Assignment 1

AI1110: Probability and Random Variables

INDIAN INSTITUTE OF TECHNOLOGY, HYDERABAD

LAHARI GUNTI
AI22BTECH11008

11.16.3.5: Given that a fair coin is marked 1 on one face and 6 on the other and a fair die are tossed.find the probability sum turns up to be 3 and 12

Solution: Let the random variable X,Y denote the toss of a coin and roll of a dice.

(a) The generating function of X is

$$M_X(z) = E\left[z^X\right] = \sum_{i=0}^{\infty} \Pr\left(X = i\right) z^{-X} \tag{1}$$

(b) Let us define a random variable Z,Let X and Y are independent random variables then

$$M_Z(z) = E\left[z^{X+Y}\right] = E\left[e^X e^Y\right] = E\left[z^X\right] E\left[z^Y\right] \tag{2}$$

$$= M_X(z) M_Y(z) \tag{3}$$

(c) We have

$$M_X(z) = \left(z^{-1}\right)\left(\frac{1}{2}\right) + \left(z^{-6}\right)\left(\frac{1}{2}\right) \tag{4}$$

$$M_Y(z) = \frac{z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6}}{6} \tag{5}$$

$$M_Z(z) = \left[\frac{z^{-1} + z^{-6}}{2}\right]\left[\frac{z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6}}{6}\right] \tag{6}$$

$$= \frac{1}{12}\left[z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6} + z^{-8} + z^{-9} + z^{-10} + z^{-11} + z^{-12}\right] + \frac{1}{6}\left[z^{-7}\right] \tag{7}$$

(d) probability of Z=i is coefficient of z^{-i} in $M_Z(z)$.Hence from eqns (3),(4),(5) we get

$$\Pr\left(Z = 3\right) = \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) \tag{8}$$

$$= \frac{1}{12} \tag{9}$$

$$\Pr\left(Z = 12\right) = \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) \tag{10}$$

$$= \frac{1}{12} \tag{11}$$