Philia 1.0. An ecological PK-SFC model.
https://github.com/lagoarde/philia

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1. Accounting structure

As shown in table A1, the model comprises seven institutional sectors: a household sector (divided into working and rentier households), a government sector (divided into the Treasury and public sector firms), a sector of social firms consisting of cooperatives owned by working households, a sector of listed non-financial corporations, a banking sector, a Central Bank, and a sector of investment funds comprising unlisted intermediaries (such as institutional investors, asset management companies, and insurance companies)

This matrix depicts a three-tiered productive structure. Public sector firms are operated by the government and their budget constraint is merged with that of the Treasury. Social firms are owned by working households and financially constrained as they only have access to retained earnings and bank loans. Listed firms are owned by investment funds and finance their investment with their retained earnings and by issuing the full range of debt market instruments (bank loans, corporate bonds, commercial paper) and equities.

The matrix also features two categories of households with segmented access to financial markets. Working households earn wages and redistributed surpluses from social firms and keep their unspent disposable income as cash and savings deposits. Rentier households, on the other hand, earn financial income only, and keep their unspent disposable income as cash, savings deposits and investment fund shares.

Available financial sector instruments include high-powered money (reserve assets and cash), advance loans from the Central Bank, savings account deposits, loans, bonds, commercial paper, equities, investment fund shares, Treasuries. The Central Bank operates a refinancing desk, a deposit facility, and holds a repurchased asset portfolio. Investment funds hold the banking sector's equity and listed corporations' equity (in addition to Treasuries and banking deposits) and distribute their entire profits to rentier households.

Finally, the model includes a full green taxonomy for all debt instruments, productive assets, and investment decisions (not shown in the matrix for clarity purposes).

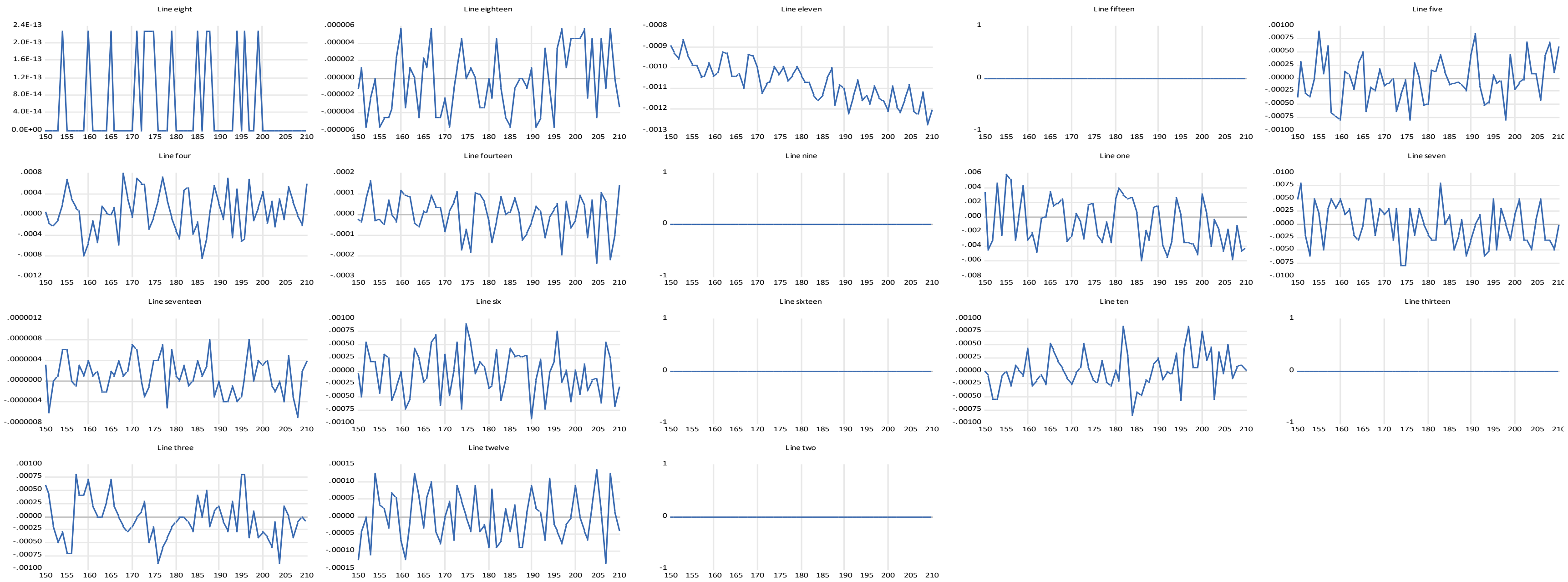
Figure A1, A2 and A3 check that the 28 rows and 15 columns of table 1 sum to zero. As a robustness check, each flow with its predicted transactional counterpart – the latter being computed using the relevant sector's balance sheet constraint. For example, taking line 10 of Table A1, we check that Central Bank profits (as stated in the Central Bank's income statement) are equal to Central Bank profit (as deducted from the Treasury's budget constraint) i.e. that $\hat{F}_{cb} + \Delta\widehat{GB} + \widehat{DEF}_G - \hat{I}_p + \widehat{DA}_p + \hat{F}_p + \Delta\hat{K}_{cb} = 0$. In the case of column 1 in Table 1, we check that $\left(\widehat{WB}_c + \widehat{WB_p} + \widehat{WB}_k + \widehat{FD}_c + i_d\widehat{M}_w - \hat{T}_w - \hat{C}_w - \Delta\widehat{M}_h - \Delta\widehat{H}_h\right) = 0$. The 43 accounting closure tests are shown in figure A2.

Table A1 Simplified transaction matrix

	Households		Government			SME/SSE		Listed firms		Banks		Central Bank		Investment funds	
	Working	Rentiers	Treasury	Public sector											
				Current	Capital	Current	Capital	Current	Capital	Current	Capital	Current	Capital	Current	Capital
Final consumption	$-C_w$	$-C_k$		$+C_p$		$+C_c$		$+C_k$							
Public expenditure			$-G$			$+G_c$		$+G_k$							
Taxes	$-T_w$	$-T_k$	$+T$			$-T_c$		$-T_k$							
Public deficit			$-DEF_g$		$+DEF_g$										
Investment				$+I_p$	$-I_p$	$+I_c$	$-I_c$	$+I_k$	$-I_k$						
Depreciation				$-DA_p$	$+DA_p$	$-DA_c$	$+DA_c$	$-DA_k$	$+DA_k$						
Wages	$+W$			$-W_p$		$-W_c$		$-W_k$							
Entrepreneurial profits	$+FD_c$			$-F_p$	$+F_p$	$-F_c$	$+FU_c$	$-F_k$	$+FU_k$					$+Div_k$	
Bank profits										$-F_b$				$+F_b$	
Central bank profits					$+F_{Cb}$							$-F_{Cb}$			
Investment fund profits		$+F_s$												$-F_s$	
Interest paid on :															
Central bank refinancing										$-r_aA$		$+r_aA$			
Private debt instruments						$-r_{d,c}D_c$		$-r_{d,k}D_k$		$+rD$					
Bank deposits	$+i_dM_w$	$+i_dM_m$								$-i_dM$				$+i_dM_s$	
Central bank portfolio						$-r_{d,c}R_c$		$-r_{d,k}R_k$				$+rR$			
Mandatory reserves										$+r_hH$		$-r_hH$			
Excess reserves										$+r_eH_e$		$-r_eH_e$			
Treasuries			$-r_gGB$							$+r_gGB_b$		$+r_gGB_{bc}$		$+r_gGB_s$	
Δ STOCKS															
Central bank loans											$+\Delta A$		$-\Delta A$		
Private debt instruments							$+\Delta D_C$		$+\Delta D_k$		$-\Delta D$				
Bank deposits	$-\Delta M_w$	$-\Delta M_k$									$+\Delta M$				$-\Delta M_s$
Reserve currency	$-\Delta H_w$	$-\Delta H_k$									$-\Delta H_b$		$+\Delta H$		
Equities									$+\Delta E_k$						$-\Delta E_k$
Investment fund shares		$-\Delta S$													$+\Delta S$
Central bank asset purchases											$+\Delta RA$		$-\Delta RA$		
Treasuries					$+\Delta GB$						$-\Delta GB_b$		$-\Delta GB_{cb}$		$-\Delta GB_s$
Bank equity											$-\Delta BE$				$+\Delta BE$
Central bank equity					$+\Delta K_{cb}$								$-\Delta K_{cb}$		

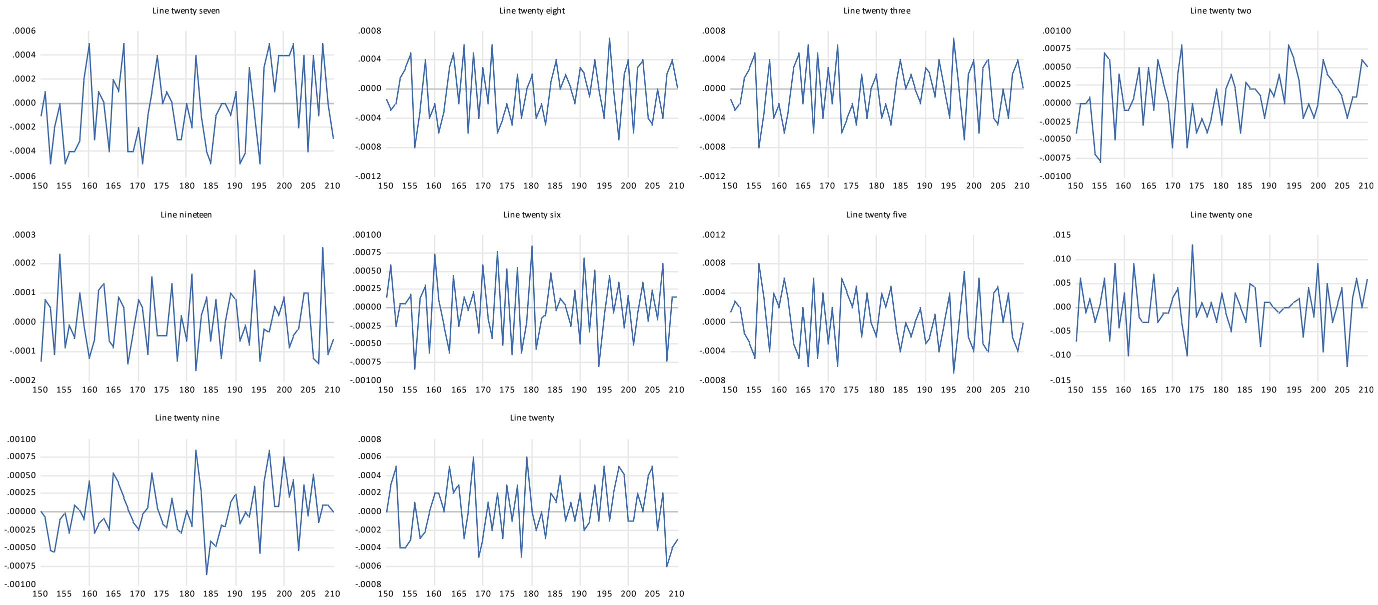
Note: To make reading easier, this matrix consolidates bank loans, private bonds and commercial paper (and their respective interest rates) into the ‘private debt instruments’ category. This matrix also does not show the green taxonomy applying to both productive and financial assets.

Figure A1 Accounting closure – income and spending flows



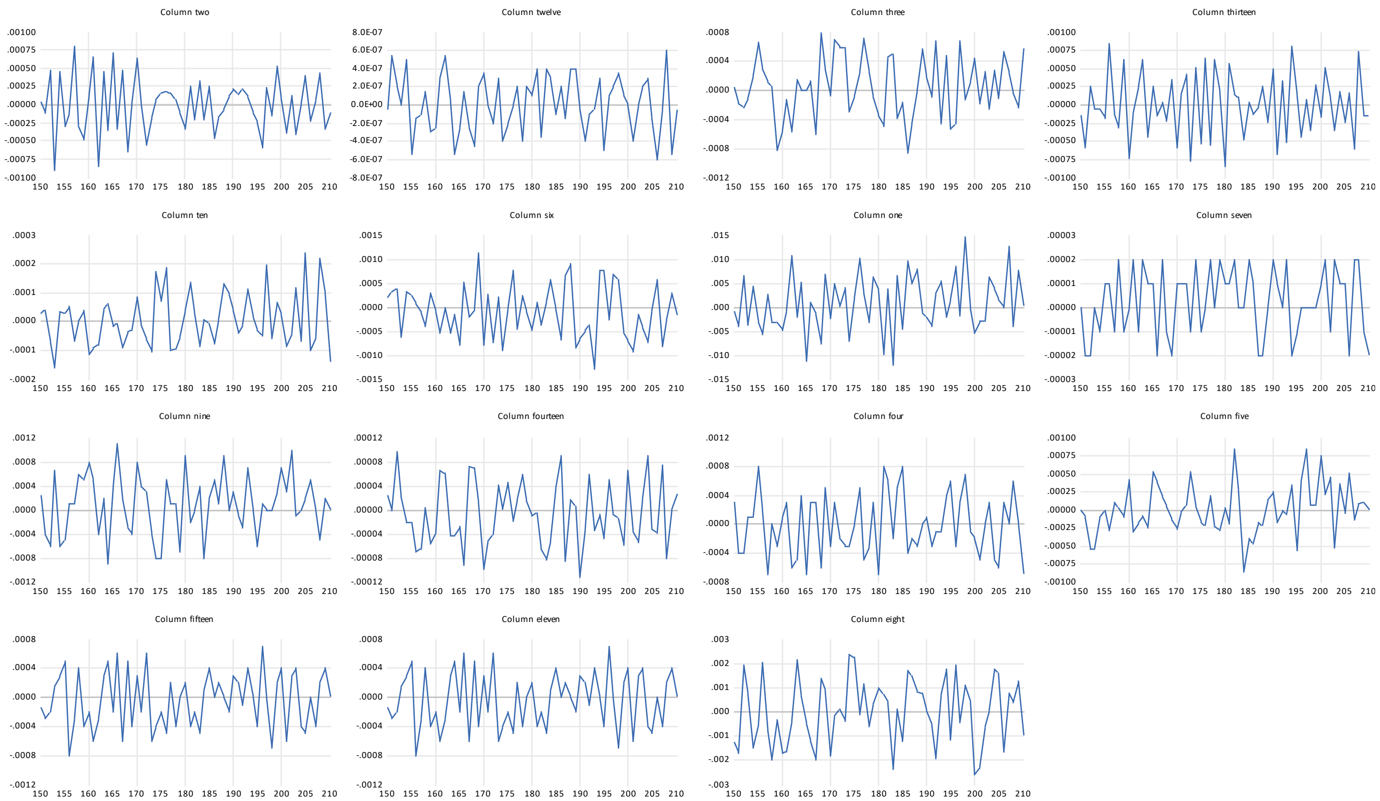
Note: this table shows the accounting closure of the income and spending flows (line 1 to 18 of table A1).

Figure A2 Accounting closure – financial assets and liabilities



Note: this table shows the accounting closure of adjustment of assets and liabilities (line 18 to 28 of table A1).

Figure A3 Accounting closure – sectoral income statement and balance sheet constraints



Note: this table shows the accounting closure of adjustment of assets and liabilities (columns 1 to 15 of table A1).

Table A2 describes the physical flow matrix, which is based on Carnevali et al. (2021). This matrix ensures first, that neither energy nor matter is created or destroyed during production (first law of thermodynamics), and second, that energy consumption dissipates in the form of heat (second law or 'entropy'). The closure of the ecosystemic blow can be checked in figures A4 and A5.

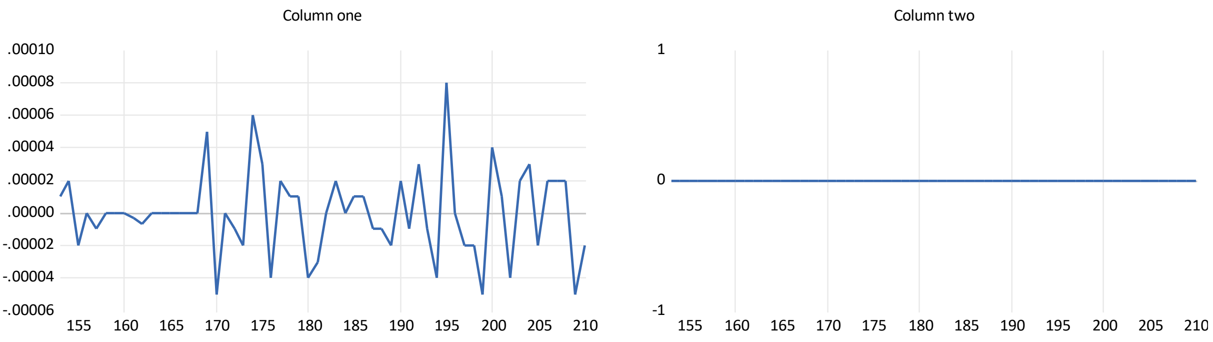
The first column contains the yearly material balance of the economy (in gigatons (Gt)). It equates material inputs (extracted carbon mass (M), non-renewable energy (CEN) and oxygen (02)) with annual material outputs (industrial CO2 emissions ($EMIS$), waste (W)), and change in the socio-economic stock (ΔSES). The second column shows the yearly energy balance (in Exajoules (E_j)). It equates annual energy inputs (renewable (RE) and non-renewable (NRE) reserves) with entropy (DE).

Table A2. Material and energy balance

	Material reserves (Gt)	Energy reserves (Ej)
Inputs		
Material extraction	$+M$	
Renewable energy		$+RE$
Non-renewable energy	$+CEN$	$+NRE$
Oxygen used in combustion	$+02$	
Outputs		
Industrial CO2 emissions	$-EMIS$	
Waste	$-W$	
Dissipated energy (entropy)		$-ED$
Variation of the socio-economic stock	$-\Delta\text{SES}$	
Total	0	0

Note: Additions are indicated by a (+) sign. Reductions are indicated by a (-) sign.

Figure A4 Geophysical closure



Note: this figure shows the value of column one and two of table A2.

Table A3 shows the physical stocks and flows matrix. It traces the joint evolution of the stock of physical reserves (REV_M), non-renewable energy reserves (REV_E), atmospheric CO2 concentration ($C02at$), and the socio-economic stock (SES). This matrix draws an important distinction between *resources* and *reserves*. Reserves are stocks of material and non-renewable energy resources that have been extracted and are available for production. The first row of the matrix contains the reserves inherited from the previous period. The last row displays the stocks available at the end of the annual production process.

Table A3. Stocks and physical flows matrix

	Material reserves	Non-renewable energy reserves	Atmospheric concentration of C02	Socio-economic stock
Initial stock	REV_{M-1}	REV_{E-1}	$C02at_{-1}$	SES_{-1}
Conversion of resources to reserves	$+CON_M$	$+CON_E$		
CO2 emissions			$+EMIS$	
Production of material goods				$+YM$
Extraction/use of material and energy	$-M$	$-NRE$		
Net transfer to the oceans and biosphere			$+(\phi_{11} - 1)C02at_{-1} + \phi_{21}CO2up_{-1}$	
Destruction of the socio-economic stock				$-DIS$
Final stock	REV_M	REV_E	$C02at$	SES

Note: Additions are indicated by a (+) sign. Reductions are indicated by a (-) sign.

Figure A5 Physical stock-flow consistency tests

Note: this figure shows the value of columns 1 to 4 of table A3.

2. Writing conventions

Real sector transaction flow variables are denoted by a capital letter. In the financial sector, a lower-case letter denotes a flow variable, and an upper-case letter denotes a stock variable. Regarding subscripts, the first subscript reads *s* or *d* to indicate whether the corresponding variable is supply or demand-side. The second subscript identifies the sector to which the variable belongs: the household sector (*h*), working households (*w*), rentier households (*r*), social firms (*c*), listed firms (*k*), banks (*b*), investment funds (*s*), the central bank (*cb*), and the public sector (*p*). The third subscript, when present, applies a green (*g*) or brown (*b*) taxonomy to the variable. A (-1) subscript indicates a lagged variable. A variable which is preceded by the sign Δ is first-differenced.

3. Macroeconomic income

Nominal GDP

Nominal GDP (Y) is the sum of expenditures on goods and services, including consumption (C), investment (I) and government spending (G) (equation 1.1). Investment (I) is the sum of listed corporations' investment (I_k), social firms' investment (I_c) and public sector firms' investment (I_p) (equation (1.2)). Macroeconomic final consumption (C) is the sum of working (C_w) and rentier households (C_r) consumption expenditures (equation (1.3)).

$$Y = C + I + G \tag{1.1}$$
$$I = I_k + I_c + I_p \tag{1.2}$$
$$C = C_w + C_r \tag{1.3}$$

Inflation and real GDP

Annual inflation (π) has three components: entrenched inflation expectations (π^a), and two idiosyncratic shocks related to the ecosystem (π^e), and firms' mark-up behavior over labor costs (π^f) (1.4). Inflation expectations (π^a) are adaptive and adjust to observed expectations errors (1.5). Firms are price-makers and adjust their prices to lagged increases in unit labor costs (π^f) (1.6). Ecosystemic inflation shocks (π^e) arise from the depletion of material reserves (dep_{l-1}), material resource (dep_{m-1}), and climate-related destruction (d_{t-1}) (1.7).

$$\pi = \pi^a + \pi^e + \pi^f \tag{1.4}$$
$$\pi^a = \pi_{-1} + o_1(\pi_{-1} - \pi^a_{-1}) \tag{1.5}$$
$$\pi^f = o_3\left(\frac{\Delta WB_{-1}}{Y_{-2}}\right) iff \ \frac{\Delta WB_{-1}}{Y_{-2}} > 0 \tag{1.6}$$
$$\pi^e = o_2(dep_{l-1} + dep_{m-1} + d_{t-1}) \tag{1.7}$$

The annual inflation rate (π) yields the macroeconomic price index (p) (equation 1.8) and real GDP $(\hat{Y})$ (1.9).

$$p = p_{-1}(1 + \pi) \tag{1.8}$$
$$\hat{Y} = \frac{Y}{p} \tag{1.9}$$

Productive capital assets

Total productive capital assets can be broken down by sector and according to the green taxonomy. The total stock of productive assets (K) is the sum of productive assets held by listed corporations (K_k), social firms (K_c) and public sector firms (K_p) (equation (1.10)). Macroeconomic depreciation expenditure (DA) is spent by listed firms (DA_k), social firms

(DA_c) and public sector firms (DA_p) (1.11). The stock of brown productive capital assets (K_b) is owned by listed corporations ($K_{k,b}$), social firms ($K_{c,b}$) and public sector firms ($K_{p,b}$) (1.12). The macroeconomic stock of green capital (K_g) is obtained with an accounting criterion (1.13).

$$K = K_k + K_c + K_p \tag{1.10}$$

$$DA = DA_k + DA_c + DA_p \tag{1.11}$$

$$K_b = K_{k,b} + K_{c,b} + K_{p,b} \tag{1.12}$$

$$K_g = K - K_b \tag{1.13}$$

Nominal and real household disposable income

Working and rentier households earn different types of income. Working households' nominal disposable income (YD_w) is the sum of the wage bill (WB), redistributed surpluses of social firms (P_c), net of self-financing (ret_c)), and the interest earned on the lagged stock of bank deposits ($i_d M_{w-1}$), minus the income tax (θ_w) (2.1).

$$YD_w = \left[WB + (1 - ret_c)P_c + i_d M_{d,w-1}\right](1 - \theta_w) \tag{2.1}$$

Rentier households' nominal income (YD_r) is the sum of the dividend payments of investment funds (Div_s), interest earned on lagged bank deposits ($i_r M_{r-1}$), minus the income tax (θ_r) (2.2).

$$YD_r = \left(Div_s + i_d M_{d,r-1}\right)(1 - \theta_r) \tag{2.2}$$

Real disposable household income is calculated by adjusting both nominal disposable income and nominal financial wealth (V) (2.3) and (2.4)[1].

$$\widehat{YD}_w = \frac{YD_w}{p} - \pi V_{w-1} \tag{2.3}$$

$$\widehat{YD}_r = \frac{YD_r}{p} - \pi V_{r-1} \tag{2.4}$$

The wage bill

The macroeconomic wage bill is determined by labor market negotiations between trade unions and firms, in which trade unions seek to keep the *ex-post* real wage share constant. The *ex-ante* wage bill (WB^a) is the product of the lagged wage share ($\iota_{wb,-1}$) and the current year's real GDP ($\hat{Y}$) (equation (2.5)).

$$WB^a = \iota_{wb,-1}\hat{Y} \tag{2.5}$$

The ex-ante nominal wage share is equal to the ex-ante wage bill (WB^a) divided by nominal GDP (Y) (2.6). In the presence of inflation, $\hat{Y} < Y$, and $\iota_{wb}^a < \iota_{wb,-1}$: the wage share decreases.

[1] See Godley and Lavoie (2012, p. 293) for a definition of inflation-accounted household real disposable income.

$$\iota_{wb}^{a} = \frac{WB^{a}}{Y} \tag{2.6}$$

Trade unions react by moving up the wage share target $(\iota_{wb}^{T)})$ during negotiations according to a parameter $(0 < \iota < 1)$ (2.7).

$$\iota_{wb}^{T} = \iota_{wb} + \iota\,(\iota_{wb} - \iota_{wb}^{a}) \tag{2.7}$$

The *ex-post* wage share (ι_{wb}) is determined by negotiations. It is equal to the ex-ante wage share (ι_{wb}^{a}) plus any positive distance between the wage share target (ι_{wb}^{T}) and the ex-ante wage share (ι_{wb}^{a}), factored by a parameter $(0 < \Omega_{wb} < 1)$ indicative of trade union's negotiating power (2.8).

$$\iota_{wb} = \iota_{wb}^{a} + (\iota_{wb}^{T} - \iota_{wb}^{a})\Omega_{wb} \tag{2.8}$$

The nominal wage bill (WB) equals the product of the lagged wage share $(\iota_{wb,-1})$ and nominal GDP (Y) (equation (2.9)).

$$WB = \iota_{wb,-1}Y \tag{2.9}$$

Tax payments

Taxes (T) are paid by working households' (T_w), rentier households (T_r), social firms (T_c) and listed firms (T_k) ((3.1) to (3.5))[2].

$$T_w = \theta_w\left[WB + (1 - ret_c)P_c + i_d M_{d,w-1}\right] \tag{3.1}$$

$$T_r = \theta_r\left[\left(Div_s + i_d M_{d,r-1}\right)\right] \tag{3.2}$$

$$T_c = \frac{\theta_c}{(1 - \theta_c)}P_c \tag{3.3}$$

$$T_k = \frac{\theta_k}{(1 - \theta_k)(1 - r_e)}P_k \tag{3.4}$$

$$T = T_w + T_r + T_c + T_k \tag{3.5}$$

4. Households

Nominal and real household consumption

[2] To understand equations (3.3) and (3.4), note that, letting S represent before-tax profit, P the after-tax profit, T tax payments, r_e the dividend payout ratio and θ the tax rate, then $P = S(1 - r_e)(1 - \theta) = S(1 - r_e) - T$. Given that $S = \frac{P}{(1-r_e)(1-\theta)}$, then $T = S - P = \frac{P}{(1-r_e)(1-\theta)} - P = \frac{\theta}{(1-r_e)(1-\theta)}P$. Equation (3.3.) is a specific case where $r_e = 0$.

Nominal working household consumption (C_w) depends on a minimum threshold (α_0), nominal annual disposable income (YD_w) (parameter α_1) and lagged financial wealth ($V_{w,-1}$) (parameter α_2) (equation (4.1)). Similar principles hold for rentier households (equation (4.2)).

$$C_w = \alpha_0 + \alpha_1 YD_w + \alpha_2(V_{w,-1}) \quad (4.1)$$
$$C_r = \alpha_0 + \alpha_1 YD_r + \alpha_2(V_{r,-1}) \quad (4.2)$$

Households react to ecosystem-caused destructions (d_{t-1}) building up precautionary savings, which reduces consumption. Household's propensity to draw on their stock of wealth (α_2) decreases with ecosystemic shocks (4.3).

$$\alpha_2 = \frac{\alpha_{2,-1}}{(1 + \vartheta d_{t-1})} \quad (4.3)$$

Real consumption $(\widehat{C_w})$ and $(\widehat{C_r})$ is a ratio between nominal consumption and the annual macroeconomic price index (p) (equations (4.4) and (4.5)).

$$\widehat{C_w} = C_w/p \quad (4.4)$$
$$\widehat{C_r} = C_r/p \quad (4.5)$$

Allocation of consumption

Nominal consumption expenditures are directed towards the social firm sector (C_c) and the public sector (C_p) as fixed proportions (($\alpha_{1,c}$) and ($\alpha_{1,p}$), respectively) of total nominal consumption ($C_{w,m} + C_{r,m}$) ((5.1) and (5.2)). Consumption expenditures allocated to the listed corporation sector (C_k) is determined with an accounting criterion (5.3).

$$C_c = \alpha_{1,c}[C_w + C_r] \quad (5.1)$$
$$C_p = \alpha_{1,p}[C_w + C_r] \quad (5.2)$$
$$C_k = C - C_c - C_p \quad (5.3)$$

Nominal and real financial wealth

Macroeconomic nominal financial wealth (V) is equal to its lagged value, plus the gap between macroeconomic disposable income (YD_w+ YD_r) and macroeconomic consumption (C) (6.1). A similar method determines working households' nominal financial wealth (V_w) (6.2). The nominal financial wealth of rentier households (V_r) is determined with an accounting criterion (6.3).

$$V = V_{-1} + (YD_w + YD_r - C) \quad (6.1)$$
$$V_w = V_{w-1} + (YD_w - C_w) \quad (6.2)$$
$$V_r = V - V_w \quad (6.3)$$

Macroeconomic real financial wealth ($\hat{V}$), working household's real financial wealth ($\widehat{V_w}$) and rentier households' real financial wealth ($\widehat{V_r}$) are obtained by dividing their nominal values by the macroeconomic price index (p) ((6.4) to (6.6)).

$$\hat{V} = \frac{V}{p} \tag{6.4}$$

$$\widehat{V_w} = \frac{V_w}{p} \tag{6.5}$$

$$\widehat{V_r} = \frac{V_r}{p} \tag{6.6}$$

Households' portfolio choice

Working households and rentier households have access to different financial assets. The only financial assets held by working households are cash and savings deposits. Working households wish to hold a baseline proportion (κ_{10}) of their wealth (V_w) as savings deposits ($M_{d,w}$) and another ($1-\kappa_{10}$) as cash ($H_{d,w}$). This baseline proportion is modulated by two elements: the deposit rate (i_d) (through parameter (κ_{11})) and the transaction motive of money demand ($\frac{YD_w}{V_{w-1}}$) (through parameter (κ_{12})). Following Godley and Lavoie (2012), we set $\kappa_{12} = -(\kappa_{11})$ (7.1).

$$\frac{M_{d,w}}{V_{w-1}} = \kappa_{10} + \kappa_{11} i_d + \kappa_{12} \frac{YD_w}{V_{w-1}} \tag{7.1}$$

Working households' demand for cash ($H_{d,w}$) is obtained with an accounting criterion (7.2).

$$H_{d,w} = V_w - M_{d,w} \tag{7.2}$$

The portfolio decision of rentier households is slightly more complex: in addition to cash and deposit accounts, rentier households can also purchase shares in investment funds ($S_{d,r}$). Their portfolio holdings depend on deposit rates (i_d), expected return on investment funds shares (r_i), and the transaction motive ($\frac{YD_r}{V_r}$) (8.1).

$$\begin{pmatrix} M_{d,r} \\ S_{d,r} \\ H_{d,r} \end{pmatrix} = \begin{pmatrix} \gamma_{10} \\ \gamma_{20} \\ \gamma_{30} \end{pmatrix} V_{r-1} + \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & \gamma_{14} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & -\gamma_{24} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} & \gamma_{34} \end{pmatrix} \begin{pmatrix} i_d \\ r_i \\ 0 \\ d_{t-1} \end{pmatrix} V_{r-1} + \begin{pmatrix} \gamma_{15} \\ \gamma_{25} \\ \gamma_{35} \end{pmatrix} YD_r \tag{8.1}$$

The elements contained in the first three columns of the above (4x3) parameter matrix are set so as to respect the two vertical constraints ($\sum_{i=1}^{I} \gamma_{i0} = 1$; $\sum_{i=1}^{I} \gamma_{ij} = 0$); as well as the symmetry constraints ($\gamma_{ij} = \gamma_{ji}$; $\forall i \neq j$) (Godley and Lavoie, 2012).
Adverse ecosystemic events affect portfolio allocations of rentier households by increasing their liquidity preference. Observing increased eco-systemic destructions (d_{t-1}) lead rentier households to hold a larger share of their wealth as cash and savings deposits. This is reflected by the last column of the parameter matrix, which states that $\gamma_{24} = -(\gamma_{14} + \gamma_{34})$. In addition, γ_{10} increases with adverse ecosystemic events, in proportion to a parameter Ω (8.2).

$$\gamma_{10} = \gamma_{10,-1}\big(1 + \Omega(d_{t-1})\big) \tag{8.2}$$

Instead of using the third line of matrix (8.1), we employ an accounting criterion. Rentier households' liquid holdings ($H_{d,r}$) equal their financial wealth (V_r) net of savings deposits ($M_{d,r}$) and portfolio of investment funds shares ($S_{d,r}$) (8.3).

$$H_{d,r} = V_r - M_{d,r} - S_{d,r} \tag{8.3}$$

Total household demand for banking deposits ($M_{d,h}$) is the sum of rentier and working households demand (9.1). Macroeconomic cash holdings ($H_{d,h}$) is obtained with an accounting criterion (9.2).

$$M_{d,h} = M_{d,w} + M_{d,r} \tag{9.1}$$
$$H_{d,h} = V - M_{d,h} - S_{d,r} \tag{9.2}$$

5. Social enterprises

Surplus

Social firms are cooperatives held by working households. Their financial surplus (P_c) is derived from their current account. It equates their annual turnover (final consumption (C_c), investment expenditure ($I_{s,c}$), plus a fixed proportion (ς_1) of central government spending (G) and public procurement of public enterprises ($I_{s,g}$)), net of the wage bill (WB_c), depreciation expenditure (DA_c), interest paid on the stock of 'brown' and 'green' bank loans ($i_{l,c,b}L_{s,c,b-1}$ and $i_{l,c,g}L_{s,c,g-1}$, respectively), and tax payments (at rate (θ_c)) (10).

$$P_c = (1-\theta_c)\left[C_c + I_{s,c} + \varsigma_1\left(G + I_{s,g}\right) - WB_c - DA_c - \left(i_{l,c,b}L_{s,c,b-1} + i_{l,c,g}L_{s,c,g-1}\right)\right] \tag{10}$$

Total investment demand

Social firms' productive assets (K_c) are divided into a green ($K_{c,g}$) and a brown ($K_{c,b}$) component (11).

$$K_c = K_{c,g} + K_{c,b} \tag{11}$$

Depreciation expenditures (DA_c) depends on the rate of obsolescence of productive capital assets. The latter is a fixed proportion ($0 < \lambda_0 < 1$) of lagged brown ($K_{c,b-1}$) and green ($K_{c,g-1}$) productive capital assets ((12.1) to (12.3)).

$$DA_c = DA_{c,b} + DA_{c,g} \tag{12.1}$$
$$DA_{c,b} = \lambda_0 K_{c,b-1} \tag{12.2}$$
$$DA_{c,g} = \lambda_0 K_{c,g-1} \tag{12.3}$$

Social firms' demand for investment follows the partial accelerator model (Mazier, 2020). In contrast to listed firms, social firms have limited market power and access to information. Their expectations are hence formed based on micro-level variables. The capital stock target (K_c^T)

increases with the lagged surplus rate ($\frac{P_{c,-1}}{K_{c,-1}}$) (according to parameter ι_1), this effect being modulated by the lagged debt to capital asset ratio ($\frac{D_{s,c,-1}}{K_{c,-1}}$) (according to a parameter ι_2) (13).

$$K_c^T = K_{c,-1}\left(1 + \iota_1\left(\frac{P_{c,-1}}{K_{c,-1}} - \iota_2\frac{D_{s,c,-1}}{K_{c,-1}}\right)\right) \tag{13}$$

Social firms' gross investment demand ($I_{d,c}$) has two components. The first is a partial adjustment ($v_{c1} < 1$) to the gap between the capital stock target (K_c^T) and the lagged capital stock ($K_{c,-1}$). The second component consists in depreciation expenditures (DA_c) (14.1).

$$I_{d,c} = v_{c1}\left(K_c^T - K_{c,-1}\right) + DA_c \tag{14.1}$$

Green structure of investment demand

The green structure of investment demand depends on three factors: sectoral adaptation efforts, a 'Mazzucato effect' (2018) linked to green public investment, and the relative cost of external finance. Social firms' baseline demand for green investment ($I_{d,c,g}$) is a proportion of total gross investment demand ($I_{d,c}$) (through parameter (ω_{c1})). This baseline demand is modulated by adaptation efforts to ecosystemic destructions ($ad \times d_{t-1}$), by increases in public green investment programs ($\frac{\Delta I_{s,p,g}}{I_{s,p,g-1}}$), and by the brown interest rate spread ($\omega_{c2} \times \left(i_{l,c,b} - i_{l,c,g}\right)$) (through a parameter ω_{c2}) ((14.1) and (14.2)).

$$I_{d,c,g} = \omega_{c1} I_{d,c}\left(1 + ad \times d_{t-1} + \frac{\Delta I_{s,g}}{I_{s,g,-1}} + \omega_{c2} \times \left(i_{l,c,b} - i_{l,c,g}\right)\right) \qquad \begin{matrix}(14.1)\\(14.2)\end{matrix}$$

Social firm's demand for brown investment $I_{d,c,b}$ is obtained with an accounting criterion (equation (14.3)).

$$I_{d,c,b} = I_{d,c} - I_{d,c,g} \tag{14.3}$$

Financial structure

Bank loans are the only form of external finance available to social firms. The demand for external finance follows the pecking order theory of corporate financing (Myers and Majluf, 1984). The demand for green ($l_{d,c,g}$) and brown ($l_{d,c,b}$) credit equals green ($I_{d,c,g}$) and brown ($I_{d,c,b}$) investment demand, minus an exogenous self-financing rate (ret_c) factored by surpluses (P_c) ((15.1) and (15.2)). The total demand for credit is obtained with an accounting criterion (15.3).

$$l_{d,c,g} = I_{d,c,g} - DA_{c,g} - P_c ret_c\left(\frac{I_{d,c,g}}{I_{d,c}}\right) \tag{15.1}$$

$$l_{d,c,b} = I_{d,c,b} - DA_{c,b} - P_c ret_c\left(\frac{I_{d,c,b}}{I_{d,c}}\right) \tag{15.2}$$

$$l_{d,c} = l_{d,c,g} + l_{d,c,b} \tag{15.3}$$

Investment spending

Brown investment expenditures ($I_{s,c,b}$) is the sum of new brown loans obtained from banks ($l_{s,c,b}$) and self-financing of brown projects ($P_c ret_c \frac{I_{d,c,b}}{I_{d,c}}$). Green investment is modeled along ((16.1) and (16.2)).

$$I_{s,c,b} = l_{s,c,b} + P_c ret_c \frac{I_{d,c,b}}{I_{d,c}} \tag{16.1}$$

$$I_{s,c,g} = l_{s,c,g} + P_c ret_c \frac{I_{d,c,g}}{I_{d,c}} \tag{16.2}$$

The above two equations distinguish the *effective* loan supply granted by the banking sector ($l_{s,c,b}$ and $l_{s,c,g}$) from the *notional*[3] demand for loans ($l_{d,c,b}$ and $l_{d,c,g}$) originating from entrepreneurial expectations. Given the fact that social firms do not issue equity, the distance between the notional demand and effective loan supply is a measure of investment rationing.

Total investment expenditure ($I_{s,c}$) is the sum of its green ($I_{s,c,g}$) and brown ($I_{s,c,b}$) components (16.3).

$$I_{s,c} = I_{s,c,g} + I_{s,c,b} \tag{16.3}$$

Financial liabilities

The effective loan supply ($l_{s,c}$) is the sum of brown ($l_{s,c,b}$) and green ($l_{s,c,g}$) credit granted by the banking sector (16.4). Stocks of green ($L_{s,c,g}$), brown ($L_{s,c,b}$) loans equal their past values, plus corresponding annual flows ((16.5) and (16.6)). The total debt ($L_{s,c}$) is given by an accounting criterion (16.7).

$$l_{s,c} = l_{s,c,b} + l_{s,c,g} \tag{16.4}$$

$$L_{s,c,g} = L_{s,c,g,-1} + l_{s,c,g} \tag{16.5}$$

$$L_{s,c,b} = L_{s,c,b,-1} + l_{s,c,b} \tag{16.6}$$

$$L_{s,c} = L_{s,c,b} + L_{s,c,g} \tag{16.7}$$

[3] While we use the term effective loan 'supply' to simplify notations, the flow of loans issued by the banking sector is always demand-driven: "*There is a notional demand curve, which corresponds to the demand for loans by entrepreneurs, according to their expectations. Then there is another demand curve, the 'effective' demand curve, which takes into account only the demand that responds to the conditions and expectations of the bankers*" (Lavoie, 2022, p. 248)

Productive assets

Social firm's stock of green and brown productive assets ($K_{c,g}$ and $K_{c,b}$, respectively) is the sum of their lagged value and new investment expenses ($I_{s,c,g}$ and $I_{s,c,b}$, respectively), net of depreciation allowances ($DA_{c,g}$ and $DA_{c,b}$, respectively) ((17.1) and (17.2)).

$$K_{c,g} = K_{c,g-1} + I_{s,c,g} - DA_{c,g} \tag{17.1}$$
$$K_{c,b} = K_{c,b-1} + I_{s,c,b} - DA_{c,b} \tag{17.2}$$

6. Listed firms

Profits and dividend payments

'Shareholder value maximization' has become the dominant governance corporate paradigm under neoliberalism (Duménil and Lévy, 2013). Equation (18) to (21) thus describe a finance-led governance for listed firms: dividend payments to outside shareholders take priority over the financing of new investment projects.

The gross profits of listed firms equals their annual turnover (i.e. the sum of final consumption (C_k), investment expenditure ($I_{s,k}$), a fixed proportion ($\varsigma_2 = 1 - \varsigma_1$) of government spending (G) and public procurement of public enterprises ($I_{s,g}$)), net of the wage bill (WB_k) depreciation expenditure (DA_k), and external financing costs. The latter include interest paid on 'brown' and 'green' bank loans (($i_{l,k,b}L_{s,k,b-1}$) and ($i_{l,k,g}L_{s,k,g-1}$), respectively), coupon payments on 'brown' and 'green' corporate bonds (($i_{b,b,-1}B_{s,b,-1}$) and $\left(i_{b,g,-1}B_{s,g,-1}\right)$, respectively), interest on 'brown' and 'green' corporate paper (($i_{cp,b}CP_{s,b-1}$) and ($i_{cp,g}CP_{s,g-1}$), respectively) and dividend payments to external shareholders. Net profits (P_k) equal gross profits, net of taxes (θ_k). Subtracting dividend payment (at rate ($r_{e,k}$)) yields listed firms' self-financing capacity (P_k) (18).

$$P_k = (1 - r_{e,k})(1 - \theta_k)\big[\, C_k + I_{s,k} + \varsigma_2\left(G + I_{s,p}\right) - WB_k - DA_k - \left(i_{l,k,b}L_{s,k,b,-1} + i_{l,k,g}L_{s,k,g,-1} + i_{b,b,-1}B_{s,b,-1} + i_{b,g,-1}B_{s,g,-1} + i_{cp,b}CP_{s,b-1} + i_{cp,g}CP_{s,g-1}\right)\big] \tag{18}$$

Listed firms are controlled by investment funds, which set an annual dividend payout ratio ($r_{e,k}$) expressed in nominal terms. The demand for dividend payments (Div_d) and the annual flow of dividend payment (Div_k) are shown in (19) and (20).

$$Div_d = \left(\frac{r_{e,k}}{1 - r_{e,k}}\right) P_k \tag{19}$$

$$Div_k = Div_d \tag{20}$$

The retained earnings (PU_k) allocated to the financing of productive investments are shown in (21).

$$PU_k = P_k \tag{21}$$

Total investment demand

Brown capital assets ($K_{k,b}$) are obtained by subtracting depreciation allowances ($DA_{k,b}$) and adding new brown investment ($I_{s,k,b}$) to the lagged stock of brown capital ($K_{k,b,-1}$) (22.2). A similar process is used to determine the stock of green capital assets (22.3). The total productive capital stock is obtained with an accounting criterion (22.1).

$$K_k = K_{k,b} + K_{k,g} \tag{22.1}$$
$$K_{k,b} = K_{k,b,-1} - DA_{k,b} + I_{s,k,b} \tag{22.2}$$
$$K_{k,g} = K_{k,g,-1} - DA_{k,g} + I_{s,k,g} \tag{22.3}$$

Depreciation and amortization expenditures (DA_k) are equal to the lagged capital stock, factored by a homogenous rate of capital obsolescence (0< $\lambda_0 < 1$) ((23.1) to (23.3)).

$$DA_k = DA_{k,b} + DA_{k,g} \tag{23.1}$$
$$DA_{k,b} = \lambda_0 K_{k,b-1} \tag{23.2}$$
$$DA_{k,g} = \lambda_0 K_{k,g-1} \tag{23.3}$$

In contrast to social businesses, listed corporations have a significant market power and access to information. They hence make productive bets based on global macroeconomic prospects. Their productive capital target (K_K^T) is set as a proportion (*k*) of nominal GDP (equation (24.1)).

$$K_K^T = kY_{-1} \tag{24.1}$$

Investment demand ($I_{d,k}$) has two components: the first is a partial adjustment ($v_{k1} < 1$) to the distance between the capital target (K_k^T) and the lagged capital stock ($K_{k,-1}$). The second component consists in the depreciation expenditures (DA_k) (24.2).

$$I_{d,k} = v_{k1}(K_k^T - K_{k-1}) + DA_k \tag{24.2}$$

Green structure of investment demand

The baseline demand for green investment ($I_{d,k,g}$) is a proportion of total gross investment demand (through a parameter (ω_{k1})). This proportion increases with ecosystemic destruction which generates an adaptive behavior ($ad \times d_{t-1}$). The scaling up of green investment by the public sector firms ($\frac{\Delta I_{s,p,g}}{I_{s,g,-1}}$) leads listed firms to increase the proportion of green investment in their total investment. This proportion increases with the relative cost of brown and green loans ($\omega_{c2} \times (i_{l,k,b} - i_{l,k,g})$) (through a parameter ω_{k2}) (equation (25.1)).

$$I_{d,k,g} = \omega_{k1} I_{d,k}\left(1 + ad \times d_{t-1} + \frac{\Delta I_{s,p,g}}{I_{s,p,-1}} + \omega_{k2} \times (i_{l,k,b} - i_{l,k,g})\right) \tag{25.1}$$

The demand for brown investment is obtained with an accounting criterion (25.2).

$$I_{d,k,b} = I_{d,k} - I_{d,k,g} \tag{25.2}$$

Investment financing

In contrast to social firms, listed firms have access to the full range of debt instruments and can also issue equity to finance their investment program. Following the pecking order theory of corporate finance (Myers and Maljuf, 1984), listed corporations prioritize retained earnings to finance their investment expenditure. Then, they turn to the credit and the debt market, and issue new equities in last resort to fill the finance gap.
This process is reflected in equations (26.1) to (26.5). Equations (26.1) gives the notional demand for credit $(f_{d,k})$ as the sum of 'green' and 'brown' demand components $((f_{d,k,g})$ and $(f_{d,k,b})$, respectively).

$$f_{d,k} = f_{d,k,g} + f_{d,k,b} \tag{26.1}$$

In turn, the notional demand for green $(f_{d,k,g})$ and brown $(f_{d,k,b})$ credit is equal to gross investment demand $((I_{d,k,g})$ and $(I_{d,k,b})$, respectively), net of depreciation expenditures $((DA_{k,g})$ and $(DA_{k,b})$, respectively), and retained earnings $((PU_k\left(\frac{I_{d,k,g}}{I_{d,k}}\right))$ and $(PU_k\left(\frac{I_{d,k,b}}{I_{d,k}}\right))$, respectively) ((26.2) and (26.3)).

$$f_{d,k,g} = I_{d,k,g} - DA_{k,g} - PU_k\left(\frac{I_{d,k,g}}{I_{d,k}}\right) \tag{26.2}$$

$$f_{d,k,b} = I_{d,k,b} - DA_{k,b} - PU_k\left(\frac{I_{d,k,b}}{I_{d,k}}\right) \tag{26.3}$$

Equity issues $(e_{s,k})$ cover the distance between the notional demand for credit $(f_{d,k})$ and the effective demand for credit $(f_{s,k} \leq f_{d,k})$ (26.4). To the extent that sufficient loanable funds are available, listed firms' investment is virtually unconstrained.

$$e_{s,k} = f_{d,k} - f_{s,k} \tag{26.4}$$

Investment spending

Investment expenditure $(I_{s,k})$ is the sum of brown and green investment expenditure $((I_{s,k,b})$ and $(I_{s,k,g})$, respectively) (27.1). Brown and green investment expenditure are the sum of corporate debt $(f_{s,k})$, self-financing (PU_k), and equity issues $(e_{s,k})$ ((27.2) and (27.3)).

$$I_{s,k} = I_{s,k,b} + I_{s,k,g} \tag{27.1}$$

$$I_{s,k,g} = f_{s,k,g} + PU_k\left(\frac{I_{d,k,g}}{I_{d,k}}\right) + e_{s,k}\left(\frac{I_{d,k,g}}{I_{d,k}}\right) \tag{27.2}$$

$$I_{s,k,b} = f_{s,k,b} + PU_k\left(\frac{I_{d,k,b}}{I_{d,k}}\right) e_{s,k}\left(\frac{I_{d,k,b}}{I_{d,k}}\right) \tag{27.3}$$

Financial liabilities

New debt liabilities include green and brown commercial paper $((cp_{s,g})$ and $(cp_{s,b})$, respectively), green and brown corporate bonds $((b_{s,g})$ and $(b_{s,b})$, respectively) and green and brown loans (($l_{s,k,g}$) and ($l_{s,k,b}$), respectively) ((28.1) to (28.7)). These flows, which permit to identify the breakdown of corporate debt, are driven by banking sector's liquidity preference (hence the 'd' suffix on the right-hand term of each equation). The behavior of banks is discussed in section 7.

$$cp_{s,b} = cp_{d,b} \tag{28.1}$$
$$cp_{s,g} = cp_{d,g} \tag{28.2}$$
$$b_{s,b} = b_{d,b} \tag{28.3}$$
$$b_{s,g} = b_{d,g} \tag{28.4}$$
$$l_{s,k,b} = l_{d,k,b} \tag{28.5}$$
$$l_{s,k,g} = l_{d,k,g} \tag{28.6}$$
$$f_{s,k} = f_{s,k,b} + f_{s,k,g} \tag{28.7}$$

Stocks of financial liabilities include equities (E_s), total, brown and green corporate paper ((CP_s), ($CP_{s,b}$) and ($CP_{s,g}$), respectively), total, brown and green corporate bonds (($B_{s,k}$), ($B_{s,b}$) and ($B_{s,g}$), respectively), total brown and green loans (($L_{s,k}$), ($L_{s,k,b}$) and($L_{s,k,g}$), respectively). These are recorded at their historical value and decomposed into a green and brown component. Each stock is equal to its lagged value, plus the annual flow ((28.8) to (28.17)).

$$E_s = E_{s,-1} + e_{s,k} \tag{28.8}$$
$$CP_{s,b} = CP_{s,b,-1} + cp_{s,b} \tag{28.9}$$
$$CP_{s,g} = CP_{s,g,-1} + cp_{s,g} \tag{28.10}$$
$$CP_s = CP_{s,b} + CP_{s,g} \tag{28.11}$$
$$B_{s,b} = B_{s,b,-1} + b_{s,b} \tag{28.12}$$
$$B_{s,g} = B_{s,g,-1} + b_{s,g} \tag{28.13}$$
$$B_{s,k} = B_{s,b} + B_{s,g} \tag{28.14}$$
$$L_{s,k,b} = L_{s,k,b,-1} + l_{s,k,b} \tag{28.15}$$
$$L_{s,k,g} = L_{s,k,g-1} + l_{s,k,g} \tag{28.16}$$
$$L_{s,k} = L_{s,k,b} + L_{s,k,g} \tag{28.17}$$

Total debt ($D_{s,k}$) is the sum of its brown and green components (($D_{s,k,b}$) and ($D_{s,k,g}$), respectively) (28.18). Each components includes corporate paper (($CP_{s,b}$) and ($CP_{s,g}$), respectively), corporate bonds (($B_{s,b}$) and ($B_{s,g}$), respectively) and loans (($L_{s,k,b}$) and ($L_{s,k,g}$), respectively) ((28.19) and (28.20)).

$$D_{s,k} = D_{s,k,b} + D_{s,k,g} \tag{28.18}$$
$$D_{s,k,b} = CP_{s,b} + B_{s,b} + L_{s,k,b} \tag{28.19}$$
$$D_{s,k,g} = CP_{s,g} + B_{s,g} + L_{s,k,g} \tag{28.20}$$

7. Banks

Cash and deposit liabilities

Commercial banks fully accommodate households' demand for cash ($H_{d,h}$) and the demand for bank deposits from households ($M_{d,h}$) and investment funds ($M_{d,s}$) ((29.1) and (29.2)).

$H_{s,h} = H_{d,h}$ (29.1)
$M_{s,h} = M_{d,h} + M_{d,s}$ (29.2)

Inside money creation

Inside money creation (m_b) is the sum effective credit granted to social firms ($l_{s,c}$) and listed firms ($f_{s,k}$) (30.1). It is decomposed into a 'green' ($m_{b,g}$) and a 'brown' ($m_{b,b}$) component ((30.2) and (30.3)). The stock of inside money is the sum of its lagged value, plus the new flow (30.4).

$m_b = l_{s,c} + f_{s,k}$ (30.1)
$m_{b,g} = l_{s,c,g} + l_{s,lcc} + f_{s,k,g}$ (30.2)
$m_{b,b} = l_{s,c,b} + f_{s,k,b}$ (30.3)
$M_b = M_{b,-1} + m_b$ (30.4)

Banking credit

Banks do not issue credit to borrowers and projects that they deem unworthy. The effective supply of credit is determined based on the post-Keynesian principle of increasing risk. Banks consider the financial position of borrowers, the project type, as well as their own expectations about the economy.
However, banks have tight links to listed firms, through the cross shareholding of investment funds. As a result, they are more eager to open credit lines to listed firms than to social firms.
Banks indeed apply strict collateral requirements to social firms. Green and brown credit of to social firms (($l_{s,c,g}$) and ($l_{s,c,b}$)) are zero when the latter's debt stock ($L_{s,c,-1}$) exceeds a given fraction (γ) of the lagged capital stock ($K_{c,-1}$). Banks then apply a sector-specific and a taxonomy-specific credit risk score ($LR_{c,g-1}$ and $LR_{c,b-1}$, respectively) to borrowers that satisfy the collateral condition ((31.1) and (31.2)).

$l_{s,c,g} = l_{d,c,g}\left(1 - LR_{c,g-1}\right)\ iff\ L_{s,c,-1} < \gamma^{-1}K_{c,-1}$ (31.1)
$l_{s,c,b} = l_{d,c,b}\left(1 - LR_{c,b-1}\right) iff\ L_{s,c,-1} < \gamma^{-1}K_{c,-1}$ (31.2)

By contrast, banks do not screen listed firms for collateral. Effective credit supply (($f_{s,k,g}$) and ($f_{s,k,b}$)) depends on sector-specific and a taxonomy-specific scores ($LR_{k,b,-1}$ and $LR_{k,g,-1}$, respectively) ((31.3) and (31.4)).

$f_{s,k,g} = f_{d,k,g}\left(1 - LR_{k,g,-1}\right)$ (31.3)
$f_{s,k,b} = f_{d,k,b}\left(1 - LR_{k,b,-1}\right)$ (31.4)

The lending risk scores for both types of borrowers (($LR_{c,g}$) and ($LR_{k,g}$)) increase with leverage ($\left(\frac{L_{s,c}}{K_c}\right)$ and $\left(\frac{D_{s,k}}{K_k}\right)$, respectively), decrease with increase with banks' expectations (observed economic growth $\left(\frac{\Delta Y}{Y_{-1}}\right)$) , and increase the Central Bank's main refinancing operations rate i_e ((32.1) and (32.2)).

$$LR_{c,g} = a_1\left(\frac{L_{s,c}}{K_c}\right) - b_1.\left(\frac{\Delta Y}{Y_{-1}}\right) + c_1.i_e \tag{32.1}$$

$$LR_{k,g} = a_1\left(\frac{D_{s,k}}{K_k}\right) - b_1.\left(\frac{\Delta Y}{Y_{-1}}\right) + c_1.i_e \tag{32.2}$$

Brown lending risk equals green lending risk, plus a risk premium which increases with eco-systemic damages (d_{t-1}), through a parameter (Ψ) ((32.3) and (32.4)). Banks' relative eagerness to provide brown credit thus internalizes the costs of ecosystemic destruction.

$$LR_{c,b} = LR_{c,g}(1 + \Psi d_{t-1}) \tag{32.3}$$

$$LR_{k,b} = LR_{k,g}(1 + \Psi d_{t-1}) \tag{32.4}$$

Debt structure

Following Le Héron and Mouakil (2008), the structure of listed firms' brown and green debt $((f_{s,k,b})$ and $(f_{s,k,g})$ respectively) is driven by banks' preference for liquidity. The latter depends on observed returns for green and brown bonds $((r_{b,b})$ and $(r_{b,g}))$, loan portfolios $((r_{l,k,b})$ and $(r_{l,k,g}))$ and interest rates on corporate paper $((i_{cp,b})$ and $(i_{cp,g}))$ ((33.1) and (33.2)).

$$\begin{pmatrix} b_{d,b} \\ l_{d,k,b} \\ cp_{d,b} \end{pmatrix} = \begin{pmatrix} \chi_{10} \\ \chi_{20} \\ \chi_{30} \end{pmatrix}.f_{s,k,b} + \begin{pmatrix} \chi_{11} & -\chi_{12} & -\chi_{13} \\ -\chi_{21} & \chi_{22} & -\chi_{23} \\ -\chi_{31} & -\chi_{32} & \chi_{33} \end{pmatrix}.\begin{pmatrix} r_{b,b} \\ r_{l,k,b} \\ i_{cp,b} \end{pmatrix} f_{s,k,b} \tag{33.1}$$

$$\begin{pmatrix} b_{d,g} \\ l_{d,k,g} \\ cp_{d,g} \end{pmatrix} = \begin{pmatrix} \chi_{10} \\ \chi_{20} \\ \chi_{30} \end{pmatrix}.f_{s,k,g} + \begin{pmatrix} \chi_{11} & -\chi_{12} & -\chi_{13} \\ -\chi_{21} & \chi_{22} & -\chi_{23} \\ -\chi_{31} & -\chi_{32} & \chi_{33} \end{pmatrix}.\begin{pmatrix} r_{b,g} \\ r_{l,k,g} \\ i_{cp,g} \end{pmatrix} f_{s,k,g} \tag{33.2}$$

The parameters of the matrix respect both *vertical* $(\sum_{i=1}^{I} \chi_{i0} = 1\ ; \sum_{i=1}^{I} \chi_{ij} = 0)$ and *symmetry* constraints: $\chi_{ij} = \chi_{ji};\ \forall i \neq j$ (Godley and Lavoie, 2012). We replace the last line of the above matrices with an accounting criteria ((34.1) and (34.2)).

$$cp_{d,b} = f_{s,k,b} - b_{d,b} - l_{d,k,b} \tag{34.1}$$

$$cp_{d,g} = f_{s,k,g} - b_{d,g} - l_{d,k,g} \tag{34.2}$$

Equations (34.3), (34.4) and (34.5) give the annual flows of corporate paper (cp_d), corporate bonds (b_d) and corporate loans $(l_{d,k})$.

$$cp_d = cp_{d,g} + cp_{d,b} \tag{34.3}$$

$$b_d = b_{d,g} + b_{d,b} \tag{34.4}$$

$$l_{d,k} = l_{d,k,g} + l_{d,k,b} \tag{34.5}$$

Quantitative easing

The outstanding stock of corporate debt, however, is not always entirely held by the banking sector. Consider for instance equation (35.1). It shows that the stock of brown corporate paper $(CP_{d,b})$ held by the banking sector is equal to listed corporations' stock of brown corporate paper liabilities ($CP_{s,b}$) net of lagged brown corporate paper repurchases by the Central Bank ($qe_{cp,b,-1}$). Stocks of green corporate paper ($CP_{d,g}$), brown ($B_{d,b}$) and green ($B_{d,g}$) bonds, and brown ($L_{d,k,b}$) and green ($L_{d,k,g}$) loans to listed firms, and brown ($L_{d,c,b}$) and green ($L_{d,c,g}$) loans to social firms are determined in the same way ((35.2) to (35.8)). The historical value of corporate paper (CP_d), corporate bonds (B_d) and loans (L_d) showing up in banks' balance sheets is shown in (35.9) to (35.11).

$$CP_{d,b} = CP_{s,b} - qe_{cp,b,-1} \tag{35.1}$$
$$CP_{d,g} = CP_{s,g} - qe_{cp,g,-1} \tag{35.2}$$
$$B_{d,b} = B_{s,b} - qe_{b,b,-1} \tag{35.3}$$
$$B_{d,g} = B_{s,g} - qe_{b,g,-1} \tag{35.4}$$
$$L_{d,k,b} = L_{s,k,b} - qe_{l,k,b,-1} \tag{35.5}$$
$$L_{d,k,g} = L_{s,k,g} - qe_{l,k,g,-1} \tag{35.6}$$
$$L_{d,c,b} = L_{s,c,b} - qe_{l,c,b,-1} \tag{35.7}$$
$$L_{d,c,g} = L_{s,c,g} - qe_{l,c,g,-1} \tag{35.8}$$
$$CP_d = CP_{d,b} + CP_{d,g} \tag{35.9}$$
$$B_d = B_{d,b} + B_{d,g} \tag{35.10}$$
$$L_d = L_{d,b} + L_{d,g} \tag{35.11}$$

The outstanding stocks of green and brown loans ($L_{s,b}$ and $L_{s,g}$, respectively), total debt instruments (D_s), and the stock of debt instruments issued by listed firms (F_s) are have their counterpart not in the balance sheet of banks but in that of the issuing sector ((35.12) to (35.15)).

$$L_{s,b} = L_{s,k,b} + L_{s,c,b} \tag{35.12}$$
$$L_{s,g} = L_{s,k,g} + L_{s,c,g} \tag{35.13}$$
$$D_s = L_{s,c} + D_{s,k} \tag{35.14}$$
$$F_s = D_{s,k} \tag{35.15}$$

Refinancing operations

Banks' mandatory holdings of Central Bank reserves ($H_{d,m}$) are a fixed ratio (ρ_1) of deposits ($M_{s,-1}$)) (36.1).

$$H_{d,m} = \rho_1 .(M_{s,-1}) \tag{36.1}$$

The banking sector's stock of Central Bank loans ($A_{d,b}$) equals the required volume of reserves ($H_{d,m}$), net of reserves supplied by the Central Bank through quantitative easing ($H_{s,qe}$) (36.2).

$$A_{d,b} = \left(H_{d,m} - H_{s,qe}\right) iff H_{d,m} - H_{s,qe} > 0 \tag{36.2}$$

The Central Bank accommodatively supplies reserve loans ($A_{s,b}$) to the banking sector (Moore, 1988)(36.3).

$$A_{s,b} = A_{d,b} \tag{36.3}$$

Banks accommodate the Central Bank's asset repurchase programs, so that the demand for reserve money ($H_{d,qe}$) issued through quantitative easing is equal to its supply ($H_{s,qe}$) (36.5).

$$H_{d,qe} = H_{s,qe} \tag{36.5}$$

Banks' holdings of excess reserves (H_{ex}) parked at the deposit facility is the gap between total reserve holdings (H_d) and the sum of mandatory reserves ($H_{d,m}$) and circulating cash ($H_{d,h}$) (equation 36.6)[4].

$$H_{ex} = H_d - (H_{d,m} + H_{d,h}) \tag{36.6}$$

Income statement

The income of the banking sector is the sum of the interest earned on Treasuries $\left(i_{gb} GB_{d,b,-1}\right)$, brown and green loans to listed firms ($\left(i_{l,k,b} L_{d,k,b,-1}\right)$ and ($i_{l,k,g} L_{d,k,g,-1}$), respectively), brown and green loans to social firms ($\left(i_{l,c,b} L_{d,c,b,-1}\right)$ and ($i_{l,c,g} L_{d,c,g,-1}$), respectively), brown and green commercial paper ($\left(i_{cp,b} CP_{d,b,-1}\right)$ and $\left(i_{cp,g} CP_{d,g,-1}\right)$, respectively), brown and green corporate bonds ($\left(i_{b,b}\ B_{d,b,-1}\right)$ and $\left(i_{b,g} B_{d,g,-1}\right)$, respectively), required reserves holdings ($i_e H_{d,m,-1}$) and excess reserves holdings ($i_{df} H_{ex,-1}$).
As in the Eurozone, required reserves holdings are remunerated at the main refinancing rate (i_e), and excess reserves are remunerated at the deposit facility rate (i_{df}). Banks' expenses include interest paid out to depositors $\left(i_d M_{h,d-1}\right)$ and to the Central Bank $\left(i_e A_{s,b-1}\right)$.
Banks' profits (P_b) are the difference between income and expenses (equation (37.1)). These profits are transferred to investments funds as dividends (Div_b) (37.2).

$$\begin{aligned} P_b = {} & i_{gb}\left(GB_{d,b,-1}\right) + i_{l,k,b}\left(L_{d,k,b,-1}\right) + i_{l,k,g}\left(L_{d,k,g,-1}\right) + i_{l,c,b}\left(L_{d,c,b,-1}\right) + \\ & i_{l,c,g}\left(L_{d,c,g,-1}\right) + i_{cp,b}\ \left(CP_{d,b,-1}\right) + i_{cp,g}\ \left(CP_{d,g,-1}\right) + i_{b,g}\ \left(B_{d,g,-1}\right) + \\ & i_{b,b}\ \left(B_{d,b,-1}\right) + i_e H_{d,m,-1} + i_{df} H_{ex,-1} - \left[i_{d-1}\left(M_{s,-1}\right) + i_e\left(A_{s,b-1}\right)\right] \end{aligned} \tag{37.1}$$

$$Div_b = \ P_b z_1; z_1 = 1\ iff P_b\ > 0 \tag{37.2}$$

Banks' balance sheet

Since the 2008 financial crisis, Central Banks frequently intervene in secondary markets for financial stabilization purposes[5]. In this model, the Central Bank undertakes quantitative easing (QE) operation, when necessary, to align the capital adequacy ratio with its regulatory target. Whenever the banking sector's capital risk-weighted ratio falls below its safe value (CAR^T),

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[4] Each individual bank may of course exchange these excess reserves for interest-bearing financial instruments. In this case, excess reserves are transferred from one bank's balance sheet to another, leaving the consolidated balance sheet of the banking sector, as well as the amount showing up as a liability on the central bank's balance sheet, unchanged.

[5] In the case of the euro area: "*monetary policy cannot turn a blind eye to rising financial stability risks. This was one of the main conclusions of our monetary policy strategy review, which we completed in July of this* year" (Isabel Schnabel, executive board of the ECB, December 2021). See https://www.ecb.europa.eu/press/key/date/2021/html/ecb.sp211208_2~97c82f5cfb.en.html

the Central Bank extracts risky assets from banks' balance sheets by creating new reserves deposits.
To calculate banks' balance sheets, one thus needs to distinguish between banks' *ex-ante* and *ex-post* asset portfolios (BA_a and BA_e, respectively) (i.e. before and after any quantitative easing intervention).
The *ex-ante* portfolio (BA_a) consists of mandatory reserve holdings ($H_{d,m}$), Treasuries holdings valued at market price ($p_{gb}GB_{d,b}$), the total stock of risky assets (also valued at market price) (38.1).

$$BA_a \;= H_{d,m} + p_{gb}GB_{d,b} + \; p_{l,k,b}\, L_{s,k,b} + \; + p_{l,k,g}\, L_{s,k,g} + p_{l,c,b}\, L_{s,c,b} + p_{l,c,g}\, L_{s,c,g} + \;\; p_{b,b}\, B_{s,b} + \;\; p_{b,g}\, B_{s,g} + p_{cp,b}CP_{s,b} + p_{cp,g}CP_{s,g} \tag{38.1}$$

The ex-post portfolio (BA_e) consists of mandatory reserve holdings ($H_{d,m}$), reserve assets accumulated as a result of quantitative easing ($H_{d,qe}$), Treasuries valued at market price ($p_{gb}GB_{d,b}$), and banks' final holding of risky assets (valued at market price) (38.2).

$$BA_e \;= H_{d,m} + H_{d,qe} + \; p_{gb}GB_{d,b} + \; p_{l,k,b}\, L_{d,k,b} + \; + p_{l,k,g}\, L_{d,k,g} + p_{l,c,b}\, L_{d,c,b} + p_{l,c,g}\, L_{d,c,g} + \;\; p_{b,b}\, B_{d,b} + \;\; p_{b,g}\, B_{d,g} + p_{cp,b}CP_{d,b} + p_{cp,g}CP_{d,g} \tag{38.2}$$

Banks' liabilities (BL) feature reserves loans from the Central Bank ($A_{s,b}$) and total deposits (M_s)) (38.3).

$$BL = \left(A_{s,b} + M_s\right) \tag{38.3}$$

The banking sector's equity, with or without Central Bank intervention ($BE_a$and BE_e, respectively) is deducted from its balance sheet (38.4) and (38.5).

$$BE_a = BA_a - BL \tag{38.4}$$

$$BE_e = BA_e - BL \tag{38.5}$$

Capital adequacy ratio (CAR)

We shall again distinguish the *ex-ante* from the *ex-post* capital adequacy ratio (CAR^a and CAR^e, respectively) (i.e. before and after any quantitative easing) ((38.6) and (38.7)).

$$CAR^a = \frac{BE^a}{WR^a} \tag{38.6}$$

$$CAR^e = \frac{BE^e}{WR^e} \tag{38.7}$$

From a prudential perspective, the upper bound for banks' weighted risky asset holdings is given by the ratio between their *ex-ante* equity (BE_a) and the regulatory CAR target (CAR^T) (equation (38.6).

$$WR^{max} = \frac{BE_a}{CAR^T} \tag{38.6}$$

Given the *ex-ante* value of banks' weighted risky assets (WR^a), the Central calibrates the size of its repurchased asset portfolio to keep the *ex-post* weighted risky assets (WR^e) below its upper bound, so that $WR^e = WR^{max}$if $WR^a > WR^{max}$.

Prudential supervisors compare the above with the *ex-ante* value of banks weighted risky assets (WR^a). The latter is the product of the market value of the stocks of loans, bonds and commercial paper showing up in the asset side of the banking sector's balance sheet and a risk parameter (η_1) (38.7).

$$WR^a = \eta_1 \left(p_{l,k,b}\, L_{d,k,b} + p_{l,k,g}\, L_{d,k,g} + p_{l,c,b}\, L_{d,c,b} + p_{l,c,g}\, L_{d,c,g} + p_{b,b}\, B_{d,b} + p_{b,g}\, B_{d,g} + p_{cp,b} CP_{d,b} + p_{cp,g} CP_{d,g}\right) \tag{38.7}$$

The Central Bank's targeted stock of quantitative easing (QE^T) realigns the ex-post value of banks risky assets (WR^e) with the upper bound (WR^{max}) ((38.9) and (38.10)).

$$QE^T = \frac{(WR^a - WR^{max})}{\eta_1} z_2\,; \;\; z_2 = 1\; iff\; WR^a > WR^T \tag{38.9}$$

$$qe^T = \Delta QE^T \tag{38.10}$$

Finally, banks' *ex-post* weighted risky asset portfolio (WR^e) is *ex-ante* weighted risky assets (WR^a) net of weighted repurchased assets ($qe^T \eta_1$) and (38.11).

$$WR^e = WR^a - qe^T \eta_1 \tag{38.11}$$

Liquidity ratio (LCR)

Banks continually adjust their Treasuries holdings to align the liquidity ratio with the regulatory target (LCR^T). We shall thus differentiate the *ex-ante* from the *ex-post* liquidity ratio.
The ex-ante liquidity ratio (LCR^a) equals the ratio of banks' class 1 assets (including total reserve assets (H_d) net of households' cash holdings ($H_{s,h}$)) and Treasury bond holdings ($GB_{d,b,-1}$)) to total deposits (M_s)factored by a parameter measuring banks' assessment of households' liquidity preference (ζ) (38.13).

$$LCR^a = \frac{GB_{d,b,-1} + H_d - H_{s,h}}{\zeta M_{s,h}} \tag{38.13}$$

(ζ) is an endogenous parameter which increases with (γ_{10}) (equation (38.14)). Therefore, the liquidity ratio target shall fall, for instance, in the event of a 'bank run' during which households' preference for liquidity (γ_{10}) increases. The latter is impacted by ecosystemic damages (see (8.2)).

$$\zeta = \left(\frac{GB_s + H_d - H_{s,h}}{M_{s,h}}\right)(1 + \gamma_{10}) \tag{38.14}$$

Banks adjust their stock of Treasuries so as to bring the *ex-post* liquidity coverage ratio (LCR^e) in line with its regulatory target (38.15) ($LCR^e = LCR^T$). The corresponding flow of Treasury purchases is given in (38.16). The *ex-post* liquidity coverage ratio is given in (38.17).

$$GB_{d,b} = \zeta M_s LCR^T - (H_d - H_{s,h}) \quad (38.15)$$
$$gb_{d,b} = \Delta GB_{d,b} \quad (38.16)$$
$$LCR^e = \frac{GB_{d,b} + (H_d - H_{s,h})}{\zeta M_{s,h}} \quad (38.17)$$

7. Investment funds

Investment funds raise capital by issuing shares (S_s) in response to the demand of rentier households ($S_{d,r}$) (39.1).

$$S_s = S_{d,r} \quad (39.1)$$

Investment funds use these 'loanable funds' portfolios to purchase equities. Funds thus hold the total stock of listed firms' equities (E_k) (39.2).
In cases where $S_s > E_k$, investments funds turn to the Treasuries market where they act as second-order buyers (the priority in the adjudication of Treasuries being given to banks). Finally, any remaining funds are held as bank deposits.

Funds' holdings of Treasuries ($GB_{d,s}$) equal its lagged value, plus annual transactions ($gb_{d,s}$) (39.3). The latter are equal to the gap between annual Treasuries issues and banking sector purchases (39.4). This implies that investment funds are second-order purchasers in Treasury issues. Finally, banking deposits are inferred from the balance sheet constraint (39.5).

$$E_s = E_k \quad (39.2)$$
$$GB_{d,s} = GB_{d,s,-1} + gb_{d,s} \quad (39.3)$$
$$gb_{d,s} = gb_s - gb_{d,b} iff S_s > E_k \quad (39.4)$$
$$M_{d,s} = S_s - E_s - BE - GB_{d,s} \quad (39.5)$$

As shown in (39.5), investment funds profits (P_s) are the sum of dividends paid by banks (Div_b) and listed firms (Div_k), and interest earned on Treasuries ($i_{gb} GB_{d,s,-1}$) and banking deposits ($i_d M_s$) (39.6). These profits are paid out in full to rentier households as fund dividends (Div_s) (39.7).

$$P_s = Div_b + Div_k + i_{gb} GB_{d,s} + i_d M_s \quad (39.6)$$
$$Div_s = P_{s,-1} \quad (39.7)$$

8. The Central Bank

High powered money

The creation of high-powered money (h_d) is the sum of new reserve loans to banks ($\Delta A_{d,b}$), flows of quantitative easing (qe^T), annual purchases of Treasuries (qe_{gb}), new cash issues in response to household demand ($\Delta H_{s,h}$) (39.8). The stock of high-powered money is equal to the stock inherited from the previous period, plus the annual flow (equation (39.9)).

$$h_d = \Delta A_{d,b} + \Delta qe^T + qe_{gb} + \Delta H_{s,h} \quad (39.8)$$
$$H_d = H_{d,-1} + h_d \quad (39.9)$$

Quantitative easing operations

Quantitative easing interventions are 'market neutral' and do not affect the green structure of bank balance sheets whatsoever.
Equations (40.1) through (40.4) decompose the flow of asset repurchases into a brown and green component for listed firm loans ($qe_{l,k}$), social firm loans ($qe_{l,c}$), bonds (qe_b) and commercial paper (qe_{cp}).

$$qe_{l,k} = qe_{l,k,g} + qe_{l,k,b} \quad (40.1)$$
$$qe_{l,c} = qe_{l,c,g} + qe_{l,c,b} \quad (40.2)$$
$$qe_b = qe_{b,g} + qe_{b,b} \quad (40.3)$$
$$qe_{cp} = qe_{cp,g} + qe_{cp,b} \quad (40.4)$$

The next equations define the Central Bank's annual transactions for each category of risky assets. Consider equation (40.5). It shows that the annual Central Bank demand for portfolios of brown loans issued to listed firms ($qe_{l,k,b}$) equals the new quantitative easing flow (qe^T) factored by the share of brown loans to listed firms in the debt market ($\frac{L_{s,k,b,-1}}{D_{d,-1}}$) (equation (40.5)). A similar procedure is used to compute the value of repurchased green loans to listed firms ($qe_{l,k,g}$), brown and green loans to social firms ($qe_{l,c,b}$ and $qe_{l,c,g}$), brown and green corporate paper ($qe_{cp,b}$ and $qe_{cp,g}$), and brown and green corporate bonds ($qe_{b,b}$and $qe_{b,g}$) ((40.5) to (40.12)).

$$qe_{l,k,b} = qe^T \frac{L_{s,k,b,-1}}{D_{d,-1}} \quad (40.5)$$
$$qe_{l,k,g} = qe^T \frac{L_{s,k,g,-1}}{D_{d,-1}} \quad (40.6)$$
$$qe_{l,c,b} = qe^T \frac{L_{s,c,b,-1}}{D_{d,-1}} \quad (40.7)$$
$$qe_{l,c,g} = qe^T \frac{L_{s,c,g,-1}}{D_{d,-1}} \quad (40.8)$$
$$qe_{cp,b} = qe^T \frac{CP_{s,b,-1}}{D_{d,-1}} \quad (40.9)$$
$$qe_{cp,g} = qe^T \frac{CP_{s,g,-1}}{D_{d,-1}} \quad (40.10)$$
$$qe_{b,b} = qe^T \frac{B_{d,b,-1}}{D_{d,-1}} \quad (40.11)$$
$$qe_{b,g} = qe^T \frac{B_{d,g,-1}}{D_{d,-1}} \quad (40.12)$$

The Central Bank's demand for Treasuries ($g_{d,cb}$) is a residual buffer between annual Treasury issues (gb_s), and net purchases of Treasuries by banks ($gb_{d,b}$) and investment funds ($gb_{d,s}$). This means that the Central Bank can either sell Treasuries when the demand exceeds supply (in this case, reserves reflux on its balance sheet) or intervene as a buyer of last resort through outright monetary transactions (this implies the creation of new reserve deposits) ((40.13) and (40.14)).

$$qe_{gb} = gb_s - gb_{d,s} - gb_{d,s} \quad (40.13)$$
$$GB_{d,cb} = GB_{d,cb,-1} + qe_{gb} \quad (40.14)$$

Equations (41.1) to (41.11) are accounting equalities defining the green structure of the Central Bank's repurchased asset portfolio. Consider for instance (41.1). It shows that the Central Bank's portfolio of repurchased brown loans to listed corporations ($QE_{l,k,b}$) equals the stock inherited from the previous period ($QE_{l,k,b,-1}$), plus the new flow ($qe_{l,k,b}$). Stocks of green loans to listed firms ($QE_{l,k,g}$), brown and green loans to social firms (($QE_{l,c,b}$) and ($QE_{l,c,g}$)) respectively), brown and green corporate paper (($QE_{cp,b}$) and ($QE_{cp,g}$)) respectively), and brown and green corporate bonds (($QE_{b,b}$) and ($QE_{b,g}$)) respectively) are determined with a similar procedure.

$$QE_{l,k,b} = QE_{l,k,b,-1} + qe_{l,k,b} \quad (41.1)$$
$$QE_{l,k,g} = QE_{l,k,g,-1} + qe_{l,k,g} \quad (41.2)$$
$$QE_{l,c,b} = QE_{l,c,b,-1} + qe_{l,c,b} \quad (41.3)$$
$$QE_{l,c,g} = QE_{l,c,g,-1} + qe_{l,c,g} \quad (41.4)$$
$$QE_{cp,b} = QE_{cp,b,-1} + qe_{cp,b} \quad (41.5)$$
$$QE_{cp,g} = QE_{cp,g,-1} + qe_{cp,g} \quad (41.6)$$
$$QE_{b,b} = QE_{b,b,-1} + qe_{b,b} \quad (41.7)$$
$$QE_{b,g} = QE_{b,g,-1} + qe_{b,g} \quad (41.8)$$
$$QE_b = QE_{b,b} + QE_{b,g} \quad (41.9)$$
$$QE_l = QE_{l,k,g} + QE_{l,c,g} + QE_{l,k,b} + QE_{l,c,b} \quad (41.10)$$
$$QE_{cp} = QE_{cp,b} + QE_{cp,g} \quad (41.11)$$

Reserve liabilities

QE-created reserve deposits ($H_{s,qe}$) are the counterpart of the stock of repurchased assets (qe^T) (41.12).

$$H_{s,qe} = qe^T \quad (41.12)$$

New reserve deposits ($H_{s,gb}$) show up as Central Bank liabilities as the counterpart of Central Bank's holdings of Treasuries ($GB_{d,cb}$) (41.13).

$$H_{s,gb} = GB_{d,cb} \quad (41.13)$$

The Central Bank's liabilities (H_s) featured on its balance sheet have four components: those issued to the banking sector through refinancing operations ($A_{s,b}$), through quantitative easing ($H_{d,qe}$), through the purchase of Treasuries ($H_{s,gb}$), and the supply of cash held by households ($H_{d,h}$) (41.14).

$$H_s = A_{s,b} + H_{d,qe} + H_{s,gb} + H_{d,h} \quad (41.14)$$

Income statement and equity

The annual net income of the Central Bank equals the sum of the interest earned on its portfolio of repurchased assets and the interest earned through refinancing operations banks $(i_e A_{s,b,-1})$, net of the remuneration of mandatory reserves at the specified rate $(i_{mr} H_{d,m-1})$ and the remuneration of excess reserves at the deposit facility rate $(i_{df} H_{ex,-1})$ (42).

$$\begin{aligned} F_{cb} = {} & i_{l,k,b} QE_{l,k,b,-1} + i_{l,k,g} QE_{l,k,g,-1} + i_{l,c,b} QE_{l,c,b,-1} + i_{l,c,g} QE_{l,c,g,-1} \\ & + \ i_{b,b} QE_{b,b,-1} + i_{b,g} QE_{b,g,-1} + i_{cp,b} QE_{cp,b,-1} + i_{cp,g} QE_{cp,g,-1} \\ & + i_{gb} QE_{gb,-1} + + i_e A_{s,b,-1} - i_e H_{d,m,-1} - i_{df} H_{ex,-1} \end{aligned} \tag{42}$$

The annual variation of the Central Bank's equity (k_{cb}) equals the difference between changes in assets (reserves loans $(\Delta A_{s,b})$, repurchased assets (Δqe^T), Treasuries holdings $(\Delta GB_{d,cb})$ and liabilities (high powered money (ΔH_s)) (equation 43.1). The stock of Central Bank equity is equal to lagged value, plus the annual variation (equation (43.2)). Both Central Bank profits and equity are transferred to the Treasury, for accounting purposes (Nersisyan and Wray, 2024).

$$k_{cb} = \Delta A_{s,b} + qe^T + \Delta GB_{d,cb} - (\Delta H_s) \tag{43.1}$$

$$K_{cb} = K_{cb,-1} + k_{cb} \tag{43.2}$$

9. Interest rates

Money market rates

The interest rate structure follows a post-Keynesian horizontalist framework (Moore, 1988). The supply of reserves is modeled as a set of horizontal lines, representing different stances of discretionary monetary policy. In addition, the transmission channel follows a pure floor framework[6], in which the interbank rate follows that of the Central Bank deposit facility rate. By supplying of reserves more than the regulatory threshold, the Central Bank corners the interbank market by adjusting the deposit facility rate at the required level. The interbank market rate (i_e) follows that of the deposit facility rate $\widehat{\iota_{df}}$ ((44.1) and (44.2)).

$$i_e = i_{df} \tag{44.1}$$

$$i_{df} = \widehat{\iota_{df}} \tag{44.2}$$

Monetary policy stances influence the credit market by affecting banks' lending risk (see (32.1) and (32.2)) as well as lending rates ((46.2) and (46.3)). As in the Eurozone, the deposit facility rate differs from that paid on mandatory reserves requirement, with $\widehat{\iota_{mr}} < \widehat{\iota_{df}}$ (44.3).

$$i_{mr} = \widehat{\iota_{mr}} \tag{44.3}$$

[6] Variants of the floor system have become the new paradigm in Central Banking since the 2008 crisis. See Schnabel (2024). The Eurosystem's operational framework. Speech by Isabel Schnabel, member of the Executive Board of the ECB.
https://www.ecb.europa.eu/press/key/date/2024/html/ecb.sp240314~8b609de772.en.html

Bank lending rates

The determination of lending rates is based on the post-Keynesian principle of increasing risk. It also takes a 'greenium' mechanism into account.
Banks determine lending rates for highly liquid commercial paper given a chosen mark-up over the Central Bank rate. The mark-up reflects lender risk (see (32.1) and (32.2)), an exogenous liquidity premium (σ_1) and a *greenium* ((46.2) and (46.3)).

$$i_{cp,b} = i_e + LR_{k,b} + \sigma_1 + greenium \quad (46.2)$$
$$i_{cp,g} = i_e + LR_{k,g} + \sigma_1 - greenium \quad (46.3)$$

The interest rate on brown and green loans to listed firms equals the coupon rate of corporate paper, plus a liquidity risk premium (σ_2) and a *greenium* differentiating brown from green projects (46.4) and (46.5)).

$$i_{l,k,g} = i_{cp,g} + \sigma_2 - greenium \quad (46.4)$$
$$i_{l,k,b} = i_{cp,b} + \sigma_2 + greenium \quad (46.5)$$

The interest rate on brown and green loans to social firm equates that charged to listed corporations, plus a lander-specific risk premium $\left(LR_{c,g} - LR_{k,g}\right)$ ((46.6) and (46.7)), and a *greenium*.

$$i_{l,c,g} = \left[i_{l,k,g} + \left(LR_{c,g} - LR_{k,g}\right)\right] - greenium \quad (46.6)$$
$$i_{l,c,b} = i_{l,k,b} + \left(LR_{c,b} - LR_{k,b}\right) + greenium \quad (46.7)$$

The coupon rate for brown ($i_{b,b}$) and green corporate bonds ($i_{b,g}$) equals to the interest rate on loans (($i_{l,k,b}$) and ($i_{l,k,g}$), respectively), plus a specific greenium (equation (49.2) and (49.3)).

$$i_{b,b} = i_{l,k,b} + greenium \quad (46.8)$$
$$i_{b,g} = i_{l,k,g} - greenium \quad (46.9)$$

The greenium is equal to a baseline parameter (σ_5), modulated by climate destruction (γd_{t-1}) (47).

$$greenium = \sigma_5(1 + \gamma d_{t-1}) \quad (47)$$

Deposit rates

Banks determine the deposit rate i_d by applying a mark-up (σ_3) to the refinancing rate (i_e) (48).

$$i_d = i_{d,-1} + \sigma_3 i_e \quad (47)$$

Treasuries rate

The interest rate on government bonds is equal to the interbank rate (i_e), plus a short-term liquidity risk premium (49).

$$i_{gb} = i_e + \sigma_1 \tag{49}$$

Given the above framework, the shape of the yield curve follows a standard term structure so that: $i_b > i_l > i_{cp} > i_{gb} > i_d > i_e$. In addition, the risk structure of interest takes a different shape for brown and green project due to the *greenium* mechanism.

10. Financial markets

Total returns

The total annual return on corporate bonds, Treasuries and securitized loan portfolios is the sum of the coupon rate and the realized capital gain. Consider for instance (50.1). It shows that the total return on brown corporate bonds ($r_{b,b}$) is the sum of the corresponding yield rate ($i_{b,b,-1}$) and the percent capital gain ($\frac{CG_b}{p_{b,b,-1}}$). This procedure also applies to green corporate bonds ($r_{b,g}$), Treasuries (r_{gb}), brown and green loans to listed firms (($r_{l,k,b}$) and ($r_{l,k,g}$), respectively), and brown and green loans to social firms (($r_{l,c,b}$) and ($r_{l,c,g}$), respectively) ((50.1) to (50.7)).

$$r_{b,b} = i_{b,b,-1} + \frac{CG_b}{p_{b,b,-1}} \tag{50.1}$$

$$r_{b,g} = i_{b,g,-1} + \frac{CG_b}{p_{b,g,-1}} \tag{50.2}$$

$$r_{gb} = i_{gb,-1} + \frac{CG_{gb}}{p_{gb,-1}} \tag{50.3}$$

$$r_{l,k,b} = i_{l,k,b,-1} + \frac{CG_{l,k,b}}{p_{l,k,b}} \tag{50.4}$$

$$r_{l,k,g} = i_{l,k,g,-1} + \frac{CG_{l,k,g}}{p_{l,k,g}} \tag{50.5}$$

$$r_{l,c,b} = i_{l,c,b,-1} + \frac{CG_{l,c,b}}{p_{l,c,b}} \tag{50.6}$$

$$r_{l,c,g} = i_{l,c,g,-1} + \frac{CG_{l,c,g}}{p_{l,c,g}} \tag{50.7}$$

Expected capital gains

Expectations are modelled with adaptive expectations. Expected capital gains equal their lagged value, plus an error correction term, where (ψ) represents the speed of expectations adjustment. For instance, the expected capital gain on brown bonds ($CG_{b,b}^e$) is equal to its lagged value ($CG_{b,b,-1}$), plus a factor (ψ) of the last observed expectation error $\left(CG_{b,b,-1} - CG_{b,b,-1}^e\right)$. A similar process describes expectations for green bonds ($CG_{b,g}^e$), brown and green loans to listed firms (($CG^e_{l,k,b}$) and ($CG^e_{l,k,g}$), respectively) , brown and green loans to social firms (($CG^e_{l,c,b}$) and ($CG^e_{l,c,g}$), respectively), Treasuries (CG^e_{gb}) and equities (CG^e_e) ((51.1) to (51.8)).

$$CG^e_{b,b} = CG_{b,b,-1} + \psi\left(CG_{b,b,-1} - CG^e_{b,b,-1}\right) \tag{51.1}$$
$$CG^e_{b,g} = CG_{b,g,-1} + \psi\left(CG_{b,g,-1} - CG^e_{b,g-1}\right) \tag{51.2}$$
$$CG^e_{l,k,b} = CG_{l,k,b,-1} + \psi\left(CG_{l,k,b,-1} - CG^e_{l,k,b,-1}\right) \tag{51.3}$$
$$CG^e_{l,k,g} = CG_{l,k,g,-1} + \psi\left(CG_{l,k,g,-1} - CG^e_{l,k,g,-1}\right) \tag{51.4}$$
$$CG^e_{l,c,b} = CG_{l,s,b,-1} + \psi\left(CG_{l,s,b,-1} - CG^e_{l,s,b,-1}\right) \tag{51.5}$$
$$CG^e_{l,c,g} = CG_{l,s,g,-1} + \psi\left(CG_{l,s,g,-1} - CG^e_{l,s,g,-1}\right) \tag{51.6}$$
$$CG^e_{gb} = CG_{gb,-1} + \psi\left(CG_{gb,-1} - CG^e_{gb,-1}\right) \tag{51.7}$$
$$CG^e_e = CG_{e,-1} + \psi\left(CG_{e,-1} - CG^e_{e,-1}\right) \tag{51.8}$$

Realized capital gains

Realized capital gains equal to the first-differenced market prices, factored by lagged outstanding asset volumes. For instance, capital gains on Treasuries are the product of price changes (Δp_{gb}) and the lagged outstanding stock of Treasuries (GB_{s-1}) (52.1). A similar logic is applied to brown and green corporate bonds (($CG_{b,b}$) and ($CG_{b,g}$), respectively), portfolios of brown and green loans to listed corporations (($CG_{l,k,b}$) and ($CG_{l,k,g}$), respectively), portfolios of brown and green loans to third sector firms (($CG_{l,c,b}$) and ($CG_{l,c,g}$), respectively), and equities (CG_e) ((52.2) to (52.8)).

$$CG_{gb} = \Delta p_{gb}(GB_{s-1}) \tag{52.1}$$
$$CG_{b,b} = \Delta p_{b,b}.(B_{s,b,-1}) \tag{52.2}$$
$$CG_{b,g} = \Delta p_{b,g}.(B_{s,g,-1}) \tag{52.3}$$
$$CG_{l,k,b} = \Delta p_{l,k,b}.(L_{s,k,b,-1}) \tag{52.4}$$
$$CG_{l,k,g} = \Delta p_{l,k,g}.(L_{s,k,g,-1}) \tag{52.5}$$
$$CG_{l,c,b} = \Delta p_{l,c,b}.(L_{s,c,b,-1}) \tag{52.6}$$
$$CG_{l,c,g} = \Delta p_{l,c,g}.(L_{s,c,g,-1}) \tag{52.7}$$
$$CG_e = \Delta p_e.(E_{s,-1}) \tag{52.8}$$

Under the steady state scenario, all market prices are stationary, i.e. $\Delta p_i = 0$ (i representing the range of available assets). However, specific shocks can be introduced on market prices in different segments of financial markets.

11. The public sector

The government's budget constraint

Government spending includes direct spending in the private sector (G), investment expenses of public-sector firms ($I_{s,g}$), and interest payment to Treasuries holders ($i_g GB_{s,-1}$)). Government revenue includes total taxes (T), public sector firms' profits (P_p) and central bank profit and equity revaluation ($F_{cb} + k_{cb}$). The latter item is included here for the purpose of accounting consistency (Nersisyan and Wray, 2024).

Fresh Treasury issues (gb_s) fill in the gap between spending and revenue (53.4). The volume of outstanding public debt equals the lagged volume of Treasuries ($GB_{s,-1}$) plus new issues (gb_s) (53.5).

$$gb_s = \ Max\ ((G + I_{s,g} + i_g GB_{s,-1}) - (T + P_g + F_{cb} + k_{cb}); 0) \tag{53.4}$$
$$GB_s = GB_{s,-1} + gb_s \tag{53.5}$$

Government spending ($\bar{G}$) in the private sector is exogenous and split across listed firms and social firms ((53.1) to (51.3)).

$$G = \bar{G} \tag{53.1}$$
$$G_c = \ \varsigma_1 G \tag{53.2}$$
$$G_k = G - G_c \tag{53.3}$$

Public sector firms

Public sector firms' profit (P_p) equals sales revenues (C_p) net of the wage bill (WB_p) and depreciation expenses (DA_p) (equation (54.1)). Public sector firms' wage bill is determined with an accounting closure criterion ((54.2)). The depreciation and amortization expenditures (DA_p) of public sector firms are a fixed proportion of the lagged capital stock (K_{p-1}) (54.3).

$$P_p = C_p - WB_p - DA_p \tag{54.1}$$
$$WB_p = WB - WB_c - WB_k \tag{54.2}$$
$$DA_p = \lambda K_{p,-1} \tag{54.3}$$

In contrast to private sector firms, investment demand in the public sector is an exogenous policy variable. Public sector firms' productive capital target (K_p^T) equals the lagged capital stock ($K_{p,-1}$) factored by a discretionary growth rate ($\overline{g_{k1}}$). Public sector firms' demand for investment ($I_{d,p}$) is the difference between capital target (K_p^T) and the lagged stock (K_{p-1}), plus depreciation expenditure ($DA_p = \lambda K_{p,-1}$) ((54.4) to (54.6)).

$$K_p^T = (1 + \overline{g_{k1}}) K_{p,-1} \tag{54.4}$$
$$K_p = K_{p,-1} + I_{s,p} - DA_p \tag{54.5}$$
$$I_{d,p} = K_p^T - K_p + DA_p \tag{54.6}$$

The green structure of public sector investment demand is also a policy variable. The effective demand for green capital goods ($I_{d,p,g}$) is an exogenous fraction ($\overline{g_{k2}}$) of the total demand for capital goods ($I_{d,p}$). The demand for brown capital goods ($I_{d,p,b}$) is determined through accounting closure (54.8). Public sector investment is unrestricted as supply ($I_{s,p}$) equals demand ($I_{d,p}$) (equation (54.9)).

$$I_{d,p,g} = \overline{g_{k2}} I_{d,p} \tag{54.7}$$
$$I_{d,p,b} = I_{d,p} - I_{d,p,g} \tag{54.8}$$
$$I_{s,p} = I_{d,p} \tag{54.9}$$

12. Hidden equation

The equality between the flow of equity purchases by investment funds (as implied by their balance sheet) and new shares issues by listed companies shows up as our redundant equation:

$$e_{s,k} = \Delta S_s - (gb_s + \Delta M_{d,s} + \Delta BE) \tag{55}$$

This equation implies that equity market financing is limited by the availability of loanable funds represented by the appetite of rentier households for investment fund shares.

13. The eco-systemic block

Material extraction

The production of material goods (YM) is a proportion (μ_e) of annual real GDP (Y) (56.1). This proportion is modulated by the productive structure of the economy, i.e. the share of green $(\mu_g \frac{K_g}{K})$ and brown capital $(\mu_s \frac{K_b}{K})$ (56.2).

$$YM = \mu_e \hat{Y} \tag{56.1}$$

$$\mu_e = \mu_g \frac{K_g}{K} + \mu_s \frac{K_b}{K} \tag{56.2}$$

Annual material extraction (M) is the difference between material production (YM) and the recycled socio-economic stock (REC) (56.3). The latter is a proportion (ρ_e) of the discarded socio-economic stock (DIS) (56.4), which itself depends on a share (μ_e) of the sum of the lagged socio-economic stock (SES_{-1}) and the total obsolescence of capital goods (DA) (56.5).

$$M = YM - REC \tag{56.3}$$

$$REC = \rho_e * DIS \tag{56.4}$$

$$DIS = \mu_e [DA + \zeta SES_{-1}] \tag{56.5}$$

The socio-economic stock (SES) is the sum of material production (YM) and the lagged socio-economic stock (SES_{-1}), net of the discarded stock (DIS) (56.6). Material waste emissions (WA) are the difference between the discarded stock (DIS) and recycling (REC) (56.7). Material reserves (REV_M) are equal to their lagged value $(REV_{M,-1})$ net of the conversion of reserves into resources (CON_M) and material extraction (M) (56.8). The conversion of natural reserves into material stocks (CON_M) is a fraction (σ_M) of lagged material resources $(RES_{M,-1})$ (56.9). Material resources (RES_M) equal their lagged value $\left(RES_{M,-1}\right)$ net of the new conversions into reserves (CON_M) (56.10).

$$SES = SES_{-1} + YM - DIS \tag{56.6}$$

$$WA = DIS - REC \tag{56.7}$$

$$REV_M = REV_{M,-1} - CON_M - M \tag{56.8}$$

$$CON_M = \sigma_M RES_{M,-1} \tag{56.9}$$

$$RES_M = RES_{M,-1} - CON_M \tag{56.10}$$

Energy consumption

Total energy consumption (E) is indexed on real GDP, with the conversion rate depending on the state of eco-efficiency technological capacities (tek) and the green energy mix (ε_y) (57.1). Total energy consumption is then broken down into renewable energy (RE) and non-renewable energy (NRE), based on a green energy mix parameter (η_e) ((57.2) and (57.3)).

$$E = \frac{1}{1 + tek} \varepsilon_{y-1} \vartheta \hat{Y} \tag{57.1}$$

$$RE = \eta_e E \tag{57.2}$$

$$NRE = E - RE \tag{57.3}$$

Total energy consumption dissipates as heat (DE) according to the second law of thermodynamics (57.4).

$$DE = RE + NRE \tag{57.4}$$

Changes in the stock of non-renewable energy reserves (REV_e) equal the conversion of energy resources into reserves (CON_e) net of non-renewable energy consumption (NRE) (57.5). The conversion (CON_e) of energy reserves into resources (RES_e) occurs at rate (σ_e) ((57.5) to (57.7)).

$$REV_e = REV_{e,-1} + CON_e - NRE \tag{57.5}$$

$$CON_e = \sigma_e RES_e \tag{57.6}$$

Emissions

Total CO2 emissions ($EMIS$) are the sum of land emissions ($EMIS_l$) emissions (declining at a rate (lr)) and industrial emissions ($EMIS_{in}$) which are indexed on the consumption of non-renewable energy (NRE) with an endogenous parameter (β_e) measuring the carbon intensity of non-renewable energy (equations (58.1) to (58.3)).

$$EMIS_{in} = \beta_0 + \beta_e NRE \tag{58.1}$$
$$EMIS_l = EMIS_{l-1}(1 - lr) \tag{58.2}$$
$$EMIS = EMIS_{in} + EMIS_l \tag{58.3}$$

The carbon mass (CEN) is a fixed ratio of CO2 emissions (58.4). Oxygen inputs are equal to the difference between total CO2 emissions and the carbon mass (58.5).

$$CEN = \frac{EMIS}{car} \tag{58.4}$$

$$02 = EMIS - CEN \tag{58.5}$$

The carbon cycle

Equations (58.6) to (58.12) describe a simplified carbon cycle: the net transfers of carbon from the atmosphere to the biosphere and ocean reservoirs $CO2_{up}$ and $CO2_{lo}$ ((58.6) and (58.7)) increase the atmospheric concentration of CO2 ($CO2_{at}$) (58.8). In turn, the accumulation of CO2 in the atmosphere then increases radiative forcing (F) and, eventually, both atmospheric and oceanic temperatures T_{at} and T_{lo} ((58.9) to (58.12)).

$$CO2_{at} = EMIS + \varphi_{11}CO2_{at,-1} + \varphi_{21}CO2_{up,-1} \tag{58.6}$$

$$CO2_{up} = \varphi_{12}CO2_{at,-1} + \varphi_{22}CO2_{up,-1} + \varphi_{32}CO2_{lo,-1} \tag{58.7}$$

$$CO2_{lo} = \varphi_{23}CO2_{up,-1} + \varphi_{33}CO2_{lo,-1} \tag{58.8}$$

$$F = F_2 log_2 \frac{C02_{at}}{CO2_{pre}} + F_{ex} \tag{58.9}$$

$$F_{ex} = F_{ex,-1} + fex \tag{58.10}$$

$$T_{at} = T_{at-1} + t_1 \left[F - \left(\frac{F_2}{sens}\right) T_{at,-1} - t_2(T_{at-1} - T_{lo-1}) \right] \tag{58.11}$$

$$T_{lo} = T_{lo-1} + t_3 \left(T_{at,-1} - T_{lo-1}\right) \tag{58.12}$$

Eco-efficiency

The level of eco-efficiency depends both on the green taxonomy of productive capital and on technology affecting total energy consumption.

The energy intensity of GDP (ε_y), the carbon intensity (β_e) of non-renewable energy, and the share of renewable energy in the energy mix (η_e) are defined as the weighted sum of the share of brown capital and green capital in the total capital stock (($\frac{K_b}{K}$) and ($\frac{K_g}{K}$), respectively) (equations 59.1 to 59.3).

$$\varepsilon_y = \varepsilon_g \frac{K_g}{K} + \varepsilon_b \frac{K_b}{K} \tag{59.1}$$

$$\beta_e = \beta_g \frac{K_g}{K} + \beta_b \frac{K_b}{K} \tag{59.2}$$

$$\eta_e = \eta_g \frac{K_g}{K} + \eta_b \frac{K_b}{K} \tag{59.3}$$

The above weights in turn, depend on technological pathways. Energy intensity of GDP and the carbon intensity of energy both decline with an exogenous parameter (tek) measuring green innovation and eco-efficiency technological change ((59.4) and (59.5)). Technological change also increases the share of renewables in the energy mix (59.6).

$$\beta_g = \beta_{g,-1}(1 - tek) \tag{59.4}$$
$$\varepsilon_g = \varepsilon_{g,-1}(1 - tek) \tag{59.5}$$
$$\eta_g = \eta_{g,-1}(1 + tek) \tag{59.6}$$

Ecosystemic retroaction

The depletion ratios of material (dep_m) and energy reserves (dep_e) are obtained by dividing material extraction (M) and non-renewable energy consumption (NRE) by lagged material (REV_{M-1}) and energy reserves (REV_{E-1}) ((59.7) and (59.8), respectively). Climate-induced economic damage (d_t) is a nonlinear function of changes in atmospheric temperature $(TEMP_{at})$ (59.8).

$$dep_m = \frac{M}{REV_{M-1}} \tag{59.7}$$

$$dep_e = \frac{NRE}{REV_{E-1}} \tag{59.8}$$

$$d_t = 1 - \frac{1}{\left[1 + \varrho_1 TEMP_{at} + \varrho_2 TEMP_{at}{}^2 + \varrho_3 TEMP_{at}{}^{\varrho_4}\right]} \tag{59.9}$$

14. Biomimicry-inspired postgrowth metrics

Monetary trophic flows

Following Ulanowicz et al. (2009), let $T_{i.}$ be the trophic monetary outflow of each sector i to any other sector; $T_{.j}$ the trophic monetary inflow of sector j from any other sector; and T_{ij} money outflow of any sector i to any sector j. The sum of all net money transfers of money between sector i and j is $T_{..} = \sum_{ij} T_{ij}$. Table A4 contains these intersectoral monetary 'trophic' flows, which are discussed in the following equations.

Equation (60.1) defines the money transfer of working household to social firms $(T_{h,w,c})$ as consumption spending $(\alpha_{1,c}C_w)$. Equation (60.2) defines the money transfer of working households to listed firms $(T_{h,w,k})$ with an accounting criterion. Equations (60.3) and (60.4) apply similar principles to define money transfer of rentier household to social firms $(T_{h,r,c})$, and listed firms $(T_{h,r,k})$. Equation (60.5) defines the money transfer of rentier households to investment funds as new share purchases ($\Delta S_{d,r}$). Equation (60.6) and (60.7) define total money transfer of working households ($T_{h,w.}$) and rentier households ($T_{h,r.}$) to the private sector.

$$T_{h,w,c} = \alpha_{1,c} C_{w,m} \tag{60.1}$$
$$T_{h,w,k} = C_w - (\alpha_{1,c} C_{w,m}) \tag{60.2}$$
$$T_{h,r,c} = \alpha_{1,c} C_{r,m} \tag{60.3}$$
$$T_{h,r,k} = C_r - (\alpha_{1,c} C_{r,m}) \tag{60.4}$$
$$T_{h,r,s} = \Delta S_{d,r} \tag{60.5}$$
$$T_{h,w.} = T_{h,w,c} + T_{h,w,k} \tag{60.6}$$
$$T_{h,r.} = T_{h,r,c} + T_{h,r,k} + T_{h,r,s} \tag{60.7}$$

Equation (61.1) defines the money transfer of social firms to working households ($T_{c,h,w}$) as the sum of the wage bill (WB_c) and distributed surpluses ($1 - ret_c P_c$). Equation (61.2) defines the money transfer of social firms to the banking sector ($T_{c,b}$) as the sum of interest payments on green and brown loans ($L_{s,c,g} i_{l,c,g} + L_{s,c,b} i_{l,c,b}$). Equation (61.3) defines the total money transfer of social firms to the rest of the private sector ($T_{c,.}$).

$$T_{c,h,w} = WB_c + (1 - ret_c P_c) \tag{61.1}$$
$$T_{c,b} = L_{s,c,g} i_{l,c,g} + L_{s,c,b} i_{l,c,b} \tag{61.2}$$
$$T_{c,.} = T_{c,h,w} + T_{c,b} \tag{61.3}$$

Equation (62.1) defines the money transfer of listed firms to working households ($T_{k,h,w}$) as the wage bill (WB_k). Equation (62.2) defines the money transfer of listed firms to the banking sector ($T_{k,b}$) as the sum of interest payments on green and brown loans ($L_{s,c,g} i_{l,c,g} + L_{s,c,b} i_{l,c,b}$), green and brown bonds ($B_{s,g} i_{b,g} + B_{s,b} i_{b,b}$) and green and brown commercial paper ($CP_{s,g} i_{cp,g} + CP_{s,b} i_{cp,b}$). Equation (62.3) defines the money transfer of listed firms to investment funds ($T_{k,s}$) as paid out dividends (Div_s). Equation (62.4) defines the total money transfer of social firms to the rest of the private sector ($T_{k,.}$).

$$T_{k,h,w} = WB_k \tag{62.1}$$
$$T_{k,b} = L_{s,k,g} i_{l,k,g} + L_{s,k,b} i_{l,k,b} + B_{s,g} i_{b,g} + B_{s,b} i_{b,b} + CP_{s,g} i_{cp,g} + CP_{s,b} i_{cp,b} \tag{62.2}$$
$$T_{k,s} = Div_s \tag{62.3}$$
$$T_{k,.} = T_{k,h,w} + T_{k,b} + T_{k,s} \tag{62.4}$$

Equation (63.1) defines the money transfer of banks to working households ($T_{b,h,w}$) as interest on deposits ($i_d M_{d,w}$). Equation (63.2) defines the money transfer of banks to rentier households ($T_{b,h,r}$) as interest on deposits ($i_d M_{d,r}$). Equation (63.3) defines the money transfer of banks to the social firm sector ($T_{b,c}$) as annual credit flows ($l_{s,c}$). Equation (63.4) defines the money transfer of banks to the social firm sector ($T_{b,k}$) as annual credit flows ($f_{s,k}$). Equation (63.5) defines the money transfer of banks to investment funds ($T_{b,s}$) as annual dividend payments (Div_b). Equation (63.6) defines the total money transfer of banks to the private sector ($T_{b,.}$).

$$T_{b,h,w} = i_d M_{d,w} \tag{63.1}$$

$$T_{b,h,r} = i_d M_{d,r} \tag{63.2}$$
$$T_{b,c} = l_{s,c} \tag{63.3}$$
$$T_{b,k} = f_{s,k} \tag{63.4}$$
$$T_{b,s} = Div_b \tag{63.5}$$
$$T_{b,.} = T_{b,h,w} + T_{b,h,r} + T_{b,c} + T_{b,k} + T_{b,s} \tag{63.6}$$

Equation (64.1) defines the money transfer of investment funds to rentier households ($T_{s,r}$) as distributed profits (P_i). Equation (64.1) defines the money transfer of investment funds to listed firms ($T_{s,k}$) as new equity purchases ($e_{s,k}$). Equation (64.3) gives the total money transfer of investment funds to the private sector ($T_{s,.}$).

$$T_{s,r} = P_i \tag{64.1}$$
$$T_{s,k} = e_{s,k} \tag{64.2}$$
$$T_{s,.} = T_{s,r} + T_{s,k} \tag{64.3}$$

Equations (65.1) to (65.7) are macro aggregates corresponding the last row and columns of table A4. They measure total money flows to the working household sector ($T_{.,h,w}$), the rentier household sector ($T_{.,h,r}$), the social firm sector ($T_{.,c}$), the listed firm sector ($T_{.,k}$), the banking sector ($T_{.,b}$), investment funds ($T_{.,s}$) and the total money transfers ($T_{.,.}$), respectively.

$$T_{.,h,w} = T_{c,h,w} + T_{k,h,w} + T_{b,h,w} \tag{65.1}$$
$$T_{.,h,r} = T_{b,h,r} + T_{s,r} \tag{65.2}$$
$$T_{.,c} = T_{h,w,c} + T_{h,r,c} + T_{b,c} \tag{65.3}$$
$$T_{.,k} = T_{h,w,k} + T_{h,r,k} + T_{b,k} \tag{65.4}$$
$$T_{.,b} = T_{c,b} + T_{k,b} \tag{65.5}$$
$$T_{.,s} = T_{h,r,s} + T_{k,s} + T_{b,s} \tag{65.6}$$
$$T_{.,.} = T_{.,h,w} + T_{h,w.} + T_{.,h,r} + T_{h,r.} + T_{.,c} + T_{c,.} + T_{.,k} + T_{k,.} + T_{.,b} + T_{b,.} + T_{.,s} + T_{s,.} \tag{65.7}$$

Table A4 Money trophic flows

i \ j	Working households	Rentier households	Social firms	Listed corporations	Banks	Investment funds	All
Working households			$\alpha_{1,c}C_{w,m} + C_{w,lcc}$	$C_w - (\alpha_{1,c}C_{w,m} + C_{w,lcc})$			C_w
Rentier households			$\alpha_{1,c}C_{r,m} + C_{r,lcc}$	$C_r - (\alpha_{1,c}C_{r,m} + C_{r,lcc})$		$\Delta S_{d,r}$	$C_r + \Delta S_{d,r}$
Social firms	$WB_c + (1 - ret_c P_c)$				$L_{s,c,g}i_{l,c,g} + L_{s,c,b}i_{l,c,b}$		$WB_c + (1 - ret_c P_c) + L_{s,c,g}i_{l,c,g} + L_{s,c,b}i_{l,c,b}$
Listed corporations	WB_k				$L_{s,k,g}i_{l,k,g} + L_{s,k,b}i_{l,k,b} + B_{s,g}i_{b,g} + B_{s,b}i_{b,b} + CP_{s,g}i_{cp,g} + CP_{s,b}i_{cp,b}$	Div_s	$WB_k + L_{s,k,g}i_{l,k,g} + L_{s,k,b}i_{l,k,b} + B_{s,g}i_{b,g} + B_{s,b}i_{b,b} + CP_{s,g}i_{cp,g} + CP_{s,b}i_{cp,b} + Div_s$
Banks	$i_d M_{d,w}$	$i_d M_{d,r}$	$l_{s,c}$	$f_{s,k}$		Div_b	$i_d M_{d,w} + i_d M_{d,r} + l_{s,c} + f_{s,k} + Div_b$
Investment funds		P_i		$e_{s,k}$			$P_i + e_{s,k}$
All	$WB_c + (1 - ret_c P_c) + WB_k + i_d M_{d,w}$	$i_d M_{d,r} + P_i$	$\alpha_{1,c}C_{w,m} + C_{w,lcc} + \alpha_{1,c}C_{r,m} + C_{r,lcc} + l_{s,c}$	$C_w - (\alpha_{1,c}C_{w,m} + C_{w,lcc}) + C_r - (\alpha_{1,c}C_{r,m} + C_{r,lcc}) + f_{s,k}$	$+L_{s,c,g}i_{l,c,g} + L_{s,c,b}i_{l,c,b} + L_{s,k,g}i_{l,k,g} + L_{s,k,b}i_{l,k,b} + B_{s,g}i_{b,g} + B_{s,b}i_{b,b} + CP_{s,g}i_{cp,g} + CP_{s,b}i_{cp,b}$	$\Delta S_{d,r} + Div_s + Div_b$	$\Delta M_{d,w} + \Delta M_{d,r} + \Delta S_{d,r} + e_{s,k} + C_w + C_r + WB_c + (1 - ret_c P_c) + L_{s,c,g}i_{l,c,g} + L_{s,c,b}i_{l,c,b} + WB_k + L_{s,k,g}i_{l,k,g} + L_{s,k,b}i_{l,k,b} + B_{s,g}i_{b,g} + B_{s,b}i_{b,b} + CP_{s,g}i_{cp,g} + CP_{s,b}i_{cp,b} + Div_s + P_i$

Note: this table shows private sector trophic flows within Philia 1.0. The first column lists the originating sector, and the first line lists the receiving sector.

Boltzmann transformation

Defining the sum of all net transfers of money between units i and j as $T_{..} = \sum_{ij} T_{ij}$, the realization probability of each transaction is given by its frequency:

$p_{ij} = \frac{T_{ij}}{T_{..}}$; $p_{i.} = \frac{T_{i.}}{T_{..}}$; $p_{.j} = \frac{T_{.j}}{T_{..}}$

Following Ulanowicz et al. (2009), the 'evolutionary capacity' of the private sector is then equal to $C = A + \Phi$, where each term is defined in equations (66.1) to (66.3):

$$C = -\sum_{ij} T_{ij} log\left(\frac{T_{ij}}{T_{..}}\right) \tag{66.1}$$

$$A = \sum_{ij} T_{ij} log\left(\frac{T_{ij}T_{..}}{T_{i.}T_{.j}}\right) \tag{66.2}$$

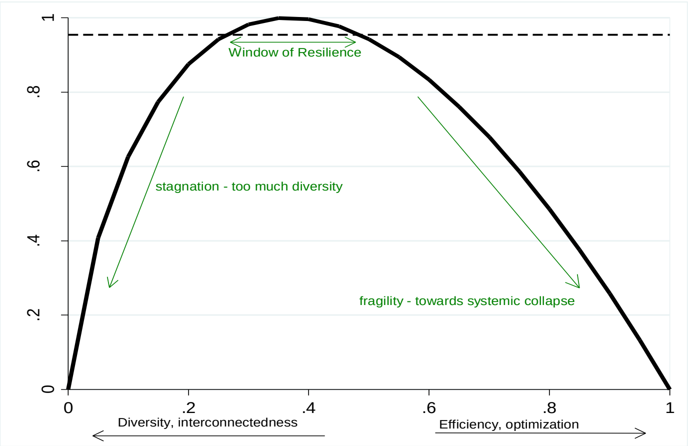
$$\Phi = -\sum_{ij} T_{ij} log\left(\frac{T_{ij}^2}{T_{i.}T_{.j}}\right) \tag{66.3}$$

Defining $a = \frac{A}{C}$ yields the economy's fitness for evolution (F) through a Boltzmann transformation (equation (67)).

$$F = -ka\log(a) \tag{67}$$

Where k is a positive scalar set to 2.71. If a =1, F tends to zero, because the ability to adapt to shocks is insufficient. If a =0, then F also tends to zero because the system is insufficiently structured. F takes significant values for intermediate values of a and reaches its maximum at $a = \frac{1}{e}$ = 0.36. (figure A3).

Figure A3 Fitness for evolution and resilience



Note: this graph represents the ability of the system to evolve F = −kalog (a) for different values of 0< a <1. The dotted line indicates values above 0.954 and the resilience window. See Ulanowicz et al (2009) for more details.

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