Assignment 1
AI1110: Probability and Random Variables
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11.16.3.5: Given that a fair coin is marked 1 on one face and 6 on the other and a fair die are tossed.find the probability sum turns up to be 3 and 12
Solution: Let the random variable X,Y denote the toss of a coin and roll of a dice.

(a) The generating function of X is

$$M_X(z) = E\left[z^{-X}\right] = \sum_{n=-\infty}^{\infty} p_X(n)\, z^{-n} \tag{1}$$

(b) Let us define a random variable Z,Let X and Y are independent random variables then

$$M_Z(z) = E\left[z^{-(X+Y)}\right] = E\left[e^{-X}e^{-Y}\right] = E\left[z^{-X}\right]E\left[z^{-Y}\right] \tag{2}$$

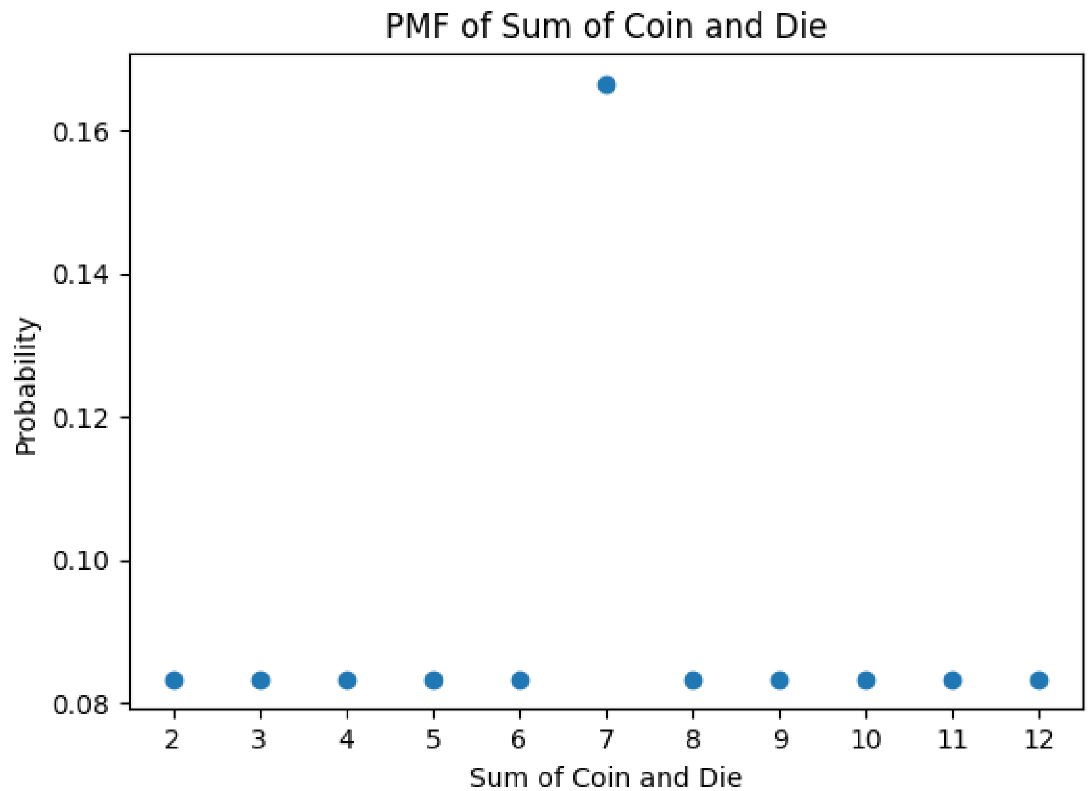
$$= M_X(z)M_Y(z) \tag{3}$$

(c) pmf of X,Y and Z

$$p_X(n) = \frac{1}{2} \qquad \text{for } n = \{1, 6\} \tag{4}$$

$$p_Y(n) = \frac{1}{6} \qquad \text{for} 1 \leqslant n < 7 \tag{5}$$

$$p_Z(n) = \begin{cases} \frac{1}{12} & \text{if } 2 \leqslant n < 7, \\ \frac{1}{6} & \text{if } n = 7, \\ \frac{1}{12} & \text{if } 8 \leqslant n < 13 \end{cases} \tag{6}$$



(d) We have

$$M_X(z) = \left(z^{-1}\right)\left(\frac{1}{2}\right) + \left(z^{-6}\right)\left(\frac{1}{2}\right) \tag{7}$$

$$M_Y(z) = \frac{z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6}}{6} \tag{8}$$

$$M_Z(z) = \left[\frac{z^{-1} + z^{-6}}{2}\right]\left[\frac{z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6}}{6}\right] \tag{9}$$

$$= \frac{1}{12}\left[z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6} + z^{-8} + z^{-9} + z^{-10} + z^{-11} + z^{-12}\right] + \frac{1}{6}\left[z^{-7}\right] \tag{10}$$

(e) probability of Z=i is coefficient of z^{-i} in $M_Z(z)$.Hence from eqn (10) we get

$$\Pr(Z = 3) = \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) \tag{11}$$

$$= \frac{1}{12} \tag{12}$$

$$\Pr(Z = 12) = \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) \tag{13}$$

$$= \frac{1}{12} \tag{14}$$