

# Assignment 1

## AI1110: Probability and Random Variables

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**11.16.3.5:** Given that a fair coin is marked 1 on one face and 6 on the other and a fair die are tossed. find the probability sum turns up to be 3 and 12

**Solution:** Let the random variable  $X, Y$  denote the toss of a coin and roll of a dice.

(a) The generating function of  $X$  is

$$M_X(z) = E[z^X] = \sum_{i=0}^{\infty} \Pr(X = i) z^{-i} \quad (1)$$

(b) Let us define a random variable  $Z$ , Let  $X$  and  $Y$  are independent random variables then

$$M_Z(z) = E[z^{X+Y}] = E[e^X e^Y] = E[z^X] E[z^Y] = M_X(z) M_Y(z) \quad (2)$$

(c) We have

$$M_X(z) = \frac{1}{2z} + \frac{1}{2z^6} \quad (3)$$

$$M_Y(z) = \sum_{n=1}^6 \frac{1}{6z^n} \quad (4)$$

$$M_Z(z) = \left( \sum_{n=1}^6 \frac{1}{6z^n} \right) \left( \frac{1}{2z} + \frac{1}{2z^6} \right) \quad (5)$$

(d) probability of  $Z=i$  is coefficient of  $z^{-i}$  in  $M_Z(z)$ . Hence from eqns (3),(4),(5) we get

$$\Pr(Z = 3) = \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) \quad (6)$$

$$= \frac{1}{12} \quad (7)$$

$$\Pr(Z = 12) = \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) \quad (8)$$

$$= \frac{1}{12} \quad (9)$$