lsg04

Aufgabe 1

```
reset()
v1 = vector([1,2,3,4,5])
v2 = vector([-1,27,26,1,-27])
v3 = vector([160,-48,112,-160,48])
v4 = vector([120,234,-23,-43,29])


X = matrix([v1,v2,v3,v4]).transpose()
X.transpose() * X
```

```
[   55     0     0   492]
[    0  2136     0  4774]
[    0     0 68352 13664]
[  492  4774 13664 72375]
```

```
v = [v1,v2,v3,v4]
matrix([[vi*vj for vi in v] for vj in v])
```

```
[   55     0     0   492]
[    0  2136     0  4774]
[    0     0 68352 13664]
[  492  4774 13664 72375]
```

Aufgabe 2

```
reset()
A = matrix([[1,8,4], [3,1,3], [2,2,2]])
B = matrix([[5,5,6], [7,9,1], [8,10,0]])
C = matrix([[1,1,2], [2,1,1], [2,2,2]])
D = A*B
E = B*C
F = B.transpose()*C
Ms = [A,B,C,D,E,F]


[X.det() for X in Ms]
```

```
[12, -22, 2, -264, -44, -44]
```

```
[X.rank() for X in Ms]
```

```
[3, 3, 3, 3, 3, 3]
```

Aufgabe 3

```
reset()

A = matrix([[var("a%i%i" % (i,j)) for j in [1..2]] for i in [1..2]])
B = matrix([[var("b%i%i" % (i,j)) for j in [1..2]] for i in [1..2]])
C = matrix([[var("c%i%i" % (i,j)) for j in [1..2]] for i in [1..2]])

x = vector([var("x%i" % i) for i in [1..2]])
y = vector([var("y%i" % i) for i in [1..2]])


print (A*B).transpose() - B.transpose()*A.transpose()
print bool((A*B).transpose() == B.transpose()*A.transpose())
```

```
[0 0]
[0 0]
True
```

```
print map_threaded(expand, A*(x+y)-A*x-A*y)
print bool(A*(x+y) == A*x + A*y)
```

```
(0, 0)
True
```

```
print expand((A*B).det() - A.det()*B.det())
print bool((A*B).det() == A.det()*B.det())
```

```
0
```

```
True
```

```
# funktioniert nicht (Elemente von (A*B)^(-1) sind gebrochen rationale
# Funktionen):
# map_threaded(expand, (A*B).inverse() - B.inverse()*A.inverse())


# Hier wird simplify_full oder simplify_rational benötigt.

# simplify_rational läßt sich als Methode auf Matrizen anwenden
( (A*B).inverse() - B.inverse()*A.inverse() ).simplify_rational()
```

```
[0 0]
[0 0]
```

```
# simplify_full nicht
( (A*B).inverse() - B.inverse()*A.inverse() ).simplify_full()
```

```
Traceback (click to the left of this block for traceback)
...
AttributeError: 'sage.matrix.matrix_symbolic_dense.Matrix_symbolic_'
object has no attribute 'simplify_full'
```

```
# Um simplify_full auf jedes Element anzuwenden, benötigt man eine
# Hilfsfunktion
def f(x): return x.simplify_full()

map_threaded(f, (A*B).inverse() - B.inverse()*A.inverse())
```

```
[0 0]
[0 0]
```

Aufgabe 4

```
reset()
A = matrix([[1,0,0], [0, cos(pi/6), -sin(pi/6)], [0, sin(pi/6), cos(pi/6)]])
B = matrix([[1,1,i], [1,i,1], [i,1,1]])
C = matrix([[0,0,1], [2/3+i/3, 2/3, 0], [2/3,-2/3-i/3,0]])
E = identity_matrix(3)
Ms = [A,B,C]


# unitär?
[(X.conjugate().transpose()*X-E).is_zero() for X in Ms]
```

```
[True, False, False]
```

```
# orthogonal?
[(X.transpose()*X-E).is_zero() for X in Ms]
```

```
[True, False, False]
```

```
# invertierbar?
[X.is_invertible() for X in Ms]
```

```
[True, True, True]
```

```
# symmetrisch?
[(X.transpose()-X).is_zero() for X in Ms]
```

```
[False, True, False]
```

```
# hermitesch?
[(X.conjugate().transpose()-X).is_zero() for X in Ms]
```

```
[False, False, False]
```

Aufgabe 5

```
reset()
V = span([vector([3,4,-5,6,-8]), vector([2,4,8,12,-5]), vector([6,5,13,21,2])])
W = span([vector([5,4,-12,2,0]), vector([3,5,2,-2,8]), vector([4,2,9,1,8])])


V.intersection(W).dimension()
# oder:
```

```
V.dimension() + W.dimension() - (V+W).dimension()
```

```
1
```

Aufgabe 6

```
reset()
A = matrix([var("a%i%i" % (i,j)) for j in [1..4] for i in [1..4]]); A
```

```
[a11 a21 a31 a41 a12 a22 a32 a42 a13 a23 a33 a43 a14 a24 a34 a44]
```

```
A.det()
```

```
a11*a22*a33*a44 - a11*a22*a34*a43 - a11*a23*a32*a44 +
a11*a23*a34*a42 + a11*a24*a32*a43 - a11*a24*a33*a42 -
a12*a21*a33*a44 + a12*a21*a34*a43 + a12*a23*a31*a44 -
a12*a23*a34*a41 - a12*a24*a31*a43 + a12*a24*a33*a41 +
a13*a21*a32*a44 - a13*a21*a34*a42 - a13*a22*a31*a44 +
a13*a22*a34*a41 + a13*a24*a31*a42 - a13*a24*a32*a41 -
a14*a21*a32*a43 + a14*a21*a33*a42 + a14*a22*a31*a43 -
a14*a22*a33*a41 - a14*a23*a31*a42 + a14*a23*a32*a41
```

Aufgabe 7

```
reset()
H = matrix([[1/(i+j-1) for i in [1..15]] for j in [1..15]]); show(H)
```

$$\begin{pmatrix}
1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} \\
\frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} \\
\frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} \\
\frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} \\
\frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} \\
\frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} & \frac{1}{20} \\
\frac{1}{7} & \frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} & \frac{1}{20} & \frac{1}{21} \\
\frac{1}{8} & \frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} & \frac{1}{20} & \frac{1}{21} & \frac{1}{22} \\
\frac{1}{9} & \frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} & \frac{1}{20} & \frac{1}{21} & \frac{1}{22} & \frac{1}{23} \\
\frac{1}{10} & \frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} & \frac{1}{20} & \frac{1}{21} & \frac{1}{22} & \frac{1}{23} & \frac{1}{24} \\
\frac{1}{11} & \frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} & \frac{1}{20} & \frac{1}{21} & \frac{1}{22} & \frac{1}{23} & \frac{1}{24} & \frac{1}{25} \\
\frac{1}{12} & \frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} & \frac{1}{20} & \frac{1}{21} & \frac{1}{22} & \frac{1}{23} & \frac{1}{24} & \frac{1}{25} & \frac{1}{26} \\
\frac{1}{13} & \frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} & \frac{1}{20} & \frac{1}{21} & \frac{1}{22} & \frac{1}{23} & \frac{1}{24} & \frac{1}{25} & \frac{1}{26} & \frac{1}{27} \\
\frac{1}{14} & \frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} & \frac{1}{20} & \frac{1}{21} & \frac{1}{22} & \frac{1}{23} & \frac{1}{24} & \frac{1}{25} & \frac{1}{26} & \frac{1}{27} & \frac{1}{28} \\
\frac{1}{15} & \frac{1}{16} & \frac{1}{17} & \frac{1}{18} & \frac{1}{19} & \frac{1}{20} & \frac{1}{21} & \frac{1}{22} & \frac{1}{23} & \frac{1}{24} & \frac{1}{25} & \frac{1}{26} & \frac{1}{27} & \frac{1}{28} & \frac{1}{29}
\end{pmatrix}$$

```
H.det().n()
```

```
1.05854274306972e-124
```

```
r = vector([1 for i in [1..15]]); y = H*r
```

```
H\y
```

```
(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)
```

```
H = matrix([[1.0/(i+j-1) for i in [1..15]] for j in [1..15]]); show(H)
```

$$\begin{pmatrix}
1.00000000000000 & 0.500000000000000 & 0.333333333333333 & 0.250000000000000 & 0.2000000000000 \\
0.500000000000000 & 0.333333333333333 & 0.250000000000000 & 0.200000000000000 & 0.1666666666666 \\
0.333333333333333 & 0.250000000000000 & 0.200000000000000 & 0.166666666666667 & 0.1428571428571 \\
0.250000000000000 & 0.200000000000000 & 0.166666666666667 & 0.142857142857143 & 0.1250000000000 \\
0.200000000000000 & 0.166666666666667 & 0.142857142857143 & 0.125000000000000 & 0.1111111111111 \\
0.166666666666667 & 0.142857142857143 & 0.125000000000000 & 0.111111111111111 & 0.1000000000000 \\
0.142857142857143 & 0.125000000000000 & 0.111111111111111 & 0.100000000000000 & 0.09090909090909 \\
0.125000000000000 & 0.111111111111111 & 0.100000000000000 & 0.0909090909090909 & 0.08333333333333 \\
0.111111111111111 & 0.100000000000000 & 0.0909090909090909 & 0.0833333333333333 & 0.07692307692307 \\
0.100000000000000 & 0.0909090909090909 & 0.0833333333333333 & 0.0769230769230769 & 0.07142857142857 \\
0.0909090909090909 & 0.0833333333333333 & 0.0769230769230769 & 0.0714285714285714 & 0.06666666666666 \\
0.0833333333333333 & 0.0769230769230769 & 0.0714285714285714 & 0.0666666666666667 & 0.06250000000000 \\
0.0769230769230769 & 0.0714285714285714 & 0.0666666666666667 & 0.0625000000000000 & 0.05882352941176 \\
0.0714285714285714 & 0.0666666666666667 & 0.0625000000000000 & 0.0588235294117647 & 0.05555555555555 \\
0.0666666666666667 & 0.0625000000000000 & 0.0588235294117647 & 0.0555555555555556 & 0.05263157894736
\end{pmatrix}$$

H\y

```
(1.00000002444278, 0.999998131330912, 1.00000095379382,
1.00116615408083, 0.978440609115038, 1.18179736092829,
0.116783239421053, 3.68652617755879, -4.25559146988044,
7.48588212305572, -3.53201215195821, 1.93152918613242,
2.04738507207912, 0.169470351533555, 1.18862431095391)
```

Aufgabe 8

```
reset()
x = 2
for i in [1..1000]:
   x = x * (2*i)^2/(2*i-1)/(2*i+1)
float(x)
```

```
3.1408077460303945
```

```
# oder:
float( 2 * prod([(2*i)^2/(2*i-1)/(2*i+1) for i in [1..1000]]) )
```

```
3.1408077460303945
```

Aufgabe 9

```
reset()
def summe(xs):
   s = 0
   for x in xs:
      s = s + x
   return s

summe([1..100])
```

```
5050
```