MT4113 - Report

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 $I\ confirm\ that\ the\ following\ report\ and\ associated\ code\ is\ my\ own\ work,\ except\ where\ clearly\ indicated.$

Abstract

More than 17 million people voted to leave the EU in 2016 Referendum in the UK. This statistical report investigated the difference of approval rates of Brexit in England and the rest of the UK by carrying out both parametric and non-parametric tests. Particularly, Monte Carlo simulation is conducted on the referendum dataset by regions, to research the properties of those statistical tests under a range of different scenarios. add a bit summary of result

Motivation

Hundreds of thousands of people marched to London's Parliament Square for a referendum on the final Brexit deal in late October(BBC News, 2018). In the EU referendum in 2016, approximately 52%, or more than 17 million people in the UK, voted to leave the European Union (Becker, Fetzer & Novy, 2017). To investigate what voters in different nations of the UK vote for, the dataset of EU referendum results by regions(Electoral Commission, 2016) are statistically analysed and tested in this report. The dateset contains numerous variates regarding the referendum outcome, yet only *Region* and *Percent Leave* are concerned in terms of our research objectives.

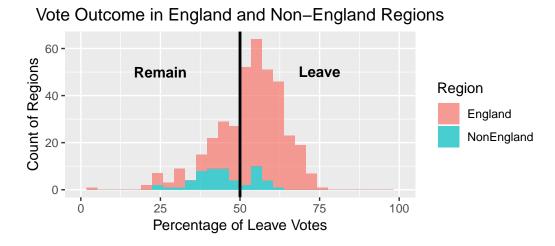


Figure 1: Vote Outcome in England and Non-England Regions

According to the histogram of leave votes, it seems that England regions tend to be more supportive of Brexit than other regions. Hence, The research question is proposed as that

• Were approval rates of Brexit Higher in England than rest of the UK in 2016 referendum?

On the basis of our research question, regions are categorized as England or Non-England, thus, One-tail Student's T test and Mann-Whitney U test are chosen to be conducted to compare the difference of mean approval rates of Brexit. Moreover, 1000-iteration Monte Carlo simulations are carried out under a range of scenarios in order to investigate power and size of these tests.

Methodology

Data Exploration and Tests Specification

By organizing and exploring dataset *Referendum*, the properties of leave vote rates are summarized in table 1. It shows that the Brexit approval rate in England regions was approximate 10% higher than in Non-England regions on average. In general, England tended to leave the EU with leave vote rate more than a half, while other nations tended to remain with leave vote rate less than 50%.

Table 1: Summary of Leave Vote Rates in Regions of England and Non-England

| Region | n | mean | sd |
|-----------------------|-----------|----------------------|-----------------------|
| England NonEngland | 327 55 | 54.34725 44.91364 | 10.382207 8.884886 |
| | | | |

What is concerned in this research is whether the leave vote rates in England regions *higher* than in Non-England regions, which involves two groups of samples. Therefore, the statistical tests to be conducted should be one-tailed test for two groups. The chosen tests are

• Parametric: One-tailed Student's t Test

• Non-parametric: One-tailed Mann-Whitney U Test

The null Hypothesis and alternative hypothesis are

• H_0 : $\mu_e - \mu_o \le 0$

• H_1 : $\mu_e - \mu_o > 0$

where μ_e is the mean leave vote rate in England, and μ_o is the mean leave vote rate in the rest of the UK.

Scenario Design

Scenario design and data simulation are grounded on the properties shown in table 1 in previous section.

To investigate the *power* of two tests, the difference of mean leave vote rates, a.k.a. the effect size, are set to be positive, where H_0 is false. The specific effect sizes of simulating data are chosen to be 1%, 3%, 6%, 10%, and 15%, covering scenarios under which the mean differences are less than, close to, or greater than the actual mean difference 9.4%. The steps between those effect sizes are incremental, because it is speculated that the power of a test is more responsive in smaller scale. The standard deviation are set to be 10% for England group and 8% for Non-England group, which is close to the true properties.

To investigate the *size* of two tests, the effect sizes are set to be 0, where H_0 is true. There are two groups of scenarios, one simulated with all percentage records rounded to 0, another as default. For each scenario of one group, the difference of standard deviation are set to be 0%, 3%, 5%, 10%.

For each attributes stated above, 10 datasets of different sample sizes from 10 to 1000 are generated to be test.

Power and Size

Two functions are created to complete the workflow. Function simulating() generates a dataset given conditions of scenario, e.g. sample size, mean and standard deviation of two groups of datasets. Function MonteCarlo() invokes simulating(), carry out specific test, store the p-value, by doing all these 1000 iterations it finally return the proportion of rejecting H_0 . The number of iteration, parametric or non-parametric test to be conducted are also arbitrary and can be specified as inputs. Also, both functions have a seed argument for reproducibility purpose.

A 10 by 5 power matrix is evaluated for T test and Mann-Whitney U Test respectively. 10 rows are for 10 different sample sizes, 5 columns are for different effect sizes. Similarly, two 10 by 8 size matrices are evaluated for two test, 8 columns are for 4 differences of standard deviation and round to 0 or not.

Result and Discussion

The power of parametric and non-parametric tests, in terms of different sample sizes and effect sizes, are calculated based on 1000 simulated datasets, and the result are shown in two tables below.

Table 2: Power of Student's T Test

| Sample Size | Effect Size | | | | | | | |
|-------------|-------------|-------|------|-------|----|--|--|--|
| | 1 | 3 | 6 | 10 | 15 | | | |
| 10 | 0.000 | 0.009 | 0.32 | 0.953 | 1 | | | |
| 50 | 0.000 | 0.510 | 1.00 | 1.000 | 1 | | | |
| 100 | 0.000 | 1.000 | 1.00 | 1.000 | 1 | | | |
| 150 | 0.000 | 1.000 | 1.00 | 1.000 | 1 | | | |
| 200 | 0.001 | 1.000 | 1.00 | 1.000 | 1 | | | |
| 300 | 0.047 | 1.000 | 1.00 | 1.000 | 1 | | | |
| 400 | 0.296 | 1.000 | 1.00 | 1.000 | 1 | | | |
| 500 | 0.720 | 1.000 | 1.00 | 1.000 | 1 | | | |
| 700 | 0.995 | 1.000 | 1.00 | 1.000 | 1 | | | |
| 1000 | 1.000 | 1.000 | 1.00 | 1.000 | 1 | | | |

Table 3: Power of Mann-Whitney U Test

| Sample Size | Effect Size | | | | | | | |
|-------------|-------------|-------|-------|------|----|--|--|--|
| | 1 | 3 | 6 | 10 | 15 | | | |
| 10 | 0.001 | 0.016 | 0.358 | 0.92 | 1 | | | |
| 50 | 0.000 | 0.565 | 1.000 | 1.00 | 1 | | | |
| 100 | 0.000 | 0.998 | 1.000 | 1.00 | 1 | | | |
| 150 | 0.003 | 1.000 | 1.000 | 1.00 | 1 | | | |
| 200 | 0.009 | 1.000 | 1.000 | 1.00 | 1 | | | |
| 300 | 0.072 | 1.000 | 1.000 | 1.00 | 1 | | | |
| 400 | 0.305 | 1.000 | 1.000 | 1.00 | 1 | | | |
| 500 | 0.634 | 1.000 | 1.000 | 1.00 | 1 | | | |
| 700 | 0.975 | 1.000 | 1.000 | 1.00 | 1 | | | |
| 1000 | 1.000 | 1.000 | 1.000 | 1.00 | 1 | | | |

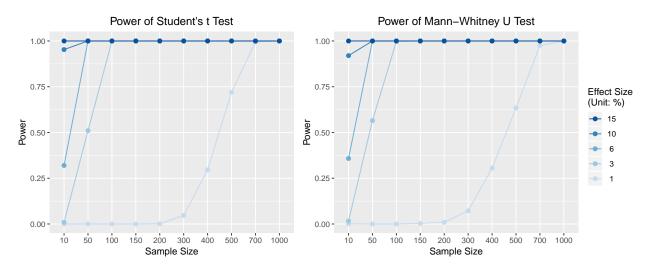


Figure 2: Power of Parametric and Non-parametric Tests under Different Scenarios

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Table 4: Size of Student's T Test

| Sample Size | SD Difference | | | | SD Difference(record round to 0) | | | |
|-------------|---------------|-------|-------|-------|----------------------------------|-------|-------|-------|
| | 0 | 3 | 5 | 10 | 0 | 3 | 5 | 10 |
| 10 | 0 | 0.017 | 0.025 | 0.032 | 0 | 0.014 | 0.025 | 0.035 |
| 50 | 0 | 0.009 | 0.022 | 0.034 | 0 | 0.011 | 0.017 | 0.035 |
| 100 | 0 | 0.012 | 0.020 | 0.023 | 0 | 0.014 | 0.020 | 0.024 |
| 150 | 0 | 0.019 | 0.027 | 0.031 | 0 | 0.019 | 0.026 | 0.030 |
| 200 | 0 | 0.009 | 0.021 | 0.025 | 0 | 0.010 | 0.017 | 0.028 |
| 300 | 0 | 0.010 | 0.022 | 0.034 | 0 | 0.011 | 0.025 | 0.036 |
| 400 | 0 | 0.019 | 0.029 | 0.038 | 0 | 0.018 | 0.030 | 0.041 |
| 500 | 0 | 0.016 | 0.025 | 0.039 | 0 | 0.015 | 0.025 | 0.038 |
| 700 | 0 | 0.013 | 0.022 | 0.034 | 0 | 0.013 | 0.020 | 0.033 |
| 1000 | 0 | 0.015 | 0.029 | 0.036 | 0 | 0.013 | 0.028 | 0.036 |

Table 5: Size of Mann-Whitney U Test

| Sample Size | SD Difference | | | | SD Difference(record round to 0) | | | |
|-------------|---------------|-------|-------|-------|----------------------------------|-------|-------|-------|
| | 0 | 3 | 5 | 10 | 0 | 3 | 5 | 10 |
| 10 | 0 | 0.029 | 0.041 | 0.049 | 0 | 0.031 | 0.047 | 0.057 |
| 50 | 0 | 0.028 | 0.046 | 0.057 | 0 | 0.031 | 0.047 | 0.058 |
| 100 | 0 | 0.028 | 0.048 | 0.065 | 0 | 0.029 | 0.045 | 0.070 |
| 150 | 0 | 0.033 | 0.049 | 0.057 | 0 | 0.032 | 0.045 | 0.057 |
| 200 | 0 | 0.026 | 0.045 | 0.061 | 0 | 0.028 | 0.047 | 0.061 |
| 300 | 0 | 0.036 | 0.051 | 0.071 | 0 | 0.034 | 0.056 | 0.075 |
| 400 | 0 | 0.043 | 0.064 | 0.080 | 0 | 0.044 | 0.064 | 0.080 |
| 500 | 0 | 0.034 | 0.051 | 0.066 | 0 | 0.027 | 0.051 | 0.069 |
| 700 | 0 | 0.029 | 0.048 | 0.057 | 0 | 0.034 | 0.050 | 0.062 |
| 1000 | 0 | 0.038 | 0.054 | 0.071 | 0 | 0.037 | 0.056 | 0.070 |

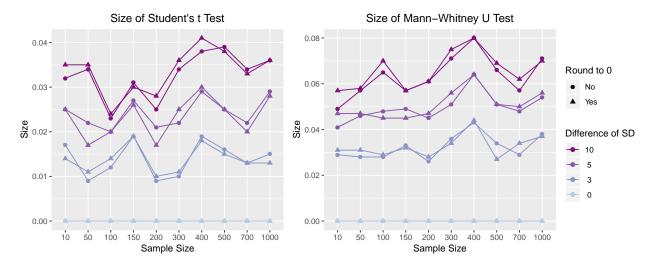


Figure 3: Size of Parametric and Non-parametric Tests Under Different Scenarios

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Conclusion

Here to draw conclusions! Here to draw conclusions! Here to draw conclusions! Here to draw conclusions!

Reference

BBC News (2018) People's Vote march: Hundreds of thousands attend London protest. [Online]. Available from: https://www.bbc.co.uk/news/uk-45925542.

Becker, S.O., Fetzer, T. & Novy, D. (2017) Who voted for Brexit? A comprehensive district-level analysis. *Economic Policy*. [Online] Available from: doi:10.1093/epolic/eix012.

Electoral Commission (2016) 2016 EU Referendum in the United Kingdom. [Online]. Available from: https://www.kaggle.com/electoralcommission/brexit-results.

Appendix