Assignment 3

AI1110: Probability and Random Variables

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exemplar 10.13.3.22: Two dice are thrown at the same time and the product of numbers appearing on them are noted.Find the probability that the product is less than 9.
Solution: Let the random variable X,Y denote the outcomes of dice.
The product of numbers appearing on them represented by XY.

(a) Finding the probability of XY less than n

$$\Pr(XY < n) = \sum_{k=1}^{6} \Pr\left(X = k, Y < \frac{n}{k}\right) \tag{1}$$

As X and Y are independent,

$$\Pr(XY < n) = \sum_{k=1}^{6} \left(\Pr(X = k) \Pr\left(Y < \frac{n}{k}\right)\right) \tag{2}$$

$$\Pr\left(Y < \frac{n}{k}\right) = \Pr\left(Y \leq \frac{n}{k}\right) - \Pr\left(Y = \frac{n}{k}\right) \tag{3}$$

(b) $\Pr\left(Y \leq \frac{n}{k}\right)$ is cdf of Y represented by $F_Y\left(\frac{n}{k}\right)$.

$$F_Y(n) = \begin{cases} 0 & \text{for } n < 1, \\ \frac{[n]}{6} & \text{for } 1 \leqslant n \leqslant 6, \\ 1 & \text{for } n > 6 \end{cases} \tag{4}$$

$\Pr(X = k)$ is pmf of X which is represented by $P_X(k)$.

$$P_X(k) = \begin{cases} \frac{k}{6} & \text{for } k \in \{1, 2, 3, 4, 5, 6\}, \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

From (3)

$$\Pr\left(Y < \frac{n}{k}\right) = F_Y\left(\frac{n}{k}\right) - \Pr\left(Y = \frac{n}{k}\right) \tag{6}$$

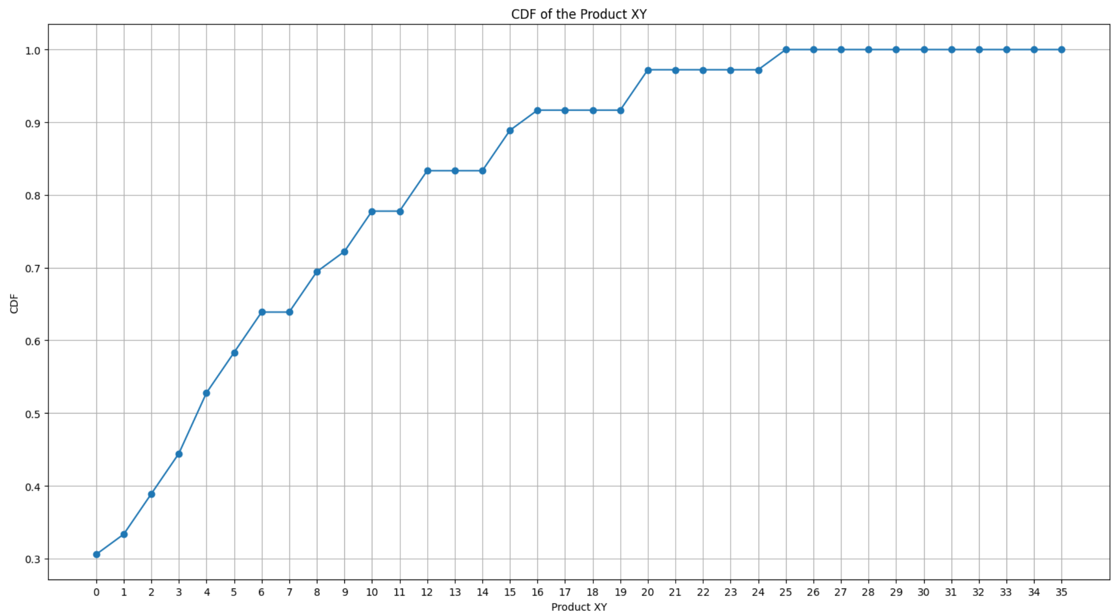

Fig. (b): cdf of product of numbers appeared on dice

(c) Keeping n = 9

$$\Pr(XY < 9) = \sum_{k=1}^{6}\left(\Pr(X = k)\Pr\left(Y < \frac{9}{k}\right)\right) \tag{7}$$

$$\begin{aligned} &= \Pr(X = 1)\Pr\left(Y < \frac{9}{1}\right) + \Pr(X = 2)\Pr\left(Y < \frac{9}{2}\right) + \Pr(X = 3)\Pr\left(Y < \frac{9}{3}\right) + \\ &\Pr(X = 4)\Pr\left(Y < \frac{9}{4}\right) + \Pr(X = 5)\Pr\left(Y < \frac{9}{5}\right) + \Pr(X = 6)\Pr\left(Y < \frac{9}{6}\right) \end{aligned} \tag{8}$$

$$= \left(\frac{1}{6}\right)\left(1 + \frac{4}{6} + \frac{2}{6} + \frac{2}{6} + \frac{1}{6} + \frac{1}{6}\right) \tag{9}$$

$$= \left(\frac{1}{6}\right)\left(\frac{16}{6}\right) \tag{10}$$

$$= \frac{16}{36} = \frac{4}{9} \tag{11}$$

Required probability is $\frac{4}{9}$