

Assignment 1

AI1110: Probability and Random Variables

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11.16.3.5: Given that a fair coin is marked 1 on one face and 6 on the other and a fair die are tossed. find the probability sum turns up to be 3 and 12

Solution: Let the random variable X, Y denote the toss of a coin and roll of a dice.

(a) The generating function of X is

$$M_X(z) = E[z^{-X}] = \sum_{n=-\infty}^{\infty} p_X(n) z^{-n} \quad (1)$$

(b) Let us define a random variable Z , Let X and Y are independent random variables then

$$M_Z(z) = E[z^{-(X+Y)}] = E[e^{-X} e^{-Y}] = E[z^{-X}] E[z^{-Y}] \quad (2)$$

$$= M_X(z) M_Y(z) \quad (3)$$

(c) pmf of X, Y and Z

$$p_X(n) = \frac{1}{2} \quad \text{for } n = \{1, 6\} \quad (4)$$

$$p_Y(n) = \frac{1}{6} \quad \text{for } 1 \leq n < 7 \quad (5)$$

$$p_Z(n) = \begin{cases} \frac{1}{12} & \text{if } 2 \leq n < 7, \\ \frac{1}{6} & \text{if } n = 7, \\ \frac{1}{12} & \text{if } 8 \leq n < 13 \end{cases} \quad (6)$$

(d) We have

$$M_X(z) = (z^{-1})\left(\frac{1}{2}\right) + (z^{-6})\left(\frac{1}{2}\right) \quad (7)$$

$$M_Y(z) = \frac{z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6}}{6} \quad (8)$$

$$M_Z(z) = \left[\frac{z^{-1} + z^{-6}}{2} \right] \left[\frac{z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6}}{6} \right] \quad (9)$$

$$= \frac{1}{12} [z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6} + z^{-8} + z^{-9} + z^{-10} + z^{-11} + z^{-12}] + \frac{1}{6} [z^{-7}] \quad (10)$$

(e) probability of $Z=i$ is coefficient of z^{-i} in $M_Z(z)$. Hence from eqn (10) we get

$$\Pr(Z = 3) = \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) \quad (11)$$

$$= \frac{1}{12} \quad (12)$$

$$\Pr(Z = 12) = \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) \quad (13)$$

$$= \frac{1}{12} \quad (14)$$