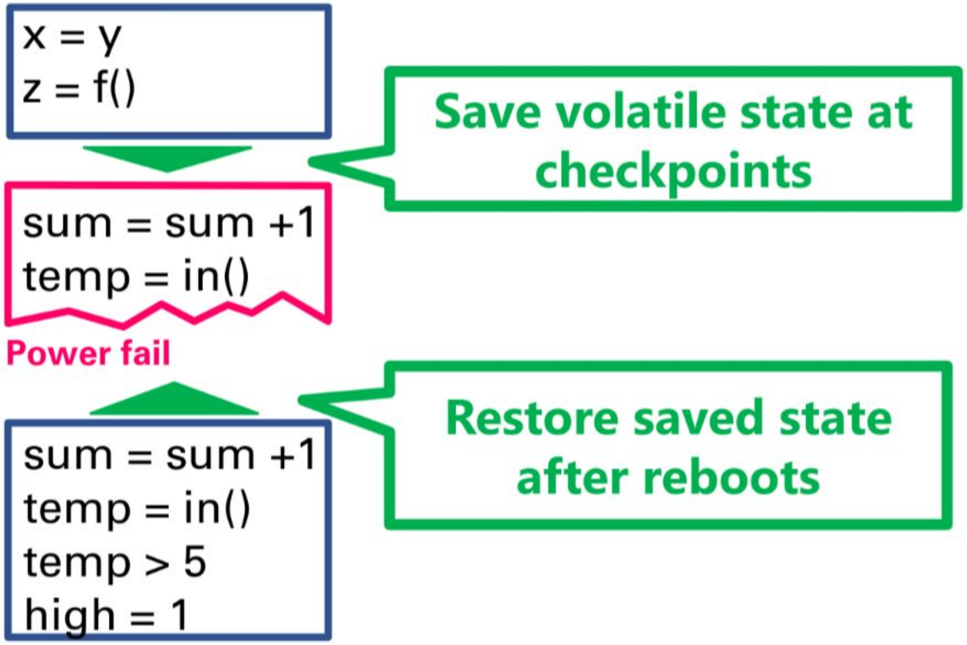
Indistinguishability Reasoning for Interaction Trees or, How to Tolerate a Lack of Productivity

Justine Frank

Crash recovery systems



Crash recovery systems

- Intermittent power harvesting systems
- Transactional file systems
- Distributed systems with persistent memory

Crashing and recovering should be *invisible to outside observers*

Atomic regions

```
atomic {
  if !x < !y
  then max := !y
  else max := !x
}
```

```
atomic {
  x := !x + 1
}
```

```
atomic (x) {
  y := !x;
  x := !y + 1
}
```

Interaction trees for crash recovery systems

- Embedding of impure non-terminating computations in proof assistants
- Has a rich theory mechanized in Rocq
- Crashing can be modeled as an effect that is nondeterministically triggered
- Crash recovery is then a handler for crash effects

```
Variant crashE : Type → Type :=
| Crash : crashE unit.
```

Interaction trees*

```
CoInductive itree (E : Type → Type) (A : Type) :=
| Ret (r : A) : itree E A
| Tau (t : itree E A) : itree E A
| Vis B (e : E B) (k : B → itree E A) : itree E A.
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```
Variant mapE : Type → Type :=
| Get : string → mapE (option nat)
| Set : string → nat → mapE unit.
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```
Variant nondetE : Type → Type :=
| Or : nondetE bool.
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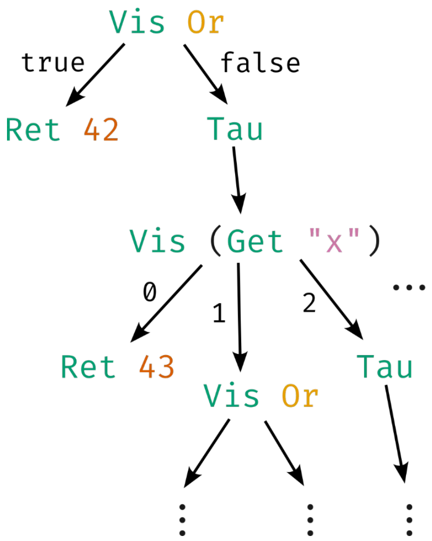
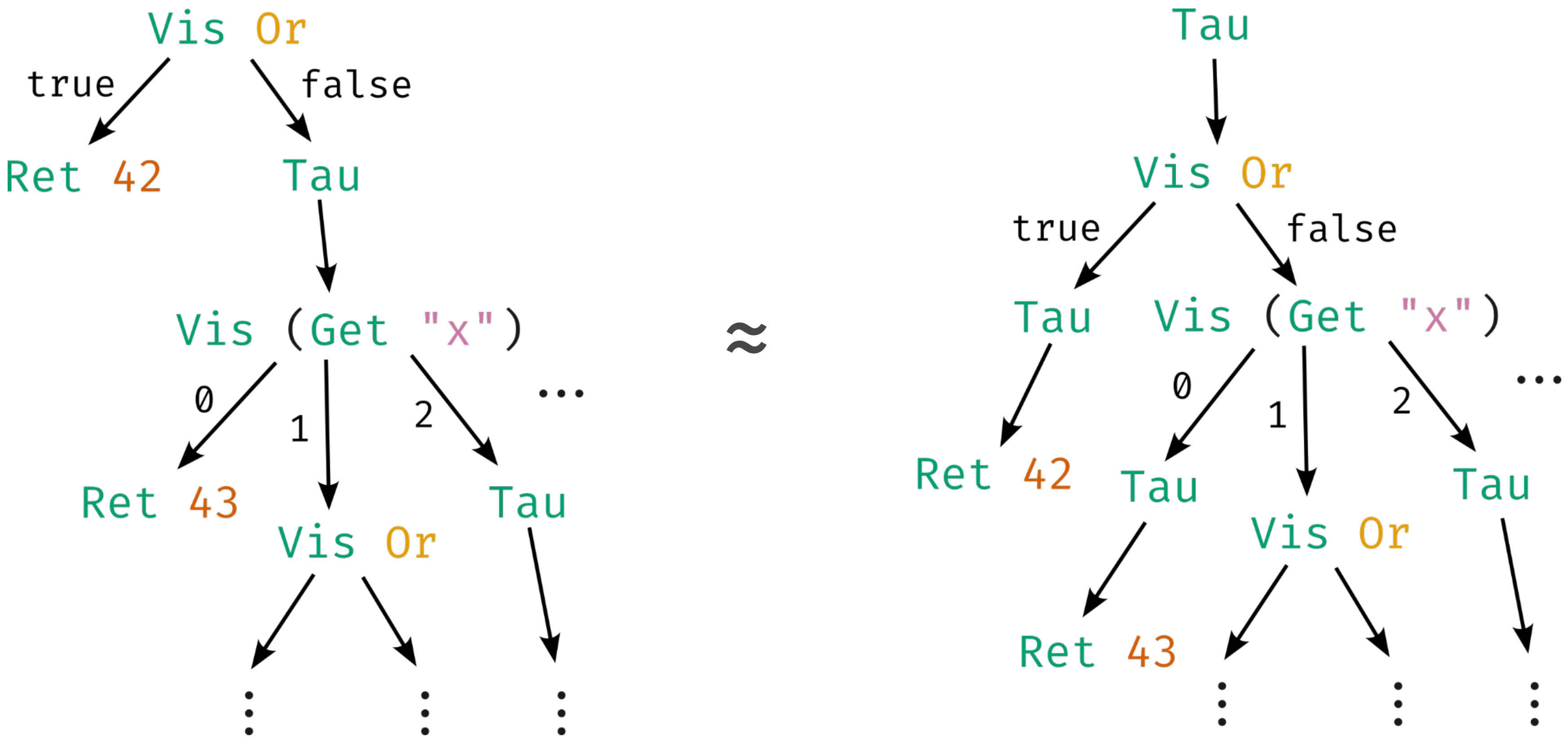


Equivalence up to taus (eutt)



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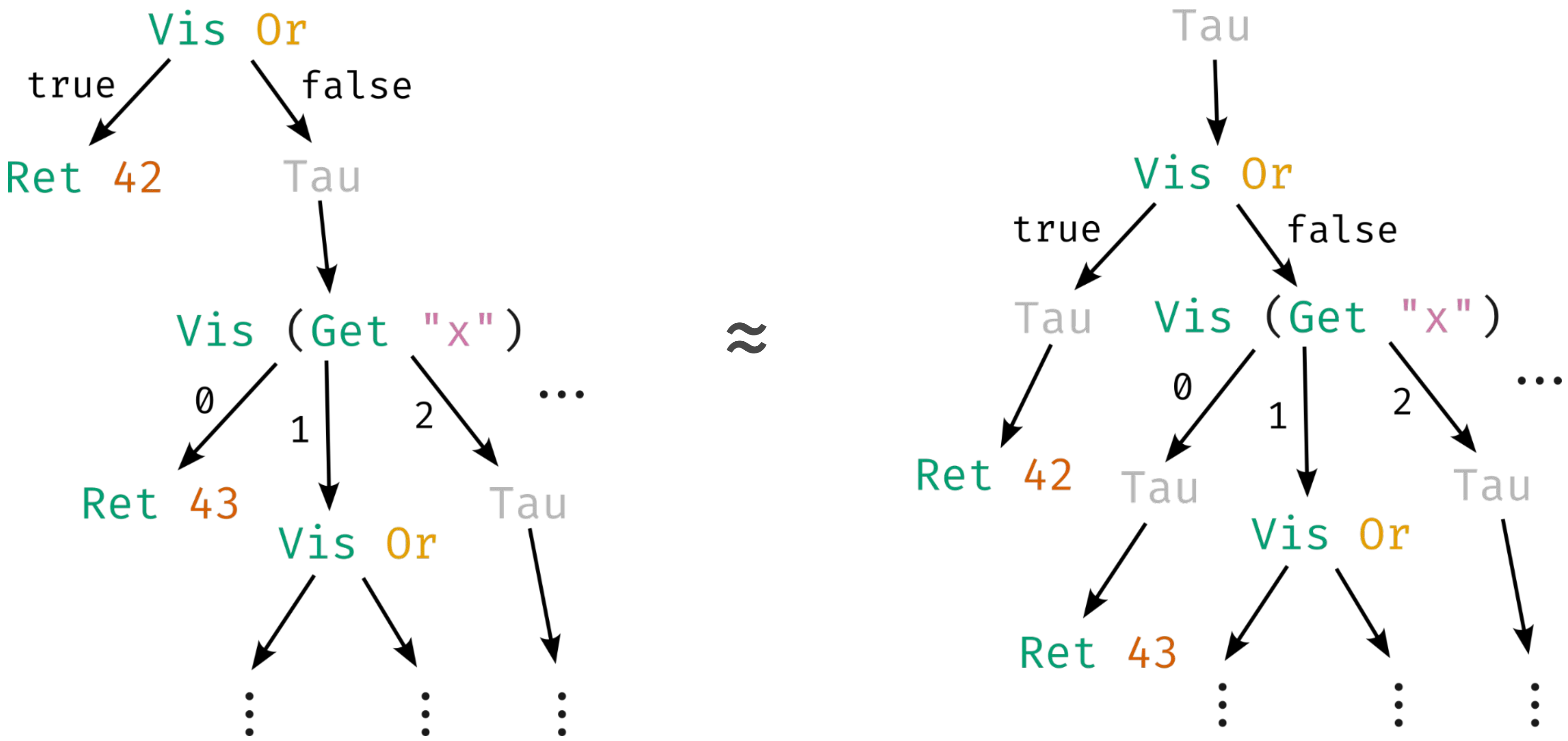

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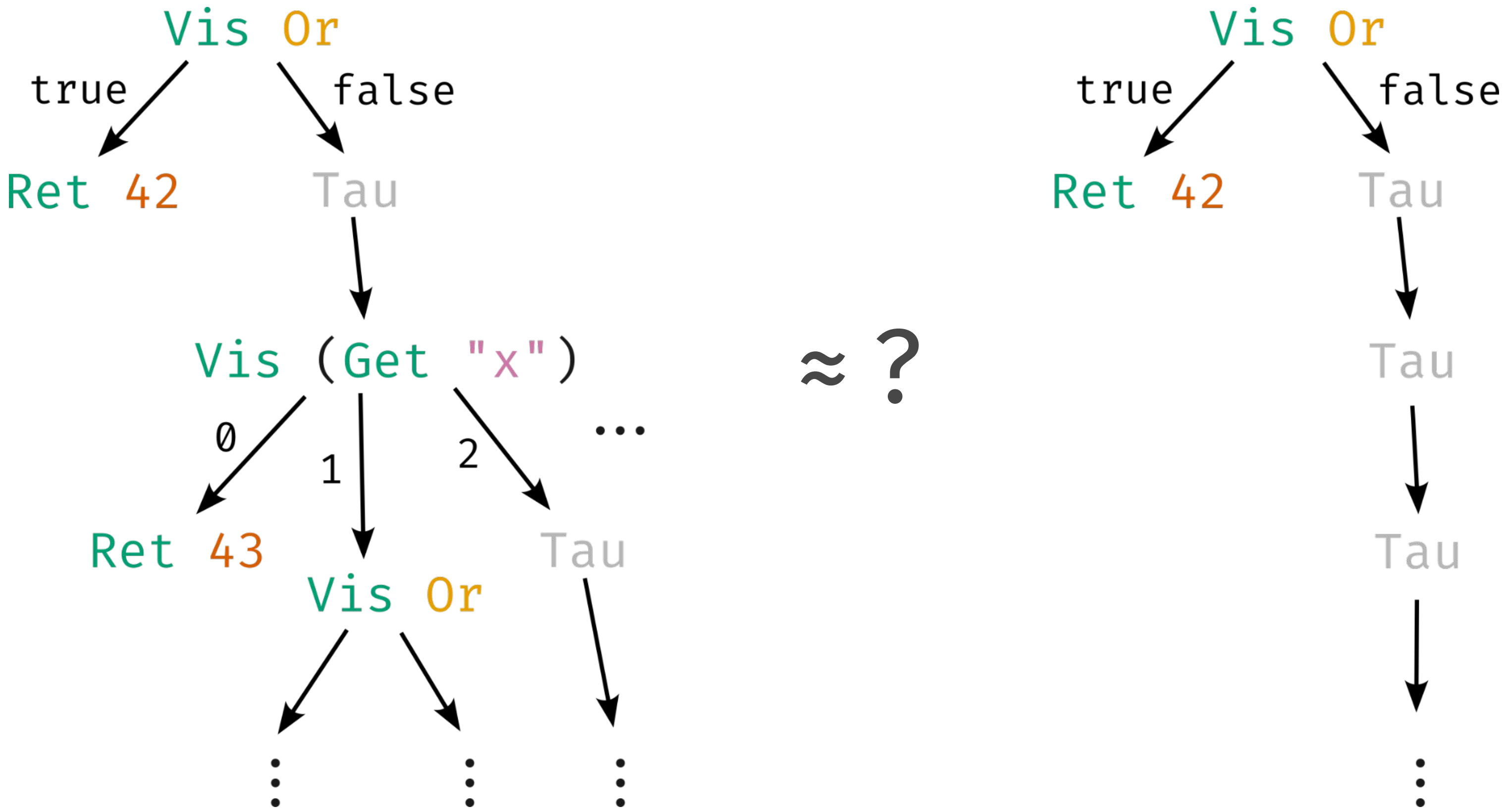

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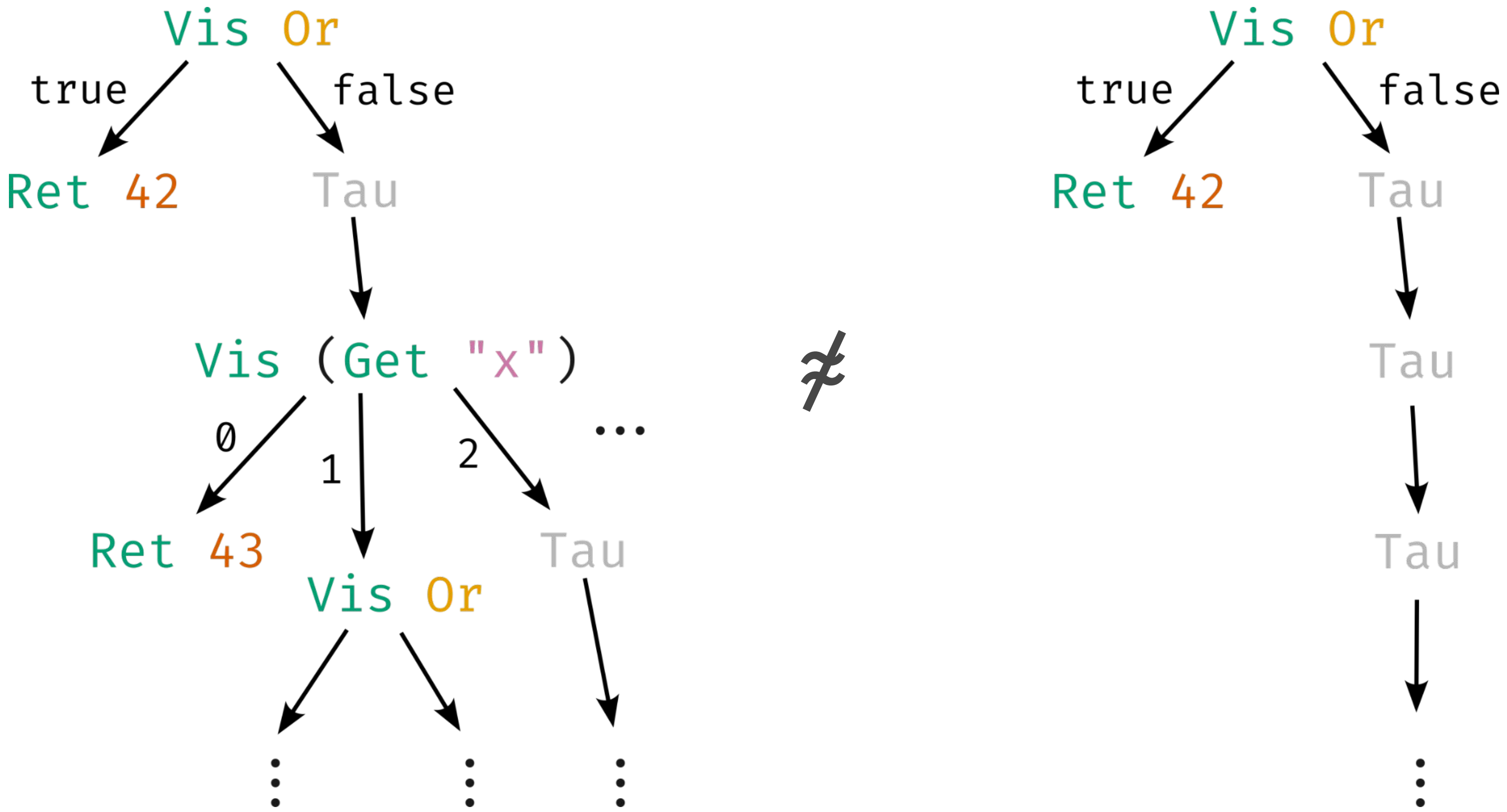
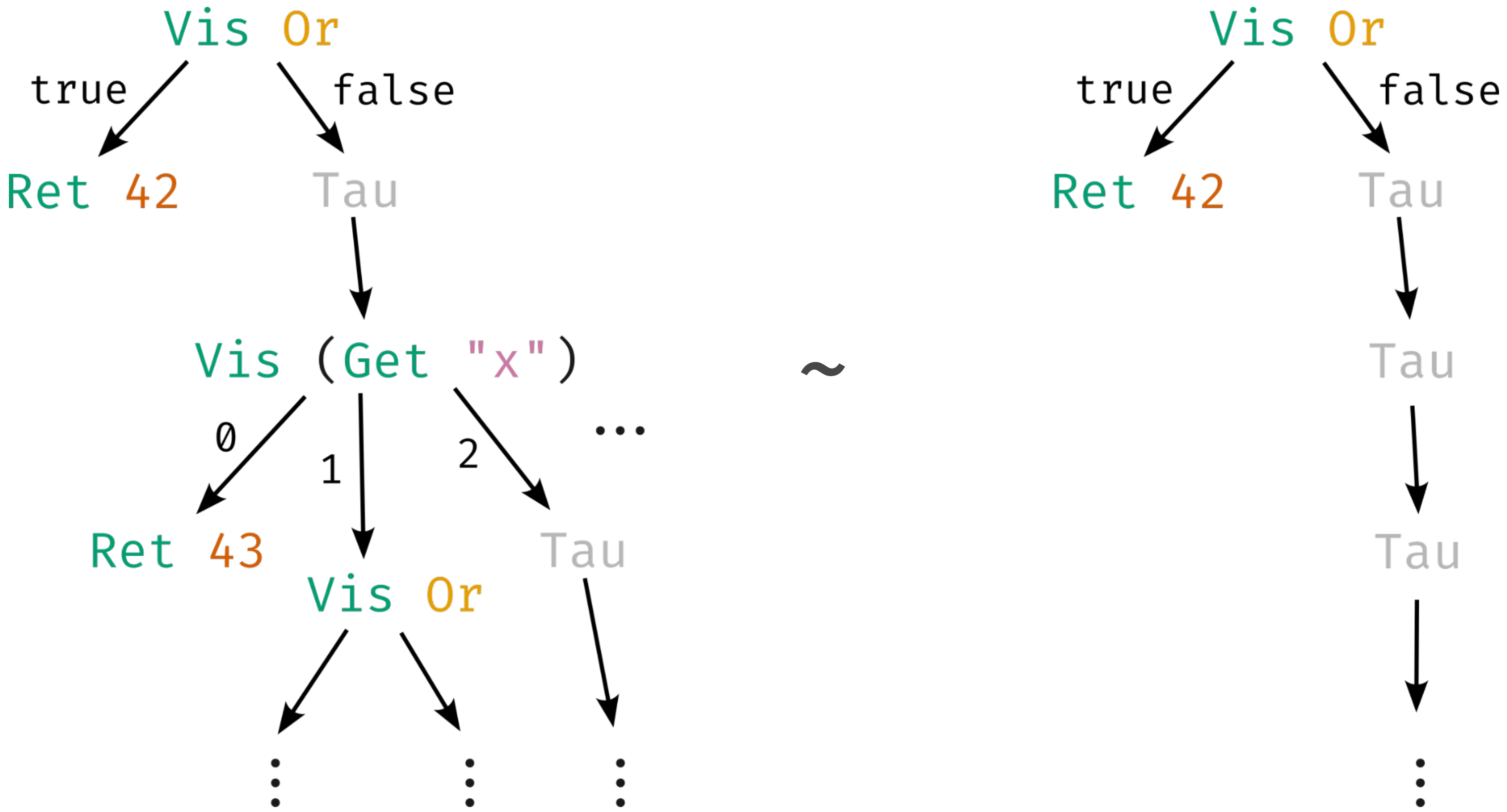
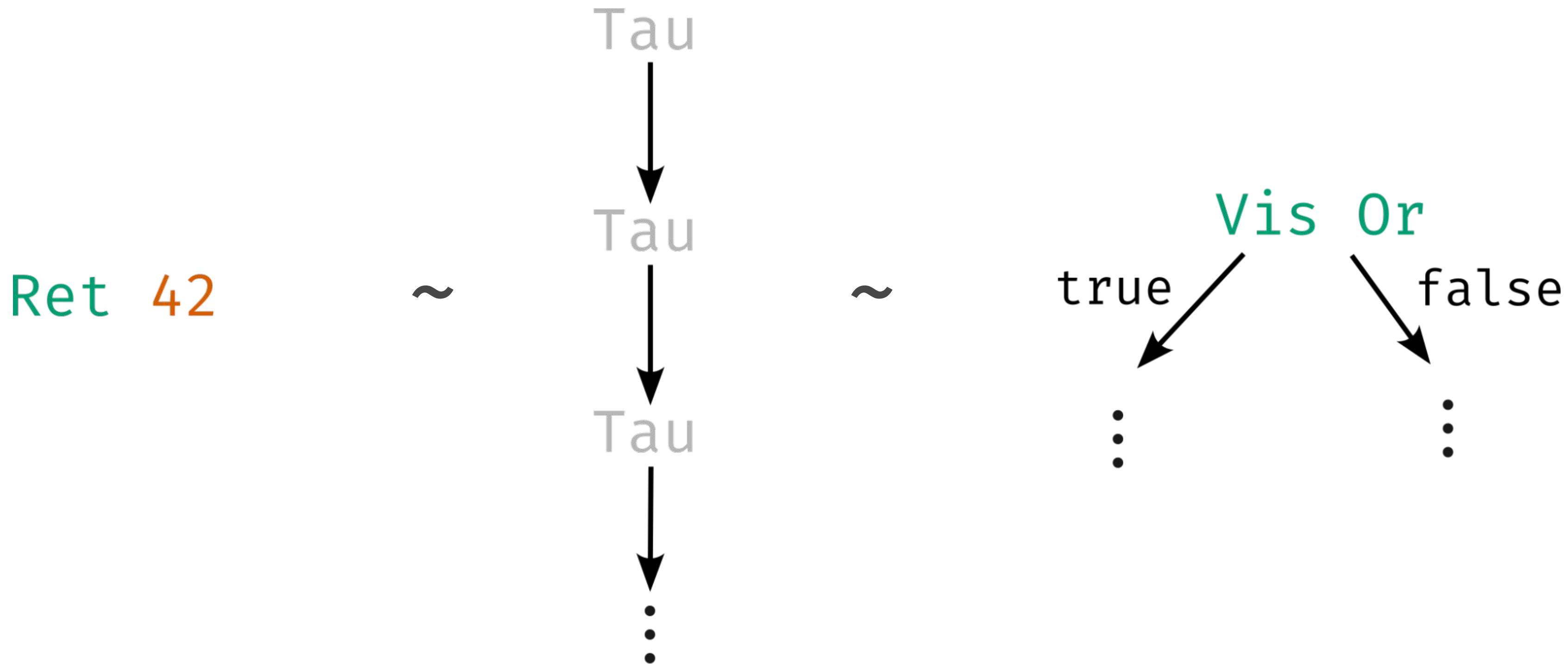


Tolerance up to taus (tutt)



Tolerance up to taus



Tolerance up to taus

$$\frac{}{t \sim t}\ \text{Refl} \qquad \frac{t_1 \sim t_2}{t_2 \sim t_1}\ \text{Sym}$$

$$\frac{t_1 \sim t_2 \quad t_1 \approx t_3 \quad t_2 \approx t_4}{t_3 \sim t_4}\ \text{EuttCompat}$$

$$\frac{t_1 \sim t_2 \quad \forall v, k_1\ v \sim k_2\ v}{t_1 \gg= k_1 \sim t_2 \gg= k_2}\ \text{Bind}$$

Proof sketch

```
atomic (x) {
  y := !x;
  x := !y + 1
}
```

Proof sketch

$$
\begin{tikzcd}
\mathrm{run}\ (y := !x;\ x := !y + 1)\ \sigma_1 \arrow[d] & \sim & \mathrm{run}\ (y := !x;\ x := !y + 1)\ \sigma_2 \arrow[d] \\
\mathrm{run}\ (x := !y + 1)\ \sigma_1[y \mapsto v_x] \arrow[d] & \sim & \mathrm{run}\ (x := !y + 1)\ \sigma_2[y \mapsto v_x] \arrow[d] \\
\mathrm{ret}\ \sigma_1[y \mapsto v_x][x \mapsto v_x + 1] \arrow[dr] & \sim & \mathrm{ret}\ \sigma_2[y \mapsto v_x][x \mapsto v_x + 1] \arrow[dl] \\
& \sigma_1[y \mapsto v_x][x \mapsto v_x + 1] \ = \ \sigma_2[y \mapsto v_x][x \mapsto v_x + 1] &
\end{tikzcd}
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Proof sketch

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\begin{tikzcd}
\mathrm{run}\ (y := !x;\ x := !y + 1)\ \sigma_1 \arrow[d] \arrow[loop left, "\lightning"] \arrow[r, phantom, "\sim"] & \mathrm{run}\ (y := !x;\ x := !y + 1)\ \sigma_2 \arrow[d] \arrow[loop right, "\lightning"] \\
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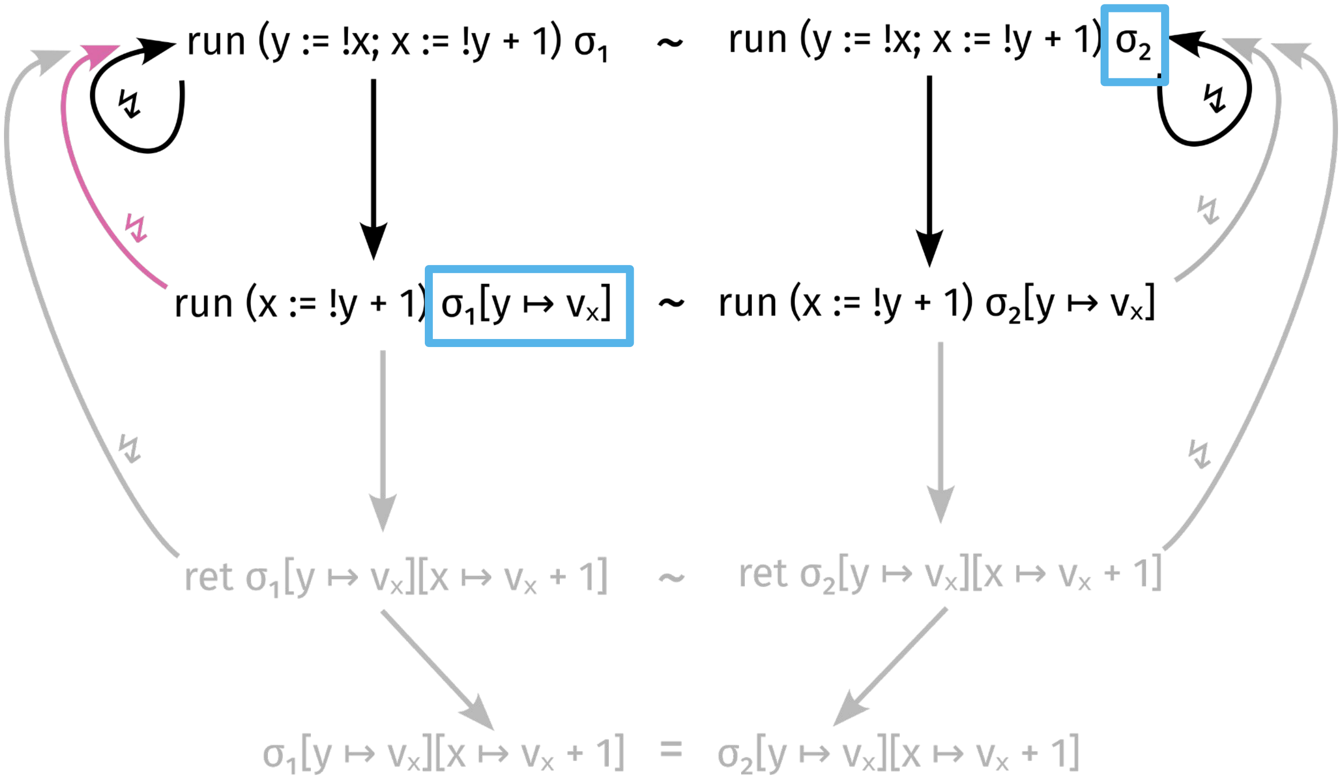
Proof sketch

$\mathrm{Inv}(\sigma_1, \sigma_2) := \forall\ k \in K, \sigma_1(k) = \sigma_2(k)?$



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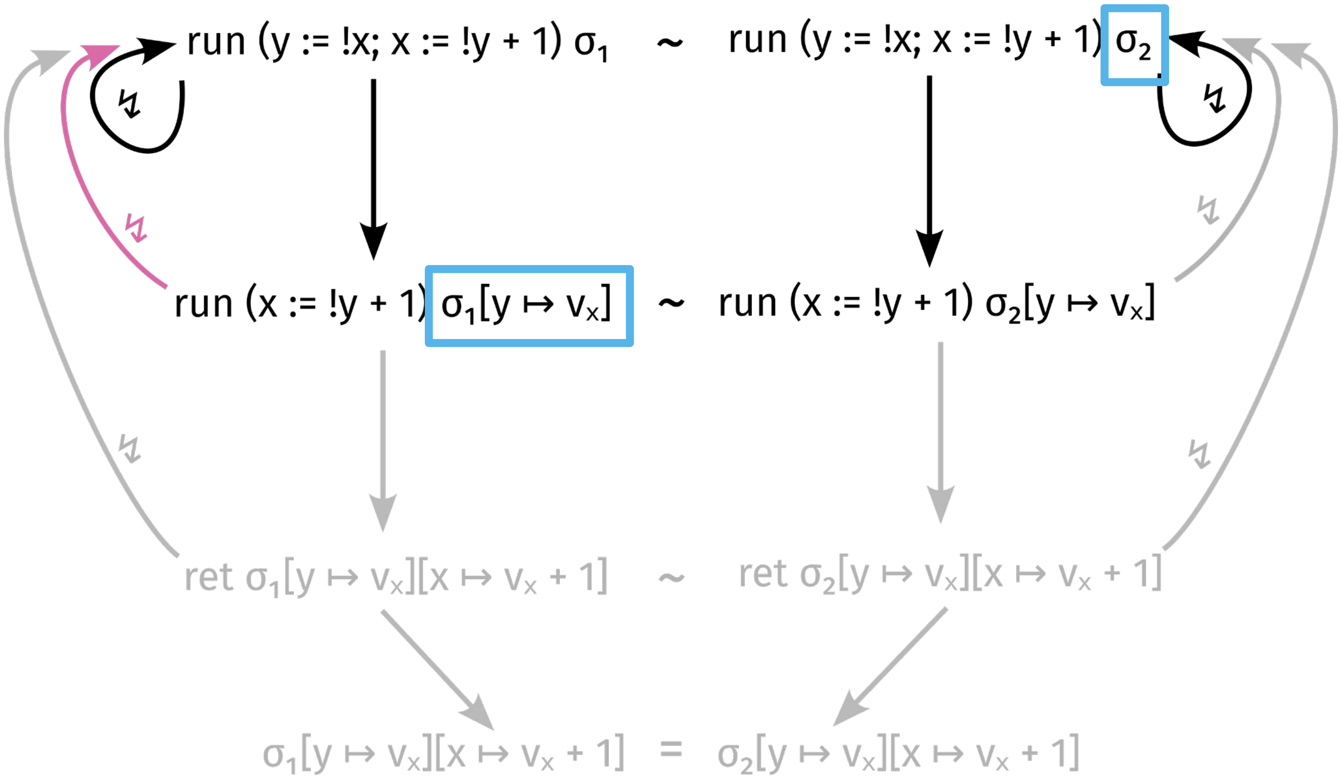

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\end{tikzcd}
$$

Tolerating a lack of productivity, in summary

- Eutt is too strict for verification of crash recovery systems.
- Tutt enables reasoning up to indistinguishability.
- Mechanized in ROCQ
- Some additional properties:
    - Tutt of two *productive* itrees implies they are eutt.
    - Can rewrite by itree equivalences while writing coinductive proofs of tutt.

Show definitions here

```
Inductive euttF sim :
    itree E R1 → itree E R2 → Prop :=
| EqRet : ∀ r1 r2,
    RR r1 r2 →
    euttF sim (Ret r1) (Ret r2)
| EqTau : ∀ m1 m2,
    sim m1 m2 →
    euttF sim (Tau m1) (Tau m2)
| EqVis : ∀ u (e : E u) k1 k2,
    (∀ v, sim (k1 v) (k2 v)) →
    euttF sim (Vis e k1) (Vis e k2)
| EqTauL : ∀ t1 t2,
    euttF sim t1 t2 →
    euttF sim (Tau t1) t2
| EqTauR : ∀ t1 t2,
    euttF sim t1 t2 →
    euttF sim t1 (Tau t2)
.
```

```
Variant tuttF sim :
    itree E R1 → itree E R2 → Prop :=
| TolRet : ∀ r1 r2,
    RR r1 r2 →
    tuttF sim (Ret r1) (Ret r2)
| TolTau : ∀ m1 m2,
    sim m1 m2 →
    tuttF sim (Tau m1) (Tau m2)
| TolVis : ∀ u (e : E u) k1 k2,
    (∀ v, sim (k1 v) (k2 v)) →
    tuttF sim (Vis e k1) (Vis e k2)
| TolTauL : ∀ t1 t2,
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    tuttF sim (Tau t1) t2
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| EqVis : ∀ u (e : E u) k1 k2,
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| TolTauL : ∀ t1 t2,
    sim t1 t2 →
    tuttF sim (Tau t1) t2
| TolTauR : ∀ t1 t2,
    sim t1 t2 →
    tuttF sim t1 (Tau t2)
.
```

Show definitions here

```
Inductive euttF sim :
    itree E R1 → itree E R2 → Prop :=
| EqRet : ∀ r1 r2,
    RR r1 r2 →
    euttF sim (Ret r1) (Ret r2)
| EqTau : ∀ m1 m2,
    sim m1 m2 →
    euttF sim (Tau m1) (Tau m2)
| EqVis : ∀ u (e : E u) k1 k2,
    (∀ v, sim (k1 v) (k2 v)) →
    euttF sim (Vis e k1) (Vis e k2)
| EqTauL : ∀ t1 t2,
    euttF sim t1 t2 →
    euttF sim (Tau t1) t2
| EqTauR : ∀ t1 t2,
    euttF sim t1 t2 →
    euttF sim t1 (Tau t2)
.
```

```
Variant tuttF sim :
    itree E R1 → itree E R2 → Prop :=
| TolRet : ∀ r1 r2,
    RR r1 r2 →
    tuttF sim (Ret r1) (Ret r2)
| TolTau : ∀ m1 m2,
    sim m1 m2 →
    tuttF sim (Tau m1) (Tau m2)
| TolVis : ∀ u (e : E u) k1 k2,
    (∀ v, sim (k1 v) (k2 v)) →
    tuttF sim (Vis e k1) (Vis e k2)
| TolTauL : ∀ t1 t2,
    sim t1 t2 →
    tuttF sim (Tau t1) t2
| TolTauR : ∀ t1 t2,
    sim t1 t2 →
    tuttF sim t1 (Tau t2)
.
```