Assignment 1

AI1110: Probability and Random Variables

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11.16.3.5: Given that a fair coin is marked 1 on one face and 6 on the other and a fair die are tossed.find the probability sum turns up to be 3 and 12

Solution: Let the random variable X,Y denote the toss of a coin and roll of a dice.

(a) The generating function of X is

$$M_X(z) = E[z^X] = \sum_{i=0}^{\infty} \Pr(X = i)\, z^{-i} \tag{1}$$

(b) Let us define a random variable Z,Let X and Y are independent random variables then

$$M_Z(z) = E[z^{X+Y}] = E[e^X e^Y] = E[z^X]E[z^Y] = M_X(z)M_Y(z) \tag{2}$$

(c) We have

$$M_X(z) = \left(z^{-1}\right)\left(\frac{1}{2}\right) + \left(z^{-1}\right)\left(\frac{1}{2}\right) \tag{3}$$

$$M_Y(z) = \sum_{n=1}^{6} \left(z^{-n}\right)\left(\frac{1}{6}\right) \tag{4}$$

$$M_Z(z) = \left[\left(z^{-1}\right)\left(\frac{1}{2}\right) + \left(z^{-1}\right)\left(\frac{1}{2}\right)\right]\left[\sum_{n=1}^{6} \left(z^{-n}\right)\left(\frac{1}{6}\right)\right] \tag{5}$$

(d) probability of Z=i is coefficient of z^{-i} in $M_Z(z)$.Hence from eqns (3),(4),(5) we get

$$\Pr(Z = 3) = \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) \tag{6}$$

$$= \frac{1}{12} \tag{7}$$

$$\Pr(Z = 12) = \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) \tag{8}$$

$$= \frac{1}{12} \tag{9}$$