dx$, como $x^2 + 2x + 1 = (x+1)^2$

$$\int \frac{x^3}{(x+1)^2}dx = \int (x-2)dx + \frac{3}{2}\int \frac{2x}{x^2+2x+1}dx + 2\int \frac{1}{(x+1)^2}dx =$$

$$\frac{x^2}{2} - 2x + \frac{3}{2}\ln|x^2 + 2x + 1| - \frac{2}{x+1} + C$$

e) $\int \frac{x^2+x+1}{x^3-4x^2+4x}\boldsymbol{dx} = \int \frac{x^2+x+1}{(x^2-4x+4)(x)}dx = \int \frac{x^2+x+1}{(x)(x-2)^2}dx$

$$\frac{x^2+x+1}{(x)(x-2)^2} = \frac{A}{x} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$$

$$x^2 + x + 1 = A(x-2)^2 + B(x)(x-2) + C(x)$$

$$x = 0;\ \ 1 = 4A \to A = \frac{1}{4}$$

$$x = 2;\ \ 7 = 2C \to C = \frac{7}{2}$$

$$x = 1;\ \ 3 = \frac{1}{4} - B + \frac{7}{2} \to \frac{-3}{4} = -B \to \frac{3}{4} = B$$

$$\int \frac{x^2+x+1}{(x)(x-2)^2}dx = \frac{1}{4}\int \frac{1}{x}dx + \frac{3}{4}\int \frac{1}{x-2}dx + \frac{7}{2}\int \frac{1}{(x-2)^2}dx \to \frac{1}{4}\ln|x| + \frac{3}{4}\ln|x-2| - \frac{7}{2}\left(\frac{1}{x-2}\right) + C$$

f) $\int \frac{3x^2+1}{(2x-1)(3x^2+2)}dx$; $\frac{3x^2+1}{(2x-1)(3x^2+2)} = \frac{A}{2x-1} + \frac{Mx+N}{3x^2+2}$

$\rightarrow 3x^2 + 1 = A(3x^2 + 2) + (Mx + N)(2x - 1)$

$x = 0 \rightarrow 1 = 2A - N \rightarrow N = 2A - 1 \rightarrow 2\left(\frac{7}{11}\right) - 1 \qquad \rightarrow N = \frac{3}{11}$

$x = 1 \rightarrow 4 = A(5) + (M + N) \rightarrow 4 = 5A + M + N \rightarrow 4 = 5A + M + 2A - 1 \quad \rightarrow M = \frac{6}{11}$

$x = 2 \rightarrow 13 = 14A + (2M + N)3 \rightarrow 13 = 14A + 6M + 3N \qquad \rightarrow A = \frac{7}{11}$

$\int \frac{3x^2+1}{(2x-1)(3x^2+2)}dx = \frac{7}{11}\int \frac{1}{2x-1}dx + \int \frac{6x+3}{11(3x^2+2)}dx = \frac{7}{22}\int \frac{2}{2x-1}dx + \frac{1}{11}\int \frac{6x+3}{3x^2+2}dx =$

$= \frac{7}{22}\ln|2x - 1| + \frac{1}{11}\int \frac{6x+3}{3x^2+2}dx = \frac{7}{22}\ln|2x - 1| + \frac{1}{11}\int \frac{6x}{3x^2+2}dx + \frac{3}{11}\int \frac{\sqrt{3}}{(\sqrt{3}x)^2+2}dx =$

$= \frac{7}{22}\ln|2x - 1| + \frac{1}{11}\ln|3x^2 + 2| + \frac{3}{11}\cdot\frac{1}{\sqrt{2}}arctan\left(\frac{\sqrt{3}x}{\sqrt{2}}\right) + C$

g) $\int \frac{x^2-2}{x^3(x^2+1)}dx \rightarrow \frac{x^2-2}{x^3(x^2+1)} = \frac{A}{x^3} + \frac{B}{x^2} + \frac{C}{x} + \frac{Dx+E}{x^2+1} \rightarrow$

$\rightarrow x^2 - 2 = A(x^2 + 1) + B\big(x(x^2 + 1)\big) + Cx^2(x^2 + 1) + (Dx + E)x^3 \rightarrow$

$x = 0;\ -2 = A$

$$\begin{cases} x = -1;\ -1 = -4 - 2B + 2C - (-D + E) \\ x = 1;\ -1 = -4 + 2B + 2C + D + E \\ x = 2; 2 = -10 + 10B + 20C + 8\,(2D + E) \\ x = -2; 2 = -10 - 10B + 20C - 8\,(-2D + E) \end{cases} \rightarrow F_1 + F_2 \rightarrow 5F_1 + F_3 \rightarrow -5F_1 + F_4$$

$$\rightarrow \begin{cases} 3 = -2B + 2C + D - E \\ 6 = 4C + 2D \\ 27 = 30C + 21D + 3E \\ -3 = 10C + 11D - 3E \end{cases} \rightarrow B = 0; C = 3; D = -3; E = 0 \rightarrow$$

$\int \frac{x^2-2}{x^3(x^2+1)}dx = \int \frac{-2}{x^3}dx + \int \frac{3}{x}dx + \int \frac{-3x}{x^2+1}dx = -2\int \frac{1}{x^3}dx + 3\int \frac{1}{x}dx - \frac{3}{2}\int \frac{2x}{x^2+1}dx =$

$= -2\frac{x^{-2}}{-2} + 3\ln|x| - \frac{3}{2}\ln|x^2 + 1| + C = \frac{1}{x^2} + 3\ln|x| - \frac{3}{2}\ln|x^2 + 1| + C$

h) $\int \frac{x^3+2x^2+5x+3}{x^2+1}dx = \qquad (x^3 + 2x^2 + 5x + 3):(x^2 + 1); \quad r(x) = 4x + 1; \quad c(x) = x + 2$

$= \int (x + 2 + \frac{4x+1}{x^2+1})dx = \int x\,dx + \int 2\,dx + \int \frac{4x}{x^2+1}dx + \int \frac{1}{x^2+1}dx =$

$= \frac{x^2}{2} + 2x + 2\,ln(x^2 + 1) + arctg(x) + C$

i) $\int \frac{x+1}{(x-1)(x+1)^2(x^2+1)}dx \rightarrow \frac{x+1}{(x-1)(x+1)^2(x^2+1)} = \frac{A}{(x-1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)} + \frac{Dx+E}{(x^2+1)} \rightarrow$

$x + 1 = A(x + 1)^2(x^2 + 1) + B(x - 1)(x^2 + 1) + C(x - 1)(x + 1)(x^2 + 1) + (Dx + E)(x - 1)(x + 1)^2$

$x = -1; \ \ 0 = 4B; \ \ \ B = 0$

$x = 1; \ \ \ \ 2 = 8A; \ \ \ A = 4$

$$\begin{cases} x = 0; \quad 1 = 4 - C - E \\ x = -2; \quad -1 = 20 - 9C + 6D - 3E \rightarrow C = -7; D = -9; E = 10 \\ x = 2; \quad 3 = 180 + 15C + 18D + 9E \end{cases}$$

$$\int \frac{x+1}{(x-1)(x+1)^2(x^2+1)} dx = \int \frac{4}{(x-1)} dx + \int \frac{-7}{(x+1)} dx + \int \frac{-9x+10}{(x^2+1)} dx =$$

$$= 4\int \frac{1}{(x-1)} dx - 7\int \frac{1}{(x+1)} dx + \int \frac{-9x}{(x^2+1)} dx + 10\int \frac{1}{(x^2+1)} dx =$$

$$= 4\ln|x-1| - 7\ln|x+1| - \frac{9}{2}\ln|x^2+1| + 10arctgx + C$$

13. Halla las siguientes primitivas:

a) $\int \textbf{sen}\left(\frac{3}{2}x + 1\right) dx$; $\quad t = \frac{3}{2}x + 1 \quad \frac{dt}{dx} = \frac{3}{2} \rightarrow dx = \frac{2 \cdot dt}{3}$

- Ahora sustituimos en la integral y la resolvemos.

$$\int \text{sen}(t) \frac{2 \cdot dt}{3} \rightarrow \frac{2}{3} \cdot \int \text{sen}(t)\, dt \rightarrow \frac{2}{3} \cdot (-\cos t) \rightarrow$$

- Sustituimos otra vez ***t*** por lo que teníamos y obtendremos el resultado.

$$\rightarrow -\frac{2 \cdot cos\left(\frac{3}{2}x + 1\right)}{3} + C$$

b) $\int \frac{sen(3x)}{\sqrt[3]{cos(3x)}} dx = \int \frac{\text{sen}\,(3x)}{(\cos\,(3x))^{\frac{1}{3}}} \text{dx}$

t = cos (3x) → $\frac{dt}{dx} = -sin(3x) \cdot 3 \ \rightarrow dx = \frac{dt}{-sen(3x) \cdot 3}$

Ahora lo sustituimos y realizamos la integral

$$\int \frac{\cancel{sen(3x)}}{t^{\frac{1}{3}}} \cdot \frac{dt}{-\cancel{sen(3x)} \cdot 3} \rightarrow \int -\frac{1}{t^{\frac{1}{3}}} \cdot \frac{1}{3}\, dt \rightarrow \int -\frac{1}{3t^{\frac{1}{3}}} dt \rightarrow -\frac{1}{3}\int \frac{1}{t^{\frac{1}{3}}} dt$$

$$\frac{1}{3}\int t^{-\frac{1}{3}}\, dt \rightarrow -\frac{1}{3} \cdot \frac{t^{\frac{2}{3}}}{\frac{2}{3}} \rightarrow -\frac{1}{3} \cdot \frac{3\sqrt[3]{\cos(3x)^2}}{2} \rightarrow -\frac{\sqrt[3]{\cos(3x)^2}}{2} + C$$

c) $\int \frac{cotg(x)}{sen^2(x)} dx = \int \frac{\frac{\cos(x)}{\text{sen}(x)}}{\text{sen}^2(x)} \text{dx} \rightarrow \int \frac{\cos(x)}{\text{sen}^3(x)} \text{dx}$

$$\text{t} = \text{sen}(\text{x}) \ ; \frac{\text{dt}}{\text{dx}} = \cos(\text{x}) \rightarrow \text{dx} = \frac{\text{dt}}{\cos(\text{x})};$$

$$\int \frac{\cancel{\cos(x)}}{t^3} \cdot \frac{dt}{\cancel{\cos(x)}} \rightarrow \int \frac{1}{t^3}\, dt \rightarrow \int t^{-3}\, dt \rightarrow \frac{t^{-2}}{-2} \rightarrow -\frac{1}{2\text{sen}(x)^2} + C$$

$$\int \frac{cotg(x)}{sen^2(x)}dx = -\frac{1}{2\text{sen}(x)^2} + C$$

d) $\int \frac{\mathbf{sen\,2x}}{(\mathbf{cos\,2x+1})^2}\,\boldsymbol{dx} = -\frac{1}{2}\int \frac{(-2)\cdot \text{sen}\,2x}{(\cos 2x+1)^2}dx = -\frac{1}{2}\cdot\frac{-1}{(\cos 2x+1)} + c = \frac{1}{2(\cos 2x+1)} + C$

$\text{t} = (\cos 2\text{x} + 1) \qquad \text{dt} = (-2)\cdot \text{sen}\,2\text{x}\,\text{dx}$

e) $\int (\boldsymbol{\tan x})^2\,\boldsymbol{dx} = \int (1 + (\tan x)^2 - 1)\,dx = \int (1 + (\tan x)^2)\,dx - \int 1\,dx = \tan x - x + C$

f) $\int ((\boldsymbol{\tan x})^2 + \boldsymbol{x} + \mathbf{1})\,\boldsymbol{dx} = \int (1 + (\tan x)^2 + x)\,dx = \int (1 + (\tan x)^2)\,dx + \int x\,dx =$

$= \tan x + \frac{x^2}{2} + C$

g) $\int \frac{\boldsymbol{\tan x}}{\boldsymbol{cos^2 x}}\boldsymbol{dx} = \int \frac{1}{cos^2 x}(\tan x)dx = \frac{tan^2 x}{2} + C$

h) $\int \frac{\boldsymbol{dx}}{\mathbf{1}-\boldsymbol{sen^2 x}}$ caso general $\left[\begin{matrix} t = tg\frac{x}{2} & \cos x = \frac{1-t^2}{1+t^2} \\ dx = \frac{2\cdot dt}{1+t^2} & senx = \frac{2t}{1+t^2} \end{matrix}\right]$

$$\int \frac{dx}{1-sen^2 x} = \int \frac{\frac{2\cdot dt}{1+t^2}}{1-\frac{2t}{1+t^2}} = \int \frac{\frac{2}{1+t^2}}{\frac{1+t^2-2t}{1+t^2}}dt = \int \frac{2}{(t-1)^2}dt = \frac{2\cdot(t-1)^{-1}}{-1} + C = \frac{-2}{tg\frac{x}{2}-1} + C$$

i) $\int \frac{\boldsymbol{dx}}{\boldsymbol{senx}} = \int \frac{1}{senx}dx$ Caso: $\frac{1}{-senx} = -\frac{1}{senx}$ Impar en seno

$cosx = t\,; \qquad senx = \sqrt{1-cos^2x} = \sqrt{1-t^2}$

$-senxdx = dt\,; \quad dx = \frac{dt}{-senx} = -\frac{dt}{\sqrt{1-t^2}}$

$\int \frac{dx}{senx} = \int \frac{1}{senx}dx = \int \frac{1}{\sqrt{1-t^2}}\cdot\frac{-dt}{\sqrt{1-t^2}} = \int \frac{1}{t^2-1}dt$

$t^2 - 1 = 0 \to t = \pm 1 \quad \to \frac{1}{t^2-1} = \frac{A}{t-1} + \frac{B}{t+1}$

$1 = A(t+1) + B(t-1)$

$t = 1 \;\; ; \;\; 1 = A2 \;\; \to A = \frac{1}{2}$

$t = -1 \;\; ; \quad 1 = -2B \;\; \to B = -\frac{1}{2}$

$\int \frac{1}{t^2-1}dt = \int \frac{\frac{1}{2}}{t-1}dt + \int \frac{-\frac{1}{2}}{t+1}dt \;\; \to \;\; \frac{1}{2}\ln|t-1| - \frac{1}{2}\ln|t+1| =$

$= \frac{1}{2}(\ln|cosx - 1| - \ln|cosx + 1|) + C = \frac{1}{2}\ln\frac{cosx-1}{cosx+1} + C$

j) $\int \mathbf{sen}^2 \boldsymbol{x} \cos \boldsymbol{x}\, \boldsymbol{dx}$; $\quad \operatorname{sen} x = t$, $\cos x\, dx = dt$;

$$\int \operatorname{sen}^2 x \cos x\, dx = \int t^2 dt = \frac{t^3}{3} + C = \frac{\operatorname{sen}^3 x}{3} + C$$

k) $\int \mathbf{sen}^2 \boldsymbol{x} \boldsymbol{dx}$; $\quad \operatorname{sen}^2 x = \frac{1-\cos 2x}{2}$;

$$\int \operatorname{sen}^2 x dx = \int \frac{1-\cos 2x}{2} dx = \frac{1}{2}\int (1-\cos 2x) dx = \frac{1}{2}\int 1 dx - \frac{1}{2}\int \cos 2x dx =$$

$$\frac{1}{2}x - \frac{1}{2}\int \cos 2x dx = \frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{2}\int \cos 2x \cdot 2dx = \frac{1}{2}x - \frac{1}{4}\operatorname{sen} 2x + C = \frac{x}{2} - \frac{\operatorname{sen} 2x}{4} + C$$

l) $\int \mathbf{sen}^4 \boldsymbol{x} \boldsymbol{dx}$; $\quad \operatorname{sen}^2 x = \frac{1-\cos 2x}{2}$; $\quad \cos^2 x = \frac{1+\cos 2x}{2}$

$$\int \operatorname{sen}^4 x dx = \int (\operatorname{sen}^2 x)^2 dx = \int \left(\frac{1-\cos 2x}{2}\right)^2 dx = \int \frac{(1-\cos 2x)^{\ 2}}{2^2} dx = \int \frac{1-2\cos 2x + (\cos 2x)^2}{4} dx =$$

$$= \frac{1}{4}\int 1dx - \frac{1}{4}\int 2\cos 2x\, dx + \frac{1}{4}\int \frac{1+\cos 4x}{2} dx = \frac{x}{4} - \frac{sen 2x}{4} + \frac{1}{4}\cdot\frac{1}{2}\int 1dx + \frac{1}{4}\cdot\frac{1}{2}\int \cos 4x\, dx =$$

$$\frac{x - sen 2x}{4} + \frac{x}{8} + \frac{1}{8}\cdot\frac{1}{4}\int \cos 4x \cdot 4\, dx = \frac{3x}{8} - \frac{sen 2x}{4} + \frac{\operatorname{sen} 4x}{32} + C$$

m) $\int (\cos x)^4 \mathbf{dx} = \int ((\cos x)^2)^2\, dx = \frac{1}{2}\int (1+\cos 2x)^2\, dx =$

$= \frac{1}{2}\int 1 + 2\cos 2x + (\cos 2x)^2 dx = \frac{1}{2}\int 1dx + \frac{1}{2}\int 2\cos 2x\, dx + \frac{1}{4}\int 1 + \cos 4x =$

$= \frac{1}{2}\int 1dx + \frac{1}{2}\int 2\cos 2x\, dx + \frac{1}{4}\int 1dx + \frac{1}{8}\int 4\cos 4x\, dx =$

$= \frac{1}{2}x + \frac{1}{2}\operatorname{sen} 2x + \frac{1}{4}x + \frac{1}{8}\operatorname{sen} 4x + C = \frac{1}{8}\operatorname{sen} 4x + \frac{1}{2}\operatorname{sen} 2x + \frac{3}{4}x + C$

n) $\int \cos(\ln x)\, dx$

u=$\cos(\ln x)$ $\qquad$ dv= dx

$$du = -\operatorname{sen}(lnx)\frac{1}{x}dx \qquad v = x$$

$$\int \cos(\ln x)\, dx = x\cdot \cos \ln x - \int \cancel{x}\left(-\operatorname{sen}(lnx)\frac{1}{\cancel{x}}\right) dx = x\cos(\ln x) + \int \operatorname{sen}(\ln x)\, dx$$

$$\int u\, dv = u\, v - \int v\, du$$

u= $\operatorname{sen}(\ln x)$ $\qquad$ dv= dx

$$du = \cos(\ln x)\frac{1}{x}dx \qquad v = x$$

$$x\cos(\ln x) + x\operatorname{sen}(\ln x) - \int \cancel{x}\cos(\ln x)\frac{1}{\cancel{x}}dx =$$

$$= x\cos(\ln x) + x\operatorname{sen}(\ln x) - \int \cos(\ln x) dx$$

$$\int \cos(\ln x)\,dx = x\cos(\ln x) + x\,\mathrm{sen}(\ln x) - \int \cos(\ln x)dx$$

$$I = x\cos(\ln x) + x\,\mathrm{sen}(\ln x) - I$$

$$2I = x\cos(\ln x) + x\,\mathrm{sen}(\ln x)\,; \qquad I = \frac{x\cos(\ln x) + x\,\mathrm{sen}(\ln x)}{2}$$

$$\int \cos(\ln x)\,dx = \frac{x\cos(\ln x) + x\,\mathrm{sen}(\ln x)}{2} + C$$

ñ) $\int \frac{(1+(\sin x)^2)}{\sin x \cos x} dx = \int \frac{1}{\mathrm{sen}\,x\cos x}dx + \int \frac{(\mathrm{sen}\,x)^{\cancel{2}}}{\cancel{\mathrm{sen}\,x}\cos x}dx$

*Hacemos la primera integral:

$$\int \frac{1}{\mathrm{sen}\,x\cos x}dx$$

Sabemos que:

$$\left[\begin{array}{l} \tan\frac{x}{2} = t \to dx = \frac{2}{1+t^2}dt\,; \quad \mathrm{sen}\,x = \frac{2t}{1+t^2}\ ; \ \cos x = \frac{1-t^2}{1+t^2} \\ \qquad\qquad \mathrm{sen}\,x\cos x = \frac{2t}{1+t^2}\cdot\frac{1-t^2}{1+t^2} = \frac{2t-2t^3}{(1+t^2)^2} \end{array}\right.$$

$$\int \frac{1}{\mathrm{sen}\,x\cos x}dx = \int \frac{1}{\frac{2t-2t^3}{(1+t^2)^2}}\cdot\frac{2}{1+t^2}\,dt = \int \frac{(1+t^2)^2}{2t-2t^3}\cdot\frac{2}{1+t^2}dt = \int \frac{2(1+t^2)^2}{2t+2t^3-2t^3-2t^5}dt =$$

$$= \frac{\cancel{1}}{\cancel{2}}\cdot\cancel{2}\int \frac{(1+t^2)^2}{t-t^5}dt = \int \frac{(1+t^2)^2}{t(1-t^4)}dt = \int \frac{(1+t^2)^{\cancel{2}}}{t(1-t^2)\cancel{(1+t^2)}}dt = \int \frac{1+t^2}{t(1-t^2)}\mathrm{dt}$$

$$\frac{1+t^2}{t(1-t^2)} = \frac{A}{t} + \frac{B}{1-t} + \frac{C}{1+t}; \qquad 1+t^2 = A(1-t^2) + Bt(1+t) + Ct(1-t)$$

*Calculamos A, B y C dando valores a t:

-Si t = 0; 1 = A$(1-0^2)$; $1 = A$

-Si t = 1; 2 = B·1·(1+1); 1 = B

-Si t=-1; 2 = 2C ; 1 = C

*Sustituimos los valores:

$$\int \left(\frac{1}{t} + \frac{1}{1-t} + \frac{1}{1+t}\right)dt = \ln|t| - \ln|1-t| + \ln|1+t|$$

*Sustituimos t por tan $\frac{x}{2}$:

$$\int \frac{1}{\mathrm{sen}\,x\cos x}dx = \ln\left|\tan\frac{x}{2}\right| - \ln\left|1-\tan\frac{x}{2}\right| + \ln\left|1+\tan\frac{x}{2}\right|$$

*Realizamos la segunda integral:

$$\int \frac{\mathrm{sen}\,x}{\cos x}dx = -ln|cosx|$$

*Por tanto:

$$\int \frac{(1+(\operatorname{sen} x)^2)}{\operatorname{sen} x \cos x} dx = \ln\left|\tan \frac{x}{2}\right| - \ln\left|1 - \tan \frac{x}{2}\right| + \ln\left|1 + \tan \frac{x}{2}\right| - \ln|\cos x| + C =$$

o) $\int \frac{dx}{1+\operatorname{sen}^2 x}$

$t = \operatorname{tg} x$	$\operatorname{sen} x = \frac{t}{\sqrt{t^2+1}}$	$\operatorname{sen}^2 x = \frac{t^2}{t^2+1}$
$dx = \frac{dt}{1+t^2}$	$\cos x = \frac{1}{\sqrt{t^2+1}}$	$\cos^2 x = \frac{1}{t^2+1}$

$$\int \frac{dx}{1+\operatorname{sen}^2 x} = \int \frac{\frac{dt}{1+t^2}}{1+\frac{t^2}{t^2+1}} = \int \frac{\frac{dt}{1+t^2}}{\frac{t^2+1+t^2}{t^2+1}} = \int \frac{dt}{2t^2+1} = \int \frac{dt}{2\left(t^2+\frac{1}{2}\right)} = \frac{1}{2}\int \frac{dt}{t^2+\frac{1}{2}} =$$

$$= \frac{1}{2}\cdot\frac{2}{\sqrt{2}}\operatorname{arctg}\frac{t}{\sqrt{\frac{1}{2}}} + C = \frac{1}{2}\cdot\frac{2}{\sqrt{2}}\operatorname{arctg}\frac{\operatorname{tg} x}{\sqrt{\frac{1}{2}}} + C = \frac{1}{2}\cdot\frac{2}{\sqrt{2}}\operatorname{arctg}\frac{\operatorname{tg} x}{\frac{1}{\sqrt{2}}} + C = \frac{1}{\sqrt{2}}\operatorname{arctg}\left(\sqrt{2}\cdot \operatorname{tg} x\right) + C$$

p) $\int \operatorname{sen} 5x \cdot \cos 4x\, dx$

$$\boxed{\operatorname{sen}\alpha\cdot\cos\beta = \frac{\operatorname{sen}(\alpha+\beta)+\operatorname{sen}(\alpha-\beta)}{2}}$$

$$\int \operatorname{sen} 5x\cdot\cos 4x\,dx = \int \frac{\operatorname{sen}(5x+4x)+\operatorname{sen}(5x-4x)}{2}dx = \frac{1}{2}\int(\operatorname{sen} 9x + \operatorname{sen} x)dx =$$

$$= \frac{1}{2}\left(\int \operatorname{sen} 9x\,dx + \int \operatorname{sen} x\,dx\right) = \frac{1}{2}\left(\frac{1}{9}\int \operatorname{sen} 9x\cdot 9\,dx + \int \operatorname{sen} x\,dx\right) =$$

$$= \frac{1}{2}\left(\frac{1}{9}\cdot(-\cos 9x) + (-\cos x)\right) + C = -\frac{1}{2}\left(\frac{\cos 9x}{9} + \cos x\right) + C$$

q) $\int \frac{dx}{13+12\cos x}$

$t = \operatorname{tg}\frac{x}{2}$	$\frac{x}{2} = \operatorname{arctg} t \to x = 2\operatorname{arctg} t$	$dx = \frac{2}{1+t^2}dt$
$\cos x = \frac{1-t^2}{1+t^2}$	$\operatorname{sen} x = \frac{2t}{1+t^2}$	$\operatorname{tg} x = \frac{2t}{1-t^2}$

$$\int \frac{dx}{13+12\cos x} = \int \frac{\frac{2dt}{1+t^2}}{13+12\left(\frac{1-t^2}{1+t^2}\right)} = \int \frac{\frac{2dt}{1+t^2}}{13+\frac{12-12t^2}{1+t^2}} = \int \frac{\frac{2dt}{1+t^2}}{\frac{13+13t^2+12-12t^2}{1+t^2}} = \int \frac{2dt}{t^2+25} = 2\int \frac{1}{t^2+25}dt =$$

$$= 2\int \frac{1}{t^2+5^2}dt = 2\cdot\frac{1}{5}\operatorname{arctg}\frac{t}{5} + C = \frac{2}{5}\operatorname{arctg}\left(\frac{\operatorname{tg}\frac{x}{2}}{5}\right) + C$$

14. Resuelve las siguientes integrales definidas:

a) $\int_0^6 (x^2+x+1)dx$ $= \frac{x^3}{3} + \frac{x^2}{2} + x\Big|_0^6 = \left(\frac{6^3}{3} + \frac{6^2}{2} + 6\right) - \left(\frac{0^3}{3} + \frac{0^2}{2} + 0\right) = 92$

b) $\int_{-1}^1 (x^2+x+1)dx$ $= \frac{x^3}{3} + \frac{x^2}{2} + x\Big|_{-1}^1 = \left(\frac{1^3}{3} + \frac{1^2}{2} + 1\right) - \left(\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + (-1)\right) = \frac{8}{3}$

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c) $\int_0^{\sqrt{3}} x\sqrt{x^2+1}\,dx$ $= \frac{1}{2}\int_0^{\sqrt{3}} 2x(x^2+1)^{\frac{1}{2}}dx = \frac{1}{2}\frac{(x^2+1)^{\frac{3}{2}}}{\frac{3}{2}}\Big|_0^{\sqrt{3}} = \frac{1}{3}\left[\left(((\sqrt{3})^2+1)^{\frac{3}{2}}\right) - \left((0^2+1)^{\frac{3}{2}}\right)\right] = \frac{7}{3}$

d) $\int_{-1}^{1} \frac{x+1}{x^2+2x+2}dx$ $= \frac{1}{2}\int_{-1}^{1} \frac{2x+2}{x^2+2x+2}dx = \frac{1}{2}ln|x^2+2x+2|\big|_{-1}^{1} = \frac{1}{2}[(ln5)-(ln1)] = \frac{ln5}{2}$

e) $\int_0^{\pi} \operatorname{sen} x\,dx$ $= -cosx|_0^{\pi} = -cos\pi + cos0 = 2$

f) $\int_1^e \ln x\,dx$ $\int lnxdx = \begin{Bmatrix} u = lnx & \rightarrow & du = \frac{1}{x}dx \\ dv = dx & \rightarrow & v = \int dx = x \end{Bmatrix} = xlnx - \int x\frac{1}{x}dx = xlnx - x + C$

$\int_1^e lnxdx = xlnx - x]_1^e = (elne - e) - (1ln1 - 1) = 1$

15.

Halla el valor de c que verifica $\int_0^5 (2x+1)dx = f(c)\cdot(5-0)$ y razona su interpretación geométrica.

$\int_0^5 (2x+1)dx = x^2 + x|_0^5 = 30$, f(c) = 2c + 1, 30 = (2c + 1)·5 , $c = \frac{25}{10} = \frac{5}{2}$

16. Sin efectuar el cálculo de la integral indefinida, calcula f'(x) si $f(x) = \int_2^{e^x} \frac{dt}{lnt}$

La función $g(t) = \frac{1}{lnt}$ es continua en [2,b] , $g(x) = e^x$ es derivable,

Por el teorema fundamental del cálculo integral:

$$f'(x) = \frac{1}{\ln(e^x)} \cdot e^x$$

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EJERCICIOS Y PROBLEMAS

1.

Sabiendo que $\int x^n dx = \frac{x^{n+1}}{n+1}+C$ y $\int f^n(x)\,f'(x)dx = \frac{f^{n+1}(x)}{n+1}+C$, calcula:

1) $\int x^5 dx = \frac{x^6}{6}+C$

2) $\int\frac{4}{x^5}dx = \int 4x^{-5}dx = 4\cdot\frac{x^{-4}}{-4} = \frac{-1}{x^4}+C$

3) $\int\frac{dx}{x^2} = \int\frac{1}{x^2}dx = \int x^{-2}dx = \frac{x^{-1}}{-1}+C$

4) $\int 37dx = 37x + C$

5) $\int 6x^7 dx = 6\cdot\frac{x^8}{8}+C$

6) $\int 5x^{\frac{1}{4}}dx = 5\cdot\frac{x^{\frac{5}{4}}}{\frac{5}{4}} = 4\sqrt[4]{x^5}+C$

7) $\int 5\cdot\sqrt{x^3}dx = \int 5x^{\frac{3}{2}}dx = 5\cdot\frac{x^{\frac{5}{2}}}{\frac{5}{2}} = 2\sqrt{x^5}+C$

8) $\int(3-2x-x^4)dx = 3x - \frac{2x^2}{2} - \frac{x^5}{5} + C = 3x - x^2 - \frac{x^5}{5} + C$

9) $\int(2x^5-5x+3)dx = \frac{2x^6}{6} - \frac{5x^2}{2} + 3x + C$

10) $\int(2+3x^3)^2dx = \int 4+9x^6+12x^3dx = 4x+9\frac{x^7}{7}+12\frac{x^4}{4}+C = 4x+\frac{9x^7}{7}+3x^4+C$

11) $\int(2\cdot(x^2+2)^3dx = \int 2\cdot((x^2)^3+(3x^2)^2\cdot 2+3x^2\cdot 2^2+2^3)dx = \int 2\cdot(x^6+18x^4+12x^2+8)dx =$

$\int(2x^6+36x^4+24x^2+16)dx = 2\cdot\frac{x^7}{7}+36\cdot\frac{x^5}{5}+24\cdot\frac{x^3}{3}+16x+C$

12) $\int(1-x^3)^2dx = \int 1-2x^3+(x^3)^2dx = x-2\cdot\frac{x^4}{4}+\frac{x^7}{7}+C$

13) $\int\frac{x^3-x+2}{x^3}dx = \int\frac{\cancel{x^3}}{\cancel{x^3}}dx - \int\frac{\cancel{x}}{x^{\cancel{2}}}dx + \int\frac{2}{x^3}dx = \int 1dx - \int\frac{1}{x^2}dx + \int\frac{2}{x^3}dx = \int(1-x^{-2}+2x^{-3})dx =$

$= x+\frac{x^{-1}}{1}+2\cdot\frac{x^{-2}}{-2} = x+\frac{x^{-1}}{1}-x^{-2}+C$

14) $\int(-4x^{\frac{2}{3}}+2x)dx = -4\cdot\frac{x^{\frac{5}{3}}}{\frac{5}{3}}+\cancel{2}\cdot\frac{x^2}{\cancel{2}}+C = -\frac{12X^{\frac{5}{3}}}{5}+x^2+C$

15) $\int(3a-\frac{1}{3e^2}+2x^a)dx = (3a-\frac{1}{3e^2})x+2\frac{x^{a+1}}{a+1}+C$

16) $\int -\frac{3}{x^3}+2-\frac{3}{\sqrt{x}}dx = \int -3x^{-3}+2-3x^{\frac{-1}{2}}dx = -3\frac{x^{-2}}{-2}+2x-3\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+C =$

$= \frac{3}{2x^2}+2x-6\sqrt{x}+C$

17) $\int(3x^5 - \frac{4}{3x^2} + 2\sqrt[5]{x^2})dx = \frac{3x^6}{6} - 12\cdot\frac{x^{-1}}{-1} + 2\cdot\frac{x^{\frac{7}{5}}}{\frac{7}{5}} = \frac{x^6}{2} - \frac{12}{x} + \frac{10x^{\frac{7}{5}}}{7} + C$

18) $\int(1-x)\sqrt{x}dx = \int\sqrt{x} - x\sqrt{x}dx = \int x^{\frac{1}{2}} - x\cdot x^{\frac{1}{2}}\,dx = \int x^{\frac{1}{2}} - x^{\frac{3}{2}}dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{5}{2}}}{\frac{5}{2}} = \frac{2x^{\frac{3}{2}}}{3} - \frac{2x^{\frac{5}{2}}}{5} + C$

19) $\int\frac{x^3+5x^2-4}{x^2}dx = \int\frac{x^{\cancel{3}}}{\cancel{x^2}}dx + \int\frac{5\cancel{x^2}}{\cancel{x^2}}dx - \int\frac{4}{x^2}dx = \int x\,dx + \int 5\,dx - \int 4x^{-2}dx =$

$= \frac{x^2}{2} + 5x - 4\cdot\frac{x^{-1}}{-1} = \frac{x^2}{2} + 5x - \frac{4}{x} + C$

20) $\int(5e^x + \frac{2x^3-3x^2+5}{4x^2})dx = \int 5e^x dx + \int\frac{2x^{\cancel{3}}}{4\cancel{x^2}}dx - \int\frac{3\cancel{x^2}}{4\cancel{x^2}}dx + \int\frac{5}{4x^2}dx =$

$= \int 5e^x dx + \int\frac{2x}{4}dx - \int\frac{3}{4}dx + \int\frac{5x^{-2}}{4}dx \quad = 5e^x + \frac{1}{4}x^2 - \frac{3}{4}x - \frac{5x^{-1}}{4} =$

$= 5e^x + \frac{1}{4}x^2 - \frac{3}{4}x + \frac{5}{4x} + C$

21) $\int\frac{(1+x)^2}{\sqrt{x}}dx \quad = \int(x^2+2x+1)\left(x^{\frac{-1}{2}}\right)dx = \int\ (x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + x^{\frac{-1}{2}})\,dx = \frac{x^{5/2}}{5/2} + 2\frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C = \frac{2\sqrt{x^5}}{5} + \frac{4\sqrt{x^3}}{3} + 2\sqrt{x} + C$

22) $\int(\sqrt{x} - \frac{1}{2}x + \frac{2}{\sqrt{x}})\,dx = \int\left(x^{\frac{1}{2}} - \frac{1}{2}x + 2x^{\frac{-1}{2}}\right)dx = \frac{x^{3/2}}{3/2} - \frac{x^2}{4} + \frac{2x^{1/2}}{1/2} + C = \frac{2\sqrt{x^3}}{3} - \frac{x^2}{4} + 4\sqrt{x} + C$

23) $\int\ \sqrt{x}\,(x^3+1)dx = \int\left(x^{\frac{1}{2}}\right)(x^3+1)dx = \int\left(x^{\frac{7}{2}}\right) + \left(x^{\frac{1}{2}}\right)dx = \frac{x^{9/2}}{9/2} + \frac{x^{3/2}}{3/2} + C =$

$= \frac{2\sqrt{x^9}}{9} + \frac{2\sqrt{x^3}}{3} + C$

24) $\int\left(\sqrt{x^5} - \frac{2}{3\sqrt{x}}\right)dx = \int\left(x^{\frac{5}{2}} - \frac{2x^{\frac{-1}{2}}}{3}\right)dx = \frac{x^{7/2}}{7/2} - \frac{2x^{1/2}}{3.{}^1/_2} + C = \frac{2\sqrt{x^7}}{7} - \frac{4\sqrt{x}}{3} + C$

25) $\int\sqrt{x}\,(3-5x)dx \quad = \int(x^{\frac{1}{2}})\,(3-5x)dx = \int\left(3x^{\frac{1}{2}} - 5x^{\frac{3}{2}}\right)dx \quad = \frac{3x^{3/2}}{3/2} - \frac{5x^{5/2}}{5/2} + C =$

$= 2\sqrt{x^3} - 2\sqrt{x^5} + C$

26) $\int\frac{(x+1)+(x-2)}{\sqrt{x}}dx \qquad \int(x^2+x-2)\left(x^{\frac{-1}{2}}\right)dx = \int\left(x^{\frac{3}{2}} + x^{\frac{1}{2}} - 2x^{\frac{-1}{2}}\right)dx = \frac{x^{5/2}}{5/2} + \frac{x^{3/2}}{3/2} - \frac{2x^{1/2}}{1/2} + C = \frac{2\sqrt{x^5}}{5} + \frac{2\sqrt{x^3}}{3} - 4\sqrt{x} + C$

27) $\int\ (3x+4)^2\,dx = \int 3(3x+4)^2dx = \frac{(3x+4)^3}{3\cdot3} + C = \frac{(3x+4)^3}{9} + C$

28) $\int(3x-7)^4dx = \frac{1}{3}\int 3(3x-7)^4dx = \frac{(3x-7)^5}{3\cdot5} + C = \frac{(3x-7)^5}{15} + C$

29) $\int x\,(x^2-4)^3dx \quad = \frac{1}{2}\int 2x(x^2-4)^3dx = \frac{(x^2-4)^4}{2\cdot4} + C = \frac{(x^2-4)^4}{8} + C$

30. $\int \boldsymbol{3x\,(x^2+2)^3\,dx} = \frac{3}{2}\int 2x\,(x^2+2)^3dx = \frac{3(x^2+2)^4}{2\cdot 4} + C = \frac{3(x^2+2)^4}{8} + C$

31. $\int \boldsymbol{(x^3+2)^2x^2\;dx} = \frac{1}{3}\int (x^3+2)^2 3x^2dx = \frac{(x^3+2)^3}{3\cdot 3} + C = \frac{(x^3+2)^3}{9} + C$

32. $\int \boldsymbol{(x^3+3)\,x^2\,dx} = \frac{1}{3}\int (x^3+3)3x^2dx = \frac{(x^3+3)^2}{3\cdot 2} + C = \frac{(x^3+3)^2}{6} + C$

33. $\int \boldsymbol{(x-2)^{3/2}dx} = \frac{2\sqrt{(x-2)^5}}{5} + C$

34. $\int \boldsymbol{(a\;+\;x)^3\,dx} = \frac{(a+x)^4}{4} + C$

35. $\int \boldsymbol{[(x+2)^3 - (x+2)^2]\,dx} = \frac{(x+2)^4}{4} - \frac{(x+2)^3}{3} + C$

36. $\int \boldsymbol{\sqrt{3x+12}\,dx} = \frac{1}{3}\int 3(3x+12)^{\frac{1}{2}}dx = \frac{(3x+12)^{3/2}}{3\cdot 3/2} + C = \frac{2\sqrt{(3x+12)^3}}{9} + C$

37. $\int \boldsymbol{\frac{dx}{\sqrt{x+3}}} = \int (x+3)^{\frac{-1}{2}}dx = \frac{(x+3)^{1/2}}{1/2} + C = 2\sqrt{x+3} + C$

38. $\int \boldsymbol{\frac{dx}{(x-1)^3}} = \int (x-1)^{-3}dx = \frac{(x-1)^{-2}}{-2} + C = \frac{-1}{2(x-1)^2} + C$

39. $\int \boldsymbol{(x^2+x)^4\,(2x+1)\,dx} = \frac{(x^2-x)^5}{5} + C$

40. $\int \boldsymbol{\frac{1}{\sqrt{x}}(1+\sqrt{x})^2dx} = 2\int \frac{1}{2\sqrt{x}}\left(1+\sqrt{x}\right)^2dx = \frac{2(1+\sqrt{x})^3}{3} + C$

41. $\int \boldsymbol{\frac{x^3}{(x^4-1)^2}\,dx} = \int x^3(x^4-1)^{-2}\,dx = \frac{1}{4}\int 4x^3(x^4-1)^{-2}\,dx = \frac{1}{4}\cdot\frac{(x^4-1)^{-1}}{-1} + c = \frac{(x^4-1)^{-1}}{-4} + c = \frac{1}{4(x^4-1)} + c$

42. $\int \boldsymbol{\frac{x}{(x^2+4)^3}\,dx} = \int x(x^2+4)^{-3}dx = \frac{1}{2}\int 2x(x^2+4)^{-3}dx = \frac{1}{2}\cdot\frac{(x^2+4)^{-2}}{-2} + c = \frac{(x^2+4)^{-2}}{-4} + c = -\frac{1}{4(x^2+4)^2} + c$

43. $\int \boldsymbol{x\sqrt{x^2-7}dx} = \int x(x^2-7)^{\frac{1}{2}}\,dx = \frac{1}{2}\int 2x\,(x^2-7)^{\frac{1}{2}}\,dx = \frac{1}{2}\cdot\frac{(x^2-7)^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{(x^2-7)^{\frac{3}{2}}}{3} + c = \frac{\sqrt{(x^2-7)^3}}{3} + c$

44. $\int \boldsymbol{(x-1)(x^2-2x+3)^4\,dx} =$

$\frac{1}{2}\int (2x-2)(x^2-2x+3)^4\,dx = \frac{1}{2}\cdot\frac{(x^2-2x+3)^5}{5} + c = \frac{(x^2-2x+3)^5}{10} + c$

45. $\int \boldsymbol{\frac{3x}{\sqrt{1+7x^2}}dx} = \int 3x(1+7x^2)^{-\frac{1}{2}}\,dx =$

$\frac{3}{14}\int 14x(1+7x^2)^{-\frac{1}{2}}\,dx = \frac{3}{14}\cdot\frac{(1+7x^2)^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{3\sqrt{1+7x^2}}{7} + c$

46. $\int \frac{8x^2}{(x^3+2)^2}\boldsymbol{dx} = \int 8x^2\,(x^3+2)^{-2}dx =$

$\frac{8}{3}\int 3x^2\,(x^3+2)^{-2}dx = \frac{8}{3}\cdot\frac{(x^3+2)^{-1}}{-1} + c = -\frac{8}{3(x^3+2)} + c$

47. $\int \frac{3x}{\sqrt[3]{x^2+3}}\boldsymbol{dx} = \int 3x(x^2+3)^{-\frac{1}{3}}\,dx = \frac{3}{2}\int 2x(x^2+3)^{-\frac{1}{3}}\,dx =$

$\frac{3}{2}\cdot\frac{(x^2+3)^{\frac{2}{3}}}{\frac{2}{3}} + c = \frac{9\sqrt[3]{(x^2+3)^2}}{4} + c$

48. $\int \boldsymbol{x\sqrt[3]{1-x^2}\,dx} = \int x(1-x^2)^{\frac{1}{3}}\,dx = \frac{1}{2}\int 2x(1-x^2)^{\frac{1}{2}}\,dx =$

$\frac{1}{2}\cdot\frac{(1-x^2)^{\frac{4}{3}}}{\frac{4}{3}} + c = \frac{3\sqrt[3]{(1-x^2)^4}}{4} + c$

49. $\int \frac{x^2}{\sqrt[4]{x^3+5}}\boldsymbol{dx} = \int x^2(x^3+5)^{-\frac{1}{4}}\,dx = \frac{1}{3}\int 3x^2(x^2+5)^{-\frac{1}{4}}\,dx = \frac{1}{3}\cdot\frac{(x^3+5)^{\frac{3}{4}}}{\frac{3}{4}} + c = \frac{4\sqrt[4]{(x^3+5)^3}}{9} + c$

50. $\int \boldsymbol{x^2(x^3-1)^{\frac{3}{5}}\,dx} = \frac{1}{3}\int 3x^2(x^3-1)^{\frac{3}{5}}dx =$

$\frac{1}{3}\cdot\frac{(x^3-1)^{\frac{3}{5}}}{\frac{3}{5}} + c = \frac{5\sqrt[5]{(x^3-1)^8}}{24} + c$

51. $\int \boldsymbol{\sqrt{x^2-2x^4}\,dx} = \int \sqrt{x^2(1-2x^2)}\,dx = \int x\sqrt{1-2x^2}dx = \int x(1-2x^2)^{\frac{1}{2}}\,dx = -\frac{1}{4}\int -4x(1-$
$2x^2)^{\frac{1}{2}}\,dx =$

$-\frac{1}{4}\cdot\frac{(1-2x^2)^{\frac{3}{2}}}{\frac{3}{2}} + c = -\frac{\sqrt{(1-2x^2)^3}}{6} + c$

52. $\int \boldsymbol{(e^x+1)^3e^x dx} = \frac{(e^x+1)^4}{4} + c$

53. $\int \boldsymbol{sen^3x(cos\,x)\,dx} = \frac{sen^4x}{4} + c$

54. $\int \boldsymbol{x(cos^4x^2)sen\,x^2\,dx} = -\frac{1}{2}\int 2x\,(cos^4x^2)(-sen\,x^2)dx = -\frac{1}{2}\cdot\frac{cos^5x^2}{5} + c = -\frac{cos^5x^2}{10} + c$

55. $\int \frac{x\cdot ln(x^2+3)}{x^2+3}\,\boldsymbol{dx} = \int ln\,(x^2+3)\cdot\frac{x}{x^2+3}dx =$

$\frac{1}{2}\int \ln(x^2+3)\cdot\frac{2x}{x^2+3}dx = \frac{1}{2}\cdot\frac{ln^2|x^2+3|}{2} + c = \frac{ln^2|x^2+3|}{4} + c$

56. $\int \frac{sen\,x}{cos^3x}\boldsymbol{dx} = \int cos^{-3}x(sen\,x)dx =$

$-1\int cos^{-3}x(-sen\,x)dx = -1\cdot\frac{cos^{-2}x}{-2} + c = \frac{cos^{-2}x}{2} + c = \frac{1}{2cos^2x} + c$

57. $\int \frac{e^x}{2e^x-3}\boldsymbol{dx} = \frac{1}{2}\int\frac{2e^x}{2e^x-3}dx = \frac{1}{2}\cdot ln|2e^x-3| + c$

58. $\int \boldsymbol{tg^5 x(sec^2 x)dx} = \frac{tg^6 x}{6} + c$

59. $\int \boldsymbol{\frac{sec^2 3x}{tg\, 3x} dx} = \frac{1}{3}\int \frac{3\cdot sec^2 3x}{tg\, 3x} dx = \frac{1}{3}\cdot ln\, |tg\, 3x| + c$

60. $\int \boldsymbol{\frac{ln(x)}{3x} dx} = \frac{1}{3}\int ln\, |x|\cdot \frac{1}{x} dx = \frac{1}{3}\cdot \frac{ln^2|x|}{2} + c = \frac{ln^2|x|}{6} + c$

2. Sabiendo que

$\int \frac{1}{x}dx = \ln|x| + C$ y $\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + C$ $\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + C$, calcula:

1) $\int \frac{\mathbf{dx}}{\mathbf{x+2}} = \int \frac{1}{x+2}dx = \ln|x+2| + C$

2) $\int \frac{\mathbf{dx}}{\mathbf{2x-3}} = \frac{1}{2}\ln|2x-3| + C$

3) $\int \frac{\mathbf{dx}}{\mathbf{x-1}} = \int \frac{1}{x-1}dx = \ln|x-1| + C$

4) $\int \frac{\mathbf{xdx}}{\mathbf{x^2-1}} = \frac{1}{2}\int \frac{2x}{x^2-1}dx = \frac{1}{2}\ln|x^2-1| + C$

5) $\int \frac{\mathbf{x}}{\mathbf{1-2x^3}}\mathbf{dx} = \frac{-1}{6}\int \frac{-6x^2}{1-2x^3} = \frac{-1}{6}\ln|2x^3-1| + C$

6) $\int \frac{\mathbf{x^2}}{\mathbf{1-x^3}}\mathbf{dx} = \frac{-1}{3}\int \frac{3x^2}{1-x^3}dx = \frac{-1}{3}\ln|x^3-1| + C$

7) $\int \frac{\mathbf{3x}}{\mathbf{x^2+2}}\mathbf{dx} = \frac{3}{2}\int \frac{2x}{x^2+2}dx = \frac{3}{2}\ln|x^2+2| + C$

8) $\int \frac{\mathbf{4}}{\mathbf{3x+5}}\mathbf{dx} = \frac{4}{3}\int \frac{3}{3x+5}dx = \frac{4}{3}\ln|3x+5| + C$

9) $\int \frac{\mathbf{x+1}}{\mathbf{x^2+2x+2}}\mathbf{dx} = \frac{1}{2}\int \frac{2(x+1)}{x^2+2x+2}dx = \frac{1}{2}\int \frac{2x+2}{x^2+2x+2}dx = \frac{1}{2}\ln|x^2+2x+2| + C$

10) $\int \left(\sqrt{\mathbf{x}} + \frac{\mathbf{1}}{\mathbf{x}}\right)\mathbf{dx} = \int \sqrt{x}dx + \int \frac{1}{x}dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \ln|x| + C$

11) $\int \left(\frac{\mathbf{3}}{\mathbf{x^2}} + \frac{\mathbf{2}}{\mathbf{x}} + \sqrt{\mathbf{x}}\right)\mathbf{dx} = \int 3x^{-2} + \frac{2}{x} + x^{\frac{1}{2}}dx = \frac{3x^{-1}}{-1} + 2\ln(x) + \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = -3x^{-1} + 2\ln(x) + \frac{2\sqrt{x^3}}{3} + C$

12) $\int \frac{\mathbf{dx}}{\mathbf{xlnx}} = \ln(|\ln|x||) + C$

13) $\int \frac{\mathbf{dx}}{\sqrt{\mathbf{x}}(\mathbf{1}-\sqrt{\mathbf{x}})} = \int \frac{1}{\sqrt{x}(1-\sqrt{x})}dx = -2\int \frac{-1}{2\sqrt{x}(1-\sqrt{x})}dx = -\,2\ln(|1-\sqrt{x}|) + C$

14) $\int \frac{\mathbf{1}}{\mathbf{2x-1}} - \frac{\mathbf{1}}{\mathbf{2x+1}}\mathbf{dx} = \frac{1}{2}(\int \frac{2}{(2x-1)}dx - \int \frac{2}{(2x-1)}dx) = \frac{1}{2}\ln(|2x-1|) - \frac{1}{2}\ln(|2x+1|) + C$

15) $\int \frac{\mathbf{e^x}}{\mathbf{e^x+1}}\mathbf{dx} = \ln(e^x+1)+C$

16) $\int \frac{\mathbf{e^{2x}}}{\mathbf{e^{2x}+3}}\mathbf{dx} = \frac{1}{2}\int \frac{2e^{2x}}{e^{2x}+3}dx = \frac{1}{2}\ln(e^{2x}+3)+C$

17) $\int \mathbf{\tan(x)\,dx} = \int \frac{\sin(x)}{\cos(x)}dx = -\ln(|\cos(x)|)+C$

18) $\int \frac{\mathbf{\cos(x)}}{\mathbf{\sin(x)}}\mathbf{dx} = \ln(|\sin(x)|)+C$

19) $\int \frac{\mathbf{5}}{\mathbf{x\ln(x)}}\mathbf{dx} = 5\ln(|\ln(x)|)+C$

20) $\int \frac{\mathbf{\sin(x)+\cos(x)}}{\mathbf{\cos(x)}}\mathbf{dx} = \int\left(\frac{\sin(x)}{\cos(x)}+\frac{\cos(x)}{\cos(x)}\right)dx = \int\left(\frac{\sin(x)}{\cos(x)}+1\right)dx = \int\frac{\sin(x)}{\cos(x)}dx+\int 1dx =$
$-\ln(|\cos(x)|)+x+C$

21) $\int \frac{\mathbf{2\sin(x)\cos(x)}}{\mathbf{1+\sin(x^2)}}\mathbf{dx} = \ln(|1+\sin(x^2)|)+C$

22) $\int \frac{\mathbf{\sin(x)-\cos(x)}}{\mathbf{\sin(x)+\cos(x)}}\mathbf{dx} = -\ln(|\sin(x)+\cos(x)|)+C$

23) $\int \mathbf{x\cot x^2\,dx} = \frac{1}{2}\int 2x\frac{\cos(x^2)}{\sin(x^2)}dx = \frac{\ln(|\sin x^2|)}{2}+C$

3. Si $\int e^x\,dx = e^x + C$, $\int e^{f(x)}\cdot f'(x)dx = e^{f(x)}+C$,

$\int a^x dx = \frac{a^x}{\ln a}+C$ **y** $\int a^{f(x)}\cdot f'(x)dx = \frac{a^{f(x)}}{\ln a}+C$, **calcula:**

1. $\int 3^x\,dx = \frac{3^x}{\ln 3}+C$
2. $\int a^{4x}dx = \frac{1}{4}\int 4(a^{4x})dx = \frac{a^{4x}}{4\ln a}+C$
3. $\int e^{-x}dx = -1\int(-1)(e^{-x})dx = \frac{-1}{e^x}+C$
4. $\int 4e^{3x}dx = 4(\frac{1}{3})\int 3(e^{3x})dx = \frac{4e^{3x}}{3}+C$
5. $\int(3x^2\cdot e^{x^3+2})\,dx = e^{x^3+2}+C$
6. $\int(4e^{4-x})dx = 4\cdot(-1)\int(-1)(e^{4-x})dx = -4(e^{4-x})+C$
7. $\int(x^2e^{x^3})\,dx = \frac{1}{3}\int\left[3x^2(e^{x^3})\right]dx = \frac{e^{x^3}}{3}+C$
8. $\int(e^x+1)^2\,dx = \int[(e^x)^2+2(e^x)(1)+1^2]dx = \int e^{2x}dx+\int 2e^xdx+\int 1dx =$
$= \frac{1}{2}\int 2e^{2x}dx+\int 2e^xdx+\int 1dx = \frac{e^{2x}}{2}+2e^x+x+C$
9. $\int\left(e^x+\frac{1}{e^x}\right)^2dx = \int(e^x+e^{-x})^2dx = \int[(e^x)^2+2(e^x)(e^{-x})+(e^{-x})^2]dx$

$= \int e^{2x}\,dx + \int 2dx + \int e^{-2x}dx = \frac{e^{2x}}{2} + 2x - \frac{e^{-2x}}{2} + C$

10. $\int (e^x + x^6)^2 dx = \int [(e^x)^2 + 2(e^x)(x^6) + (x^6)^2]dx =$

$= \frac{1}{2}\int 2e^{2x}\,dx + \int 2(e^x)(x^6)\,dx\ (por\ partes) + \int x^{12}dx =$

$= \frac{1}{2}e^{2x} + 2e^x(x^6 - 6x^5 + 30x^4 - 120x^3 + 360x^2 - 720x + 720\ ) + \frac{x^{13}}{13} + C$ -

11. $\int e^{-x^2+2}\,xdx = \frac{-1}{2}\int e^{-x^2+2}\,(-2x)dx = \frac{-\left(e^{-x^2+2}\right)}{2} + C$

12. $\int \frac{e^{\ln x}}{x}dx = \int \frac{x}{x}dx = \int dx = x + C$

13. $\int \frac{e^{\frac{1}{x^2}}}{x^3}dx = \int e^{\frac{1}{x^2}} \cdot \frac{1}{x^3}dx = \frac{-1}{2}\int e^{\frac{1}{x^2}} \cdot \frac{-2}{x^3}dx = \frac{-1}{2}e^{\frac{1}{x^2}} + C$

14. $\int xe^{\sin x^2}\cos x^2\,dx = \frac{1}{2}\int 2x\,e^{\sin x^2}\cos x^2\,dx = \frac{e^{\sin x^2}}{2} + C$

15. $\int (e^{3\cos 2x}\sin 2x)dx = \frac{-1}{6}\int(-6)\,(e^{3\cos 2x}\sin 2x)dx = \frac{-e^{3\cos 2x}}{6} + C$

16. $\int \frac{e^{\sqrt{x}}}{5\sqrt{x}}dx = \frac{1}{5}\cdot 2\int \frac{e^{\sqrt{x}}}{2\sqrt{x}}dx = \frac{2e^{\sqrt{x}}}{5} + C$

17. $\int e^{\cos x}\sin x\,dx = -\int e^{\cos x}\,(-\sin x)\,dx = -e^{\cos x} + C$

18. $\int \left(\frac{\sqrt{1+e^2}}{2e} - e^{x-3}\right)dx = \frac{\sqrt{1+e^2}}{2e}x - e^{x-3} + C$

19. $\int e^{\tan 2x}\sec^2 2x\,dx = \frac{1}{2}\int 2e^{\tan 2x}\sec^2 2x\,dx = \frac{e^{\tan 2x}}{2} + C$

20. $\int \frac{2x}{3}(3^{3+5x^2})dx = \frac{2}{3}\int x(3^{3+5x^2})dx = \frac{2}{3}\left(\frac{1}{10}\right)\int 10x(3^{3+5x^2})dx = \frac{1}{15}\left(\frac{3^{3+5x^2}}{\ln 3}\right) + C$

$$\int \frac{x}{2}(2^{3-5x^2})dx = \frac{1}{2}\int x(2^{3-5x^2})dx = \frac{1}{2}\cdot(-\frac{1}{10})\int(-10x)(2^{3-5x^2})dx = \frac{-1}{20}(\frac{2^{3-5x^2}}{\ln 2}) + C$$

4. Sabiendo que $\int \sin x\,dx = -\cos x + C,\quad \int f'(x)\cdot\sin f(x) = -\cos f(x) + C,$

$\int \cos x dx = \sin x + C \quad y \quad \int \cos f(x)\cdot f'(x) = \sin f(x) + C$ **calcula:**

1. $\int \sin(2x+8)dx = \frac{1}{2}\int \sin(2x+8)2dx = \frac{-\cos(2x+8)}{2} + C$
2. $\int \sin\frac{x}{2}dx = 2\int \sin(\frac{x}{2})(\frac{1}{2})dx = -2\cos\frac{x}{2} + C$
3. $\int \cos 3x\,dx = \frac{1}{3}\int 3(\cos(3x)dx) = \frac{\sin 3x}{3} + C$
4. $\int x\sin x^2 dx = \frac{1}{2}\int 2x(\sin x^2)\,dx = \frac{-\cos x^2}{2} + C$
5. $\int (\frac{3\sin x - 2\cos x}{4})dx = \frac{1}{4}\int(3\sin x - 2\cos x)dx = \frac{-3\cos x}{4} - \frac{\sin x}{2} + C$
6. $\int \sin 2x\,dx = \frac{1}{2}\int 2\,(\sin 2x)dx = \frac{-\cos 2x}{2} + C$

7. $\int e^x \cos e^x \, dx = \sin e^x + C$

8. $\int x\cos(2x^2)\cdot\sin(2x^2)dx = \frac{1}{4}\int 4x\cos(2x^2)\cdot\sin(2x^2)dx = -\frac{1}{4}\cos(2x^2) + C$

9. $\int \frac{\sin(\ln x)}{x}dx = \int \frac{1}{x}\cdot\sin(lnx)\,dx = -\cos(lnx) + C$

7. Si

$\int \frac{1}{\cos^2 x}dx = \int\left(1+\operatorname{tg}^2 x\right)dx = \operatorname{tg} x + C$ y $\int \frac{f'(x)}{\cos^2 f(x)}dx = \int\left[1+\operatorname{tg}^2 f(x)\right]\cdot f'(x)dx = \operatorname{tg} f(x) + C$, calcula:

1) $\int x(1+\tan x^2)\,dx = \frac{1}{2}\int 2x(1+\tan x^2)\,dx = \frac{1}{2}tan(x^2) + \text{C}$

2) $\int(1+\tan x)^2\,dx = \int(1+tan^2x + 2\tan x)dx = \int((1+tan^2x)+2\tan x)dx = \tan x -$

$2\int\frac{-\sin x}{\cos x}dx = \tan x - 2\ln|\cos x| + \text{C}$

3) $\int tan^2\, 3x\,dx = \int(1-1+tan^2\, 3x\,)dx = \frac{1}{3}\int 3(1+tan^2\, 3x)\,dx - \int 1\,dx =$

$= \frac{1}{3}\cdot\tan 3x - x + \text{C}$

6. Halla el valor de las siguientes integrales, usando un cambio de variable:

1) $\int(2+5x)^4dx = \int t^4\cdot\frac{1}{5}dt = \frac{1}{5}\int t^4dt = \frac{1}{5}\cdot\frac{t^5}{5} = \frac{t^5}{25} = \frac{(2+5x)^5}{25} + c$

t = 2+5x , dt = 5 dx $\rightarrow dx = \frac{1}{5}dt$

2) $\int(3+4x)^6dx = \int t^6\cdot\frac{1}{4}dt = \frac{1}{4}\int t^6dt = \frac{1}{4}\cdot\frac{t^7}{7} = \frac{t^7}{28} = \frac{(3+4x)^7}{28} + c$

t = 3+4x , dt = 4x dx$\rightarrow dx = \frac{1}{4}dt$

3) $\int 6x(3+x^2)^5dx = \int 6x\cdot t^5\cdot\frac{1}{2x}dt = \int 3t^5dt = 3\int t^5dt = \frac{3t^6}{6} = \frac{t^6}{2} = \frac{(3+x^2)^6}{2} + c$

t =$3+x^2$, dt = 2x dx$\rightarrow dx = \frac{1}{2x}dt$

4) $\int\left[\frac{3}{5+4x} + \frac{3}{(5+4x)^3}\right]dx = \int\left[\frac{3}{t}\cdot\frac{1}{4}\right]dt + \int\left[\frac{3}{t^3}\cdot\frac{1}{4}\right]dt = \int\left[\frac{3}{4t}\right]dt + \int\left[\frac{3}{4t^3}\right]dt =$

$\frac{3}{4}\int\left[\frac{1}{t}\right]dt + \frac{3}{4}\int\left[\frac{1}{t^3}\right] = \frac{3}{4}\int\left[\frac{1}{t}\right]dt + \frac{3}{4}\int\left[t^{-3}\right]dt = \frac{3}{4}\ln|t| + \frac{3}{4}\cdot\frac{t^{-2}}{-2} = \frac{3}{4}\ln|t| - \frac{3t^{-2}}{8} =$

$\frac{3}{4}\ln|t| - \frac{3}{8t^2} = \frac{3}{4}\ln|5+4x| - \frac{3}{8(5+4x)^2} + c$

t = 5+4x , dt = 4 dx→$dx=\frac{1}{4}dt$

5) $\int(\sqrt{3+2x}+\sqrt[3]{3+2x})dx=\int(\sqrt{t}+\sqrt[3]{t})\cdot\frac{1}{2}dt=\frac{1}{2}\int(t^{\frac{1}{2}}+t^{\frac{1}{3}})dt=\frac{1}{2}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+\frac{1}{2}\cdot\frac{t^{\frac{4}{3}}}{\frac{4}{3}}=$

$=\frac{t^{\frac{3}{2}}}{3}+\frac{3t^{\frac{4}{3}}}{8}=\frac{\sqrt{t^3}}{3}+\frac{3\sqrt[3]{t^4}}{8}=\frac{\sqrt{(3+2x)^3}}{3}+\frac{3\sqrt[3]{(3+2x)^4}}{8}+c$

t = 3+2x , dt = 2 dx→$dx=\frac{1}{2}dt$

6) $\int(\frac{e^x-4}{e^{2x}})dx=\int(\frac{t-4}{t^2})\cdot\frac{1}{t}dt=\int(\frac{t-4}{t^3})dt=\int(\frac{t}{t^3})dt-\int(\frac{4}{t^3})dt=$

$=\int(\frac{1}{t^2})dt-4\int(\frac{1}{t^3})dt=\int t^{-2}dt-4\int t^{-3}dt=\frac{t^{-1}}{-1}-\frac{4t^{-2}}{-2}=\frac{-1}{t}+\frac{2}{t^2}=\frac{-1}{e^x}+\frac{2}{e^{2x}}+c$

$t=e^x$, $dt=e^x dx$→$dx=\frac{1}{e^x}dt=\frac{1}{t}dt$

7) $\int\sin^3(x)\cdot\cos(x)dx=\int t^3\cdot\cos(x)\cdot\frac{1}{\cos(x)}dt=\int t^3dt=\frac{t^4}{4}=\frac{\sin^4(x)}{4}+c$

$t=\sin(x)$, $dt=\cos(x)dx$→$dx=\frac{1}{\cos(x)}dt$

8) $\int(\frac{\sin(x)}{\cos(x)})dx=\int(\frac{\sin(x)}{t})\cdot\frac{-1}{\sin(x)}dt=\int\frac{-1}{t}dt=-\ln|t|=-\ln|\cos(x)|+c$

$t=\cos(x)$, $dt=-\sin(x)dx$→$dx=\frac{-1}{\sin(x)}dt$

9) $\int(\frac{\cos(x)}{\sin^4(x)})dx=\int(\frac{\cos(x)}{t^4})\cdot\frac{1}{\cos(X)}dt=\int(\frac{1}{t^4})dt=\int t^{-4}dt=\frac{t^{-3}}{-3}=\frac{-1}{3t^3}=\frac{-1}{3\sin^3(x)}+c$

$t=\sin(x)$, $dt=\cos(x)dx$→$dx=\frac{1}{\cos(x)}dt$

10) $\int x\sqrt{x^2+4}dx=\int x\sqrt{t}\cdot\frac{1}{2x}dt=\int(\frac{\sqrt{t}}{2})dt=\int(\frac{t^{\frac{1}{2}}}{2})dt=\frac{t^{\frac{3}{2}}}{\frac{3}{2}\cdot 2}=\frac{2t^{\frac{3}{2}}}{6}=\frac{t^{\frac{3}{2}}}{3}=\frac{\sqrt[3]{(x^2+4)^2}}{3}+c$

$t=x^2+4$, dt = 2x dx→$dx=\frac{1}{2x}dt$

11) $\int(\frac{e^x+3}{e^{2x}})dx=\int(\frac{t+3}{t^2}\cdot\frac{1}{t})dt=\int(\frac{t+3}{t^3})dt=\int(\frac{t}{t^3})dt+\int(\frac{3}{t^3})dt=$

$\int t\cdot t^{-3}dt+\int 3t^{-3}dt=\int t^{-2}dt+\int 3t^{-3}dt=\frac{t^{-1}}{-1}+\frac{3t^{-2}}{-2}=\frac{-1}{t}-\frac{3}{2t^2}=\frac{-1}{e^x}-\frac{3}{2e^{2x}}+c$

$$t = e^x \quad , \quad dt = e^x dx \to dx = \frac{1}{e^x} dt = \frac{1}{t} dt$$

12) $\int \left(\frac{e^{-x}+2}{e^{3x}}\right)dx = \int \left(\frac{-t+2}{t^3}\cdot\frac{1}{t}\right)dt = \int \left(\frac{-t+2}{t^4}\right)dt = \int \left(\frac{-t}{t^4}\right)dt + \int \left(\frac{2}{t^4}\right)dt =$

$$\int (-t)\cdot t^{-4}dt + \int 2t^{-4}dt = \int (-t^{-3})dt + \int 2t^{-4}dt = \frac{t^{-2}}{2} - \frac{2t^{-3}}{3} = \frac{e^{-2x}}{2} - \frac{2e^{-3x}}{3} + c$$

$$t = e^x \quad , \quad dt = e^x dx \to dx = \frac{1}{e^x} dt = \frac{1}{t} dt$$

13) $\int \frac{(x-2\sqrt{x})^2}{3x^2}\mathbf{dx}$

$$\int \frac{(x-2\sqrt{x})^2}{3x^2}dx = \frac{1}{3}\int \left(\frac{x^2-4\sqrt{x}+4x}{x^2}\right)dx = \frac{1}{3}\int \left(\frac{x^2}{x^2} - \frac{4\sqrt{x}}{x^2} + \frac{4x}{x^2}\right)dx = \frac{1}{3}\left(\int 1dx - 4\int x^{-\frac{3}{2}}dx + 4\int \frac{1}{x}dx\right) =$$

$$= \frac{1}{3}\left(x - 4\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + 4ln|x|\right) + C = \frac{1}{3}\left(x + \frac{8}{\sqrt{x}} + 4ln|x|\right) + C$$

14) $\int \frac{(2+3\sqrt{x})^2}{4x}\mathbf{dx}$

$$\int \frac{(2+3\sqrt{x})^2}{4x}dx = \frac{1}{4}\int \left(\frac{2^2+12\sqrt{x}+9x}{x}\right)dx = \frac{1}{4}\int \left(\frac{4}{x} - \frac{12\sqrt{x}}{x} + \frac{9x}{x}\right)dx = \frac{1}{4}\left(\int \frac{4}{x}dx + 12\int x^{-\frac{1}{2}}dx + \int 9dx\right) =$$

$$= \frac{1}{4}\left(4ln|x| + 12\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 9x\right) + C = ln|x| + 6\sqrt{x} + \frac{9x}{4} + C$$

15) $\int \mathbf{sen}\sqrt{x}\,\boldsymbol{dx}$; $x = t^2$; $dx = 2tdt$; $t = \sqrt{x}$

$\int sen\sqrt{t^2}2tdt = \int sent2tdt$ u = 2t , du = 2dt ; dv = sentdt , v = -cost

$$\int sent2tdt = -2tcost + \int cost2dt = -2tcost + 2sent + C = 2\sqrt{x}cos\sqrt{x} + 2sen\sqrt{x} + C$$

7. Halla el valor de las siguientes integrales, usando el método de integración por partes.

1) **∫3x cosx dx =** (dv = cos x dx u = 3x ; v = sen x du = 3 dx)

=3x · (sen x) - ∫ sen x · 3 dx = 3x · sen x + 3 · cos x + C

2) ∫x² · sen x dx = (dv = sen x dx u = x² ; v = -cos x du = 2x dx)

=x² · (-cos x) - ∫ (- cos x)· 2x dx = (dv = -cos x dx u = 2x ; v = - sen x du = 2 dx)

= (x² · (- cos x)) – [2x · (- sen x) - ∫ (- sen x) · 2 dx]=

= x² · (- cos x) + 2x · (sen x) - 2 · (-cos x) + C =

= - x² cos x + 2x sen x + 2 cos x + C

IES ATENEA Ciudad Real
Revisor: Luis Carlos Vidal del Campo
Ilustraciones: Creadas con GeoGebra

3) $\int x^2 \ln x \, dx =$ (dv = x^2 dx u = lnx ; v = $\frac{x^3}{3}$ du = $\frac{1}{x}$)

$= \ln x \cdot \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} dx = \ln x \cdot \frac{x^3}{3} - \int \frac{x^3}{3x} dx =$

$= \ln x \cdot \frac{x^3}{3} - \frac{1}{3} \int x^2 \, dx = \ln x \cdot \frac{x^3}{3} - \frac{x^3}{9} + C$

4) $\int \sqrt{x} \cdot \ln x \, dx =$ (dv = $\sqrt{x}$ dx = $x^{\frac{1}{2}}$ dx u = lnx; v = $=\frac{x^{\frac{3}{2}}}{\frac{3}{2}}$ du = $\frac{1}{x}$dx)

$= \ln x \cdot \frac{x^{\frac{3}{3}}}{\frac{3}{2}} - \int \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \cdot \frac{1}{x} dx = \ln x \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \int \frac{2x^{\frac{1}{2}}}{3} dx =$

$= \ln x \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{2}{3} \int x^{\frac{1}{2}} dx = \ln x \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{2}{3} \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \ln x \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{4x^{\frac{3}{2}}}{9} + C$

5) $\int \frac{\ln x}{x^2} dx =$ (dv = x^{-2} dx u = lnx ; v = $\frac{x^{-1}}{-1}$ du = $\frac{1}{x}$ dx)

$= \int \ln x \cdot x^{-2} dx = \ln x \cdot \frac{x^{-1}}{-1} - \int \frac{x^{-1}}{-1} \cdot \frac{1}{x} dx = \ln x \cdot \frac{x^{-1}}{-1} - \int \frac{x^{-1}}{-x} dx =$

$= \ln x \cdot \frac{x^{-1}}{-1} + \int \frac{1}{x^2} dx = -\ln x \, \frac{1}{x} + \int \frac{1}{x^2} dx = -\ln x \, \frac{1}{x} + \frac{x^{-1}}{-1} + C =$

$= -\ln x \, \frac{1}{x} - \frac{1}{x} + C$

6) $\int 2e^x \cdot \cos x \, dx =$

Cambios 1	Cambios 2
dv = cos x dx v = sen x u = $2e^x$ du = $2e^x$dx	dv = sen x v = - cos x u = $2e^x$ du = $2e^x \, dx$

(cambios 1) $= 2e^x \cdot \text{sen } x - \int \text{sen } x \cdot 2e^x dx =$

(cambios 2) $= 2e^x \cdot \text{sen } x - [2e^x \cdot (-\cos x) - \int (-\cos x) \cdot 2e^x \, dx \,] =$

$= 2e^x \cdot \text{sen } x - [2e^x \cdot (-\cos x) + \int \cos x \cdot 2e^x \, dx \,] =$

$= 2e^x \cdot \text{sen } x + 2e^x \cdot \cos x - \int \cos x \cdot 2e^x \, dx \,]$ (haciendo $\int \cos x \cdot 2e^x dx = \text{I}$)

$\text{I} = 2e^x \cdot \text{sen } x + 2e^x \cdot \cos x - \text{I}$

$2\text{I} = 2e^x \cdot \text{sen } x + 2e^x \cdot \cos x$

$\text{I} = e^x \cdot \text{sen } x - e^x \cdot \cos x + C$

7) $\int 2e^x \cdot \text{senx} \, dx$

- Fórmula: $\int u \cdot dv = u \cdot v - \int v \cdot du$
- $u = 2e^x \to du = 2e^x dx$
- $dv = senxdx \to v = \int senx \, dx = -cosx$

$\int 2e^x \cdot \text{senx} \, dx = 2e^x \cdot (-\text{cosx}) - \int -\cos x \cdot 2e^x dx = 2e^x \cdot (-\text{cosx}) + \int \cos x \cdot 2e^x dx =$

Repetimos el método de integración por partes:

- $u = 2e^x \rightarrow du = 2e^x dx$
- $dv = cosxdx \rightarrow v = \int cosx\, dx = senx$

$= 2e^x \cdot (-cosx) + 2e^x \cdot senx - \int senx \cdot 2e^x dx$

Hacemos $\int \mathrm{senx} \cdot 2\mathrm{e}^{\mathrm{x}} = \mathrm{I}$ de donde,

$I = 2e^x \cdot (-cosx) + 2e^x \cdot senx - I$

$2I = 2e^x \cdot (-cosx) + 2e^x \cdot senx$

$I = e^x \cdot (-cosx) + e^x \cdot senx$

$I = e^x \cdot (senx - cosx) + C$

8) $\int \boldsymbol{e^x \cdot cos3x\, dx}$

- $u = e^x \rightarrow du = e^x dx$
- $dv = cos3xdx \rightarrow v = \int cos3x\, dx = \frac{1}{3} \cdot sen3x$

$\int \mathrm{e}^{\mathrm{x}} \cdot \cos 3\mathrm{x}\, \mathrm{dx} = \frac{\mathrm{e}^{\mathrm{x}}}{3} \mathrm{sen3x} - \int \frac{\mathrm{e}^{\mathrm{x}}}{3}\, \mathrm{sen3x}\, \mathrm{dx} = \frac{\mathrm{e}^{\mathrm{x}}}{3} \mathrm{sen3x} - \frac{1}{3} \int \mathrm{e}^{\mathrm{x}}\, \mathrm{sen3x}\, \mathrm{dx} =$

Repetimos el método de integración por partes:

- $u = e^x \rightarrow du = e^x dx$
- $dv = sen3xdx \rightarrow v = \int sen3x\, dx = -\frac{1}{3} cos3x$

$= \frac{e^x}{3} sen3x - \frac{1}{3}(-\frac{e^x}{3} \cdot cos3x - \frac{1}{3} \cdot \int e^x(-cos3x)\, dx)$

Hacemos $\int \mathrm{e}^{\mathrm{x}} \cos 3\mathrm{x}\, \mathrm{dx} = \mathrm{I}$, de donde,

$I = \frac{e^x}{3} sen3x + \frac{e^x}{9} cos3x - \frac{1}{9} \cdot I$

$\frac{10}{9} \cdot I = \frac{e^x}{3} \cdot sen3x + \frac{e^x}{9} cos3x$

$I = \frac{e^x}{10} \cdot \left(3sen3x + \frac{cos3x}{3}\right) + C$

9) $\int \boldsymbol{\frac{4-2\,x^2}{x} \cdot ln(x) dx}$

- $u = \ln(x) \rightarrow du = \frac{1}{x} dx$
- $\mathrm{dv} = \frac{4-2\,\mathrm{x}^2}{\mathrm{x}} \mathrm{dx} \rightarrow \mathrm{v} = \int \left(\frac{4}{\mathrm{x}} - 2\mathrm{x}\right) \mathrm{dx} = 4 \cdot \ln(\mathrm{x}) - \mathrm{x}^2$

$\int \frac{4-2\,x^2}{x} \cdot ln(x)\, dx = \ln(x) \cdot (4 \cdot \ln(x) - x^2) - \int \frac{4}{x} \cdot (\ln(x) - x) dx =$

$=\ln(x) \cdot (4 \cdot \ln(x) - x^2) - (-4x + \int \frac{4}{x} \cdot \ln(x)\, dx) =$

$= \ln(x) \cdot (4 \cdot \ln(x) - x^2) + 4x - 4 \int \frac{\ln(x)}{x} dx =$

$=4ln^2(x)-x^2\ln(x)+4x-\frac{4ln^2(x)}{2}=2ln^2(x)-x^2\ln(x)+4x+C$

8. Halla el valor de las siguientes integrales racionales:

1) $\int\frac{2}{x^2+1}dx$

Las raíces del denominador son raíces complejas.

$x^2+1=0;\quad x=\pm i$ luego:

$\int\frac{2}{x^2+1}dx=2\int\frac{1}{x^2+1}dx=2arctan(x)+C$

2) $\int\frac{3}{2x^2+2}dx$

Las raíces del denominador son raíces complejas.

$2x^2+2=0;\quad x=\ \pm i$ luego:

$\int\frac{3}{2x^2+2}\mathrm{dx}=\int\frac{3}{2(x^2+1)}\mathrm{dx}=\frac{3}{2}\int\frac{1}{x^2+1}\mathrm{dx}=\frac{3}{2}\arctan(x)+C=\frac{3\arctan(x)}{2}+C$

3) $\frac{3}{x-3}dx$

La raíz del denominador es una raíz real simple.

$x-3=0;\quad x=3$ luego:

$\int\frac{3}{x-3}dx=3\int\frac{1}{x-3}dx=3ln|x-3|+C$

4) $\int\frac{2}{3x^2+3}dx$

$$\int\frac{2}{3x^2+3}dx=\int\frac{2}{3(x^2+1)}dx=\frac{2}{3}\int\frac{1}{x^2+1}dx=\frac{2}{3}\arctan(x)+\ C=\frac{2\arctan(x)}{3}+C$$

5) $\int\frac{5x}{x^2+3}dx$

$$\int\frac{5x}{x^2+3}dx=\frac{5}{3}\int\frac{3x}{(x^2+3)}dx=\frac{5}{3}\ln|x^2+3|+C$$

6) $\int\frac{3x-2}{x^2+1}dx$

$\int\frac{3x-2}{x^2+1}dx=\int\left(\frac{3x}{x^2+1}-\frac{2}{x^2+1}\right)dx=\frac{3}{2}\int\frac{2x}{x^2+1}dx-2\int\frac{1}{x^2+1}dx=\frac{3\ln|x^2+1|}{2}-2\arctan(x)+C$

7. $\int\frac{(2x-3)^2}{3x^2}dx$

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Revisor: Luis Carlos Vidal del Campo
Ilustraciones: Creadas con GeoGebra

$\int \frac{(2x-3)^2}{3x^2} dx = \int \frac{4x^2+9-12x}{3x^2} dx = \int \frac{4}{3} dx + \int \frac{-12x}{3x^2} dx + \int \frac{9}{3x^2} dx =$

$= \frac{4}{3} \int dx - 4 \int \frac{1}{x} dx + 3 \int \frac{1}{x^2} dx = \frac{4}{3} x - 4 \ln|x| + 3 \int x^{-2} dx =$

$= \frac{4}{3} x - 4 \ln|x| + 3 \frac{x^{-1}}{-1} + C = \frac{4}{3} x - 4 \ln|x| - \frac{3}{x} + C$

8. $\int \frac{x+2}{x+1} dx$

Según la división euclídea, obtenemos:

C(x)=1 R(x)=1 Q(x)=x+1

$\int \frac{x+2}{x+1} dx = \int \left(1 + \frac{1}{x+1}\right) dx = \int 1dx + \int \frac{1}{x+1} dx = x + \ln|x+1| + C$

9. $\int \frac{x-1}{x+1} dx$

Según la división euclídea, obtenemos:

C(x)=1 R(x)=-2 Q(x)=x+1

$\int \frac{x-1}{x+1} dx = \int (1 + \frac{-2}{x+1}) dx = \int 1dx + \int \frac{-2}{x+1} dx = x - 2 \ln|x+1| + C$

10) $\int \frac{3x-1}{x+3} dx$;

Efectuando la división obtenemos: C(x): 3 R(x): -10 Q(x): $x+3$

$$\int \frac{P(x)}{Q(x)} dx = \int C(x)\, dx + \int \frac{R(x)}{Q(x)} dx$$

$$\int \frac{3x-1}{x+3} dx = \int 3\, dx + (-10) \int \frac{1}{x+3} dx;\ 3x - 10 \ln(|x+3|) + C$$

11) $\int \frac{3x^3}{x^2-4} dx$; C(x): $3x$ R(x): $12x$ Q(x): x^2-4

$$\int \frac{3x^3}{x^2-4} dx = \int 3x\, dx + 6 \int \frac{\frac{12x}{6}}{x^2-4} dx;\ \frac{3x^2}{2} + 6 \ln(|x^2-4|) + C$$

12) $\int \frac{3x^3}{x^2-1} dx$; C(x): $3x$ R(x): $3x$ Q(x): x^2-1

$\int \frac{3x^3}{x^2-1} dx = \int 3x\, dx + \int \frac{3x}{x^2-1} dx;\ \int 3x\, dx + \frac{3}{2} \int \frac{2x}{(x^2-1)} dx;\ \frac{3x^2}{2} + \frac{3}{2} \ln(|x^2-1|) + C$

13) $\int \frac{x^2+2x+2}{x+2} dx$; efectuando la división obtenemos C(x) = x; R(x) = 2; de donde

$\int \frac{x^2+2x+2}{x+2}dx = \int (x)dx + \int \frac{2}{x+2}dx = \int (x)dx + 2\int \frac{1}{x+2}dx = \frac{x^2}{2} + 2\ln|x+2| + C$

14) $\int \frac{\mathbf{x^3+4x^2-2x+5}}{\mathbf{x-2}}\mathbf{dx}$; efectuando la división, C(x) = $x^2+6x+10$; R(x) = -25; de donde

$\int \frac{x^3+4x^2-2x+5}{x-2}dx = \int (x^2+6x+10)dx + \int \frac{-25}{x-2}dx =$

$= \int (x^2+6x+10)dx - 25\int \frac{1}{x-2}dx = \frac{x^3}{3} + 3x^2 + 10x - 25\ln|x-2| + C$

15) $\int \frac{\mathbf{2}}{\mathbf{x^2-4}}\mathbf{dx}$; hacemos la descomposición en fracciones simples,

$\frac{2}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2}$ 2 = A(x+2) + B(x-2), sustituyendo por x = y x = -2 , $A = \frac{1}{2}$, $B = -\frac{1}{2}$

$\int \frac{2}{(x-2)(x+2)}dx = \int \frac{A}{x-2}dx + \int \frac{B}{x+2}dx = \int \frac{1}{2(x-2)}dx + \int -\frac{1}{2(x+2)}dx = \int \frac{1}{2x-2}dx - \int \frac{1}{2x+2}dx =$
$\frac{1}{2}\int \frac{2}{2x-2}dx - \frac{1}{2}\int \frac{2}{2x+2}dx = \frac{1}{2}\ln|x-2| - \frac{1}{2}\ln|x+2| + C \quad = \frac{1}{2}\ln\left|\frac{x-2}{x+2}\right| + C$

16) $\int \frac{\boldsymbol{3x+2}}{\boldsymbol{x^2+3x}}\,\boldsymbol{dx}$

1. Se analizan las raíces del denominador: $x^2+3x=0; x(x+3)=0 \rightarrow x=0; x=-3$

2. Se escribe la descomposición en fracciones simples:
$$\frac{3x+2}{x^2+3x} = \frac{A}{x} + \frac{B}{(x+3)}$$

3. Para hallar A y B, se sustituye la x por el valor de las raíces:

$$3x+2 = A(x+3) + Bx$$

$$x=0 \rightarrow 2=3A \rightarrow A=\frac{2}{3}$$
$$x=-3 \rightarrow -7=-3B \rightarrow B=\frac{7}{3}$$

4. Se escribe la integral como suma de integrales:

$$\int \frac{3x+2}{x^2+3x}\,dx = \int \frac{2/3}{x}dx + \int \frac{7/3}{(x+3)}dx = \boldsymbol{\frac{2}{3}\ln|x| + \frac{7}{3}\ln|x+3| + C}$$

17) $\int \frac{\boldsymbol{4x+3}}{\boldsymbol{x^2-1}}\,\boldsymbol{dx}$

1. Se analizan las raíces del denominador: $x^2-1=0; x=\sqrt{1} \rightarrow x=1; x=-1$

2. Se escribe la descomposición en fracciones simples:
$$\frac{4x+3}{x^2-1} = \frac{A}{(x-1)} + \frac{B}{(x+1)}$$

3. Para hallar A y B, se sustituye la x por el valor de las raíces:

$$4x+3=A(x+1)+B(x-1)$$

$$x=1\rightarrow\ 7=2A\rightarrow A=\frac{7}{2}$$
$$x=-1\rightarrow -1=-2B\rightarrow B=\frac{1}{2}$$

4. Se escribe la integral como suma de integrales:

$$\int\frac{4x+3}{x^2-1}\,dx=\int\frac{7/2}{(x-1)}dx+\int\frac{1/2}{(x+1)}dx=\frac{7}{2}\ln|x-1|+\frac{1}{2}\ln|x+1|+C$$

18) $\int\frac{3x^2}{x^2+6x+9}dx$

1. Al ser el grado del polinomio del numerador igual que el del denominador, debemos realizar la división de estos, $3x^2:(x^2+6x+9)\rightarrow$ y obtenemos como cociente 3 y como resto $-18x-27$
   a. Obtenemos la siguiente integral: $\int 3dx+\int\frac{-18x-27}{x^2+6x+9}dx\rightarrow 3x+\int\frac{-18x-27}{x^2+6x+9}dx$
2. Con la nueva integral obtenida, realizamos el procedimiento habitual para integrales racionales:
   a. Igualamos el polinomio del divisor a cero: $x^2+6x+9=0\rightarrow x=-3$ (doble)
   b. Con las soluciones obtenidas calculamos: $\frac{-18x-27}{x^2+6x+9}=\frac{A}{x+3}+\frac{B}{(x+3)^2}\rightarrow$

   $-18x-27=A(x+3)^2+B(x+3)$

   Sustituimos x por un valor cualquiera:

   $x=-2\rightarrow 9=A+B\ ;\quad x=-1\rightarrow -9=4A+4B$

   Resolvemos el sistema de ecuaciones obtenido: $A=-\frac{27}{2};\quad B=\frac{45}{2}$

3. Gracias a este procedimiento hemos obtenido:

$$3x+\int\frac{\frac{-27}{2}}{x+3}\,dx+\int\frac{\frac{45}{2}}{(x+3)^2}dx=3x-\frac{27}{2}\int\frac{dx}{x+3}+\frac{45}{2}\int\frac{dx}{(x+3)^2}=$$

$$3x-\frac{27}{2}ln|x+3|+\frac{45}{2}\cdot\frac{(x-3)^{-1}}{-1}=3x-\frac{27}{2}ln|x+3|-\frac{45}{2}\cdot\frac{1}{x+3}+C$$

IES ATENEA Ciudad Real

Revisor: Luis Carlos Vidal del Campo

Ilustraciones: Creadas con GeoGebra

19) $\int \frac{x+2}{x^2-5x+6} dx$

1. Como el grado del polinomio del denominador es superior al del polinomio del numerador, utilizamos el procedimiento habitual para estos casos:
   a. Igualamos el polinomio del denominador a cero: $x^2 - 5x + 6 = 0 \rightarrow x_1 = 3; x_2 = 2$
   b. Con las soluciones obtenidas calculamos:

   $$\frac{x+2}{x^2-5x+6} = \frac{A}{x-3} + \frac{B}{x-2} \rightarrow x + 2 = A(x-2) + b(x-3)$$

   Sustituimos x por un valor cualquiera:

   $x = 2 \rightarrow 4 = -B \rightarrow B = -4;$ $x = 3 \rightarrow 5 = A$

2. Gracias a este procedimiento obtenemos:

   $$\int \frac{5}{x-3} dx + \int \frac{-4}{x-2} dx = 5ln|x-3| - 4ln|x-2| + C$$

20) $\int \frac{3x-2}{x^2-4x+4} dx$

1. Como el grado del polinomio del denominador es superior al del polinomio del numerador, utilizamos el procedimiento habitual para estos casos:
   a. Igualamos el polinomio del divisor a cero: $x^2 - 4x + 4 = 0 \rightarrow x = 2$ (doble)
   b. Con las soluciones obtenidas calculamos:

   $$\frac{3x-2}{x^2-4x+4} = \frac{A}{x-2} + \frac{B}{(x-2)^2} \rightarrow 3x - 2 = A(x-2)^2 + B(x-2)$$

   Sustituimos x por un valor cualquiera: $x = 3 \rightarrow 7 = A + B; x = 1 \rightarrow 1 = A - B$

   Resolvemos el sistema de ecuaciones obtenido: $A = 4; B = 3$

2. Gracias a este procedimiento obtenemos:

   $$\int \frac{4}{x-2} dx + \int \frac{3}{(x-2)^2} dx = 4ln|x-2| + 3 \cdot \frac{(x-2)^{-1}}{-1} = 4ln|x-2| - \frac{3}{x-2} + C$$

21) $\int \frac{3x+1}{x^3-4x^2+3x} dx$

$$\frac{3x+1}{x^3-4x^2+3x} = \frac{3x+1}{x(x-3)(x-1)} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{x-1} \Longrightarrow \frac{3x+1}{x(x-3)(x-1)} = \frac{A(x-3)(x-1)+Bx(x-1)+Cx(x-3)}{x(x-3)(x-1)}$$

$3x + 1 = (Ax - 3A)(x - 1) + Bx^2 - Bx + Cx^2 - 3Cx$

$3x + 1 = Ax^2 - Ax - 3Ax + 3A + Bx^2 - Bx + Cx^2 - 3Cx$

$3x + 1 = (A+B+C)x^2 - (4A + B + 3C)x + 3A$

$\mathrm{A}+\mathrm{B}+\mathrm{C}=0$

$-(4\mathrm{A}+\mathrm{B}+3\mathrm{C})=3$

$3A=1 \Longrightarrow \mathbf{A}=\mathbf{\frac{1}{3}}$

$\begin{cases} \frac{1}{3}+B+C=0 \\ \frac{4}{3}+\mathrm{B}+3\mathrm{C}=3 \end{cases} \begin{cases} \mathrm{B}+\mathrm{C}=-\frac{1}{3} \\ \mathrm{B}+3\mathrm{C}=3-\frac{4}{3} \end{cases} \begin{cases} \mathrm{B}+\mathrm{C}=-\frac{1}{3} \\ -\mathrm{B}-3\mathrm{C}=3+\frac{4}{3} \end{cases} \{-2\mathrm{C}=4 \Longrightarrow \mathrm{C}=\frac{4}{-2} \Longrightarrow \mathbf{C}=\mathbf{-2}$

$\mathrm{B}=-\mathrm{A}-\mathrm{C}=-\frac{1}{3}-(-2) \Longrightarrow \mathbf{B}=\mathbf{\frac{5}{3}}$

$\int \frac{3x+1}{x^3-4x^2+3x}dx=\int \frac{\frac{1}{3}}{x}dx+\int \frac{\frac{5}{3}}{x-3}dx+\int \frac{-2}{x-1}dx=\frac{1}{3}\int \frac{dx}{x}+\frac{5}{3}\int \frac{dx}{x-3}-2\int \frac{dx}{x-1}=$

$=\mathbf{\frac{1}{3}ln(x)+\frac{5}{3}ln(x-3)-2ln(x-1)+C}$

22) $\int \mathbf{\frac{2x^2-1}{x^2+3x+2}dx}$ efectuando la división obtenemos, C(x) = 2; R(x) = - 6x – 5 , de donde,

$\frac{D}{d}=C+\frac{R}{d} \Longrightarrow \int \frac{2x^2-1}{x^2+3x+2}dx=\int 2dx+\int \frac{-(6x+5)}{x^2+3x+2}dx=2\int dx-\int \frac{6x+5}{x^2+3x+2}dx=I_1+I_2$

$I_1=2\int dx=2x+C$

$I_2=\int \frac{6x+5}{x^2+3x+2} \Longrightarrow \frac{6x+5}{(x+2)(x+1)}=\frac{A}{x+2}+\frac{B}{x+1}$

$\frac{6x+5}{(X+2)(x+1)}=\frac{A(x+1)+B(x+2)}{(x+2)(x+1)} \Longrightarrow 6x+5=Ax+A+Bx+2B$

$6x+5=(A+B)x+A+2B$

$A+B=6$

$A+2B=5$

$B=-1$

$A=7$

$\int \frac{7}{x+2}dx+\int \frac{-1}{x+1}dx=7\int \frac{dx}{x+2}-\int \frac{dx}{x+1}=7\ln(x+2)-\ln(x+1)+C$

$\int \frac{2x^2-1}{x^2+3x+2}dx=2x-7\ln(x+2)-\ln(x+1)+C$

23) $\int \mathbf{\frac{x-1}{x^2-4x+4}dx}$

$\frac{x-1}{x^2-4x+4}=\frac{x-1}{(x-2)^2}=\frac{A}{(x-2)^2}+\frac{B}{(x-2)} \Longrightarrow \frac{x-1}{(x-2)^2}=\frac{A+B(x-2)}{(x-2)^2} \Longrightarrow x-1=A+Bx-2B$

$B=1$

$A-2B=-1 \Longrightarrow A-2(1)=-1 \Longrightarrow A=1$

$\int \frac{x-1}{x^2-4x+4}dx=\int \frac{1}{(x-2)^2}dx+\int \frac{1}{x-2}dx=I_1+I_2$

$I_1 \Longrightarrow \int \frac{dx}{(x-2)^2}\,dx \begin{cases} z = x-2 \\ dz = dx \end{cases} = \int \frac{dz}{z^2} = \int z^{-2}dz = \frac{z^{-1}}{-1} + C = -\frac{1}{z} + C = -\frac{1}{x-2} + C$

$$I_2 \Longrightarrow \int \frac{dx}{x-2} \begin{cases} z = x-2 \\ dz = dx \end{cases} = \int \frac{dz}{z} = \ln(z) + C = \ln|x-2| + C$$

$\int \frac{x-1}{x^2-4x+4}dx = -\frac{1}{x-2} + \ln(x-2) + C$

24) $\int \frac{\mathbf{3x-1}}{\mathbf{x^2+6x+9}}\mathbf{dx} = \int \frac{3x-1}{(x+3)(x+3)}dx = \int \frac{3x-1}{(x+3)^2}dx$

$\frac{3x-1}{(x+3)^2} = \frac{A}{(x+3)^2} + \frac{B}{(x+3)} \rightarrow \frac{3x-1}{(x+3)^2} = \frac{A+B(x+3)}{(x+3)^2}$

$3x - 1 = A + B(x+3)$

$x = -3;\ 3\cdot(-3) - 1 = A + B(-3+3) \rightarrow A = 10$

$$x = 3;\ 3\cdot 3 - 1 = -10 + B(3+3) \rightarrow 8 = -10 + 6B \rightarrow B = \frac{18}{6} = 3$$

$\int \frac{3x-1}{x^2+6x+9}dx = \int \frac{-10}{(x+3)^2}dx + \int \frac{3}{(x+3)}dx = -10\int \frac{1}{(x+3)^2}dx + 3\int \frac{1}{(x+3)}dx =$

$= -10\int (x+3)^{-2}\,dx + 3\int \frac{1}{(x+3)}dx = -10\cdot\frac{(x+3)^{-1}}{-1} + 3\ln|x+3| + C = \frac{10}{(x+3)} + 3\ln|x+3| + C$

9. Halla el valor de las siguientes integrales definidas

1) $\int_1^3 \frac{dx}{2x} = \int_1^3 \frac{1}{2x}dx = \frac{1}{2}\int_1^3 \frac{1}{x}dx = \frac{1}{2}\ln(|x|)\Big|_1^3 = \frac{1}{2}\ln(|3|) - \frac{1}{2}\ln(|1|) = \frac{1}{2}\ln(3)$

2) $\int_2^3 \frac{x}{x^2-1}\mathbf{dx} = \int_2^3 \frac{1}{2t}dt = \frac{1}{2}\int_2^3 \frac{1}{t}dt = \frac{1}{2}\ln(|t|)\Big|_2^3 = \frac{1}{2}\ln(|x^2-1|)\Big|_2^3 = \frac{1}{2}\ln(|3^2-1|) - \frac{1}{2}\ln(|2^2-1|) = \frac{1}{2}\ln\left(\frac{8}{3}\right)$

3) $\int_{\frac{\pi}{4}}^{\frac{5\pi}{3}} \sin(x)\,dx = -\cos(x)|_{\frac{\pi}{4}}^{\frac{5\pi}{3}} = -\cos\left(\frac{5\pi}{3}\right) - \left(-\cos\left(\frac{\pi}{4}\right)\right) = \frac{-1+\sqrt{2}}{2}$

4) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin(3x)\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\sin(t)}{3}dt = \frac{1}{3}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin(t)\,dt = \frac{1}{3}(-\cos(t))\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} = \frac{1}{3}(-\cos(3x))\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} = -\frac{\cos(3x)}{3}\Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} = -\frac{\cos\left(3\times\frac{\pi}{4}\right)}{3} - \left(-\frac{\cos\left(3\times\frac{\pi}{6}\right)}{3}\right) = \frac{\sqrt{2}}{6}$

5) $\int_{-4}^{4} |x|\,dx = \int_{-4}^{0}(-x)\,dx + \int_0^4 x\,dx = -\frac{x^2}{2}\Big|_{-4}^{0} + \frac{x^2}{2}\Big|_0^4 = 8 + 8 = 16$

6) $\int_{-1}^{1}\left(3x^2 - 2x + \frac{1}{2}\right)dx = \int_{-1}^{1} 3x^2\,dx - \int_{-1}^{1} 2x\,dx + \int_{-1}^{1}\frac{1}{2}dx = \left(x^3 - x^2 + \frac{1}{2}x\right)\Big|_{-1}^{1} = \left(1^3 - 1^2 + \frac{1}{2}\cdot 1\right) - \left((-1)^3 - (-1)^2 + \frac{1}{2}\cdot(-1)\right) = 3$

7) $\int_{-1}^{2}\left(\frac{2}{x+2}-\frac{3}{x-3}\right)dx = \int_{-1}^{2}\frac{2}{x+2}dx - \int_{-1}^{2}\frac{3}{x-3}dx = (2\ln(|x+2|)-3\ln(|x-3|))|_{-1}^{2} = 10\ln(2)$

8) $\int_{-2}^{2}\left(\frac{3a}{5}-\frac{x}{2}\right)dx = \int_{-2}^{2}\frac{3a}{5}dx - \int_{-2}^{2}\frac{x}{2}dx = \left(\frac{3ax}{5}-\frac{x^2}{4}\right)\Big|_{-2}^{2} = \frac{3a\cdot 2}{5}-\frac{2^2}{4}-\left(\frac{3a\cdot(-2)}{5}-\frac{(-2)^2}{4}\right) = \frac{12}{5}a$

9) $\int_{2}^{3}\frac{1}{x\cdot(\ln x)^3}dx$

$\int\frac{1}{x\cdot(\ln x)^3}dx = \int(\ln x)^{-3}\cdot\frac{1}{x}dx = \frac{(\ln x)^{-2}}{-2} = -\frac{1}{2\cdot(\ln x)^2}+C$

$\int_{2}^{3}\frac{1}{x\cdot(\ln x)^3}dx = -\frac{1}{2\cdot(\ln x)^2}\Big|_{2}^{3} = -\frac{1}{2\cdot(\ln 3)^2}+\frac{1}{2\cdot(\ln 2)^2} = -0{,}414+1{,}04 = 0{,}626$

10) $\int_{-2}^{0}\left(e^{2x}+\frac{3}{e^{3x}}\right)dx$

$\int\left(e^{2x}+\frac{3}{e^{3x}}\right)dx = \int e^{2x}dx + \int e^{-3x}\cdot 3dx = \frac{1}{2}\int e^{2x}\cdot 2\,dx - \int e^{-3x}\cdot(-3)\,dx$

$= \frac{1}{2}e^{2x} - e^{-3x} + C$

$\int_{-2}^{0}\left(e^{2x}+\frac{3}{e^{3x}}\right)dx = \frac{1}{2}e^{2x}-e^{-3x}\Big|_{-2}^{0} = \left(\frac{1}{2}e^{2\cdot 0}-e^{-3\cdot 0}\right)-\left(\frac{1}{2}e^{2\cdot(-2)}-e^{-3\cdot(-2)}\right) = 403{,}91$

11) $\int_{\frac{\pi}{4}}^{\frac{5\pi}{3}}(\operatorname{sen} x-\cos x)^2\,dx$

$\int_{\frac{\pi}{4}}^{\frac{5\pi}{3}}(\operatorname{sen} x-\cos x)^2\,dx = \int_{\frac{\pi}{4}}^{\frac{5\pi}{3}}(\operatorname{sen}^2 x - 2\operatorname{sen} x\cos x + \cos^2 x)dx =$

$= \int_{\frac{\pi}{4}}^{\frac{5\pi}{3}}(1-\cos^2 x - 2\operatorname{sen} x\cos x+\cos^2 x)dx = (x-\operatorname{sen}^2 x)|_{\frac{\pi}{4}}^{\frac{5\pi}{3}} =$

$= \left[\frac{5\pi}{3}-\operatorname{sen}^2\frac{5\pi}{3}\right]-\left[\frac{\pi}{4}-\operatorname{sen}^2\frac{\pi}{4}\right] = [5{,}23]-[0{,}79] = 4{,}4\text{u}^2$

10. Halla el valor de b para que se cumpla $\int_{-1}^{b}(2bx-3x^2)dx = -12$.

1. Se resuelve la integral con la incógnita b:

$$\int_{-1}^{b}(2bx-3x^2)dx = 2b\frac{x^2}{2}-3\frac{x^3}{3}\Big]_{-1}^{b} = bx^2-x^3]_{-1}^{b}$$

2. Sustituimos los límites de integración:

$$(b\cdot b^2-b^3)-(b\cdot(-1)^2-(-1)^3) = b^3-b^3-b-1 = -b-1$$

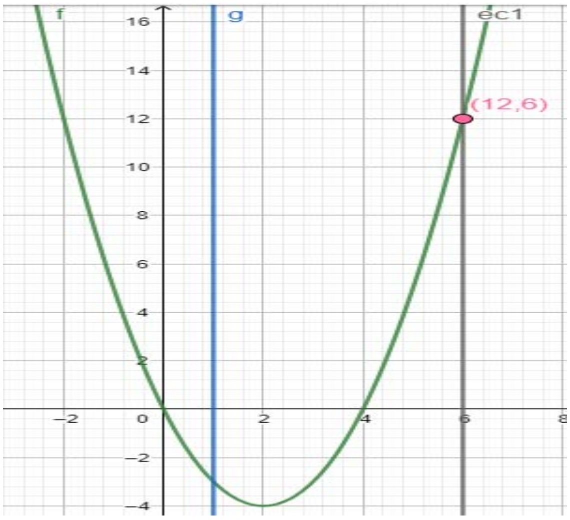
3. Igualamos el resultado a -12:

$$-b-1=-12 \rightarrow -b=-12+1 \rightarrow -b=-11 \rightarrow b=11$$

Resultado: b=11

11. Halla el área entre la función $f(x)=x^2-4x$, el eje de abscisas, y las rectas x=1 y x=6.

1. Hacemos el gráfico:



2. Hallamos los cortes con el eje x de la función:

$$f(x)=x^2-4x \rightarrow x^2-4x=0 \rightarrow x(x-4)=0 \rightarrow x_1=0, \qquad x_2=4$$

3. Hallamos el área de las dos zonas de áreas obtenidas; de x=1 a x=4, y de x=4 a x=6:

$$\int_1^4 (x^2-4x)dx = \left.\frac{x^3}{3}-4\frac{x^2}{2}\right]_1^4 = \left.\frac{x^3}{3}-2x^2\right]_1^4 = \left(\frac{4^3}{3}-2\cdot 4^2\right)-\left(\frac{1^3}{3}-2\cdot 1^2\right)=-9$$

Como es un área tomamos su valor positivo, 9.

$$\int_4^6 (x^2-4x)dx = \left.\frac{x^3}{3}-4\frac{x^2}{2}\right]_4^6 = \left.\frac{x^3}{3}-2x^2\right]_4^6 = \left(\frac{6^3}{3}-2\cdot 6^2\right)-\left(\frac{4^3}{3}-2\cdot 4^2\right)=\frac{32}{3}$$

4. Sumamos ambas áreas:

$$\frac{32}{3}+(9)=\frac{59}{3}\ u.a.$$

Resultado: El área es $\frac{59}{3}$ u.a.

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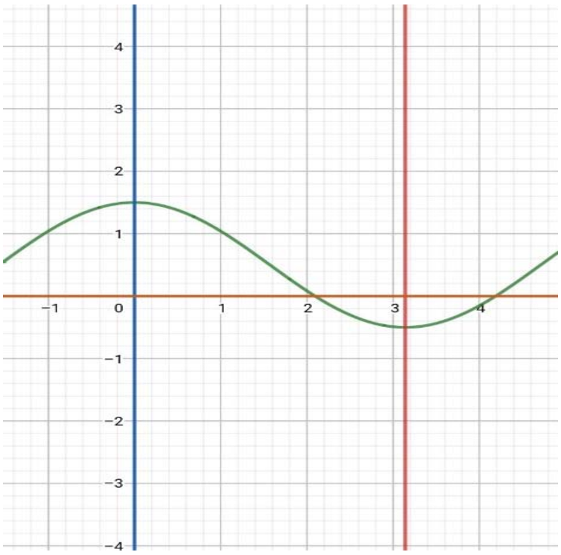
12. Halla el área limitada por la función f(x) = 0,5 + cosx, el eje de abcisas y las rectas x = 0 y x = π.



a=0

b=π

c = $0{,}5+\cos x=0; \quad \cos x=-0{,}5$

$x=arc\cos(-0{,}5)=\frac{2\pi}{3}$

At=A1+A2;

A1= $\int_a^c f(x)dx = \int_0^{\frac{2\pi}{3}}(0{,}5+\cos x)dx = \int_0^{\frac{2\pi}{3}} 0{,}5dx + \int_0^{\frac{2\pi}{3}}\cos x dx = (0{,}5x+\sin x)\begin{cases}\frac{2\pi}{3}\\ 0\end{cases} =$

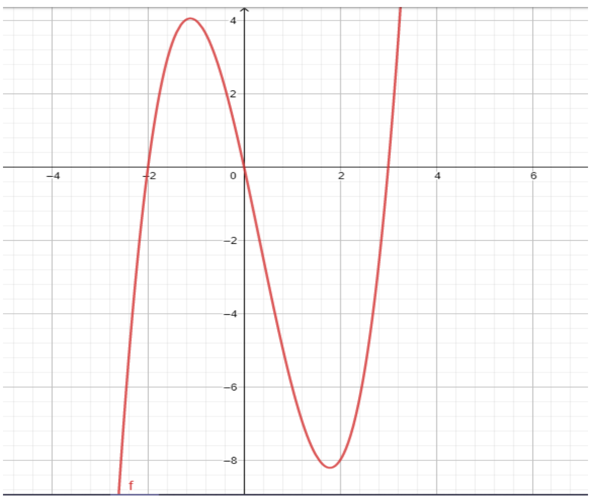
$=\left[0{,}5\left(\frac{2\pi}{3}\right)+\sin\left(\frac{2\pi}{3}\right)\right]-[0{,}5(0)+\sin(0)]=1{,}08-0=1{,}08u^2$

A2=$\left|\int_c^b f(x)dx\right| = \left|\int_{\frac{2\pi}{3}}^{\pi}(0{,}5+\cos x)dx\right| = \quad \left|(0{,}5x+\operatorname{sen} x)\begin{cases}\pi\\ \frac{2\pi}{3}\end{cases}\right| = \left|[0{,}5(\pi)+\operatorname{sen}(\pi)]-\left[0{,}5\left(\frac{2\pi}{3}\right)+\right.\right.$
$\left.\left.\sin(\frac{2\pi}{3})\right]\right| = 0{,}55$

At= 1,08+0,55= 1,63u^2

13. Halla el área de la región limitada por la función $f(x)=x^3-x^2-6x$ y el eje de abscisas.

1. Hacemos el gráfico:



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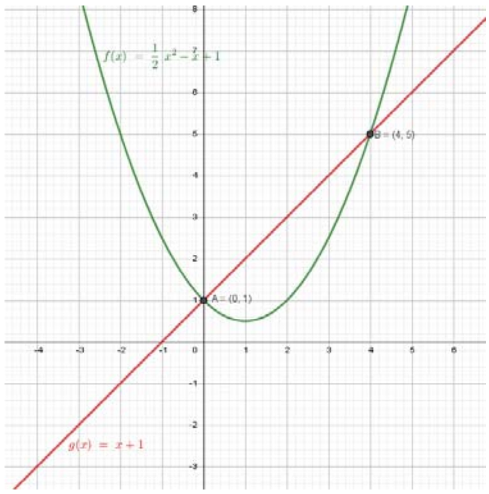
2. Hallamos los cortes con el eje x de la función:
$f(x) = x^3 - x^2 - 6x \rightarrow x^3 - x^2 - 6x = 0 \rightarrow x(x^2 - x - 6) = 0 \rightarrow \quad x_1 = 0, x_2 = -2, x_3 = 3$

3. Hallamos el área de las dos zonas obtenidas; de x=-2 a x=0, y de x=0 a x=3:
$$\int_{-2}^{0}(x^3 - x^2 - 6x)dx - \int_{0}^{3}(x^3 - x^2 - 6x)dx =$$
$$\left.\frac{x^4}{4} - \frac{x^3}{3} - 3x^2\right]_{-2}^{0} - \left.\left(\frac{x^4}{4} - \frac{x^3}{3} - 3x^2\right)\right]_{0}^{3} =$$
$$\left(\frac{(-2)^4}{4} - \frac{(-2)^3}{3} - 3\cdot(-2)^2\right) - \left(\frac{3^4}{4} - \frac{3^3}{3} - 3\cdot 3^2\right) = 10{,}42u^2$$

Resultado: El área es $\frac{125}{12}$ u.a.

14. Calcula el área de la porción del plano que limitan las curvas $y = \frac{1}{2}x^2 - x + 1$ e y – x – 1 = 0



$$\text{Área} = \int_0^4 \left[(x+1) - \left(\frac{1}{2}x^2 - x + 1\right)\right] dx = \int_0^4 \left(\frac{1}{2}x^2 + 2x\right) dx = \left.\left(\frac{1}{2}\cdot\frac{x^3}{3} + x^2\right)\right]_0^4 = \frac{80}{3} u.a.$$

15. Halla el área delimitada por las gráficas:

a) $(x) = \sqrt{x}$ y $g(x) = x^2$

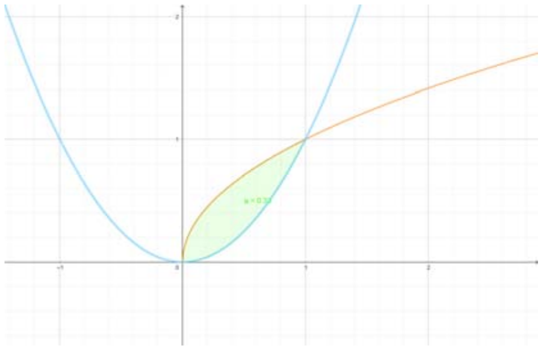
Igualamos f(x) y g(x) para hallar los puntos de corte:

$\sqrt{x} = x^2$; $(\sqrt{x})^2 = (x^2)^2$; $x = x^4$; $x^4 - x = 0$; $x(x^3 - 1) = 0$

$x = 0$

$(x^3 - 1) = 0$; $x^3 = 1$; $x = \sqrt[3]{1}$; $x = 1$

IES ATENEA Ciudad Real
Revisor: Luis Carlos Vidal del Campo
Ilustraciones: Creadas con GeoGebra



Los puntos de corte son x=0 y x=1.

Área=$\int_0^1 (f(x)-g(x))\,dx = \int_0^1 (\sqrt{x}-x^2)\,dx = \int_0^1 (x^{1/2}-x^2)\,dx = \frac{2\sqrt{x^3}}{3}-\frac{x^3}{3}\Big]_0^1 = \frac{2\sqrt{x^3}-x^3}{3}\Big]_0^1 = \left(\frac{2\sqrt{1^3}-1^3}{3}\right)-\left(\frac{2\sqrt{0^3}-0^3}{3}\right) = \left(\frac{1}{3}\right)-(0) = \frac{1}{3}\,u.a.$

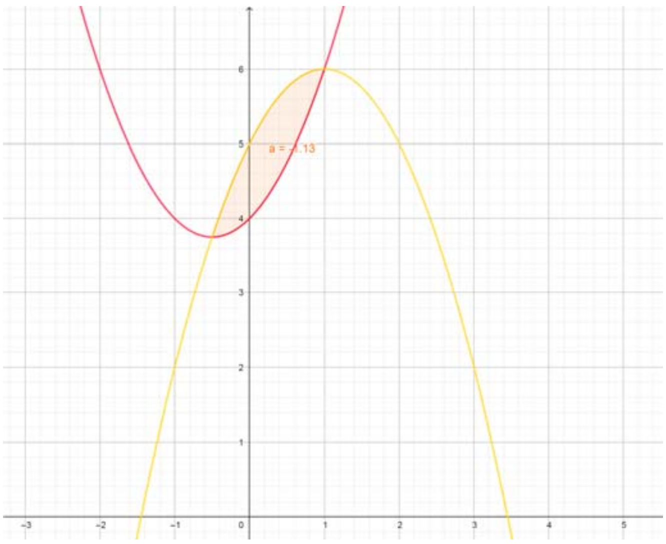
b) $\mathbf{f(x) = x^2 + x + 4 \quad y \quad g(x) = -x^2 + 2x + 5}$

Igualamos f(x) y g(x) para hallar los puntos de corte:

$x^2 + x + 4 = -x^2 + 2x + 5 \quad ; \quad 2x^2 - x - 1 = 0 \quad ; \quad x = 1 \quad ; \quad x = -\frac{1}{2}$

Los puntos de corte son x=1 y x=$-\frac{1}{2}$

Área=$\left|\int_{-\frac{1}{2}}^{1} (f(x)-g(x))dx\right| = \left|\int_{-\frac{1}{2}}^{1} ((x^2+x+4)-(-x^2+2x+5))\,dx\right| = \left|\int_{-\frac{1}{2}}^{1} (x^2+x+4+x^2-2x-5)\,dx\right| = \left|\int_{-\frac{1}{2}}^{1} (2x^2-x-1)\,dx\right| = \left|\frac{2x^3}{3}-\frac{x^2}{2}-x\Big]_{-\frac{1}{2}}^{1}\right| = \left|\left(\frac{2\cdot 1^3}{3}-\frac{1^2}{2}-1\right)-\left(\frac{2\cdot\left(-\frac{1}{2}\right)^3}{3}-\frac{\left(-\frac{1}{2}\right)^2}{2}-\left(-\frac{1}{2}\right)\right)\right| = \left|\left(-\frac{5}{6}\right)-\left(\frac{7}{24}\right)\right| = \left|-\frac{5}{6}-\frac{7}{24}\right| = \left|-\frac{9}{8}\right| = \frac{9}{8}\,u.a. = 1{,}125\,u.a.$



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Ejercicios Autoevaluación

1) Los valores de a, b y c para los que $\mathrm{F}(x) = ax^3 - be^x + c\sin x$ es una primitiva de la función $f(x) = 3x^2 - 7e^x + 5\cos x$ son:

$\mathrm{F}(x) = ax^3 - be^x + c\sin x$
$f(x) = 3x^2 - 7e^x + 5\cos x$

F'(x)= f(x)

$$\mathrm{F}'(x) = 3ax^2 - be^x + c\cos x$$

a= 1 b=7 c=5

La respuesta correcta es la b)

2) La integral indefinida $\int x\sqrt{2x^2+3}\,dx$ vale:

$$\int x\sqrt{2x^2+3}\,dx = \int x(2x^2+3)^{\frac{1}{2}}\,dx = \frac{1}{4}\int 4x(2x^2+3)^{\frac{1}{2}}\,dx =$$

$$\frac{1}{4}\cdot\frac{(2x^2+3)^{\frac{3}{2}}}{\frac{3}{2}} + \mathrm{C} = \frac{\sqrt{(2x^2+3)^3}}{6} + \mathrm{C}$$

La respuesta correcta es la b)

3) La integral $\int \frac{sen2x}{sen^4x + cos^4x}dx$

$$hacemos\ el\ cambio\ t = sen^2x \quad dt = 2senxcosxdx = sen2xdx$$

$$cos^4x = (cos^2x)^2 = (1-sen^2x)^2 = (1-t)^2 = 1-2t+t^2$$

$$\int \frac{\mathrm{sen2x}}{\mathrm{sen^4x+cos^4x}}\mathrm{dx} = \int\frac{\mathrm{dt}}{\mathrm{t^2+1-2t+t^2}} = \int\frac{1}{2\mathrm{t}^2+1-2\mathrm{t}}dt = \int\frac{1}{2\left(\mathrm{t}^2+\frac{1}{2}-\mathrm{t}\right)}\mathrm{dt} =$$

$$\int\frac{1}{2\left(t^2-t+\frac{1}{2}\right)}dt = \int\frac{1}{2\left(t^2-t+\frac{1}{4}+\frac{1}{4}\right)}dt = \int\frac{1}{2\left(\left(t-\frac{1}{2}\right)^2+\frac{1}{4}\right)}dt = \frac{1}{2}\int\frac{1}{\left(t-\frac{1}{2}\right)^2+\frac{1}{4}}dt =$$

$$\frac{1}{2}\cdot\frac{1}{\frac{1}{2}}\arctan\left(\frac{t-\frac{1}{2}}{\frac{1}{2}}\right) = \frac{1}{2}\cdot\frac{1}{\frac{1}{2}}\arctan\left(\frac{sen^2x-\frac{1}{2}}{\frac{1}{2}}\right) = -\boldsymbol{\arctan(\cos 2x) + C}$$

La respuesta correcta a este apartado es la d)

4) Al integrar por partes $\int \frac{x\cdot e^{\mathrm{arcsenx}}}{\sqrt{1-x^2}}dx\ \ se\ obtiene$:

$$\mathrm{x = senu\,,\quad u = arcsenx,\quad du = \frac{1}{\sqrt{1-x^2}}dx}$$

$$\int e^u\sin u\ du = -e^u\cos u - \int(-e^u\cos u)du = -e^u\cos u - \left(-\int e^u\cos u\,du\right)$$

$$= -e^u\cos u - \left(-\left(e^u\sin u - \int e^u\sin u\,du\right)\right)$$

Por lo tanto $\int e^u\sin u\ du = -e^u\cos u - (e^u\sin u - \int e^u\sin u\,du)$

Despejamos $\int e^u\sin u\ du = -\frac{e^u\cos u}{2} + \frac{e^u\sin u}{2}\ \ sustituimos\ en\ u = \mathbf{arcsen\ x}$

$$-\frac{e^{\text{arcsen x}}\cos \text{arcsen x}}{2}+\frac{e^{\text{arcsen x}}\sin \text{arcsen x}}{2}\ simplificamos = -\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{e}^{\textbf{arcsen x}}\left(\sqrt{\mathbf{1}-\boldsymbol{x}^{\mathbf{2}}}-\boldsymbol{x}\right)+\boldsymbol{C}$$

La respuesta a este apartado es la d)

5) La integral $\int \frac{2x+2}{x^2+4x+13}$ vale

$$\int \frac{(2x+4)-2}{x^2+4x+13}dx = \int \frac{(2x+4)}{x^2+4x+13} - \frac{2}{x^2+4x+13}dx = \int \frac{(2x+4)}{x^2+4x+13}dx - \int \frac{2}{(x+2)^2+9}dx =$$

$$= \mathbf{ln}(\boldsymbol{x}^{\mathbf{2}}+\mathbf{4}\boldsymbol{x}+\mathbf{13})-\frac{\mathbf{2}}{\mathbf{3}}\mathbf{arctan}\frac{\boldsymbol{x}+\mathbf{2}}{\mathbf{3}}+\boldsymbol{C}$$

La respuesta correcta a este apartado es la a)

6) La integral $\int \frac{dx}{sen^2xcos^2x}$ vale

$\mathrm{t = tgx}$	$\mathrm{sen\ x} = \frac{\mathrm{t}}{\sqrt{\mathrm{t}^2+1}}$	$\mathrm{sen}^2\mathrm{x} = \frac{\mathrm{t}^2}{\mathrm{t}^2+1}$
$\mathrm{dx} = \frac{\mathrm{dt}}{1+\mathrm{t}^2}$	$\cos \mathrm{x} = \frac{1}{\sqrt{\mathrm{t}^2+1}}$	$\cos^2\mathrm{x} = \frac{1}{\mathrm{t}^2+1}$

$$\int \frac{dx}{sen^2xcos^2x} = \int \frac{\frac{\mathrm{dt}}{1+\mathrm{t}^2}}{\frac{\mathrm{t}^2}{\mathrm{t}^2+1}\frac{1}{\mathrm{t}^2+1}} = \int \frac{\mathrm{t}^2+1}{\mathrm{t}^2}dt = \int \left(1+\frac{1}{\mathrm{t}^2}\right)dt = t-\frac{1}{t}+C = tgx - cotgx + C$$

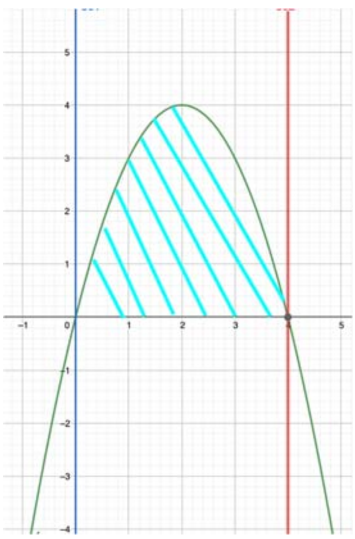
La respuesta a este apartado es la d)

7) La integral definida $\int_0^{\pi} \cos x\, dx$ vale:

$$\int_0^{\pi} \cos x\, dx = \left.\sin x\right]_0^{\pi} = \sin \pi - (\sin 0) = 0$$

La respuesta correcta es la c)

8) Para hallar el área comprendida entre la función $f(x) = -x^2+4x$, el eje de abscisas y las rectas x=0 y x=4, debemos representar dicha función y ver el área que comprende:



Una vez que tenemos la gráfica, y vemos donde corta la función con el eje y con las rectas, comenzamos a aplicar la regla de Barrow para obtener el área.

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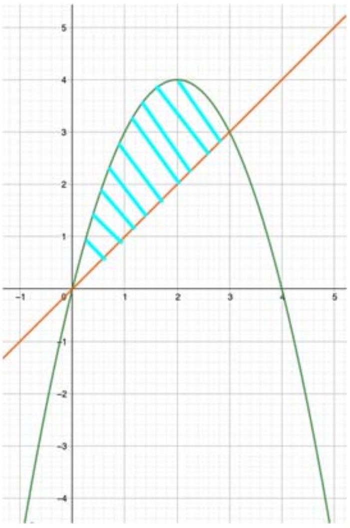
$$\int_0^4 (-x^2 + 4x\,)dx = -\frac{x^3}{3} + \frac{4x^2}{2}\Bigg]_0^4$$

$$\left(-\frac{4^3}{3} + \frac{4 \cdot 4^2}{2}\right) - (0) = \frac{32}{3}$$

La respuesta correcta es la b)

9) Para hallar el área comprendida entre las funciones $f(x) = -x^2 + 4x$ y $g(x) = x$, debemos representar ambas funciones y ver el área que comprenden:



Una vez que tenemos la gráfica, y vemos el dónde corta f(x) con g(x), debemos sacar los puntos de corte y, una vez hallados comenzamos a aplicar la regla de Barrow para obtener el área.

Puntos de corte: Para hallarlos debemos igualar las funciones y despejar la incógnita "x".

$$-x^2 + 4x = x$$

$$0 = x^2 - 3x$$

$$0 = x(x - 3)$$

$$x = 0$$

$$x = 3$$

Ahora ya podemos aplicar la regla de Barrow:

$$\int_0^3 [(-x^2 + 4x) - (x)]\,dx = \int_0^3 (-x^2 + 3x)dx = -\frac{x^3}{3} + \frac{3x^2}{2}\Bigg]_0^3$$

$$\left(-\frac{3^3}{3} + \frac{3 \cdot 3^2}{2}\right) - (0) = \frac{9}{2}$$

La respuesta correcta es la a)

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10) El volumen del sólido de revolución generado por $y = x^2$, entre 0 y 2, al girar en torno al eje de abcisas es:

$$V = \pi \int_0^2 (x^2)^2 dx = \pi \left(\frac{x^5}{5}\right)\Big]_0^2 = \pi \frac{32}{5}$$

La respuesta a este apartado es la d)