Assignment 3

AI1110: Probability and Random Variables

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exemplar 10.13.3.22: Two dice are thrown at the same time and the product of numbers appearing on them are noted.Find the probability that the product is less than 9.
Solution: Let the random variable X,Y denote the outcomes of dice.
The product of numbers appearing on them represented by XY.

TABLE (): Events and their probabilities

Parameter	Description	Value
X	outcome when first die is rolled	{1,2,...,6}
Y	outcome when second die is rolled	{1,2,...,6}
XY	product of outcomes of two dice	{1,2,.....,36}

(a) Finding the probability of XY less than n

$$\Pr(A|B) = \frac{\Pr(AB)}{B} \tag{1}$$

Taking $A = XY < n$ and $B = Y < \frac{n}{k}$ we get

$$\Pr(A) = \sum_{k=1}^{6} \Pr(X = k) \Pr(B|X = k) \tag{2}$$

$$\Pr(B|X = k) = \frac{\Pr(B.X = k)}{\Pr(X = k)} \tag{3}$$

keeping (3) in (2) (4)

$$\Pr(A) = \sum_{k=1}^{6} \Pr(B.X = k) \tag{5}$$

As X and Y are independent,

$$\Pr(A) = \sum\nolimits_{k} = 1^{6} \Pr(B) \,.\, \Pr(X = k) \tag{6}$$

$$\Pr(XY < n) = \sum_{k=1}^{6} \left(\Pr(X = k) \Pr\left(Y < \frac{n}{k}\right)\right) \tag{7}$$

$$\Pr\left(Y < \frac{n}{k}\right) = \Pr\left(Y \leq \frac{n}{k}\right) - \Pr\left(Y = \frac{n}{k}\right) \tag{8}$$

(b) $\Pr\left(Y \leq \frac{n}{k}\right)$ is cdf of Y represented by $F_Y\left(\frac{n}{k}\right)$.

$$F_Y(n) = \begin{cases} 0 & \text{for } n < 1, \\ \frac{[n]}{6} & \text{for } 1 \leqslant n \leqslant 6, \\ 1 & \text{for } n > 6 \end{cases} \tag{9}$$

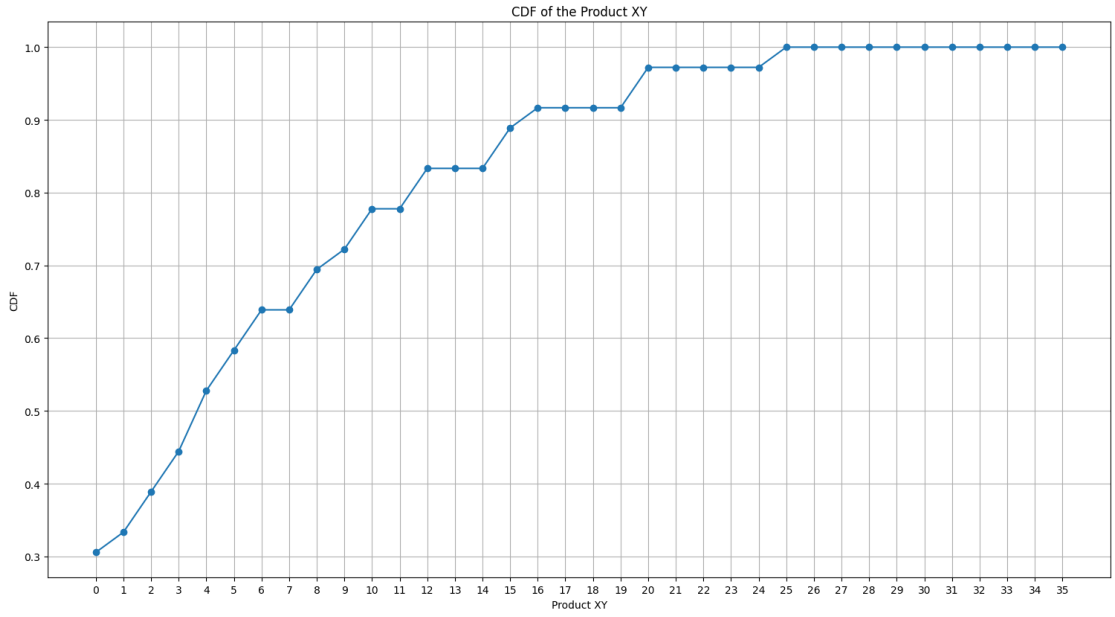

Fig. (b): cdf of product of numbers appeared on dice

$\Pr(X = k)$ is pmf of X which is represented by $p_X(k)$.

$$p_X(k) = \begin{cases} \frac{k}{6} & \text{for } k \in \{1,2,3,4,5,6\}, \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

From (3)

$$\Pr\left(Y < \frac{n}{k}\right) = F_Y\left(\frac{n}{k}\right) - \Pr\left(Y = \frac{n}{k}\right) \tag{11}$$

$$\Pr(XY < n) = \sum_{k=1}^{6} \frac{1}{6} \Pr\left(Y < \frac{n}{k}\right) \tag{12}$$

(c) Keeping n = 9

$$\Pr(XY < 9) = \sum_{k=1}^{6} \left(\frac{1}{6} \Pr\left(Y < \frac{9}{k}\right)\right) \tag{13}$$

$$\begin{aligned} &= \frac{1}{6}\Pr\left(Y < \frac{9}{1}\right) + \frac{1}{6}\Pr\left(Y < \frac{9}{2}\right) + \frac{1}{6}\Pr\left(Y < \frac{9}{3}\right) + \\ &\frac{1}{6}\Pr\left(Y < \frac{9}{4}\right) + \frac{1}{6}\Pr\left(Y < \frac{9}{5}\right) + \frac{1}{6}\Pr\left(Y < \frac{9}{6}\right) \end{aligned} \tag{14}$$

$$= \left(\frac{1}{6}\right)\left(1 + \frac{4}{6} + \frac{2}{6} + \frac{2}{6} + \frac{1}{6} + \frac{1}{6}\right) \tag{15}$$

$$= \left(\frac{1}{6}\right)\left(\frac{16}{6}\right) \tag{16}$$

$$= \frac{16}{36} = \frac{4}{9} \tag{17}$$

Required probability is $\frac{4}{9}$