

Assignment 3

AI1110: Probability and Random Variables

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exemplar 10.13.3.22: Two dice are thrown at the same time and the product of numbers appearing on them are noted. Find the probability that the product is less than 9.

Solution: Let the random variable X, Y denote the outcomes of dice. The product of numbers appearing on them represented by XY .

(a) Finding the probability of XY less than n

$$\Pr(XY < n) = \sum_{k=1}^6 \Pr\left(X = k, Y < \frac{n}{k}\right) \quad (1)$$

As X and Y are independent,

$$\Pr(XY < n) = \sum_{k=1}^6 \left(\Pr(X = k) \Pr\left(Y < \frac{n}{k}\right) \right) \quad (2)$$

$$\Pr\left(Y < \frac{n}{k}\right) = \Pr\left(Y \leq \frac{n}{k}\right) - \Pr\left(Y = \frac{n}{k}\right) \quad (3)$$

(b) $\Pr\left(Y \leq \frac{n}{k}\right)$ is cdf of Y represented by $F_Y\left(\frac{n}{k}\right)$.

$$F_Y(n) = \begin{cases} 0 & \text{for } n < 1, \\ \frac{[n]}{6} & \text{for } 1 \leq n \leq 6, \\ 1 & \text{for } n > 6 \end{cases} \quad (4)$$

$\Pr(X = k)$ is pmf of X which is represented by $p_X(k)$.

$$p_X(k) = \begin{cases} \frac{1}{6} & \text{for } k \in \{1, 2, 3, 4, 5, 6\}, \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

From (3)

$$\Pr\left(Y < \frac{n}{k}\right) = F_Y\left(\frac{n}{k}\right) - \Pr\left(Y = \frac{n}{k}\right) \quad (6)$$

$$\Pr(XY < n) = \sum_{k=1}^6 \frac{1}{6} \Pr\left(Y < \frac{n}{k}\right) \quad (7)$$

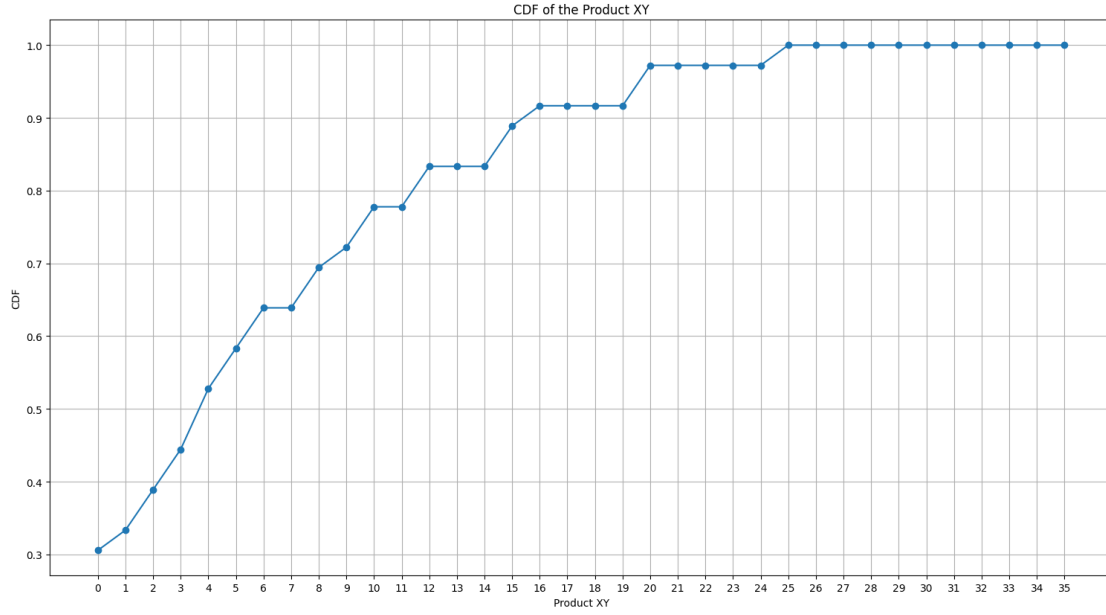


Fig. (b): cdf of product of numbers appeared on dice

(c) Keeping $n = 9$

$$\Pr(XY < 9) = \sum_{k=1}^6 \left(\frac{1}{6} \Pr\left(Y < \frac{9}{k}\right) \right) \quad (8)$$

$$= \frac{1}{6} \Pr\left(Y < \frac{9}{1}\right) + \frac{1}{6} \Pr\left(Y < \frac{9}{2}\right) + \frac{1}{6} \Pr\left(Y < \frac{9}{3}\right) + \quad (9)$$

$$\frac{1}{6} \Pr\left(Y < \frac{9}{4}\right) + \frac{1}{6} \Pr\left(Y < \frac{9}{5}\right) + \frac{1}{6} \Pr\left(Y < \frac{9}{6}\right)$$

$$= \left(\frac{1}{6}\right) \left(1 + \frac{4}{6} + \frac{2}{6} + \frac{2}{6} + \frac{1}{6} + \frac{1}{6}\right) \quad (10)$$

$$= \left(\frac{1}{6}\right) \left(\frac{16}{6}\right) \quad (11)$$

$$= \frac{16}{36} = \frac{4}{9} \quad (12)$$

Required probability is $\frac{4}{9}$