

MT4113: Assignment 3, Monte Carlo Simulation

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I confirm that the following report and associated code is my own work, except where clearly indicated.

1 Abstract

More than 17 million people voted to leave the EU in 2016 Referendum in the UK. This statistical report investigated the difference of approval rates of Brexit in England and the rest of the UK by carrying out both parametric and non-parametric tests. Particularly, Monte Carlo simulation is conducted on the referendum dataset by regions, to research the properties of those statistical tests under a range of different scenarios.

2 Motivation

Hundreds of thousands of people marched to London's Parliament Square for a referendum on the final Brexit deal in late October(BBC News, 2018). In the EU referendum in 2016, approximately 52%, or more than 17 million people in the UK, voted to leave the European Union (Becker, Fetzter & Novy, 2017). To investigate what voters in different nations of the UK vote for, the dataset of EU referendum results by regions(Electoral Commission, 2016) are statistically analysed and tested in this report. The dataset contains numerous variates regarding the referendum outcome, yet only *Region* and *Percent Leave* are concerned in terms of our research objectives.

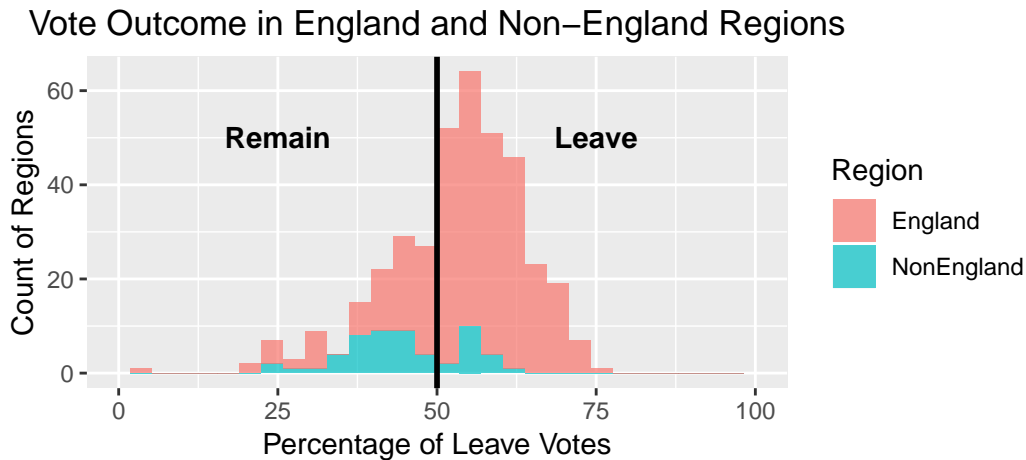


Figure 1: Vote Outcome in England and Non-England Regions

According to the histogram of leave votes, it seems that England regions tend to be more supportive of Brexit than other regions. Hence, The research question is proposed as that

- *Were approval rates of Brexit Higher in England than rest of the UK in 2016 referendum?*

On the basis of our research question, regions are categorized as England or Non-England, thus, One-tail Student’s T test and Mann-Whitney U test are chosen to be conducted to compare the difference of mean approval rates of Brexit. Moreover, 1000-iteration Monte Carlo simulations are carried out under a range of scenarios in order to investigate power and size of these tests.

3 Methodology

3.1 Data Exploration and Tests Specification

By organizing and exploring dataset *Referendum*, the properties of leave vote rates are summarized in table 1. It shows that the Brexit approval rate in England regions was approximate 10% higher than in Non-England regions on average. In general, England tended to leave the EU with leave vote rate more than a half, while other nations tended to remain with leave vote rate less than 50%.

Table 1: Summary of Leave Vote Rates in Regions of England and Non-England

Region	n	mean	sd
England	327	54.35	10.38
NonEngland	55	44.91	8.88

What is concerned in this research is whether the leave vote rates in England regions *higher* than in Non-England regions, which involves two groups of samples. Therefore, the statistical tests to be conducted should be one-tailed test for two groups. The chosen tests are

- Parametric: One-tailed Student’s t Test
- Non-parametric: One-tailed Mann-Whitney U Test

The null Hypothesis and alternative hypothesis are

- $H_0: \mu_e - \mu_o \leq 0$
- $H_1: \mu_e - \mu_o > 0$

where μ_e is the mean leave vote rate in England, and μ_o is the mean leave vote rate in the rest of the UK.

3.2 Scenario Design

Scenario design and data simulation are grounded on the properties shown in table 1 in previous section.

To investigate the *power* of two tests, the difference of mean leave vote rates, a.k.a. the effect size, are set to be positive, where H_0 is false. The specific effect sizes of simulating data are chosen to be 1%, 3%, 6%, 10%, and 15%, covering scenarios under which the mean differences are less than, close to, or greater than the actual mean difference 9.4%. The steps between those effect sizes are incremental, because it is speculated that the power of a test is more responsive in smaller scale. The standard deviation are set to be 10% for England group and 8% for Non-England group, which is close to the true properties.

To investigate the *size* of two tests, the effect sizes are set to be 0, where H_0 is true. There are two groups of scenarios, one simulated with all percentage records rounded to 0, another as default. For each scenario of one group, the difference of standard deviation are set to be 0%, 3%, 5%, 10%.

For each attributes stated above, 10 datasets of different sample sizes from 10 to 1000 are generated to be test.

3.3 Monte Carlo Simulation

Two functions are created to complete the workflow. Function `simulating()` generates a dataset given conditions of scenario, e.g. sample size, mean and standard deviation of two groups of datasets. Function `MonteCarlo()` invokes `simulating()`, carries out specific test and stores the p -value, by doing all these 1000 iterations it finally return the proportion of rejecting H_0 . The number of iteration, parametric or non-parametric test to be conducted are also arbitrary and can be specified as inputs. Also, both functions have a *seed* argument for reproducibility purpose.

A 10 by 5 *power* matrix is evaluated for t test and Mann-Whitney U test respectively. 10 rows are for 10 different sample sizes, 5 columns are for different effect sizes. Similarly, two 10 by 8 *size* matrices are evaluated for two tests, 8 columns are for 4 differences of standard deviation and round to 0 or not.

4 Result and Discussion

Each cell of the *power* matrix is calculated based on 1000 times of test, and the outcomes for are t test and Mann-Whitney U test are respectively shown in table 2 and table 3.

Table 2: Power of Student's T Test

Sample Size	Effect Size (Unit: %)				
	1	3	6	10	15
10	0.000	0.009	0.32	0.953	1
50	0.000	0.510	1.00	1.000	1
100	0.000	1.000	1.00	1.000	1
150	0.000	1.000	1.00	1.000	1
200	0.001	1.000	1.00	1.000	1
300	0.047	1.000	1.00	1.000	1
400	0.296	1.000	1.00	1.000	1
500	0.720	1.000	1.00	1.000	1
700	0.995	1.000	1.00	1.000	1
1000	1.000	1.000	1.00	1.000	1

Table 3: Power of Mann-Whitney U Test

Sample Size	Effect Size (Unit: %)				
	1	3	6	10	15
10	0.001	0.016	0.358	0.92	1
50	0.000	0.565	1.000	1.00	1
100	0.000	0.998	1.000	1.00	1
150	0.003	1.000	1.000	1.00	1
200	0.009	1.000	1.000	1.00	1
300	0.072	1.000	1.000	1.00	1
400	0.305	1.000	1.000	1.00	1
500	0.634	1.000	1.000	1.00	1
700	0.975	1.000	1.000	1.00	1
1000	1.000	1.000	1.000	1.00	1

The result suggests that, for both parametric and non-parametric tests, small effect size like 1 results in low power, even 0 power. With sample size increasing, the power of test also increases. Under scenarios with small sample size, the power of test is positively related to the effect size. Even sample size is as small as 10, the power can be appreciably high when effect size reach 10 or greater.

By plotting the power over sample size(fig. 2), it is clear to see that sample size is also positively related to power of tests. Besides, the plots indicate that there is no sensible power difference between parametric and non-parametric tests under same scenario. The sample sizes of two groups in the true dataset are respectively 327 and 55, and the mean difference of two samples are round 10. In this case, both t test and Mann-Whitney test have significant power estimates.

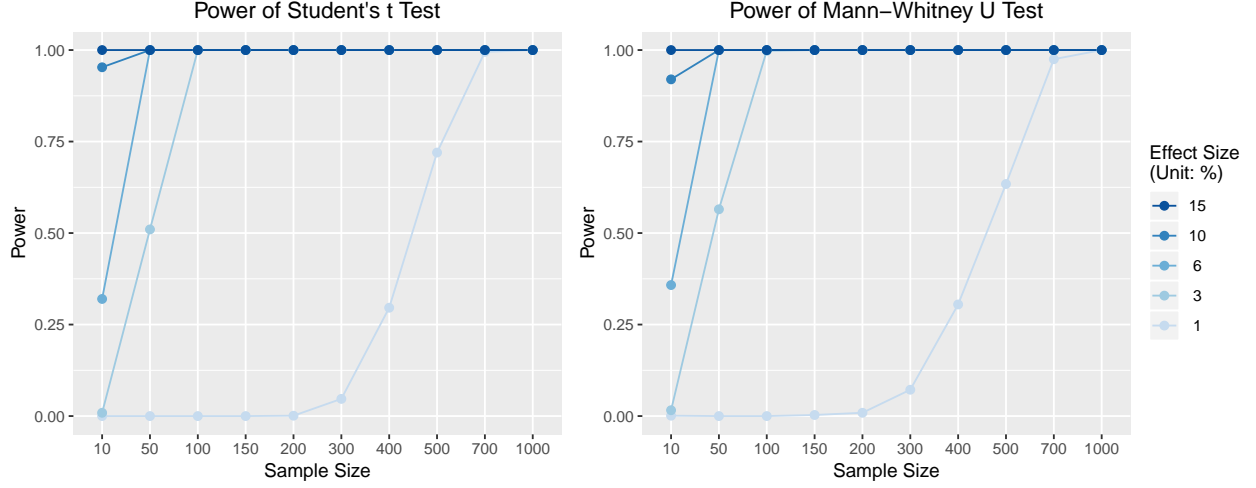


Figure 2: Power of Parametric and Non-parametric Tests under Different Scenarios

Likewise, each cell of the *size* matrix is calculated based on 1000 times of test, and the outcomes for are t test and Mann-Whitney U test are respectively shown in table 4 and 5. The *size* matrix suggest that, the larger the difference of standard deviation is, the higher the sizes of tests are, given identical sample size. When the difference is 0, the size is always zero.

Table 4: Size of Student's T Test

Sample Size	SD Difference				SD Difference(rounded)			
	0	3	5	10	0	3	5	10
10	0	0.017	0.025	0.032	0	0.014	0.025	0.035
50	0	0.009	0.022	0.034	0	0.011	0.017	0.035
100	0	0.012	0.020	0.023	0	0.014	0.020	0.024
150	0	0.019	0.027	0.031	0	0.019	0.026	0.030
200	0	0.009	0.021	0.025	0	0.010	0.017	0.028
300	0	0.010	0.022	0.034	0	0.011	0.025	0.036
400	0	0.019	0.029	0.038	0	0.018	0.030	0.041
500	0	0.016	0.025	0.039	0	0.015	0.025	0.038
700	0	0.013	0.022	0.034	0	0.013	0.020	0.033
1000	0	0.015	0.029	0.036	0	0.013	0.028	0.036

Table 5: Size of Mann-Whitney U Test

Sample Size	SD Difference				SD Difference(rounded)			
	0	3	5	10	0	3	5	10
10	0	0.029	0.041	0.049	0	0.031	0.047	0.057
50	0	0.028	0.046	0.057	0	0.031	0.047	0.058
100	0	0.028	0.048	0.065	0	0.029	0.045	0.070
150	0	0.033	0.049	0.057	0	0.032	0.045	0.057
200	0	0.026	0.045	0.061	0	0.028	0.047	0.061
300	0	0.036	0.051	0.071	0	0.034	0.056	0.075
400	0	0.043	0.064	0.080	0	0.044	0.064	0.080

Table 5: Size of Mann-Whitney U Test (*continued*)

Sample Size	SD Difference				SD Difference(rounded)			
	0	3	5	10	0	3	5	10
500	0	0.034	0.051	0.066	0	0.027	0.051	0.069
700	0	0.029	0.048	0.057	0	0.034	0.050	0.062
1000	0	0.038	0.054	0.071	0	0.037	0.056	0.070

By visualizing the two verbose tables(see fig. 3), it is obvious that, in contrast to power, the size of test does not vary over sample size. The true difference of sample standard deviations is 1.5%, falling between 0% and 3%, resulting relatively low size for both tests. Also, there is no distinct effect of rounding percentage of vote rate to integer. However, the different scale of y-axis is one thing worth noticing, which indicates that the size of Mann-Whitney test is greater than t test in general.

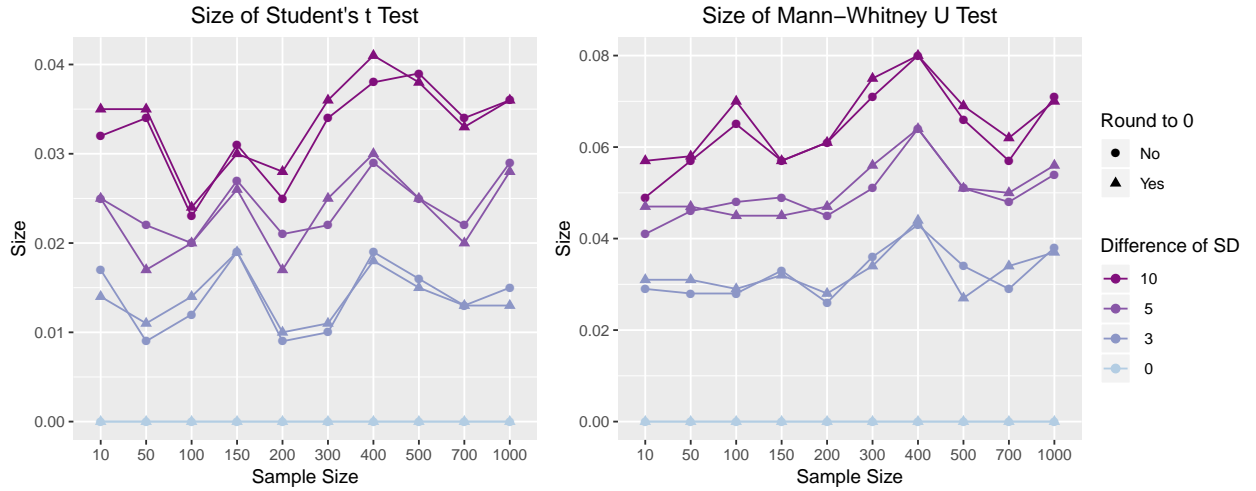


Figure 3: Size of Parametric and Non-parametric Tests Under Different Scenarios

In summary, effect size and variance have considerable impacts on power and size of tests, but there is not much can be done to the properties of population. Nevertheless, the result analysis does indicate that larger sample size leads to greater power of test, and parametric test shows better performance with smaller size under identical scenario.

5 Conclusion

Both t test and Mann-Whitney U test rejected the null hypothesis, and these tests has convinible power and size according to the result analysis of Monte Carlo simulation. We can statistically infer that the approval rates of Brexit were higher in England than rest of the UK in 2016 referendum.

Generally, we can also conclude that it is prudent to enlarge the sample size to reach higher power of test and choose parametric test when we can assume the distribution of population.

6 Reference

BBC News (2018) *People's Vote march: Hundreds of thousands attend London protest*. [Online]. Available from: <https://www.bbc.co.uk/news/uk-45925542>.

Becker, S.O., Fetzer, T. & Novy, D. (2017) Who voted for Brexit? A comprehensive district-level analysis. *Economic Policy*. [Online] Available from: [doi:10.1093/epolic/eix012](https://doi.org/10.1093/epolic/eix012).

Electoral Commission (2016) *2016 EU Referendum in the United Kingdom*. [Online]. Available from: <https://www.kaggle.com/electoralcommission/brexit-results>.

7 Appendix