

$$1.46 \quad N_{ph}^2 = T \int d\omega \frac{d\mathcal{L}}{d\omega} T \int d\omega' \frac{d\mathcal{L}}{d\omega'}$$

$$N_{ph}^2 = T^2 \left(\frac{\sqrt{2\pi}}{\sigma_\omega} N_e^2 \right) \left(\frac{\sqrt{2\pi}}{\sigma_{\omega'}} N_e^2 \right)$$

$$N_{ph}^2 = T^2 (2\pi) N_e^4$$

$$\langle N_{ph}^2 \rangle = \frac{T^2 (2\pi) N_e^3}{\sigma_\omega \sigma_{\omega'}}$$

$$\langle N_{ph} \rangle^2 = \frac{T^2 (2\pi) N_e^2}{\sigma_\omega^2}$$

$$\langle N_{ph}^2 \rangle = \langle N_{ph} \rangle^2 N_e$$

Q.E.D.

$$\langle N_{ph}^2 \rangle \approx \langle N_{ph} \rangle^2 \left\langle 1 + \frac{1}{N_e^2} \sum_{j \neq k} e^{-\sigma_\omega^2 (t_j - t_k)^2} \right\rangle$$