Question 01

Finding the general solution of the equation (2)

$$V_1(x) = A_1 e^{-x} + B_1 e^{x} \qquad 0 \le x \le L_1 \quad \text{——— (A)}$$
$$V_{21}(x) = A_{21} e^{-x} + B_{21} e^{x} \qquad L_1 \le x \le L_{21} \quad \text{——— (B)}$$
$$V_{22}(x) = A_{22} e^{-x} + B_{22} e^{x} \qquad L_1 \le x \le L_{22} \quad \text{——— (C)}$$

Boundary condition I

$$\left.\frac{dV_1}{dx}\right|_{x=0} = -(r_i + d_c) I_{app}$$

Differentiating the equation (A)

$$\frac{dV_1}{dx} = -A_1 e^{-x} + B_1 e^{x}$$

$$\left.\frac{dV_1}{dx}\right|_{x=0} = -A_1 + B_1 = -(r_i + d_c) I_{app}$$

$$\boxed{A_1 - B_1 = (r_i + d_c) I_{app}}$$ //

Boundary condition II

$$V_{21}(L_{21}) = V_{22}(L_{22}) = 0$$

By (B),

$$V_{21}(x) = A_{21} e^{-x} + B_{21} e^{x}$$
$$V_{21}(L_{21}) = A_{21} e^{-L_{21}} + B_{21} e^{-L_{21}} = 0$$

$$\boxed{A_{21} e^{-L_{21}} + B_{21} e^{-L_{21}} = 0}$$ //

By ©,

$$V_{22}(x) = A_{22}e^{-x} + B_{22}e^{x}$$

$$V_{22}(L_{22}) = A_{22}e^{-L_{22}} + B_{22}e^{L_{22}} = 0$$

$$\boxed{A_{22}e^{-L_{22}} + B_{22}e^{L_{22}} = 0}$$

Nordal Condition I

$$V_1(L_1) = V_{21}(L_1) = V_{22}(L_1)$$

$$V_1(L_1) = V_{21}(L_1)$$

$$A_1e^{-L_1} + B_1e^{L_1} = A_{21}e^{-L_1} + B_{21}e^{L_1}$$

$$\boxed{A_1e^{-L_1} + B_1e^{L_1} - A_{21}e^{-L_1} - B_{21}e^{L_1} = 0}$$

$$V_{21}(L_1) = V_{22}(L_1)$$

$$A_{21}e^{-L_1} + B_{21}e^{-L_1} = A_{22}e^{-L_1} + B_{22}e^{L_1}$$

$$\boxed{A_{21}e^{-L_1} + B_{21}e^{-L_1} - A_{22}e^{-L_1} - B_{22}e^{L_1} = 0}$$

Nordal Condition II

$$\frac{-1}{(n^2 A c)_1}\left.\frac{dV_1}{dx}\right|_{x=L_1} = \frac{-1}{(n^2 A c)_{21}}\left.\frac{dV_{21}}{dx}\right|_{x=L_1} + \frac{-1}{(n^2 A c)_{22}}\left.\frac{dV_{22}}{dx}\right|_{x=L_1} \quad \text{——— (D)}$$

$$V_1(x) = A_1e^{-x} + B_1e^{x}$$

$$\frac{dV_1}{dx} = -A_1e^{-x} + B_1e^{x}$$

$$\left.\frac{dV_1}{dx}\right|_{x=L_1} = -A_1e^{-L_1} + B_1e^{L_1} \quad \text{——— (1)}$$

$$V_{21}(x) = A_{21}e^{-x} + B_{21}e^{x}$$

$$\frac{dV_{21}}{dx} = -A_{21}e^{-x} + B_{21}e^{x}$$

$$\left.\frac{dV_{21}}{dx}\right|_{x=L_1} = -A_{21}e^{-L_1} + B_{21}e^{L_1} \quad \text{——} \quad (2)$$

$$V_{22}(x) = A_{22}e^{-x} + B_{22}e^{x}$$

$$\frac{dV_{22}}{dx} = -A_{22}e^{-x} + B_{22}e^{x}$$

$$\left.\frac{dV_{22}}{dx}\right|_{x=L_1} = -A_{22}e^{-L_1} + B_{22}e^{L_1} \quad \text{——} \quad (3)$$

Substituting (1), (2) and (3) in the equation (4)

$$\frac{-1}{(r_i d_c)_1}\left(-A_1 e^{-L_1} + B_1 e^{L_1}\right) = \frac{-1}{(r_i d_c)_{21}}\left(-A_{21}e^{-L_1} + B_{21}e^{L_1}\right) + \frac{-1}{(r_i d_c)_{22}}\left(-A_{22}e^{-L_1} + B_{22}e^{L_1}\right)$$

$$\boxed{\frac{-A_1 e^{-L_1}}{(r_i d_c)_1} + \frac{B_1 e^{L_1}}{(r_i d_c)_1} + \frac{A_{21}e^{-L_1}}{(r_i d_c)_{21}} - \frac{B_{21}e^{L_1}}{(r_i d_c)_{21}} + \frac{A_{22}e^{-L_1}}{(r_i d_c)_{22}} - \frac{B_{22}e^{L_1}}{(r_i d_c)_{22}} = 0}$$

Question 02

$$Ax = b$$

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-\omega_1} & e^{\omega_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-\omega_2} & e^{\omega_2} \\ e^{-\mu} & e^{\mu} & -e^{-\mu} & -e^{\mu} & 0 & 0 \\ 0 & 0 & e^{-\mu} & e^{\mu} & -e^{-\mu} & -e^{\mu} \\ \frac{-e^{-\mu}}{(ridc)_1} & \frac{e^{\mu}}{(ridc)_1} & \frac{e^{-\mu}}{(ridc)_{21}} & \frac{-e^{\mu}}{(ridc)_{21}} & \frac{e^{-\mu}}{(ridc)_{22}} & \frac{-e^{\mu}}{(ridc)_{22}} \end{pmatrix} \begin{pmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{pmatrix} = \begin{pmatrix} (ridc)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} A_1 & -B_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{21}e^{-\omega_1} & B_{21}e^{\omega_1} & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{22}e^{-\omega_2} & B_{22}e^{\omega_2} \\ A_1 e^{-\mu} & B_1 e^{\mu} & -A_{21}e^{-\mu} & -B_{21}e^{\mu} & 0 & 0 \\ 0 & 0 & A_{21}e^{-\mu} & B_{21}e^{\mu} & -A_{22}e^{-\mu} & -B_{22}e^{\mu} \\ \frac{-A_1 e^{-\mu}}{(ridc)_1} & \frac{B_1 e^{\mu}}{(ridc)_1} & \frac{A_{21}e^{-\mu}}{(ridc)_{21}} & \frac{-B_{21}e^{\mu}}{(ridc)_{21}} & \frac{A_{22}e^{-\mu}}{(ridc)_{22}} & \frac{-B_{22}e^{\mu}}{(ridc)_{22}} \end{pmatrix}$$

$$A_1 - B_1 = (ridc)_1 I_{app}$$

$$A_{21}e^{-\omega_1} + B_{21}e^{\omega_1} = 0$$

$$A_{22}e^{-\omega_2} + B_{21}e^{\omega_2} = 0$$

$$A_1 e^{-\mu} + B_1 e^{\mu} - A_{21}e^{-\mu} - B_{21}e^{\mu} = 0$$

$$A_{21}e^{-\mu} + B_{21}e^{\mu} - A_{22}e^{-\mu} - B_{22}e^{\mu} = 0$$

$$\frac{-A_1 e^{-\mu}}{(ridc)_1} + \frac{B_1 e^{\mu}}{(ridc)_1} + \frac{A_{21}e^{-\mu}}{(ridc)_{21}} - \frac{B_{21}e^{\mu}}{(ridc)_{21}} + \frac{A_{22}e^{-\mu}}{(ridc)_{22}} - \frac{B_{22}e^{\mu}}{(ridc)_{22}} = 0$$

```
% electrical constants and derived quantities for typical
% mammalian dendrite

% Dimensions of compartments

d1 = 75e-4;             % cm
% d21 = 30e-4;          % cm
% d22 = 15e-4;          % cm
d21 = 47.2470e-4;       % E9 cm
d22 = d21;               % E9 cm

l1 = 1.5;           % dimensionless
l21 = 3.0;          % dimensionless
l22 = 3.0;          % dimensionless

% Electrical properties of compartments

Rm = 6e3;           % Ohms cm^2
Rc = 90;            % Ohms cm
Rs = 1e6;           % Ohms

c1 = 2*(Rc*Rm)^(1/2)/pi;

rl1 = c1*d1^(-3/2);     % Ohms
rl21 = c1*d21^(-3/2);       % Ohms
rl22 = c1*d22^(-3/2);       % Ohms


% Applied current

iapp = 1e-9;    % Amps

% Coefficient matrices

A = [1 -1 0 0 0 0;
     0 0 exp(-l21) exp(l21) 0 0;
     0 0 0 0 exp(-l22) exp(l22);
     exp(-l1) exp(l1) -exp(-l1) -exp(l1) 0 0;
     0 0 exp(-l1) exp(l1) -exp(-l1) -exp(l1);
     -exp(-l1) exp(l1) rl1*exp(-l1)/rl21 -rl1*exp(l1)/rl21 rl1*exp(-l1)/rl22 -rl1*exp(-l1)/rl22];

b = [rl1*iapp 0 0 0 0 0]';
```

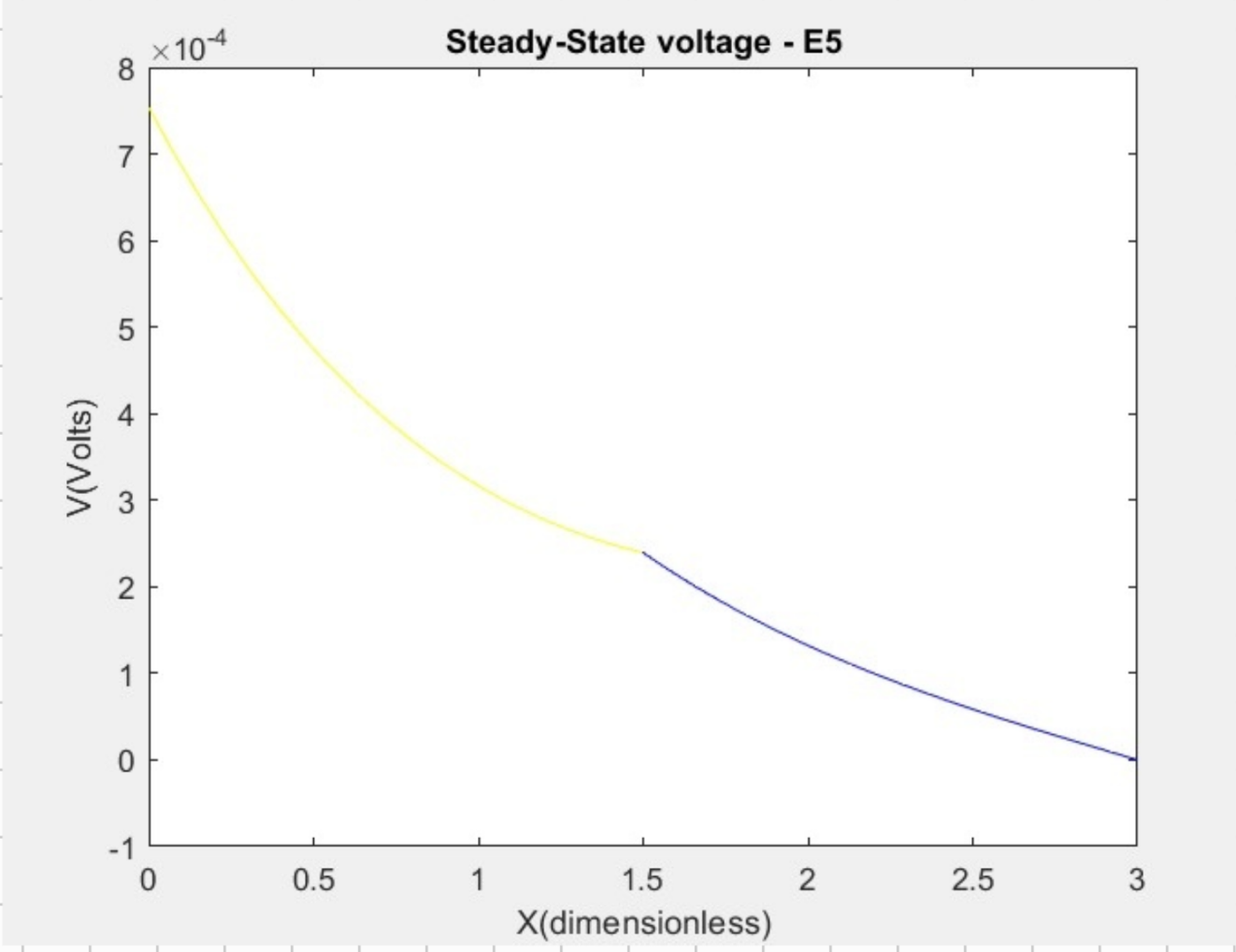
Question 03

```
x = A\b;
display(x)
```

```
x =

   1.0e-03 *

    0.7189
   -0.0014
    0.7275
   -0.0018
    0.7275
   -0.0018
```

Question 04

```
y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);
v1 = x(1)*exp(-y1) + x(2)*exp(-y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot(y1,v1,'y-',y21,v21,'r-',y22,v22,'b-');
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady-State voltage - E5');
```



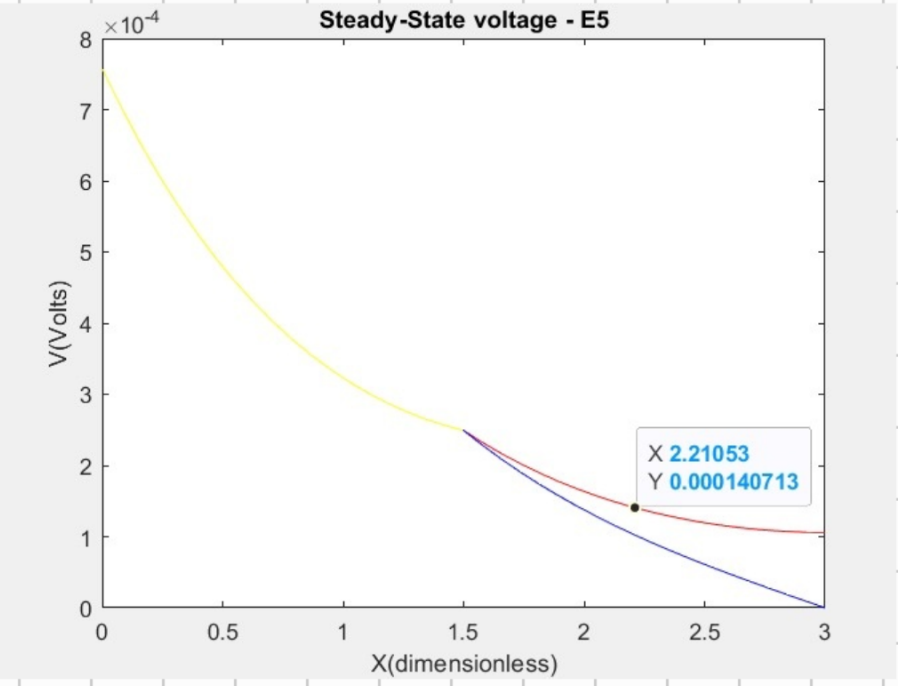
Explanation

Here, the red line is not visible, which means it is either equal or very close to the blue line. Yellow line shows the membrane potential of the parent branch and it has no influence on the two daughter branches. ∴ It is safe to assume that the red and blue lines are equal. This means that the steady state voltages of the 2 daughter branches are identical.

Question 05.

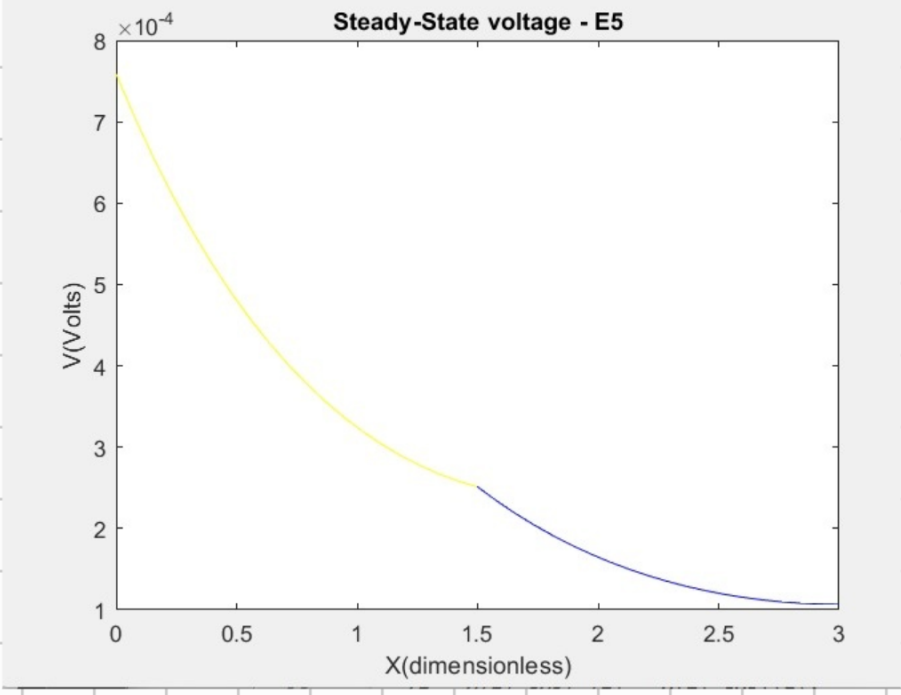
(a)

```
A(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];

x = A\b;

y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);
v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot(y1,v1,'y-',y21,v21,'r-',y22,v22,'b-');
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady-State voltage - E5');
```



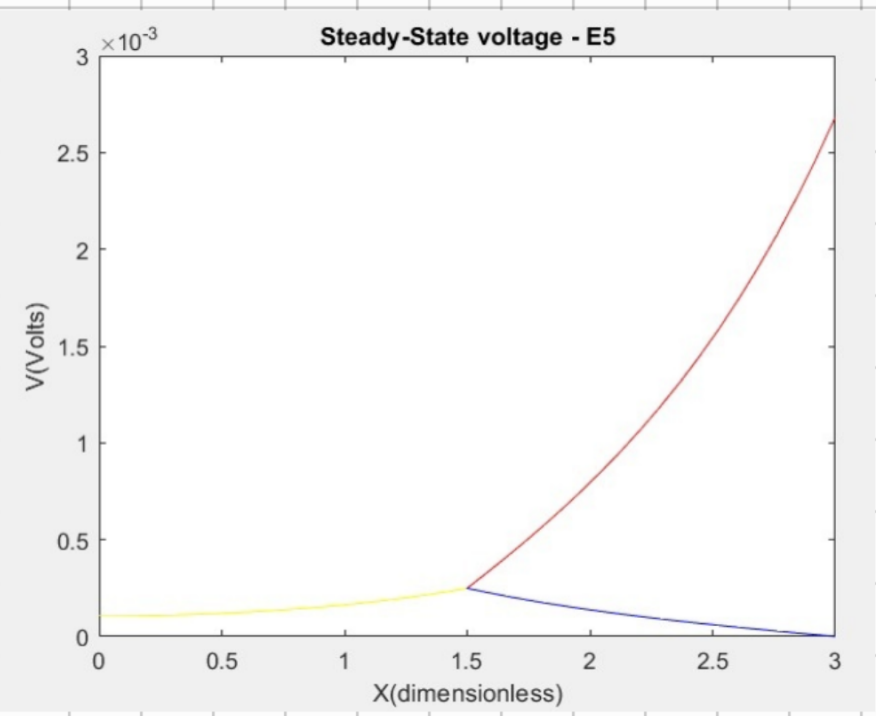
(b)

```
A(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];
A(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];

x = A\b;

y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);
v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot(y1,v1,'y-',y21,v21,'r-',y22,v22,'b-');
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady-State voltage - E5');
```



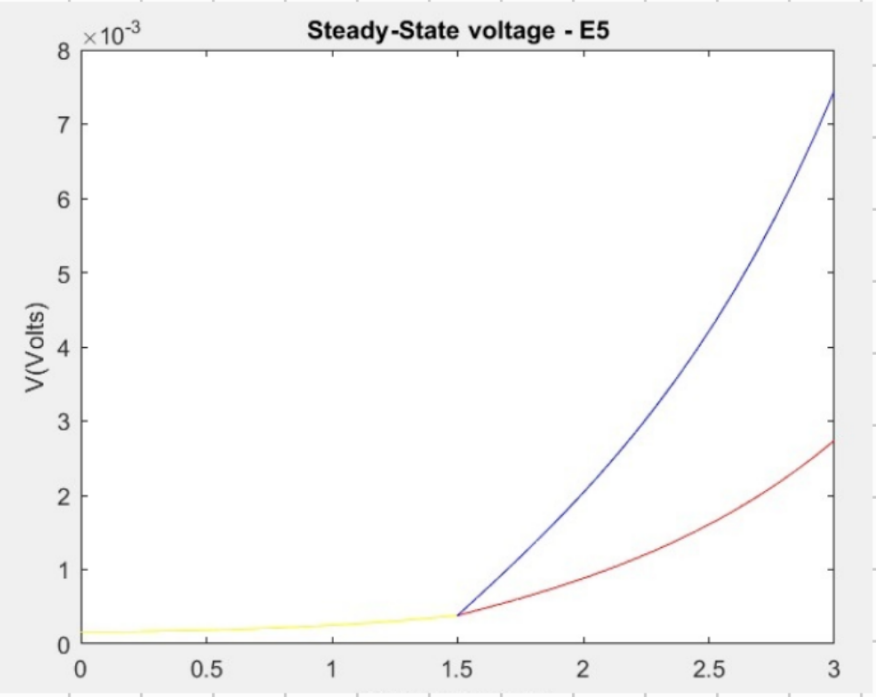
(c)

```
A(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];
%A(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];

b(1) = 0;
b(2) = rl21*iapp;

x = A\b;

y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);
v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot(y1,v1,'y-',y21,v21,'r-',y22,v22,'b-');
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady-State voltage - E5');
```



(d)

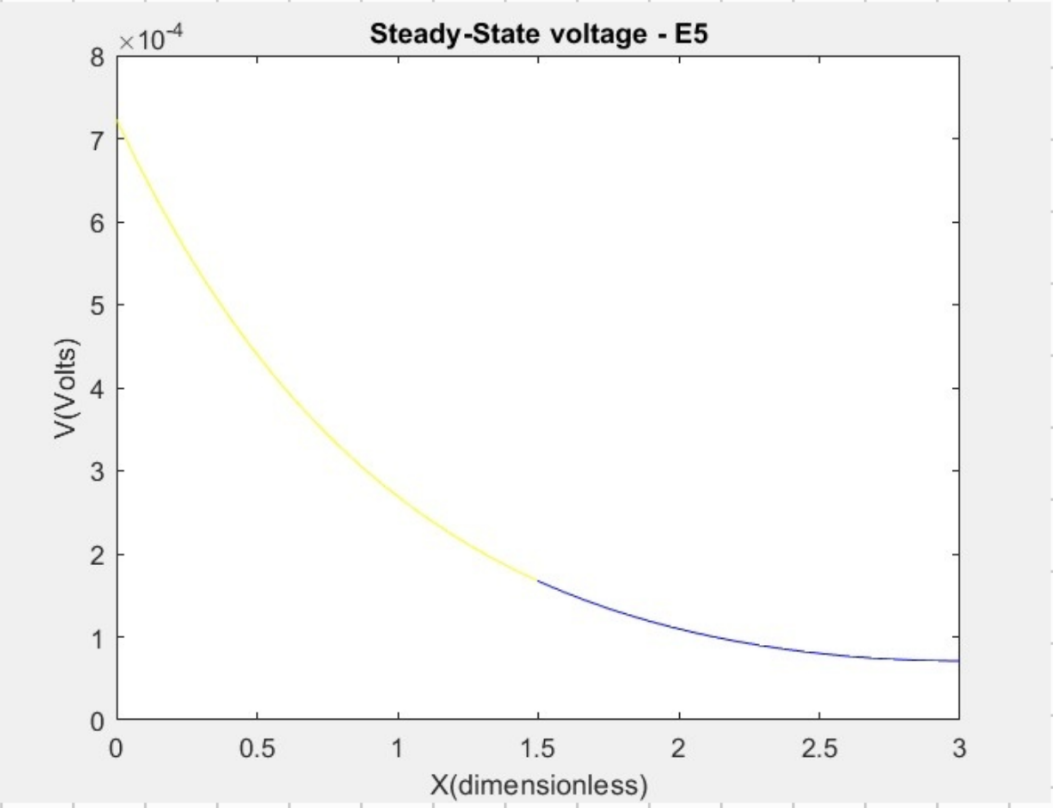
```
A(2,:) = [0 0 -exp(-l21) exp(l21) 0 0];
A(3,:) = [0 0 0 0 -exp(-l22) exp(l22)];

b(1) = 0;
b(2) = rl21*iapp;
b(3) = rl22*iapp;

x = A\b;
display(x);

y1 = linspace(0,l1,20);
y21 = linspace(l1,l21,20);
y22 = linspace(l1,l22,20);
v1 = x(1)*exp(-y1) + x(2)*exp(y1);
v21 = x(3)*exp(-y21) + x(4)*exp(y21);
v22 = x(5)*exp(-y22) + x(6)*exp(y22);
plot(y1,v1,'y-',y21,v21,'r-',y22,v22,'b-');
xlabel('X(dimensionless)');
ylabel('V(Volts)');
title('Steady-State voltage - E5');
```



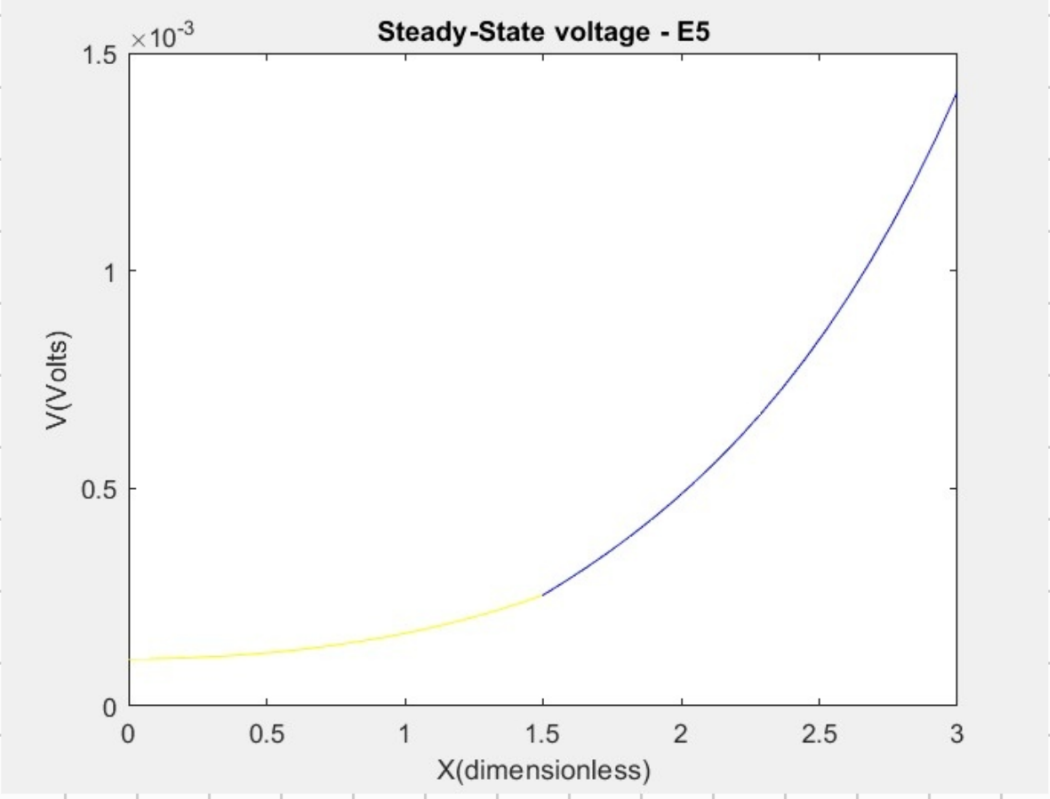
A positive terminal is generated due to the current flowing through an impedence. Therefore the voltage gradient is more positive on the right side.

Question 06

(b)



(d)



Here the transition from parent brach to daughter branches is smooth.

Also, because of the equal diameter both daughter branches overlaps.