School of Computing and Information Technology

ASSIGNMENT 1
(Individual)

CSIT113 – Problem Solving

Session 1, January – March 2024

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Total No of Questions: Eight (8) questions

Total Marks: 100 marks

Weightage: 8% of total subject mark

Objective

In this assignment, students are assessed on the understanding of the materials from Unit 1 to Unit 4 in the lecture notes. Students are required to apply the appropriate strategies and methods discussed in these units for each of the problems stated for the questions in this assignment.

Question 1 [10 marks]:

In applying **pure brute force** to arrange the numbers in a sequence in non-decreasing order, in the worst case, in total what are the number of possible solutions we have to consider for the following cases:

(a) when sequence-size = 4

[2 mark]

(b) when sequence-size = 5

[2 mark]

(c) when sequence-size = 6

[2 mark]

(d) when sequence-size = n

[4 mark]

Answer:

(a)

$4 \times 3 \times 2 \times 1 = 24$

(b)

$5 \times 4 \times 3 \times 2 \times 1 = 120$

(c)

$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

(d)

$n \times (n-1) \times (n-2) \times (n-3) \times \ldots\ldots \times .2 \times 1 = n!$

Question 2 [20 marks]:

A swimming team consists of 4 swimmers. The following table tabulates the time it took for each swimmer to swim a standard length (50 meter) with different technique at practice.

Technique / Swimmer	Free style (1)	Breast stroke (2)	Butterfly (3)	Back stroke (4)
Andrew	44	44	41	43
Bernard	41	47	42	42

Chester	40	43	44	46
Daniel	46	44	45	48

The team is entering a relay team contest, where in a team each of the swimmers must swim 50 meters using a different technique listed in the table. Using **brute force** strategy, assign the four techniques to the 4 swimmers such that the total time for the team is optimal.

(a) How many possible assignments (solutions) in total in using **pure brute force** (that is, brute force without considering any constraint).

[5 mark]

(b) How many possible assignments (solutions) in total in using **brute force** with the constraint that each swimmer must use a different technique taken into consideration

[5 mark]

(c) Show all the possible combinations of assigning the techniques to the four swimmers according to Q1(b) and indicate the total time for each of the assignments.

[5 mark]

(d) Which combination(s) of swimming technique yields the lowest (optimal) total team time?

[5 mark]

Answer:

(a)

In total there are 4×4×4×4 = 256 possible assignments.

(b)

In total there are 4! = 24 possible ways of assigning the 4 different techniques to the four swimmers.

(c)

State of an assignment: {Andrew, Bernard, Chester, Daniel, sA-time, sB-time, sC-time, sD-time }. That is, {n1, n2, n3, n4, t1, t2, t3, t4} means Andrew, Benard, Chester and Daniel are assigned to technique n1, n2 and n3 respectively, and their time achieved are t1, t2, t3 and t4 respectively.

Answer:

Andrew	Bernard	Chester	Danie l	A time	B time	C time	D time	Total time
1	2	3	4	44	47	44	48	183

1	2	4	3	44	47	46	45	182
1	3	2	4	44	42	43	48	177
1	3	4	2	44	42	46	44	176
1	4	2	3	44	42	43	45	174
1	4	3	2	44	42	44	44	174
2	1	3	4	44	41	44	48	177
2	1	4	3	44	41	46	45	176
2	3	1	4	44	42	40	48	174
2	3	4	1	44	42	46	46	178
2	4	3	1	44	42	44	46	176
2	4	1	3	44	42	40	45	171
3	1	2	4	41	41	43	48	173
3	1	4	2	41	41	46	44	172
3	2	1	4	41	47	40	48	176
3	2	4	1	41	47	46	46	180
3	4	1	2	41	42	40	44	167
3	4	2	1	41	42	43	46	172
4	1	2	3	43	41	43	45	172
4	1	3	2	43	41	44	44	172
4	2	1	3	43	47	40	45	175
4	2	3	1	43	47	44	46	180
4	3	1	2	43	42	40	44	169
4	3	2	1	43	42	43	46	174

(d)

Andrew	Bernard	Chester	Daniel	A time	B time	C time	D time	Total time

3	4	1	2	41	42	40	44	167

The lowest (optimal) total team time is achieved when Butterfly technique is assigned to Andrew, Back stoke technique is assigned Bernard, Free style is assigned to Chester and Breast stroke is assigned to Daniel. The assignment yields a total team time of 167.

Question 3 [10 marks]:

Alice, Bob, Carol and Dave want to cross a river. They have a boat with a capacity of 100Kg. Alice weighs 46Kg, Bob 49Kg, Carol 52Kg and Dave 100Kg. Except Bob, all of them can row. Using abstraction and invariants to find a way to get all of them across the river with the minimum number of rows.

Answer:
We shall adopt some notations used in Example 2, Unit 2. We shall use {l || r} and {l | b, x | r} to represent a state and a move respectively for this problem, where l and r represent the current contents at the left and right bank respectively and b represents the current content of the boat, and x represents the row's direction. Furthermore, we shall use "p m q" to represent a row from state p to state q via move m.

Let A, B, C, and D be Alice, Bob, Carol and Dave. The following shows the minimum number of rows that get all across by showing the from-state, move and to-state (state transition):

{ABCD||} {CD|AB, →|} {CD||AB}
{CD||AB} {CD|A, ←|B} {CDA||B}
{CDA||B} {D|AC, →|B} {D||ABC}
{D||ABC} {D|C, ←|AB} {DC||AB}
{DC||AB} {C|D, → |AB} {C||ABD}
{C||ABD} {C|A, ←|BD} {CA||BD}
{CA||BD} { |AC, → |BD} { ||ABCD}

Question 4 [10 marks]:

Prove or disprove that $p \wedge (q \equiv r) \equiv ((p \wedge q) \equiv (p \wedge r))$.

Answer:
We construct the following truth table:

p	q	r	$q \equiv r$	$p \wedge (q \equiv r)$	$p \wedge q$	$p \wedge r$	$(p \wedge q) \equiv (p \wedge r)$.
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	F	F	T	F
T	F	F	T	T	F	F	T
F	T	T	T	F	F	F	T

F	T	F	F	F	F	F	T
F	F	T	F	F	F	F	T
F	F	F	T	F	F	F	T

From the above truth table, we can see that the truth values of $p\wedge(q \equiv r)$ and $a(p\wedge q)) \equiv (p\wedge r)$ are different under the last four rows. Hence, $p\wedge(q \equiv r) \equiv ((p\wedge q)) \equiv (p\wedge r))$.is not valid.

Question 5 [10 marks]:

Prove that $(p \Rightarrow q)\wedge(q \Rightarrow r) \Rightarrow (p \Rightarrow r)$.

Answer:
We construct the following truth table:

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q)\wedge(q \Rightarrow r)$	$p \Rightarrow r$	$(p \Rightarrow q)\wedge(q \Rightarrow r) \Rightarrow (p \Rightarrow r)$.
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

From the truth table, the truth value of $(p \Rightarrow q)\wedge(q \Rightarrow r) \Rightarrow (p \Rightarrow r)$ is always true, therefore, $p \Rightarrow q)\wedge(q \Rightarrow r) \Rightarrow (p \Rightarrow r)$.

Question 6 [15 marks]:

On Knights and Knave Island, all natives are either knights, who always tell the truth, or knaves, who always tell lies.
You meet five islanders, Alice, Bob, Carol, Steven and John, who make the following statements:

Alice: “Bob and Carol are of the same type.”
Bob: “Carol is a knave.”

Answer the following questions;
(a) Is Alice a knight or a knave?
(b) What question can you ask Steven to identify exactly what John is?

Answer:

a)

Let A represents the proposition, ”Alice is a knight”
B represents the proposition, ”Bob is a knight”
C represents the proposition, ”Carol is a knight”

Then, from the given statements, we have:

$A \equiv (B \equiv C)$

$B \equiv \neg C$

Hence, $A \equiv (\neg C \equiv C) \equiv$ false

b) Let J represents the proposition, ”John is a knight”
Let S represents the proposition, ”Steven is a knight”
Let Q be the question to ask Steven to identify exactly what John is.

$(S \equiv Q) \equiv J$

$Q \equiv (S \equiv J)$

Hence, the question Q to ask Steven is “Is John the same type as you?”

Question 7 [10 marks]:

Let A be a sequence with n nonzero numbers. Design an algorithm by using induction to compute the sum of the reciprocal of the numbers in A. Specify your algorithm using recursive implementation. **Note that for a number x, its reciprocal is 1/x.**

Answer:

```
SumRecpRecursive(A, n) {
    If n = 1
        return 1/A[0]
    else
        return ProdRecursive(A, n-1) + 1/A[n-1]
}
```

Question 8 [15 marks]:

Prove by Mathematical Induction that for all n ≥ 1, $1^3 + 2^3 + 3^3 + \cdots + n^3 = n^2(n+1)^2/4$.

Proof:

When n = 1:

$$LHS = 1^3 = 1^2(1+1)^2/4 = RHS$$

Hence, the formula holds when n = 1.

Assume that the formula is true when n = k. That is, we have:

$$1^3 + 2^3 + 3^3 + \cdots + k^3 = k^2(k+1)^2/4$$

Then, when n = k+1:

$$1^3 + 2^3 + 3^3 + \cdots + k^3 + (k+1)^3$$

$$= k^2(k+1)^2/4 + (k+1)^3 \qquad \text{from the assumption}$$

$$= (k+1)^2(k^2/4 + (k+1))$$
$$= (k+1)^2(k^2 + 4k + 4)/4$$
$$= (k+1)^2(k+2)^2/4$$

Hence, the formula holds when n = k+1.

Therefore, from Mathematical Induction, the formula holds.

