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Problem 1

1. Let $\mathbf{u} = (1, -2, 4)$ and $\mathbf{v} = (3, 5, 1)$ and $\mathbf{w} = (2, 1, -3)$ find:

a. $\mathbf{3u} - \mathbf{2v}$

**3u** - **2v**$= 3(1, -2, 4) - 2(3, 5, 1) = (3, -6, 12) + (-6, -10, -2) = (-3, -16, 10)$

b. $\mathbf{5u} + \mathbf{3v} - \mathbf{4w}$

**5u** + **3v** - **4w**$= 5(1, -2, 4) + 3(3, 5, 1) - 4(2, 1, -3) = (5, -10, 20) + (9, 15, 3) + (-8, -4, 12) = (6, 1, 35)$

c. $\mathbf{u} \cdot \mathbf{v}, \mathbf{v} \cdot \mathbf{w}, \mathbf{u} \cdot \mathbf{w}$

$\mathbf{u} \cdot \mathbf{v} = 1(3) - 2(5) + 4(1) = -3$
$\mathbf{v} \cdot \mathbf{w} = 3(2) + 5(1) + 1(-3) = 8$
$\mathbf{u} \cdot \mathbf{w} = 1(2) - 2(1) + 4(-3) = -12$

d. $||\mathbf{u}||, ||\mathbf{v}||, ||\mathbf{w}||$

$||\mathbf{u}|| = \sqrt{(1)^2 + (-2)^2 + (4)^2} = \sqrt{21}$
$||\mathbf{v}|| = \sqrt{(3)^2 + (5)^2 + (1)^2} = \sqrt{35}$
$||\mathbf{w}|| = \sqrt{(2)^2 + (1)^2 + (-3)^2} = \sqrt{14}$

e. If $\theta$ is the angle between $\mathbf{u}$ and $\mathbf{v}$, find $\cos\theta$.

$\cos\theta = \frac{1(3)-2(5)+4(1)}{\sqrt{(1)^2+(-2)^2+(4)^2}\sqrt{(3)^2+(5)^2+(1)^2}} = \frac{-3}{\sqrt{21}\sqrt{35}}$

f. If $\alpha$ is the angle between $\mathbf{v}$ and $\mathbf{w}$, find $\cos\alpha$.

$\cos\alpha = \frac{3(2)+5(1)+1(-3)}{\sqrt{(3)^2+(5)^2+(1)^2}\sqrt{(2)^2+(1)^2+(-3)^2}} = \frac{8}{\sqrt{35}\sqrt{14}}$

g. If $\beta$ is the angle between $\mathbf{u}$ and $\mathbf{w}$, find $\cos\beta$.

$\cos\beta = \frac{1(2)-2(1)+4(-3)}{\sqrt{(1)^2+(-2)^2+(4)^2}\sqrt{(2)^2+(1)^2+(-3)^2}} = \frac{-12}{\sqrt{21}\sqrt{14}}$

h. Find $\mathbf{d}(\mathbf{u},\mathbf{v})$.

$\mathbf{d}(\mathbf{u},\mathbf{v}) = \sqrt{(1-3)^2+(-2-5)^2+(4-1)^2} = \sqrt{4+49+9} = \sqrt{62}$

i. Find $\mathbf{d}(\mathbf{w},\mathbf{v})$.

$\mathbf{d}(\mathbf{w},\mathbf{v}) = \sqrt{(2-3)^2+(1-5)^2+(-3-1)^2} = \sqrt{1+16+16} = \sqrt{33}$

j. Find $\mathbf{d}(\mathbf{u},\mathbf{w})$.

$\mathbf{d}(\mathbf{u},\mathbf{w}) = \sqrt{(1-2)^2+(-2-1)^2+(4-(-3))^2} = \sqrt{1+9+49} = \sqrt{59}$

k. Find $\mathbf{proj}(\mathbf{u},\mathbf{v})$.

$\mathbf{proj}(\mathbf{u},\mathbf{v}) = \frac{1(3)-2(5)+4(1)}{\left(\sqrt{(3)^2+(5)^2+(1)^2}\right)^2}(3,5,1) = \frac{-3}{35}(3,5,1) = (\frac{-9}{35},\frac{-15}{35},\frac{-3}{35})$

l. Find $\mathbf{proj}(\mathbf{w},\mathbf{v})$.

$\mathbf{proj}(\mathbf{w},\mathbf{v}) = \frac{2(3)+1(5)-3(1)}{\left(\sqrt{(3)^2+(5)^2+(1)^2}\right)^2}(3,5,1) = \frac{8}{35}(3,5,1) = (\frac{24}{35},\frac{40}{35},\frac{8}{35}) = (\frac{24}{35},\frac{8}{7},\frac{8}{35})$

m. Find $\mathbf{proj}(\mathbf{u},\mathbf{w})$.

$\mathbf{proj}(\mathbf{u},\mathbf{w}) = \frac{1(2)-2(1)+4(-3)}{\left(\sqrt{(2)^2+(1)^2+(-3)^2}\right)^2}(2,1,-3) = \frac{-12}{14}(2,1,-3) = \frac{-6}{7}(2,1,-3) =$
$(\frac{-12}{7},\frac{-6}{7},\frac{18}{7})$

Problem 2

1. Let $\mathbf{u} = \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix}$

a. $\mathbf{3u - 2v}$

$$= 3 \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix} - 2 \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 9 \\ -12 \end{bmatrix} + \begin{bmatrix} -4 \\ -2 \\ -10 \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \\ -22 \end{bmatrix}$$

b. $\mathbf{5u + 3v - 4w}$

$$= 5 \begin{bmatrix} 1 \\ 3 \\ -4 \end{bmatrix} + 3 \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} - 4 \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \\ -20 \end{bmatrix} + \begin{bmatrix} 6 \\ 3 \\ 15 \end{bmatrix} + \begin{bmatrix} -12 \\ 8 \\ -24 \end{bmatrix} = \begin{bmatrix} -1 \\ 26 \\ -29 \end{bmatrix}$$

c. $\mathbf{u \cdot v}, \mathbf{v \cdot w}, \mathbf{u \cdot w}$

$\mathbf{u} \cdot \mathbf{v} = 1(2) + 3(1) - 4(5) = -15$
$\mathbf{v} \cdot \mathbf{w} = 2(3) + 1(-2) + 5(6) = 34$
$\mathbf{u} \cdot \mathbf{w} = 1(3) + 3(-2) - 4(6) = -27$

d. $||\mathbf{u}||, ||\mathbf{v}||, ||\mathbf{w}||$

$||\mathbf{u}|| = \sqrt{(1)^2 + (3)^2 + (4)^2} = \sqrt{26}$
$||\mathbf{v}|| = \sqrt{(2)^2 + (1)^2 + (5)^2} = \sqrt{30}$
$||\mathbf{w}|| = \sqrt{(3)^2 + (-2)^2 + (6)^2} = \sqrt{49} = 7$

e. If $\theta$ is the angle between $\mathbf{u}$ and $\mathbf{v}$, find $\cos\theta$.

$$\cos\theta = \frac{1(2)+3(1)-4(5)}{\sqrt{(1)^2+(3)^2+(4)^2}\sqrt{(2)^2+(1)^2+(5)^2}} = \frac{-15}{\sqrt{26}\sqrt{30}}$$

f. If $\alpha$ is the angle between $\mathbf{v}$ and $\mathbf{w}$, find $\cos\alpha$.

$$\cos\alpha = \frac{2(3)+1(-2)+5(6)}{\sqrt{(2)^2+(1)^2+(5)^2}\sqrt{(3)^2+(-2)^2+(6)^2}} = \frac{34}{\sqrt{30}(7)}$$

g. If $\beta$ is the angle between $\mathbf{u}$ and $\mathbf{w}$, find $\cos\beta$.

$\cos\beta = \frac{1(3)+3(-2)-4(6)}{\sqrt{(1)^2+(3)^2+(4)^2}\sqrt{(3)^2+(-2)^2+(6)^2}} = \frac{-27}{\sqrt{26}(7)}$

h. Find $\mathbf{d}(\mathbf{u},\mathbf{v})$.

$\mathbf{d}(\mathbf{u},\mathbf{v}) = \sqrt{(1-2)^2+(3-1)^2+(-4-5)^2} = \sqrt{1+4+81} = \sqrt{86}$

i. Find $\mathbf{d}(\mathbf{w},\mathbf{v})$.

$\mathbf{d}(\mathbf{w},\mathbf{v}) = \sqrt{(3-2)^2+(-2-1)^2+(6-5)^2} = \sqrt{1+9+1} = \sqrt{11}$

j. Find $\mathbf{d}(\mathbf{u},\mathbf{w})$.

$\mathbf{d}(\mathbf{u},\mathbf{w}) = \sqrt{(1-3)^2+(3-(-2))^2+(-4-6)^2} = \sqrt{4+25+100} = \sqrt{129}$

k. Find $\mathbf{proj}(\mathbf{u},\mathbf{v})$.

$\mathbf{proj}(\mathbf{u},\mathbf{v}) = \frac{1(2)+3(1)-4(5)}{\left(\sqrt{(2)^2+(1)^2+(5)^2}\right)^2}(2,1,5) = \frac{-15}{30}(2,1,5) = \frac{-1}{2}(2,1,5) = (-1,\frac{-1}{2},\frac{-5}{2})$

l. Find $\mathbf{proj}(\mathbf{w},\mathbf{v})$.

$\mathbf{proj}(\mathbf{w},\mathbf{v}) = \frac{3(2)-2(1)+6(5)}{\left(\sqrt{(2)^2+(1)^2+(5)^2}\right)^2}(2,1,5) = \frac{34}{30}(2,1,5) = \frac{17}{15}(2,1,5) = (\frac{34}{15},\frac{17}{15},\frac{85}{15})$

m. Find $\mathbf{proj}(\mathbf{u},\mathbf{w})$.

$\mathbf{proj}(\mathbf{u},\mathbf{w}) = \frac{1(3)+3(-2)-4(6)}{\left(\sqrt{(3)^2+(-2)^2+(6)^2}\right)^2}(3,-2,6) = \frac{-27}{49}(3,-2,6) = (\frac{-81}{49},\frac{54}{49},\frac{-162}{49})$

Problem 3 - Let $\mathbf{u} = (2,-5,4,6,-3), \mathbf{v} = (5,-2,1,-7,-4)$, and $\mathbf{w} = (2,1,-3)$

a. $\mathbf{4u} - \mathbf{3v}$

**4u** - **3v**$= 4(2,-5,4,6,-3) - 3(5,-2,1,-7,-4) = (8,-20,16,24,-12) + (-15,6,-3,21,12) = (-7,-14,13,45,0)$

b. $\mathbf{5u} + \mathbf{2v} - \mathbf{2w}$

**5u** + **2v** - **2w**$= 5(2,-5,4,6,-3) + 2(5,-2,1,-7,-4) - 2(2,1,-3,0,0) =$ $(10,-25,20,30,-15)+(10,-4,2,-14,-8)+(-4,-2,6,0,0) = (16,-31,28,16,-23)$

c. $\mathbf{u}\cdot\mathbf{v}, \mathbf{v}\cdot\mathbf{w}, \mathbf{u}\cdot\mathbf{w}$

$\mathbf{u}\cdot\mathbf{v} = 2(5) - 5(-2) + 4(1) + 6(-7) - 3(-4) = -6$
$\mathbf{v}\cdot\mathbf{w} = 5(2) - 2(1) + 1(-3) - 7(0) - 4(0) = 5$
$\mathbf{u}\cdot\mathbf{w} = 2(2) - 5(1) + 4(-3) + 6(0) - 3(0) = -13$

d. $||\mathbf{u}||, ||\mathbf{v}||, ||\mathbf{w}||$

$||\mathbf{u}|| = \sqrt{(2)^2 + (-5)^2 + (4)^2 + (6)^2 + (-3)^2} = \sqrt{90} = 3\sqrt{10}$
$||\mathbf{v}|| = \sqrt{(5)^2 + (-2)^2 + (1)^2 + (-7)^2 + (-4)^2} = \sqrt{95}$
$||\mathbf{w}|| = \sqrt{(2)^2 + (1)^2 + (-3)^2 + (0)^2 + (0)^2} = \sqrt{14}$

e. If $\theta$ is the angle between $\mathbf{u}$ and $\mathbf{v}$, find $\cos\theta$.

$\cos\theta = \frac{2(5)-5(-2)+4(1)+6(-7)-3(-4)}{\sqrt{(2)^2+(-5)^2+(4)^2+(6)^2+(-3)^2}\sqrt{(5)^2+(-2)^2+(1)^2+(-7)^2+(-4)^2}} = \frac{-6}{3\sqrt{10}\sqrt{95}} =$ $\frac{-6}{3\sqrt{950}} = \frac{-6}{(3)(5\sqrt{38})} = \frac{-2}{5\sqrt{38}}$

f. If $\alpha$ is the angle between $\mathbf{v}$ and $\mathbf{w}$, find $\cos\alpha$.

$\cos\alpha = \frac{5(2)-2(1)+1(-3)-7(0)-4(0)}{\sqrt{(5)^2+(-2)^2+(1)^2+(-7)^2+(-4)^2}\sqrt{(2)^2+(1)^2+(-3)^2+(0)^2+(0)^2}} = \frac{5}{\sqrt{95}\sqrt{14}} =$

g. If $\beta$ is the angle between $\mathbf{u}$ and $\mathbf{w}$, find $\cos\beta$.

$\cos\beta = \frac{2(2)-5(1)+4(-3)+6(0)-3(0)}{\sqrt{(2)^2+(-5)^2+(4)^2+(6)^2+(-3)^2}\sqrt{(2)^2+(1)^2+(-3)^2+(0)^2+(0)^2}} = \frac{-13}{3\sqrt{10}\sqrt{14}} =$ $\frac{-13}{3\sqrt{140}} = \frac{-13}{(3)(2)\sqrt{35}} = \frac{-13}{6\sqrt{35}}$

h. Find $\mathbf{d}(\mathbf{u},\mathbf{v})$.

$\mathbf{d}(\mathbf{u},\mathbf{v}) = \sqrt{(2-5)^2+(-5-(-2))^2+(4-1)^2+(6-(-7))^2+(-3-(-4))^2} = \sqrt{9+9+9+169+1} = \sqrt{197}$

i. Find $\mathbf{d}(\mathbf{w},\mathbf{v})$.

$\mathbf{d}(\mathbf{w},\mathbf{v}) = \sqrt{(5-2)^2+(-2-1)^2+(1-(-3))^2+(-7-0)^2+(-4-0)^2} = \sqrt{9+9+16+49+16} = \sqrt{99} = 3\sqrt{11}$

j. Find $\mathbf{d}(\mathbf{u},\mathbf{w})$.

$\mathbf{d}(\mathbf{u},\mathbf{w}) = \sqrt{(2-2)^2+(-5-1)^2+(4-(-3))^2+(6-0)^2+(-3-0)^2} = \sqrt{0+36+49+36+9} = \sqrt{130}$

k. Find $\mathbf{proj}(\mathbf{u},\mathbf{v})$.

$\mathbf{proj}(\mathbf{u},\mathbf{v}) = \frac{2(5)-5(-2)+4(1)+6(-7)-3(-4)}{\left(\sqrt{(5)^2+(-2)^2+(1)^2+(-7)^2+(-4)^2}\right)^2}(5,-2,1,-7,-4) = \frac{-6}{95}(5,-2,1,-7,-4) = (\frac{-30}{95}, \frac{12}{95}, \frac{-6}{95}, \frac{42}{95}, \frac{24}{95})$

l. Find $\mathbf{proj}(\mathbf{w},\mathbf{v})$.

$\mathbf{proj}(\mathbf{w},\mathbf{v}) = \frac{5(2)-2(1)+1(-3)-7(0)-4(0)}{\left(\sqrt{(5)^2+(-2)^2+(1)^2+(-7)^2+(-4)^2}\right)^2}(5,-2,1,-7,-4) = \frac{5}{95}(5,-2,1,-7,-4) = \frac{1}{19}(5,-2,1,-7,-4) = (\frac{5}{19}, \frac{-2}{19}, \frac{1}{19}, \frac{-7}{19}, \frac{-4}{19})$

m. Find $\mathbf{proj}(\mathbf{u},\mathbf{w})$.

$\mathbf{proj}(\mathbf{u},\mathbf{w}) = \frac{2(2)-5(1)+4(-3)+6(0)-3(0)}{\left(\sqrt{(2)^2+(1)^2+(-3)^2+(0)^2+(0)^2}\right)^2}(2,1,-3,0,0) = \frac{-13}{14}(2,1,-3,0,0) = (\frac{-26}{14}, \frac{-13}{14}, \frac{39}{14}, 0, 0)$

Problem 4 - Normalize each vector:

a. $\mathbf{u} = (5,-7)$

$||\mathbf{u}|| = \sqrt{(5)^2 + (-7)^2} = \sqrt{74}$
**u'** $= \left(\frac{5}{\sqrt{74}}, \frac{-7}{\sqrt{74}}\right)$

b. $\mathbf{v} = (1, 2, -2, 4)$

$||\mathbf{v}|| = \sqrt{(1)^2 + (2)^2 + (-2)^2 + (4)^2} = \sqrt{25} = 5$
**v'** $= (\frac{1}{5}, \frac{2}{5}, \frac{-2}{5}, \frac{4}{5})$

c. $\mathbf{w} = (\frac{1}{2}, \frac{-1}{3}, \frac{3}{4})$

$||\mathbf{w}|| = \sqrt{(\frac{1}{2})^2 + (\frac{-1}{3})^2 + (\frac{3}{4})^2} = \sqrt{(\frac{1}{4}) + (\frac{1}{9}) + (\frac{9}{16})} = \sqrt{(\frac{13}{36}) + (\frac{9}{16})} = \sqrt{\frac{133}{144}}$
**w'** $= \left(\frac{\frac{1}{2}}{\sqrt{\frac{133}{144}}}, \frac{\frac{-1}{3}}{\sqrt{\frac{133}{144}}}, \frac{\frac{3}{4}}{\sqrt{\frac{133}{144}}}\right)$
**w'** $= \left(\frac{1}{2\sqrt{\frac{133}{144}}}, \frac{-1}{3\sqrt{\frac{133}{144}}}, \frac{3}{4\sqrt{\frac{133}{144}}}\right)$

Problem 5

Let $\mathbf{u} = (1, 2, -2)$, $\mathbf{v} = (3, -12, 4)$ and a scalar $\mathbf{k} = -3$.
Find : $||\mathbf{u}||, ||\mathbf{v}||, ||\mathbf{u}+\mathbf{v}||, ||\mathbf{ku}||$

$||\mathbf{u}|| = \sqrt{(1)^2 + (2)^2 + (-2)^2} = \sqrt{9} = 3$
$||\mathbf{v}|| = \sqrt{(3)^2 + (-12)^2 + (4)^2} = \sqrt{169} = 13$
$||\mathbf{u}+\mathbf{v}|| = \sqrt{(1+3)^2 + (2 + (-12))^2 + (-2+4)^2} = \sqrt{120}$
$||\mathbf{ku}|| = \sqrt{(-3(1))^2 + (-3(2))^2 + (-3(-2))^2} = \sqrt{81} = 9$

Problem 6 - Find k so that **u** and **v** are orthogonal

a. $u = (3, k, -2), v = (6, -4, -3)$

$3(6) + k(-4) + (-2)(-3) = 0$
$18 - 4k + 6 = 0$
$-4k + 24 = 0$
$-4k = -24$
$\frac{-4k}{-4} = \frac{-24}{-4}$

$k = 6$

b. $u = (5, k, -4, 2), v = (1, -3, 2, 2k)$

$5(1) + k(-3) + (-4)(2) + 2(2k) = 0$
$5 - 3k - 8 + 4k = 0$
$k - 3 = 0$
$k = 3$

c. $u = (1, 7, k + 2, -2), v = (3, k, -3, k)$

$1(3) + 7(k) + (k + 2)(-3) + (-2)(k) = 0$
$3 + 7k + (-3k - 6) - 2k = 0$
$3 + 5k - 3k - 6 = 0$
$-3 + 2k = 0$
$2k = 3$
$\frac{2k}{2} = \frac{3}{2}$
$k = \frac{3}{2}$