

Practice quiz on Bayes Theorem and the Binomial Theorem

TOTAL POINTS 9

1. A jewelry store that serves just one customer at a time is concerned about the safety of its isolated customers.

1 / 1 point

The store does some research and learns that:

- 10% of the times that a jewelry store is robbed, a customer is in the store.
- A jewelry store has a customer on average 20% of each 24-hour day.
- The probability that a jewelry store is being robbed (anywhere in the world) is 1 in 2 million.

What is the probability that a robbery will occur while a customer is in the store?

- ☐ $\frac{1}{500000}$
- ☐ $\frac{1}{2000000}$
- ☒ $\frac{1}{4000000}$
- ☐ $\frac{1}{5000000}$

✓ **Correct**

What is known is:

A : "a customer is in the store," $P(A) = 0.2$

B : "a robbery is occurring," $P(B) = \frac{1}{2,000,000}$

$P(\text{a customer is in the store} \mid \text{a robbery occurs}) = P(A \mid B)$

$P(A \mid B) = 10\%$

What is wanted:

$P(\text{a robbery occurs} \mid \text{a customer is in the store}) = P(B \mid A)$

By the product rule:

$$P(B \mid A) = \frac{P(A, B)}{P(A)}$$

and $P(A, B) = P(A \mid B)P(B)$

2. If I flip a fair coin, with heads and tails, ten times in a row, what is the probability that I will get exactly six heads?

1 / 1 point

- ☐ 0.021
- ☐ 0.187
- ☒ 0.2051
- ☐ 0.305

✓ **Correct**

By Binomial Theorem, equals

$$\binom{10}{6} (0.5^{10})$$

$$= \left(\frac{10!}{4! \times 6!} \right) \left(\frac{1}{1024} \right)$$
$$= 0.2051$$

3. If a coin is bent so that it has a 40% probability of coming up heads, what is the probability of getting *exactly* 6 heads in 10 throws?

1 / 1 point

- ☐ 0.0974
- ☐ 0.1045
- ☒ 0.1115
- ☐ 0.1219

✓ **Correct**

$$\binom{10}{6} \times 0.4^6 \times 0.6^4 = 0.1115$$

4. A bent coin has 40% probability of coming up heads on each independent toss. If I toss the coin ten times, what is the probability that I get at least 8 heads?

1 / 1 point

- ☐ 0.0312
- ☐ 0.0132
- ☒ 0.0123
- ☐ 0.0213

✓ **Correct**

The answer is the sum of three binomial probabilities:

$$\left(\binom{10}{8} \times (0.4^8) \times (0.6^2) \right) + \left(\binom{10}{9} \times (0.4^9) \times (0.6^1) \right) +$$

$$\left(\binom{10}{10} \times (0.4^{10}) \times (0.6^0) \right)$$

5. Suppose I have a bent coin with a 60% probability of coming up heads. I throw the coin ten times and it comes up heads 8 times.

1 / 1 point

What is the value of the "likelihood" term in Bayes' Theorem -- the conditional probability of the data given the parameter.

- ☐ 0.043945
- ☐ 0.168835
- ☒ 0.120932
- ☐ 0.122885

✓ **Correct**

Bayesian "likelihood" --- the $p(\text{observed data} \mid \text{parameter})$ is

$p(8 \text{ of } 10 \text{ heads} \mid \text{coin has } p = .6 \text{ of coming up heads})$

$$\binom{10}{8} \times (0.6^8) \times (0.4^2) = 0.120932$$

6. We have the following information about a new medical test for diagnosing cancer.

1 / 1 point

Before any data are observed, we know that 5% of the population to be tested actually have Cancer.

Of those tested who do have cancer, 90% of them get an accurate test result of "Positive" for cancer. The other 10% get a false test result of "Negative" for Cancer.

Of the people who do not have cancer, 90% of them get an accurate test result of "Negative" for cancer. The other 10% get a false test result of "Positive" for cancer.

What is the conditional probability that I have Cancer, if I get a "Positive" test result for Cancer?

**Formulas in the feedback section are very long, and do not fit within the standard viewing window. Therefore, the font is a bit smaller and the word "positive test" has been abbreviated as PT.

- ☐ 9.5%
- ☐ 4.5%
- ☒ 32.1% probability that I have cancer
- ☐ 67.9%

✓ **Correct**

I still have a more than $\frac{2}{3}$ probability of not having cancer

Posterior probability:

$p(\text{I actually have cancer} \mid \text{receive a "positive" Test})$

7. We have the following information about a new medical test for diagnosing cancer.

1 / 1 point

Before any data are observed, we know that 8% of the population to be tested actually have Cancer.

Of those tested who do have cancer, 90% of them get an accurate test result of "Positive" for cancer.

The other 10% get a false test result of "Negative" for Cancer.

Of the people who do not have cancer, 95% of them get an accurate test result of "Negative" for cancer.

The other 5% get a false test result of "Positive" for cancer.

What is the conditional probability that I have cancer, if I get a "Negative" test result for Cancer?

- ☐ 88.2%
- ☐ 99.1%
- ☒ 0.9%
- ☐ .80%

✓ **Correct**

$$p(\text{cancer} \mid \text{negative test}) =$$

$$\frac{p(\text{negative test} \mid \text{Cancer}) p(\text{Cancer})}{p(\text{negative test} \mid \text{cancer}) p(\text{cancer}) + p(\text{negative test} \mid \text{no cancer}) p(\text{no cancer})}$$

$$\frac{(10\%)(8\%)}{(10\%)(8\%) + (95\%)(92\%)}$$

$$\frac{0.8\%}{0.8\% + 87.4\%}$$

$$\frac{0.8\%}{88.2\%}$$

$$= 0.9\%$$

8. An urn contains 50 marbles – 40 blue and 10 white. After 50 draws, exactly 40 blue and 10 white are observed.

1 / 1 point

You are not told whether the draw was done “with replacement” or “without replacement.”

What is the probability that the draw was done with replacement?

- ☐ 13.98%
- ☐ 87.73%
- ☒ 12.27%
- ☐ 1

✓ **Correct**

$p(40$
blue and 10 white | draws without replacement) = 1 [this is the only possible outcome when 50 draws are made without replacement]

$p(40$ blue and 10 white | draws
with replacement)

$S = 40$

$N = 50$

$P = .8$ [for draws with replacement] because 40 blue of 50 total means $p(\text{blue}) = 40/50 = .8$

$$\binom{50}{40} (0.8^{40}) (0.2^{10})$$

$$= 13.98\%$$

By Bayes' Theorem:

$p(\text{draws with replacement} | \text{observed data}) =$

$$\frac{13.98\%(.5)}{(13.98\%(.5) + (1)(.5))}$$

$$= \frac{0.1398}{1.1398}$$

$$= 12.27\%$$

9. According to Department of Customs Enforcement Research: 99% of people crossing into the United States are not smugglers. **1 / 1 point**

The majority of all Smugglers at the border (65%) appear nervous and sweaty.

Only 8% of innocent people at the border appear nervous and sweaty.

If someone at the border appears nervous and sweaty, what is the probability that they are a Smuggler?

- ☐ 8.57%
- ☐ 7.92%
- ☐ 92.42%
- ☒ 7.58%

✓ **Correct**

By
Bayes' Theorem, the answer is

$$\frac{(.65)(.01)}{((.65)(.01) + (.08)(.99))}$$

$$= 7.58\%$$