CSIT 113 (Problem Solving)

Assignment 1

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Question 1)

a) 4! = 4 x 3 x 2 x 1 = 24
b) 5! = 5 x 4 x 3 x 2 x 1 = 120
c) 6! = 6 x 5 x 4 x 3 x 2 x 1 = 720
d) n! = n x (n-1) x (n-2) x … x 3 x 2 x 1

Question 2)

a) Number of total possible outcomes (without constraint)
   = 4^4
   = 256
b) Number of total possible outcomes (with constraint)
   = 4!
   = 4 x 3 x 2 x 1
   24
c) Legend:
   Freestyle →1
   Breaststroke →2
   Butterfly →3
   Backstroke →4

Andrew	1	1	1	1	1	1	2	2	2	2	2	2	3	3	3	3	3	3	4	4	4	4	4	4
Bernard	2	3	2	4	4	3	1	3	1	4	4	3	1	2	1	4	4	2	1	2	1	3	3	2
Chester	3	2	4	2	3	4	3	1	4	1	3	4	2	1	4	1	2	4	2	1	3	1	2	3
Daniel	4	4	3	3	2	2	4	4	3	3	1	1	4	4	2	2	1	1	3	3	2	2	1	1
Outcomes	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24

d) All combinations

1 – 183	7 – 177	13 – 173	19 – 172
2 – 177	8 – 174	14 – 176	20 – 175
3 – 182	9 – 176	15 – 172	21 – 172
4 – 174	10 – 171	16 – 167	22 – 169
5 – 174	11 – 176	17 – 172	23 – 174
6 - 176	12 - 178	18 - 180	24 - 180

Best combination for optimal time is:

Andrew – Butterfly (3)

Bernard – Backstroke (4)

Chester – Freestyle (1)

Daniel – Breaststroke (2)

Question 3)

1. { A B C D }{ C D | A B , → | }{ CD | | AB } : Initially all at left bank, then, **Alice** and **Bob** row over to right bank, now, **Carol** and **Dave** at left bank and **Alice** and **Bob** at right bank
2. { CD | | AB }{ C D | A , ← | B }{ ACD | | B } : **Carol** and **Dave** at left bank, **Alice** row from right bank to left bank, and now **Alice**, **Carol** and **Dave** at left bank and **Bob** at right bank
3. { ACD | | B }{ D | AC , → | B }{ D | | ABC } : **Alice**, **Carol** and **Dave** at left bank, **Alice** and **Carol** row to right bank, and now **Alice**, **Bob** and **Carol** at right bank while **Dave** is at left bank.
4. { D | | ABC }{ D | A , ← | BC }{ AD | | BC } : **Dave** is at left bank, **Alice** row back to left bank, and now, **Alice** and **Dave** at left bank, while **Bob** and **Carol** at right bank.
5. { AD | | BC }{ A | D , → | BC }{ A | | BCD } : **Alice** at **Dave** at left bank, then **Dave** row to right bank leaving **Alice** at left bank and **Bob**, **Carol** and **Dave** at right bank.
6. { A | | BCD }{ A | C , ← | BD }{ A C | | BD } : Since **Alice** is at left bank, **Carol** has to row back from right bank to left bank. Now **Alice** and **Carol** at left bank, **Bob** and **Dave** at right bank.
7. { AC | | BD }{ | A C , → | BD }{ | | ABCD } : **Alice** and **Carol** row from left bank to right bank and now everyone has crossed over to right bank safely. **Alice**, **Bob**, **Carol** and **Dave** are at right bank and no one is at left bank.

Minimum number of **rows is 7**.

Question 4)

$p \wedge (q \equiv r) \equiv ((p \wedge q) \equiv (p \wedge r))$

p	q	r	$(q \equiv r)$	$p \wedge (q \equiv r)$	$p \wedge q$	$p \wedge r$	$((p \wedge q) \equiv (p \wedge r))$	$p \wedge (q \equiv r) \equiv ((p \wedge q) \equiv (p \wedge r))$
T	T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F	T
T	F	T	F	F	F	T	F	T
F	T	T	T	F	F	F	T	F
F	F	F	T	F	F	F	T	F
F	F	T	F	F	F	F	T	F
F	T	F	F	F	F	F	T	F
T	F	F	T	T	F	F	T	T

In this case, $p \wedge (q \equiv r) \equiv ((p \wedge q) \equiv (p \wedge r))$ is only true under 4 scenarios and is false under 4 scenarios. Hence $p \wedge (q \equiv r) \equiv ((p \wedge q) \equiv (p \wedge r))$ is not always true.

Question 5)

$(p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$p \Rightarrow r$	$(q \Rightarrow r) \Rightarrow (p \Rightarrow r)$	$(p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	T
T	F	T	F	T	T	T	T
F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	T	F	T	F	T	T	T
T	F	F	F	T	F	F	T

Hence it is proven that the formula is always true for $(p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$

Question 6)

a) Given, $A \equiv (B \equiv C)$ and $B \equiv \sim C$
Substitute $B \equiv \sim C$ into $A \equiv (B \equiv C)$

$A \equiv (\sim C \equiv C)$
Since $(\sim C \equiv C)$, C is not true.
If C is not true, then C is a knave, hence proves that Bob is a knight.
And Alice says that Bob and Carol are the same type.
Since we know Bob is a knight and Carol is a knave, we can conclude that **Alice is a knave.**

b)
1. Let J represent the proposition "John is a knight".
2. Let S represent the proposition "Steven is a knight".
3. Let Q represent the proposition "The question to ask steven to identify exactly what John is"

$(S \equiv Q) \equiv J$
$Q \equiv (S \equiv J)$

Therefore, the question to ask Steven is "Is John a knight?"
If Steven say yes, it proves that John is a knight as we have assumed Steven is saying the truth. If Steven say no, then we know that John is a knave assuming that Steven is saying the truth.

Question 7)

Recursive Algorithm for

```
SumOfR(A,n) {
        for n ≥ 1
        if n = 1
                sum = 1/A[0]
        else
                sum = SumOfR(A,n-1) + 1/A[n-1]
```

Question 8)

8) $1^3 + 2^3 + 3^3 + \ldots + n^3 = [n^2(n+1)^2]/4$

Base step: $1^3 = (1^2(1+1)^2)/4$

$1 = 1$

LHS = RHS, hence the formula holds for base step,,

Inductive step

Assume formula holds for $n = k$

$1^3 + 2^3 + 3^3 + \ldots + k^3 = [k^2(k+1)^2]/4$

for $n = k+1$,

$$\begin{aligned} LHS &= 1^3 + 2^3 + 3^3 + \ldots + k^3 + (k+1)^3 \\ &= \frac{k^2(k+1)^2}{4} + (k+1)^3 \\ &= \frac{k^2(k+1)^2 + 4(k+1)^3}{4} = \frac{(k+1)^2(k^2 + 4(k+1))}{4} \\ &= \frac{(k+1)^2(k^2+4k+4)}{4} \\ &= \frac{[(k+1)^2(k+2)^2]}{4} \end{aligned}$$

for $n = k+1$,

$$\begin{aligned} RHS &= \frac{[(k+1)^2((k+1)+1)^2]}{4} \\ &= \frac{[(k+1)^2(k+2)^2]}{4} \end{aligned}$$,,

$\therefore$ LHS = RHS as proven above, for all $n \geq 1$ the formula holds.