Simulation Toolbox Job Description: Evolution in a 1D Quartic Potential

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Simulate the quantum dynamics of a particle in a 1D quartic potential. One considers a particle of mass m initially prepared in a thermal state of a harmonic potential given by $U_0(x) = m\Omega^2 x^2/2$ with phonon mean number occupation $\bar{n}$. At time $t \geq 0$, the potential is changed to a combined harmonic and quartic potential. The quartic term is centered at a position $x = x_s$, and its strength is parametrized with the dimensionless parameter α via

$$U_q(x) = \alpha \frac{\hbar\Omega}{4} \left(\frac{x - x_\mathrm{s}}{x_0} \right)^4 . \tag{1}$$

Here, $x_0^2 = \hbar/(2m\Omega)$ is the zero point motion. The overall potential determining the dynamics for $t \geq 0$ will be given by

$$U(x) = \beta U_0(x) + U_q(x), \tag{2}$$

where $\beta \in [0, 1)$ is a dimensionless parameter used to model the influence of a residual harmonic potential.

One can anticipate that the maximum amount of motional squeezing (using the variance in units of zero-point motion fluctuations) during the dynamics is given by

$$\mathcal{S} = -10 \log_{10} \left(\sqrt{\alpha} \right) . \tag{3}$$

We will concentrate in the XS/S parameter regime given by $\alpha \in [10^{-3}, 1]$.

The goal of the simulator is to produce the Wigner function, in units of zero-point motion, as a function of time in units of $1/\Omega$, as a function of $\bar{n}$, α, β and x_s/x_0. The simulator will also output mean value of position, momentum, and their variances, and the marginal probability distribution in position and momentum, everything in units of zero-point motion.

The regime of the free parameters for which the simulator will be guaranteed to work properly will be:

$$\begin{aligned} \beta &\in [0, 1) \\ \alpha &\in [10^{-3}, 1] \\ \bar{n} &< 5 \\ |x_s|/x_0 &< 10 \end{aligned} \tag{4}$$

The total time of evolution will be left unrestricted and we will guarantee that the simulation works properly until the first partial re-compression.