SUPERVISED LEARNING PROJECT

COST OF LIVING MODEL IN UNITED STATES USING MACROECONOMICS INDICATORS

The purpose of this project is to build a Supervised Machine Learning model that is using the US macroeconomic data to predict the cost of living for US householders. The cost of living target indicator that we will use is the total annual expenditure, which is published by the U.S. Bureau of Labor Statistics (BLS). The macroeconomy data that will be considered in this model is the inflation rate (Annual consumer product index (CPI)), Employment / Unemployment statistics by the U.S. Census Bureau and the Housing prices provided by the leading housing portal Zillow.

Here are the sources and reports extracted to build the dataset used in this project:

U.S. Bureau of Labor Statistics:

https://www.bls.gov/cex/tables.htm

- 1. All Urban Consumers (Consumer Price Index CPI)
- 2. Consumer Expenditure Survey (CE)

U.S. Census Bureau :

https://data.census.gov/

1. Employment rate by BLS region

Zillow housing data:

https://www.zillow.com/research/data/

1. Zillow Home Value Index (ZHVI) all homes by state

The data from these websites has been downloaded in Excel format and different formatting operations had to be performed (picking certain cells, transposing, converting, merging) in order to produce csv files ready to be imported in this notebook.

```
In [1]: import scipy as sp
import scipy.stats as stats
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
```

```
import copy
# Set color map to have light blue background
sns.set()
import statsmodels.formula.api as smf
import statsmodels.api as sm
%matplotlib inline
```

EDA Procedure

Data Loading and Cleaning

In this section, we will load the different data set, inpect the data and make any needed transformations.

In [2]: # This file contains the details on the US household expending per year and
It contains also the income data as well as the breadown of the expending
df_expenditure = pd.read_csv('Expenditure_data.csv')

In [3]: df_expenditure.head()

Region Income_before_taxes Income_after_taxes Expenditure_total Out[3]: Year Food Hou 0 2022 Northeast 79741 10199 108768 93870 **1** 2022 Midwest 91430 81174 69870 8827 **2** 2022 South 83346 75370 65576 8443 2 **3** 2022 West 83317 10699 2 103418 90354 4 2021 Northeast 100103 87948 72678 9334 2

In [4]: df_expenditure.info()

<class 'pandas.core.frame.DataFrame'>
RangeIndex: 44 entries, 0 to 43
Data columns (total 14 columns):

#	Column	Non-	Non-Null Count		
0	Year	44 n	on-null	int64	
1	Region	44 n	on-null	object	
2	<pre>Income_before_taxes</pre>	44 n	on-null	int64	
3	<pre>Income_after_taxes</pre>	44 n	on-null	int64	
4	Expenditure_total	44 n	on-null	int64	
5	Food	44 n	on-null	int64	
6	Housing	44 n	on-null	int64	
7	Apparel	44 n	on-null	int64	
8	Transportation	44 n	on-null	int64	
9	Healthcare	44 n	on-null	int64	
10	Entertainment	44 n	on-null	int64	
11	Education	44 n	on-null	int64	
12	Insurance_pensions	44 n	on-null	int64	
13	Others	44 n	on-null	int64	
dtvp	es: int64(13), object	(1)			

dtypes: int64(13), object(1)
memory usage: 4.9+ KB

In [5]: # This file contains the macroeconomic information (employment and unemploym
df_employment = pd.read_csv('Employment_data.csv')

In [6]: df_employment.head()

Out [6]: Year Region Employment_rate Unemployment_rate O 2022 Northeast 61.00% 4.60%

0	2022	Northeast	61.00%	4.60%
1	2022	Midwest	61.70%	3.80%
2	2022	South	59.20%	4.20%
3	2022	West	60.40%	4.60%
4	2021	Northeast	59.00%	7.40%

In [7]: df_employment.info()

<class 'pandas.core.frame.DataFrame'>

RangeIndex: 44 entries, 0 to 43
Data columns (total 4 columns):

Column Non-Null Count Dtype
--- ----- 44 non-null int64
1 Region 44 non-null object
2 Employment_rate 44 non-null object
3 Unemployment rate 44 non-null object

dtypes: int64(1), object(3)

memory usage: 1.5+ KB

In [8]: # Since the rates are loaded as object, we need to convert these two attribut
df_employment['Employment_rate'] = df_employment['Employment_rate'].str.rstr
df_employment['Unemployment_rate'] = df_employment['Unemployment_rate'].str.

```
df employment.head()
 Out[9]:
             Year
                    Region Employment_rate Unemployment_rate
          0 2022 Northeast
                                     0.610
                                                       0.046
          1 2022
                   Midwest
                                     0.617
                                                       0.038
          2 2022
                                     0.592
                                                       0.042
                     South
          3 2022
                      West
                                     0.604
                                                       0.046
          4 2021 Northeast
                                     0.590
                                                       0.074
In [10]: # This file contains the evolution of the Consumer Price Index (CPI) by year
         df CPI = pd.read csv('CPI data.csv')
In [11]: df_CPI.head()
Out[11]:
             Year
                    Region Annual_CPI
          0 2012 Northeast
                              245.698
          1 2013 Northeast
                              249.038
          2 2014 Northeast
                              252.463
          3 2015 Northeast
                              252.185
          4 2016 Northeast
                              254.850
In [12]: df_CPI.info()
         <class 'pandas.core.frame.DataFrame'>
         RangeIndex: 44 entries, 0 to 43
          Data columns (total 3 columns):
          #
                           Non-Null Count
               Column
                                            Dtype
                                            ____
                           44 non-null
          0
               Year
                                            int64
          1
               Region
                           44 non-null
                                            object
               Annual_CPI 44 non-null
                                            float64
         dtypes: float64(1), int64(1), object(1)
         memory usage: 1.2+ KB
In [13]: # Last, this is the evolution of the average house price per year and region
         df_housing = pd.read_csv('Housing_data.csv')
In [14]: df housing.head()
```

```
Out[14]:
            Year
                  Region House_price
          0 2012 Midwest
                         131822.1934
          1 2013 Midwest 139825.3620
          2 2014 Midwest 145371.2890
          3 2015 Midwest 153770.0406
          4 2016 Midwest 159955.5674
In [15]: df_housing.info()
         <class 'pandas.core.frame.DataFrame'>
          RangeIndex: 44 entries, 0 to 43
          Data columns (total 3 columns):
               Column
                            Non-Null Count
                                             Dtype
           0
               Year
                            44 non-null
                                             int64
                            44 non-null
          1
               Region
                                             object
               House_price 44 non-null
                                             float64
          dtypes: float64(1), int64(1), object(1)
         memory usage: 1.2+ KB
In [16]: # We will not merge the 4 files into a single dataframe by joining the dataf
         #Perform an inner join based on equality between Year and Region
         merged df1 = pd.merge(df expenditure, df employment, left on=['Year', 'Regio
         merged_df2 = pd.merge(df_CPI, df_housing, left_on=['Year', 'Region'], right_
          df = pd.merge(merged df1, merged df2, left on=['Year', 'Region'], right on=[
In [17]: df.head()
Out[17]:
             Year
                    Region Income_before_taxes Income_after_taxes Expenditure_total
                                                                                 Food Hou
          0 2022 Northeast
                                       108768
                                                         93870
                                                                          79741 10199
                                                                                        2
          1 2022
                   Midwest
                                        91430
                                                          81174
                                                                                 8827
                                                                         69870
          2 2022
                                                                                8443
                     South
                                       83346
                                                         75370
                                                                         65576
                                                                                        2
          3 2022
                      West
                                       103418
                                                         90354
                                                                         83317 10699
          4 2021 Northeast
                                                                                        2
                                       100103
                                                         87948
                                                                         72678 9334
```

Data visualization

Once the data is loaded in a single dataframe, we are now in the position to visualize the data to gain insights and determine already what could be the most appropriate models and features. We will start by visualizing the **expenditure by region over the years**.

```
In [18]: # Create a figure and axis to plot on
fig, ax = plt.subplots()

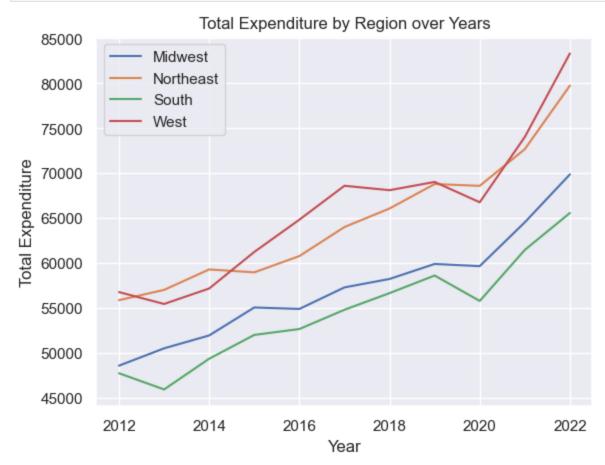
# Iterate through unique regions and plot each series separately
for region, data in df.groupby('Region'):
```

```
ax.plot(data['Year'], data['Expenditure_total'], label=region)

# Set labels and title
ax.set_xlabel('Year')
ax.set_ylabel('Total Expenditure')
ax.set_title('Total Expenditure by Region over Years')

# Show a legend
ax.legend()

# Show the plot
plt.show()
```



As expected the trend of the expenditure is overall ascending for all regions. However it is curious to see that in 2020 the expenditure decreased in all regions, clearly an effect of the COVID19 pandemic.

We will now go on with the visualization of the total expenditure versus the Macroeconomic data. We will start by plotting the *Average expenditure vs Unemployment rate.*

Unemployment rate value has been multiplied by 10^6 to be able to represent both values in the same range.

```
In [19]: # Grouping the DataFrame by 'Year' and 'Region' and calculating the mean for
yearly_region_means = df.groupby(['Year', 'Region'])[['Expenditure_total', '
```

```
# Calculating the average values for each year after grouping by 'Region'
yearly_means = yearly_region_means.groupby('Year')[['Expenditure_total', 'Ur

# Create a figure and axis to plot on
fig, ax = plt.subplots()

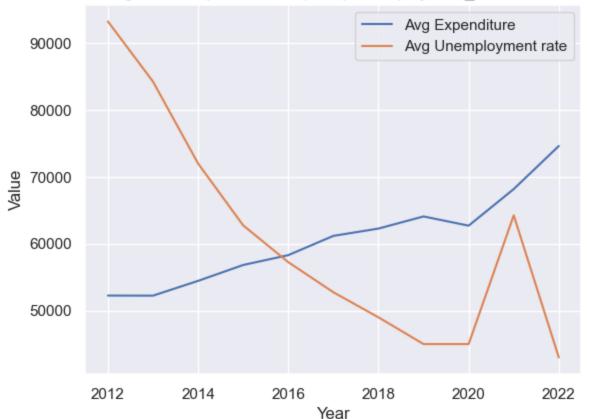
# Plotting 'Expenditure_total' and 'Unemployment_rate' averaged over regions
ax.plot(yearly_means['Year'], yearly_means['Expenditure_total'], label='Avg
ax.plot(yearly_means['Year'], 10000000*yearly_means['Unemployment_rate'], lab

# Set labels and title
ax.set_xlabel('Year')
ax.set_ylabel('Yalue')
ax.set_title('Average Total Expenditure vs (10^6) Unemployment_rate over Yea

# Show the legend
ax.legend()

# Show the plot
plt.show()
```





As it could be anticipated, a decline in unemployement rate for 2012 to 2022 translates in an increase of average expenditure.

Because of the pandemic the unemployment data for 2020 was not available and hence the 2019 value was used. One can see the effect of the pandemic in 2021 as the

unemployment rate peak due to the lockdowns and freeze of the economy. Also it rapidly went down again in 2020 to lower values than 2019.

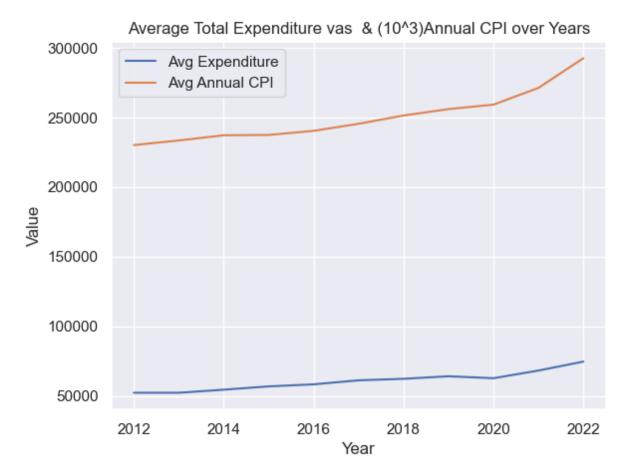
It is also noticeable that the expenditure only decrease slightly in 2020 and swiftly recover the ascending trend in 2021 and 2022 thanks to the government stimuli and low interest rates.

Because of this anomaly in the behavior in 2021, one can expect that the unemployment rate will not be the best predictor for average expenditure.

Let's explore now the effect of inflation with the graph showing *Average Total Expenditure vs Annual CPI.*

Again, CPI values have been multiplied by 10^3 to keep both variables in the same range st

```
In [20]: # Grouping the DataFrame by 'Year' and 'Region' and calculating the mean for
         yearly_region_means = df.groupby(['Year', 'Region'])[['Expenditure_total',
         # Calculating the average values for each year after grouping by 'Region'
         yearly_means = yearly_region_means.groupby('Year')[['Expenditure_total', 'Ar
         # Create a figure and axis to plot on
         fig, ax = plt.subplots()
         # Plotting 'Expenditure_total' and 'Annual_CPI' averaged over regions agains
         ax.plot(yearly means['Year'], yearly means['Expenditure total'], label='Avg
         ax.plot(yearly_means['Year'], 1000*yearly_means['Annual_CPI'], label='Avg Ar
         # Set labels and title
         ax.set xlabel('Year')
         ax.set ylabel('Value')
         ax.set title('Average Total Expenditure vas & (10^3)Annual CPI over Years')
         # Show the legend
         ax.legend()
         # Show the plot
         plt.show()
```



Here there is no surprise. Both the Average expenditure and annual CPI show a clear ascending trend. Higher CPI (and therefore higher prices) will imply higher average expenditure. For this reason, we expect the Annual CPI to be a good predictor of the expenditure. It is also curious to see how in 2021-2022 the CPI (and hence the inflation) has skyrocked well above the trend over the past years. The expenditure also increase but not at the same pace, which indicates that households are loosing buying power due to the high inflation. The next and last macroeconomy indicator displayed will be the ***Average House price vs Total Expenditure.*** *Again, for visualization purposes, we had to take 10% of the house pricing to make sure that both values stay in the same range.*

```
In [21]: # Grouping the DataFrame by 'Year' and 'Region' and calculating the mean for
    yearly_region_means = df.groupby(['Year', 'Region'])[['Expenditure_total', '
    # Calculating the average values for each year after grouping by 'Region'
    yearly_means = yearly_region_means.groupby('Year')[['Expenditure_total', 'Ho

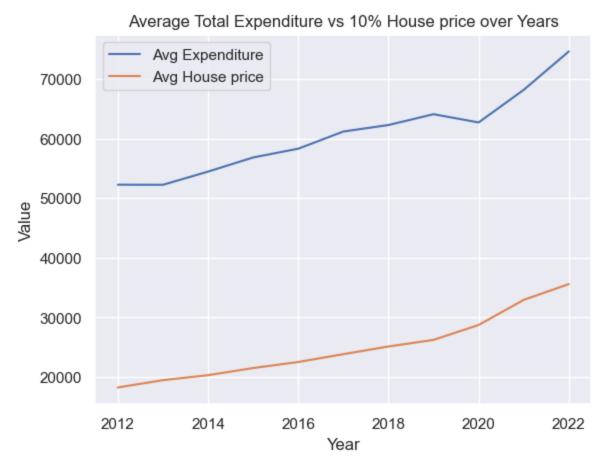
# Create a figure and axis to plot on
    fig, ax = plt.subplots()

# Plotting 'Expenditure_total' and 'House_price' averaged over regions again
    ax.plot(yearly_means['Year'], yearly_means['Expenditure_total'], label='Avg
    ax.plot(yearly_means['Year'], 0.1*yearly_means['House_price'], label='Avg Ho

# Set labels and title
    ax.set_xlabel('Year')
    ax.set_ylabel('Value')
    ax.set_title('Average Total Expenditure vs 10% House price over Years')

# Show the legend
```

```
ax.legend()
# Show the plot
plt.show()
```



The trend of both expenditure and house price is clearly ascending as housing is one of the most important expenditures for houshold. For that reason we expect it to be a good predictor of total expending. We se that, again in 2020 because of the the pandemic, the expenditure decrease slightly while house prices actually increase at a faster rate. This was a well known event that was caused by the fact that fewer houses were put in the market because of the "stay home" policies and, because of the low supply the house prices increased

Model building

We are now in a position to start building our model (or models) to predict the total expenditure based on the data and insights obtained in the visualization.

Data adaptation

As and additional data manipulation, we will convert the Region attribute (that has text values) into numeric. This is mandatory if we want the Region to play a role in the

numeric regression and models and therefore maintain the influence of the region in the overall prediction.

```
In [22]: # Convert 'Region' column to categorical type
          # Define a mapping dictionary for string to numeric representation
          region mapping = {'Northeast': 100, 'Midwest': 200, 'South': 300, 'West': 40
          # Convert categorical variable to numeric
          df['Region_num'] = df['Region'].map(region_mapping)
In [23]: df.head()
Out[23]:
             Year
                    Region Income_before_taxes Income_after_taxes Expenditure_total
                                                                                  Food Hou
          0 2022 Northeast
                                       108768
                                                          93870
                                                                           79741 10199
          1 2022
                                                                                  8827
                   Midwest
                                        91430
                                                           81174
                                                                          69870
          2 2022
                     South
                                        83346
                                                          75370
                                                                          65576
                                                                                 8443
                                                                                         2
          3 2022
                      West
                                        103418
                                                          90354
                                                                           83317 10699
                                                                                         2
          4 2021 Northeast
                                                          87948
                                                                          72678 9334
                                                                                         2
                                       100103
```

Data trimming and sorting

As a final data cleaning, we are going to remove most of the attributes that represent a breakdown of the total expenses (food, apparel, etc) since these are highly correlated with the total.

Also we will remove the Income_after_taxes which is also highly correlated with the Income_before_taxes.

Finally, the Region will also be removed since it is already converted to a numeric value for regressions.

The resulting dataframe will be sorted by Year and Region_num for visualization purposes.

```
In [24]: # drop unnecessary features, replace df
df = df.drop(columns=['Region', 'Food', 'Apparel', 'Others', 'Housing', 'Insurance
df = df.sort_values(by=['Year', 'Region_num'])
```

Inspection, correlation and pair plot

In the following section we will perform analysis and visualization for the target data point (Expenditure_total) and the rest of feature in order to determine which one are the best predictor for the supervised models.

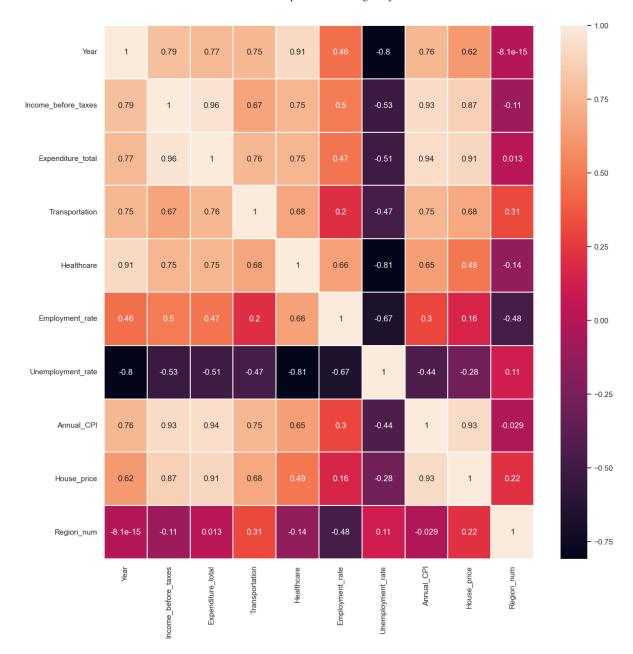
```
In [25]: df.corr()
```

Out[25]:

	Year	Income_before_taxes	Expenditure_total	Transportation
Year	1.000000e+00	0.788233	0.766206	0.754656
Income_before_taxes	7.882331e-01	1.000000	0.959763	0.672039
Expenditure_total	7.662065e-01	0.959763	1.000000	0.758304
Transportation	7.546557e-01	0.672039	0.758304	1.000000
Healthcare	9.132629e-01	0.751645	0.747309	0.676912
Employment_rate	4.550436e-01	0.498494	0.470976	0.202244
Unemployment_rate	-7.969976e- 01	-0.529885	-0.509381	-0.471205
Annual_CPI	7.589880e-01	0.926128	0.935513	0.749910
House_price	6.193570e-01	0.866853	0.905532	0.684320
Region_num	-8.119262e-15	-0.107786	0.013115	0.313945

Correlation Matrix using a Heatmap

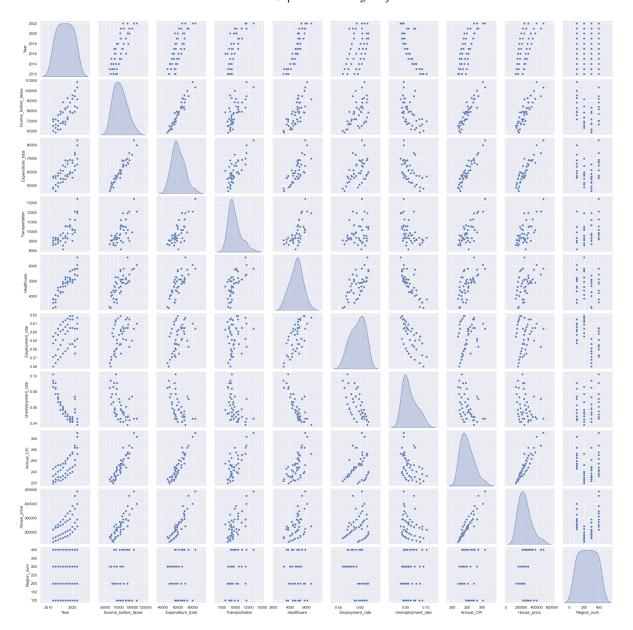
```
In [26]: # visualizing correlation matrix using a heatmap
fig, ax = plt.subplots (figsize=(14, 14))
sns.heatmap (df.corr(), linewidths=.3,annot=True)
plt.show ()
```



Pair Plot

```
In [27]: df_10 = df.iloc[:, : 10]
sns.pairplot(df_10, diag_kind="kde")
```

Out[27]: <seaborn.axisgrid.PairGrid at 0x17d776350>



Based on the results above, the feature Income_before_taxes seems to be the best predictor (coef = 0.959763) to determine the Expenditure_total, which makes a lot of sense as households spend in accordance to their income.

Other features will also show good correlation with the target variable and will be considered in the next steps.

Simple linear regression

Model & training

In this section we will use a simple linear regression model to fit the data.

First we will split the dataset in train and test partition. Since this is a time evolution, we will use the data from years 2012 to 2020 as training and the last two years (2021,2022)

as test.

This way using past values of the target variable as features can help the model learn temporal patterns.

Below, we will first attempt each of the features as predictors and select the one that obtains best results, i.e., the maximum \mathbb{R}^2 score.

```
In [28]: #simple one variable regression
         X_{train} = df[(df['Year'] >= 2012) & (df['Year'] <= 2020)]
         X \text{ test} = df[(df['Year'] >= 2021) & (df['Year'] <= 2022)]
         print("X_train:", X_train.shape)
         print("X_test:", X_test.shape)
         Y_test = X_test['Expenditure_total'] # Target variable
         Y train = X train['Expenditure total'] # Target variable
         best_feature = None
         max r2 score = -float('inf')
         columns = X_train.columns
         variables = []
         form =''
         for c in columns:
             if c != 'Expenditure total' :
                     form = 'Expenditure_total' + ' ~ ' + c
                     model = smf.ols(formula=form, data=X_train)
                     res = model.fit()
                     variables.append([c,res.rsquared])
                     if res.rsquared > max_r2_score:
                          max r2 score = res.rsquared
                          best_feature = c
                          best_mod = res
         print('Best predictor : ', best_feature)
         print('Max R2 score :', max_r2_score)
         print(best mod.summary())
```

X_train: (36, 10) X_test: (8, 10)

Best predictor : Income_before_taxes
Max R2 score : 0.8798355503489361

OLS Regression Results

	ULS 			кеѕиттѕ 		
=== Dep. Variable:	Expenditure	total	R-sc	nuared:		0.
880	_x,p 0a _ ca. c			100.00.		•
Model:	0L		Adj	R-squared:		0.
876 Method:	least S	auarec	F_c1	tatistic:		24
8.9	Least 3	quares	1-5	latistic.		24
Date:	Sun, 10 De	c 2023	Prob	(F-statisti	ic):	3.28e
-17 - ·						
Time: 7.95	20	:51:38	Log-	-Likelihood:		-32
No. Observations:		36	AIC:	:		65
9.9				•		
Df Residuals:		34	BIC	:		66
3.1		1				
<pre>Df Model: Covariance Type:</pre>	non	1 robust				
=======================================			=====	-=======		
=========						
0.0751	coef	std	err	t	P> t	[0.025
0.975]						
Intercept	1.272e+04	2910.	479	4.371	0.000	6806.182
1.86e+04						
<pre>Income_before_taxes 0.687</pre>	0.6089	0.	039	15.778	0.000	0.530
0.007 =========		======	=====	-=======	=======	:=======
===						
Omnibus:		1.785	Durk	oin-Watson:		1.
868		0 440	-	D (3D)		
Prob(Omnibus): 905		0.410	Jaro	que-Bera (JB)	:	0.
Skew:		0.349	Prob	o(JB):		0.
636						
Kurtosis:		3.340	Cond	d. No.		5.85e
+05						
=======================================	========	======	=====		=======	========

Notes:

- $\[1\]$ Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 5.85e+05. This might indicate that there are
- strong multicollinearity or other numerical problems.

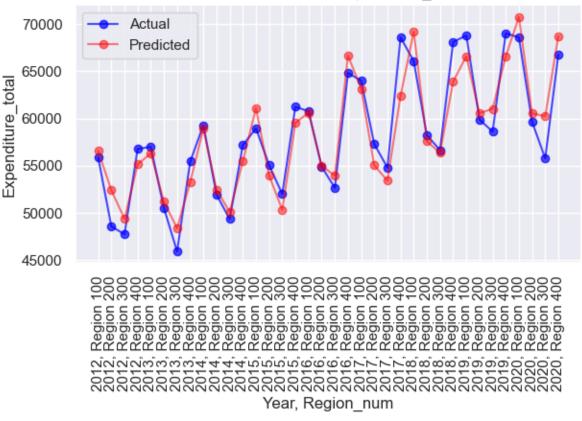
Results

as already anticipated, the best predictor turns out to be the Income_before_taxes. It delivers a 0.880 R-squared value, which is already not bad.

Now we will use the model generated with simple predictor and plot the actual results versus the ones predicted by the simple linear model using the single best predictor.

```
In [29]: # Predict using the test data
         model = best_mod
         Y_pred = model.predict(X_train)
         # Extracting the 'Year', 'Expenditure_total', and 'Region_num' values for th
         years = X_train['Year'].values
         region_nums = X_train['Region_num'].values
         # Combine 'Year' and 'Region num' to form a combined label for x-axis
         x_ticks_labels = [f"{year}, Region {region_num}" for year, region_num in zip
         # Scatter plot for actual 'Expenditure_total' values with combined 'Year' ar
         plt.plot(x_ticks_labels, Y_train, color='blue', label='Actual', marker='o',
         # Scatter plot for predicted 'Expenditure total' values with combined 'Year'
         plt.plot(x_ticks_labels, Y_pred, color='red', label='Predicted', marker='o',
         # Rotate x-axis labels for better readability
         plt.xticks(rotation=90)
         # Labels and title
         plt.xlabel('Year, Region num')
         plt.ylabel('Expenditure_total')
         plt.title('Actual vs Predicted Expenditure total')
         # Show legend
         plt.legend()
         # Show plot
         plt.tight_layout()
         plt.show()
```

Actual vs Predicted Expenditure_total



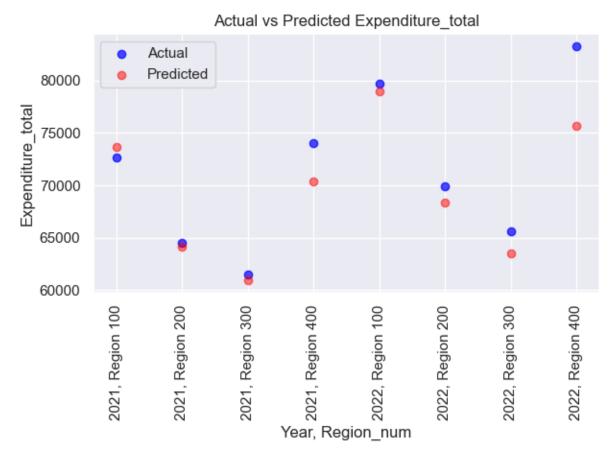
Now we will use that same model to predict the target variable using the test data.

We will plot the results and compare the accuracy of the prediction versus the actuals.

```
In [30]: # Predict using the test data
         model = best_mod
         Y pred = model.predict(X test)
         # Extracting the 'Year', 'Expenditure_total', and 'Region_num' values for th
         years = X_test['Year'].values
         region nums = X test['Region num'].values
         # Combine 'Year' and 'Region_num' to form a combined label for x-axis
         x_ticks_labels = [f"{year}, Region {region_num}" for year, region_num in zip
         # Scatter plot for actual 'Expenditure_total' values with combined 'Year' ar
         plt.scatter(x ticks labels, Y test, color='blue', label='Actual', alpha=0.7)
         # Scatter plot for predicted 'Expenditure_total' values with combined 'Year'
         plt.scatter(x_ticks_labels, Y_pred, color='red', label='Predicted', alpha=0.
         # Rotate x-axis labels for better readability
         plt.xticks(rotation=90)
         # Labels and title
         plt.xlabel('Year, Region_num')
         plt.ylabel('Expenditure_total')
         plt.title('Actual vs Predicted Expenditure_total')
```

```
# Show legend
plt.legend()

# Show plot
plt.tight_layout()
plt.show()
```



As it can be observed already, the predicted values is quite accurate with the exception of the value for 2022 in the Region 400 (West).

In the previous data visualization, we observed that in 2020 for the West Region, there was a spike of expenditure, surpassing for the first time the Northeast region. So, since the model was built in "past" data, it is normal that the value was not accurately predicted as it really constituted a change of tendency in 2022.

We will finalize this section by determining what are the top 5 predictors in the dataset for usage in next analysis.

The results are displayed below.

```
model = smf.ols(formula=form, data=X_train)
    res = model.fit()
    variables.append([c,res.rsquared_adj])

# uncomment and update top_five
sorted_data = sorted(variables, key=lambda x: x[1], reverse=True)
top_all = sorted_data[:5]
top_all = [item[0] for item in top_all]
print(top_all)
```

['Income_before_taxes', 'Annual_CPI', 'House_price', 'Year', 'Healthcare']

Multi linear regression

Model and training

I this section we will use a simple linear regression model to fit the data.

Again, we will split the dataset in train and test partition. Since this is a time evolution, we will use the data from years 2012 to 2020 as training and the last two years (2021,2022) as test.

We will then remove the target variable from the features files (X) and add it to the target variable file both for the training and test datasets.

```
In [32]: # Filter the DataFrame for the required years
         train data = df[(df['Year'] >= 2012) & (df['Year'] <= 2020)]
         test_data = df[(df['Year'] >= 2021) & (df['Year'] <= 2022)]
         # Define features and target variable for training data
         X_train = train_data.drop(['Expenditure_total'], axis=1) # Features
         Y train = train data['Expenditure total'] # Target variable
         # Define features and target variable for test data
         X_test = test_data.drop(['Expenditure_total'], axis=1) # Features
         Y test = test data['Expenditure total'] # Target variable
         # Display the shapes of the resulting datasets
         print("Shapes of the datasets:")
         print("X_train:", X_train.shape)
         print("Y_train:", Y_train.shape)
         print("X_test:", X_test.shape)
         print("Y_test:", Y_test.shape)
         Shapes of the datasets:
         X_train: (36, 9)
         Y train: (36,)
         X_test: (8, 9)
         Y_test: (8,)
```

We will now create a multi-linear regression model (using all the features) and trained it with the data obtained in the section above.

The key parameters of the multi-linear regression are returned

```
In [33]: from sklearn.linear model import LinearRegression
         from sklearn.metrics import mean squared error, r2 score
         # Create a linear regression model
         model = LinearRegression()
         # Fit the model using the training data
         m = model.fit(X train, Y train)
         # Predict using the test data
         Y_pred = model.predict(X_test)
         # Evaluate the model
         mse = mean_squared_error(Y_test, Y_pred)
         r2 = r2 score(Y test, Y pred)
         print("Mean Squared Error:", mse)
         print("R-squared Score:", r2)
         # Access model coefficients and intercept
         coefficients = model.coef
         intercept = model.intercept_
         print("Coefficients:", coefficients)
         print("Intercept:", intercept)
         Mean Squared Error: 7440270.739206381
```

```
Mean Squared Error: 7440270.739206381
R-squared Score: 0.8519313745112188
Coefficients: [-8.71469593e+02 1.76221305e-01 1.56241287e+00 3.78099464e+00
2.19130164e+04 -3.76598334e+04 -1.40063857e+02 9.37754623e-02
-1.05566467e+01]
Intercept: 1774492.9737614421
```

The resulting model is quite accurate as the R-squared Score is 0.85.

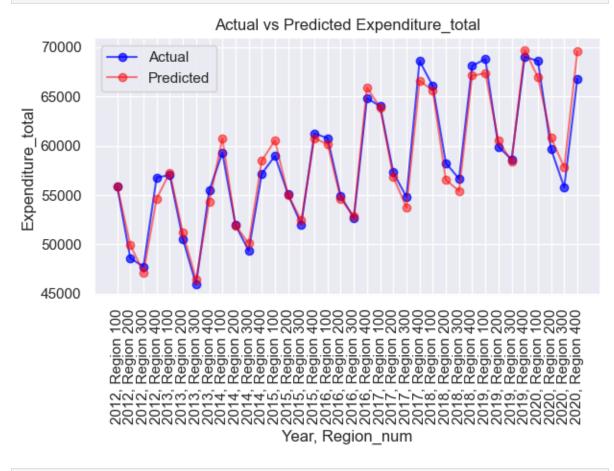
In the sections below, we will plot the actual vs predicted both for the training and test data.

```
In [34]: # Predict using the test data
model = m
Y_pred = model.predict(X_train)

# Extracting the 'Year', 'Expenditure_total', and 'Region_num' values for the
years = X_train['Year'].values
region_nums = X_train['Region_num'].values

# Combine 'Year' and 'Region_num' to form a combined label for x-axis
x_ticks_labels = [f"{year}, Region {region_num}" for year, region_num in zip
```

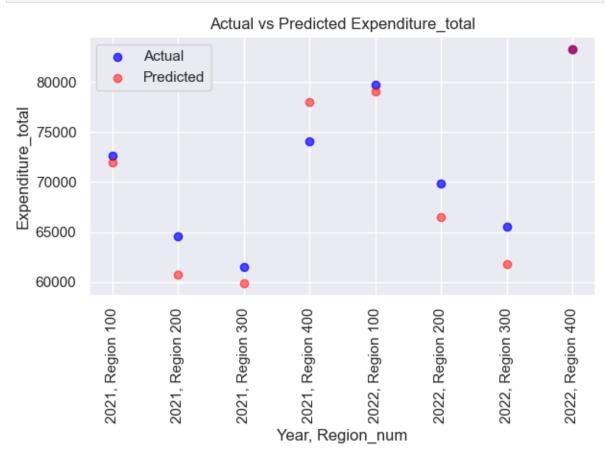
```
# Scatter plot for actual 'Expenditure_total' values with combined 'Year' ar
#plt.scatter(x_ticks_labels, Y_train, color='blue', label='Actual', alpha=0.
plt.plot(x_ticks_labels, Y_train, color='blue', label='Actual', marker='o',
# Scatter plot for predicted 'Expenditure_total' values with combined 'Year'
#plt.scatter(x_ticks_labels, Y_pred, color='red', label='Predicted', alpha=0
plt.plot(x_ticks_labels, Y_pred, color='red', label='Predicted',marker='o',
# Rotate x-axis labels for better readability
plt.xticks(rotation=90)
# Labels and title
plt.xlabel('Year, Region num')
plt.ylabel('Expenditure total')
plt.title('Actual vs Predicted Expenditure total')
# Show legend
plt.legend()
# Show plot
plt.tight_layout()
plt.show()
```



```
import matplotlib.pyplot as plt

# Extracting the 'Year', 'Expenditure_total', and 'Region_num' values for the
years = test_data['Year'].values
region_nums = test_data['Region_num'].values
```

```
Y_pred = model.predict(X_test)
# Combine 'Year' and 'Region num' to form a combined label for x-axis
x_ticks_labels = [f"{year}, Region {region_num}" for year, region_num in zip
# Scatter plot for actual 'Expenditure_total' values with combined 'Year' ar
plt.scatter(x_ticks_labels, Y_test, color='blue', label='Actual', alpha=0.7)
# Scatter plot for predicted 'Expenditure total' values with combined 'Year'
plt.scatter(x_ticks_labels, Y_pred, color='red', label='Predicted', alpha=0.
# Rotate x-axis labels for better readability
plt.xticks(rotation=90)
# Labels and title
plt.xlabel('Year, Region_num')
plt.ylabel('Expenditure_total')
plt.title('Actual vs Predicted Expenditure_total')
# Show legend
plt.legend()
# Show plot
plt.tight_layout()
plt.show()
```



This model does a better job in "fitting" the traning data and predicts the 2022 spike in Expenditure for the region 400 (West).

However, some of the other predictions seems to be worse than the simple linear model, which could be a result of "overfitting" on the train data and then augmenting the variance in the "never seen" test data.

Forward Stepwise Refinement & Multi-predictor model

We will now use other features in the dataset to gradually add the "best predictor" up to a list of 6. We are using the features that are clearly not part of the expenditure breakdown and coming from other macroeconomic datasets. See the allowed_factors list.

In order to do that , we will set up a regression model with k factors that involves the fixed predictors from the previous step k-1

- Try all p predictors in the new kth position
- Select the best parameter using $Adjusted-R^2$ (e.g. ${\tt model.rsquared_adj}$) given your training data
- Fix the new parameter and continue the process for k+1

Model and training

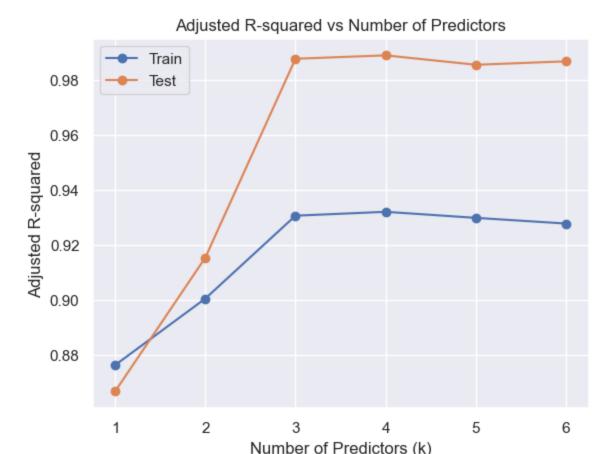
```
In [36]: allowed factors = ['Year', 'Income before taxes', 'Annual CPI', 'House price
                 'Region_num', 'Employment_rate']
In [37]: formula_chain = 'Expenditure_total ~'
         X_{train} = df[(df['Year'] >= 2012) & (df['Year'] <= 2020)]
         X_{\text{test}} = df[(df['Year'] >= 2021) & (df['Year'] <= 2022)]
         print("X_train:", X_train.shape)
         print("X_test:", X_test.shape)
         Y test = X test['Expenditure total'] # Target variable
         Y_train = X_train['Expenditure_total'] # Target variable
         train =[]
         test = []
         for i in range(len(allowed factors)) :
             best = ['',0]
             for p in allowed factors:
                 model = smf.ols(formula= formula_chain + str(' + ') + p, data=X_tra
                 if model.rsquared>best[1]:
                     best = [p, model.rsquared]
             print('best:',best)
             formula chain = formula chain + str(' + ') + str(best[0])
             model = smf.ols(formula= formula chain, data=X train).fit()
             train.append(model)
             model = smf.ols(formula= formula_chain, data=X_test).fit()
             test.append(model)
```

```
X_train: (36, 10)
X_test: (8, 10)
best: ['Income_before_taxes', 0.8798355503489361]
best: ['House_price', 0.9061379883738019]
best: ['Employment_rate', 0.9365174480827176]
best: ['Region_num', 0.9397437616537183]
best: ['Year', 0.9398059918720612]
best: ['Annual_CPI', 0.9400968830089957]
```

Listed below are the 6 best predictors resulting from the iterative process to determine the best combination.

Now we will plot the $Adjusted-R^2$ from the train and test models to determine what is the optimal combination of predictors.

```
In [38]: # plot resulting adjusted rsquared vs number of predictors (k=1,2,3,4,5,6)
# overlay the adjusted rsquared for the test data
adjr2_train = []
adjr2_test = []
for i in range(len(train)):
    adjr2_train.append(train[i].rsquared_adj)
for j in range(len(test)):
    adjr2_test.append(test[j].rsquared_adj)
plt.plot(range(1, len(adjr2_train) + 1), adjr2_train, marker='o', label='Traplt.plot(range(1, len(adjr2_test) + 1), adjr2_test, marker='o', label='Test'
plt.xlabel('Number of Predictors (k)')
plt.ylabel('Adjusted R-squared')
plt.title('Adjusted R-squared vs Number of Predictors')
plt.legend()
plt.show()
```



The "sweet spot" seems with the inclusion of 3 predictors. Up to that value both the accuracy of the training and test models improve notably and then stay flat or even decline.

For that reason, for our analysis we will *select as "best model" the one resulting from the combination of the first 3 best predictors* as it delivers the best balance between complexity and accuracy limiting in this way the risk of overfitting future data.

Below we will plot both the most simple model with one predictor and our "best model" with 3 predictors both for the training and test data and will come to conclusion on the results.

Results

1-Predictor Model

```
In [39]: # Predict using the test data
model = train[0]
print(model.summary())
Y_pred = model.predict(X_train)

# Extracting the 'Year', 'Expenditure_total', and 'Region_num' values for the years = X_train['Year'].values
region_nums = X_train['Region_num'].values
```

```
# Combine 'Year' and 'Region num' to form a combined label for x-axis
x_ticks_labels = [f"{year}, Region {region_num}" for year, region_num in zip
# Scatter plot for actual 'Expenditure_total' values with combined 'Year' ar
#plt.scatter(x_ticks_labels, Y_train, color='blue', label='Actual', alpha=0.
plt.plot(x_ticks_labels, Y_train, color='blue', label='Actual', marker='o',
# Scatter plot for predicted 'Expenditure_total' values with combined 'Year'
#plt.scatter(x_ticks_labels, Y_pred, color='red', label='Predicted', alpha=@
plt.plot(x_ticks_labels, Y_pred, color='red', label='Predicted',marker='o',
# Rotate x-axis labels for better readability
plt.xticks(rotation=90)
# Labels and title
plt.xlabel('Year, Region_num')
plt.ylabel('Expenditure total')
plt.title('Actual vs Predicted Expenditure_total')
# Show legend
plt.legend()
# Show plot
plt.tight_layout()
plt.show()
```

OLS Regression Results

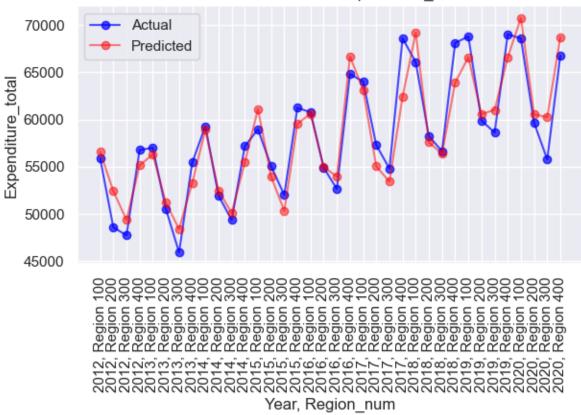
=======================================		======	=====			
=== Dep. Variable:	Expenditure	total	R_s	quared:		0.
880	Expendicare		1, 3	quai ca:		0.
Model:		0LS	Adj	. R-squared:		0.
876 Method:	Loast S	auarac	E 6	tatistic:		24
8.9	Least 3	quares	1 -5	tatistic.		24
Date:	Sun, 10 De	c 2023	Pro	b (F-statisti	ic):	3.28e
-17						
Time: 7.95	20	:51:40	Log	-Likelihood:		-32
No. Observations:		36	AIC	:		65
9.9		2.4	5.7.0			
<pre>Df Residuals: 3.1</pre>		34	BIC	:		66
Df Model:		1				
Covariance Type:	non	robust				
=======================================		======	=====	========		
=========	coef	c+d	orr	t	D~ +	[0.025
0.975]	coei	Stu	CII	C	1/[1	[0:023
 Intercept	1 2720±0/	2010	<i>1</i> 70	4.371	0 000	6806.182
1.86e+04	112/20104	23101	7/3	71371	0.000	00001102
<pre>Income_before_taxes</pre>	0.6089	0.	039	15.778	0.000	0.530
0.687						
=======================================	=======	======	=====	========		=======
Omnibus:		1.785	Dur	bin-Watson:		1.
868 Prob(Omnibus):		0.410	lar	que-Bera (JB)		0.
905		0.410	Jai	que-bera (5b)	•	0.
Skew:		0.349	Pro	b(JB):		0.
636		2 242	6			
Kurtosis: +05		3.340	Con	d. No.		5.85e
=======================================	========	======	=====	=========	=======	========
===						

Notes:

- $\[1\]$ Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 5.85e+05. This might indicate that there are

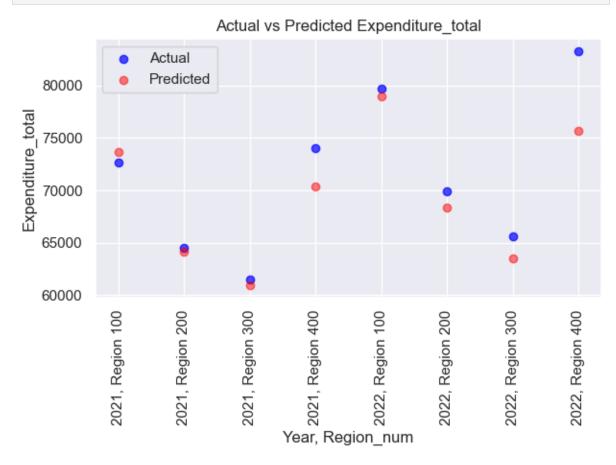
strong multicollinearity or other numerical problems.

Actual vs Predicted Expenditure total



```
In [40]: # Predict using the test data
         model = train[0]
         Y_pred = model.predict(X_test)
         # Extracting the 'Year', 'Expenditure_total', and 'Region_num' values for th
         years = X test['Year'].values
         region_nums = X_test['Region_num'].values
         # Combine 'Year' and 'Region num' to form a combined label for x-axis
         x_ticks_labels = [f"{year}, Region {region_num}" for year, region_num in zir
         # Scatter plot for actual 'Expenditure_total' values with combined 'Year' ar
         plt.scatter(x_ticks_labels, Y_test, color='blue', label='Actual', alpha=0.7)
         # Scatter plot for predicted 'Expenditure total' values with combined 'Year'
         plt.scatter(x_ticks_labels, Y_pred, color='red', label='Predicted', alpha=0.
         # Rotate x-axis labels for better readability
         plt.xticks(rotation=90)
         # Labels and title
         plt.xlabel('Year, Region num')
         plt.ylabel('Expenditure_total')
         plt.title('Actual vs Predicted Expenditure_total')
         # Show legend
         plt.legend()
         # Show plot
```

```
plt.tight_layout()
plt.show()
```



Results

3-Predictors Model

```
In [41]: # Predict using the test data
model = train[2]
print(model.summary())
Y_pred = model.predict(X_train)

# Extracting the 'Year', 'Expenditure_total', and 'Region_num' values for the
years = X_train['Year'].values
region_nums = X_train['Region_num'].values

# Combine 'Year' and 'Region_num' to form a combined label for x-axis
x_ticks_labels = [f"{year}, Region {region_num}" for year, region_num in zig

# Scatter plot for actual 'Expenditure_total' values with combined 'Year' and "plt.scatter(x_ticks_labels, Y_train, color='blue', label='Actual', alpha=0.
plt.plot(x_ticks_labels, Y_train, color='blue', label='Actual', marker='o',

# Scatter plot for predicted 'Expenditure_total' values with combined 'Year'
#plt.scatter(x_ticks_labels, Y_pred, color='red', label='Predicted', alpha=0.
plt.plot(x_ticks_labels, Y_pred, color='red', label='Predicted', alpha=0.
```

```
# Rotate x-axis labels for better readability
plt.xticks(rotation=90)

# Labels and title
plt.xlabel('Year, Region_num')
plt.ylabel('Expenditure_total')
plt.title('Actual vs Predicted Expenditure_total')

# Show legend
plt.legend()

# Show plot
plt.tight_layout()
plt.show()
```

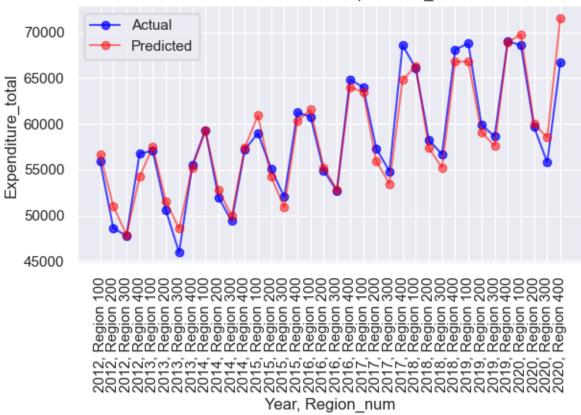
OLS Regression Results

=======================================	========	======	======		=======	=======
=== Dep. Variable:	Expenditure	_total	R-so	quared:		0.
937 Model:		0LS	Adi.	R-squared:		0.
931						
Method: 7.4	Least S	quares	F-s1	tatistic:		15
Date: -19	Sun, 10 De	c 2023	Prob	(F—statist	ic):	3.12e
Time:	20	:51:40	Log-	-Likelihood:		-31
<pre>6.46 No. Observations:</pre>		36	AIC	:		64
<pre>0.9 Df Residuals:</pre>		32	BIC	:		64
7.3						
Df Model:		3				
Covariance Type:						
=========						
0.975]				t		[0.025
Intercept -5466.218	-3.243e+04	1.326	e+04	-2.450	0.020	-5.94e+04
<pre>Income_before_taxes 0.354</pre>	0.1781	0.	.086	2.059	0.048	0.002
House_price	0.0582	0.	011	5.309	0.000	0.036
<pre>0.081 Employment_rate 1.64e+05</pre>		2.76	e+04	3.913	0.000	5.17e+04
===	========	======	======	========	=======	========
Omnibus: 996		5.320	Durk	oin-Watson:		0.
<pre>Prob(Omnibus):</pre>		0.070	Jaro	que-Bera (JB):	4.
105 Skew:		-0.544	Prob	o(JB):		0.
128 Kurtosis: +07		4.246	Cond	d. No.		2 . 69e
===	=======	======	======	========	======	=======

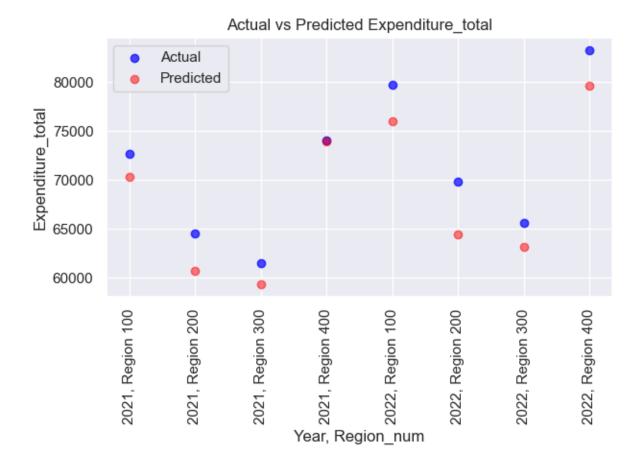
Notes:

- $\[1\]$ Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 2.69e+07. This might indicate that there are $\ensuremath{\text{are}}$
- strong multicollinearity or other numerical problems.

Actual vs Predicted Expenditure total



```
In [42]: # Predict using the test data
         model = train[2]
         Y_pred = model.predict(X_test)
         # Extracting the 'Year', 'Expenditure_total', and 'Region_num' values for th
         years = X test['Year'].values
         region_nums = X_test['Region_num'].values
         # Combine 'Year' and 'Region_num' to form a combined label for x—axis
         x_ticks_labels = [f"{year}, Region {region_num}" for year, region_num in zip
         plt.scatter(x_ticks_labels, Y_test, color='blue', label='Actual', alpha=0.7)
         plt.scatter(x ticks labels, Y pred, color='red', label='Predicted', alpha=0.
         # Rotate x-axis labels for better readability
         plt.xticks(rotation=90)
         # Labels and title
         plt.xlabel('Year, Region num')
         plt.ylabel('Expenditure_total')
         plt.title('Actual vs Predicted Expenditure_total')
         # Show legend
         plt.legend()
         # Show plot
         plt.tight_layout()
         plt.show()
```



Results discussion

The comparaison between the 1-predictor model with the 3-predictor model is quite interesting.

Obviously, the R-squared for the 3-predictor model is better that 1-predictor model (0.880 vs 0.937), but overall the 1-predictor model is more accurate in more of its predictions but very bad at predicting the value for the Region 400 (West).

The 3-predictor model predictions doe not come as close as the 1-predictor model in most values but correctly predicts the values for the Region 400, so it is consistent accros the different data points.

Before concluding, we will do a final comparaison between the two models and use them to predict the total Expenditure for 2023 for the South Region assuming that the Income before taxes will increase slightly. I have also gathered the current (as of Oct 2023) CPI and House prices for the region to make the test more realistic.

Here are the predicted values by the two models:

In [43]: X_test

Out[43]:		Year	Income_before_taxes	Expenditure_total	Transportation	Healthcare	Employment_r
	4	2021	100103	72678	9918	5759	3.0
	5	2021	84454	64542	10165	5834	0.6
	6	2021	79148	61473	11203	5107	0.1
	7	2021	94666	74033	12083	5447	9.0
	0	2022	108768	79741	12093	6069	0.0
	1	2022	91430	69870	11912	6569	0.
	2	2022	83346	65576	11932	5376	3.0
	3	2022	103418	83317	13420	5828	0.6
	<pre># data entered manually with assumptions in income before taxes. # Employment rate, CPI and House_price are the year-to-date (Oct23) values. data = { 'Year': [2023], 'Income_before_taxes': [85000], 'Expenditure_total': [0], 'Transportation': [0], 'Healthcare': [0], 'Employment_rate': [0.6125], 'Unemployment_rate': [0], 'Annual_CPI': [300], 'House_price': [296500], 'Region_num': [300] } entry_data = pd.DataFrame(data) entry_data</pre>						
Out[44]:		Year	Income_before_taxes	Expenditure_total	Transportation	Healthcare	Employment_r
	0	2023	85000	0	0	0	0.6
In [45]:	<pre>model = train[0] Y_pred = model.predict(entry_data) formatted_amount = '\${:,.2f}'.format(Y_pred[0]) print('Predicted Expenditure for Region South in 2023',formatted_amount) value_v = X_test.loc[(X_test['Year'] == 2022) & (X_test['Region_num'] == 300 value = value_v.values percentage = pd.to_numeric((Y_pred[0] - value)*100/ value) print('This is a ',percentage,'% variation')</pre>						
			ed Expenditure for a [-1.67622938] %	-	2023 \$64,476	.80	
In [46]:	Y_ fo pr va	pred : rmatte int('l	<pre>train[2] = model.predict(ent ed_amount = '\${:,.2} Predicted Expenditu = X_test.loc[(X_text) value_v.values</pre>	2f}'.format(Y_pr ure for Region S	outh in 2023'		_

```
percentage = pd.to_numeric((Y_pred[0] - value )*100/ value)
print('This is a ',percentage,'% variation')
```

```
Predicted Expenditure for Region South in 2023 $66,015.77 This is a [0.67063319] % variation
```

The *1-predictor* model predicts a total expenditure of 64,476.80 for 2023 in the South Region while the *3-predictor* model outputs a total expenditure of 66,015.77.

While the first one is predicting a small decline of -1.64% the second predict a small increase of 0.67% as compared to 2022.

At first sight, it looks like the results seem to confirm that the 3-predictor model is actually the best fit as it is expected that this year the expenditure increases (as it has done over the years) accentuated by the raise of inflation.

Obviously, this is just a data point and we still need to have all the parameters to confirm the adequacy of our choice but it is confirming that we are in the right direction.

We will need to the end of 2023 to have the final confirmation ;-)

Conclusion

In this project, we have collected data from public sources to build a supervised learning model that can predict the evolution of the annual cost of living for U.S. Households based on the US macroeconomic indicators.

The selected target variable that better represent the cost of living is the annual total expenditure. The features tested as predictors reflect the inflation (Annual CPI), unemployment (employment/unemployment rates) and housing prices.

A first correlation analysis of the different features imported along with these dataset has determine that there is enough correlation between the target variable and the features coming from the macroeconomic indicators.

Thanks to data visualization and plots we have confirmed the correlation analysis mentioned below, with one anomaly to be taken into account around the year 2020. The COVID pandemic impacted negatively the employment but, the effect in the expenditure was not as acute thanks to the government subsidies and policies. Also the house prices actually increased more than the trend in 2020-2021 as a consequence of the low supply. After pandemic, the inflation (CPI) surged and we could clearly see that the index slope was much steeper than the expenditure in the years after 2020. These are "one off" effects make the linear predictions challenging to fit these irregularities and our model will have to deal with it.

A first simple linear model was attempted using the best predictor (Income_before_taxes) that showed the R-squared score of 0.880 is already quite good.

We attempted as well a multi-linear model taking into consideration all availables features to obtained actually a worse R-squared score of 0.850. However, this model managed to better fit some of the data points (region 400) that were clearly badly predicted by the simple linear model.

In order to come up with a better model, we use the forward stepwise refinement technique to come up with a multi-predictor model that could delivered better results. We recursively attempted a combination of up to 6 "best preditors" to finally come to the conclusion that the optimal model efficiency for the training and test data was achieved with a 3-predictor model.

By visualizing the results and comparing with the 1-predictor model, the 3-predictor model (besides obviously having a better R-square of 0.937) does a better job of fitting and predicting the overall test data, hence confirming the conclusion achieved by analytical methods.

According to the optimal model the 2013 total expenditure in the South Region will beo 66,015.77 ... let's see how that compares to reality when the year comes to an end.

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