1.4A $$E_\omega = \frac{E_0\sqrt{\pi}}{\sigma_\omega}\sum_{a=1}^{N_e}\exp\left[\frac{-(\omega-\omega_1)^2}{4\sigma_\omega^2} + i\omega t_a\right]$$

$$N_{ph} = \frac{4\pi\epsilon_0 c\lambda^2}{\hbar\omega}\int d\omega\,|E_\omega|^2 = \frac{4\pi\epsilon_0 c\lambda^2}{\hbar\omega}\int d\omega\left(E_\omega\cdot E_\omega^*\right)$$

$$E_\omega\cdot E_\omega^* = \frac{\pi\epsilon_0^2}{\sigma_\omega^2}\sum_{a,b=1}^{N_e}\exp\left[\frac{-(\omega-\omega_1)^2}{2\sigma_\omega^2} + i\omega(t_a+t_b)\right]$$

$$N_{ph} = \frac{4\pi^2\epsilon_0^3 c\lambda^2}{\hbar\omega\sigma_\omega^2}\sum_{a,b=1}^{N_e}\int\exp\left[\frac{-(\omega-\omega_1)^2}{2\sigma_\omega^2} + i\omega(t_a+t_b)\right]d\omega$$

$$\bar{\omega} = \omega-\omega'$$

$$N_{ph} = \frac{4\pi^2\epsilon_0^3 c\lambda^2}{\hbar\omega\sigma_\omega^2}\sum_{a,b=1}^{N_e}\exp\left[i\omega.(t_a-t_b)\right]\int\exp\left(\frac{-\omega^2}{2\sigma_\omega^2}\right)d\bar{\omega}$$

$$\int\exp\left(-ax^2-2bx\right)dx = \sqrt{\frac{\pi}{a}}\exp\left(\frac{b^2}{a}\right)\qquad a=\frac{1}{2\sigma_\omega^2}\qquad b=\frac{-i(t_a-t_b)}{2}$$

$$N_{ph} = \frac{4\pi^2\epsilon_0^3 c\lambda^2}{\hbar\omega\sigma_\omega^{\cancel{2}}}\sqrt{2\cancel{\sigma_\omega^2}\pi}\sum_{a,b=1}^{N_e}\exp\left[i\omega(t_a-t_b)\right]\exp\left[\frac{-\sigma_\omega^2(t_a-t_b)^2}{2}\right]$$

$$N_{ph} = \frac{4\pi^2\epsilon_0^3 c\lambda^2\sqrt{2\pi}}{\hbar\omega\sigma_\omega}\sum_{a,b=1}^{N_e}\exp\left[i\omega\overset{0}{(t_a-t_b)} - \frac{\sigma_\omega^2}{2}\overset{0}{(t_a-t_b)}^2\right]$$

$$N_{ph} = \frac{4\pi^2\epsilon_0^3 c\lambda^2\sqrt{2\pi}}{\hbar\omega\sigma_\omega}\sum_{a=1}^{N_e}\sum_{b=1}^{N_e}1$$

$$N_{ph} = \frac{4\pi^2\epsilon_0^3 c\lambda^2\sqrt{2\pi}}{\hbar\omega\sigma_\omega}\sum_{a=1}^{N_e}N_e$$

$$N_{ph} = \frac{4\pi^2\epsilon_0 c\lambda^2\sqrt{2\pi}}{\hbar\omega\sigma_\omega}N_e^2 = \frac{T\sqrt{2\pi}\,N_e^2}{\sigma_\omega}$$

$$\langle N_{ph}\rangle = \frac{1}{N_e}N_{ph} = \frac{4\pi^2\epsilon_0 c\lambda^2\sqrt{2\pi}}{\hbar\omega\sigma_\omega}N_e$$

$$T = \frac{4\pi\epsilon_0 c\lambda^2}{\hbar\omega}\qquad \langle N_{ph}\rangle = \frac{T\sqrt{2\pi}}{\sigma_\omega}N_e$$

$$\langle N_{ph}\rangle^2 = (2\pi)\frac{N_e^2T^2}{\sigma_\omega^2}$$