

5.3 Part 3:

$$A_{\ell,m}(\vec{r}) = r^{m/2} e^{-r/2} \alpha_{\ell,m}(r)$$

$$r \alpha''_{\ell,m}(r) + (1+m-r) \alpha'_{\ell,m}(r) - \frac{1}{2} \left[ m+1 + i\mu_{\ell} \hat{\sigma}_x \left( \mu_{\ell} - \frac{\Delta V}{2\rho} - \frac{1}{\mu_{\ell}^2} \right) \right] \alpha_{\ell,m}(r) = 0$$

eigenvalue must satisfy:

$$\mu_{\ell}^2 \left( \mu_{\ell} - \frac{\Delta V}{2\rho} \right) - 1 = \frac{i\mu_{\ell}}{\hat{\sigma}_x} (2\ell+m+1), \text{ for } \ell \geq 0, \ell \in \mathbb{Z}, m \in \mathbb{Z}$$

eigenvalues given by:

$$\mu_{\ell}^2 \left( \mu_{\ell} - \frac{\Delta V}{2\rho} \right) - 1 = \frac{i\mu_{\ell}}{\hat{\sigma}_x} (2\ell+1)$$

$$\text{if } \sigma_x \rightarrow \infty \Rightarrow \boxed{\mu_{\ell}^2 = \frac{1}{\mu_{\ell} - \frac{\Delta V}{2\rho}}}$$

$$A_{\ell,0}(\hat{r}) = L_{\ell}^0 \left( \frac{i\hat{r}^2}{\mu_{\ell} \hat{\sigma}_x} \right) \exp \left( -\frac{i\hat{r}^2}{2\mu_{\ell} \hat{\sigma}_x} \right)$$

$$L=0 \Rightarrow A_{00}(\hat{r}) = \exp \left( -\frac{i\hat{r}^2}{2\sigma_x^2} \right)$$