Quantitative Relational Synthesis With Semantic Preference Objectives

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M.Tech Defense

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Relational Property as a Hoare Triple.

$$\{\mathsf{Pre}(\vec{x_1},\ \vec{x_2})\}\ \mathcal{P}_1, \mathcal{P}_2\ \{\mathsf{Post}(y_1,\ y_2)\}$$

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When both programs $\mathcal{P}_1$ and $\mathcal{P}_2$ are the same programs, i.e $\mathcal{P}$, *relational properties* become **hyper-properties**.

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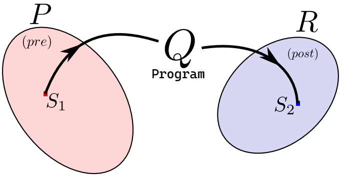
Specification as a **Hoare Triple**,

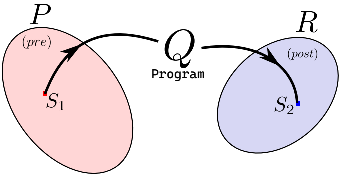
$$\{P\}Q\{R\}$$



What is program sketching?

A partial program (referred to as a *sketch*), which leaves out certain $\boxed{holes}$ for the synthesizer to fill such that the completed program satisfies the required specification.

Specification as a **Hoare Triple**, $\{P\}Q\{R\}$




```
int 𝒫(int n){
    // PRE : assume(n > 1);
    int i = 0, x = 0;
    while(i < n){
        i = i + 1;
        x = [·]
    }
    // POST : assert(x > 2 * n);
    return x;
}
```

Given partial programs $\mathcal{P}_1^{[\cdot]}$ and $\mathcal{P}_2^{[\cdot]}$, find completion $\mathcal{E}$, where $\mathcal{E}.H$ and $\mathcal{E}.G$ respective completions of $\mathcal{P}_1^{[\cdot]}$ and $\mathcal{P}_2^{[\cdot]}$ that satisfy a specification.

Lifting Program Sketching to Relational Sketching

Given partial programs $\mathcal{P}_1^{[\cdot]}$ and $\mathcal{P}_2^{[\cdot]}$, find completion $\mathcal{E}$, where $\mathcal{E}.H$ and $\mathcal{E}.G$ respective completions of $\mathcal{P}_1^{[\cdot]}$ and $\mathcal{P}_2^{[\cdot]}$ that satisfy a specification.

Relational Synthesis

$$\exists \mathcal{E}.\ \{\mathsf{Pre}(\vec{x_1},\ \vec{x_2})\}\ \mathcal{P}_1^{[\mathcal{E}.H]}, \mathcal{P}_2^{[\mathcal{E}.G]}\ \{\mathsf{Post}(y_1,\ y_2)\}$$

Program equivalence requires that, any two executions of a pair of programs, $\mathcal{P}_1$ and $\mathcal{P}_2$ on same the same input $\vec{x}$, must yield the same outputs.

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A Verification Problem

$$\forall \vec{x}.\ \mathcal{P}_1(\vec{x}) = \mathcal{P}_2(\vec{x})$$

Given a reference program $\mathcal{P}_1$ and a *partial* program $\mathcal{P}_2$ that has a hole $\boxed{\cdot}$, we are interested in *completing* the partial program by synthesizing an expression to fill the hole such that the two programs are rendered semantically equivalent.

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Formal Definition

$$\exists \mathcal{E} \in \mathcal{L}(\mathcal{G}).\ \forall \vec{x}.\ \mathcal{P}_1(\vec{x}) = \mathcal{P}_2^{[\mathcal{E}]}(\vec{x})$$

```
int P1(int n){
    assume(n > 1);
    int i = 0, ans = 0;
    while(i < (n - 1)){
        i = i + 1;
        ans = ans + (5 * i) + 1;
    }
    return ans + 1;
}
```

(a) Program 1

```
int P2[·](int n){
    assume(n > 1);
    int x = 0, y = 0, z = n;
    while(z ≠ 0){
        z = z - 1;
        x = [·];
        y = y + 1;
    }
    return x + y;
}
```

(b) Program 2 (with hole $\boxed{\cdot}$)

```
int P1(int n){
    assume(n > 1);
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    }
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}
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(b) Program 2 (with hole $\boxed{\cdot}$)

Post condition for sketching.

$$\exists \mathcal{E} \in \mathcal{L}(\mathcal{G}).\ \forall \vec{x}.\ \mathcal{P}_1(\vec{x}) = \mathcal{P}_2^{[\mathcal{E}]}(\vec{x})$$

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(b) Program 2 (with hole $\boxed{\cdot}$)

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$$\exists \mathcal{E} \in \mathcal{L}(\mathcal{G}).\ \forall \vec{x}.\ \mathcal{P}_1(\vec{x}) = \mathcal{P}_2^{[\mathcal{E}]}(\vec{x})$$

$$assert(ans + 1 = x + y)$$

```
int P_1(int n){
    assume(n > 1);
    int i = 0, ans = 0;
    while(i < (n - 1)){
        i = i + 1;
        ans = ans + (5 * i) + 1;
    }
    return ans + 1;
}
```

(a) Program 1

```
int P_2^[.](int n){
    assume(n > 1);
    int x = 0, y = 0, z = n;
    while(z ≠ 0){
        z = z - 1;
        x = x + 6 · y − n;
        y = y + 1;
    }
    return x + y;
}
```

(b) Program 2 (with completion)

Program equivalence posed as a **relational** property.

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Relational Synthesis for Program Equivalence

$$\exists \mathcal{E}.\ \{\vec{x_1} = \vec{x_2}\}\ \ \mathcal{P}_1^{[\mathcal{E}.H]}, \mathcal{P}_2^{[\mathcal{E}.G]}\ \ \{\mathcal{P}_1^{[\mathcal{E}.H]}(\vec{x_1}) = \mathcal{P}_2^{[\mathcal{E}.G]}(\vec{x_2})\}$$

Strict Non-Interference

Given public inputs, $(\vec{x_1}, \vec{x_2})$ and secret inputs (s_1 , s_2), program $(\mathcal{P})$ does not reveal any information about the secret inputs.

$$\exists \mathcal{E}.\ \{\vec{x_1} = \vec{x_2}\}\ \mathcal{P}^{[\mathcal{E}]}\ \ \{\mathcal{P}^{[\mathcal{E}]}(s_1 , \vec{x_1}) = \mathcal{P}^{[\mathcal{E}]}(s_2 , \vec{x_2})\}$$

Weak Equivalence

$$\exists \mathcal{E}.\ \{\vec{x_1} = \vec{x_2}\}\ \ \mathcal{P}_1^{[\mathcal{E}.H]}, \mathcal{P}_2^{[\mathcal{E}.G]}\ \{||\mathcal{P}_1^{[\mathcal{E}.H]}(\vec{x_1}) - \mathcal{P}_2^{[\mathcal{E}.G]}(\vec{x_2})|| \leq c\}$$

Weak Non-Interference

$$\exists \mathcal{E}.\ \{\vec{x_1} = \vec{x_2}\}\ \mathcal{P}^{[\mathcal{E}]}\ \ \{||\mathcal{P}^{[\mathcal{E}]}(\ s_1\ , \vec{x_1}) - \mathcal{P}^{[\mathcal{E}]}(\ s_2\ , \vec{x_2})|| \leq c\}$$

Robustness

Small changes in the inputs must not lead to large difference in the responses of the program, $\mathcal{P}$.

$$\exists \mathcal{E}.\ \{||\vec{x_1} - \vec{x_2}|| \leq d_1\}\ \mathcal{P}^{[\mathcal{E}]}\ \ \{||\mathcal{P}^{[\mathcal{E}]}(\vec{x_1}) - \mathcal{P}^{[\mathcal{E}]}(\vec{x_2})|| \leq f(\vec{x_1}, \vec{x_2})\}$$

Program, $\mathcal{P}$ must not use the *sensitive attribute* (s) to be unfair to the minority population.

$$\exists \mathcal{E}.\ \{s_1 \leq s_2 \wedge \vec{x_2} \sqsubseteq \vec{x_1}\}\ \mathcal{P}^{[\mathcal{E}]}\ \ \{\mathcal{P}^{[\mathcal{E}]}(\vec{x_2}, s_2) \leq \mathcal{P}^{[\mathcal{E}]}(\vec{x_1}, s_1)\}$$

Monotonicity

For any two executions of the program, $\mathcal{P}$, if the inputs are ordered, so must be the outputs.

$$\exists \mathcal{E}.\ \{\vec{x_1} \sqsubseteq \vec{x_2}\}\ \mathcal{P}^{[\mathcal{E}]}\ \ \{\mathcal{P}^{[\mathcal{E}]}(\vec{x_1}) \sqsubseteq \mathcal{P}^{[\mathcal{E}]}(\vec{x_2})\}$$

Quantitative objectives on the synthesis problem that defines the *preference* among the feasible completions, $\mathcal{E}$.

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Quantitative Objective, Γ

$$\Gamma(\ \mathcal{P}_1^{[\mathcal{E}.H]}(\vec{x_1}),\ \mathcal{P}_2^{[\mathcal{E}.H]}(\vec{x_2})\ )$$

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Quantitative Objective, Γ

$$\Gamma(\ \mathcal{P}_1^{[\mathcal{E}.H]}(\vec{x_1}),\ \mathcal{P}_2^{[\mathcal{E}.H]}(\vec{x_2})\ )$$

The most preferred completion can be found by **minimizing** the quantitative objective.

The ordering relation $\mathcal{E}$ is a partial order over the completions, defined by the relation, $\sqsubseteq$, and the *strict* ordering $\sqsubset$

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Dominance

A completion $\mathcal{E}'$ dominates a completion $\mathcal{E}$ if the value of Γ score with $\mathcal{E}'$ is smaller or equals to $\mathcal{E}$ across all input pairs $\langle \vec{x_1}, \vec{x_2} \rangle$

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Strict Dominance

Additionally requires the existence of at least one input-pair where the Γ score of $\mathcal{E}'$ is strictly smaller than that of $\mathcal{E}$

Monotonicity, Robustness, Weak Non-Interference, Weak Equivalence

Completions that minimize the distance between any two responses of the program.

$$\Gamma(\mathcal{P}^{[\mathcal{E}]}(\vec{x_1}), \mathcal{P}^{[\mathcal{E}]}(\vec{x_2})) \triangleq ||\mathcal{P}^{[\mathcal{E}]}(\vec{x_1}) - \mathcal{P}^{[\mathcal{E}]}(\vec{x_2})||$$

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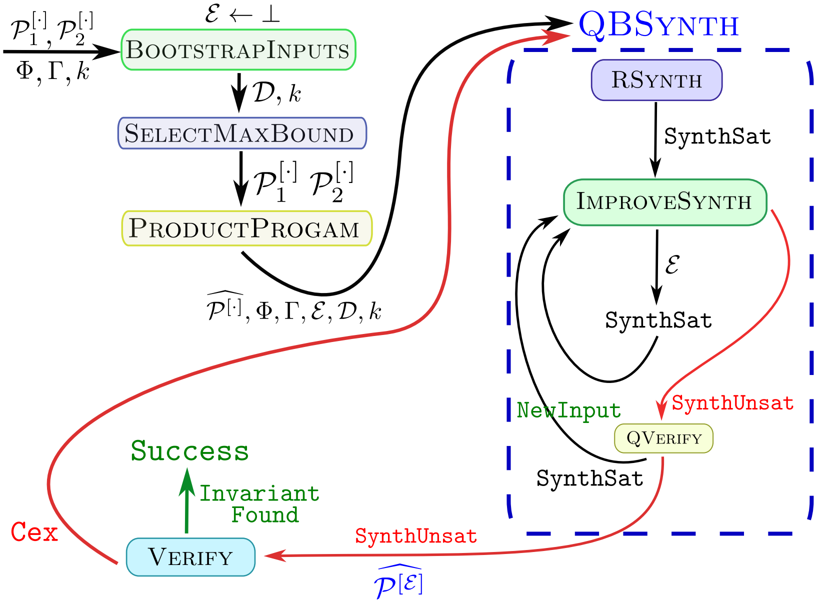
Group Fairness

Prefer completions where the deviation in responses between candidates of two populations is small for similar candidates.

$$\begin{aligned}\Gamma(\mathcal{P}^{[\mathcal{E}]}(\vec{x_1}, s_1), \mathcal{P}^{[\mathcal{E}]}(\vec{x_2}, s_2)) \triangleq & \text{ if } (s_1 < s_2 \land \vec{x_1} \sim \vec{x_2}) \\ & \text{then } ||\mathcal{P}^{[\mathcal{E}]}(\vec{x_1}, y_1) - \mathcal{P}^{[\mathcal{E}]}(\vec{x_2}, y_2)|| \text{ else } 0\end{aligned}$$

Monotonicity Example

```
// PRE: (a2 < a1 ∧ b2 > b1)
int P^[·] (int a, int b){
    assume((0 < a) && (a < b));
    while (a < b) {
        c = c + [·];
        a = a + 1;
    }
    return c;
}
// POST: (c2 > c1)
```


$\mathcal{E} \leftarrow \perp$
$\mathcal{P}_1^{[\cdot]}, \mathcal{P}_2^{[\cdot]}$
Φ, Γ, k
BootstrapInputs
$\mathcal{D}, k$
SelectMaxBound
$\mathcal{P}_1^{[\cdot]}$ $\mathcal{P}_2^{[\cdot]}$
ProductProgam
$\widehat{\mathcal{P}^{[\cdot]}}, \Phi, \Gamma, \mathcal{E}, \mathcal{D}, k$
QBSynth
RSynth
SynthSat
ImproveSynth
$\mathcal{E}$
SynthSat
NewInput
SynthUnsat
QVerify
SynthSat
Success
Invariant Found
Cex
Verify
SynthUnsat
$\widehat{\mathcal{P}^{[\mathcal{E}]}}$

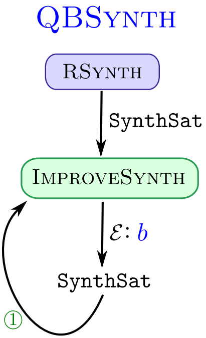
Monotonicity Example: Product Program

```
int P̂[·](int a1, int b1, int a2, int b2){

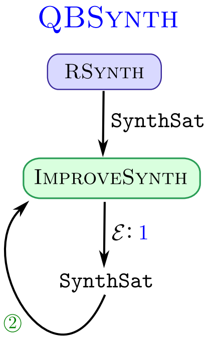
    assume((0 < a1) && (a1 < b1));
    assume((0 < a2) && (a2 < b2));
    assume((a2 < a1) && (b2 > b1));

    int c1 = 0, c2 = 0;
    while ( (a1 < b1) || (a2 < b2) ) {
        if ( (a1 < b1) ) {
            c1 = c1 + [·];
            a1 = a1 + 1;
        }
        if ( (a2 < b2) ) {
            c2 = c2 + [·];
            a1 = a1 + 1;
        }
    }
    return c1, c2;
}
```

```
// PRE: (a2 < a1 ∧ b2 > b1)
int P[·] (int a, int b){
    assume((0 < a) && (a < b));
    while (a < b) {
        c = c + [b];
        a = a + 1;
    }
    return c;
}
// POST: (c2 > c1)
```

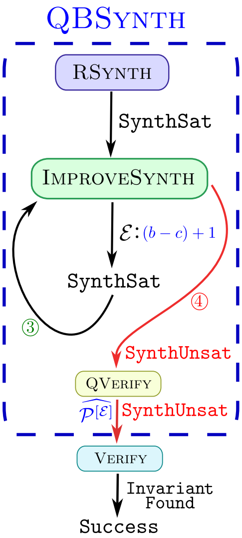


```
// PRE: (a2 < a1 ∧ b2 > b1)
int P[.] (int a, int b){
    assume((0 < a) && (a < b));
    while (a < b) {
        c = c + [1];
        a = a + 1;
    }
    return c;
}
// POST: (c2 > c1)
```



Monotonicity Example: Third Completion

```
// PRE: (a₂ < a₁ ∧ b₂ > b₁)
int P^[·] (int a, int b){
    assume((0 < a) && (a < b));
    while (a < b) {
        c = c + [(b - c) + 1];
        a = a + 1;
    }
    return c;
}
// POST: (c₂ > c₁)
```



Experiments

RQ1. Relational synthesis **without** quantitative objectives

RQ2. Relational synthesis **with** quantitative objectives

Grammar to describe the expressions to be instantiated for the holes in the program sketches; $\langle \texttt{var} \rangle$ and $\langle \texttt{num} \rangle$ refer to program variables and numbers respectively.

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Domain Specific Language

$$E ::= E + E \mid E - E \mid E * E \mid \langle \texttt{var} \rangle \mid \langle \texttt{num} \rangle$$

Bench	Property	Time(s)
b26	**Strict Equivalence**	4
b10	**Strict Equivalence**	3
b18	**Strict Equivalence**	2
b16	**Strict Equivalence**	1
b21	**Strict Equivalence**	3
b27	**Strict Equivalence**	4
b04	**Strict Equivalence**	3
b34	**Strict Equivalence**	10
b05	**Strict Equivalence**	7
nonintf01	**Strict Non-Interference**	7
nonintf02	**Strict Non-Interference**	8
nonintf05	**Strict Non-Interference**	6

Bench	Property	Time(s)	Best?
mono01	**Monotonicity**	329	✔
mono02	**Monotonicity**	311	✔
mono02	**Monotonicity**	310	✔
weak01	**Weak Equivalence**	210	✔
weak02	**Weak Equivalence**	198	✔
weak03	**Weak Equivalence**	128	✔
weak04	**Weak Equivalence**	168	✔
robust01	**Robustness**	95	✔
robust02	**Robustness**	102	✔
fair01	**Group Fairness**	82	✔
nonintf03	**Weak Non-Interference**	70	✔
nonintf04	**Weak Non-Interference**	75	✔

New Synthesis Problem!

Relational synthesis with semantic quantitative objectives—and designed an algorithm to solve the same.

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Conclusion

- QRS instantiates this algorithm and solves non-trivial problems in a reasonable time.
- Our solutions are optimal only for a k-bounded setting, however, we do achieve the globally optimal completions (empirically) for all benchmarks in our suite.

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Future Directions

- Synthesis for global optimality.
- Synthesis problems for those beyond 2-safety.

Thank You!

Questions?