Assignment 1

AI1110: Probability and Random Variables

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11.16.3.5: Given that a fair coin is marked 1 on one face and 6 on the other and a fair die are tossed.find the probability sum turns up to be 3 and 12

Solution: Let the random variable A,B denote the toss of a coin and roll of a dice.

(a) The moment generating function of random variable is

$$MGF_X(s) = E(e^{sX}) \tag{1}$$

$$= \sum_{i=1}^{\infty} \Pr(X = i)\, e^{sX} \tag{2}$$

(b) Let us define a random variable Z

$$MGF_Z(s) = E(e^{sA+sB}) \tag{3}$$

$$= E(e^{sA}e^{sB}) \tag{4}$$

(c) Let A and B are independent random variables, then

$$MGF_Z(s) = MGF_A(s)MGF_B(s) \tag{5}$$

(d) We have

$$MGF_A(s) = \frac{e^s}{2} + \frac{e^{6s}}{2} \tag{6}$$

$$MGF_B(s) = \frac{\sum_{n=1}^{6} e^{ns}}{6} \tag{7}$$

$$MGF_B(s) = (\frac{\sum_{n=1}^{6} e^{ns}}{6})(\frac{e^s + e^{6s}}{2}) \tag{8}$$

(e) probability of Z=z is its coefficient in MGF

$$\Pr(Z = 3) = (\frac{1}{6})(\frac{1}{2}) \tag{9}$$

$$= \frac{1}{12} \tag{10}$$

$$\Pr(Z = 12) = (\frac{1}{6})(\frac{1}{2}) \tag{11}$$

$$= \frac{1}{12} \tag{12}$$